EE 2372 Test 2

9 problems, 100 points.

November 24, 1998

SOLUTIONS

NAME

Closed book, closed notes, no calculators. Scratch paper will be provided, so do not use any of your own.

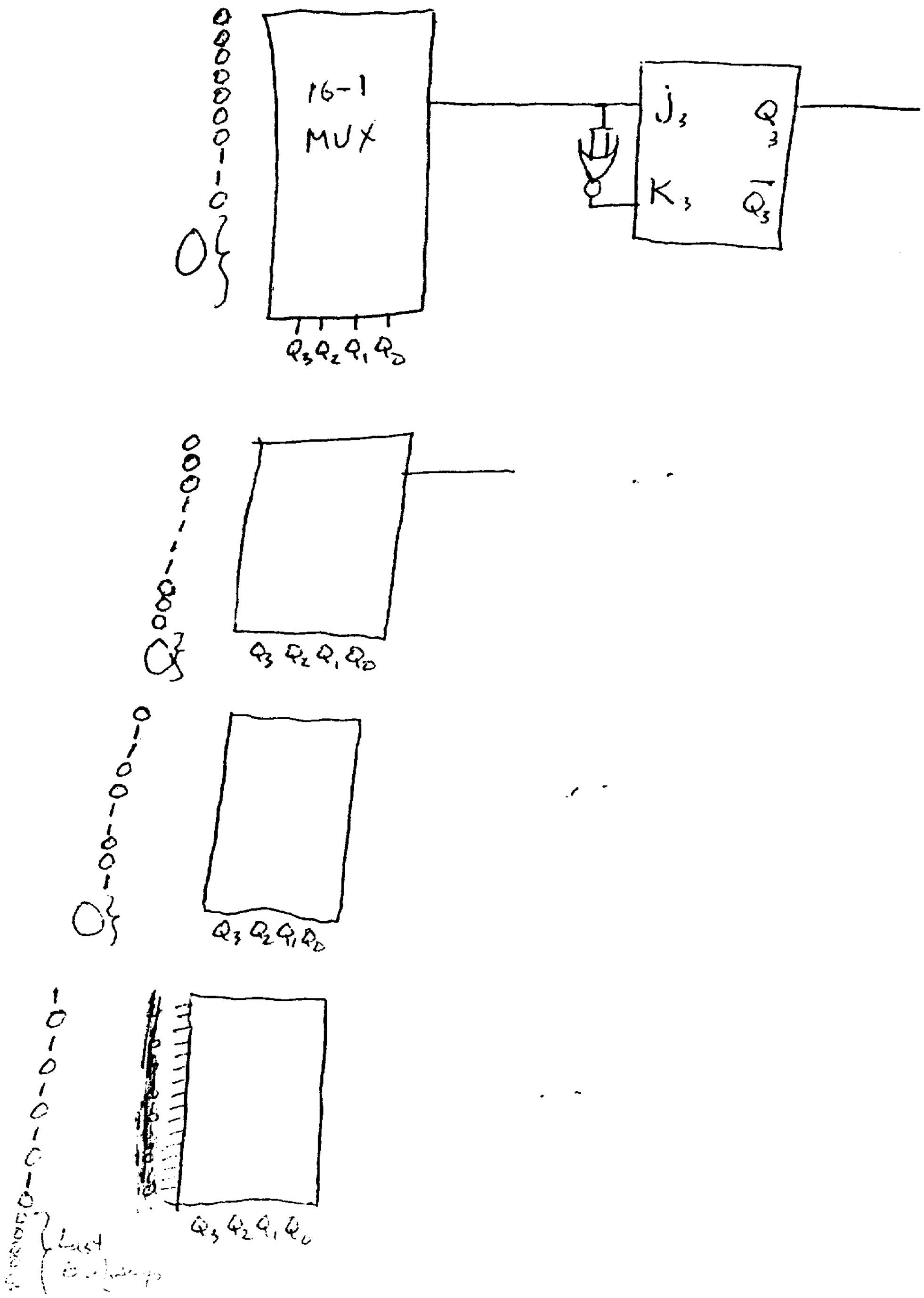
You are permitted pens or pencils, erasers, and a (non-calculator) watch. All other items are to be placed underneath your desk.

Please read the entire exam before beginning, and note point values. Some problems are more worthwhile than others.

Do not turn this page until instructed to do so.

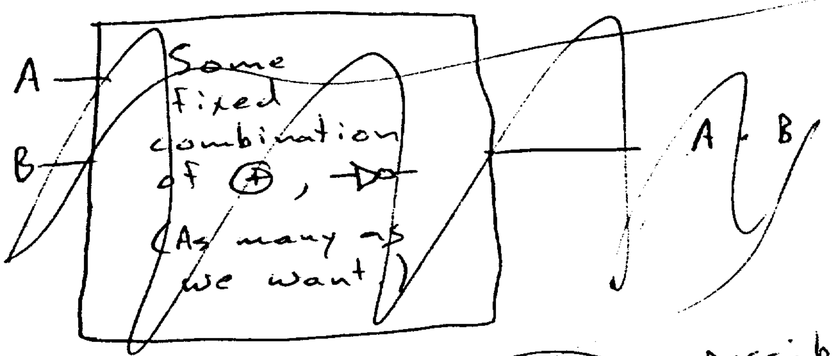
Good luck!

1. Design a synchronous decade (0–9) counter using four JK flip-flops, four NOR gates, and four 16–1 multiplexers. Any erroneous (glitch) state must be reset to zero on the next clock cycle. The states are $\underset{(MSB)}{Q_3} Q_2 Q_1 \underset{(LSB)}{Q_0}$.



2. No, we can't make any gate we want, using only XOR and inverters.

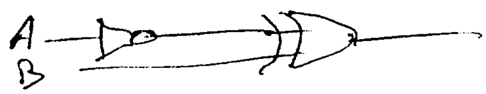
~~If we could, we could make an AND gate. It would look like this:~~



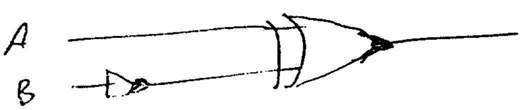
Erase in Photoshop

~~We can see this is impossible by considering the case of A AND A. Of course A·A = A. But consider what can be done with XOR and inverters~~

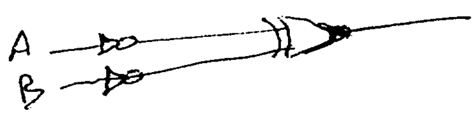
We have these possibilities



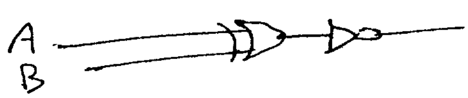
$\bar{A} \oplus B = AB + \bar{A}\bar{B}$



$A \oplus \bar{B} = AB + \bar{A}\bar{B}$



$\bar{A} \oplus \bar{B} = \bar{A}B + A\bar{B}$



$\overline{A \oplus B} = AB + \bar{A}\bar{B}$

These all reduce to $\overline{\oplus}$ or $\oplus$ and we're stuck in a closed set of possibilities.

3.

J K	Q^t
00	Q^t
01	0 reset
10	1 set
11	toggle

Q^t Q^{t+1}	JK
0 0	0X
0 1	1X
1 0	X1
1 1	X0

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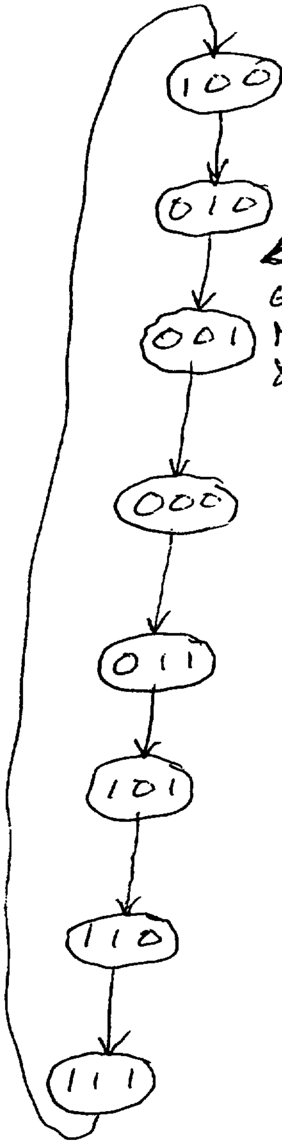
solve backwards to get this

cur $Q_2 Q_1 Q_0$	next $Q_2 Q_1 Q_0$	$J_2 K_2$	$J_1 K_1$	$J_0 K_0$
000	011	0X	1X	$Q_0 \bar{Q}_0$
001	000	0X	0X	
010	001	0X	X1	
011	101	1X	X1	
100	010	X1	1X	
101	110	X0	1X	
110	111	X0	X0	
111	100	X0	X1	

Get Moore diagram



Like a twisted ring counter with transition states (000, 111)

$J_2 = Q_1 Q_0$

Q_2 \ Q_1Q_0	00	01	11	10
0	0	0	1	0
1	X	X	X	X

$K_2 = \bar{Q}_1 \bar{Q}_0$

	X	X	X	X
	1	0	0	0

$J_1 = \bar{Q}_0 + Q_2$

1	0	X	X
1	1	X	X

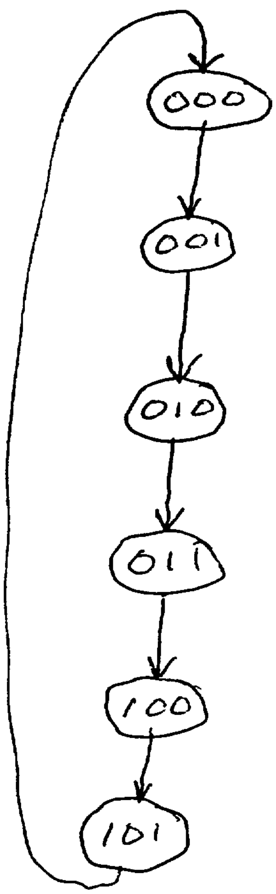
$K_1 = \bar{Q}_2 + Q_0$

X	X	1	1
X	X	1	0

$J_0 = \bar{Q}_2 \bar{Q}_0 + \bar{Q}_2 Q_1 + Q_1 \bar{Q}_0$

Q_2 \ Q_1Q_0	00	01	11	10
0	1	0	1	1
1	0	0	0	1

4. Design a 3-bit synchronous Modulo-6 counter, using D flip flops: D_2 D_1 D_0.
(It counts 0-5 and starts over.)
Use a Moore diagram



Q_2	Q_1	Q_0	D_2	D_1	D_0
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	X	X	X
1	1	1	X	X	X

$D_2 = Q_1 Q_0 + Q_2 \overline{Q_0}$

D_2

Q_2 \ Q_1Q_0	00	01	11	10
0	0	0	1	0
1	1	0	X	X

$D_1 = \overline{Q_2}\,\overline{Q_1} Q_0 + Q_1 \overline{Q_0}$

D_1

Q_2 \ Q_1Q_0	00	01	11	10
0	0	1	0	1
1	0	0	x	x

$D_0 = \overline{Q_0}$

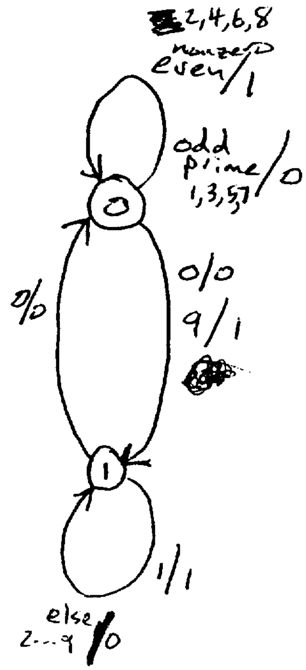
D_0

Q_2 \ Q_1Q_0	00	01	11	10
0	1	0	0	1
1	1	0	x	x

[redacted]

5.

Q	WXYZ	D	S	D(z)	S(z)
0	0000	x	x	d	d
	0001	x	x		
	0010	x	x	$z+\bar{z}d$	$\bar{z}d$
	0011	1	0		
	0100	0	0	0	z
	0101	0	1		
	0110	0	0	0	z
	0111	0	1		
	1000	0	0	0	z
	1001	0	1		
	1010	0	0	0	z
	1011	0	1		
	1100	1	1	$\bar{z}+zd$	$\bar{z}+zd$
	1101	x	x		
	1110	x	x	d	d
	1111	x	x		
1	0000	x	x	d	d
	0001	x	x		
	0010	x	x	$\bar{z}d$	$\bar{z}d$
	0011	0	0		
	0100	1	1	$z+\bar{z}$	$\bar{z}$
	0101	1	0		
	0110	1	0	$z+\bar{z}$	0
	0111	1	0		
	1000	1	0	$z+\bar{z}$	0
	1001	1	0		
	1010	1	0	$z+\bar{z}$	0
	1011	1	0		
	1100	1	0	$\bar{z}+zd$	zd
	1101	x	x		
	1110	x	x	d	d
	1111	x	x		



D

QW \ XY	00	01	11	10
00	d	$z+\bar{z}d$	0	0
01	0	0	d	$\bar{z}+zd$
11	$z+\bar{z}$	$z+\bar{z}$	d	$\bar{z}+zd$
10	d	$\bar{z}d$	$z+\bar{z}$	$z+\bar{z}$

$\bar{Q}\bar{W}\bar{X}$, WX, QW, QX

$$D = \bar{Q}\bar{W}\bar{X} + WX + QW + QX$$
$$S = \bar{Q}XZ + \bar{Q}WX + QWZ + Q\bar{W}\bar{Z}$$

Minimal SOP, solved separately

S

QW \ XY	00	01	11	10
00	d	$\bar{z}d$	z	z
01	z	z	d	$\bar{z}+zd$
11	0	0	d	zd
10	d	$\bar{z}d$	0	$\bar{z}$

$\bar{Q}XZ$, $\bar{Q}WX$, $Q\bar{W}\bar{Z}$

$$D = \bar{Q}\bar{W}\bar{X} + \bar{Q}WX + QW + QX$$

S = same as above

Minimal gate count, by using $\bar{Q}WX$ term from S twice, ie re-using it in D.

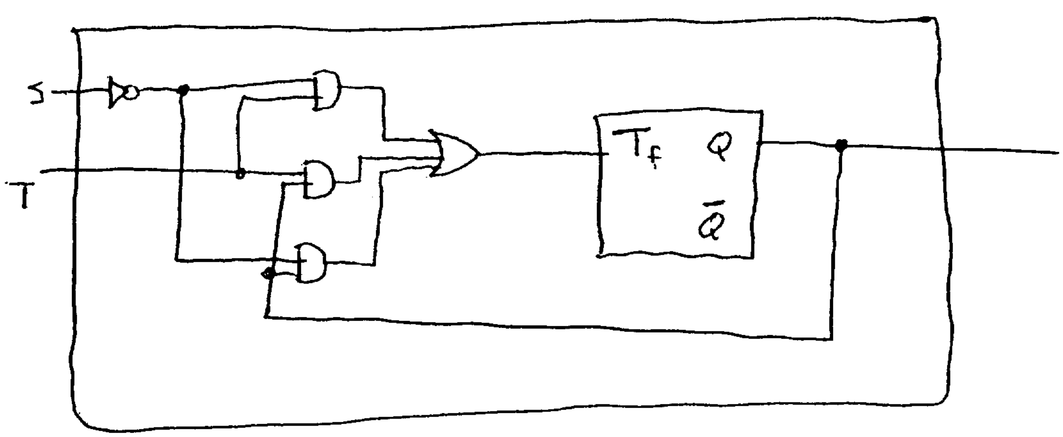
6. Make a Store-Toggle Flip flop with the following excitation table

S T	Q^{t+1}	
0 0	0	reset
0 1	$\overline{Q^t}$	Toggle
1 0	Q^t	Store
1 1	0	reset

Q^t S T	Q^{t+1}	T_f
0 0 0	0	0
0 0 1	1	1
0 1 0	0	0
0 1 1	0	0
1 0 0	0	1
1 0 1	0	1
1 1 0	1	0
1 1 1	0	1

T_f: Q^t \ ST	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$$T_f = \bar{S}T + Q^t\bar{S} + Q^tT$$



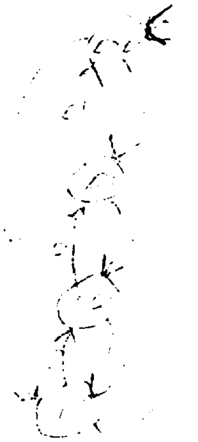
Q^t Q^{t+1}	S T
0 0	Anything except 01
0 1	0 1
1 0	Anything except 10
1 1	1 0

7. 2 bit up/down counter

Input x: $\begin{cases} 1 & \text{count up} \\ 0 & \text{count down} \end{cases}$

Call $Q_A = A$ $Q_B = B$ Use SR FF

State Diagram



A^t	B^t	x	A^{t+1}	B^{t+1}	S_A	R_A	S_B	R_B
0	0	0	1	1	1	0	1	0
0	0	1	0	1	0	x	1	0
0	1	0	0	0	0	x	0	1
0	1	1	1	0	1	0	0	1
1	0	0	0	1	0	1	1	0
1	0	1	1	1	x	0	1	0
1	1	0	1	0	x	0	0	1
1	1	1	0	0	0	1	0	1

Q^t	Q^{t+1}	S	R
0	0	0	x
0	1	1	0
1	0	0	1
1	1	x	0

$$S_A = \overline{A^t}\,\overline{B^t}\,\overline{x} + \overline{A^t}\,B^t x = \overline{A^t \odot B^t \odot x}$$

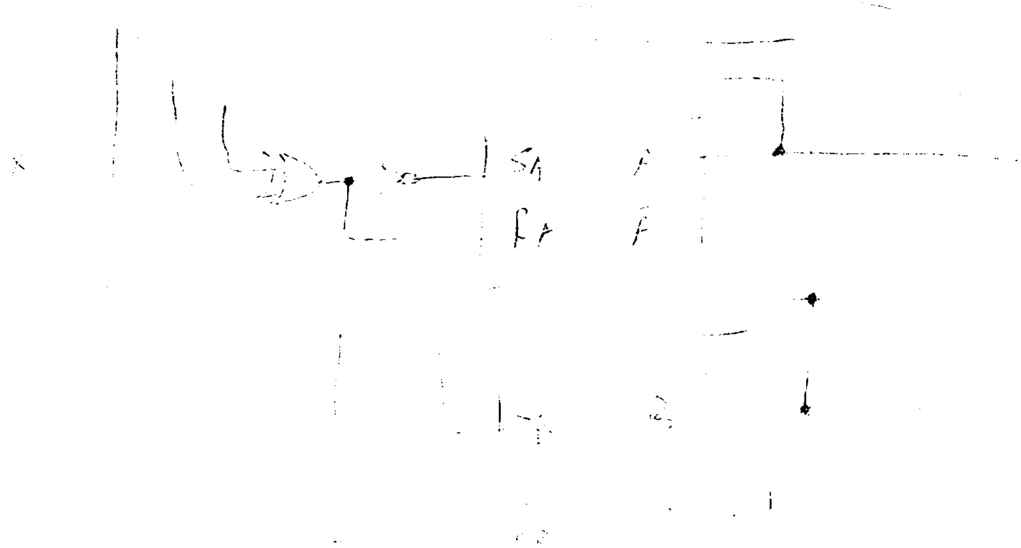
$A^t \backslash B^t x$	00	01	11	10
0	1	0	1	0
1	0	x	0	x

$$R_A = A^t\,\overline{B^t}\,\overline{x} + A^t B^t x = A^t \odot B^t \odot x$$

$A^t \backslash B^t x$	00	01	11	10
0	0	x	0	x
1	1	0	1	0

$S_B = \overline{B^t}$

$R_B = B^t$



8. Design a 4-bit ring counter with initialize and error correction. The only inputs are the clock, and I, the initialize signal.
The system outputs are:

1000
0100
0010
0001
1000
⋮

If the system enters any invalid state, it must return to 1000 on the next clock cycle. The signal to do this is called E.
Use D flip flops: $D_3\ D_2\ D_1\ D_0$.
Just write the equations for each D input — you don't need to draw them. Also write the equation for E.

E = You don't have an odd # of 1's + You have 3 1's

$$\therefore\ E = \overline{Q_4 \oplus Q_3 \oplus Q_2 \oplus Q_1} + Q_3 Q_2 Q_1 + Q_4 Q_2 Q_1 + Q_4 Q_3 Q_1 + Q_4 Q_3 Q_2$$

$$D_3 = I + E + Q_0$$

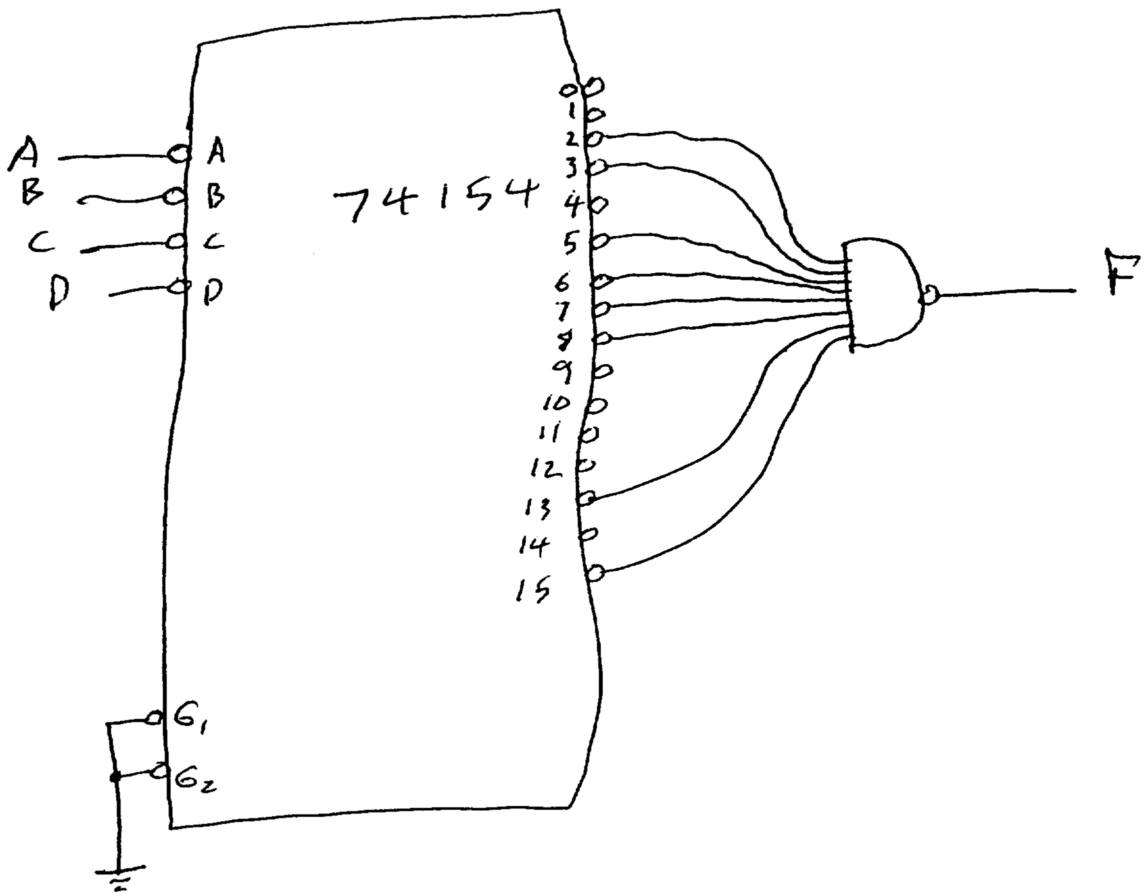
$$D_2 = \bar{I}\,\bar{E}\,Q_3$$

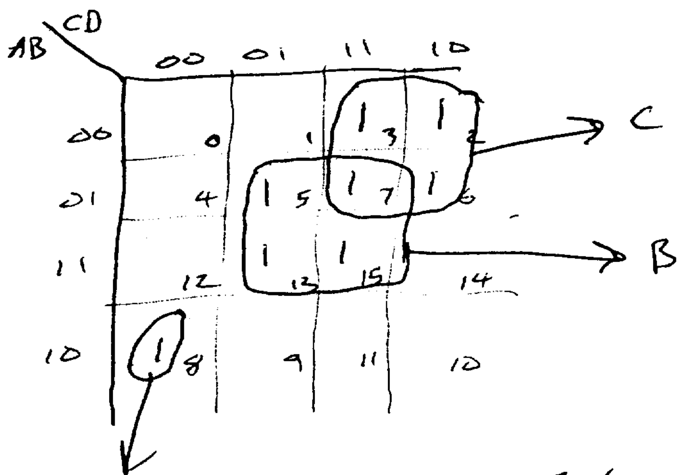
$$D_1 = \bar{I}\,\bar{E}\,Q_2$$

$$D_0 = \bar{I}\,\bar{E}\,Q_1$$

$$E = Q_3 Q_2 Q_1 Q_0 + Q_3 Q_2$$

9. $F = A\bar{B}\bar{C}\bar{D} + B + C$





$A\bar{B}\bar{C}\bar{D}$

$\therefore F = \sum(2, 3, 5, 6, 7, 8, 13, 15)$