Modeling and Analysis of Time Series Data

Chapter 10: Forecasting

Edward L. Ionides

Contents

1 Introduction

Model-based forecasts

- Data, $y^*_{1:N}$, and a model $Y_{1:N+h}$ with joint density $f_{Y_{1:N+h}}(y_{1:N+h}|\theta)$ can be used to **forecast** future values $y_{N+1:N+h}$ up to a **horizon**, h.
- A model-based **probabilistic forecast** of the not-yet-observed values $y_{N+1:N+h}$ is
$$f_{Y_{N+1:N+h}|Y_{1:N}}\big(y_{N+1:N+h}|y^*_{1:N};\hat\theta\big), \tag{1}$$
where $\hat\theta$ is a point estimate such as an MLE.
- A model-based **point forecast** of $y_{N+1:N+h}$ is
$$\mathbb{E}\big[Y_{N+1:N+h}\big|Y_{1:N}=y^*_{1:N};\hat\theta big]. \tag{2}$$
- Point forecasts and probabilistic forecasts have many applications in business and elsewhere.

Evaluating forecasts

- Point forecasts could be evaluated by squared error, absolute error, relative squared error, relative absolute error, etc.
- Probabilistic forecasts are naturally evaluated by the forecast log-density,
$$\log f_{Y_{N+1:N+h}|Y_{1:N}}\big(y_{N+1:N+h}|y^*_{1:N};\hat\theta\big), \tag{3}$$
evaluated at the data, $y^*_{N+1:N+h}$, once it is collected.
- Due to time dependence, and limited amounts of data, it can be problematic to evaluate by cross-validation.
- Note that log-likelihood can be written as a sum of one-step forecast log-densities:
$$\log f_{Y_{1:N}}(y^*_{1:N};\theta)=\sum_{n=1}^{N}\log f_{Y_n|Y_{1:n-1}}(y^*_n|Y^*_{1:n-1};\theta) \tag{4}$$

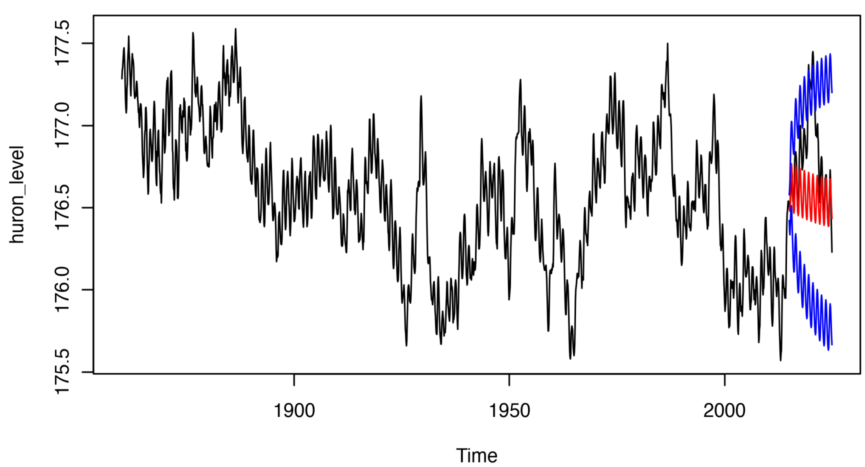
2 ARIMA forecasting

ARIMA forecasting

`predict.Arima()` computes the conditional Gaussian distribution for forecasting an ARIMA model.

```
dat <- read.table(file="huron_level.csv",sep=",",header=TRUE)
huron_level <- ts(as.vector(t(dat[,2:13])),start=1860,freq=12)
time <- rep(dat$Year,each=12)+ rep(0:11,nrow(dat))/12
huron_old <- window(huron_level,end=2014.99)
sarma <- arima(huron_old,order=c(1,0,1),
  seasonal=list(order=c(1,0,1),period=12))
f.sarma <- predict(sarma,n.ahead=120)
f.val <- as.vector(f.sarma$pred)
f.se <- as.vector(f.sarma$se)
f.time <- as.vector(time(f.sarma$pred))
plot(huron_level)
lines(f.time,f.val,col="red")
lines(f.time,f.val+1.96*f.se,col="blue")
lines(f.time,f.val-1.96*f.se,col="blue")
```

95% prediction interval from December 2014



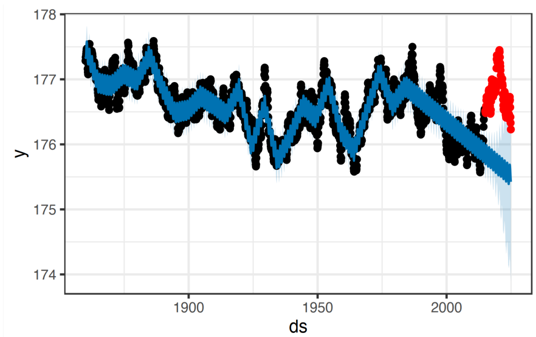
3 Prophet

Facebook Prophet

- ARIMA models are good for relatively short time series.
- SARIMA is good for monthly and quarterly data, but less so for daily or hourly.
- You may have already experienced this. Large-scale forecasting competitions confirm it (Makridakis *et al.*, 2020).
- Prophet was designed for high-frequency (daily, hourly) business forecasting tasks at Facebook, and is widely used for similar tasks elsewhere.

- Prophet does penalized regression estimating trend and seasonality components. It can also do Bayesian fitting.
- Unlike ARIMA, Prophet cannot describe general covariance structures.

```
library(prophet)
library(ggplot2)
history <- data.frame(y = huron_old,
  ds = seq(as.Date('1860-01-01'), as.Date('2014-12-01'), by = 'm'))
fit <- prophet(history)
future <- make_future_dataframe(fit, periods = 10*12,freq='month')
forecast <- predict(fit,future)
plot(fit,forecast)+
  geom_point(data=data.frame(ds=future$ds[-(1:1860)],y=huron_level[-(1:1860)]),color="red")
```



4 Forecasting vs modeling

Forecasting versus model fitting

- A good model should imply a good model-based forecast.
- Long-term forecasting is extrapolation. The model may be unreliable far from the timeframe used to build it.
- Without evidence to support a model for long-term forecasts, uncertainty estimates should be high. Uncertainty estimates are also uncertain!
- Deep learning methods need large amounts of data. They are not yet standard for forecasting. Prophet uses automatic differentiation techniques that enable deep learning.

Forecasting with trends and covariates

- A model with trends and covariates must project those into the future in order to forecast.
- Uncertainty about future trends may be captured by "stochastic trend" models. Prophet does this.
- We've seen the difficulty assessing stationarity vs slowly varying trend. The same issue arises with forecasting. How do we know if a trend will continue, or if it will change in future?

Further reading

- Section 3.5 of Shumway and Stoffer (2017) covers ARIMA forecasting.
- Hyndman and Khandakar (2008) introduces the `forecast` R package.
- Taylor and Letham (2018) presents the Facebook Prophet forecasting algorithm.

Acknowledgments

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- We acknowledge previous versions of this course.

References

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Taylor SJ, Letham B (2018). "Forecasting at scale." *The American Statistician*, **72**(1), 37–45. 9