

Section 1

Models with WIMPs

There are several examples of the models that contain WIMP DM candidates. In this section, two of them (**♣ Really? ♣**) are briefly reviewed. (**♣ EWIMP and WIMP?? ♣**)

1.1 Minimally supersymmetric standard model

The minimally supersymmetric standard model (MSSM) is the simple extension of the SM with $\mathcal{N} = 1$ supersymmetry (SUSY).^{‡1} One of the motivations to introduce SUSY is to solve the so-called hierarchy (or naturalness) problem [1–3] in the SM. The problem is related to the quantum correction to the SM Higgs boson mass from heavy new physics particles. For example, we can consider the one-loop correction to the Higgs mass from a Weyl fermion f and a complex scalar S as illustrated in Fig. 1. The corrections to the Higgs mass is given by

$$\Delta m_h^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_f^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_f} \right) + \dots \right] \quad (\text{fermion}), \quad (1.1)$$

$$\Delta m_h^2 = \frac{\lambda_S}{16\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_S^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_S} \right) + \dots \right] \quad (\text{scalar}), \quad (1.2)$$

(**♣ Check this! ♣**) where λ_f and m_f are the Higgs-fermion coupling constant and the fermion mass, respectively, and λ_S and m_S are those for the scalar S . We take the cut-off scale of the theory to be Λ_{UV} to regularize the otherwise divergent loop integral and neglect

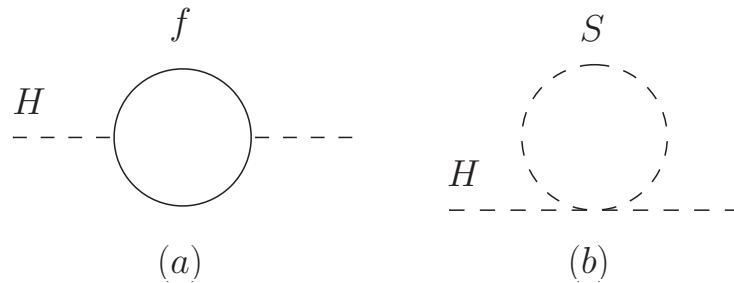


Figure 1: One-loop correction to the Higgs mass from (a) a Weyl fermion f and (b) a complex scalar S .

^{‡1}For a brief review of the $\mathcal{N} = 1$ SUSY, see Sec. ??.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{Q}_i	3	2	1/6
\hat{L}_i	1	2	-1/2
\hat{U}_i	$\bar{\mathbf{3}}$	1	-2/3
\hat{D}_i	$\bar{\mathbf{3}}$	1	1/3
\hat{E}_i	1	1	1
\hat{H}_u	1	2	1/2
\hat{H}_d	1	2	-1/2

Table 1: Notations and quantum numbers of the chiral superfields in the MSSM.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{g}	8	1	0
\hat{W}	1	3	0
\hat{B}	1	1	0

Table 2: Notations and quantum numbers of the vector superfields in the MSSM.

the lower order terms of Λ_{UV} . Eqs. (1.1) and (1.2) show the quadratic dependence of Δm_H^2 on Λ_{UV} , which means that the Higgs mass is sensitive to the energy scale of the beyond the SM physics. However, there is at least one extremely high energy scale physics in the nature, gravity at the Planck scale $M_{\text{pl}} \sim 10^{18-19}$ GeV. By substituting $\Lambda_{UV} = M_{\text{pl}}$ in Eqs. (1.1) and (1.2) and assuming $\lambda_f \sim \lambda_S \sim \mathcal{O}(1)$, we notice that orders-of-magnitude fine-tuning is required to obtain the correct Higgs mass $m_h = 125.09$ GeV [4], which is unnatural.

SUSY provides a nice solution to this fine-tuning problem. As is summarized in Appendix ??, (**♣ summarize later ♣**) each Weyl fermion in a supersymmetric model has two complex scalars with the same mass $m_f = m_S$. In addition, their coupling constants should have a relationship $|\lambda_f|^2 = \lambda_S$ due to the fact that λ_S is a coupling constant in the F-term potential sourced by a superpotential term proportional to λ_f . (**♣ description of F-term and D-term ♣**) By using both equations and summing the corrections (1.1) and (1.2) with factor of two multiplied to the latter, we obtain a result independent of the cut-off scale Λ_{UV} without fine-tuning. This cancellation is ensured by the so-called non-renormalization theorem. [5, 6]

We now summarize the notations and quantum numbers of the chiral and vector superfields in the MSSM in Table 1 and 2, respectively. The supersymmetric part of the MSSM lagrangian is described by the superpotential

$$W = Y_u^{ij} U_i Q_j H_u - Y_d^{ij} D_i Q_j H_d - Y_e^{ij} E_i L_j H_d + \mu H_u H_d, \quad (1.3)$$

Value	Description	Reference
$M_W = 80.384 \pm 0.014 \text{ GeV}$	Pole mass of the W boson	[7, 8]
$M_Z = 91.1876 \pm 0.0021 \text{ GeV}$	Pole mass of the Z boson	[9]
$M_h = 125.15 \pm 0.24 \text{ GeV}$	Pole mass of the Higgs	[10, 11]
$M_t = 173.34 \pm 0.82 \text{ GeV}$	Pole mass of the top quark	[12]
$(\sqrt{2}G_\mu)^{-1/2} = 246.21971 \pm 0.00006 \text{ GeV}$	Fermi constant for μ decay	[13]
$\alpha_3(M_Z) = 0.1184 \pm 0.0007$	$\overline{\text{MS}}$ $SU(3)_C$ gauge coupling	[14]

Table 3: Experimentally measured SM parameters used for the derivation of Eq. (1.6).

where $i, j = 1, 2, 3$ labels the quark and lepton generation, while Q, L, U, D, E are superfields that contain the left-handed quark, left-handed lepton, right-handed up-type quark, right-handed down-type quark, and right-handed charged lepton, respectively. In Eq. (1.3), proper contraction of $SU(3)_C$ and $SU(2)_L$ indices is assumed. Note that two Higgs doublets H_u and H_d with opposite values of $U(1)_Y$ hypercharges are introduced, which is needed to cancel the contributions to the gauge anomaly from Higgs superpartners, Higgsinos.

Since no superpartner of any SM particle is observed yet, SUSY should be broken and superpartners should obtain the SUSY breaking masses. (**♣ ref: boson and fermion obtain equal mass ♣**) The SUSY breaking part of the lagrangian is expressed as

$$\begin{aligned}
\mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right) \\
& - \left(A_u^{ij} \tilde{U}_i \tilde{Q}_j H_u - A_d^{ij} \tilde{D}_i \tilde{Q}_j H_d - A_e^{ij} \tilde{E}_i \tilde{L}_j H_d \right) \\
& - m_Q^{2ij} \tilde{Q}_i^\dagger \tilde{Q}_j - m_L^{2ij} \tilde{L}_i^\dagger \tilde{L}_j - m_U^{2ij} \tilde{U}_i^\dagger \tilde{U}_j - m_D^{2ij} \tilde{D}_i^\dagger \tilde{D}_j - m_E^{2ij} \tilde{E}_i^\dagger \tilde{E}_j \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}), \tag{1.4}
\end{aligned}$$

where the tilde is used to express the superpartner of the SM particle contained in a superfield, while a field without a hat nor tilde denotes the other component.

Higgs mass in the MSSM

Under the spontaneously broken SUSY, the cancellation of the quantum correction to the Higgs boson discussed above is not exact. One obvious consequence of the SUSY breaking in Eqs. (1.1) and (1.2) is the hierarchy between m_f and m_S that appear in the second term of each contribution. In the case of the MSSM, the largest contribution comes from the superpartner of the top quark, stop, that have the largest Yukawa coupling with the Higgs boson.

When there is a large hierarchy between the SUSY breaking scale M_S , which is comparable with stop masses, and the top mass M_t , the stop contributions to the Higgs mass contains a large logarithm of the form of $\log(M_S^2/M_t^2)$. **(♣ Consistency with above equations. Maybe MSbar better ♣)** To resum this large logarithm and obtain a precise result, we have to rely on the renormalization group equation (RGE). In this framework, the value of the Higgs self coupling λ at the electroweak scale is closely related to the Higgs mass. We assume the SM parameters summarized in Table 3 and the definition of the SM Higgs potential

$$V(H) = -\frac{m^2}{2}|H|^2 + \lambda|H|^4, \quad (1.5)$$

with H being the SM Higgs doublet. Then, according to [15], we obtain the relationship ^{‡2}

$$\lambda(M_t) = 0.12604 + 0.00206 \left(\frac{M_h}{\text{GeV}} - 125.15 \right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.34 \right). \quad (1.6)$$

In the MSSM, the value of λ at the SUSY breaking scale M_S is given by

$$\lambda(M_S) = \frac{g_1^2(M_S) + g_2^2(M_S)}{8} \cos^2 2\beta + \delta\lambda, \quad (1.7)$$

where g_1 and g_2 are $U(1)_Y$ and $SU(2)_L$ gauge coupling constants, respectively, while β parametrizes the ratio of the vacuum expectation values

$$\frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle} = \tan \beta, \quad (1.8)$$

with H_u^0 and H_d^0 being electromagnetically neutral components of the corresponding Higgs doublets. In Eq. (1.7), the first term shows the tree-level contribution from the D-term potential and $\delta\lambda$ denotes the threshold correction from heavy superpartners. Once the spectrum of the MSSM particles is fixed, we can evaluate the Higgs self coupling using Eq. (1.7), calculate its running according to the RGE, and obtain the prediction for the Higgs mass through Eq. (1.6).

(♣ Contour plot of m_h in $\tan\beta$ vs. M_S plane here. ♣) (♣ Simply assume $m_{Q3} = m_{U3}$ and compare minimal and maximal mixings ♣) (♣ Discussion and confirm that heavy SUSY is motivated ♣)

1.2 Need review

(♣ Relationship between λ parameter above should be clearer ♣) WIMPs with

^{‡2}Although the values listed in Table 3 are different from the latest ones given in [4], we use older ones because the change in input values may cause the slight change in coefficients of second and third terms of Eq. (1.6). The latest central values of the Higgs and top masses are $M_h = 125.09 \text{ GeV}$ and $M_t = 173.1 \text{ GeV}$, with which we can estimate $\lambda(M_t) = 0.12593$.

WIMP DM candidate	Quntum numbers			Masses	
	$SU(2)_L$	$U(1)_Y$	Spin	m_χ/TeV	$\Delta m_\chi/\text{MeV}$
Higgsino	2	1/2	Dirac fermion	1.1	341
Wino	3	0	Majorana fermion	2.9	166
5-plet scalar	5	0	real scalar	9.4	166
5-plet fermion	5	0	Majorana fermion	10	166

Table 4: Table of properties of popular WIMP DM candidates [16–21]. The $SU(2)_L$ electroweak charge, $U(1)_Y$ hypercharge, spin nature, mass, and mass difference compared with a charged component of the multiplet are shown. See Sec. ?? (♣ **Caution!!** ♣) for the details of the last column.

mass around or just above the electroweak scale are theoretically well-motivated in connection with problems of the SM such as the naturalness problem. For example, the minimal supersymmetric extension of the SM (the so-called MSSM) contains several WIMP DM candidate such as Higgsino and Wino.^{‡3} Another example is the minimal dark matter (MDM) model [19, 23, 24], which is a simple extension of the SM with an $SU(2)_L$ electroweak multiplet such as a 5-plet scalar / fermion. In these models, the stability of the DM is ensured by the R -parity (for the MSSM case) and by high dimensionality of the operator that describes the decay of the DM (for the MDM case). The properties of these WIMP DM candidates are summarized in Table 4. The required masses to explain the DM relic abundance through the freezeout mechanism are also shown. Since the non-relativistic annihilation cross section of TeV mass particles is significantly enhanced by the Sommerfeld enhancement effect [18, 25], there are deviations from the rough estimation formula Eq. (??). We will return to this point later in Sec. ??. (♣ **Caution!!** ♣) In addition, in the last column there are mass differences Δm_χ between the DM and its charged counterpart that will be explained in detail in Sec. ??. (♣ **Caution!!** ♣)

1.3 Minimal dark matter model

References

- [1] S. Weinberg, Implications of Dynamical Symmetry Breaking, Phys. Rev. D13 (1976) 974–996, [Addendum: Phys. Rev.D19,1277(1979)]. doi:10.1103/PhysRevD.19.1277, 10.1103/PhysRevD.13.974.

^{‡3}For a review of the MSSM, see for example [22].

-
- [2] E. Gildener, Gauge Symmetry Hierarchies, Phys. Rev. D14 (1976) 1667. [doi:10.1103/PhysRevD.14.1667](#).
- [3] L. Susskind, Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory, Phys. Rev. D20 (1979) 2619–2625. [doi:10.1103/PhysRevD.20.2619](#).
- [4] M. Tanabashi, et al., Review of Particle Physics, Phys. Rev. D98 (3) (2018) 030001. [doi:10.1103/PhysRevD.98.030001](#).
- [5] A. Salam, J. A. Strathdee, On Superfields and Fermi-Bose Symmetry, Phys. Rev. D11 (1975) 1521–1535. [doi:10.1103/PhysRevD.11.1521](#).
- [6] M. T. Grisaru, W. Siegel, M. Rocek, Improved Methods for Supergraphs, Nucl. Phys. B159 (1979) 429. [doi:10.1016/0550-3213\(79\)90344-4](#).
- [7] T. E. W. Group, 2012 Update of the Combination of CDF and D0 Results for the Mass of the W Boson (2012). [arXiv:1204.0042](#).
- [8] J. Alcaraz, P. Azzurri, A. Bajo-Vaquero, E. Barberio, A. Blondel, D. Bourilkov, P. Checchia, R. Chierici, R. Clare, J. D’Hondt, G. Della Ricca, M. Dierckxsens, D. Duchesneau, G. Duckeck, M. Elsing, M. W. Grünewald, A. Gurtu, J. B. Hansen, R. Hawkings, S. Jezequel, R. W. L. Jones, T. Kawamoto, E. Lançon, W. Liebig, L. Malgeri, S. Mele, M. N. Minard, K. Mönig, C. Parkes, U. Parzefall, B. Pietrzyk, G. Quast, P. B. Renton, S. Riemann, K. Sachs, D. Strom, A. Strässner, R. Tenchini, F. Teubert, M. A. Thomson, S. Todorova-Nová, A. Valassi, A. Venturi, H. Voss, C. P. Ward, N. K. Watson, P. S. Wells, S. Wynhoff, P. de Jong, B. de la Cruz, [A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, 2006](#), Tech. Rep. hep-ex/0612034. ALEPH-2006-001 PHYSICS-2006-001. CERN-L3-310. CERN-PH-EP-2006-042. DELPHI-2006-014 PHYS-948. L3-Note-2833. LEPEWWG-2006-01. OPAL-PR-419, CERN, Geneva, preprint not submitted to publication (Dec 2006). URL <https://cds.cern.ch/record/1016509>
- [9] J. Beringer, et al., Review of Particle Physics (RPP), Phys. Rev. D86 (2012) 010001. [doi:10.1103/PhysRevD.86.010001](#).
- [10] G. Aad, et al., Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC, Phys. Lett. B726 (2013) 88–119, [Erratum: Phys. Lett.B734,406(2014)]. [arXiv:1307.1427](#), [doi:10.1016/j.physletb.2014.05.011](#), [10.1016/j.physletb.2013.08.010](#).
- [11] S. Chatrchyan, et al., Measurement of the properties of a Higgs boson in the four-lepton final state, Phys. Rev. D89 (9) (2014) 092007. [arXiv:1312.5353](#), [doi:10.1103/PhysRevD.89.092007](#).

-
- [12] First combination of Tevatron and LHC measurements of the top-quark mass (2014). [arXiv:1403.4427](#).
- [13] V. Tishchenko, et al., Detailed Report of the MuLan Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant, Phys. Rev. D87 (5) (2013) 052003. [arXiv:1211.0960](#), [doi:10.1103/PhysRevD.87.052003](#).
- [14] S. Bethke, World Summary of α_s (2012)[Nucl. Phys. Proc. Suppl.234,229(2013)] (2012). [arXiv:1210.0325](#), [doi:10.1016/j.nuclphysbps.2012.12.020](#).
- [15] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, Investigating the near-criticality of the Higgs boson, JHEP 12 (2013) 089. [arXiv:1307.3536](#), [doi:10.1007/JHEP12\(2013\)089](#).
- [16] M. Farina, D. Pappadopulo, A. Strumia, A modified naturalness principle and its experimental tests, JHEP 08 (2013) 022. [arXiv:1303.7244](#), [doi:10.1007/JHEP08\(2013\)022](#).
- [17] N. Arkani-Hamed, A. Delgado, G. F. Giudice, The Well-tempered neutralino, Nucl. Phys. B741 (2006) 108–130. [arXiv:hep-ph/0601041](#), [doi:10.1016/j.nuclphysb.2006.02.010](#).
- [18] J. Hisano, S. Matsumoto, M. Nagai, O. Saito, M. Senami, Non-perturbative effect on thermal relic abundance of dark matter, Phys. Lett. B646 (2007) 34–38. [arXiv:hep-ph/0610249](#), [doi:10.1016/j.physletb.2007.01.012](#).
- [19] M. Cirelli, A. Strumia, M. Tamburini, Cosmology and Astrophysics of Minimal Dark Matter, Nucl. Phys. B787 (2007) 152–175. [arXiv:0706.4071](#), [doi:10.1016/j.nuclphysb.2007.07.023](#).
- [20] T. Moroi, M. Nagai, M. Takimoto, Non-Thermal Production of Wino Dark Matter via the Decay of Long-Lived Particles, JHEP 07 (2013) 066. [arXiv:1303.0948](#), [doi:10.1007/JHEP07\(2013\)066](#).
- [21] M. Beneke, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, Relic density of wino-like dark matter in the MSSM, JHEP 03 (2016) 119. [arXiv:1601.04718](#), [doi:10.1007/JHEP03\(2016\)119](#).
- [22] S. P. Martin, A Supersymmetry primer (1997) 1–98[Adv. Ser. Direct. High Energy Phys.18,1(1998)]. [arXiv:hep-ph/9709356](#), [doi:10.1142/9789812839657_0001](#), [10.1142/9789814307505_0001](#).
- [23] M. Cirelli, N. Fornengo, A. Strumia, Minimal dark matter, Nucl. Phys. B753 (2006) 178–194. [arXiv:hep-ph/0512090](#), [doi:10.1016/j.nuclphysb.2006.07.012](#).

- [24] M. Cirelli, A. Strumia, Minimal Dark Matter: Model and results, New J. Phys. 11 (2009) 105005. [arXiv:0903.3381](#), [doi:10.1088/1367-2630/11/10/105005](#).
- [25] J. Hisano, S. Matsumoto, M. M. Nojiri, O. Saito, Non-perturbative effect on dark matter annihilation and gamma ray signature from galactic center, Phys. Rev. D71 (2005) 063528. [arXiv:hep-ph/0412403](#), [doi:10.1103/PhysRevD.71.063528](#).