Section 1

Properties of the transverse mass

In this appendix, we summarize the properties of the transverse mass, which is used for the analysis of the mono-lepton final state in Sec. **??**. The transverse mass is useful when there is a unique invisible particle (which we will call I) such as a neutrino in the final state. As already mentioned in Sec. **??**, the transverse mass m_T is defined event by event using the measured value of the missing transverse momentum $\not{E}_T$ as

$$m_T^2 \equiv 2p_T \not{E}_T \left(1 - \cos\phi\right), \tag{1.1}$$

where p_T denotes the transverse momentum of a visible final state particle (which will call P) and ϕ is the difference between the azimuth angles of visible and missing transverse momenta. It is important that we can infer the invariant mass of particles P and I with m_T, if both P and I are (approximately) massless.

Let p_P and p_I be the four momenta of P and I, respectively. When there is only one invisible particle in the event, the transverse momentum of I is roughly identified with $\not{E}_T$. Hereafter, we assume the exact equality among them just for simplicity, which corresponds to neglect the detector errors, transverse momentum of initial partons, soft emissions that are invisible for detectors, and so on. Then, we can write the components of four momenta as

$$p_P = \left(E_P, p_T \cos\phi_P, p_T \sin\phi_P, p_{Pz}\right), \tag{1.2}$$

$$p_I = \left(E_I, \not{E}_T \cos\phi_I, \not{E}_T \sin\phi_I, p_{Iz}\right), \tag{1.3}$$

with $\phi \equiv \phi_P - \phi_I$. Note that massless conditions are satisfied, namely $E_P^2 = p_T^2 + p_{Pz}^2$ and $E_I^2 = \not{E}_T^2 + p_{Iz}^2$. We can derive a relation between m_T and $m_{PI} \equiv \sqrt{(p_P + p_I)^2}$

$$m_T \leq m_{PI}, \tag{1.4}$$

where the equation holds when

$$p_{Pz}\not{E}_T - p_T p_{Iz} = 0. \tag{1.5}$$

When the above equation roughly holds, $m_{PI} - m_T$ is proportional to $(p_{Pz}\not{E}_T - p_T p_{Iz})^2$.

It is more intuitive to understand the situation in the center-of-mass system (CMS), focusing on the pair production process of particles P and I. In this case, the transverse momentum of the event is simply given by

$$m_T^{(\mathrm{CMS})} = m_{PI} \sin\theta^{(\mathrm{CMS})}, \tag{1.6}$$

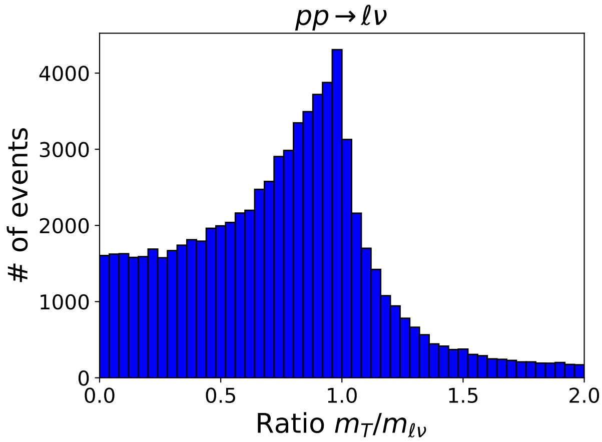

Figure 1: Distribution of $m_T/m_{\ell\nu}$ for the pair production process of $P = \ell$ and $I = \nu$. Figure for $\sqrt{s} = 100\,\mathrm{TeV}$ and $\mathcal{L} = 1\,\mathrm{ab}^{-1}$.

where $\theta^{(\mathrm{CMS})}$ is the angle between the momentum of P and the beamline in the CMS. Although the definition of m_T is not Lorentz invariant and generally $m_T^{(\mathrm{CMS})} \neq m_T$, the former gives a good approximation of the latter when the two-particle system is not highly boosted. Let us simply assume $m_T = m_T^{(\mathrm{CMS})}$ and consider the repeated production of P and I with fixed m_{PI}. When we postulate the uniform distribution of the production cross section against $\cos\theta^{(\mathrm{CMS})}$ for simplicity, the distribution of the transverse mass $f(m_T)$ calculated according to Eq. (1.6) possesses a sharp peak at $m_T = m_{PI}$, described by

$$f(m_T) = \frac{m_T}{m_{PI}} \cos^{-1}\left[\arcsin\left(\frac{m_T}{m_{PI}}\right)\right]. \tag{1.7}$$

This peak, often called the Jacobian peak, enables us to estimate m_{PI} from the distribution of m_T.

In Fig. 1, we show the distribution of the ratio $m_T/m_{\ell\nu}$ for the pair production process of $P = \ell$ and $I = \nu$. We use the setup of $\sqrt{s} = 100\,\mathrm{TeV}$ and $\mathcal{L} = 1\,\mathrm{ab}^{-1}$. To evaluate the missing transverse momentum $\not{E}_T$ for each event, we have performed the detector simulation using `Delphes` similar to the analysis in Sec. **??**. We can clearly see the peak at $m_T/m_{\ell\nu} = 1$, though it is somewhat smeared compared with Eq. (1.7) due to the effect of the Lorentz boost and the non-trivial angular dependence of the production cross section. Besides, the small tail of the distribution for $m_T > m_{\ell\nu}$ can be understood as the effects we have neglected so far, such as the detector errors.