Homework #2: Due Tuesday, Feb. 11

Problem 2.1
Compute the Fourier coefficients for the function f(t) = t (0≤ t ≤ 1).

2.1

$f(t)=t \quad 0\le t\le 1$

$T=1, \quad f=1/T=1$

$$\boxed{\begin{array}{l}\text{Fourier coefficients}\\ c=1,\ a_n=\frac{-1}{\pi n},\ b_n=0\end{array}}$$

$$c=\frac{2}{T}\int_0^T g(t)\,dt=\frac{2}{1}\int_0^1 t\,dt=2\left(\tfrac{1}{2}t^2\right)\Big|_0^1=1-0=1$$

$$a_n=\frac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt=\frac{2}{1}\int_0^1 t\sin(2\pi nt)\,dt$$

Integration by parts

$\int u\,dv = uv-\int v\,du$

$u=t \quad du=dt$

$dv=\sin(2\pi nt)dt \quad v=\frac{-1}{2\pi n}\cos(2\pi nt)+c$

$$\frac{2}{1}\int_0^1 t\sin(2\pi nt)\,dt=2\left(\frac{-t}{2\pi n}\cos(2\pi nt)\Big|_0^1-\int_0^1\frac{-1}{2\pi n}\cos(2\pi nt)\,dt\right)$$

$$=2\left(\frac{-1}{2\pi n}-0\right)+2\left(\frac{1}{4\pi^2n^2}\sin(2\pi n)\Big|_0^1\right)$$

$$=\frac{-1}{\pi n}+2(0-0)=\boxed{-\frac{1}{\pi n}=a_n}$$

$$b_n=\frac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt=\frac{2}{1}\int_0^1 t\cos(2\pi nt)\,dt$$

Again, apply integration by parts.

$b_n=0$

Problem 2.4
If a binary signal is sent over a 3 kHz channel whose signal-to-noise ratio is 20dB, what is the maximum achievable data rate?

A signal-to-noise ratio of 20 dB means *S/N* = 100. Since $\log_2 101$ is about 6.658, the Shannon limit is about 19.975 kbps. The Nyquist limit is 6 kbps. The bottleneck is therefore the Nyquist limit, giving a maximum channel capacity of 6 kbps.

Problem 2.22
A modem constellation diagram similar to Fig. 2-25 has data points at the following coordinates: (1,1), (1,-1),(-1,1),(-1,-1). How many bps can a modem with these parameters achieve at 1200 baud?

There are four legal values per baud, so the bit rate is twice the baud rate. At 1200 baud, the data rate is 2400 bps.

Problem 2.30
What is the percent overhead on a T1 carrier; that is, what percent of the 1.544 Mbps are not delivered to the end user?

The end users get $7 \cdot 24 = 168$ of the 193 bits in a frame. The overhead is therefore 25/193 = 13%. There are a couple of other answers assuming different modes of operation.

Problem 2.41
Three packet-switching networks each contain n nodes. The first network has a star topology with a central switch, the second is a (bidirectional) ring, and the third is fully interconnected, with a wire from every node to every other node. What are the best-, average-, and worst-case transmission paths in hops?

The three networks have the following properties:
star: best case = 2, average case = 2, worst case = 2
ring: best case = 1, average case = $n/4$, worst case = $n/2$
full interconnect: best case = 1, average case = 1, worst case = 1

Problem 2.42
Compare the delay in sending an x-bit message over a k-hop path in a circuit-switched network and in a (lightly loaded) packet-switched network. The circuit setup time is s sec, the propagation delay is d sec per hop, the packet size is p bits, and the data rate is b bps. Under what conditions does the packet network have a lower delay?

With circuit switching, at $t = s$ the circuit is set up; at $t = s + x/b$ the last bit is sent; at $t = s + x/b + kd$ the message arrives. With packet switching, the last bit is sent at $t = x/b$. To get to the final destination, the last packet must be retransmitted $k - 1$ times by intermediate routers, each retransmission taking p/b sec, so the total delay is $x/b + (k - 1)p/b + kd$. Packet switching is faster if $s > (k - 1)p/b$.