

Section 1

Models with WIMPs

There are several examples of the models that contain WIMP DM candidates. In this section, two of them (**♣ Really? ♣**) are briefly reviewed. (**♣ EWIMP and WIMP?? ♣**)

1.1 Minimally supersymmetric standard model

(**♣ Summary of SUSY particle names somewhere ♣**)

The minimally supersymmetric standard model (MSSM) is the simple extension of the SM with $\mathcal{N} = 1$ supersymmetry (SUSY).^{‡1} One of the motivations to introduce SUSY is to solve the so-called hierarchy (or naturalness) problem $[?, ?, ?]$ in the SM. The problem is related to the quantum correction to the SM Higgs boson mass from heavy new physics particles. For example, we can consider the one-loop correction to the Higgs mass from a Weyl fermion f and a complex scalar S as illustrated in Fig. 1. The corrections to the Higgs mass is given by

$$\Delta m_h^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_f^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_f} \right) + \dots \right] \quad (\text{fermion}), \quad (1.1)$$

$$\Delta m_h^2 = \frac{\lambda_S}{16\pi^2} \left[\Lambda_{\text{UV}}^2 - 2m_S^2 \ln \left(\frac{\Lambda_{\text{UV}}}{m_S} \right) + \dots \right] \quad (\text{scalar}), \quad (1.2)$$

(**♣ Check this! ♣**) where λ_f and m_f are the Higgs-fermion coupling constant and the fermion mass, respectively, and λ_S and m_S are those for the scalar S . We take the cut-off

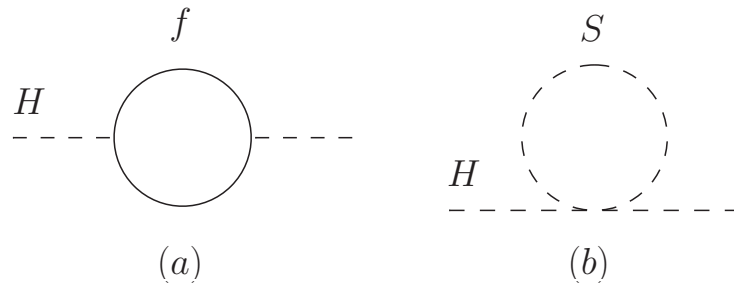


Figure 1: One-loop correction to the Higgs mass from (a) a Weyl fermion f and (b) a complex scalar S .

^{‡1}For a brief review of the $\mathcal{N} = 1$ SUSY, see Sec. ??.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{Q}_i	3	2	1/6
\hat{L}_i	1	2	-1/2
\hat{U}_i	$\bar{\mathbf{3}}$	1	-2/3
\hat{D}_i	$\bar{\mathbf{3}}$	1	1/3
\hat{E}_i	1	1	1
\hat{H}_u	1	2	1/2
\hat{H}_d	1	2	-1/2

Table 1: Notations and quantum numbers of the chiral superfields in the MSSM.

Notation	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\hat{g}	8	1	0
\hat{W}	1	3	0
\hat{B}	1	1	0

Table 2: Notations and quantum numbers of the vector superfields in the MSSM.

scale of the theory to be Λ_{UV} to regularize the otherwise divergent loop integral and neglect the lower order terms of Λ_{UV} . Eqs. (1.1) and (1.2) show the quadratic dependence of Δm_H^2 on Λ_{UV} , which means that the Higgs mass is sensitive to the energy scale of the beyond the SM physics. However, there is at least one extremely high energy scale physics in the nature, gravity at the Planck scale $M_{\text{pl}} \sim 10^{18-19} \text{ GeV}$. By substituting $\Lambda_{UV} = M_{\text{pl}}$ in Eqs. (1.1) and (1.2) and assuming $\lambda_f \sim \lambda_S \sim \mathcal{O}(1)$, we notice that orders-of-magnitude fine-tuning is required to obtain the correct Higgs mass $m_h = 125.10 \text{ GeV}$ [?], which is unnatural.

SUSY provides a nice solution to this fine-tuning problem. As is summarized in Appendix ??, (**♣ summarize later ♣**) each Weyl fermion in a supersymmetric model has two complex scalars with the same mass $m_f = m_S$. In addition, their coupling constants should have a relationship $|\lambda_f|^2 = \lambda_S$ due to the fact that λ_S is a coupling constant in the F-term potential sourced by a superpotential term proportional to λ_f . (**♣ description of F-term and D-term ♣**) By using both equations and summing the corrections (1.1) and (1.2) with factor of two multiplied to the latter, we obtain a result independent of the cut-off scale Λ_{UV} without fine-tuning. This cancellation is ensured by the so-called non-renormalization theorem. [?, ?]

We now summarize the notations and quantum numbers of the chiral and vector superfields in the MSSM in Table 1 and 2, respectively. The supersymmetric part of the MSSM

lagrangian is described by the superpotential ^{‡2}

$$W = Y_u^{ij} \hat{U}_i \hat{Q}_j \hat{H}_u - Y_d^{ij} \hat{D}_i \hat{Q}_j \hat{H}_d - Y_e^{ij} \hat{E}_i \hat{L}_j \hat{H}_d + \mu \hat{H}_u \hat{H}_d, \quad (1.3)$$

where $i, j = 1, 2, 3$ labels the quark and lepton generation, while Q, L, U, D, E are superfields that contain the left-handed quark, left-handed lepton, right-handed up-type quark, right-handed down-type quark, and right-handed charged lepton, respectively. In Eq. (1.3), proper contraction of $SU(3)_C$ and $SU(2)_L$ indices is assumed. Note that two Higgs doublets H_u and H_d with opposite values of $U(1)_Y$ hypercharges are introduced, which is needed to cancel the contributions to the gauge anomaly from fermionic partners of the Higgs doublets.

Postulating SM gauge symmetries as a unique guideline to construct a model, there are a few more terms allowed in the superpotential:

$$W_{\Delta L=1} = \lambda^{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'^{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k + \mu^i \hat{L}_i \hat{H}_u, \quad (1.4)$$

$$W_{\Delta B=1} = \lambda''^{ijk} \hat{U}_i \hat{D}_j \hat{D}_k. \quad (1.5)$$

Terms in Eqs. (1.4) and (1.5) are phenomenologically problematic since they lead to the lepton and baryon number violating operators, respectively, which may cause a too fast proton decay, depending on parameters (see for example [?]). To avoid this problem, we often rely on a symmetry called the R-parity [?] or the matter parity [?, ?, ?, ?]. Charges of the R-parity, which is basically a Z_2 symmetry, are calculated as

$$P_R = (-1)^{3(B-L)+2s}, \quad (1.6)$$

where B , L , and s are the baryon number, lepton number, and spin of the particle, respectively. According to the definition, we can see that all the SM particles have even parity ($P_R = +1$), while all the supersymmetric particles have odd parity ($P_R = -1$). Then it is easy to check that Eqs. (1.4) and (1.5) lead to the R-parity violating terms in the Lagrangian and thus are forbidden, while all the terms in Eq. (1.3) remain allowed. From now on, we only focus on the R-parity conserving MSSM.

Since no superpartner of any SM particle is observed yet, SUSY should be broken and superpartners should obtain the SUSY breaking masses. (**♣ ref: boson and fermion obtain equal mass ♣**) The SUSY breaking part of the lagrangian is expressed as

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right) \\ & - \left(A_u^{ij} \tilde{U}_i \tilde{Q}_j H_u - A_d^{ij} \tilde{D}_i \tilde{Q}_j H_d - A_e^{ij} \tilde{E}_i \tilde{L}_j H_d \right) \\ & - m_Q^{2ij} \tilde{Q}_i^\dagger \tilde{Q}_j - m_L^{2ij} \tilde{L}_i^\dagger \tilde{L}_j - m_U^{2ij} \tilde{U}_i^\dagger \tilde{U}_j - m_D^{2ij} \tilde{D}_i^\dagger \tilde{D}_j - m_E^{2ij} \tilde{E}_i^\dagger \tilde{E}_j \\ & - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}), \end{aligned} \quad (1.7)$$

^{‡2}For a more detailed review of the MSSM, see for example [?].

where the tilde is used to express the superpartner of the SM particle contained in a superfield, while a field without a hat nor tilde denotes the other component. An exception is two Higgs doublets, where H_u and H_d express the scalar components, while \tilde{H}_u and \tilde{H}_d express their superpartners called Higgsinos. The SM-like Higgs doublet H corresponds to a linear combination of H_u and H_d .

It is known that, within the MSSM, almost all SUSY breaking mechanisms, such as the F-term (O’Raifeartaigh) [?] or D-term (Fayet-Iliopoulos) SUSY breaking [?, ?], fail to generate masses of superpartners with remaining the SM gauge group in the low energy effective theory. Thus, we need a so-called hidden sector in addition to the MSSM sector, in which SUSY is spontaneously broken. In order for the MSSM sector to have Lagrangian terms (1.7), we also need some mediation mechanism of the SUSY breaking. The relative size of the SUSY breaking parameters in Eq. (1.7) highly depends on the mediation mechanism. Among many mediation mechanisms of SUSY breaking, the anomaly mediated SUSY breaking [?, ?] leads to an interesting phenomenology with relatively light WIMPs, so it will be reviewed later.

Dark matter candidate in the MSSM

There is another motivation to consider the (R-parity preserving) MSSM: the existence of the DM. Since there is a sizable amount of DM in the current universe, a DM candidate should be stable or have sufficiently small decay width. In many models, the stability of DM is ensured by imposing a symmetry and/or by kinematically forbidding the DM decay. In the MSSM, the role of stabilizer can be played by the R-symmetry described above. Recalling that all the SM (supersymmetric) particles have even (odd) parity, each interaction vertex in the MSSM Lagrangian should contain an even number of supersymmetric particles. If we consider the lightest supersymmetric particle (LSP), such vertices can not construct the kinematically allowed LSP decay chain and, as a result, the LSP becomes a stable DM candidate.

The DM phenomenology, such as the production and annihilation of DM in the universe and processes that allow us to efficiently detect it, highly depends on which species of supersymmetric particle becomes the LSP. Hereafter, we only focus on the cases where one of the gauginos and Higgsinos becomes the LSP with some motivations described below. In addition, all the LSP candidates to be described have non-zero electroweak charges and they can be viewed as examples of the WIMPs.

Higgs mass in the MSSM

Under the spontaneously broken SUSY, the cancellation of the quantum correction to the Higgs boson discussed above is not exact. One obvious consequence of the SUSY breaking in Eqs. (1.1) and (1.2) is the hierarchy between m_f and m_S that appear in the second term

Value	Description	Reference
$M_W = 80.384 \pm 0.014 \text{ GeV}$	Pole mass of the W boson	[?, ?]
$M_Z = 91.1876 \pm 0.0021 \text{ GeV}$	Pole mass of the Z boson	[?]
$M_h = 125.15 \pm 0.24 \text{ GeV}$	Pole mass of the Higgs	[?, ?]
$M_t = 173.34 \pm 0.82 \text{ GeV}$	Pole mass of the top quark	[?]
$(\sqrt{2}G_\mu)^{-1/2} = 246.21971 \pm 0.00006 \text{ GeV}$	Fermi constant for μ decay	[?]
$\alpha_3(M_Z) = 0.1184 \pm 0.0007$	$\overline{\text{MS}}$ $SU(3)_C$ gauge coupling	[?]

Table 3: Experimentally measured SM parameters used for the derivation of Eq. (1.9).

of each contribution. In the case of the MSSM, the largest contribution comes from the superpartner of the top quark, stop, that have the largest Yukawa coupling with the Higgs boson.

When there is a large hierarchy between the SUSY breaking scale M_S , which is comparable with stop masses, and the top mass M_t , the stop contributions to the Higgs mass contains a large logarithm of the form of $\log(M_S^2/M_t^2)$. (**♣ Consistency with above equations. Maybe MSbar better ♣**) To resum this large logarithm and obtain a precise result, we have to rely on the renormalization group equation (RGE). In this framework, the value of the Higgs self coupling λ at the electroweak scale is closely related to the Higgs mass. We assume the SM parameters summarized in Table 3 and the definition of the SM Higgs potential

$$V(H) = -\frac{m^2}{2}|H|^2 + \lambda|H|^4, \quad (1.8)$$

with H being the SM Higgs doublet. Then, according to [?], we obtain the relationship ^{‡3}

$$\lambda(M_t) = 0.12604 + 0.00206 \left(\frac{M_h}{\text{GeV}} - 125.15 \right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.34 \right), \quad (1.9)$$

where the $\overline{\text{MS}}$ scheme is used to renormalize the divergence of loop integrals.

In the MSSM, the value of λ at the SUSY breaking scale M_S is given by

$$\lambda(M_S) = \frac{g_1^2(M_S) + g_2^2(M_S)}{8} \cos^2 2\beta + \delta\lambda, \quad (1.10)$$

^{‡3}Although the values listed in Table 3 are different from the latest ones given in [?], we use older ones because the change in input values may cause the slight change in coefficients of second and third terms of Eq. (1.9). The latest central values of the Higgs and top masses are $M_h = 125.10 \text{ GeV}$ and $M_t = 173.1 \text{ GeV}$, with which we can estimate $\lambda(M_t) = 0.12595$.

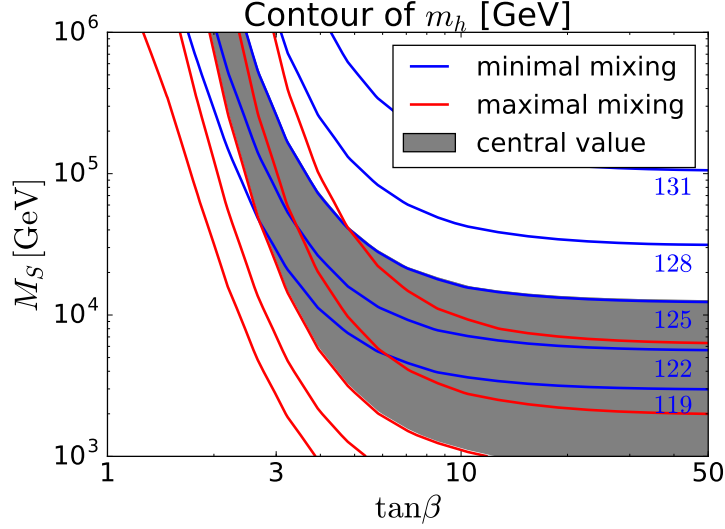


Figure 2: Contour of the Higgs mass m_h in the $\tan\beta$ vs. M_S plane. The universal mass M_S is assumed for all the SUSY particles. Blue (red) lines correspond from top to bottom to the contours of $m_h = 131, 128, 125, 122, 119$ GeV for the minimal (maximal) stop mixing. Gray shade corresponds to the region where $m_h = 125.10$ GeV can be explained.

where g_1 and g_2 are $U(1)_Y$ and $SU(2)_L$ gauge coupling constants, respectively, while β parametrizes the ratio of the vacuum expectation values

$$\frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle} = \tan \beta, \quad (1.11)$$

with H_u^0 and H_d^0 being electromagnetically neutral components of the corresponding Higgs doublets. In Eq. (1.10), the first term shows the tree-level contribution from the D-term potential and $\delta\lambda$ denotes the threshold correction from heavy superpartners. M_S is often chosen to be the geometric mean of stop masses to minimize the largest contribution to $\delta\lambda$ from stops. Once the spectrum of the MSSM particles is fixed, we can evaluate the Higgs self coupling using Eq. (1.10), calculate its running according to the RGE, and obtain the prediction for the Higgs mass through Eq. (1.9).

In Fig. 2, we show the contour plot of the Higgs mass m_h in the $\tan\beta$ vs. M_S plane. We assume the universal mass M_S for all the SUSY particles. Under this assumption, the largest contribution to the threshold correction $\delta\lambda$ from stops can be written as

$$\delta\lambda \simeq \frac{9y_t^2(M_S)}{16\pi^2} \tilde{X}_t \left[1 - \frac{\tilde{X}_t}{12} \right], \quad (1.12)$$

$$\tilde{X}_t \equiv \frac{(A_t - \mu \cot \beta)^2}{M_S^2}, \quad (1.13)$$

with $y_t \equiv Y_u^{33}$ and $A_t \equiv A_u^{33}$. It is obvious from Eq. (1.12) that, for a moderate value of $\tilde{X}_t \lesssim \mathcal{O}(1)$, $\tilde{X}_t = 0$ ($\tilde{X}_t = 6$) corresponds to the case with minimum (maximum) threshold correction, often called as the minimal (maximal) stop mixing. ⁴⁴

The red (blue) lines in Fig. 2 denote from top to bottom the contours of $m_h = 131, 128, 125, 122, 119$ GeV for the minimal (maximal) stop mixing. Gray shade corresponds to the region where the central value of the observation $m_h = 125.10$ GeV can be explained. From the figure, we can see that the discovery of the Higgs with $m_h = 125.10$ GeV may indicate a somewhat heavy SUSY breaking scale $M_S \gtrsim 10$ TeV for the case with a small stop mixing or a small $\tan \beta$. Combined with the fact that there is still no sign of the superpartners at the collider experiment, this motivates us to consider a heavy SUSY scenario.

Light Higgsino and its relation to the naturalness

When we consider a heavy SUSY model in relation to the Higgs mass, there is another problem called the little hierarchy problem. This mentions the hierarchy between the electroweak scale and the heavy SUSY breaking scale and an accompanying fine-tuning. Although the degree of the required fine-tuning is several orders of magnitude smaller than that for the large hierarchy between the electroweak and Planck scales, it will be more acceptable if some mechanism relieves the fine-tuning. The problem can be summarized in the equation

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan \beta^2 - 1} - \mu^2, \quad (1.14)$$

where the right-handed side is the MSSM prediction for the Z -boson mass assuming the successful electroweak symmetry breaking. If some of the MSSM parameters m_{H_d} , m_{H_u} , μ are much larger than m_Z , there should be some amount of fine-tuning to satisfy the equation.

There is a measure of the fine-tuning in this sence, proposed in [?, ?]:

$$\Delta_{a_i} \equiv \frac{a_i}{m_Z^2} \frac{\partial m_Z^2}{\partial a_i}, \quad (1.15)$$

where a_i is a MSSM model parameter. In order for the model to be “natural”, we require $|\Delta_{a_i}| < \Delta$ for any a_i with a typical choice of $\Delta \sim \mathcal{O}(10-100)$. (♣ Typical?? ♣) Since m_Z is sensitive to the Higgsino mass μ , this gives an upper bound on the “natural” choice of the Higgsino mass

$$\mu^2 < \frac{m_Z^2}{2} \Delta, \quad (1.16)$$

⁴⁴Eq. (1.12) shows that $\delta\lambda < 0$ for $\tilde{X}_t > 12$, resulting in the prediction of a lighter Higgs mass than the minimal stop mixing case. However, the parameter space with $\tilde{X}_t \gtrsim 6$ is severely constrained by the requirement of the stability of the electroweak vacuum (♣ Reference!! ♣) and is not considered here.

predicting the (sub-)TeV scale Higgsino. As we will see in Sec. ??, (**♣ Caution!! ♣**) this light Higgsino is also fascinating as a dark matter candidate.

Even when the SUSY breaking scale is much higher than the electroweak scale, it is not strange for Higgsino to be around the electroweak scale since it is protected by an R-symmetry and a Peccei Quinn symmetry. (**♣ Reference!! ♣**) This symmetry protection is also important for a solution to the so-called “ μ -problem” [?], where the large hierarchy between the SUSY conserving parameter μ and the cut-off scale of the MSSM itself is discussed. When we consider the low energy effective field theory in which SUSY is broken and all the squarks and sleptons are decoupled, a unique linear combination of the R-symmetry and the Peccei Quinn symmetry is enhanced only if both gauginos and Higgsinos are massless. This fact leads to the framework of the split SUSY [?], in which there is a hierarchy between the masses of Higgsinos / gauginos and the other SUSY particles. In this framework, the phenomenology highly depends on the ordering and hierarchy of Higgsino and gaugino masses. In particular, the phenomenology of Higgsino will be summarized in Sec. ??? (**♣ Caution!! ♣**) for the case when gauginos are heavier than Higgsino.

Finally, the naturalness requirement discussed above also impose an upper bound on other parameters, in particular on $m_{H_u}^2$ for $\tan^2 \beta \gg 1$. The small value of $m_{H_u}^2$ can be realized by the focus point mechanism [?, ?, ?], where the choice of the SM parameters in our universe, in particular that of y_t , allows $m_{H_u}^2$ at the low energy scale to be insensitive to its boundary condition at the high energy scale.

(**♣ Dirac nature of Higgsino? ♣**)

Light Wino in the anomaly mediated SUSY breaking model

Among many mediation models compatible with the heavy SUSY, the anomaly mediated SUSY breaking [?, ?] or the pure gravity mediation scenario [?, ?, ?] is of particular interest since it naturally predicts the existence of WIMPs (basically Winos denoted as \tilde{W}) in the TeV range. In this scenario, the SUSY breaking effect is directly mediated to the quark and lepton supermultiplets, and they obtain masses comparable to the scale of the SUSY breaking, which is approximated by the gravitino mass $m_{3/2}$. Higgsino is also consider to be heavy contrary to the model described above. Actually, It is easy to realize the hidden sector dynamics that generates the μ -term of $\mathcal{O}(m_{3/2})$. On the other hand, the superpartners of gauge bosons, gauginos, is affected only through a one-loop diagram, which is related to the conformal anomaly. As a result, gaugino mass parameters in Eq. (1.7) are one-loop suppressed compared with other mass parameters as

$$M_i(M_S) = - \left. \frac{\beta_i}{2g_i^2} \right|_{M_S} m_{3/2}, \quad (1.17)$$

where $i = 1, 2, 3$ is a gauge index and β_i denote the beta functions of gauge coupling constants. (**♣ All-order expression or not? M_S ? M_{GUT} ? ♣**) At the one-loop level, this gives

$$M_1(M_S) = \frac{11g_1^2(M_S)}{16\pi^2}m_{3/2}, \quad (1.18)$$

$$M_2(M_S) = \frac{g_2^2(M_S)}{16\pi^2}m_{3/2}, \quad (1.19)$$

$$M_3(M_S) = -\frac{3g_3^2(M_S)}{16\pi^2}m_{3/2}. \quad (1.20)$$

(♣ Normalization of g_1 ? ♣)

Since Higgsinos are assumed to have a mass comparable to $m_{3/2} \sim M_S$, they decouple from the effective theory below the scale M_S . To take account of the correction to the gaugino masses from the Higgs-Higgsino loop, one has to include the threshold correction at M_S

$$\Delta M_1 = \frac{g_1^2(M_S)}{16\pi^2}L, \quad \Delta M_2 = \frac{g_2^2(M_S)}{16\pi^2}L, \quad (1.21)$$

with

$$L \equiv \mu \sin 2\beta \frac{m_A^2}{|\mu|^2 - m_A^2} \ln \frac{|\mu|^2}{m_A^2}, \quad (1.22)$$

where m_A is the mass of the heavy CP-odd Higgs which is given by a linear combination of H_u^0 and H_d^0 .

Below M_S , gaugino mass parameters further run towards the gaugino mass scale $M_{\tilde{G}}$, where the physical gaugino masses are determined. Note that the bino and wino masses are roughly given by $|M_1(M_{\tilde{G}})|$ and $M_2(M_{\tilde{G}})$, while the gluino pole mass $m_{\tilde{g}}$ includes a sizable effect from the threshold correction as [?]

$$m_{\tilde{g}} = |M_3(M_{\tilde{G}})| \left[1 + \frac{g_3^2}{16\pi^2} \left(12 + 9 \ln \frac{M_{\tilde{G}}^2}{|M_3|^2} \right) \right]. \quad (1.23)$$

The gaugino scale is often defined through $M_3(M_{\tilde{G}}) = M_{\tilde{G}}$ to minimize the size of the logarithmic term in Eq. (1.23).

In Fig. 3, we show the dependence of gaugino masses on $m_{3/2}$ and L . In the left panel, we take $\tan \beta = 2.5$ and $L = 0$, and the $m_{3/2}$ dependence is shown. Blue, green, and red lines denote the masses of Bino, Wino, and gluino, respectively. We can see that, throughout the parameter region used here, Wino becomes the lightest gaugino, or the LSP that can be a dark matter candidate. In this choice of parameters, $m_{3/2} = 10^6 \text{ GeV}$ roughly corresponds to the observed value of the Higgs mass $m_h \sim 125 \text{ GeV}$, which at the same time realizes

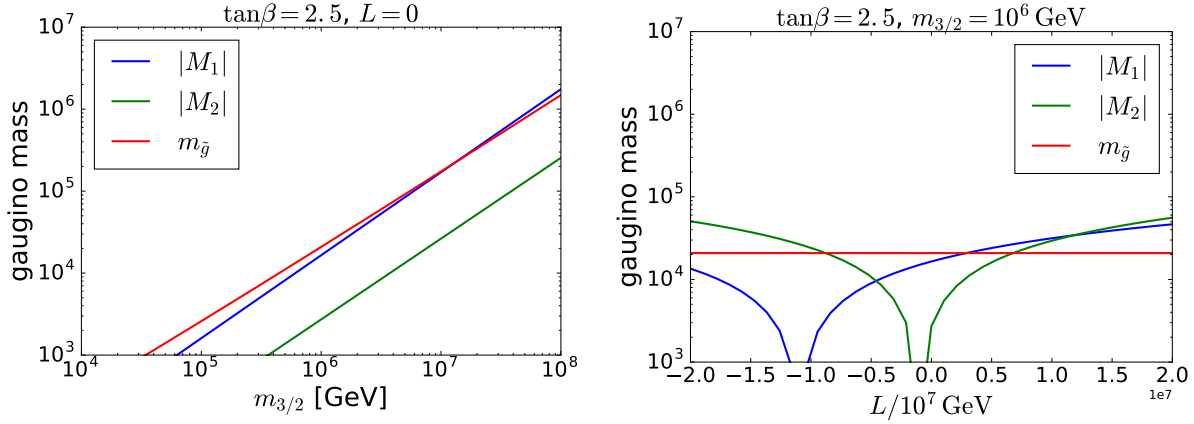


Figure 3: Gaugino masses as a function of $m_{3/2}$ with fixing $L = 0$ (left) and that of L with fixing $m_{3/2} = 10^6$ GeV (right). Blue, green, and red lines denote the masses of Bino, Wino, and gluino, respectively. $\tan \beta = 2.5$ is used in both figures.

the $\mathcal{O}(1)$ TeV mass for Wino. As we will see in Sec. ??, (**♣ Caution!! ♣**) the Wino dark matter in this mass range is well-motivated since it gives us a collect relic abundance of the dark matter.

In the right panel of Fig. 3, we also show the L dependence of gaugino masses for $\tan \beta = 2.5$ and $m_{3/2} = 10^6$ GeV. For simplicity, we neglect the relative phase of $m_{3/2}$ and L and only consider the relative sign of them. It can be seen that the hierarchy between gaugino masses is changed when a large value of $|L|$ is considered. However, we can safely say that when the threshold correction is sufficiently small, $|L| \lesssim \mathcal{O}(m_{3/2})$, Wino remains to be the LSP. In addition, dependence of m_h on L is negligibly small and m_h changes only $\mathcal{O}(0.1)$ GeV within the parameter choice of the right panel.

(**♣ Majorana nature of Wino? ♣**)

1.2 Minimal dark matter model

The minimal dark matter (MDM) [?, ?, ?] is another example model that contains a WIMP DM candidate. This model attempts to explain the existence of the stable DM by extending the SM as simply as possible. More specifically, we just assume the same gauge groups as the SM and add only one $SU(2)_L$ n -plet with/without $U(1)_Y$ hypercharge Y in the model. ⁵ Y is chosen such that one component of the multiplet, after the electroweak symmetry breaking, has no $U(1)_{\text{EM}}$ charge, and thus can be a DM candidate. This condition leaves

⁵This new particle, when it is a fermion, does not contribute to the $SU(2)_L^2 U(1)_Y$, $U(1)_Y^3$, nor $U(1)_Y \text{grav}^2$ anomalies if $Y = 0$. When $Y \neq 0$, we always consider a vector-like pair of Weyl fermions, which as a whole consists of a Dirac fermion and cancels the contributions to the gauge anomalies.

only n discrete choices of Y for an $SU(2)_L$ n -plet.

In the $SU(2)_L$ limit, masses of all the components in the multiplet are the same. Since $SU(2)_L$ symmetry is spontaneously broken, the mass difference among them is generated at one-loop level and a heavy component can decay into a lighter component. In order for the multiplet to explain the DM in the current universe, the $U(1)_{\text{EM}}$ neutral component should have the lowest mass. We will return to this point and check that this is the case in Sec. ??? **(♣ Caution!! ♣)**, related to the collider search for MDMs using the disappearing track signal.

In some sence, the MDM is just a generalization of the WIMP described above. Actually, if we choose the set of $SU(2)_L$ and $U(1)_Y$ charges as $(n, Y) = (2, \pm 1/2)$ and $(3, 0)$, they correspond to the Higgsino and Wino, respectively. However, for these choices, the stability of the $U(1)_{\text{EM}}$ neutral component is not automatically ensured, and some extra symmetry (in this case the R-parity) is needed for the DM to survive until now. The important point of the new framework MDM is that, when we use large $n \geq 5$, there are examples of multiplets that automatically contain a sufficiently long-lived DM candidate.

The stability of such multiplets can be understood through a simple group theoretical argument. To write down the effective operator that describes the decay of a n -plet field to SM particles, we have to make a n -plet representation out of several SM fields. However, since the largest $SU(2)_L$ representation in the SM is doublet, we need at least $n - 1$ SM fields in the operator. The operator made out of this large number of fields should be suppressed by a power of the cutoff scale Λ , at least by Λ^{4-n} (Λ^{3-n}) for a scalar (fermion) MDM, and results in a small decay rate. Since the well-motivated DM mass is of $\mathcal{O}(\text{TeV})$ as we will see in Sec. ?? **(♣ Caution!! ♣)**, the resulting lifetime of the DM candidate is estimated as $\tau \sim \Lambda^{-2p}(\text{TeV})^{2p-1}$ for an operator with a suppression factor Λ^{-p} . By demanding τ to be larger than the age of the universe under the assumption for the cut off scale $\Lambda < M_{\text{pl}}$, we can conclude that the operator of the DM decay should have a dimension larger than five. Then, we recast this condition to that for n and obtain

$$n \geq \begin{cases} 6 & \text{scalar MDM,} \\ 5 & \text{fermion MDM.} \end{cases} \quad (1.1)$$

Since we consider large $SU(2)_L$ multiplets, the RGE running of the $SU(2)_L$ gauge structure constant α_2 above the MDM mass is drastically modified. At the one-loop level, we have (see for example [?])

$$\alpha_2^{-1}(Q) = \alpha_2^{-1}(M_{\text{MDM}}) - \frac{b_2}{2\pi} \ln \frac{Q}{M_{\text{MDM}}}, \quad (1.2)$$

$$b_2 \equiv -\frac{19}{6} + c \frac{n^3 - n}{18}, \quad (1.3)$$

$SU(2)_L$	Quantum numbers		DM stability	Not excluded by direct detection	Examples
	$U(1)_Y$	Spin			
2	1/2	Scalar			
2	1/2	Fermion			Higgsino
3	0	Scalar		✓	[?]
3	0	Fermion		✓	Wino
3	1	Scalar/Fermion			[?]
4	1/2	Scalar/Fermion			[?]
4	3/2	Scalar/Fermion			[?]
5	0	Scalar		✓	[?]
5	0	Fermion	✓	✓	[?, ?, ?]
5	1	Scalar			
5	1	Fermion	✓		
5	2	Scalar			
5	2	Fermion	✓		
6	1/2, 3/2, 5/2	Scalar	✓		
7	0	Scalar	✓	✓	[?, ?, ?]
7	1, 2, 3	Scalar	✓		

Table 4: Table of the MDM properties. In the first three columns, we show the quantum numbers of our choice. In the next two columns, the DM stability (the checkmark means "stable") and its status under the DM direct detection experiment (the checkmark means it is still alive) are shown. The last column is devoted to the examples in the literature.

with $c = 1$ (1/4) for a Majorana/Weyl fermion (real scalar). Note that the first and second term of Eq. (1.3) represent the contributions from SM particles and the MDM, respectively. Then, assuming the perturbativity of $SU(2)_L$ gauge coupling upto M_{pl} , this relationship puts an upper bound on the choice of n . According to the strong dependence on n of b_2 , a strong bound is obtained,

$$n \leq \begin{cases} 8 & \text{real scalar MDM,} \\ 5 & \text{Majorana fermion MDM.} \end{cases} \quad (1.4)$$

In Table 4, we summarize the properties of MDMs for several different choices of (n, Y) . Throughout the table, the checkmark represents a suitable property as a DM candidate.

WIMP DM candidate	Quantum numbers			Masses	
	$SU(2)_L$	$U(1)_Y$	Spin	m_χ/TeV	$\Delta m_\chi/\text{MeV}$
Higgsino	2	1/2	Dirac fermion	1.1	341
Wino	3	0	Majorana fermion	2.9	166
5-plet scalar	5	0	real scalar	9.4	166
5-plet fermion	5	0	Majorana fermion	10	166

Table 5: Table of properties of WIMPs discussed in this thesis. In the “Quantum numbers” block, the $SU(2)_L$ and $U(1)_Y$ charges and spin nature are shown. In the “Masses” block, the proper mass of the thermally produced DM m_χ and mass difference between the neutral and charged components of the multiplet Δm_χ are shown. See Sec. ?? (♣ **Caution!!** ♣) for the descriptions and implications of m_χ and Sec. ?? (♣ **Caution!!** ♣) for those of Δm_χ .

In the first three columns, we show the quantum numbers of our choice, namely the set of (n, y) and spin. In the next column, we show the condition for the DM stability, namely, whether all the DM decay operators have dimensions larger than five or not. The checkmarks correspond to the automatically stable DM candidates. The next column shows if the DM direct detection experiment has already excluded these DM candidates or not. Since the non-zero value of Y usually leads to the large cross section as will be discussed in Sec. ??? (♣ **Caution!!** ♣), only $Y = 0$ candidates are associated with checkmarks. (♣ **Correct? in particular for 5- and 7-plets? References??** ♣) However, note that these properties may be changed due to a small modification to the model, such as imposition of an extra symmetry or the mixing between other new physics particles. The final column shows examples of the viable DM candidates analyzed in the literature.

From the table, we can see that there are two fascinating targets, a 5-plet fermion and a 7-plet scalar both of which has $Y = 0$. Among them, we neglect the latter possibility because it has been pointed out [?, ?] that a dimension five operator combined with a loop consisted of the TeV scale 7-plet induces a sizable decay rate for the $U(1)_{\text{EM}}$ neutral component. Instead, we will take a 5-plet scalar with $Y = 0$ just as a working example, assuming that its stability is ensured by some other mechanism.

1.3 Summary

In Table 5, we summarize the properties of WIMPs discussed in this thesis. In the first block named “Quantum numbers”, we show the $SU(2)_L$ electroweak charge, $U(1)_Y$ hypercharge, and spin nature. In the second block named “Masses”, two types of masses are shown. m_χ is the required masses to explain the DM relic abundance without non-thermal production

(see Sec. ?? **(♣ Caution!! ♣)** for the detail). Δm_χ is the mass difference between the electromagnetically neutral and charged components of the multiplet, as discussed in Sec. ?? **(♣ Caution!! ♣)**. Values are taken from [?, ?, ?, ?, ?, ?].