Section 1

Review of supersymmetry

(♣ More later ♣)

In this appendix, we briefly review the $\mathcal{N}=1$ supersymmetry, which is an essential element of the MSSM explained in Sec. **??**. Our argument is based on [**?**, **?**].

The $\mathcal{N}=1$ supersymmetry is

First example is the MSSM, extension of the SM with the so-called $\mathcal{N}=1$ supersymmetry (SUSY) [**?**, **?**] that relates a bosonic particle and a fermionic particle. The supersymmetry transformations for a complex scalar ϕ and its "superpartner" Weyl fermion ψ are defined as

$$\delta\phi = (\epsilon\psi)\,, \quad \delta\phi^* = \left(\epsilon^\dagger\psi^\dagger\right), \tag{1.1}$$

$$\delta\psi = -i\left(\sigma^\mu\epsilon^\dagger\right)\partial_\mu\phi, \quad \delta\psi^\dagger = i\left(\epsilon\sigma^\mu\right)\partial_\mu\phi^*, \tag{1.2}$$

where $\sigma^\mu \equiv (\mathbf{1}, \boldsymbol{\sigma})$ with $\boldsymbol{\sigma}$ being Pauli matrices, while ϵ is an anti-commuting Weyl fermionic object that parameterizes the SUSY transformation. The summation over the spinor indices is assumed inside each parenthesis. These transformations, if denoted by operators ϵQ and $\epsilon^\dagger Q^\dagger$, are known to form a closed algebra

$$\left[Q, Q^\dagger\right] = 2i\sigma^\mu\partial_\mu, \tag{1.3}$$

$$[Q, Q] = \left[Q^\dagger, Q^\dagger\right] = 0, \tag{1.4}$$

when fields are on-shell.[♮1]

(♣ chiral and vector superfield ♣)

(♣ $m_f = m_S$ for each multiplet ♣)

(♣ F-term and D-term potential ♣)

[♮1] In order for the algebra to be closed off-shell, one can introduce a new scalar field F without a kinetic term that is often called as an *auxiliary* field. F works as a Lagrange multiplier whose equation of motion