An Improved Analytical Model for RF-DC Conversion Efficiency in Microwave Rectifiers

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***Abstract*—In this paper, an improved analytical model for diode efficiency in microwave rectifier is presented. This model provides a simple calculation routine to determine the input power level at which the peak reverse voltage across the diode starts to exceed the diode breakdown voltage when the diode efficiency starts to decrease. After studying the origins of the power losses in a shunt connected diode rectifier carefully, closed-form equations are derived to calculate the diode efficiencies at various input power levels. A 2.45 GHz microwave rectifier is designed and measured. The experimental results agree well with the proposed model prediction.**

***Index Terms*—Microwave rectifier, Schottky diode, conversion efficiency, diode breakdown.**

I. Introduction

The idea of wireless power transfer (WPT) can date back to the work done by Nikola Tesla in 1910's. Modern applications of WPT include space-based solar power transmission [1] and RF energy harvesting [2]. A WPT system usually consists of a microwave power amplifier, a transmitting antenna, a receiving antenna and a microwave rectifier. In order for the WPT system to operate cost effectively, the total system efficiency must be as high as possible, which is mainly due to the conversion efficiency of microwave power amplifiers and rectifiers. Therefore, a high RF-DC conversion efficiency of microwave rectifiers is crucial in WPT system design. The reported highest RF-DC conversion efficiency records at different frequencies are 92.5% at 2.45 GHz, 82% at 5.8 GHz, and 60% at 35 GHz [1].

The shunt diode configuration is widely used in rectifier design to achieve a high conversion efficiency and a simple circuit implementation as well. A microwave rectifier based on the shunt diode configuration usually consists of four main components: matching network, shunt Schottky diode, DC pass filter and resistive load, as shown in Fig. 1. All the discussion in this paper is based on this circuit topology.

The conversion efficiency of the rectifier is largely determined by the losses of both diode and impedance mismatch at various input power levels. For a well matched rectifier, the diode loss is dominant. The development of an accurate analytical model for diode efficiency in rectifiers will be useful in both diode selection and rectifier design. In paper [3], an analytical model for diode efficiency was derived through calculating the major losses due to the diode junction and the parasitic resistance, and a monotonic increase trend of diode efficiency was predicted as the input power increased.

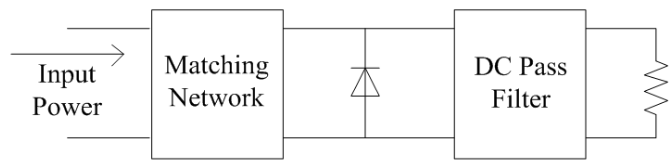

Fig. 1. The circuit topology of microwave rectifiers using a shunt diode.

However, a trend of significant conversion efficiency decrease was observed when the input power increased beyond some specific value in [4], [5].

This conversion efficiency decrease is believed to be caused by the power loss when the diode starts to operate in the breakdown region. As the input power increases, the RF swing and the rectified DC voltage across the diode will increase accordingly. When the input power is larger than some specific value, the reverse peak voltage (DC plus RF) across the diode will be larger than the diode breakdown voltage. In this situation, a large reverse bias current will pass through the diode, causing a significant amount of power loss. Therefore, the resultant conversion efficiency is greatly degraded when the input power exceeds some specific value. Further increase of the input power will damage the diode permanently which should be avoided in all situations.

To accommodate the breakdown effect in the diode efficiency analysis, an improved analytical model is presented in this paper. This improved model gives closed-form equations for diode efficiencies at various input power levels. A 2.45 GHz microwave rectifier is designed to verify the accuracy of the proposed model.

II. Analytical Model for Diode Efficiency in Rectifiers

A diode equivalent circuit model is shown in Fig. 2 for our analysis, which consists of a series resistance (R_S), a nonlinear junction resistance (R_j) and a nonlinear junction capacitance (C_j). For an ideal diode I-V curve in Fig. 2, R_j is assumed to be infinity when the voltage across the diode is larger than the breakdown voltage (-V_{br}) and smaller than the built-in voltage (V_{bi}), and to be zero when the diode voltage is beyond this region. Due to the self-biasing nature of the shunt connected diode, C_j is dependent on the bias voltage across the diode

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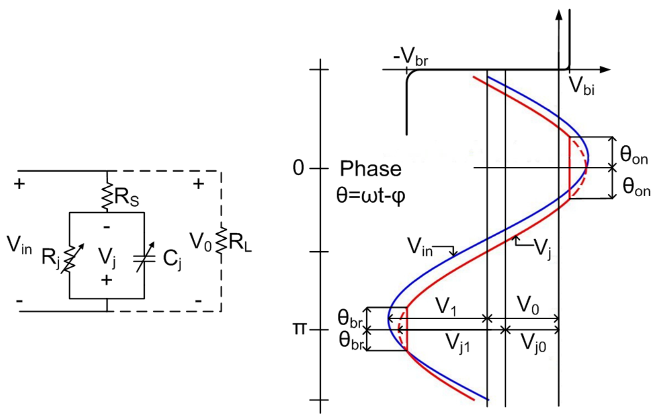

Fig. 2. Equivalent circuit model of the rectifying diode and DC load resistor (left) and the voltage waveforms of V_{in} and V_j (right).

by (1)

$$C_j = C_{j0}\sqrt{\frac{V_{bi}}{V_{bi}+V_0}} \tag{1}$$

where C_{j0} is diode's zero bias junction capacitance and V_0 is the DC output voltage. An ideal DC pass filter is assumed here in order to avoid any RF power leakage to the load.

Fig. 2 shows the voltage waveforms across the diode (V_{in}) and the diode junction (V_j). V_{in} consists of two components: an input RF signal (V_1) and a DC voltage (V_0) produced by the diode rectification. V_j is different than V_{in} due to the voltage division between the parasitic resistor and the diode itself. Furthermore, V_j is clamped to V_{bi} or -V_{br} when V_j is larger than V_{bi} or smaller than -V_{br} respectively due to the diode I-V characteristics. Both V_{in} and V_j are explicitly expressed in (2) and (3)

$$V_{in} = -V_0 + V_1\cos(\omega t) \tag{2}$$

$$V_j = \begin{cases} -V_{j0} + V_{j1}\cos(\omega t - \varphi) & (-V_{br} < V_j < V_{bi}) \\ V_{bi} & (V_j > V_{bi}) \\ -V_{br} & (V_j < -V_{br}) \end{cases} \tag{3}$$

where V_{j0} is the DC voltage while V_{j1} is peak RF voltage of V_j respectively; and φ is the phase difference between V_{in} and V_j. The approximations of $V_{j0} = V_0$ and $\varphi = 0$ are proved to be valid and used in the following derivation.

The diode efficiency is determined by analyzing various sources of the power loss on the diode during one period. When the input power increases to certain level such that the reverse peak voltage on the diode exceeds the diode breakdown voltage, there exist five sources of the diode loss: the loss on R_S and diode junction when $V_{in} > V_{bi}$; the loss on R_S when $-V_{br} < V_{in} < V_{bi}$, while the loss on diode junction can be neglected because R_j is infinity; the loss on R_S and diode junction when $V_{in} < -V_{br}$. Each loss (L_i) is calculated by integrating the instantaneous power over its specific non-zero interval in one period(from θ_{i1} to θ_{i2}) using (4)

$$L_i = \frac{1}{2\pi}\int_{\theta_{i1}}^{\theta_{i2}} I_i V_i d\theta. \tag{4}$$

The exact expressions of the five losses are not shown here due to the space limitation. The diode efficiency can then be expressed as

$$\eta = \frac{P_{out}}{P_{out} + \sum_{i=1}^{5} L_i} = \frac{1}{1+A+B+C+D+E} \tag{5}$$

where P_{out} is the DC output power and equal to $\frac{V_0^2}{R_L}$, and

$$A = R_L[\sin(2\theta_{on})V_{j1}^2 + 4(V_{bi}+V_0)^2\theta_{on} + 2V_{j1}^2\theta_{on} \\ -8(V_{bi}+V_0)\sin(\theta_{on})V_{j1}]/(4\pi R_S V_0^2) \tag{6}$$

$$B = \frac{R_L V_{bi}[\sin(\theta_{on})V_{j1} - (V_{bi}+V_0)\theta_{on}]}{\pi R_S V_0^2} \tag{7}$$

$$C = \frac{\omega^2 C_j^2 R_L R_S V_{j1}^2[2\pi + \sin(2\theta_{br}) + \sin(2\theta_{on}) - 2\theta_{br} - 2\theta_{on}]}{4\pi V_0^2} \tag{8}$$

$$D = R_L[\sin(\theta_{br})V_{j1}(4V_0 - 4V_{br} + \cos(\theta_{br})V_{j1}) \\ +2(V_{br}-V_0)^2\theta_{br} + V_{j1}^2\theta_{br}]/(2\pi R_S V_0^2) \tag{9}$$

$$E = \frac{R_L V_{br}[\sin(\theta_{br})V_{j1} - (V_{br}-V_0)\theta_{br}]}{\pi R_S V_0^2} \tag{10}$$

where ω is the angular frequency, θ_{on} is the turn-on angle, and θ_{br} is the breakdown angle (shown in Fig. 2).

To calculate the diode efficiency from (5), four unknowns of V_0, V_{j1}, θ_{on} and θ_{br} shall be determined. Applying Kirchhoff's voltage law to the diode equivalent circuit in Fig. 2, the relationship between V_0 and the average DC voltage on the diode junction $V_{j,dc}$ can be found in (11)

$$V_{j,dc} = \frac{1}{2\pi}\int_0^{2\pi} V_j = -\frac{(R_L+R_S)V_0}{R_L}. \tag{11}$$

From the voltage waveform of V_j in Fig. 2, (12), (13) can be geometrically acquired when V_j first reaches V_{bi} and $-V_{br}$

$$-V_{j0} + V_{j1}\cos(-\theta_{on}) = V_{bi} \tag{12}$$

$$-V_{j0} + V_{j1}\cos(\pi - \theta_{br}) = -V_{bi}. \tag{13}$$

For a given input power (P_{in}), the conversion efficiency (η) by definition can also be expressed as (14)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_0^2}{R_L P_{in}}. \tag{14}$$

Inserting (5) into (14) and calculating (11)-(14) together, the value of V_0, V_{j1}, θ_{on} and θ_{br} can be obtained. Then the diode efficiency can be calculated at each input power.

Under the condition of $\theta_{br} = 0$, the four unknowns V_0, V_{j1}, θ_{on} and P_{in} can be determined using (11)-(14). This

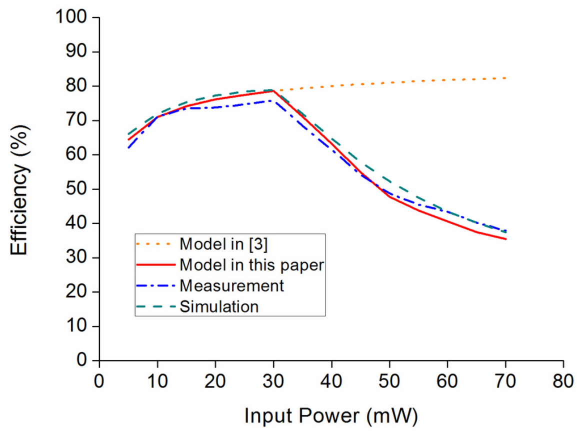

Fig. 3. Comparison of the RF-DC conversion efficiencies.

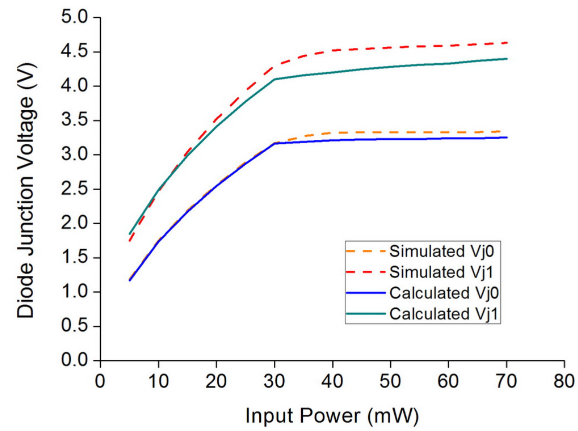

Fig. 5. Comparison of the calculated and simulated DC and RF voltages on the diode junction.

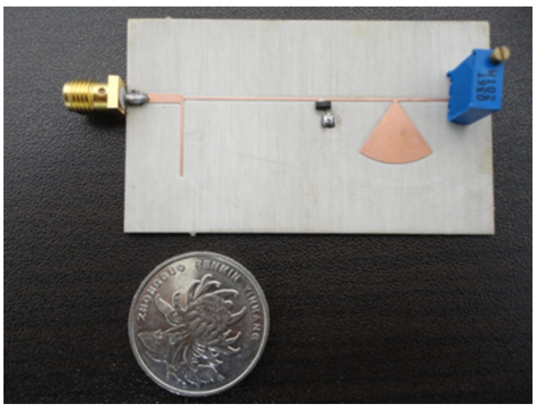

Fig. 4. Photograph of the fabricated microwave rectifier.

calculated P_{in} is the threshold input power level at which the reverse peak voltage on the diode starts to be larger than the breakdown voltage and the resultant conversion efficiency begins to decrease. When the input power is larger than this power level, the diode efficiency can be calculated using (5)-(14). When the input power is less than this power level, this model is in consistence with the model in [3].

III. Rectifier Design and Measurement

To verify the accuracy of the proposed model, a commercial Schottky diode (HSMS-8202) is used for a microwave rectifier at 2.45 GHz. The diode efficiency is first calculated using the proposed model and shown in Fig. 3. This model predicts that an input power of 30 mW will push the diode into the breakdown region, thus the diode efficiency will drop significantly when the input power exceeds 30 mW. Then a microwave rectifier using this diode is designed in Agilent ADS and fabricated on a 32-mil-thick RO4003C printed circuit board ($\epsilon = 3.55$) with a single stub matching network, a radial stub DC pass filter and a 425 Ω resistive load (shown in Fig. 4). The fabricated rectifier is tested with an input power ranging from 5 to 70 mW. Both the simulated and the measured conversion efficiencies agree well with the proposed model, while the model in [3] overestimates the conversion efficiency when the input power exceeds 30 mW (shown in Fig. 3). Furthermore, the V_{j0} and V_{j1} are also calculated and they also show good agreement with the simulation results (shown in Fig. 5).

IV. Conclusion

An improved analytical model for the diode efficiency in microwave rectifiers is derived in this paper. This model provides equations to analytically determine the input power level at which the diode efficiency starts to decrease. This calculated value provides microwave rectifier designers a good guide in the selection of the usable input power range to maintain a high conversion efficiency of rectifiers. Closed-form equations are given for diode efficiency at each input power level. A 2.45 GHz microwave rectifier is designed and fabricated, and the experiment results demonstrated a good accuracy in this improved model for diode efficiency calculation.

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