Digital Signal Analysis

ASSIGNMENT - 1

FOURIER SERIES :

Question 1

Plot the following functions and find the fourier series representations of the same -

i) $f(x) = x^2$, $-\pi < x < \pi$

ii) $f(x) = |x|$, $-\pi < x < \pi$

iii) $f(x) = e^{2x}$, $-\pi < x < \pi$.

Question 2

classify the following functions as odd or even or neither-

i) $f(x) = \begin{cases} 2 & 0 < x < 3 \\ -2 & -3 < x < 0 \end{cases}$, period = 6

ii) $f(x) = \begin{cases} \cos x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}$, period = 2π

iii) $f(x) = \begin{cases} 2x, & x < 5 \\ 15 - x, & x \geq 5. \end{cases}$

Question 3

Let $f(x) = x$, $-\pi < x < \pi$, Period $= 2\pi$.

i) Plot $f(x)$ in the interval $-2\pi < x < 2\pi$

ii) Find the fourier series representation of $f(x)$.

iii) Show that $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots$, using the above result.

Question 4

Let $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, period $= 2\pi$

i) Plot $f(x)$ in $-2\pi < x < 2\pi$

ii) Find the fourier series representation of $f(x)$

iii) Show that $\pi^2/8 = 1 + 1/3^2 + 1/5^2 + \cdots$, using the above result.

Fourier Transform

Question 5

Find the fourier transform of the function $x(t)$, where

$$x(t) = \begin{cases} 1 & 1 \le |t| \le 3 \\ -1 & |t| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Question 6

Find and sketch magnitude and phase spectrum of the fourier transform of the following -

a) $x(t) = \begin{cases} a, & -T \le t \le T \\ 0, & \text{otherwise} \end{cases}$

b) $x(t) = \delta(t-a)$, where a is real.

Question 7 :

Find the following for i) $x(t) = e^{-|at|} \cdot u(t)$

ii) $x(t) = e^{(-1+2j)t} \cdot u(t)$

a) $|x(w)|$

b) $\angle x(w)$

c) $Re\{x(w)\}$

d) $Im\{x(w)\}$.

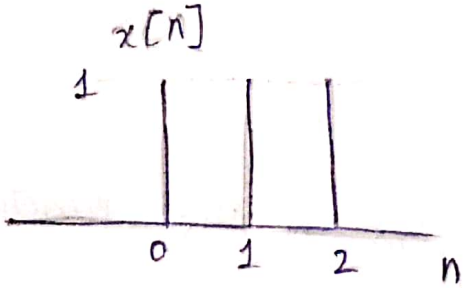
DFT & DTFT.

Question 8 :

Compute the DTFT for the following signals

i) $x[n] = (1/5)^n u(n+1)$

ii)



iii) $x[n] = (1/2)^{n+2} u(n)$.

iv) $x(n) = (1/2)^n u(n-4)$.

Question 9:

Compute the 8-point DFT for the following:

i) $x(n) = 1/4\,\delta(n)$.

ii) $x[n] = \{1, -1, 1, -1, 1\}$

iii) $x(n) = \cos\left(\frac{\pi n}{4}\right)$

iv) $x[n] = \{j, 0, -j, 1\}$.

Question 10:

Determine the inverse fourier transform of the following-

i) $x[e^{jw}] = \dfrac{e^{-jw} - (1/4)}{1 - (1/4)e^{-jw}}$

ii) $x[e^{jw}] = \dfrac{1 - (1/2)^4 e^{-4jw}}{1 - (1/2)e^{-jw}}$

iii) $x(e^{jw}) = \cos^2 3w + \sin^2 w$.

Question 11:

Find $x(\omega)$ when $x[n] = (n-1)\left(\frac{1}{a}\right)^{|n|}$, using the properties of fourier transform.

Question 12:

Evaluate:

a) $\int_{-\pi}^{\pi} |x(e^{j\omega})|^2 d\omega$

b) $\int_{-\pi}^{\pi} \left| \frac{d\,x(e^{j\omega})}{d\omega} \right|^2 d\omega.$

when $x[n] \Rightarrow$