

*Ph.D. Thesis*

**Indirect study of electroweakly  
interacting particles  
at 100 TeV hadron colliders**

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December 2019

## Abstract

(♣ To be written ♣)

# Acknowledgments

(♣ To be written ♣)

# Contents

|          |   |          |
|----------|---|----------|
| <b>1</b> | <b>Introduction</b>                         | <b>1</b> |
| <b>2</b> | <b>Weakly interacting massive particles</b> | <b>1</b> |
| 2.1      | WIMPs as a dark matter candidate . . . . .  | 1        |

# Section 1

## Introduction

- (♣ Unit:  $\hbar = c = k_B = 1$  ♣)
- (♣ Convention: dot as time derivative ♣)
- (♣ Definition of “SM” ♣)
- (♣ Definition of “WIMP” ♣)

## Section 2

### Weakly interacting massive particles

#### 2.1 WIMPs as a dark matter candidate

One of the most important evidences of the beyond SM is the existence of dark matter (DM) [1]. DM is an unknown object that occupies a non-negligible ratio of the total energy of our universe, but has not yet been directly observed because of its weak interaction with the SM particles.<sup>‡1</sup> In spite of its invisibility, the existence of DM is confirmed by several astrophysical observations such as the mass measurement using the gravitational lensing effect caused by galaxies and clusters [2, 3], the flatness of galactic rotation curves further the optical radius [4, 5], the measurement of the power spectrum of the cosmic microwave background (CMB), and so on. In particular, the observation of CMB allows us the precise determination of various cosmological parameters [6, 7] including the density of the non-relativistic matter and baryon, which is currently determined as [8]

$$\Omega_m h^2 = 0.1430 \pm 0.0011, \quad (2.1)$$

$$\Omega_b h^2 = 0.02237 \pm 0.00015, \quad (2.2)$$

where  $h \sim \mathcal{O}(1)$  is the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The difference between  $\Omega_m h^2$  and  $\Omega_b h^2$  implies the existence of DM and its abundance  $\Omega_\chi h^2 \simeq 0.12$ .

In cosmology, DM production mechanisms that try to explain the DM abundance are divided into two main categories: thermal and non-thermal production. The former assumes the equilibrium between the DM and the thermal bath in the early universe. As the universe expands, the interaction rate that maintains the thermal equilibrium becomes smaller and

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<sup>‡1</sup> At worst DM interacts with the SM particles through the gravity, which is considerably weaker than all the other known interactions. (♣ Mention to Ema paper?? ♣)

the DM decouples from the thermal bath at some time, which is the so-called *freezeout*. As we will see below, the resulting abundance of the DM in this scenario is mainly controlled by the temperature of the thermal bath  $T_f$  when the freezeout occurs. On the other hand, non-thermal production assumes the DM production by some processes irrespective of the thermal bath such as decay of a heavy particle. Since the thermal production scenario can be realized in relatively simple setup and WIMPs are well motivated in connection with this kind of scenario, we focus on it.

We assume the stable DM particle  $\chi$  with mass  $m_\chi$  can pair annihilate into SM particles with some cross section  $\sigma$ . When DM is in thermal equilibrium with the thermal bath of temperature  $T$ , DM has some fixed velocity distribution. **(♣ More clearly ♣)** Let  $v$  be the relative velocity of annihilating DM particles and  $\langle\sigma v\rangle$  be the thermal average of the product of  $\sigma$  and  $v$ . By using this quantity, we can write down the Boltzmann equation for the DM number density  $n_\chi$  as

$$\frac{d(n_\chi a^3)}{dt} = -a^3 \langle\sigma v\rangle (n_\chi^2 - n_{\text{eq}}^2), \quad (2.3)$$

where  $t$  and  $a$  are the time coordinate and the scale factor, respectively, of the Friedmann Robertson Walker metric

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, \quad (2.4)$$

while  $n_{\text{eq}}$  denotes the number density of DM in equilibrium. The first term of the right handed-side of Eq. (2.3) represents the annihilation rate of DM pairs that should be proportional to  $n_\chi^2$ , while the second term describes the DM creation through the inverse process. As desired, the number density does not change in time if  $n_\chi = n_{\text{eq}}$ . Recalling the total entropy conservation in a comoving volume  $sa^3 = (\text{const})$ , it turns out to be convenient to define the ratio  $Y \equiv n_\chi/s$ . In fact, this modification cancels the effect of the expansion of the universe  $\dot{a} > 0$  from Eq. (2.3), leading to a simpler equation

$$\frac{dY}{dt} = -s \langle\sigma v\rangle (Y^2 - Y_{\text{eq}}^2), \quad (2.5)$$

with  $Y_{\text{eq}} \equiv n_{\text{eq}}/s$ .

Here we assume that the freezeout occurs when the relativistic radiation dominates the total energy of the universe, which will be verified to be correct later. In this case, we can derive  $a \propto T^{-1}$  from the entropy conservation with  $s \propto T^3$ . For the numerical calculation, we define a dimensionless parameter  $x \equiv m_\chi/T$ . Since the time dependence of  $x$  is the same as  $a$ ,  $\dot{x} = xH$  with  $H$  being the Hubble parameter. In addition,  $H \propto \rho^{1/2} \propto x^{-2}$  and  $s \propto x^{-3}$  allows us to extract the  $x$  dependence as  $H(x) = H(x=1)x^{-2}$  and  $s(x) = s(x=1)x^{-3}$ , respectively. Finally, we can rewrite Eq. (2.5) as

$$\frac{dY}{dx} = -\frac{\lambda}{x^2} (Y^2 - Y_{\text{eq}}^2), \quad (2.6)$$

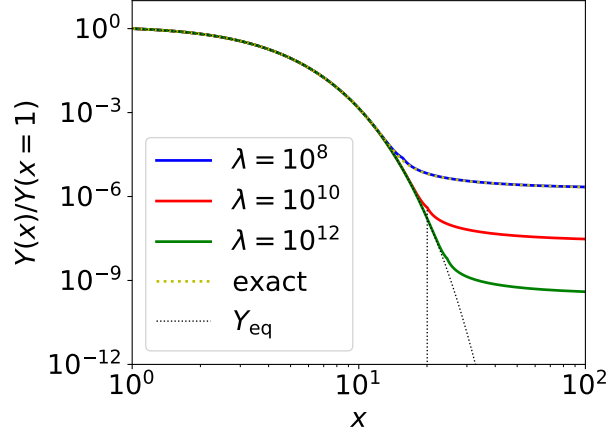


Figure 1: Plot of  $Y(x)/Y(x=1)$  with  $Y(x)$  being a solution of the evolution equation Eq. (2.6). The yellow dotted line is the solution for  $\lambda = 10^8$ , while the black dotted line denotes  $Y_{\text{eq}}$ . The solid lines are the approximation to the solutions described in the text. The blue, red, and green colors correspond to  $\lambda = 10^8$ ,  $10^{10}$ , and  $10^{12}$ , respectively. The vertical dotted line denotes the freezeout temperature  $x_f$  for the  $\lambda = 10^{10}$  case.

with

$$\lambda \equiv \frac{\langle \sigma v \rangle s(x=1)}{H(x=1)}. \quad (2.7)$$

**(♣ Please mention s-wave dominance ♣)**

In Fig. 1, we show the solution of Eq. (2.6) for  $\lambda = 10^8$  by the yellow dotted line. In the calculation, we use the boundary condition  $Y(x=1) = Y_{\text{eq}}(x=1)$  and plot the normalized value  $Y(x)/Y(x=1)$ . We also plot the function  $Y_{\text{eq}}/Y_{\text{eq}}(x=1)$  by the black dotted line.

Unfortunately, it is computationally hard to solve Eq. (2.6) for larger values of  $\lambda$  because of the almost complete cancellation between two terms of the right handed side for small  $x \sim \mathcal{O}(1)$  and its amplification caused by large  $\lambda$ . We adopt instead to use an approximation that is same with the one adopted in the public code `MicrOMEGAs` [9, 10]. For the small  $x$  region, temperature is still high enough to maintain the equilibrium  $Y \simeq Y_{\text{eq}}$ , which means that  $d\Delta Y/dx \ll dY_{\text{eq}}/dx$  with  $\Delta Y \equiv Y - Y_{\text{eq}}$ . From this approximation we obtain a formula

$$\Delta Y \simeq -\frac{x^2}{2\lambda} \frac{d \ln Y_{\text{eq}}}{dx}. \quad (2.8)$$

Then we define the time  $x_f$ , or equivalently the so-called freezeout temperature  $T_f$ , when the approximation becomes invalid through the equation

$$\Delta Y(x_f) = 2.5 Y_{\text{eq}}(x_f). \quad (2.9)$$

After the freezeout  $x > x_f$ , the annihilation of the DM pairs rapidly slows down and the DM abundance far exceeds its equilibrium value:  $Y \gg Y_{\text{eq}}$ . (**♣ Comma? ♣**) Then we can neglect the second term of the right hand of Eq. (2.6) and obtain the analytical solution

$$Y(x) \simeq -\frac{x}{c_1 x + \lambda}, \quad (2.10)$$

where  $c_1$  is a integration constant. In Fig. 1, we show results obtained with these two approximations Eqs. (2.8) and (2.10) for  $\lambda = 10^8$  (blue),  $10^{10}$  (red), and  $10^{12}$  (green). In particular, the blue and the yellow lines almost completely overlaps with each other, which proves the validity of the approximations. The vertical dotted line in the figure shows the freezeout temperature  $x_f \simeq 20$  for the case of  $\lambda = 10^{10}$ . (**♣ Plot all  $x_f$ s!! with corresponding colors!! ♣**) It can be seen from the figure that  $x = x_f$  does correspond to the time when  $Y$  starts to deviate from  $Y_{\text{eq}}$ . Note that as  $\lambda \propto \langle \sigma v \rangle$  becomes larger, the freezeout time becomes later and the late time relic abundance becomes smaller.

When the DM properties (*i.e.*, the mass  $m_\chi$  and the annihilation cross section  $\langle \sigma v \rangle$ ) are given, corresponding relic abundance can be calculated using above procedure. In particular,  $m_\chi$  determines the normalization of the figure, namely  $Y_{\text{eq}}(x = 1) = Y_{\text{eq}}(T = m_\chi)$ , and  $\langle \sigma v \rangle$  determines the freezeout temperature through the combination of Eq. (2.7). Assuming the absence of non-thermal effect, only some good combination of these two values should explain the current relic abundance of the DM.



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