

Appendix:

Assume that $\{l_{ij}\}$ is transformed weighted matrix and $\{d_{ij}\}$ is shortest path matrix.

Then we have:

$$E(G) = \frac{\sum \frac{1}{d_{ij}}}{N(N-1)}$$

But $E(G)$ is in range 0 to infinity, so we can define the ideal network which shortest path there, is equal to weight between nodes (the network is fully connected)

$$E(G^{ideal}) = \frac{\sum \frac{1}{l_{ij}}}{N(N-1)}$$

So, the normalized value of efficiency obtains from $\frac{E(G)}{E(G^{ideal})}$ and it varies from 0 to 1.

Now if the normalized value of efficiency exceeds from 1, it means that one of constraint is not satisfied.

First, we assumed that:

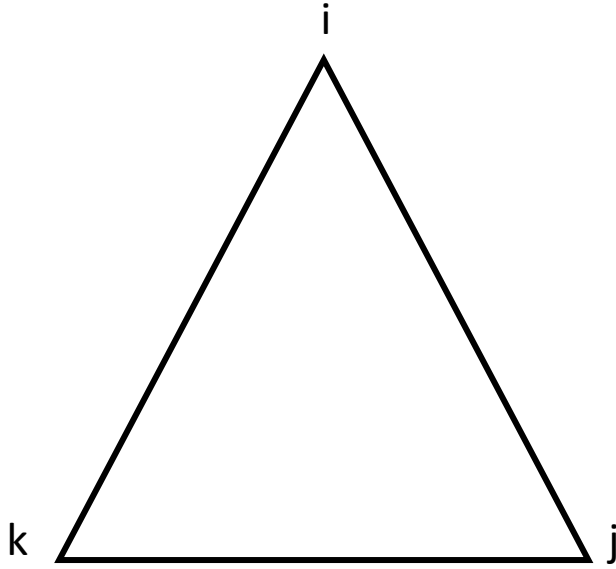
$$d_{ij} \geq l_{ij} \text{ and } \frac{1}{l_{ij}} \geq \frac{1}{d_{ij}}$$

But in this case:

$$\frac{1}{d_{ij}} \geq \frac{1}{l_{ij}} \text{ which means triangle inequality is not satisfied.}$$

I transformed all weights based on the maximum number in the initial adjacency matrix $\{w_{ij}\}$.

Transformation: $t = \max(\{w_{ij}\}) + 1$ and $\{l_{ij}\} = t - \{w_{ij}\}$



Assume 3 nodes i, j and k making a triangle. We expect:

$$l_{ij} \leq l_{jk} + l_{ik} \text{ or } t - w_{ij} \leq t - w_{jk} + t - w_{ik}$$

So, the value of the must be:

$$w_{jk} + w_{ik} - w_{ij} \leq t \text{ and we know that: } w_{jk} + w_{ik} - w_{ij} \leq 2 \times \max(\{w\}) - \min(\{w\})$$

so, transformation should be in this range:

$$2 \times \max(\{w\}) - \min(\{w\}) \leq t$$

And the first transformation doesn't satisfy the above constraint.