Probability and Statistics

Monsoon 2021
Lec 01 Notes

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1 Counting

Why counting?

- counting is a basic task in mathematics
- Our objective: To tell how many objects there aare without actually counting
- Applications of counting :
  - number of steps of an algorithm
  - estimating probability of occurrence of an event (in this course)
  - proofs such as Pigeon Hole Principle (PHP)

Sum Rule - If there are m objects ogf the first type and n objects of second type then there are m+n objects of one of the two types. // Following are few examples :-

- Number of food items in 7 Pizzas and 5 burgers
- A oiece stays in the bottom left corner of a chessboard. In one move it can move one step to the right or one step up. How many moves are needed to reach a certain box.

Note - Sum rule fails when there is an intersection between the two sets.

2 Set

A set is a collection of distinct things (or elements). Remarks and examples-

- Ordering of elements in a set is unimportant
- set of natural numbers N.
- set of integers Z
- Set of rational numbers Q, set of real numbers R

- A closed interval implies that the end points are included in the set
- Open interval (or parentheses) means that the endpoints are excluded from the set
- A set A is called **subset** of a set B if every element of A is also an element of B. It is denoted by $A \sqsubseteq B$
- Two sets A and B are said to be equal if $A \sqsubseteq B$ and $B \sqsubseteq A$
- A set with no elements is called **empty set or null set**, denoted by ϕ
- The **universal set** is the set of all the things that we could possibly consider in the context we are studying. [Every set A is a subset of universal set]
- A set S is called **countable**, if there exists a **bijective function** [i.e. the function is both one-one and onto]
  $f : S \to N$
  $Injective + Surjective \to Bijective$
- The sets $\mathbb{N}, \mathbb{Q}$ are countable
- A set with finite number of elements is called a **finite** set.
- A set which is countable and not finite is called **countably infinite**

3 Set Theory

- Venn diagrams - They are useful in analysing the relationship between sets
- Union -
  - The union of two sets is a set containing all the leements that are in A or in B. Here A union B is denoted by $A \cup B$. (eg. $\{1, 2\} \cup \{2, 3\}) = \{$1,2,3$\}$
  - Similarly, we define the union of three or more sets as follows

$$\boxed{A_1 \cup A_2 \cup A_3 .... \cup A_k = \bigcup_{i=1}^{k} A_i}$$

  - If A and B are countable, then $A \cup B$ is also countable
  - Countable union of countable sets is also countable

Intersection -

- The intersection of two sets is a set containing all the elements that are in A **and** in B.
- Here A intersection B is denoted by $A \cap B$
- Similaraly we define intersection as follows -

$$\boxed{A_1 \cap A_2 \cap A_3 .... \cap A_k = \bigcap_{i=1}^{k} A_i}$$

- Complement -
  - The **complement** of a set A denoted by A^c is the set of all elements that are in the universal set S, but **not** in A
- Set Difference -
  - The **set diffenrence** denoted by $A - B$ consists of all the elements that are in A, but **not** in B
  - For example $A = \{1, 2, 3\}$ and $B = \{3, 5\}$, then A-B = $\{1, 2\}$
  - Two sets A and B are **mutually exclusive or disjoint** if

$$\boxed{A \cap B = \phi}$$