



# UNIVERSITEIT VAN AMSTERDAM

INFORMATIE EN COMMUNICATIE

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## HW-3

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1. (a)  $H(f(Y)) \leq H(Y)$

$$H(Y) = H(Y) + H[f(Y)|Y] \quad (1)$$

$$= H(Y, f(Y)) \quad (2)$$

$$= H(f(Y)) + H(Y|f(Y)) \quad (3)$$

$$\geq H(f(Y)) \quad (4)$$

$$H(f(Y)) \leq H(Y) \quad (5)$$

(b)  $H(X|f(Y)) \leq H(X|Y)$

$$H(X|f(Y)) \leq H(X|Y) \quad (6)$$

$$H(X, f(Y)) - H(f(Y)) \leq H(X, Y) - H(Y) \quad (7)$$

$$H(f(Y)|X) + H(X) - H(f(Y)) \leq H(Y|X) + H(X) - H(Y) \quad (8)$$

$$H(f(Y)|X) - H(f(Y)) \leq H(Y|X) - H(Y) \quad (9)$$

$$\leq H(Y, f(Y)) \quad (10)$$

$$\leq H(f(Y)) + H(Y|f(Y)) \quad (11)$$

$$\geq H(f(Y)) \quad (12)$$

$$H(X|f(Y)) \leq H(X|Y) \quad (13)$$

NEEDS WORK

(c)  $I(X, Z|Y) = 0 \implies I(X; Z) \leq I(X; Y) \text{ and } I(X; Z) \leq I(Y; Z)$

$$I(X; Z|Y) = I(X; Y, Z) - I(X; Y) = 0 \quad (14)$$

$$= I(X; Y|Z) + I(X; Z) - I(X; Y) = 0 \quad (15)$$

$$= I(X; Y|Z) + I(X; Z) = I(X; Y) \implies I(X; Z) \leq I(X; Y) \quad (16)$$

$$I(X; Z|Y) = I(X; Y, Z) - I(X; Y) = 0 \quad (17)$$

$$= I(X; Y|Z) + I(X; Z) - I(X; Y) = 0 \quad (18)$$

$$= I(X; Z) = I(X; Y) - I(X; Y|Z) \quad (19)$$

Chain rule for mutual information

$$= I(X; Z) = I(X; Y) - (I(X; Y) + H(Z|X) + H(Z|Y) - H(Z|X, Y) - H(Z)) \quad (20)$$

$$= I(X; Z) = -H(Z|X) - H(Z|Y) + H(Z|X, Y) + H(Z) \quad (21)$$

$$\begin{aligned}
I(X; Y) &= H(X, Y) - H(X|Y) - H(Y|X) \\
&= I(X; Z) + H(Z, Y) - H(Y|Z) = I(Z; Y) - H(Z|X) + H(Z|X, Y) + H(Z) \quad (22) \\
&= I(X; Z) + H(Z, Y) - H(Y|Z) + H(Z|X) - H(Z|X, Y) + H(Z) = I(Z; Y) \quad (23) \\
&= I(X; Z) + H(Z, Y) - (H(Y|Z) - H(Z)) + H(Z|X) - H(Z|X, Y) = I(Z; Y) \quad (24) \\
&= I(X; Z) + H(Z, Y) - H(Z, Y) + H(Z|X) - H(Z|X, Y) = I(Z; Y) \quad (25) \\
&= I(X; Z) + H(Z|X) - H(Z|X, Y) = I(Z; Y) \quad (26) \\
&= I(X; Z) - (-H(Z|X) + H(Z|X, Y)) = I(Z; Y) \quad (27) \\
&= I(X; Z) - (H(Z|X, Y) - H(Z|X)) = I(Z; Y) \quad (28) \\
&= I(X; Z) - H(Y|Z|X) = I(Z; Y) \quad (29) \\
&= I(X; Z) = I(Z; Y) + H(Y|Z|X) \quad (30) \\
&= I(X; Z) = I(Z; Y) + H(Y|Z|X) \implies I(X; Z) \geq I(Z; Y) \quad (31)
\end{aligned}$$

Too long but correct.

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