## In general

Optimal parameters to a model f:

$$\hat{b} = \underset{b \in \mathbb{R}^d}{\arg\min} \sum_{i=1}^n \rho_f(X, b),$$

where for least squares regression

$$\rho_f(X,b) = [Y - f(X,b)]^2 \quad \Longrightarrow \quad \frac{\partial \rho_f}{\partial b_j} = 2 \left[ y_i - f(X,b) \right] \cdot \frac{\partial f(X,b)}{\partial b_j}.$$

## **Squared function**

Let 
$$f$$
 be  $Q(X,b) = (b_0 + \sum_{j>0} b_j x_j)^2$ : 
$$\begin{cases} \frac{\partial Q}{\partial b_0} = 2(b_0 + \sum_{j>0} b_j x_j) &= 2\sqrt{Q(X,b)} \\ \frac{\partial Q}{\partial b_{j\neq 0}} = 2(b_0 + \sum_{j>0} b_j x_j) \cdot x_j = 2\sqrt{Q(X,b)} \cdot x_j \end{cases}$$

## **Exponential function**

Let f be  $E(X,b) = b_0 + b_1 \cdot x_1 \cdot e^{b_2 x_2}$ :

$$\begin{cases} \frac{\partial E}{\partial b_0} = 1 & = 1\\ \frac{\partial E}{\partial b_1} = x_1 \cdot e^{b_2 x_2} & = [E(X, b) - b_0]/b_1\\ \frac{\partial E}{\partial b_2} = x_1 \cdot e^{b_2 x_2} \cdot b_1 \cdot x_2 = [E(X, b) - b_0] \cdot x_2 \end{cases}$$

## **Power function**

Let f be  $P(X,b) = b_0 \cdot x_1^{b_1 + b_2 x_2}$ :

$$\begin{cases} \frac{\partial P}{\partial b_0} = x_1^{b_1 + b_2 x_2} & = P(X, b)/b_0 \\ \frac{\partial P}{\partial b_1} = x_1^{b_1 + b_2 x_2} \cdot b_0 \cdot \ln(x_1) & = P(X, b) \cdot \ln(x_1) \\ \frac{\partial P}{\partial b_2} = x_1^{b_1 + b_2 x_2} \cdot b_0 \cdot \ln(x_1) \cdot x_2 = P(X, b) \cdot \ln(x_1) \cdot x_2 \end{cases}$$

So, we have the condition  $x_1 > 0$ ,  $\forall i$ .