# Quiz 1, STATS/DATASCI 531/631 W25

In class on 2/17

**This document produces different random quizzes each time the source code generating it is run. The actual quiz will be a realization generated by this random process, or something similar.**

**This version lists all the questions currently in the quiz generator. The actual quiz will have one question sampled from each of the 6 question categories.**

**Instructions. You have a time allowance of 30 minutes. The quiz may be ended early if everyone is done. The quiz is closed book, and you are not allowed access to any notes. Any electronic devices in your possession must be turned off and remain in a bag on the floor. For each question, circle one letter answer and provide some supporting reasoning.**

---

**Q1. Stationarity and unit roots.**

**Q1-01.**

Suppose that a dataset $y^*_{1:N}$ is well described by the statistical model

$$Y_n = a + bn + \epsilon_n,$$

where $\epsilon_n$ is white noise and $b \neq 0$. Which of the following is the best approach to time series modeling of $y^*_{1:N}$?

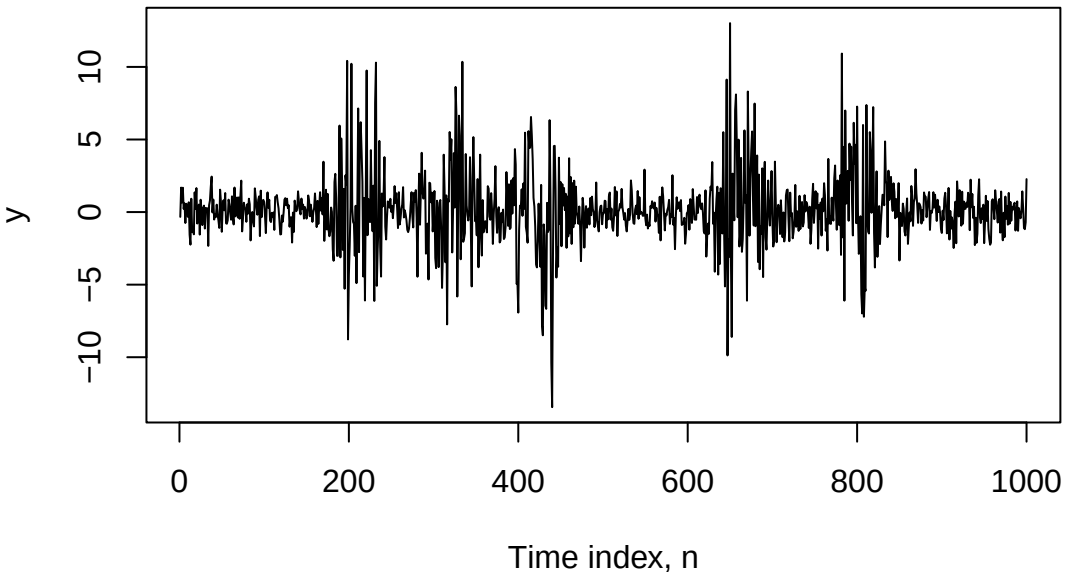
A. The data are best modeled as non-stationary, so we should take differences. The differenced data are well described by a stationary ARMA model.

B. The data are best modeled as non-stationary, and we should use a trend plus ARMA noise model.

C. The data are best modeled as non-stationary. It does not matter if we difference or model as trend plus ARMA noise since these are both linear time series models which become equivalent when we estimate their parameters from the data.

D. We should be cautious about doing any of A, B or C because the data may have nonstationary sample variance in which case it may require a transformation before it is appropriate to fit any ARMA model.

**Q1-02.**




Consider the time series plotted above. Which of the below is the most accurate statement about stationarity?

A. The plot shows that the data are clearly non-stationary. We could make a formal hypothesis test to confirm that, but it would not be insightful. To describe the data using a statistical model, we will need to develop a model with non-constant variance.

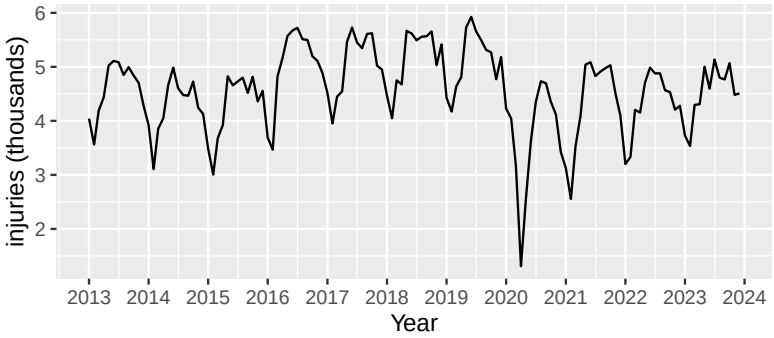
B. The sample variance is evidently different in different time intervals. However, we should not conclude that the underlying data generating mechanism is non-stationary before making a formal statistical test of equality of variances between the time regions that have lower sample variance and the regions that have higher sample variance. Visual impressions without a formal hypothesis test can be deceptive.

C. A model with randomly changing variance looks appropriate for these data. Since the variance for such a model is time-varying, the model must be non-stationary.

D. A model with randomly changing variance looks appropriate for these data. Despite the variance for such a model being time-varying, the model is stationary.

E. The sample variance is evidently different in different time intervals. An appropriate next step to investigate stationary would be to plot the sample autocorrelation function for different intervals to see if the dependence between time points is also time-varying.

**Q1-03.**




Above are monthly injuries from motor vehicle collisions in New York City. An augmented Dickey-Fuller test, `tseries::adf.test(injuries)`, gives a p-value of 0.01. Which is the best way to proceed:

A: The time plot indicates a non-constant mean function describing a major dip due to the COVID-19 pandemic and an increasing trend at other times. The ADF test does not support or refute that model.

B: The ADF test suggests the series is stationary, supporting a decision to fit a SARMA model.

C: The ADF test suggests the series is non-stationary; it should be differenced before fitting a SARMA.

D: The ADF test indicates that the series is non-stationary, supporting the use of a non-constant mean function to describe a major dip due to the COVID-19 pandemic and an increasing trend at other times.

**Q2. Calculations for ARMA models**

**Q2-01.**

Let $Y_n = \phi Y_{n-1} + \epsilon_n$ for $n = 1, 2, ...$ with $\epsilon_n \sim \text{iid}N[0, \sigma^2]$ and $Y_0 = 0$. The covariance of $Y_n$ with $Y_{n+k}$ for $k \geq 0$ is

A. $\sigma^2\phi^k/(1-\phi^2)$

B. $\sigma^2\phi^{2k}/(1-\phi^2)$

C. $\sigma^2\phi^k/(1-\phi)$

D. $\sigma^2\phi^{2k}/(1-\phi)$

E. None of the above.

**Q2-02.**

Let $Y_n$ be an ARMA model solving the difference equation

$$Y_n = (1/4)Y_{n-2} + \epsilon_n + (1/2)\epsilon_{n-1}.$$

This is equivalent to which of the following:

A. $Y_n = (1/2)Y_{n-1} + \epsilon_n$

B. $Y_n = -(1/2)Y_{n-1} + \epsilon_n$

C. $Y_n = (1/2)Y_{n-2} - (1/16)Y_{n-4} + \epsilon_n + \epsilon_{n-1} + (1/4)\epsilon_{n-2}$

D. $Y_n = -(1/2)Y_{n-2} - (1/16)Y_{n-4} + \epsilon_n + \epsilon_{n-1} + (1/4)\epsilon_{n-2}$

E. None of the above

**Q2-03.**

Is it possible for an $AR(2)$ model to have a finite moving average representation, so that it is equivalent to some $MA(q)$ model for $q < \infty$?

A. No. Any moving average representation of any $AR(2)$ model is $MA(\infty)$

B. Yes. Although it is not true for any $AR(2)$ process, it is possible to find particular choices of the autoregressive coefficients, $p_1$ and $p_2$, that lead to a finite $MA(q)$ representation.

C. It is not possible for any real-valued $p_1$ and $p_2$, but it is possible if you permit $p_1$ and $p_2$ to be complex-valued.

**Q3. Likelihood-based inference for ARMA models**

**Q3-01.**

The following table of AIC values results from fitting ARMA(p,q) models to a time series $y_{1:415}$ where $y_n$ is the time, in miliseconds, between the $n$th and $(n+1)$th firing event for a monkey neuron. The experimental details are irrelevant here. You are asked to check how many adjacent pairs of AIC values in this table are inconsistent, such that they could mathematically arise only from a numerical error? Adjacent pairs of models are those directly above or below or left or right of each other in the table.

| | MA0 | MA1 | MA2 | MA3 |
|---|---|---|---|---|
| AR0 | 3966.0 | 3961.5 | 3962.7 | 3964.7 |
| AR1 | 3961.1 | 3962.6 | 3964.6 | 3966.6 |
| AR2 | 3962.7 | 3960.5 | 3959.8 | 3961.7 |
| AR3 | 3964.6 | 3965.5 | 3962.6 | 3968.4 |

A: 0, so the table is mathematically plausible.

B: 1

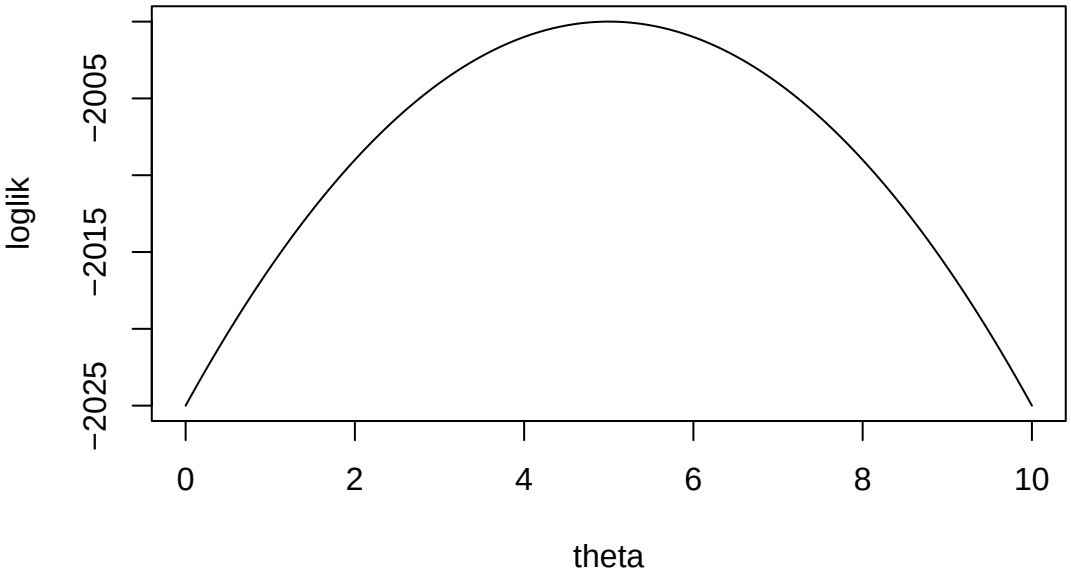
C: 2

D: 3

E: 4 or more

**Q3-02.**

The R function `arima()` provides standard errors calculated using observed Fisher information. This question tests your understanding of what that means. Suppose a parametric model has a single parameter, $\theta$, and the log-likelihood function when fitting this model to dataset is as follows:




What is the observed Fisher information ($I_{obs}$) for $\theta$?

It may be helpful to note that the observed Fisher information is accumulated over the whole dataset, not calculated per observation, so we don't have to know the number of observations, $N$.

Also, for time series models, we do not usually model observations as independent. Thus, the log-likelihood is not the sum of the log-likelihood for each observation. Its calculation will involve consideration of the dependence, and usually the job of calculating the log-likelihood is left to a computer.

For checking your answer, it may help to know that the usual variance estimate for the maximum likelihood estimate, $\hat{\theta}$, is $\text{Var}(\hat{\theta}) \approx 1/I_{obs}$.

A: $I_{obs} = 2$

B: $I_{obs} = 1$

C: $I_{obs} = 1/2$

D: $I_{obs} = 1/4$

E: None of the above

**Q3-03.**

```
##
## Call:
## arima(x = huron_level, order = c(2, 0, 1))
##
## Coefficients:
##          ar1     ar2     ma1  intercept
##       0.3388  0.4092  0.6320   176.4821
## s.e.  0.4646  0.4132  0.4262     0.1039
##
## sigma^2 estimated as 0.04479:  log likelihood = 21.42,  aic = -32.84
```

```
##
## Call:
## arima(x = huron_level, order = c(2, 0, 2))
##
## Coefficients:
##           ar1     ar2     ma1     ma2  intercept
##       -0.1223  0.7646  1.1310  0.1310   176.4815
## s.e.   0.0682  0.0550  0.1084  0.1004     0.1004
##
## sigma^2 estimated as 0.04364:  log likelihood = 22.64,  aic = -33.28
```

The R output above uses `stats::arima` to fit ARMA(2,1) and ARMA(2,2) models to the January level (in meters above sea level) of Lake Huron from 1860 to 2024. Residual diagnostics (not shown) show no major violation of model assumptions. We aim to choose one of these as a null hypothesis of no trend for later comparison with models including a trend.

Which is the best conclusion from the available evidence:

A: The ARMA(2,2) model has a lower AIC so it should be preferred.

B: We cannot reject the null hypothesis of ARMA(2,1) since the ARMA(2,2) model has a likelihood less than 1.92 log units higher than ARMA(2,1). Since there is not sufficient evidence to the contrary, it is better to select the simpler ARMA(2,1) model.

C: Since the comparison of AIC values and the likelihood ratio test come to different conclusions in this case, it is more-or-less equally reasonable to use either model.

D: When the results are borderline, numerical errors in the `stats::arima` optimization may become relevant. We should check using optimization searches from multiple starting points in parameter space, for example, using `arima2::arima`.

**Q3-04.**

Suppose model $M_0$ is nested within a larger model $M_1$ which has one additional parameter. Suppose that the AIC for $M_1$ is 0.5 units lower than the AIC for $M_0$. Which of the following is a correct expression for the p-value of a likelihood ratio test for $M_1$ against the null hypothesis $M_0$, supposing that a Wilks approximation is accurate? Here, $\chi^2_1$ is a chi-square random variable on 1 degree of freedom.

A: $P(\chi^2_1 > 0.5)$

B: $P(\chi^2_1 > 1)$

C: $P(\chi^2_1 > 1.5)$

D: $P(\chi^2_1 > 2)$

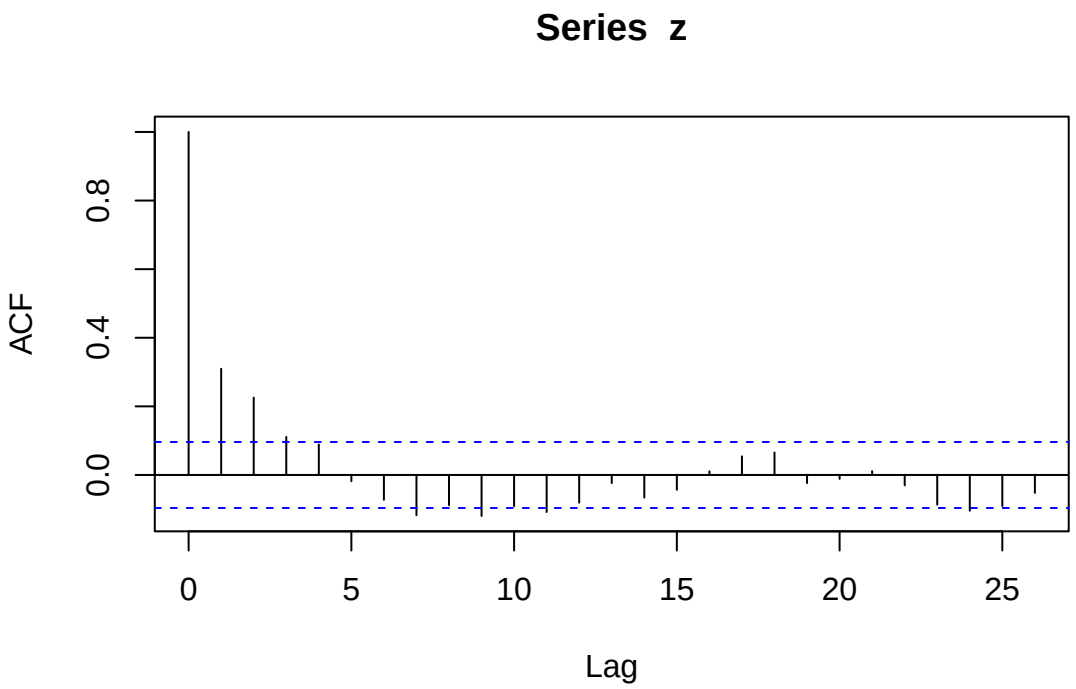
E: $P(\chi^2_1 > 2.5)$

F: $P(\chi^2_1 > 3)$

**Q4. Interpreting diagnostics**

**Q4-01.**

We consider data $y_{1:415}$ where $y_n$ is the time, in miliseconds, between the $n$th and $(n+1)$th firing event for a monkey neuron. Let $z_n = \log(y_n)$, with log being the natural logarithm. The sample autocorrelation function of $z_{1:415}$ is shown below.



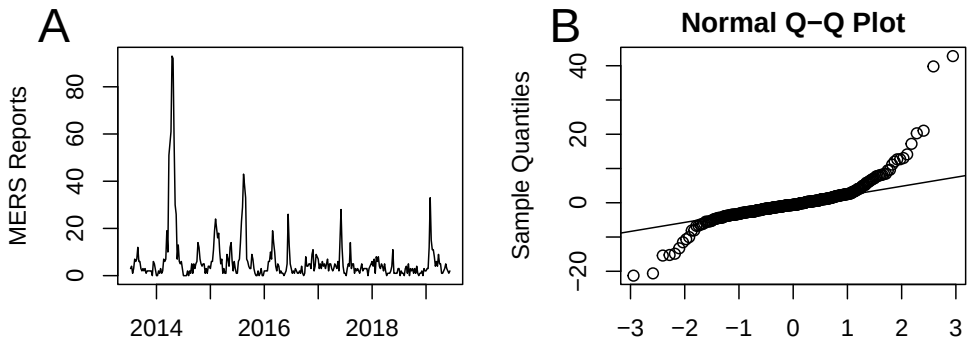

We are interested about whether it is appropriate to model the time series as a stationary causal ARMA process. Which of the following is the best interpretation of the evidence from these plots:

A. There is clear evidence of a violation of stationarity. We should consider fitting a time series model, such as ARMA, and see if the residuals become stationary.

B. This plot suggests there would be no benefit from detrending or differencing the time series before fitting a stationary ARMA model. It does not rule out a sample covariance that varies with time, which is incompatible with ARMA.

C. This plot is enough evidence to demonstrate that a stationary model is reasonable. We should proceed to check for normality, and if the data are also not far from normally distributed then it is reasonable to fit an ARMA model by Gaussian maximum likelihood.

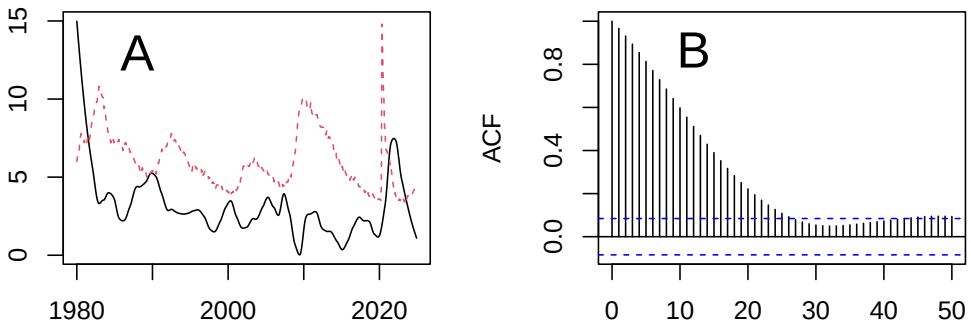
**Q4-02.**




(A) Weekly cases of Middle East Respiratory Syndome (MERS) in Saudi Arabia. (B) a normal quantile plot of the residuals from fitting an ARMA(2,2) model to these data using `arima()`. What is the best interpretation of (B)?

A: We should consider fitting a long-tailed error distribution, such as the t distribution.
B: The model is missing seasonality, which could be critical in this situation.
C: For using ARMA methods, these data should be log-transformed to make a linear Gaussian approximation more appropriate.
D: The normal quantile plot shows a long-tailed distribution, but this is not a major problem. We have over 300 data points, so the central limit theorem should hold for parameter estimates.
E: The normal quantile plot shows long tails, but with the right tail noticeably longer than the left tail. We should consider an asymmetric error distribution.
F: We should not interpret (B) before testing for stationarity. First run `adf.test()` and, if the null hypothesis is not rejected, recalculate (B) when fitting to the differenced data.

**Q4-03.**




(A) Inflation (black) and unemployment (red) for the USA, 1980-2024. (B) Sample autocorrelation function of the residuals from a least square regression, 'lm(inflation~unemployment)', with estimated coefficients below. Which is the best interpretation of these graphs and fitted model?

```
##  (Intercept) unemployment
##   2.87056052   0.04543759
```

A: 0.05 is a reasonable estimate for the additional unemployment caused by one percentage point of additional inflation. We should not trust the uncertainty estimate (not shown), since our model does not allow for autocorrelation of the residuals.

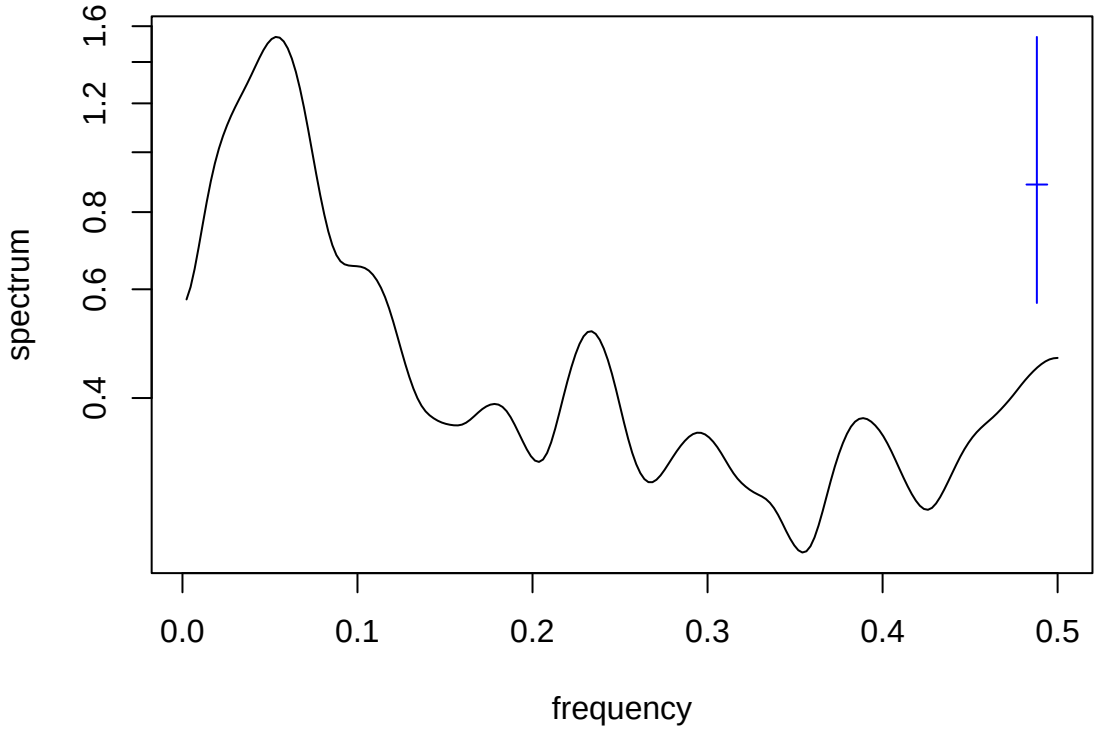
B: 0.05 is a reasonable estimate for the association between inflation and unemployment. We should not assume there is a causal relationship. We should not trust the uncertainty estimate (not shown), since our model does not allow for autocorrelation of the residuals.

C: 0.05 is a reasonable estimate for the association between inflation and unemployment. We should make an additional assumption that there are no confounding variables, and then we can interpret this association to be causal. We should not trust the uncertainty estimate (not shown), since our model does not allow for autocorrelation of the residuals.

**Q5. The frequency domain**

**Q5-01.**

We consider data $y_{1:415}$ where $y_n$ is the time interval, in milliseconds, between the $n$th and $(n+1)$th firing event for a monkey neuron. Let $z_n = \log(y_n)$, with log being the natural logarithm. A smoothed periodogram of $z_{1:415}$ is shown below. Units of frequency are the default value in R, i.e., cycles per unit observation. We see a peak at a frequency of approximately 0.07.



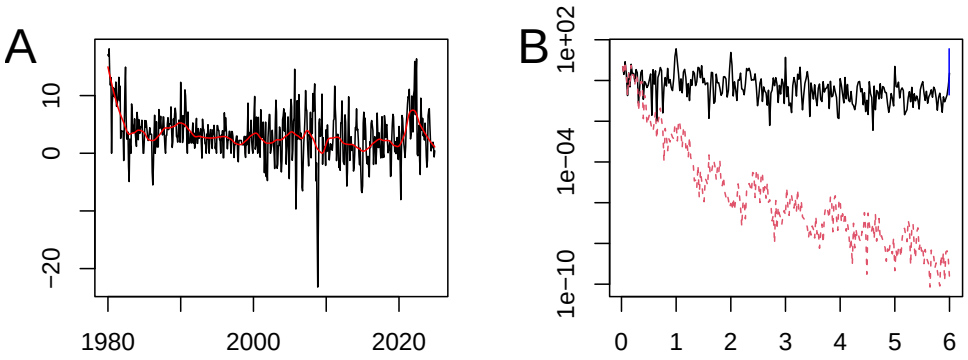

Which if the following is the best inference from this figure

A. Rapid neuron firing events (i.e., short intervals between firing events) come in groups with a typical size of $1/0.07 \approx 14$.

B. The neuron has a characteristic duration between firing events of $1/0.07 \approx 14$ milliseconds.

C. The neuron has a characteristic duration between firing events of $1/\exp(0.07) \approx 0.9$ milliseconds.

**Q5-02.**




The monthly US consumer price index (CPI) combines the price of a basket of products, such as eggs and bread and gasoline. (A) Annualized monthly percent inflation, i.e., the difference of log-CPI multiplied by

$12 \times 100$ (black line); a smooth estimate via local linear regression (red line). (B) The periodogram of inflation and its smooth estimate. Which best characterizes the behavior of the smoother?

A: Cycles longer than 2 months are removed
B: Cycles shorter than 2 months are removed
C: Cycles longer than 2 year are removed
D: Cycles shorter than 2 year are removed
E: Cycles longer than (1/2) year are removed
F: Cycles shorter than (1/2) year are removed

**Q6. Scholarship for time series projects**

**Q6-01.**

This question on citing references applies to any statistics report, but it is particularly relevant here since we are learning proper use of sources in order to write open-access midterm and final projects.

Suppose that the midterm project P1 cites a past project, P2, in the reference list. P1 refererences P2 at one point, mentioning that the projects have similarities. When you look at the source code and the writing, you find various points where P1 and P2 are almost identical, though at other points the projects are entirely different. What do you infer?

A: The authors of P1 have done enough to honestly disclose the relationship with P2. After all, there is sufficient information provided for any reader to track down the exact relationship.

B: The authors of P1 have misrepresented the relationship with P2 by appearing to take credit for some original work which was in fact heavily dependent on a source. This is a serious offence which should be reported to Rackham and/or the Associate Chair for Graduate Programs in Statistics as a violation of academic integrity.

C: There is not enough information to tell the actual story for certain. The authors of P2 may or may not have done something wrong, depending on information that is not available to us, but they did cite P2 so they should be given the benefit of the doubt and should not loose any scholarship points.

D: The authors of P1 have misrepresented the relationship with P2 by appearing to take credit for some original work which was in fact heavily dependent on a source. This is a moderately severe offence, partly offset by including P2 in the reference list. A substantial number of scholarship points should be subtracted.

E: P1 evidently has not shown perfect scholarship, but this is a small issue that could easily be an honest mistake given that the authors were not trying to hide the fact that they had studied P2. It is appropriate to substract, say, 1 point for scholarship for this mistake.

**Q6-02.** Four people in a team collaborate on a project. After the project is submitted, a reader identifies that part of the project is adapted from an unreferenced source, i.e., plagiarized. The team worked using git and cooperates on tracking down the issue, and the commit history clearly reveals who wrote the problematic part of the project. What is the most appropriate course of action:

A. The guilty coauthor should be penalized heavily for poor scholarship, and the the other coauthors should have a minor penalty for failing to check their colleague's work.

B. All coauthors should share the same penalty, since this is a team project and all coauthors share equal responsibility for the submitted report.

C. The guilty coauthor should be penalized heavily for poor scholarship. The other coauthors have demonstrated strong scholarship by following good transparent working practices that enabled this issue to get quickly resolved, so they should not receive any penalty.

D. It is necessary to collect more information before coming to a decision. For example, the team may argue that the source is well known to all readers so did not have to be cited.

**Q6-03.**

You discover that your team-mate is using Google Translate to carry out their share of the writing. The translation looks poorly done, similar in quality to ChatGPT, and does not use technical time series terminology correctly. What is the best course of action among the options below

A. Alert the instructor that you have a team mate adopting questionable scholarship strategies, in order to make sure you are not personally held responsible.

B. Ask ChatGPT to rewrite this problematic section to improve its quality

C. Help your team mate to rewrite the section in their own voice (shared with your voice).

**Q6-04.**

Why is it helpful for a course such as DATASCI/STATS 531, that permits the use of internet resources including GenAI and past solutions, to require students to say explicitly say when they do not use sources?

A. Failure to give credit to sources is against the academic integrity rules of Rackham, the graduate school at University of Michigan.

B. It helps the GSI to grade the homework when they know exactly what sources have been used and for what question.

C. Students whose solution is more dependent on sources than they want to admit are reluctant to explicitly deny using sources.

D. The GSI has the task of evaluating whether the student has demonstrated thought about the homework task beyond collecting material from sources into a solution. This is not an easy task even when the sources are clearly listed and referenced at the point (or points) where they are used.

---



---