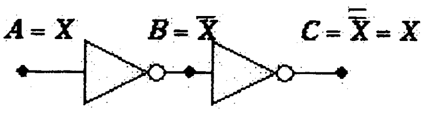
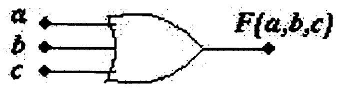
Chapter 2 Boolean Algebra and Logic Gates

[2.1]



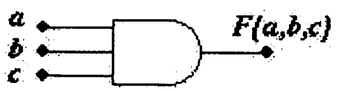
$A = X$	$B = \overline{X}$	$C = \overline{(\overline{X})} = X$
0	1	0
1	0	1

[2.2]



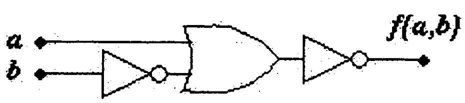
a	b	c	F(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

[2.3]



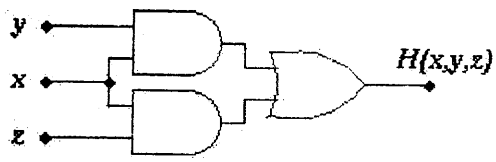
a	b	c	F(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

[2.4] $f = \overline{a + \overline{b}}$

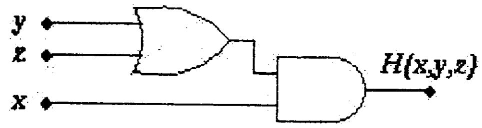


a	b	f
0	0	0
0	1	1
1	0	0
1	1	0

[2.5] $H(x,y,z) = x \bullet y + x \bullet z$

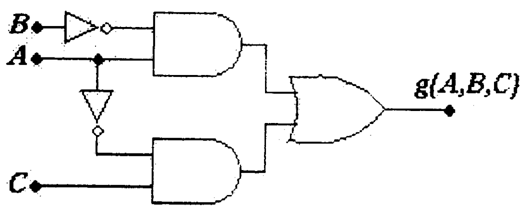


The following simplification can reduce the number of gates required for the same function:
$H(x,y,z) = x \bullet y + x \bullet z = x \bullet (y + z)$



[2.6] Same as 2.5 by replacing w(a,b,c) for H (x,y,z), b for x, and a and c for y and z respectively.

[2.7] $g(A,B,C) = A \bullet \overline{B} + C \bullet \overline{A}$



[2.8] $f = \overline{(\overline{a} + b)} = a \bullet \overline{b}$

[2.9] $g = \overline{(\overline{(x \bullet y)} + z)} = (x \bullet y) \bullet \overline{z} = x \bullet y \bullet \overline{z}$

[2.10]

$f_1 = \overline{(\overline{u} \bullet w)} = u + \overline{w}$

$f_2 = u + v$

[2.11]

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

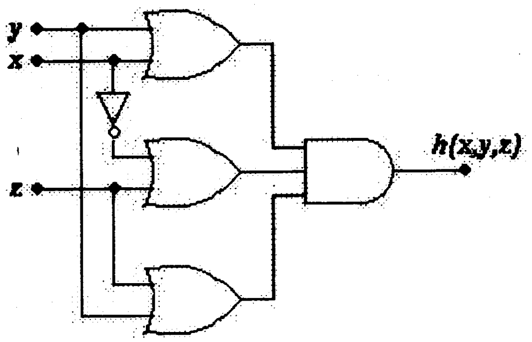
From the truth table above, F can be simplified as follow:

$$F = \overline{A} \bullet B + A \bullet \overline{B} + \overline{A} \bullet \overline{B}$$
$$= \overline{A} \bullet B + (A + \overline{A}) \bullet \overline{B}$$
$$= \overline{A} \bullet B + \overline{B}$$
$$= \overline{A} + \overline{B} = \overline{(A \bullet B)}$$

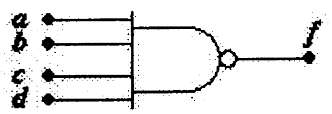
[2.12] $T(a,b,c) = a \bullet b + \bar{b} \bullet (a + c)$

a	b	c	T(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

[2.13] $h(x,y,z) = (x + y) \bullet (\bar{x} + z) \bullet (y + z)$

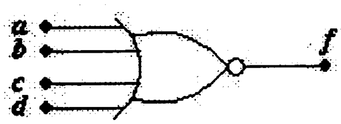


[2.14] A 4-input NAND gate is shown below:



If (any one of the inputs is 0)
then
$f = 1$
else
$f = 0$

[2.15] A 4-input NOR gate is shown below:



If (any one of the inputs is 1)
then
$f = 0$
else
$f = 1$

[2.16] If $(\phi = 1)$
then
$f = a \bullet b$
else if
$(\phi = 0)$
then
$f = 0$

The output, f, only depends on the inputs when the clock signal ϕ is 1.

[2.17] $Q = x \bullet y + z$

x	y	z	Q
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[2.18] $h = a \bullet b + \bar{a} \bullet b$

a	b	h
0	0	0
0	1	1
1	0	0
1	1	1

$$h = a \bullet b + \bar{a} \bullet b = (a + \bar{a}) \bullet b = 1 \bullet b = b$$

The function h can be simplified as shown above using the **complementary** property,

$$a + \bar{a} = 1$$

[2.19] $F = \bar{A} \bullet B + A \bullet \bar{B} \bullet C$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

The above function is at the simplest form. Thus, no simplification is possible.

[2.20] $F = (A\cdot B + (A\cdot B) + C) = \overline{(A\cdot B + A + \overline{B} + C)}$

$(A + B + C) = \overline{A}\cdot B\cdot\overline{C}$

[2.21] $F = (X+\overline{Y})\cdot(X+Y)\cdot Z = (X+(\overline{Y}\cdot Y))\cdot Z = X\cdot Z$

[2.22] $Z = \overline{((a\cdot b)\cdot d\cdot(\overline{c}+c))} = \overline{(a\cdot b\cdot d)} = \overline{a}+\overline{b}+\overline{d}$

[2.23]

A	B	$X=\overline{A}\cdot B + A\cdot\overline{B}$	A+B
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	1

A	B	$Y=A\cdot B+\overline{A}\cdot\overline{B}$	$\overline{A+B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	1	0

X = Exclusive-OR (XOR)
Y = Exclusive-NOR (XNOR)
$Y=\overline{X}$

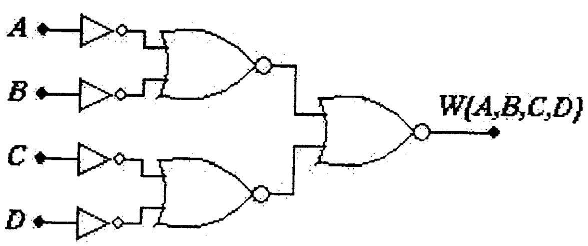
[2.24]
$f=a\cdot b+a\cdot b\cdot\overline{c}+a\cdot b\cdot c=a\cdot b\cdot(1+\overline{c}+c)=a\cdot b$

[2.25]
$g=x+y+\overline{x}\cdot y+\overline{y}=x+\overline{x}\cdot y+(y+\overline{y})$
$=x+\overline{x}\cdot y=x+y$

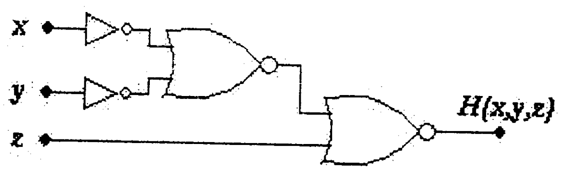
[2.26]
$J=\overline{A}\cdot B+A\cdot\overline{B}+A\cdot B+A=\overline{A}\cdot B+A\cdot(\overline{B}+B+1)$
$=\overline{A}\cdot B+A=A+B$

[2.27]
$K=X+Y\cdot Z+\overline{X}\cdot Y+\overline{X}\cdot\overline{Y}=X+Y\cdot Z+\overline{X}\cdot(Y+\overline{Y})$
$=X+Y\cdot Z+\overline{X}=(X+\overline{X})+Y\cdot Z=Y\cdot Z$

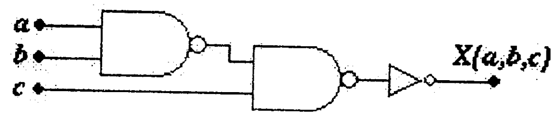
[2.28]
$W=\overline{A\cdot B}\cdot\overline{C\cdot D}=\overline{A\cdot B+C\cdot D}=\overline{\overline{(\overline{A}+\overline{B})}+\overline{(\overline{C}+\overline{D})}}$



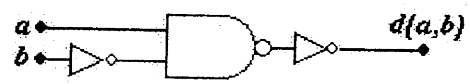
[2.29] $H=\overline{(x\cdot y)}\cdot\overline{z}=\overline{x\cdot y+z}=\overline{\overline{(\overline{x}+\overline{y})}+z}$



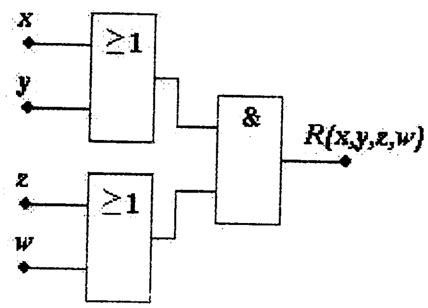
[2.30] $X=\overline{(\overline{a}+\overline{b})\cdot c}=\overline{\overline{(a\cdot b)}\cdot c}=\overline{\overline{\overline{\overline{(a\cdot b)}\cdot c}}}$



[2.31] $d=(a+b)\cdot\overline{b}=a\cdot\overline{b}+b\cdot\overline{b}=a\cdot\overline{b}=\overline{\overline{a\cdot\overline{b}}}$



[2.32] $R=(x+y)\cdot(z+w)$



[2.33] $f=\overline{a}\cdot\overline{(b+c)}=\overline{a+(b+c)}$