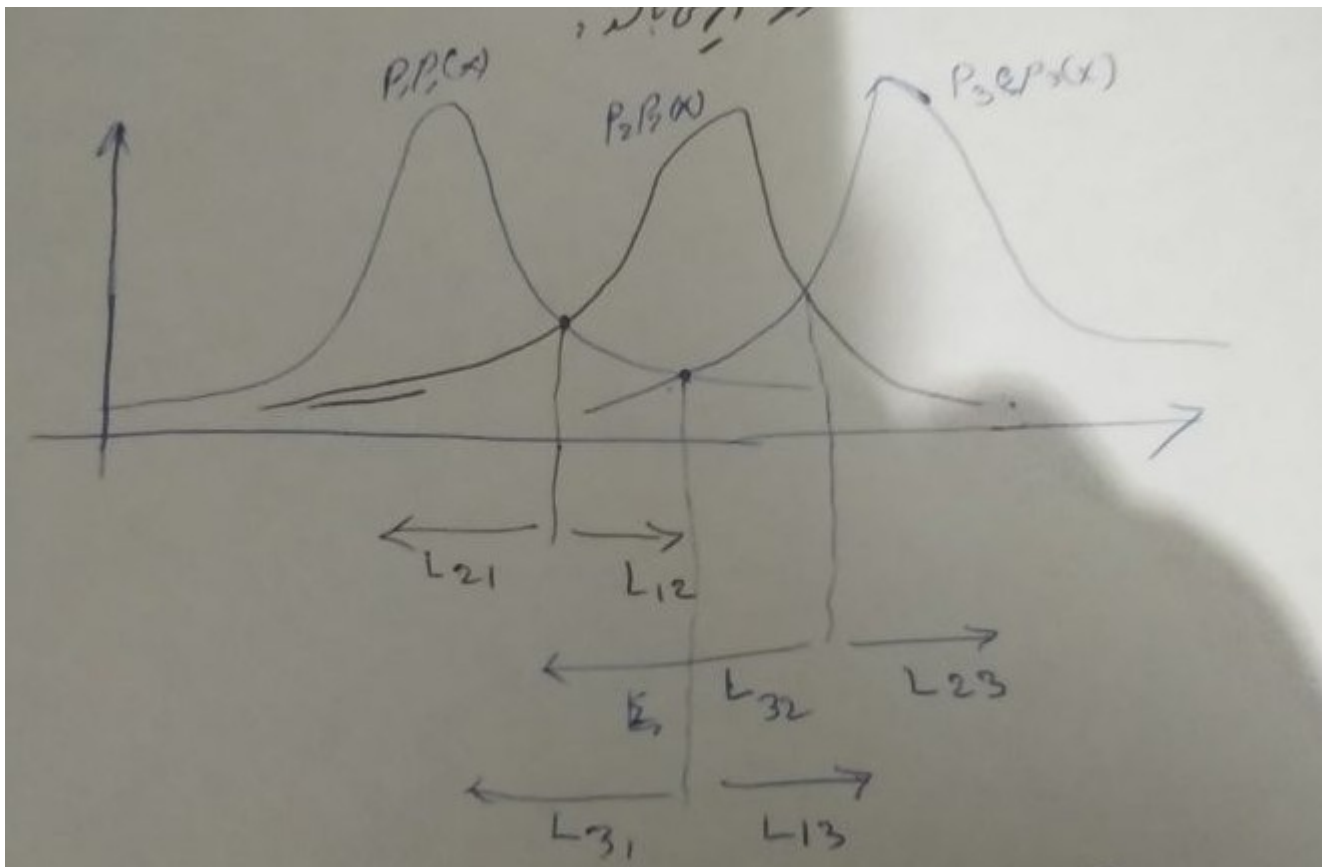


بخش اول تمرین شماره ۱۰ فصل سوم

تابع هزینه برای سه وضعیت تصمیم‌گیری  $W1, W2$  و  $W3$ ، به صورت زیر می باشد

$$R = E[r(x)] = \int \min [r_1(x), r_2(x), r_3(x)] p(x) dx$$

همچنین نمودار خط های تصمیم‌گیری بر اساس قضیه بیز برای کمترین خطا با فرض توزیع نرمال برای سه کلاس تصمیم‌گیری در تصویر زیر قابل مشاهده می باشد:



بر این اساس و با کمک فرمول زیر:

Not secure | web.khu.ac.kr/~tskim/PatternClass%20Lec%20Note%2009.pdf

h Torrent Sci-Hub محقق BCI Rehab Speed Test Mi... MATLAB kbTest Best Text-to-S... Arduino ESP8... SpeechTexter... DAT

Word - PatternClass Lec Note 09 5 / 19 175%

KHU-BME  
Pattern Classification  
Lecture 09

2. Probability of Error  $P_e$

2-class  

$$P_e = \int_{\Gamma_2} p(\underline{x} | S_1) P(S_1) d\underline{x} + \int_{\Gamma_1} p(\underline{x} | S_2) P(S_2) d\underline{x}$$

For Multiclass

$$P_e = 1 - P\{\text{correct}\} \quad \Leftarrow \text{This expression is much easier to understand.}$$

$$P\{\text{correct}\} = \sum_{i=1}^K \int_{\Gamma_i} p(\underline{x} | S_i) P(S_i) d\underline{x}$$

خطای ریسک تصمیم گیری بیز، برای سه کلاس به صورت زیر می باشد:

$$R_C(w_k | x) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} p(x|w_1)P(w_1) \\ p(x|w_2)P(w_2) \\ p(x|w_3)P(w_3) \end{bmatrix}$$

Bayes min Error  $\Rightarrow \int \min[R_C(w_k | x)] =$

$$\min \left[ \int_{\Gamma_1} \underbrace{\begin{bmatrix} c_{11} & c_{12} & c_{13} \end{bmatrix}}_B \underbrace{\begin{bmatrix} p(x|w_1)P(w_1) & p(x|w_2)P(w_2) & p(x|w_3)P(w_3) \end{bmatrix}^T}_A \right. \\ \left. \int_{\Gamma_2} \begin{bmatrix} c_{21} & c_{22} & c_{23} \end{bmatrix} \cdot A + \int_{\Gamma_3} \begin{bmatrix} c_{31} & c_{32} & c_{33} \end{bmatrix} \cdot A \right]$$

$$\Rightarrow \text{Min}[R_C] \cdot \sqrt{\int_{L_{12}} [0 \ 1 \ 1] \underbrace{\left[ \overset{(9-82P)}{\mu_1(\mu_1, \delta_1)} \mu_2(\mu_1, \delta_1) \mu_3(\mu_2, \delta_2) \right]^T}_{\wedge} }_{L_{21}} \int_{L_{21}} [1 \ 0 \ a] \cdot [A]^T + \int_{L_{23}} [1 \ 0 \ a] \cdot [A]^T$$

$$\int_{L_{32}} [1 \ a \ 0] \cdot A$$

بر این اساس با استفاده از توابع جدا ساز برای توزیع نرمال بالا شکل ۳ بعدی محدوده های بیز، آن به صورت زیر می باشد:

transformation — a projection  $\mathbf{P}$  onto line  $\mathbf{a}$  — leads to  $N(\mu, \sigma^2)$  measured along  $\mathbf{a}$ . While the transforms yield distributions in a different space, we show them superimposed on the original  $x_1 - x_2$  space. A whitening transform leads to a circularly symmetric Gaussian, here shown displaced.

Mahalanobis Distance =  $d_M$  (DHS p. 36)

$$d_M^2(\mathbf{x}, \mathbf{m}) = (\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m})$$

is the squared Mahalanobis distance from  $\mathbf{x}$  to  $\mathbf{m}$

The contours of constant density are hyperellipsoids of constant Mahalanobis distance to  $\mathbf{m}$  in Fig. 2.9

$$\Sigma = E\{(\mathbf{x} - \mathbf{m})(\mathbf{x} - \mathbf{m})^T\} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$

8

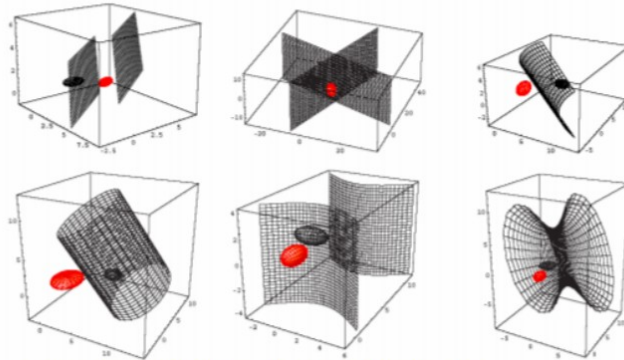
### Case 3: $\Sigma_i = \text{arbitrary}$

The discriminant surface

$$g(\mathbf{x}) = -(\mathbf{x} - \mu_1)^T \Sigma_1^{-1} (\mathbf{x} - \mu_1) + (\mathbf{x} - \mu_2)^T \Sigma_2^{-1} (\mathbf{x} - \mu_2) + c'$$

is a conic (2D) or quadric (nD).

e.g. in 3D



The surface can be a hyperboloid, i.e. it need not be closed

transformation — a projection  $\mathbf{P}$  onto line  $\mathbf{a}$  — leads to  $N(\mu, \sigma^2)$  measured along  $\mathbf{a}$ . While the transforms yield distributions in a different space, we show them superimposed on the original  $x_1 - x_2$  space. A whitening transform leads to a circularly symmetric Gaussian, here shown displaced.

Mahalanobis Distance =  $d_M$  (DHS p. 36)

$$d_M^2(\mathbf{x}, \mathbf{m}) = (\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m})$$

is the squared Mahalanobis distance from  $\mathbf{x}$  to  $\mathbf{m}$

The contours of constant density are hyperellipsoids of constant Mahalanobis distance to  $\mathbf{m}$  in Fig. 2.9

$$\Sigma = E\{(\mathbf{x} - \mathbf{m})(\mathbf{x} - \mathbf{m})^T\} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$

8

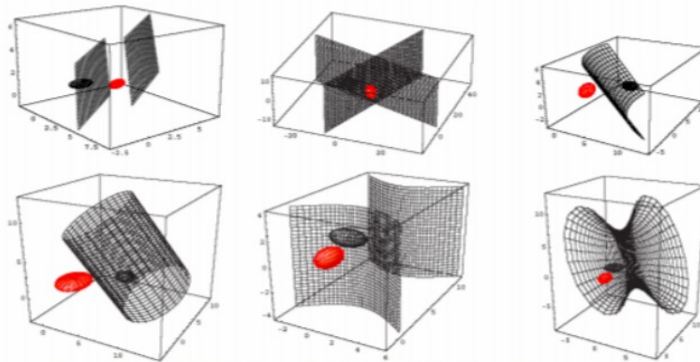
### Case 3: $\Sigma_i = \text{arbitrary}$

The discriminant surface

$$g(\mathbf{x}) = -(\mathbf{x} - \mu_1)^T \Sigma_1^{-1} (\mathbf{x} - \mu_1) + (\mathbf{x} - \mu_2)^T \Sigma_2^{-1} (\mathbf{x} - \mu_2) + c'$$

is a conic (2D) or quadric (nD).

e.g. in 3D



The surface can be a hyperboloid, i.e. it need not be closed