Assignment 1: Due 14 Aug 2023 before midnight

1. Estimation is an important aspect of modeling in sciences. Do the following exercises to test your ability to estimate quantities :
   (a) The distance between the classroom (H-205) and the institute main gate $\approx n \times 100$ m. Estimate n as an integer.
   (b) The time required to walk from the classroom to the main gate at an average pace $\approx n$ m. Estimate n as an integer.
   (c) If a thin conducting cable (say, copper or aluminum, used for fittings in homes) was laid from the classroom to the main gate on the regular path that you use for walking, and typical current (as in wires in home fittings) made to flow through it, what would be the time taken by an individual electron to go from one to the other end on average? Assume each atom in the cable to contribute one electron for conduction.
   (d) If the electron in the ground state of hydrogen atom were to go around the hydrogen nucleus following Bohr model, approximately how many rounds will it make in the time estimated in the question above (c).
   (e) Estimate the power consumed for the flow of current for the time estimated above (c).

2. Draw phase plane trajectories with three different values of total energies for a dynamical (mechanical) system
   (i) exhibiting SHM and
   (ii) subject to an anharmonic potential with the energy in one case higher than the dissociation limit.

3. The logistic map $x_{n+1} = \alpha x_n(1 - x_n)$ is usually studied for the growth parameter, α, in the range $0 \leq \alpha \leq 4$. Why? Write a small code for the logistic map and compare the results graphically for $\alpha = 0.8$ and $\alpha = 1.5$ for initial conditions $x_0 = 0.1$ and 0.5. Take another case with $\alpha = 3.2$ and $x_0 \gtrsim 0$ and plot the results.

4. Show that in the limit of large N, the binomial distribution of n out of N objects becomes a Gaussian distribution [Hint: Write $P(N|n) = \begin{pmatrix} N \\ n \end{pmatrix}$
   Take logarithms of both sides and Use Stirling's approximation $\ln(N!) = N\ln(N) - N$;
   Keeping in mind that the distribution has a maximum, find by differentiation $n = n^*$, where the peak is located.

   Expand $P(N|n)$ around $n = n^*$using Taylor's expansion

   $f(x) = f(x_{0)} + (x - x_0)f'(x)|_{x=x_0} + \frac{1}{2}(x - x_0)^2 f"(x)|_{x=x_0}$

   At the maximum the first derivative disappears. Find the value of the second derivative and take antilog on both sides].