

# Section 1

## Conventions and notations

In this appendix, we summarize the conventions and notations used throughout the thesis. Firstly, we use the natural units with

$$c = \hbar = k_B = 1, \quad (1.1)$$

where  $c$ ,  $\hbar$ , and  $k_B$  are the speed of light, the reduced Planck constant, and the Boltzmann constant, respectively.

Our convention of the four-dimensional Lorentzian metric is  $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . We sometimes use the Pauli matrices defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (1.2)$$

with  $i$  being the imaginary unit. The slash on any character denotes the so-called Feynmann slash, defined as  $\not{p} \equiv p^\mu \gamma_\mu$  with four-by-four gamma matrices given by

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma_i \\ -\sigma_i & \mathbf{0} \end{pmatrix}, \quad (1.3)$$

The unique exception of this rule is  $\not{E}_T$ , which is used to denote the missing transverse momentum in hadron collider experiments.

We use the notation  $g_1$ ,  $g_2$ , and  $g_3$  for the gauge coupling constant of the SM  $U(1)_Y$ ,  $SU(2)_L$ , and  $SU(3)_c$  gauge group. We use the so-called grand unified theory normalization of  $g_1$ ; the corresponding charge assignment is different from the conventional assignment of  $U(1)_Y$  (with, *e.g.*, charge  $-1/2$  for the left-handed leptons) by a factor of  $\sqrt{3/5}$ .