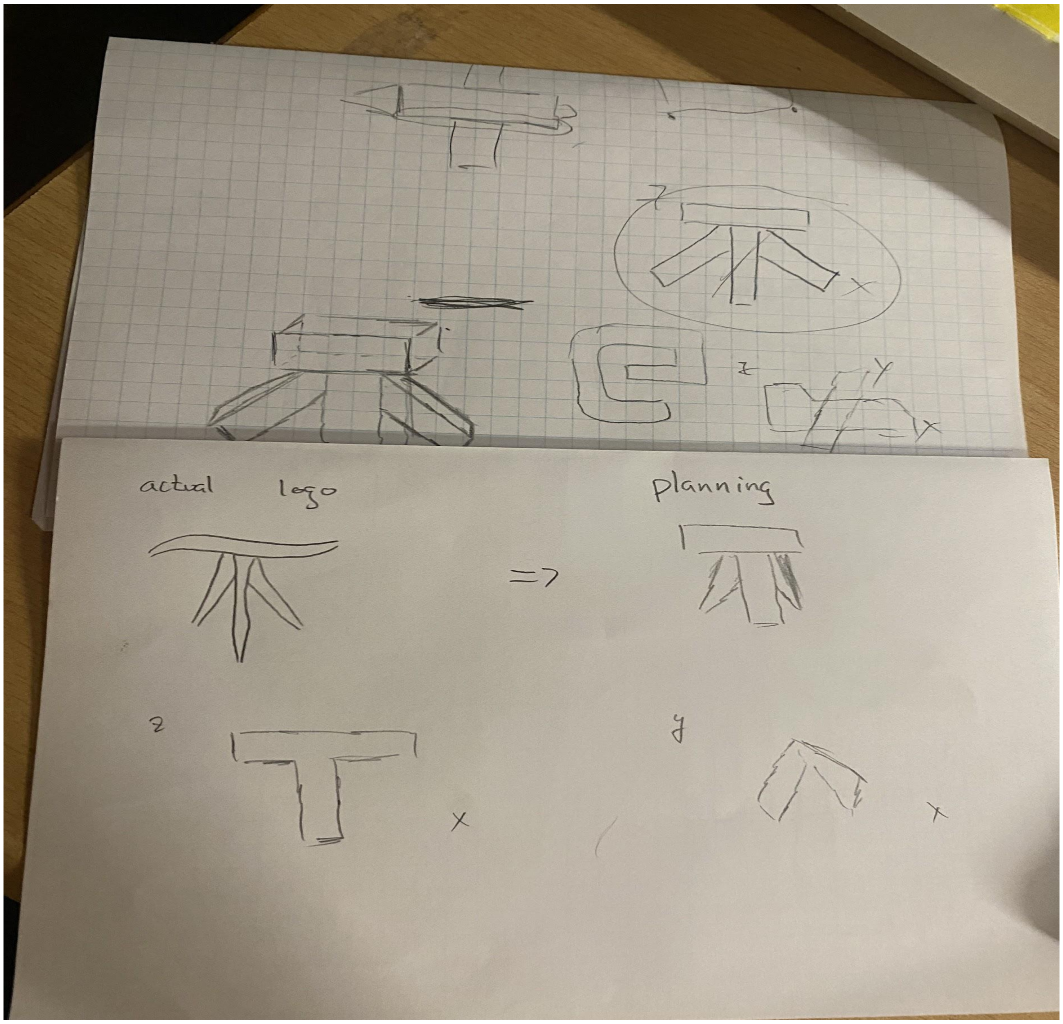
1)Design your movie

For the design of my model and movie, I chose my own logo called “”piTe” which I created when I was in high-school. I wanted the logo to be displayed as if it was like the logo of the batman in batman movies.



The main body of the logo rests on the “z” and “x” whereas the wings of the logo rests on the other axises. I am proud of the outcome which came closer to my expectations.

3) Order of rotations

The order of $R = R_{yaw} R_{pitch} R_{roll}$ matter in contrast to matrices being not commutative. We can simply prove it.

Proof:

$$(AB)^{-1} = B^{-1}A^{-1}$$

Whereas:

$$AB(AB)^{-1} = ABB^{-1}A^{-1}$$
$$= AA^{-1}B^{-1}A^{-1}$$

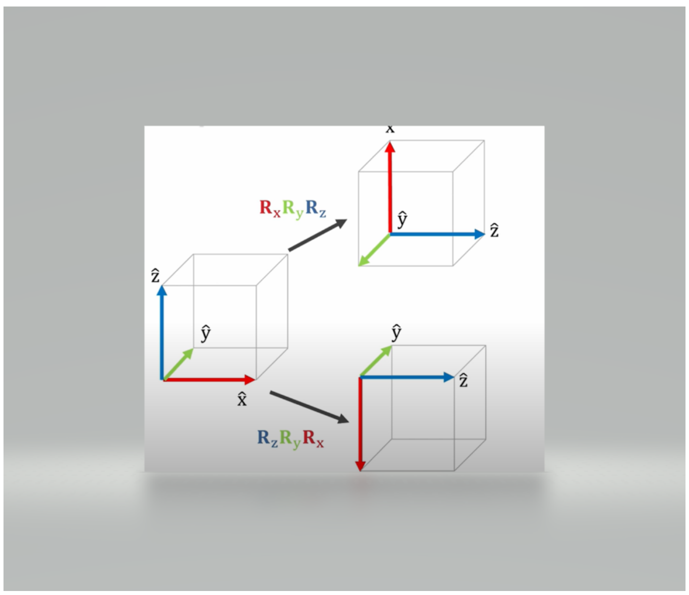
$$= AA^{-1}$$
$$= I$$

But:

$BA(AB)^{-1} = BAB^{-1}A^{-1}$ is not the same.

With the rotation matrix having in all 12 possible variations, we follow Euler's angles of the order 3-2-1(yaw,pitch,roll) or 3-1-3(yaw,pitch,yaw).

To say that we have a spacecraft and want to know how its body reference frame is oriented with respect to say the earth's center inertial frame or earth centered fixed frame if you want to look at specific points on the earth.However, if we were to change the order, it would mean that we are potentially changing the axises of x, y and z whereas the object would rotate in respect to the wrong axis, thus giving us error to calculate and visualize the pattern. In consequence, turning the matrices into the transpose of each other.



4) Prove

4)

A rotation matrix R, the tranpose of R is equal to R^{-1} such that

$$R^T = R^{-1} \rightarrow R^T \cdot R = I$$

let there be basis vectors $\{\hat{e}_i\} \rightarrow \hat{e}_i \cdot \hat{e}_j = \delta_{ij}$
(assumed to orthogonal)

$$\begin{pmatrix}\hat{e}'_1\\ \hat{e}'_2\\ \hat{e}'_3\end{pmatrix} = R\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} \Rightarrow \hat{e}'_i = R_{ij}\cdot\hat{e}_j$$

(New basis vector in rotated system.)

In rotations, we perserve length and angles since when we rotate the system, we don't want any measurement to change, or it will become a variation of stretching.

After rotation our basis vectors normalized and be orthogonal.

$$\hat{e}'_i \cdot \hat{e}'_j = \delta_{ij}$$

$$\hat{e}'_i \cdot \hat{e}'_j = (R_{ik}\hat{e}_k)(R_{jl}\cdot\hat{e}_l)$$

Scalar numbers

$$\hat{e}'_i \cdot \hat{e}'_j = R_{ik}\cdot R_{jl}\,(\hat{e}_k)(\hat{e}_l)$$
$$= R_{ik}\,(R_{jl}\,\delta_{kl})$$
$$= R_{ik}\cdot R_{jk}$$
$$(R)^T_{ki}\cdot R_{jk}$$

$$R^T\cdot R = \delta_{ij} \Rightarrow I$$

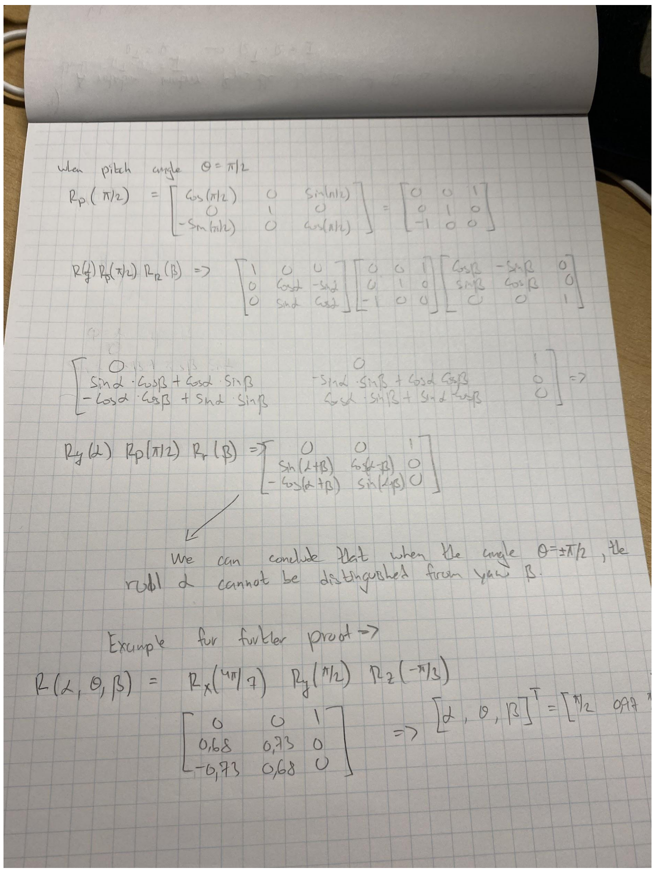
$$\underbrace{R\cdot R^T}_{I}\cdot R\cdot R^{-1} = R\cdot R^{-1}$$

$$I = R\cdot R^T$$

$$R^T\cdot R = R^{-1}\cdot R = I \quad \blacktriangle$$

5)What is gimbal lock?

The gimbal lock occurs when the pitch angle is $\pm\ 90$. degrees whereas the object will lose one degree of freedom in a three-dimensional figure. In other words, it can be referred to as two of the circular arms of the gimbal lock doing a parallel movement(rotation) such that it can lock the system degenerate into two-dimensional from three dimensions. (Picture of proof that the gimbal lock occurs)



when pitch angle $\theta = \pi/2$

$$R_p(\pi/2) = \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) \\ 0 & 1 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R(\alpha) R_p(\pi/2) R_R(\beta) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta & -\sin\alpha \cdot \sin\beta + \cos\alpha \cos\beta & 0 \\ -\cos\alpha \cdot \cos\beta + \sin\alpha \sin\beta & \cos\alpha \sin\beta + \sin\alpha \cos\beta & 0 \end{bmatrix} \Rightarrow$$

$$R_y(\alpha) R_p(\pi/2) R_r(\beta) \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 \\ -\cos(\alpha+\beta) & \sin(\alpha+\beta) & 0 \end{bmatrix}$$

We can conclude that when the angle $\theta = \pm\pi/2$, the roll α cannot be distinguished from yaw β.

Example for further proof =>

$$R(\alpha, \theta, \beta) = R_x(4\pi/7)\ R_y(\pi/2)\ R_z(-\pi/3)$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0{,}68 & 0{,}73 & 0 \\ -0{,}73 & 0{,}68 & 0 \end{bmatrix} \Rightarrow [\alpha, \theta, \beta]^T = [\pi/2\ \ 0{,}97$$

To resolve this error, we can either set the standard operating conditions away from the pitch angle of $\pm\ 90$.degree or avoid manoeuvring near the angle of $\pm\ 90$.