

# Section 1

## Review of supersymmetry

In this appendix, we briefly review the  $\mathcal{N} = 1$  supersymmetry, which is an essential element of the MSSM reviewed in Sec. ???. Our argument is based on [?, ?].

The  $\mathcal{N} = 1$  supersymmetry is

First example is the MSSM, extension of the SM with the so-called  $\mathcal{N} = 1$  supersymmetry (SUSY) [?, ?] that relates a bosonic particle and a fermionic particle. The supersymmetry transformations for a complex scalar  $\phi$  and its “superpartner” Weyl fermion  $\psi$  are defined as

$$\delta\phi = (\epsilon\psi), \quad \delta\phi^* = (\epsilon^\dagger\psi^\dagger), \quad (1.1)$$

$$\delta\psi = -i(\sigma^\mu\epsilon^\dagger)\partial_\mu\phi, \quad \delta\psi^\dagger = i(\epsilon\sigma^\mu)\partial_\mu\phi^*, \quad (1.2)$$

where  $\sigma^\mu \equiv (\mathbf{1}, \boldsymbol{\sigma})$  with  $\boldsymbol{\sigma}$  being Pauli matrices, while  $\epsilon$  is an anti-commuting Weyl fermionic object that parameterizes the SUSY transformation. The summation over the spinor indices is assumed inside each parenthesis. These transformations, if denoted by operators  $\epsilon Q$  and  $\epsilon^\dagger Q^\dagger$ , are known to form a closed algebra

$$[Q, Q^\dagger] = 2i\sigma^\mu\partial_\mu, \quad (1.3)$$

$$[Q, Q] = [Q^\dagger, Q^\dagger] = 0, \quad (1.4)$$

when fields are on-shell.<sup>[b1](#)</sup>

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<sup>b1</sup>In order for the algebra to be closed off-shell, one can introduce a new scalar field  $F$  without a kinetic term that is often called as an *auxiliary* field.  $F$  works as a Lagrange multiplier whose equation of motion