Section 1

Collider simulation of the MDM model

In this appendix, we explain the procedure we have adopted to implement the MDM model into the numerical calculation performed in Sec. ??. We first make the `FeynRules v2.3` [?] model file for the MDM model by modifying the SM model file $sm.fr$.[♮1] By running `FeynRules`, we can convert the model file to the Universal `FeynRules` Format (UFO) [?], which can be used as a model file for `MadGraph5`.[♮2]

In the Listing 1, we show a part of our `FeynRules` model file `mdm.fr`. This corresponds to the lines that contain additions and modifications to `sm.fr` to take account of the 5-plet fermion as an example of the MDM. In the following, ... denotes a description that is the same as `sm.fr` and thus omitted in the listing.

Listing 1: `mdm.fr`

```
(* ****************************************** *)
(* ***** SU(2)L representation matrix ****** *)
(********************************************)

replaceMDM = {repMDM[a_,b_,c_] :>
  {{{0,1,0,0,0},{1,0,Sqrt[3/2],0,0},{0,Sqrt[3/2],0,Sqrt[3/2],0},{0,0,Sqrt[3/2],0,1},{0,0,0,1,0}},
  {{0,-I,0,0,0},{I,0,-I Sqrt[3/2],0,0},{0,I Sqrt[3/2],0,-I Sqrt[3/2],0},{0,0,I Sqrt[3/2],0,-I},{0,0,0,I,0}},
  {{2,0,0,0,0},{0,1,0,0,0},{0,0,0,0,0},{0,0,0,-1,0},{0,0,0,0,-2}}}[[a[[2]],b[[2]],c[[2]]]]
};

(* ************************* *)
(* ****** Gauge groups ****** *)
(* ************************* *)

M$GaugeGroups = {
  U1Y == {...},
  SU2L == {
    Abelian -> False,
    CouplingConstant -> gw,
    GaugeBoson -> Wi,
    StructureConstant -> Eps,
    Representations -> {{Ta,SU2D},{TM,SU2M}},
```

♮1 Model files for the SM and several relatively simple extensions of the SM are found in the model database equipped in the official wiki [?].

♮2 The use of collider physics public codes such as `FeynRules` and `MadGraph5` can be systematically learned by referring the well-summarized lecture notes provided by Sho Iwamoto [?].

```
    Definitions -> {Ta[a_,b_,c_]->PauliSigma[a,b,c]/2, FSU2L[i_,j_,k_]:> I Eps[i,j,k], TM[a_,b_,c_]->repMDM[a,b,c]}
  },
  SU3C == {...}
};

(* ************************** *)
(* ******** Indices ********* *)
(* ************************** *)

IndexRange[Index[SU2W ]] = Unfold[Range[3]];
IndexRange[Index[SU2D ]] = Unfold[Range[2]];
IndexRange[Index[SU2M ]] = Unfold[Range[5]];
IndexRange[Index[Gluon ]] = NoUnfold[Range[8]];
IndexRange[Index[Colour ]] = NoUnfold[Range[3]];
IndexRange[Index[Generation]] = Range[3];

IndexStyle[SU2W, j];
IndexStyle[SU2D, k];
IndexStyle[SU2M, l];
IndexStyle[Gluon, a];
IndexStyle[Colour, m];
IndexStyle[Generation, f];

(* ************************** *)
(* **** Particle classes **** *)
(* ************************** *)

M$ClassesDescription = {
  ...

(* Physical MDM Dirac components *)
  F[5] == {
    ClassName -> chi0,
    SelfConjugate -> True,
    WeylComponents -> chi0w,
    Mass -> {mMDM0, 1000},
    Width -> 0,
    MajoranaPhase -> 0,
    PropagatorLabel -> "chi0",
    PropagatorType -> Straight,
    PropagatorArrow -> None,
    ParticleName -> "chi0",
    FullName -> "chi0"
  },
  F[6] == {
    ClassName -> chi1,
    SelfConjugate -> False,
    WeylComponents -> {chi1pw, chi1mwbar},
    Mass -> {mMDM1, 1000},
```

```
    Width -> 0,
    MajoranaPhase -> 0,
    QuantumNumbers -> {Q -> 1},
    PropagatorLabel -> "chi1",
    PropagatorType -> Straight,
    PropagatorArrow -> Forward,
    ParticleName -> "chi+",
    AntiParticleName -> "chi-",
    FullName -> "chi1"
  },
  F[7] == {
    ClassName -> chi2,
    SelfConjugate -> False,
    WeylComponents -> {chi2pw, chi2mwbar},
    Mass -> {mMDM2, 1000},
    Width -> 0,
    MajoranaPhase -> 0,
    QuantumNumbers -> {Q -> 2},
    PropagatorLabel -> "chi2",
    PropagatorType -> Straight,
    PropagatorArrow -> Forward,
    ParticleName -> "chi++",
    AntiParticleName -> "chi--",
    FullName -> "chi2"
  },

(* Fermions: unphysical fields *)
  F[11] == {
    ClassName -> LL,
    Unphysical -> True,
    Indices -> {Index[SU2D], Index[Generation]},
    FlavorIndex -> SU2D,
    SelfConjugate -> False,
    QuantumNumbers -> {Y -> -1/2},
    Definitions -> { LL[sp1_,1,ff_] :> Module[{sp2}, ProjM[sp1,sp2] vl[
       sp2,ff]], LL[sp1_,2,ff_] :> Module[{sp2}, ProjM[sp1,sp2] l[sp2,ff
       ]] }
  },
  F[12] == {
    ClassName -> lR,
    Unphysical -> True,
    Indices -> {Index[Generation]},
    FlavorIndex -> Generation,
    SelfConjugate -> False,
    QuantumNumbers -> {Y -> -1},
    Definitions -> { lR[sp1_,ff_] :> Module[{sp2}, ProjP[sp1,sp2] l[sp2,
       ff]] }
  },
  F[13] == {
    ClassName -> QL,
```

```
    Unphysical -> True,
    Indices -> {Index[SU2D], Index[Generation], Index[Colour]},
    FlavorIndex -> SU2D,
    SelfConjugate -> False,
    QuantumNumbers -> {Y -> 1/6},
    Definitions -> {
      QL[sp1_,1,ff_,cc_] :> Module[{sp2}, ProjM[sp1,sp2] uq[sp2,ff,cc]],
      QL[sp1_,2,ff_,cc_] :> Module[{sp2,ff2}, CKM[ff,ff2] ProjM[sp1,sp2]
          dq[sp2,ff2,cc]] }
  },
  F[14] == {
    ClassName -> uR,
    Unphysical -> True,
    Indices -> {Index[Generation], Index[Colour]},
    FlavorIndex -> Generation,
    SelfConjugate -> False,
    QuantumNumbers -> {Y -> 2/3},
    Definitions -> { uR[sp1_,ff_,cc_] :> Module[{sp2}, ProjP[sp1,sp2] uq[
        sp2,ff,cc]] }
  },
  F[15] == {
    ClassName -> dR,
    Unphysical -> True,
    Indices -> {Index[Generation], Index[Colour]},
    FlavorIndex -> Generation,
    SelfConjugate -> False,
    QuantumNumbers -> {Y -> -1/3},
    Definitions -> { dR[sp1_,ff_,cc_] :> Module[{sp2}, ProjP[sp1,sp2] dq[
        sp2,ff,cc]] }
  },

(* Unphysical MDM multiplet *)
  W[1] == {
    ClassName -> MDM,
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False,
    Indices -> {Index[SU2M]},
    FlavorIndex -> SU2M,
    Definitions -> {
      MDM[sp1_,1] -> chi2pw[sp1],
      MDM[sp1_,2] -> chi1pw[sp1],
      MDM[sp1_,3] -> chi0w[sp1],
      MDM[sp1_,4] -> chi1mw[sp1],
      MDM[sp1_,5] -> chi2mw[sp1] }
  },

(* Unphysical MDM Weyl components *)
  W[2] == {
    ClassName -> chi0w,
```

```
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False
  },
  W[3] == {
    ClassName -> chi1pw,
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False,
    QuantumNumbers -> {Q -> 1}
  },
  W[4] == {
    ClassName -> chi1mw,
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False,
    QuantumNumbers -> {Q -> -1}
  },
  W[5] == {
    ClassName -> chi2pw,
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False,
    QuantumNumbers -> {Q -> 2}
  },
  W[6] == {
    ClassName -> chi2mw,
    Unphysical -> True,
    Chirality -> Left,
    SelfConjugate -> False,
    QuantumNumbers -> {Q -> -2}
  }
};

(* ************************* *)
(* ******* Parameters ******* *)
(* ************************* *)

M$Parameters = {
  ...

  mmm == {
    ParameterType -> External,
    BlockName -> MDMBLOCK,
    OrderBlock -> 1,
    Value -> 1000,
    TeX -> Subscript[m, MDM],
    Description -> "MDM␣mass"
  }
};
```

```

(* ************************* *)
(* ******* Lagrangian ******* *)
(* ************************* *)

  ...

LMDM := Block[{mu},
  ExpandIndices[I MDMbar.sibar[mu].DC[MDM, mu] - mmm/2 chi0bar.chi0 - mmm
      chi1bar.chi1 - mmm chi2bar.chi2,
  FlavorExpand->{SU2W,SU2M}]/.replaceMDM//WeylToDirac];

Lagrangian:= LGauge + LFermions + LMDM + LHiggs + LYukawa + LGhost;
```