

Fantasy Team

Q1) Your friend Tony likes playing fantasy basketball team mobile games and he needs your help to beat his friend at it. Although his friend has 150 points on his fantasy team, Tony thinks he can beat him on the software. The program algorithm rules points, steals, rebounds and assists as 1, 1.2, 1.5 and 3 respectively for each player's team but due to some regularations there are restricted rules that the players also follow. However since the NBA has five rules, they let the players choose 4 rules out of the 5 and compete with their score. They are

$$1) p + 3s + 2r + 1a \leq 150$$

$$2) -p - 2s + 4r + 7a \leq 30$$

$$3) p + s + 2r - 1a \leq 70$$

$$4) 3p + 2s - r + 2a \leq 100$$

$$5) p + 4s - 2r - a \leq 90$$

But all variables should be counted and non-negative.

From analyzing the point system, the score that he will get is

$$p + 1.2 * s + 1.5 * r + 3 * a$$

P for points, s for steals, r for rebounds and a for assist. But he has the following constraints.

$$1) p + 3s + 2r + 1a \leq 150$$

$$2) -p - 2s + 4r + 7a \leq 30$$

$$3) p + s + 2r - 1a \leq 70$$

$$4) 3p + 2s - r + 2a \leq 100$$

$$5) p + 4s - 2r - a \leq 90$$

$$p, s, r, a \geq 0 \text{ //All the score should be negative or zero}$$

Q5) From reading and searching from google and other resources, I found out that every linear program has an extreme point or also known as a corner point that would be an optimal solution in the feasible region. However, there is a special case where if the feasible area has the point(0,0) then we must test the case, just to be sure that we are checking every possible answer for the problem. Thus, if we had multiple variables unlike the example problem, then we would have trouble graphing it. Therefore, we can use the substitution method to rule out the extra work that is needed to graph which we can possibly graph the constraint on an x and y coordinate and find the extreme points. The extreme point method was found on google and the video on youtube proves the method(<https://www.youtube.com/watch?v=TsWFZipuKi8>).

Q4)

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Since A and A' are almost the same except for last row, we can transform the equation

$$A' = A + uv^T$$

when we transform $u \cdot v^T = A - A'$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \dots & \dots & \dots \\ a_{31} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$A' = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \dots & \dots & \dots \\ a_{31} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{m1} + b_1 & \dots & \dots & a_{mn} + b_n \end{bmatrix}$$

$$A' - A \Rightarrow \begin{bmatrix} a_{11} - a_{11} & a_{12} - a_{12} & \dots & a_{1n} - a_{1n} \\ a_{21} - a_{21} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{m1} + b_1 - a_{m1} & \dots & \dots & a_{mn} + b_n - a_{mn} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & \dots & 0 \\ \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ b_1 & b_2 & \dots & b_n \end{bmatrix}$$

$$\text{Since } A' - A \Rightarrow \begin{bmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots \\ b_1 & \dots & b_n \end{bmatrix} \text{ we can say } uv^T = \begin{bmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots \\ b_1 & \dots & b_n \end{bmatrix}$$

And from this, it will be a column vector.

$$\text{Prove } \Rightarrow A'^{-1} = (A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} uv^T A^{-1}}{1 + v^T A^{-1} u}$$

$$A'^{-1} \cdot A' = I$$

if we times A'^{-1} by A' , we will get I which will make our life easier to prove above

$$(A + uv^T) \cdot \left(A^{-1} - \frac{A^{-1} uv^T A^{-1}}{1 + v^T A^{-1} u} \right) = A'^{-1} \cdot A'$$

$$\underbrace{A \cdot A^{-1}}_I - \frac{uv^T A^{-1} \cdot uv^T A^{-1}}{1 + v^T A^{-1} u} = \frac{A \cdot A^{-1} \cdot uv^T A^{-1}}{1 + v^T A^{-1} u} + uv^T A^{-1}$$

$$I + uv^T A^{-1} = \frac{uv^T A^{-1} + uv^T A^{-1} \cdot uv^T A^{-1}}{(1 + v^T A^{-1} u)}$$

$$I + uv^T A^{-1} = u \left(\frac{(1 + v^T A^{-1} u) v^T A^{-1}}{1 + v^T A^{-1} u} \right) \Rightarrow I + uv^T A^{-1} = uv^T A^{-1} \Rightarrow I$$

Since we proved that A' times A'^{-1} is a I matrix by Sherman-Morrison formula, it can be computed by equality.