

Section 1

Models with WIMPs

There are several examples of the models that contain WIMP DM candidates. In this section, two of them (**♣ Really? ♣**) are briefly reviewed. (**♣ EWIMP and WIMP?? ♣**)

1.1 Minimally supersymmetric standard model

The minimally supersymmetric standard model (MSSM) is the simple extension of the SM with $\mathcal{N} = 1$ supersymmetry (SUSY).^{b1} One of the motivations to introduce SUSY is to solve the so-called hierarchy (or naturalness) problem [1–3] in the SM. The problem is related to the quantum correction to the SM Higgs boson mass from heavy new physics particles. For example, we can consider the one-loop correction to the Higgs mass from a Weyl fermion f and a complex scalar S as illustrated in Fig. 1. The corrections to the Higgs mass is given by

$$\Delta m_h^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_f^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_f} \right) + \dots \right] \quad (\text{fermion}), \quad (1.1)$$

$$\Delta m_h^2 = \frac{\lambda_S}{16\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_S^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_S} \right) + \dots \right] \quad (\text{scalar}), \quad (1.2)$$

(**♣ Check this! ♣**) where λ_f and m_f are the Higgs-fermion coupling constant and the fermion mass, respectively, and λ_S and m_S are those for the scalar S . We take the cut-off scale of the theory to be Λ_{UV} to regularize the otherwise divergent loop integral and neglect

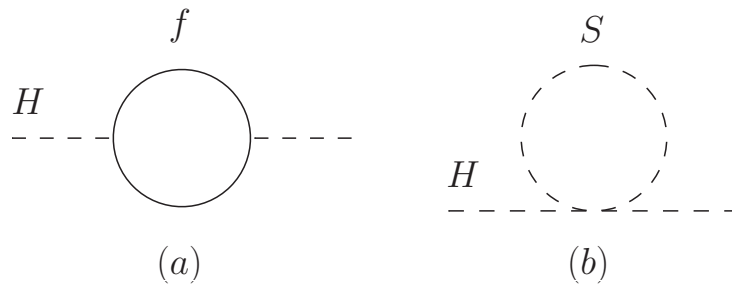


Figure 1: One-loop correction to the Higgs mass from (a) a Weyl fermion f and (b) a complex scalar S .

^{b1} For a brief review of the $\mathcal{N} = 1$ SUSY, see Sec. ??.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{Q}_i	3	2	1/6
\hat{L}_i	1	2	-1/2
\hat{U}_i	$\bar{3}$	1	-2/3
\hat{D}_i	$\bar{3}$	1	1/3
\hat{E}_i	1	1	1
\hat{H}_u	1	2	1/2
\hat{H}_d	1	2	-1/2

Table 1: Notations and quantum numbers of the chiral superfields in the MSSM.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{g}	8	1	0
\hat{W}	1	3	0
\hat{B}	1	1	0

Table 2: Notations and quantum numbers of the vector superfields in the MSSM.

the lower order terms of Λ_{UV} . Eqs. (1.1) and (1.2) show the quadratic dependence of Δm_H^2 on Λ_{UV} , which means that the Higgs mass is sensitive to the energy scale of the beyond the SM physics. However, there is at least one extremely high energy scale physics in the nature, gravity at the Planck scale $M_{\text{pl}} \sim 10^{18-19} \text{ GeV}$. By substituting $\Lambda_{UV} = M_{\text{pl}}$ in Eqs. (1.1) and (1.2) and assuming $\lambda_f \sim \lambda_S \sim \mathcal{O}(1)$, we notice that orders-of-magnitude fine-tuning is required to obtain the correct Higgs mass $m_h = 125.10 \text{ GeV}$ [4], which is unnatural.

SUSY provides a nice solution to this fine-tuning problem. As is summarized in Appendix ??, (**♣ summarize later ♣**) each Weyl fermion in a supersymmetric model has two complex scalars with the same mass $m_f = m_S$. In addition, their coupling constants should have a relationship $|\lambda_f|^2 = \lambda_S$ due to the fact that λ_S is a coupling constant in the F-term potential sourced by a superpotential term proportional to λ_f . (**♣ description of F-term and D-term ♣**) By using both equations and summing the corrections (1.1) and (1.2) with factor of two multiplied to the latter, we obtain a result independent of the cut-off scale Λ_{UV} without fine-tuning. This cancellation is ensured by the so-called non-renormalization theorem. [5, 6]

We now summarize the notations and quantum numbers of the chiral and vector superfields in the MSSM in Table 1 and 2, respectively. The supersymmetric part of the MSSM lagrangian is described by the superpotential

$$W = Y_u^{ij} U_i Q_j H_u - Y_d^{ij} D_i Q_j H_d - Y_e^{ij} E_i L_j H_d + \mu H_u H_d, \quad (1.3)$$

where $i, j = 1, 2, 3$ labels the quark and lepton generation, while Q, L, U, D, E are superfields that contain the left-handed quark, left-handed lepton, right-handed up-type quark, right-handed down-type quark, and right-handed charged lepton, respectively. In Eq. (1.3), proper contraction of $SU(3)_C$ and $SU(2)_L$ indices is assumed. Note that two Higgs doublets H_u and H_d with opposite values of $U(1)_Y$ hypercharges are introduced, which is needed to cancel the contributions to the gauge anomaly from fermionic partners of the Higgs doublets.

Since no superpartner of any SM particle is observed yet, SUSY should be broken and superpartners should obtain the SUSY breaking masses. (**♣ ref: boson and fermion obtain equal mass ♣**) The SUSY breaking part of the lagrangian is expressed as

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right) \\ & - \left(A_u^{ij} \tilde{U}_i \tilde{Q}_j H_u - A_d^{ij} \tilde{D}_i \tilde{Q}_j H_d - A_e^{ij} \tilde{E}_i \tilde{L}_j H_d \right) \\ & - m_Q^{2ij} \tilde{Q}_i^\dagger \tilde{Q}_j - m_L^{2ij} \tilde{L}_i^\dagger \tilde{L}_j - m_U^{2ij} \tilde{U}_i^\dagger \tilde{U}_j - m_D^{2ij} \tilde{D}_i^\dagger \tilde{D}_j - m_E^{2ij} \tilde{E}_i^\dagger \tilde{E}_j \\ & - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}), \end{aligned} \quad (1.4)$$

where the tilde is used to express the superpartner of the SM particle contained in a superfield, while a field without a hat nor tilde denotes the other component. An exception is two Higgs doublets, where H_u and H_d express the scalar components, while \tilde{H}_u and \tilde{H}_d express their superpartners called Higgsinos. The SM-like Higgs doublet H corresponds to a linear combination of H_u and H_d .

It is known that, within the MSSM, almost all SUSY breaking mechanisms, such as the F-term (O’Raifeartaigh) [7] or D-term (Fayet-Iliopoulos) SUSY breaking [8, 9], fail to generate masses of superpartners with remaining the SM gauge group in the low energy effective theory. Thus, we need a so-called hidden sector in addition to the MSSM sector, in which SUSY is spontaneously broken. In order for the MSSM sector to have Lagrangian terms (1.4), we also need some mediation mechanism of the SUSY breaking. The relative size of the SUSY breaking parameters in Eq. (1.4) highly depends on the mediation mechanism. Among many mediation mechanisms of SUSY breaking, the anomaly mediated SUSY breaking [10, 11] leads to an interesting phenomenology with relatively light WIMPs, so it will be reviewed later.

Higgs mass in the MSSM

Under the spontaneously broken SUSY, the cancellation of the quantum correction to the Higgs boson discussed above is not exact. One obvious consequence of the SUSY breaking in Eqs. (1.1) and (1.2) is the hierarchy between m_f and m_S that appear in the second term of each contribution. In the case of the MSSM, the largest contribution comes from the superpartner of the top quark, stop, that have the largest Yukawa coupling with the Higgs boson.

Value	Description	Reference
$M_W = 80.384 \pm 0.014 \text{ GeV}$	Pole mass of the W boson	[12, 13]
$M_Z = 91.1876 \pm 0.0021 \text{ GeV}$	Pole mass of the Z boson	[14]
$M_h = 125.15 \pm 0.24 \text{ GeV}$	Pole mass of the Higgs	[15, 16]
$M_t = 173.34 \pm 0.82 \text{ GeV}$	Pole mass of the top quark	[17]
$(\sqrt{2}G_\mu)^{-1/2} = 246.21971 \pm 0.00006 \text{ GeV}$	Fermi constant for μ decay	[18]
$\alpha_3(M_Z) = 0.1184 \pm 0.0007$	$\overline{\text{MS}}$ $SU(3)_C$ gauge coupling	[19]

Table 3: Experimentally measured SM parameters used for the derivation of Eq. (1.6).

When there is a large hierarchy between the SUSY breaking scale M_S , which is comparable with stop masses, and the top mass M_t , the stop contributions to the Higgs mass contains a large logarithm of the form of $\log(M_S^2/M_t^2)$. **(♣ Consistency with above equations. Maybe MSbar better ♣)** To resum this large logarithm and obtain a precise result, we have to rely on the renormalization group equation (RGE). In this framework, the value of the Higgs self coupling λ at the electroweak scale is closely related to the Higgs mass. We assume the SM parameters summarized in Table 3 and the definition of the SM Higgs potential

$$V(H) = -\frac{m^2}{2}|H|^2 + \lambda|H|^4, \quad (1.5)$$

with H being the SM Higgs doublet. Then, according to [20], we obtain the relationship ^{‡2}

$$\lambda(M_t) = 0.12604 + 0.00206 \left(\frac{M_h}{\text{GeV}} - 125.15 \right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.34 \right), \quad (1.6)$$

where the $\overline{\text{MS}}$ scheme is used to renormalize the divergence of loop integrals.

In the MSSM, the value of λ at the SUSY breaking scale M_S is given by

$$\lambda(M_S) = \frac{g_1^2(M_S) + g_2^2(M_S)}{8} \cos^2 2\beta + \delta\lambda, \quad (1.7)$$

where g_1 and g_2 are $U(1)_Y$ and $SU(2)_L$ gauge coupling constants, respectively, while β parametrizes the ratio of the vacuum expectation values

$$\frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle} = \tan \beta, \quad (1.8)$$

^{‡2} Although the values listed in Table 3 are different from the latest ones given in [4], we use older ones because the change in input values may cause the slight change in coefficients of second and third terms of Eq. (1.6). The latest central values of the Higgs and top masses are $M_h = 125.10 \text{ GeV}$ and $M_t = 173.1 \text{ GeV}$, with which we can estimate $\lambda(M_t) = 0.12595$.

with H_u^0 and H_d^0 being electromagnetically neutral components of the corresponding Higgs doublets. In Eq. (1.7), the first term shows the tree-level contribution from the D-term potential and $\delta\lambda$ denotes the threshold correction from heavy superpartners. Once the spectrum of the MSSM particles is fixed, we can evaluate the Higgs self coupling using Eq. (1.7), calculate its running according to the RGE, and obtain the prediction for the Higgs mass through Eq. (1.6).

(♣ Contour plot of m_h in $\tan\beta$ vs. M_S plane here. ♣) (♣ Simply assume $m_{Q3} = m_{U3}$ and compare minimal and maximal mixings ♣) (♣ Discussion and confirm that heavy SUSY is motivated ♣)

Light Wino in the anomaly mediated SUSY breaking model

The anomaly mediated SUSY breaking [10,11] or the pure gravity mediation scenario [21–23], combined with the observed value of the Higgs mass, naturally predicts the existence of WIMPs (basically Winos denoted as \tilde{W}) in the TeV range. In this scenario, the SUSY breaking effect is directly mediated to the quark and lepton supermultiplets, and they obtain masses comparable to the scale of the SUSY breaking, which is approximated by the gravitino mass $m_{3/2}$. On the other hand, the superpartners of gauge bosons, gauginos, is affected only through a one-loop diagram, which is related to the scale anomaly. (♣ Correct? ♣) As a result, gaugino mass parameters in Eq. (1.4) are one-loop suppressed compared with other mass parameters as

$$M_i(M_S) = - \left. \frac{\beta_i}{2g_i^2} \right|_{M_S} m_{3/2}, \quad (1.9)$$

where $i = 1, 2, 3$ is a gauge index and β_i denote the beta functions of gauge coupling constants. (♣ All-order expression or not? M_S ? M_{GUT} ? ♣) At the one-loop level, this gives

$$M_1(M_S) = \frac{11g_1^2(M_S)}{16\pi^2} m_{3/2}, \quad (1.10)$$

$$M_2(M_S) = \frac{g_2^2(M_S)}{16\pi^2} m_{3/2}, \quad (1.11)$$

$$M_3(M_S) = -\frac{3g_3^2(M_S)}{16\pi^2} m_{3/2}. \quad (1.12)$$

(♣ Normalization of g_1 ? ♣)

1.2 Need review

(♣ Relationship between λ parameter above should be clearer ♣) WIMPs with mass around or just above the electroweak scale are theoretically well-motivated in connection with problems of the SM such as the naturalness problem. For example, the minimal

WIMP DM candidate	Quantum numbers			Masses	
	$SU(2)_L$	$U(1)_Y$	Spin	m_χ/TeV	$\Delta m_\chi/\text{MeV}$
Higgsino	2	1/2	Dirac fermion	1.1	341
Wino	3	0	Majorana fermion	2.9	166
5-plet scalar	5	0	real scalar	9.4	166
5-plet fermion	5	0	Majorana fermion	10	166

Table 4: Table of properties of popular WIMP DM candidates [24–29]. The $SU(2)_L$ electroweak charge, $U(1)_Y$ hypercharge, spin nature, mass, and mass difference compared with a charged component of the multiplet are shown. See Sec. ?? (♣ **Caution!!** ♣) for the details of the last column.

supersymmetric extension of the SM (the so-called MSSM) contains several WIMP DM candidate such as Higgsino and Wino.^{‡3} Another example is the minimal dark matter (MDM) model [27, 31, 32], which is a simple extension of the SM with an $SU(2)_L$ electroweak multiplet such as a 5-plet scalar / fermion. In these models, the stability of the DM is ensured by the R -parity (for the MSSM case) and by high dimensionality of the operator that describes the decay of the DM (for the MDM case). The properties of these WIMP DM candidates are summarized in Table 4. The required masses to explain the DM relic abundance through the freezeout mechanism are also shown. Since the non-relativistic annihilation cross section of TeV mass particles is significantly enhanced by the Sommerfeld enhancement effect [26, 33], there are deviations from the rough estimation formula Eq. (??). We will return to this point later in Sec. ??. (♣ **Caution!!** ♣) In addition, in the last column there are mass differences Δm_χ between the DM and its charged counterpart that will be explained in detail in Sec. ??. (♣ **Caution!!** ♣)

1.3 Minimal dark matter model

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^{‡3} For a review of the MSSM, see for example [30].

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