Assignment 2: Due 23 Aug 2023 before midnight

1. The compression factor, Z, is the ratio of the measured molar volume of a gas, $V_m = \frac{V}{n}$, to the molar volume of a perfect gas, V_m° , at the same pressure (p) and temperature (T); V = volume of n moles of gas. .

   (a) Show that $pV_m = RTZ$.

   (b) Most ordinary gases have $Z > 1$ at very high pressures. At intermediate pressures, most gases have $Z < 1$, and at very low pressures, $Z \approx 1$. Explain why.

   (c) For a real gas, a general equation of state is $pV_m = RT\left(1 + B'p + C'p^2 + \cdots\right)$. Find $\frac{dZ}{dp}$ at very low pressures ($p \to 0$).

   (d) The temperature (> 0) at which $Z \to 1$ is called the Boyle temperature, T_B. For helium, $T_B = 22.64$ K; Compare the state of helium gas at temperatures 20K , 22.64 K and 25K.

   (e) The second virial coefficient of methane can be approximated by the empirical equation $B(T) = a + be^{-\frac{c}{T^2}}$, where $a = -0.1993$ bar $^{-1}$, $b = 0.2002$ bar^{-1} , and $c = 1131\text{K}^2$ with 300 K $< T <$ 600 K. What is the Boyle temperature of methane?

   (f) Show that the van der Waals equation leads to values of $Z < 1$ and $Z > 1$, and identify the conditions for which these values are obtained.

2. Derive the critical constants for a van der Waals gas in terms of the a and b coefficients.

3. Obtain the equation for a van der Waals gas in terms of the reduced variables $\frac{X}{X_C}$,where $X \in \{p, V, T\}$ and $X_C$is the corresponding critical value.