## Section 1

## Review of supersymmetry

## ( More later )

In this appendix, we briefly review the  $\mathcal{N}=1$  supersymmetry, which is an essential element of the MSSM explained in Sec. ??. Our argument is based on [?,?].

The  $\mathcal{N} = 1$  supersymmetry is

First example is the MSSM, extension of the SM with the so-called  $\mathcal{N}=1$  supersymmetry (SUSY) [?,?] that relates a bosonic particle and a fermionic particle. The supersymmetry transformations for a complex scalar  $\phi$  and its "superpartner" Weyl fermion  $\psi$  are defined as

$$\delta \phi = (\epsilon \psi), \quad \delta \phi^* = (\epsilon^{\dagger} \psi^{\dagger}), \qquad (1.1)$$

$$\delta\psi = -i \left(\sigma^{\mu} \epsilon^{\dagger}\right) \partial_{\mu} \phi, \quad \delta\psi^{\dagger} = i \left(\epsilon \sigma^{\mu}\right) \partial_{\mu} \phi^{*}, \tag{1.2}$$

where  $\sigma^{\mu} \equiv (\mathbf{1}, \boldsymbol{\sigma})$  with  $\boldsymbol{\sigma}$  being Pauli matrices, while  $\epsilon$  is an anti-commuting Weyl fermionic object that parameterizes the SUSY transformation. The summation over the spinor indices is assumed inside each parenthesis. These transformations, if denoted by operators  $\epsilon Q$  and  $\epsilon^{\dagger} Q^{\dagger}$ , are known to form a closed algebra

$$[Q, Q^{\dagger}] = 2i\sigma^{\mu}\partial_{\mu}, \tag{1.3}$$

$$[Q,Q] = [Q^{\dagger}, Q^{\dagger}] = 0, \tag{1.4}$$

when fields are on-shell.<sup>‡1</sup>

- (\$\dagger\$ chiral and vector superfield \$\dagger\$)
- ( $\clubsuit m_f = m_S \text{ for each multiplet } \clubsuit$ )
- ( F-term and D-term potential )

 $<sup>^{\</sup>natural 1}$ In order for the algebra to be closed off-shell, one can introduce a new scalar field F without a kinetic term that is often called as an *auxiliary* field. F works as a Lagrange multiplier whose equation of motion ( $\clubsuit$  What?  $\clubsuit$ )