Math 507, Fall 2024.

Homework 4

Due: Thu, Dec 05, 2024, <u>NO LATER than 11:30am</u>.

For each ticker in the file 'TechTickers.scv', download its adjusted closing price for each business day between Jan 1, 2021, and Dec 31, 2021. This is your sample. All questions listed below must be answered using this sample. Assume zero riskless return. All exercises must be solved using only the modules listed in the file 'Template.ipynb'.

1. In this exercise, you test the performance of a basic pairs trading strategy, using the back-testing method with the estimation window of size $N = 60$ days.

   The cointegration constant C^{ij} is determined according to the method by Soren Johansen, as described in Chapter 3. Note that Soren Johansen's method may yield two spreads, corresponding to the real eigenvalues of $(\Phi + 1)^\top$, in the notation of Chapter 3. In such a case, we pick a spread with the smallest p-value of the DF test (among the spreads with positive cointegration constants).

   A pair is considered cointegrated if the following three conditions are satisfied: (i) the cointegration constant is positive, (ii) the p-value of the Dickey-Fuller test of order 2, applied to the associated spread, does not exceed $p^* = 0.01$ and (iii) the sum of the two auto-regressive coefficients, estimated via the Yule-Walker method of order 2, does not exceed $\rho^* = 0.7$.

   For the stop-loss condition, use the maximum holding time of $\bar{T} = 20$ days and $4\tilde{\sigma}$ as the maximum deviation of the spread from its mean, where $\tilde{\sigma}$ is the standard deviation of the spread (computed when the pair is opened).

   A pair is opened if it is cointegrated and if the value of its spread is at least $\tilde{\sigma}$ away from the mean μ of the spread. A pair is closed whenever the stop-loss condition is triggered or whenever the spread of the pair crosses onto the opposite side of μ.

   When opening a pair, determine the size of your investment in each of the two associated assets so that the maximum of the dollar amounts invested on the long and on the short sides of the pair (these amounts are considered in absolute value, without taking the signs into account) is equal to $\$1,000,000$.

   (a) (40 pts) Back-test the basic pairs trading strategy for the cointegrated prices using your sample.

   Compute and plot the PnL of the strategy, along with the two PnLs coming from its long and short positions. Produce the plots of the total capital invested in the long and in the short positions of the strategy. Print the mean, standard deviation, and the Sharpe ratio, of the absolute returns of the PnL.

   Hedging with the SP500 index, produce a plot of the PnL of the market-neutral version of this strategy. Print the mean, standard deviation, and the Sharpe ratio, of the absolute returns of the PnL.

   Compute how many opened pairs were closed due to the stop-loss and due to reaching a desired target. Produce two histograms of the holding times: for the pairs that were closed due to the stop-loss and for those closed due to reaching the target.

   (b) (60 pts) Repeat part (a) assuming proportional transaction costs of size 0.001 for trading in stocks and zero cost of trading in the index.