Probability and Statistics

Lec 02 Notes

20-08-21

1 Cartesian Product of Sets

Definition - Cartesian Product of two sets $A = \{a_1, a_2, ..., a_m\}$ and $B = \{b_1, b_2, ..., b_n\}$ denoted by A*B is deined as follows

$$\boxed{A * B = \bigcup i, j\{(a_i, b_j)\}}$$

2 Partition of a Set

A collection of non-empty sets $A_1, A_2, ...,$ is a **partition** of a set A if they are **disjoint** and their union is A

3 Demorgan and Distributive Laws

3.1 De Morgan's Laws

- For any sets $A_1, A_2, ..., A_n$, we have

$$(A_1 \cup A_2..... \cup A_n)^c = A_1^c \cap A_2^c \cap .... \cap A_n^c$$
$$(A_1 \cap A_2 \cap ..... \cap A_n)^c = A_1^c \cup A_2^c \cup .... \cup A_n^c)$$

3.2 Distributive Law

- $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$
- $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

3.3 Rule of Sum using Set language

Rule of sum- If there is a set A with k elements, a set B with n elements and these sets do not hve common elements, then the set $A \cup B$ has n+k elements

Remark of Rule of Sum -

- if we consider $|A| + |B|$ as in sum rule, then we will be wrong

- We wil count elements that belong to both A and B twice
- $|A \cup B| = |A| + |B| - |A \cap B|$ (By the principle of inclusion-exclusion)

3.3.1 Example

Number of Paths -
Suppose there are several points connected by arrows. There is a starting point s (called source) and a final point t (called sink). How amny different ways are there to get from s to t?

Solution -
Counting can be done recursively
for each node count the number of paths from s to this node (sum rule will be used)

4 Product Rule

Definition - If there are k objects of the first type and n objects of the second type then there are k*n pairs of objects, the forst of first type and the second of the second type

4.1 Rule of Products using Sets

If there is a finite set A and a finite set B, then there are $|A| * |B|$ pairs of objects, the first from A and the second from B

4.2 Tuples : Application of Product Rule

How many different 5-symbol passwords can we create using lower case Latin letters only? (the size of the alphabet)

- How many different 1-letter passwords are possible?
- What about 2-letters?
- How many different 3-letter words are possible

4.2.1 Tuples with restrictions

Combine Sum and Product rule -
Question: How many integer numbers are there between 0 and 9999 that have only one 5 digit?

5 Permutations

Definition - Tuples of length k without repetitions are called **k-permutations**

Question - Suppose we have a set of n symbols. How many different sequences of length k we can form out of these symbols if we are not allowed to use the same symbol twice?

- Observe that if n¡k then there are no k-permutations: there are simply not enough different letters
- So it is enough to solve the problem for the case k≤ n

1 2 3 ... k
* * *... *
n n-1 n-2 ...

- Hence there are

  n*(n-k)*...(n-k+1)

  k-permutations, which is $n!/(n-k)!$
- Examples - In how many ways we can arrange n different books in n different bins on shelf?

6 Combinations

Definition - For a set S, a **k-combination** is a subset of size k
The **number of k-combinations** of an n element set is denoted by $\binom{n}{k}$
Pronounced: "n choose k"
The number of k-combinations of an n-element set is given by

$$\frac{n!}{(n-k)!}$$

7 Pascal Triangle

Question - There are n students. What is the number of ways of forming a team of k students out of them?
Answer - $\binom{n}{k}$
A result - $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$