Consider a model $Y_{1:N}$ for data $y_{1:N}^*$, with a latent variable $X_{0:N}$, and a statistical model defined by a joint density $f_{X_{0:N}, Y_{1:N}}(x_{0:N}, y_{1:N}; \theta)$. The likelihood function is

$$L(\theta) = f_{Y_{1:N}}(y_{1:N}^*; \theta).$$

Are the following identities (A) true for all statistical models; (B) true for general POMP models but not all models; (C) true for linear Gaussian POMP models but not general POMP models; (D) generally false.

$$L(\theta) = \int f_{Y_{1:N}|X_{0:N}}(y_{1:N}^*|x_{0:N}; \theta) \, f_{X_{0:N}}(x_{0:N}; \theta) \, dx_{0:N} \tag{1}$$

$$L(\theta) = \prod_{n=1}^{N} f_{Y_n|Y_{1:n-1}}(y_n^*|y_{1:n-1}^*; \theta) \tag{2}$$

$$\mathrm{Var}\{X_{n+1} \,|\, Y_{1:n}\} = E\big[\mathrm{Var}\{X_{n+1} \,|\, X_n\} \,\big|\, Y_{1:n}\big] + \mathrm{Var}\big\{E[X_{n+1} \,|\, X_n] \,\big|\, Y_{1:n}\big\} \tag{3}$$

$$L(\theta) = \int \left[\prod_{n=1}^{N} f_{Y_n|X_n}(y_n^*|x_n; \theta) \right] f_{X_{0:N}}(x_{0:N}; \theta) \, dx_{0:N} \tag{4}$$