Modeling and Analysis of Time Series Data

Chapter 10: Forecasting

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Contents

1 Introduction

Model-based forecasts

- Data, $y^*_{1:N}$, and a model $Y_{1:N+h}$ with joint density $f_{Y_{1:N+h}}(y_{1:N+h}|\theta)$ can be used to **forecast** future values $y_{N+1:N+h}$ up to a **horizon**, h.
- A model-based **probabilistic forecast** of the not-yet-observed values $y_{N+1:N+h}$ is

$$f_{Y_{N+1:N+h}|Y_{1:N}}\big(y_{N+1:N+h}|y^*_{1:N};\hat{\theta}\big), \tag{1}$$

  where $\hat{\theta}$ is a point estimate such as an MLE.
- A model-based **point forecast** of $y_{N+1:N+h}$ is

$$\mathbb{E}\big[Y_{N+1:N+h}\big|Y_{1:N}=y^*_{1:N};\hat{\theta}big\big]. \tag{2}$$

- Point forecasts and probabilistic forecasts have many applications in business and elsewhere.

Evaluating forecasts

- Point forecasts could be evaluated by squared error, absolute error, relative squared error, relative absolute error, etc.
- Probabilistic forecasts are naturally evaluated by the forecast density,

$$f_{Y_{N+1:N+h}|Y_{1:N}}\big(y_{N+1:N+h}|y^*_{1:N};\hat{\theta}\big), \tag{3}$$

  evaluated at the data, $y^*_{N+1:N+h}$, once it is collected.
- Due to time dependence, and limited amounts of data, it can be problematic to evaluate by cross-validating.

ARIMA forecasting
likelihood and one-step forecasting
exponential weights forecasting (introduced as a stochastic model)
prophet
forecasting vs model fitting
deep learning?
evaluation of forecasts
example: huron water level (forecasting from 2014)

Further reading

- Section 3.5 of Shumway and Stoffer (2017) covers ARIMA forecasting.
- Hyndman and Khandakar (2008) introduces the `forecast` R package.
- Taylor and Letham (2018) presents the Facebook Prophet forecasting algorithm.

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References

Hyndman RJ, Khandakar Y (2008). "Automatic time series forecasting: The forecast package for R." *Journal of Statistical Software*, **27**, 1–22.

Shumway RH, Stoffer DS (2017). *Time Series Analysis and its Applications: With R Examples*. 4th edition. Springer. 3

Taylor SJ, Letham B (2018). "Forecasting at scale." *The American Statistician*, **72**(1), 37–45.