Math 507, Fall 2024.

Homework 3

Due: Thursday, Nov 14, 2024, NO LATER than 11:30am.

The objective of the homework is to compute the optimal trading strategies in the Almgren-Criss transaction cost model. We denote S_t the unaffected prices of the asset and x_t the position of the trading agent. For simplicity, the unaffected price satisfies

$$dS_t = \sigma dW_t.$$

Given the agents position x_t, the transaction price is

$$\tilde{S}_t = S_t + \eta x_t'$$

where η is the temporary impact parameter. If the agent is purchasing a lot of shares very fast x_t' is large and the transaction price is above the unaffected prices.

The objective of the agent is to liquidate X shares which would require us to put the constraints $x_0 = X$ and $x_T = 0$ on the strategy x. However, we have not studied this type of problem in class. We will obtain this condition by penalizing the terminal position of the agent with the term $c(x_T)^2$ for some constant $c > 0$ that we will send to ∞

1. Question 1(100 pts)

   (a) The cost of the trading strategy is $\mathcal{C}(x) := \int_0^T \tilde{S}_t x_t' dt$. Using the martingality of S_t express $\mathbb{E}[\mathcal{C}(x)]$ as a function of S_0, X, η, x_T, S_T and $\int \mathbb{E}[(x_t')^2]dt$.

   (b) The agent aims to minimize $\mathbb{E}[\mathcal{C}(x)+c(x_T)^2]$. Write the dynamic programming principle and the HJB corresponding to this problem.

   (c) Solve the HJB by making an ansatz of form $\alpha_t s^2 + \beta_t s x + \gamma_t x^2 + \delta_t$ and find the optimal strategy.

   (d) Show that the optimal strategy admits a limit (which is constant) as $c \to \infty$ (This is tantamount optimizing among all strategy satisfying $x_0 = X$ and $x_T = 0$)