Analytical Models for Low-Power Rectenna Design

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***Abstract*—The design of a low-cost rectenna for low-power applications is presented. The rectenna is designed with the use of analytical models and closed-form analytical expressions. This allows for a fast design of the rectenna system. To acquire a small-area rectenna, a layered design is proposed. Measurements indicate the validity range of the analytical models.**

***Index Terms*—Microstrip patch antenna, rectenna, rectifier, wireless power transmission.**

I. INTRODUCTION

WIRELESS power transfer is a promising technique for the long-term power supply of wireless applications [1]. In this paper, we will focus on wireless power transfer via electromagnetic radiation, as opposed to inductive power transfer for example. The key component for this type of wireless power transfer is the rectenna. A rectenna is a combination of a *rect*ifying circuit and an ant*enna*. The antenna receives the electromagnetic power and the rectifying circuit converts it to electric power.

The amount of power that can be transferred is limited. The transmitted power is limited by regulations and the received power is attenuated, mainly due to free-space path loss. In general, portable devices have small dimensions. Therefore, the rectenna should have small dimensions as well. This results in a small antenna area and, consequently, a low amount of received power. Because of these limitations, wireless power transfer is mainly suitable for low-power applications, e.g., a low-power wireless sensor.

To be able to use wireless power transfer, an efficient rectenna is needed. In the presented work, the rectenna is modeled with analytical and closed-form analytical models. This allows for a fast design of the rectenna system and it provides a good insight in the effect of the several parameters on the performance of the rectenna.

In the experiments described in the remainder of this paper, we assume a received RF power level of 0 dBm at a central operating frequency of 2.45 GHz. This frequency lies in a license-free industrial, scientific, and medical band where the maximum radiated power is in general 20 dBm. Received RF power levels of 0 dBm and less are expected for practical applications in this frequency band.

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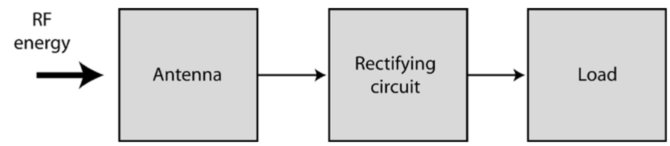

Fig. 1. Schematic rectenna system.

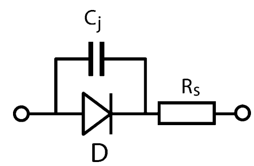

Fig. 2. Electric circuit for the Schottky diode.

II. RECTENNA SETUP

The general setup of a rectenna is shown in Fig. 1. This setup is different from the setup proposed by Heikkinen *et al.* [2], which has an impedance matching circuit between antenna and rectifying circuit. This matching circuit can be avoided when the antenna and rectifying circuit are designed such that they are already matched. For this purpose the behavior of the antenna and rectifying circuit have to be modeled at and around the operating frequency.

A. Antenna

A probe-fed microstrip patch antenna is utilized as an antenna. The advantages of this type of antenna are its small dimensions and the ease of manufacturing, making it a low-cost antenna. An advantage of the feeding technique is the separation of the electric circuit and the antenna via the ground plane. The input impedance of the microstrip patch antenna is calculated with a cavity model [3]. Effective dimensions are introduced to incorporate the effect of fringe fields at the edges of the patch. The input impedance varies with the probe position. This property is utilized to acquire a matched design.

B. Rectifying Circuit

The main component of the rectifying circuit is the Schottky diode, which is modeled by the electric circuit shown in Fig. 2. The model for the Schottky diode consists of a substrate resistance R_s, a junction capacitance C_j, which is assumed to have a linear voltage-current relation, and a nonlinear diode D, which has a voltage-current characteristic given by

$$i_D = I_s\left(e^{\alpha v_D} - 1\right). \tag{1}$$

Here, i_D is the current through the nonlinear diode, v_D is the voltage over the nonlinear diode, I_s is the saturation current, and $\alpha = q/nkT$ is the reciprocal of the thermal voltage. Here, q is the charge of an electron, T is the temperature in Kelvin, and n is the diode ideality factor.

The input impedance of the Schottky diode has to be determined to be able to match the rectifying circuit to the antenna. Therefore, the voltage over and current through the diode have to be evaluated. A possible approach is to solve the differential equations of the total electric circuit in the time domain. The drawback of this approach is that the system of differential equations can become rather large when the Schottky diode is mounted in an extensive electric circuit. Therefore, we follow a different approach. The linear parts of the circuit are evaluated in the frequency domain whereas the nonlinear diode is evaluated in the time domain.

During stationary opeartion, the current through the nonlinear diode can be represented by a sum of harmonics, i.e.,

$$i_{hN} = i_0 + \sum_{n=1}^{N} \left[\kappa_n \cos(n\omega_0 t) + \varsigma_n \sin(n\omega_0 t)\right]. \tag{2}$$

Here, i_0 represents the direct current and κ_n, ς_n represent the amplitudes of the harmonic terms, operating at the radial frequency $n\omega_0$. For each of the harmonic terms, the equivalent Thévenin circuit of the linear circuit is determined. If the current i_h is given, the voltage at the output of the linear circuit is known. This voltage is used to determine the current i_D through the diode. Since the current at the output of the linear network is equal to the current through the diode, we can find the coefficients κ_n and ς_n by minimizing the square of the error ϵ between i_D and i_{hN} over one period of time T [4], i.e.,

$$\epsilon = \frac{1}{T}\int_0^T \left[i_D - i_{hN}\right]^2 dt. \tag{3}$$

The coefficients κ_n, ς_n can be found via a multidimensional minimization method, e.g., a simplex method [5].

Once the coefficients κ_n and ς_n are known, the input impedance of the diode can be determined. The number of harmonics N that should be taken into account to obtain an accurate input impedance depends on the signal level. For large signals, more harmonics should be taken into account than for small signals, since the nonlinearity of the diode has a larger impact for these signals.

If it is allowed to consider only a limited number of harmonics N, we can find an analytical expression for (3). In the simplest case, only the dc term of the current is determined. For this case, a closed-form algebraic relation between input voltage and dc input current can be found [6]. In our case, this analysis is not sufficient, since it does not give an expression for the input impedance of the diode at RF frequencies. Therefore, we have to include at least one harmonic term ($N = 1$)

$$i_{h1} = i_0 + \kappa_1 \cos(\omega_0 t) + \sigma_1 \sin(\omega_0 t). \tag{4}$$

The voltage over the diode D in the time domain can be modeled using Thévenin's theorem as

$$v_{D1} = a\cos(\omega_0 t) + b\sin(\omega_0 t) - R_g i_{h1} - X_g \frac{\partial}{\partial t} i_{h1}. \tag{5}$$

Here, a and b determine the amplitude and phase of the equivalent Thévenin voltage source. R_g and X_g determine the output impedance of the source, where R_g represents the resistance and X_g is a measure for the inductance or capacitance. The resulting error term is given by

$$\begin{aligned}\epsilon_1 &= \frac{1}{T}\int_0^T \left[I_s\left(e^{\alpha v_{D1}} - 1\right) - i_{h1}\right]^2 dt \\ &= \frac{1}{T}\int_0^T I_s^2 e^{2\alpha v_{D1}} - 2(I_s + i_{h1}) I_s e^{\alpha v_{D1}} + (I_s + i_{h1})^2 dt.\end{aligned} \tag{6}$$

The closed-form solution of this integral can be found in [7, eq. 3.937] and is given by

$$\begin{aligned}\epsilon_1 = {} & I_s^2 e^{-2\alpha i_0 R_g} \mathrm{I}_0(2\alpha\sqrt{p^2+q^2}) \\ & - 2(I_s + i_0) I_s e^{-\alpha i_0 R_g} \mathrm{I}_0(\alpha\sqrt{p^2+q^2}) \\ & - 2I_s e^{-\alpha i_0 R_g} \frac{\kappa_1 p + \sigma_1 q}{\sqrt{p^2+q^2}} \mathrm{I}_1(\alpha\sqrt{p^2+q^2}) \\ & + (I_s + i_0)^2 + \frac{1}{2}(\kappa_1^2 + \sigma_1^2)\end{aligned} \tag{7}$$

with

$$\begin{aligned} p &= a - \kappa_1 R_g - \sigma_1 \omega X_g \\ q &= b - \sigma_1 R_g + \kappa_1 \omega X g.\end{aligned} \tag{8}$$

Here, I_0 and I_1 are the modified Bessel functions of the first kind of order 0 and 1, respectively.

When higher harmonics ($N > 1$) have to be taken into account, the integral of (3) is evaluated numerically.

C. Filter

To avoid unwanted power dissipation, the harmonics generated by the diode need extra attention. To avoid dissipation of the harmonics in the load, a radial stub that is placed between the Schottky diode and the load is utilized as a bandstop filter. An analytical model of the radial stub is presented by Giannini *et al.* [8]. A model based on this theory is used in our design.

III. Verification

To verify the combined analytical and closed-form analytical models of the antenna, Schottky diode and filter, a rectenna is constructed. This rectenna is not optimized for conversion efficiency, but only serves the purpose of validation of the analytical models described sofar. A schematic layout is shown in Fig. 3. The rectifying circuit is placed perpendicular to the patch antenna for ease of construction. The radial stub is intended to block the signal on the operating frequency such that it is not dissipated into the load. The coil is added to provide a dc short-circuit for the antenna. It is modeled as a parallel circuit of a resistance, capacitance, and inductance of which the values have been obtained from measurements.

The most interesting properties of the rectenna are the dc output voltage and the input impedance of the antenna and the rectifying circuit. The analytical model for the input impedance of the antenna is quite accurate as shown in [3]. The rectifying circuit is modeled with $N = 1$ and $N = 2$ harmonic terms taken into account. The modeled and measured results for the

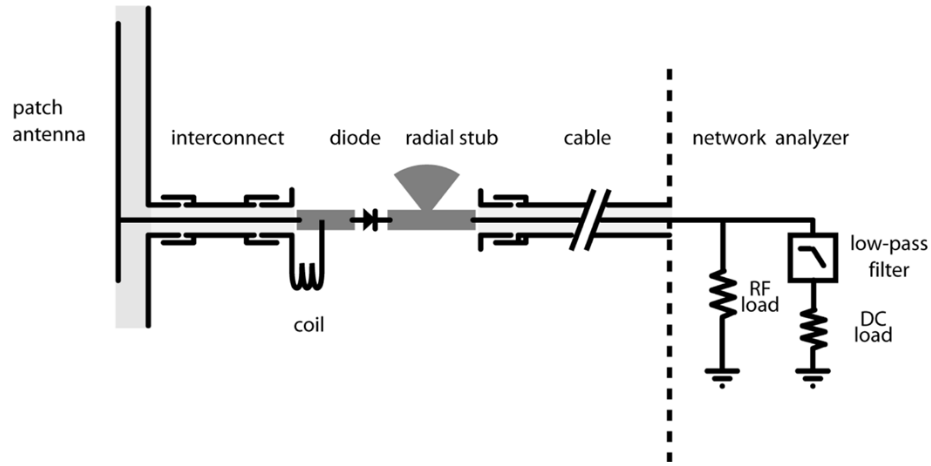

Fig. 3. Schematic layout of the rectenna.

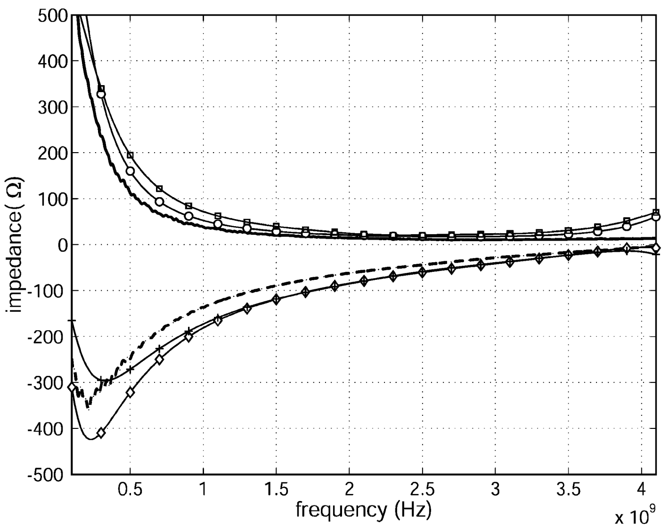

Fig. 4. Input impedance of the rectifying circuit. $\mathrm{Re}\{Z_{in}\}$: model $N = 1$ (○), model $N = 2$ (□), measurement (solid). $\mathrm{Im}\{Z_{in}\}$: model $N = 1$(◇), model $N = 2$ (+), measurement (dash).

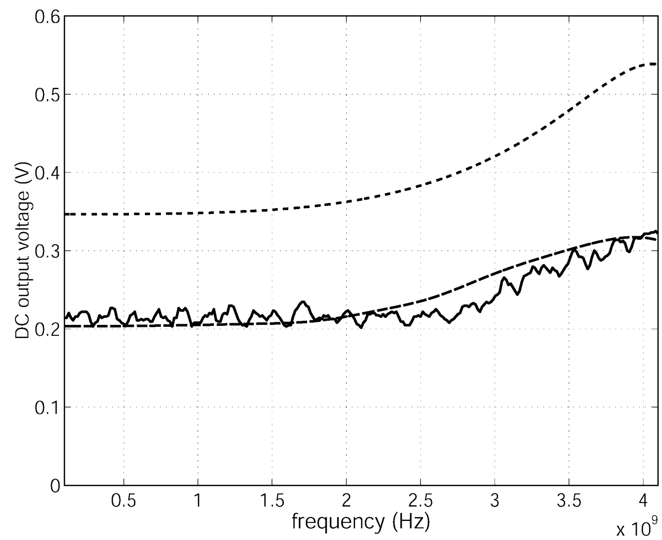

Fig. 5. DC output voltage rectifying circuit. Model $N = 1$ (dot), model $N = 2$ (dash), measurement (solid).

input impedance of the rectifying circuit are shown in Fig. 4 for an input signal with a power level of 0 dBm and a dc load $R_{\mathrm{dc}} = 470\,\Omega$. The choice for R_{dc} is arbitrary, but lies in the range of expected values for practical applications. For the Schottky diode we used the HSMS-2852 from Agilent Technologies [9]. The corresponding dc output voltage is shown in Fig. 5.

From Fig. 4 we observe that both models demonstrate an acceptable agreement between calculations and measurements of the input impedance of the rectifying circuit at the frequency range of interest (2–3 GHz). Fig. 5, however, clearly shows the limited accuracy of the simpler model for the dc output voltage. It demonstrates that at least two harmonic terms should be included to acquire an accurate prediction of the dc output voltage.

IV. Rectenna Design

The main design parameters of the rectenna are the dimensions and the conversion efficiency. To acquire a small-area rectenna, a layered design is proposed. The backside of the patch antenna is used for the rectifying circuit. As a result, the ground plane of the antenna is used as a ground plane for the rectifying circuit as well. The conversion efficiency η is defined as the ratio of dc power to received power, i.e.,

$$\eta = \frac{P_{\mathrm{dc}}}{P_{\mathrm{inc}}}. \tag{9}$$

In our design, additional radial stubs are placed between the antenna and the rectifier to prevent the first harmonic from being reradiated by the antenna. This is necessary because the harmonics of the operating frequency correspond closely to radiating modes of the patch antenna. Higher harmonics are excited as well, but not as significant as the first harmonic. The filter structure between the antenna and the rectifying circuit can be simplified when the harmonics generated by the rectifier do not correspond to radiating modes of the antenna. This property is employed in [10], where circular patch antennas are used.

Between the rectifier and the load two radial stubs are also placed. One radial stub prevents the signal at the operating frequency from being dissipated in the load and one radial stub prevents the signal at the first harmonic frequency from being dissipated.

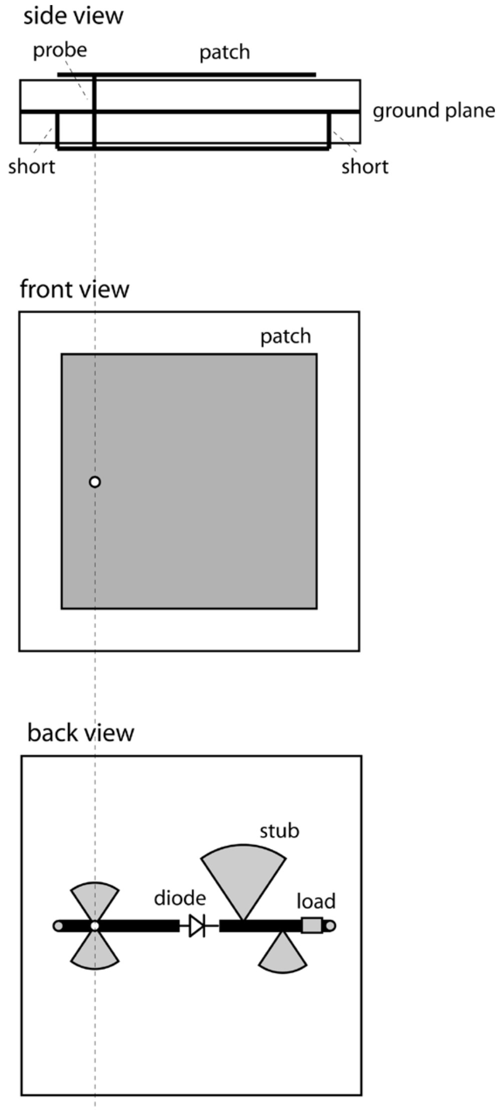

Fig. 6. Layout of the stacked rectenna.

The analytical models are employed to acquire a match between microstrip patch antenna and the rectifying circuit, including the radial stubs. The conversion efficiency is increased by choosing a probe position that corresponds to a high output impedance of the antenna. In this way the output voltage of the antenna increases for the same incident power level, which is favorable for the conversion efficiency of the Schottky diode.

A. Results

The layout of the rectenna is given in Fig. 6. The layered rectenna is photo-etched from copper-clad FR4 material, which allowed for a fast realization of the product. To model the dc output voltage, the incident power received by the patch antenna is measured for a fixed transmit-receive setup. This data is used in the model to predict the dc output voltage of the rectenna. The result is shown in Fig. 7. The conversion efficiency η equals 40%. This is comparable to the efficiency of 50% presented in [2].

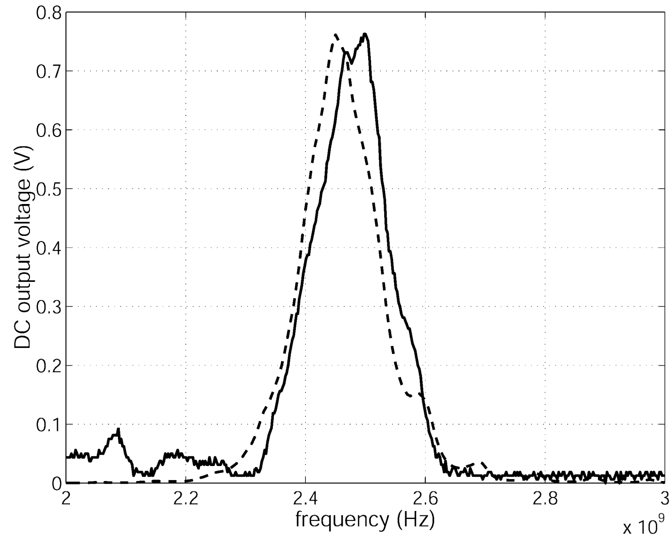

Fig. 7. DC output voltage of the stacked rectenna. Measurement (solid) and model (dash).

V. CONCLUSION

Rectenna design with the use of analytical models was presented. These models allow for a fast design of rectenna systems and enable an optimization of the rectenna layout for a large number of parameters. The models were used to design a small and efficient rectenna. The validity of the models was shown by comparison with measurements.

Future work will include the following aspects. The loss tangent of the used FR4 dielectric is relatively high compared to other dielectric materials, e.g., microwave laminate. The use of low-loss materials decreases the dielectric losses and is, therefore, beneficial for the efficiency. Finally, the dc output voltage of the rectenna can be doubled if two diodes are used in the rectifying circuit. A higher output voltage can increase the applicability of the rectenna.

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