北京交通大学

能源系统优化与控制

课程简介

课程组：许寅、王颖、吴翔宇、刘曌
北京交通大学电气工程学院

课程目标

- 适应电力与能源技术高速发展：智能电网、综合能源电力系统、能源互联网、以新能源为主体的新型电力系统
- 掌握基本的优化和控制理论并用于解决以电力为核心的能源系统的决策和控制问题（填补数学理论与工程应用之间的gap）
- 培养文献调研、发现问题、解决问题、成果总结与发表的能力
- 初步培养将理论成果转化为实用化软件的能力

教学团队

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课程内容与学时：共32学时，2学分

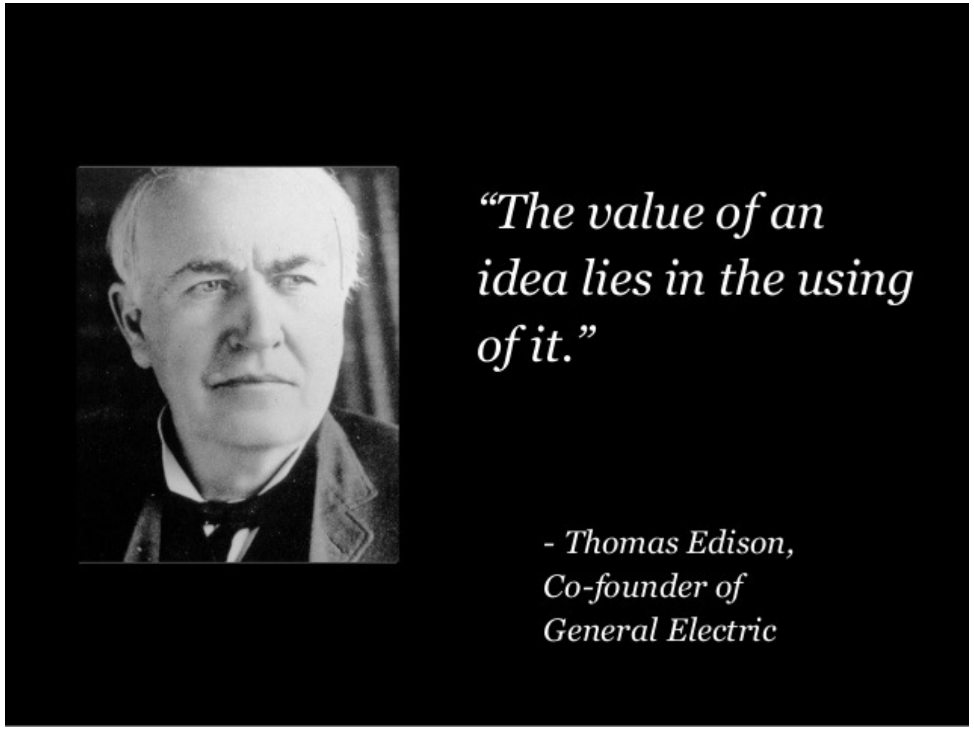
- 线性规划及其在能源系统优化中的应用（4学时）- 许寅
- 锥规划及其在能源系统优化中的应用（4学时）- 许寅
- 配电网最优潮流及相关优化问题（4学时） - 王颖
- 电力系统机组组合问题（4学时） - 王颖
- 韧性电网中的协同优化问题（2学时）- 王颖
- 人工智能及其在综合能源系统优化中的应用（4学时）- 刘曌
- 具有主动支撑能力的电力电子接口电源构网控制方法（4学时）- 吴翔宇
- 多能互补微电网（群）的协调控制方法（4学时）- 吴翔宇
- 应对极端事件的极限生存控制技术（2学时）- 吴翔宇

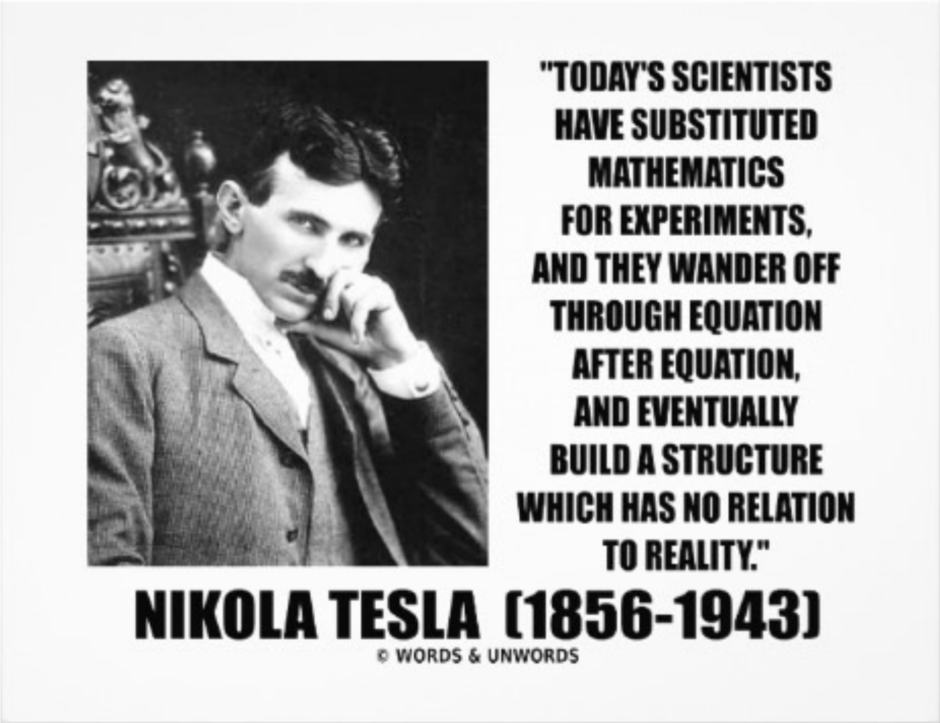
考核方式

- **平时作业：60%**
  - 每周提交1次，含理论作业和编程练习
- **大作业：40%**
  - 结合课程内容开展前沿探索，形成一份前沿研究报告，或
  - 形成一款具有一定实用性的软件

- **作业提交方式：pdf文档+程序源代码或仿真源文件打包发至助教邮箱**

与大家共勉~


"The value of an idea lies in the using of it."
- Thomas Edison,
Co-founder of
General Electric


"TODAY'S SCIENTISTS HAVE SUBSTITUTED MATHEMATICS FOR EXPERIMENTS, AND THEY WANDER OFF THROUGH EQUATION AFTER EQUATION, AND EVENTUALLY BUILD A STRUCTURE WHICH HAS NO RELATION TO REALITY."
NIKOLA TESLA (1856-1943)
© WORDS & UNWORDS

第1讲：线性规划及其在能源系统优化中的应用．Part I

许　寅

北京交通大学电气工程学院

往年教学录屏：https://www.bilibili.com/video/BV1G7411K7we/

线性规划(Linear programming)的基本概念

$$
\begin{aligned}
\text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{a}_i'\boldsymbol{x} \geq b_i, \quad i \in M_1 \\
& \boldsymbol{a}_i'\boldsymbol{x} \leq b_i, \quad i \in M_2 \\
& \boldsymbol{a}_i'\boldsymbol{x} = b_i, \quad i \in M_3 \\
& x_j \geq 0, \quad j \in N_1 \\
& x_j \leq 0, \quad j \in N_2
\end{aligned}
$$

$\boldsymbol{x} \in \mathcal{R}^n$ **decision variables**

optimization variables

$\boldsymbol{c}'\boldsymbol{x}$ **objective function**

cost function

---

constraints

If j is in neither N_1 nor N_2, x_j is a *free* or *unrestricted* variable

线性规划问题的解

feasible solution: a vector $\boldsymbol{x}$ satisfying all of the constraints

feasible set/region: the set of all feasible solutions

(optimal) solution $\boldsymbol{x}^*$: a feasible solution that minimizes the objective function, that is, $\boldsymbol{c}'\boldsymbol{x}^* \leq \boldsymbol{c}'\boldsymbol{x}$, for all feasible $\boldsymbol{x}$

optimal cost/value: the value of $\boldsymbol{c}'\boldsymbol{x}^*$

线性规划的标准形式

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}$$

Reduction to standard form

(1) Elimination of free variables

$$x_j \Rightarrow x_j^+ - x_j^-$$

$$x_j^+, x_j^- \geq 0$$

(2) Elimination of inequality constraints

$$\boldsymbol{a}_i'\boldsymbol{x} \leq b_i \Rightarrow \begin{aligned} \boldsymbol{a}_i'\boldsymbol{x} + s_i &= b_i \\ s_i &\geq 0 \end{aligned}$$

练习题

请将以下线性规划问题转化为标准形式：

$$
\begin{aligned}
\text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b}
\end{aligned}
$$

解答

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b} \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{s} = \boldsymbol{b} \\ & \boldsymbol{s} \ge \boldsymbol{0} \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{c}'(\boldsymbol{x}^+ - \boldsymbol{x}^-) \\ \text{subject to} \quad & \boldsymbol{A}(\boldsymbol{x}^+ - \boldsymbol{x}^-) + \boldsymbol{s} = \boldsymbol{b} \\ & \boldsymbol{s}, \boldsymbol{x}^+, \boldsymbol{x}^- \ge \boldsymbol{0} \end{aligned}$$

应用1：发电机容量规划

负荷预测：未来T年的负荷(MW)为$d_t, t = 1, \dots, T$

已有发电机：均为火电机组，且第t年的可用容量(MW)为e_t(发电机到期须退役)

新增装机的成本：第t年新增火电机组的成本(￥/MW)为c_t，第t年新增核电机组的成本(￥/MW)为n_t

发电机使用年限：火电机组20年，核电机组15年

安全政策：核电机组容量占比不得超过20%

问题：如何规划未来T年的新增机组容量，总成本最低？

应用1：发电机容量规划

决策变量：第t年新增火电和核电机组的容量分别为x_t和y_t

第1至t年新增且可用的火电和核电机组总容量分别为w_t和z_t

目标函数：发电机容量扩增的总成本

$$\sum_{t=1}^{T}(c_t x_t + n_t y_t)$$

应用1：发电机容量规划

约束条件：(1) 决策变量之间的关系

$$w_t = \sum_{s=\max\{1,t-19\}}^{t} x_s\,, \qquad t = 1, \dots, T$$

$$z_t = \sum_{s=\max\{1,t-14\}}^{t} y_s\,, \qquad t = 1, \dots, T$$

应用1：发电机容量规划

约束条件：

(2) 发电容量应不小于负荷需求

$$w_t + z_t + e_t \ge d_t, \qquad t = 1, \ldots, T$$

(3) 核电机组容量不超过发电总容量的20%

$$\frac{z_t}{w_t + z_t + e_t} \le 0.2 \quad \Rightarrow \quad 4z_t - w_t \le e_t$$

应用1：发电机容量规划

线性规划模型：minimize $\sum_{t=1}^{T}(c_t x_t + n_t y_t)$

subject to

$$w_t - \sum_{s=\max\{1,t-19\}}^{t} x_s = 0, \qquad t = 1, \dots, T$$

$$z_t - \sum_{s=\max\{1,t-14\}}^{t} y_s = 0, \qquad t = 1, \dots, T$$

$$w_t + z_t \geq d_t - e_t, \qquad t = 1, \dots, T$$

$$4z_t - w_t \leq e_t, \qquad t = 1, \dots, T$$

$$x_t, y_t, w_t, z_t \geq 0, \qquad t = 1, \dots, T$$

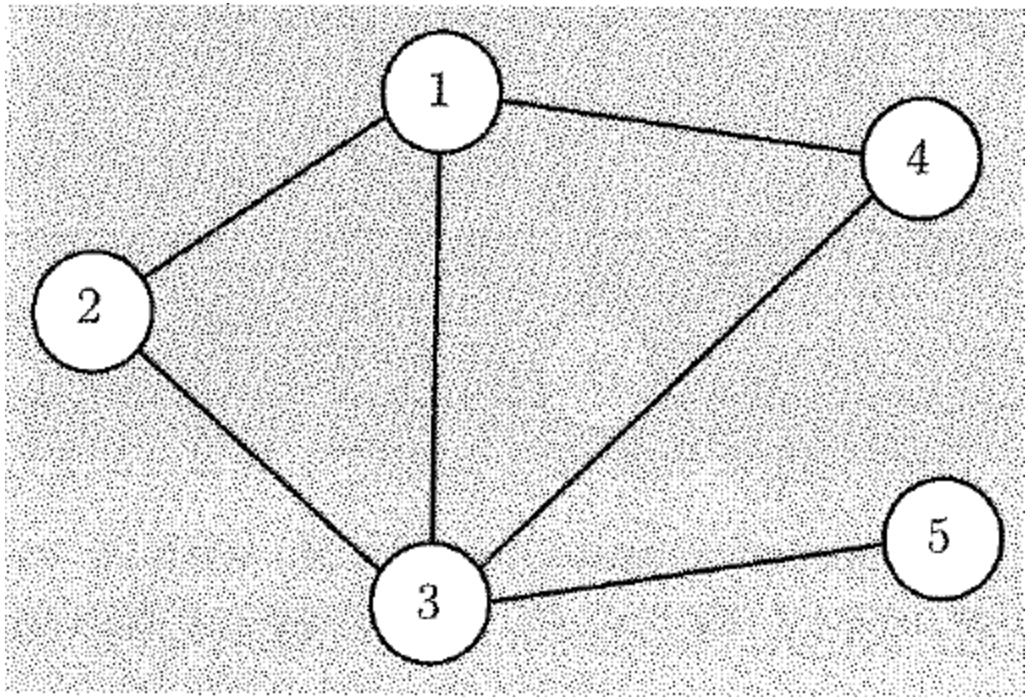
思考题

若要求电力系统始终保持至少10%的备用容量，哪些约束条件需要修改？如何修改？

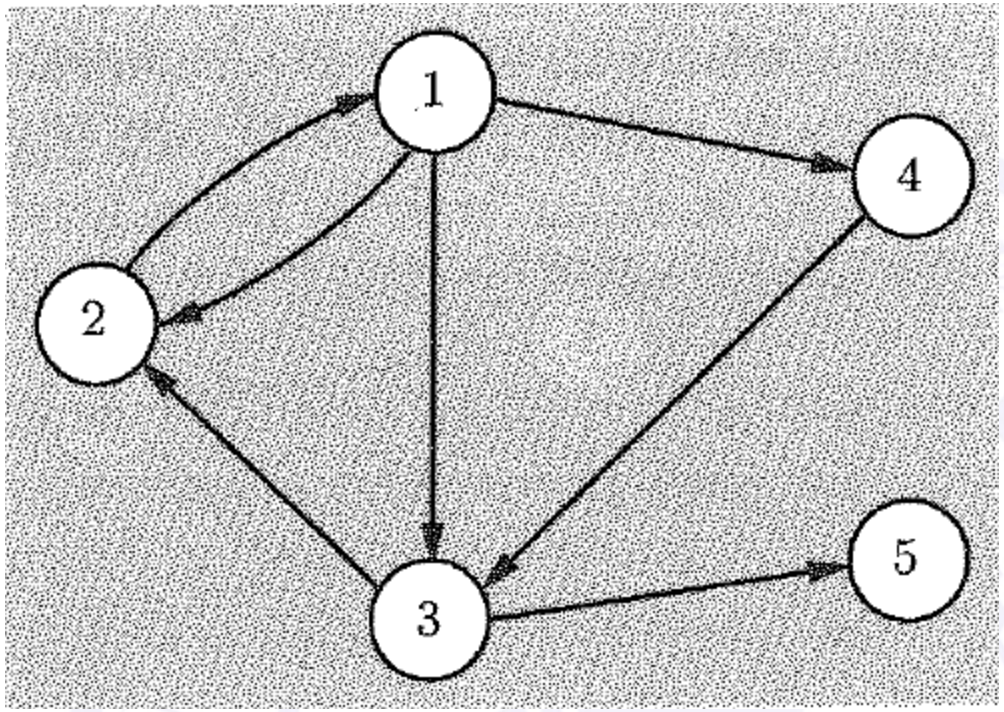
$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{t=1}^{T}(c_t x_t + n_t y_t) & \\
\text{subject to} & \displaystyle w_t - \sum_{s=\max\{1,t-19\}}^{t} x_s = 0, & t = 1, \dots, T \\
& \displaystyle z_t - \sum_{s=\max\{1,t-14\}}^{t} y_s = 0, & t = 1, \dots, T \\
& w_t + z_t \geq d_t - e_t, & t = 1, \dots, T \\
& 4z_t - w_t \leq e_t, & t = 1, \dots, T \\
& x_t, y_t, w_t, z_t \geq 0, & t = 1, \dots, T
\end{array}
$$

应用2：网络流问题(Network Flow Problems)

Undirected graphs



Directed graphs



应用2：网络流问题

Network: a directed graph $G = (\mathcal{N}, \mathcal{A})$ together with some additional numerical information

b_i: external **supply** to each node $i \in \mathcal{N}$.
Node i is called a source if $b_i > 0$, and a sink if $b_i < 0$

u_{ij}: **capacity** of each arc $(i, j) \in \mathcal{A}$

c_{ij}: cost per unit of flow along arc (i, j)

f_{ij}: the amount of **flow** through arc (i, j) ➡ 决策变量

应用2：网络流问题

General minimum cost network flow problem

$$\text{minimize} \quad \sum_{(i,j)\in\mathcal{A}} c_{ij} f_{ij}$$

$$\text{subject to} \quad b_i + \sum_{j\in\mathcal{I}(i)} f_{ji} = \sum_{j\in\mathcal{O}(i)} f_{ij} \quad \forall i \in \mathcal{N}$$

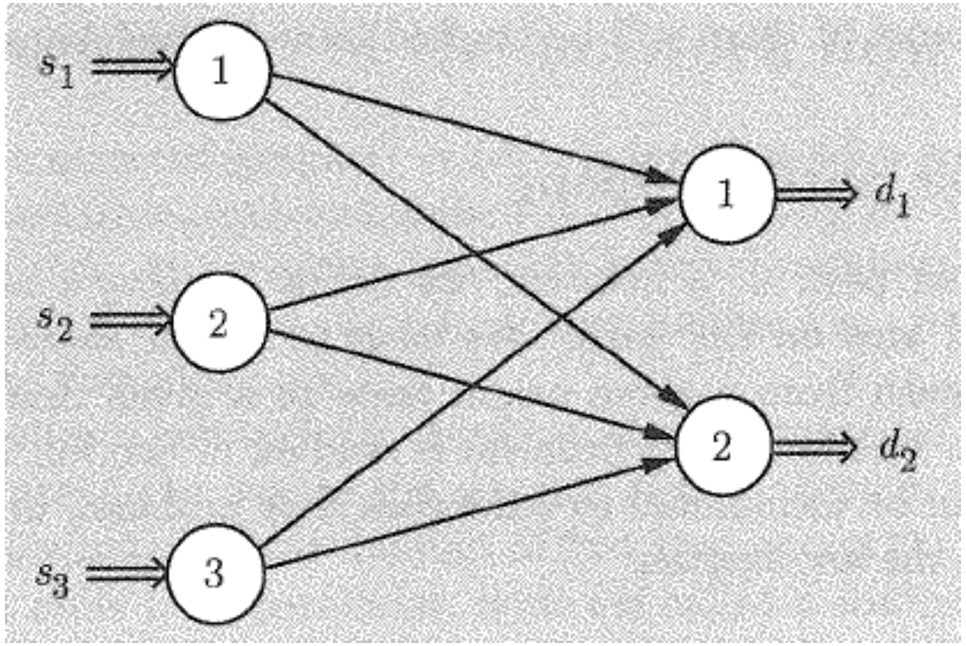
流守恒定律

$$0 \le f_{ij} \le u_{ij} \quad \forall (i,j) \in \mathcal{A}$$

边容量约束

应用2：网络流问题

The transportation problem



minimize $$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} f_{ij}$$

subject to $$\sum_{i=1}^{m} f_{ij} = d_j, \qquad j = 1, \ldots, n$$

$$\sum_{j=1}^{n} f_{ij} = s_i, \qquad i = 1, \ldots, m$$

$$f_{ij} \geq 0, \qquad \forall i, j$$

应用2：网络流问题

Multi-commodity flow problem

Network $G = (\mathcal{N}, \mathcal{A})$

$\mathcal{K}$ origin-destination pairs of nodes

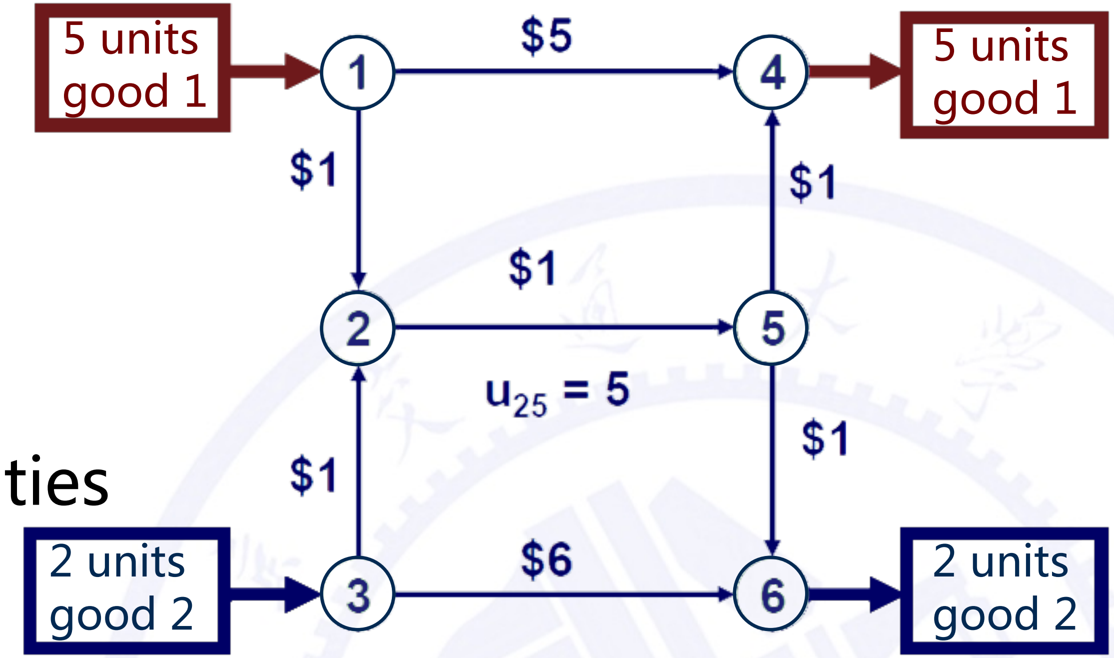
$$(s_1, t_1), (s_2, t_2), \ldots \ldots, (s_k, t_k)$$

d_k = amount of flow that must be sent from s_k to t_k

u_{ij} = capacity on (i, j) shared by all commodities

c_{ij}^k = cost of sending 1 unit of commodity k in (i, j)

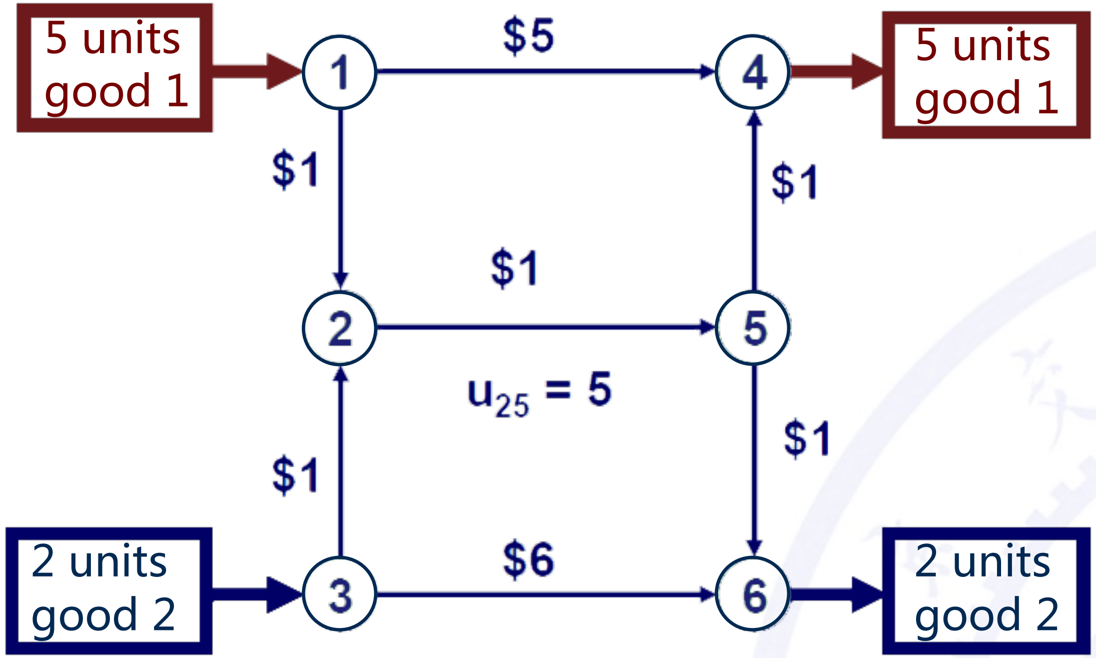
f_{ij}^k = flow of commodity k in (i, j)



Determine the optimal multi-commodity flow

练习题

以下多商品流问题的最优值（即最小运输成本）是多少？



应用2：网络流问题

Multi-commodity flow problem

$$\text{minimize} \quad \sum_{(i,j)\in\mathcal{A}} \sum_{k} c_{ij}^{k} f_{ij}^{k}$$

$$\text{subject to} \quad \sum_{j\in\mathcal{O}(i)} f_{ij}^{k} - \sum_{j\in\mathcal{I}(i)} f_{ji}^{k} = \begin{cases} d_k, & \text{if } i = s_k \\ -d_k, & \text{if } i = t_k \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N} \quad \forall k \in \mathcal{K}$$

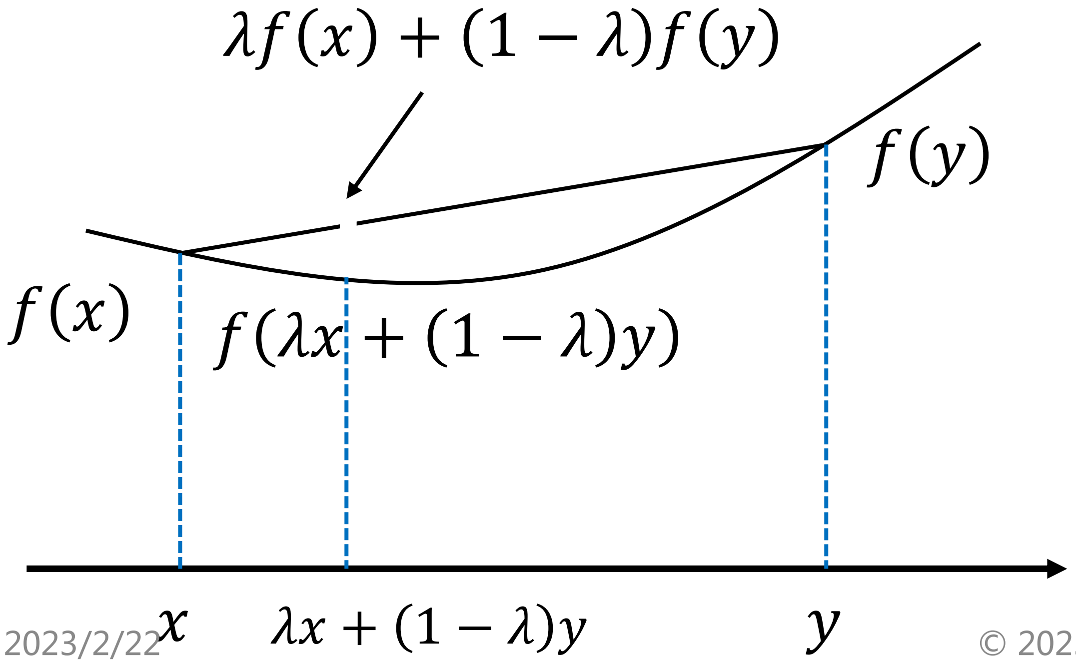
$$\sum_{k} f_{ij}^{k} \le u_{ij} \qquad \forall (i,j) \in \mathcal{A}$$

$$f_{ij}^{k} \ge 0 \qquad \forall (i,j) \in \mathcal{A} \quad \forall k \in \mathcal{K}$$

凸函数

A function $f: \mathcal{R}^n \longmapsto \mathcal{R}$ is called **convex** if for every $x, y \in \mathcal{R}^n$, and every $\lambda \in [0,1]$, we have

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y)$$



关于凸函数的判别方法，请参阅Stephen Boyd《凸优化》第3章

练习题

以下哪些说法是正确的？

A 线性函数$\boldsymbol{c}_1'\boldsymbol{x} + d_1$是凸函数

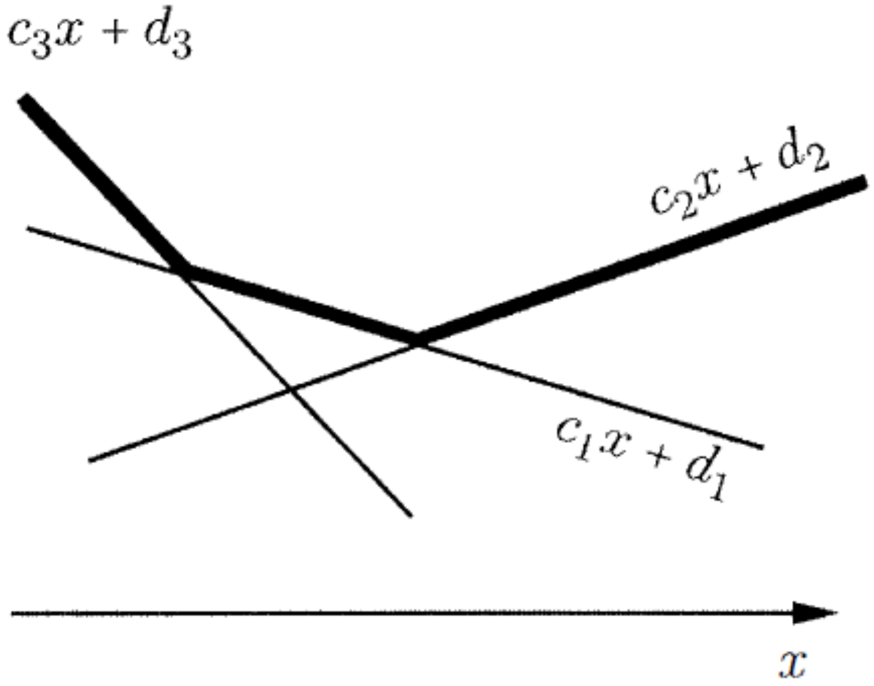
B 指数函数e^{at}是凸函数

C $|x|^3$是凸函数

D $\max_{i=1,\ldots,m} (\boldsymbol{c}_i'\boldsymbol{x} + d_i)$是凸函数

分段线性凸函数

A function of the form $\max_{i=1,\ldots,m} (\boldsymbol{c}_i'\boldsymbol{x} + d_i)$ is called a **piecewise linear convex function**



Piecewise-linear minimization

minimize $\quad \max_{i=1,\ldots,m} (\boldsymbol{c}_i'\boldsymbol{x} + d_i)$

subject to $\quad \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b}$

minimize $\quad z$

subject to $\quad z \geq \boldsymbol{c}_i'\boldsymbol{x} + d_i, \quad i = 1, \ldots, m$

$\boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b}$

含绝对值的优化问题

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} c_i |x_i| \\ \text{subject to} \quad & \boldsymbol{Ax} \geq \boldsymbol{b} \end{aligned}$$

(参数 $c_i > 0$)

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} c_i z_i \\ \text{subject to} \quad & \boldsymbol{Ax} \geq \boldsymbol{b} \\ & x_i \leq z_i, \qquad i = 1, \dots, n \\ & -x_i \leq z_i, \qquad i = 1, \dots, n \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} c_i (x_i^+ + x_i^-) \\ \text{subject to} \quad & \boldsymbol{Ax}^+ - \boldsymbol{Ax}^- \geq \boldsymbol{b} \\ & \boldsymbol{x}^+, \boldsymbol{x}^- \geq 0 \end{aligned}$$

可证明最优解必满足 $x_i^+ = 0$ 或 $x_i^- = 0$ (反证法)

练习题

以下优化问题是否能转换为线性规划问题？如果可以，如何转换？

$$
\begin{aligned}
\text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & |\boldsymbol{a}_1'\boldsymbol{x} + b_1| \le d \\
& \max_{i=2,\dots,m} (\boldsymbol{a}_i'\boldsymbol{x} + b_i) \le \boldsymbol{p}'\boldsymbol{x} + q
\end{aligned}
$$

解答

$$\text{minimize} \quad \boldsymbol{c}'\boldsymbol{x}$$

$$\text{subject to} \quad |\boldsymbol{a}_1'\boldsymbol{x} + b_1| \leq d \qquad \max_{i=2,\dots,m} (\boldsymbol{a}_i'\boldsymbol{x} + b_i) \leq \boldsymbol{p}'\boldsymbol{x} + q$$

$$\text{minimize} \quad \boldsymbol{c}'\boldsymbol{x}$$

$$\text{subject to} \quad \boldsymbol{a}_1'\boldsymbol{x} + b_1 \leq d \qquad \boldsymbol{a}_1'\boldsymbol{x} + b_1 \geq -d$$

$$\boldsymbol{a}_i'\boldsymbol{x} + b_i \leq \boldsymbol{p}'\boldsymbol{x} + q \qquad i = 2, \dots, m$$

线性整数规划(LIP)与混合整数线性规划(MILP)

- LIP: Linear Integer Programming
- MILP: Mixed Integer Linear Programming

线性整数规划

$$
\begin{aligned}
\text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\
& \boldsymbol{x} \geq \boldsymbol{0}
\end{aligned}
$$

$\boldsymbol{x}$ 的元素为整数

混合整数线性规划

$$
\begin{aligned}
\text{minimize} \quad & \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\
& \boldsymbol{x} \geq \boldsymbol{0}
\end{aligned}
$$

$\boldsymbol{x}$ 部分元素为整数

二次规划 (Quadratic programming)

二次规划的标准形式

$$
\begin{aligned}
\text{minimize} \quad & \frac{1}{2}\boldsymbol{x}'\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c}'\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\
& \boldsymbol{x} \geq \boldsymbol{0}
\end{aligned}
$$

其中，$\boldsymbol{Q}$为半正定矩阵

示例：简化的经济调度问题

$$\text{minimize} \quad \sum_{i=1}^{N}\left(a_i P_i^2 + b_i P_i + c_i\right)$$

发电成本

$$\text{subject to} \quad \sum_{i=1}^{N} P_i = P_D$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, i = 1, \ldots, N$$

应用数学优化解决工程问题的一般思路

建立实际工程问题的数学规划模型

采用建模语言描述数学规划模型并调用求解器求解优化问题

分析求解结果，给出工程问题的解决方案

Julia编程语言

- The **Julia Programming Language**: https://julialang.org/
- Julia中文社区：https://cn.julialang.org
- 基础教学视频：https://www.bilibili.com/video/BV1bK4y1d7bd/
- 清华大学开源软件镜像站：
  - Julia安装包下载：https://mirrors.tuna.tsinghua.edu.cn/help/julia-releases/
  - Julia镜像使用帮助：https://mirrors.tuna.tsinghua.edu.cn/help/julia/

建模语言：JuMP 优化工具包

- An algebraic **modeling language** for linear, quadratic, and nonlinear constrained optimization problems
- **JuMP**: https://github.com/JuliaOpt/JuMP.jl
- Documentation: http://www.juliaopt.org/JuMP.jl/stable/

JuliaOpt

Optimization packages for the Julia language.

JUMP

线性规划求解器

- **商业软件：IBM® CPLEX® Optimizer**
  - CPLEX Optimizer 为线性规划、混合整数规划、二次规划和二次约束规划问题提供灵活的高性能数学规划求解器。
  - https://www.ibm.com/cn-zh/analytics/cplex-optimizer
- **开源软件：GLPK (GNU Linear Programming Kit)**
  - The GLPK package is intended for solving large-scale linear programming (LP), mixed integer programming (MIP), and other related problems.
  - https://www.gnu.org/software/glpk/

示例：求解线性规划问题

$$
\begin{aligned}
\text{maximize} \quad & 5x + 3y \\
\text{subject to} \quad & x + 5y \le 3 \\
& 0 \le x \le 2 \\
& 0 \le y \le 30
\end{aligned}
$$

教学视频：https://www.bilibili.com/video/BV1F7411A7N5/

示例：求解线性规划问题

```julia
using JuMP, GLPK
testmodel = Model(GLPK.Optimizer)
@variable(testmodel,0<=x<=2)
@variable(testmodel,0<=y<=30)
@objective(testmodel,Max,5x+3y)
@constraint(testmodel,x+5y<=3)
print(testmodel)
optimize!(testmodel)
objective_value(testmodel)
value.(x)
value.(y)
```

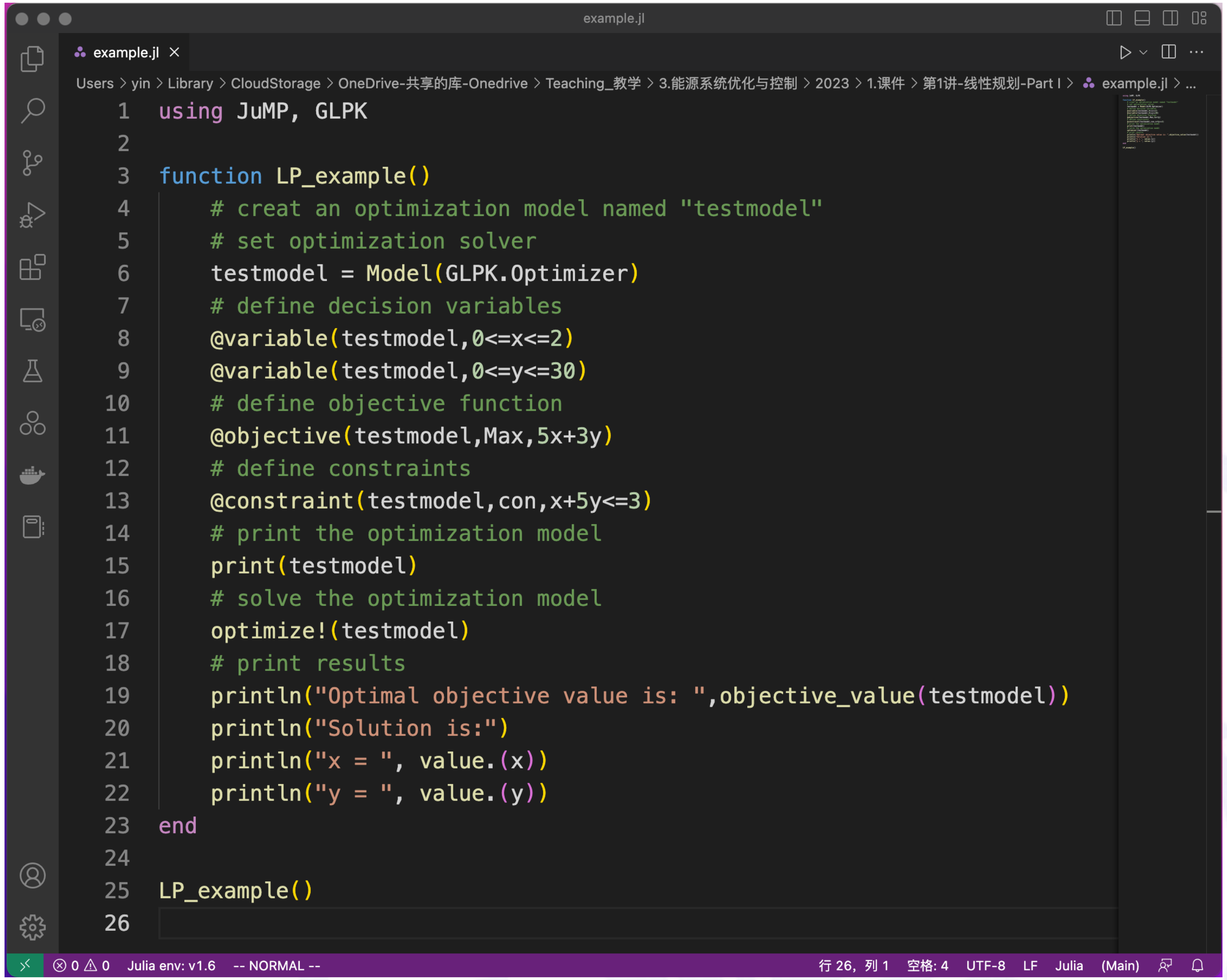
示例：求解线性规划问题

```
julia> using JuMP, GLPK

julia> testmodel = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK

julia> @variable(testmodel,0<=x<=2)
x

julia> @variable(testmodel,0<=y<=30)
y

julia> @objective(testmodel,Max,5x+3y)
5 x + 3 y

julia> @constraint(testmodel,x+5y<=3)
x + 5 y ≤ 3.0
```

```
julia> print(testmodel)
Max 5 x + 3 y
Subject to
 x + 5 y ≤ 3.0
 x ≥ 0.0
 y ≥ 0.0
 x ≤ 2.0
 y ≤ 30.0

julia> optimize!(testmodel)

julia> objective_value(testmodel)
10.6

julia> value.(x)
2.0

julia> value.(y)
0.2
```

Julia语言脚本与VS Code集成开发环境

- Julia语言脚本文件：*.jl
- Visual Studio Code: https://code.visualstudio.com
- Julia for Visual Studio Code: https://www.julia-vscode.org



作业

1. 将以下线性规划问题转化为标准形式

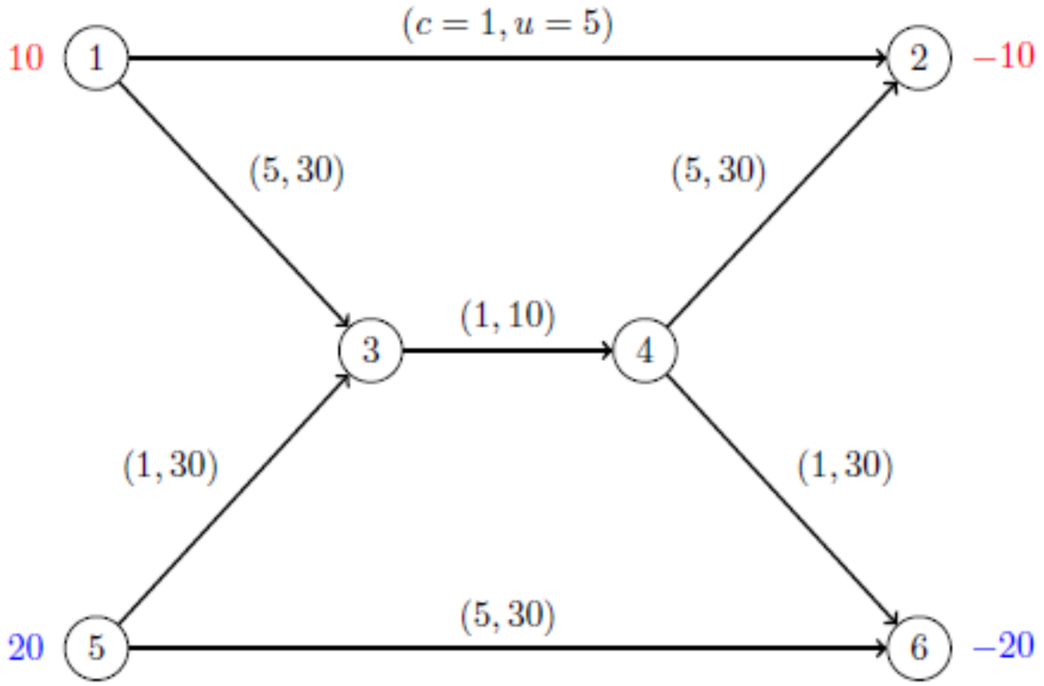
$$
\begin{array}{ll}
\text{minimize} & 2x_1 + 4x_2 \\
\text{subject to} & x_1 + x_2 \geq 3 \\
& 3x_1 + 2x_2 = 14 \\
& x_1 \geq 0
\end{array}
$$

2. 将以下优化问题转化为线性规划问题

$$
\begin{array}{ll}
\text{minimize} & 2x_1 + 3|x_2 - 10| \\
\text{subject to} & |x_1 + 2| + |x_3| \leq 5
\end{array}
$$

作业

3. 编程求解以下多商品流问题



作业【选做】

4. **Choosing paths in a communication network**

Consider a communication network consisting of n nodes. Nodes are connected by communication links. A link allowing one-way transmission from node i to node j is described by an ordered pair (i, j). Let $\mathcal{A}$ be the set of all links. We assume that each link $(i, j) \in \mathcal{A}$ can carry up to u_{ij} bits per second. There is a positive charge c_{ij} per bit transmitted along that link. Each node k generates data, at the rate of $b^{k\ell}$ bits per second, that have to be transmitted to node ℓ, either through a direct link (k, ℓ) or by tracing a sequence of links. The problem is to choose paths along which all data reach their intended destinations, while minimizing the total cost. We allow the data with the same origin and destination to be split and be transmitted along different paths. 写出上述问题的线性规划模型