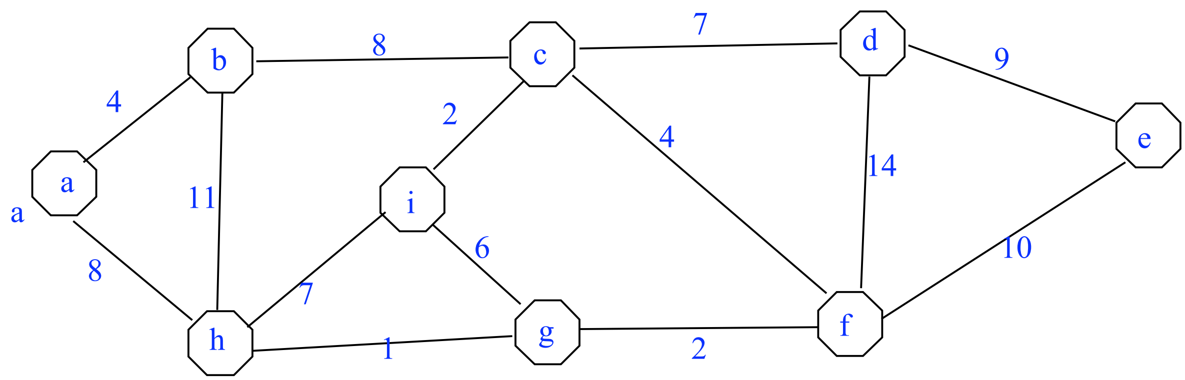
Graph Algorithms
Minimum Spanning trees.
Network flow Problem
Traveling Salesman problem

For connected undirected graph $G=(V,E)$, how does a spanning tree differ from the graph?

Define a Minimum Spannig tree T.

Define Weight of the tree T.

Use Kruskal's algorithm to create minimum spanning tree for the following graph?



Fill in the invariants in the algorithm

MST(G,T)
initialization
E_T= null, count=0;
Invariant:
For k =1, m
Invariant:
$V_k=\{v_k\}$;
For j = 1, |E| in sorted order -- O(|E|)
Invariant:
Let $e_j=(v_{j1},v_{j2})$;
If v_{j1} and v_{j2} are in different sets say V_x and V_y, O(lg|V|)
replace them with a single subtree with vertices $V_x \cup V_y$ and insert the edge e_j in the set E_T.
$E_T = E_T \cup \{e_j\}$,
count++;
If count=|V|-1 return E_T
Invariant:
either $e_j=(v_{j1},v_{j2}) \in E_T$ or e_j is not included because it would have formed a loop in E_T.

E_TReturn E_T
Invariant:

What is the Complexity of Kruskal's algorithm.

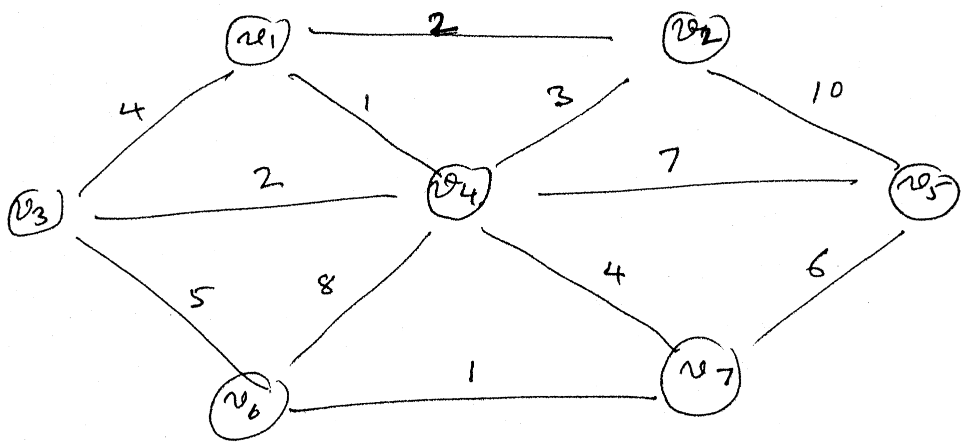
Dijkstra MST-PRIM(G,w,r) --
inititalization
For each u $\in$ V -- O(|V|)
    d(u) = ;// local distance from tree nodes
    p(u)= null// parent of node
//r -- root is arbitrarily selected vertex
d(r) = 0
E_T = null
V_T = null
H = V(G); min heap with values associated with vertices
Invariant:
For k=0 to |V|-1 O(|V|)
Invariant:
    u = Extract-min(H) -- O(lg |V|)
    if k>0, $E_T = E_T$ U {(p(u),u)}

    for each v in adj(u) --O(|E|)
        if v $\in$ H and w(u,v) < d(v)
            parent(v) = u
            d[v]=w(u,v)
            adjust heap H from v upward. -- O(lg |V|)

Invariant:

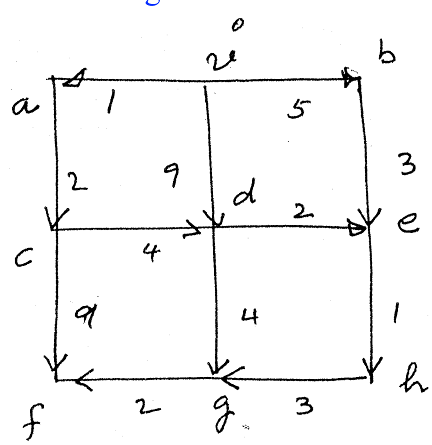
What is the complexity of Dijkstra's algorithm?

Is minimum spanning tree unique? Justify your answer with an example.
Is minimum spanning tree unique? Justify your answer.

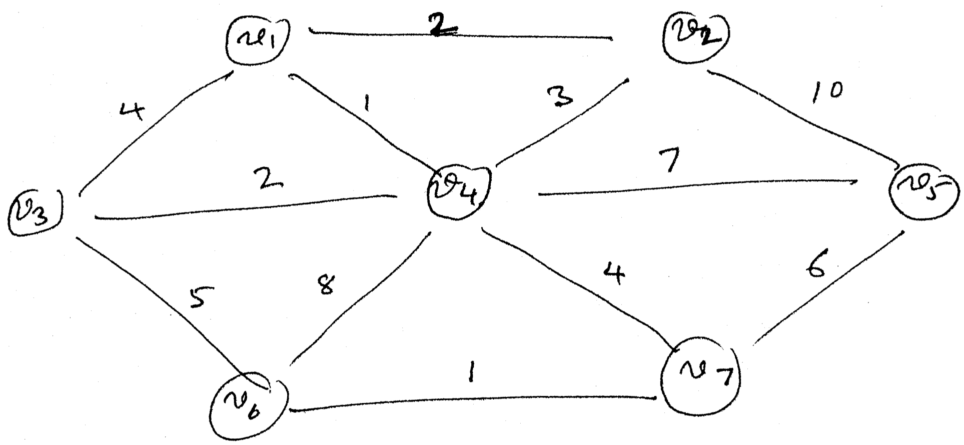


Define single source shortest paths problem.
Define all pairs shortest paths problem.

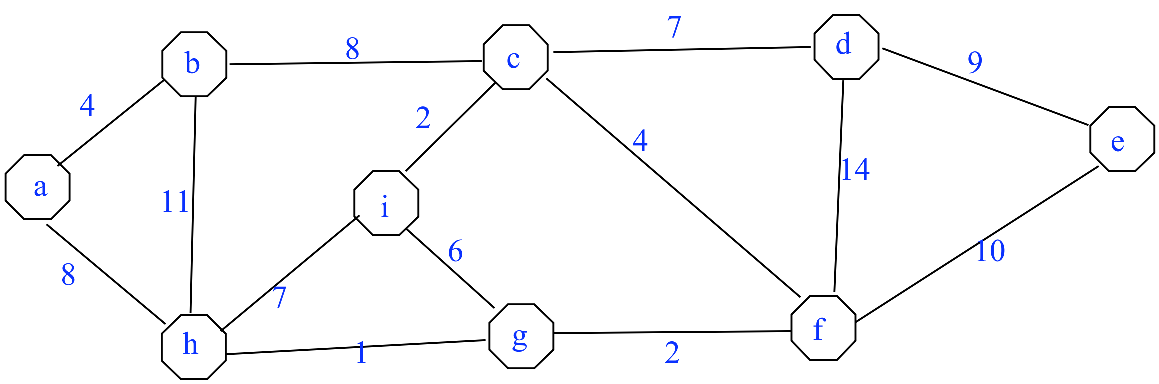
For the follwing tree create SSSPaths starting at vertex v.



Find SSSP starting at v_1



Find SSSP starting at vertex a



Give invriants for SSSP algorithm

Dijkstra MST-PRIM(G,w,r) -- Single Source Shortest Paths.
initialization
For each u $\in$ V
    d(u) = ;// absolute distance of tree nodes
    p(u)= null// parent of node
//r -- root is arbitrarily selected vertex
d(r) = 0
E_T = null
V_T = null
Invariant:
For k=0 to |V|-1 -- O(|V|)
Invariant:
    u = Extract-min(H) --O(lg |V|)
    if k>0, $E_T = E_T$ U {(p(u),u)}

    for each v in adj(u) O(|E|)

if v ∈ H and d(u) + w(u,v) < d(v)
parent(v) = u
d[v]=w(u,v)
adjust heap H from v upward. – O(lg |V|)

Invariant:

Post Condition:
Invariant:

Give the complexity of SSSP algorithm and justify your answer.

APSP: All Pairs Shortest Paths due to Dijkstra

What is the complexity of Dijkstra's APSP algorithm

Give Floyed's APSP_lengths algorithm with invariants.
Create A matrix of shortest paths between the vertices M[i,j]
Initially
M[i,i] = 0
M[i,j] = w(i,j) $i \neq j$
Invariant:
For k=1 to n
Invariant:
For i=1 to n,
For j=1 to n
M[i,j] = min (M[i,j] , M[i,k] + M[k,j])
Invariant:

Post condition:
Invariant:

What is the interpretation of above looping on index k?

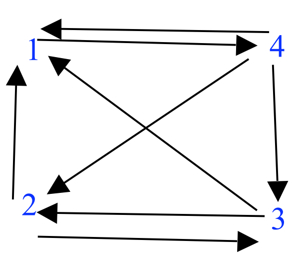
What is the complexity of Floyd's APSP algorithm

The above algorithm gives only the shortest distance. How do you determine the shortest path between v_i and v_j?

	1	2	3	4
1	0	x	x	1
2	3	0	1	x
3	5	1	0	x
4	8	1	4	0

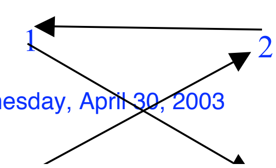
1 4

2 3



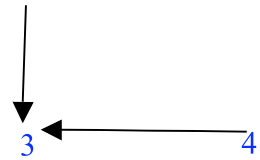
1 2 3 4

1 2



1	0	x	3	1
2	4	0	x	x
3	x	2	0	x
4	x	x	1	0

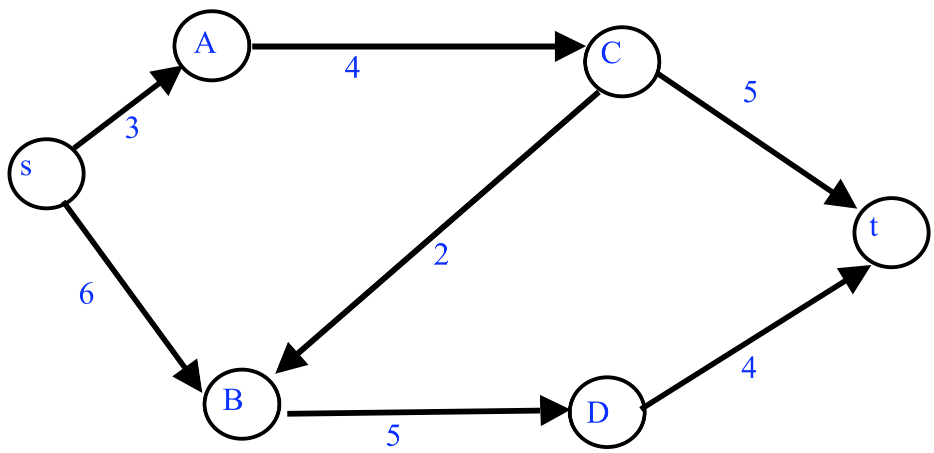
3 4

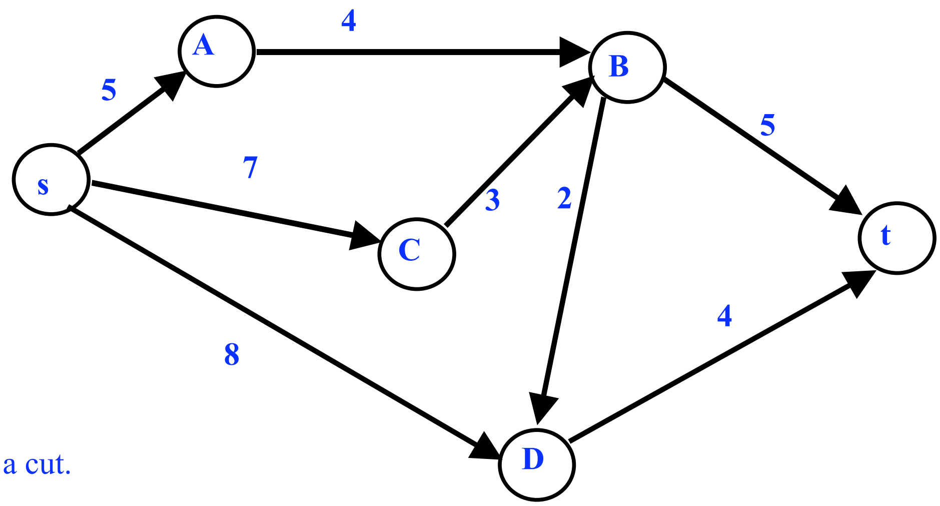


For the network flow function f and capacity function c define the properties:
feasibility condition
conservation condition.

Describe the MinCut MaxFlow problem.

Define an Augmented path.
Define slack in an edge.
Define slack in a path.
Determine the max flow and min cut for the following graphs

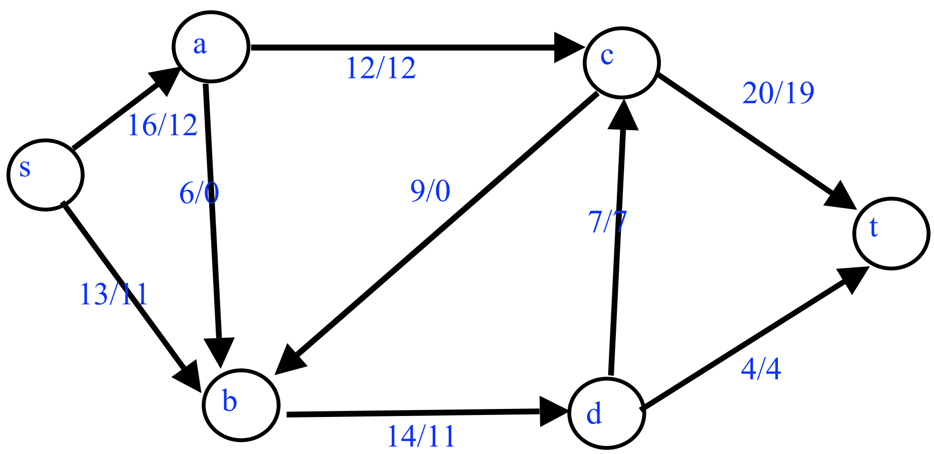




Define a cut.

Define MinCut.
Define MaxFlow.

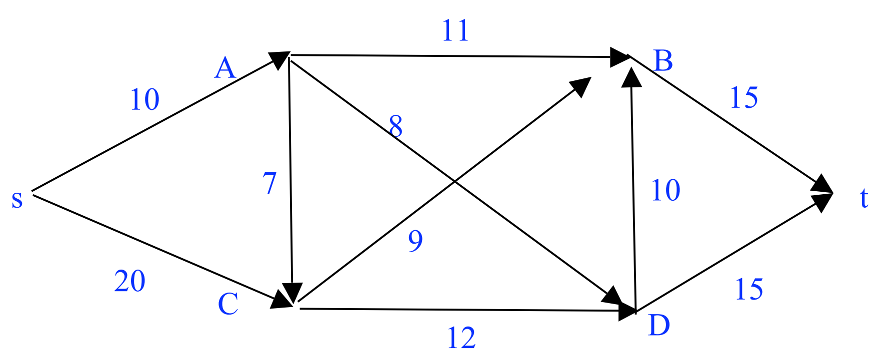
Find the min cuts. Using the following two graphs, justify that capacity of a forward edge is positive, capacity of backward is zero (implying only forward edges on paths from s towards t are used to find the capcity of the cut.)



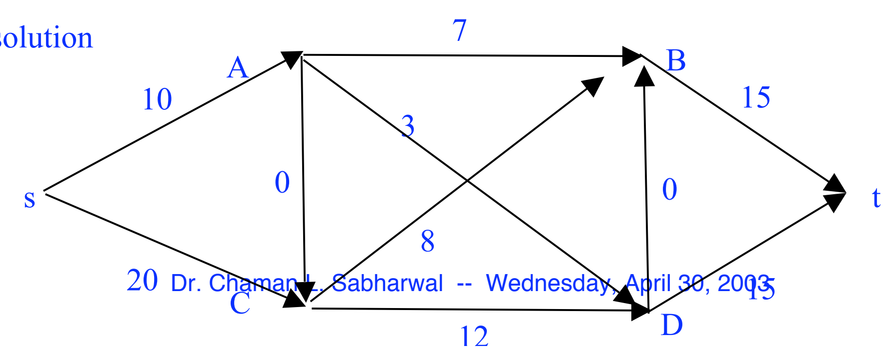
Describe Ford-Fulkerson algorithm.What is the complexity of this algorithm? Show by an example.

Describe EdmondKarp algorithm? What is the Complexity of EdmondKarp algorithm.

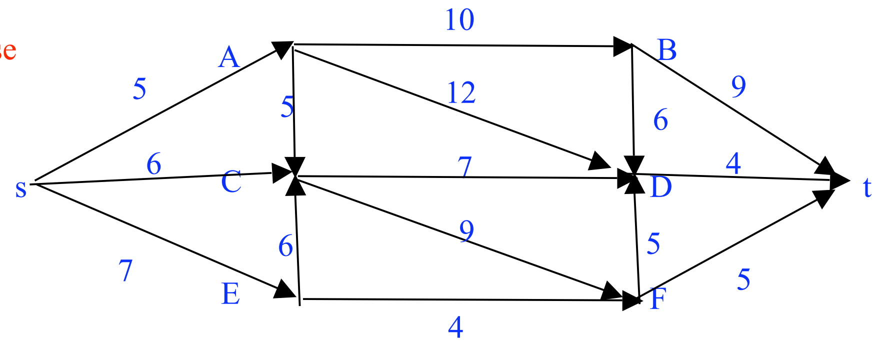
Find MinCutMaxFlow for the following graph



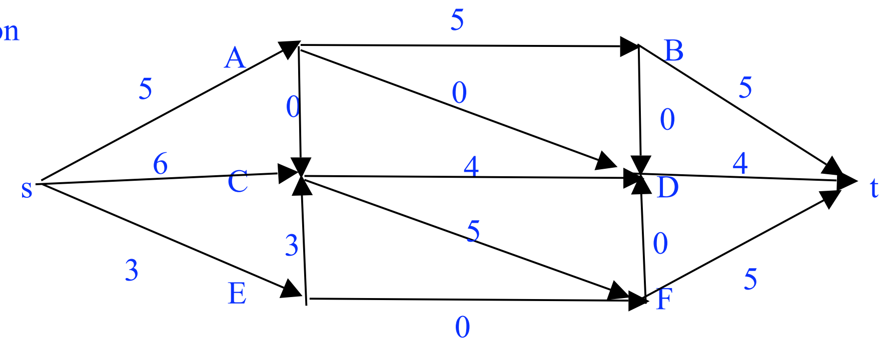
solution



Exercise



Solution



Define Hamilton cycle.

Give a criteria for determining the solution to **Traveling Salesman Problem**.

Find the solution to traveling salesman problems represented by the matrices of graphs as below

	1	2	3	4
1	x	2	13	6
2	21	x	45	12
3	11	12	x	34
4	16	17	18	x

	1	2	3	4
1	x	10	15	20
2	5	x	9	10
3	6	13	x	12
4	8	8	9	x

	1	2	3	4
1	x	1	5	9
2	4	x	11	6
3	8	12	x	2
4	10	3	7	x