H.W. #3

Chapter 3 Combinational Logic Design

[3.1] In the canonical SOP form,

$$f(a,b,c)=a\bullet c=a\bullet 1\bullet c=a\bullet(b+\overline{b})\bullet c$$
$$=a\bullet b\bullet c+a\bullet\overline{b}\bullet c$$

[3.2] In the canonical SOP form,

$$g(x,y)=x+y=x\bullet 1+1\bullet y$$
$$=x\bullet(y+\overline{y})+(x+\overline{x})\bullet y$$
$$=x\bullet y+x\bullet\overline{y}+x\bullet y+\overline{x}\bullet y$$
$$=x\bullet y+x\bullet\overline{y}+\overline{x}\bullet y$$

[3.3] In the canonical SOP form,

$$f(u,v,w)=u\bullet v\bullet\overline{w}+u\bullet v$$
$$=u\bullet v\bullet\overline{w}+u\bullet v\bullet 1$$
$$=u\bullet v\bullet\overline{w}+u\bullet v\bullet(w+\overline{w})$$
$$=u\bullet v\bullet\overline{w}+u\bullet v\bullet w+u\bullet v\bullet\overline{w}$$
$$=u\bullet v\bullet w+u\bullet v\bullet\overline{w}$$

[3.4] In the canonical SOP form,

$$g(A,B,C)=\overline{A}\bullet B\bullet\overline{C}+\overline{A}\bullet B\bullet C+A\bullet\overline{B}\bullet\overline{C}$$
$$+A\bullet B\bullet\overline{C}+A\bullet B\bullet C$$

[3.5] In the canonical POS form,

$$g(A,B,C)=(A+B+C)\bullet(A+B+\overline{C})\bullet(\overline{A}+B+\overline{C})$$

[3.6]

a	b	c	g(a,b,c)	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\Leftarrow \overline{a}\bullet b\bullet c$
1	0	0	1	$\Leftarrow a\bullet\overline{b}\bullet\overline{c}$
1	0	1	1	$\Leftarrow a\bullet\overline{b}\bullet c$
1	1	0	1	$\Leftarrow a\bullet b\bullet\overline{c}$
1	1	1	0	

[3.7]

x	y	z	F(x,y,z)	
0	0	0	1	$\Leftarrow \overline{x}\bullet\overline{y}\bullet\overline{z}$
0	0	1	0	
0	1	0	1	$\Leftarrow \overline{x}\bullet y\bullet\overline{z}$
0	1	1	0	
1	0	0	1	$\Leftarrow x\bullet\overline{y}\bullet\overline{z}$
1	0	1	0	
1	1	0	0	
1	1	1	1	$\Leftarrow x\bullet y\bullet z$

[3.8]

x	y	z	f(x,y,z)	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\Leftarrow \overline{x}\bullet y\bullet z$
1	0	0	1	$\Leftarrow x\bullet\overline{y}\bullet\overline{z}$
1	0	1	1	$\Leftarrow x\bullet\overline{y}\bullet z$
1	1	0	1	$\Leftarrow x\bullet y\bullet\overline{z}$
1	1	1	1	$\Leftarrow x\bullet y\bullet z$

In the canonical SOP form,

$$f(x,y,z)=\overline{x}\bullet y\bullet z+x\bullet\overline{y}\bullet\overline{z}+x\bullet\overline{y}\bullet z+x\bullet y\bullet\overline{z}+x\bullet y\bullet z$$

[3.9] In the canonical SOP form,

$$f(x,y,z)=\overline{x}\bullet\overline{y}\bullet\overline{z}+\overline{x}\bullet\overline{y}\bullet z+\overline{x}\bullet y\bullet\overline{z}+x\bullet y\bullet z$$

[3.10] In the canonical SOP form,

$$g(a,b,c)=\overline{a}\bullet\overline{b}\bullet c+\overline{a}\bullet b\bullet\overline{c}+a\bullet\overline{b}\bullet\overline{c}+a\bullet\overline{b}\bullet c+a\bullet b\bullet c$$

[3.11] (a) In the canonical SOP form,

$$B(u,v,w)=\overline{u}\bullet\overline{v}\bullet\overline{w}+\overline{u}\bullet\overline{v}\bullet w+u\bullet\overline{v}\bullet\overline{w}+u\bullet\overline{v}\bullet w+u\bullet v\bullet w$$

(b) In the canonical SOP form,

$$R(a,b,c)=\overline{a}\bullet\overline{b}\bullet\overline{c}+\overline{a}\bullet b\bullet c+a\bullet\overline{b}\bullet\overline{c}+a\bullet\overline{b}\bullet c+a\bullet b\bullet\overline{c}+a\bullet b\bullet c$$

[3.12]

$$r(u,v,w)=u\bullet v\bullet w+(\overline{u}+v)\bullet w=u\bullet v\bullet w+\overline{u}\bullet w+v\bullet w$$
$$=u\bullet v\bullet w+\overline{u}\bullet v\bullet w+\overline{u}\bullet\overline{v}\bullet w+u\bullet v\bullet w+\overline{u}\bullet v\bullet w$$
$$=u\bullet v\bullet w+\overline{u}\bullet v\bullet w+\overline{u}\bullet\overline{v}\bullet w$$

u	v	w	r(u,v,w)	
0	0	0	0	
0	0	1	1	$\Leftarrow \overline{u}\bullet\overline{v}\bullet w$
0	1	0	0	
0	1	1	1	$\Leftarrow \overline{u}\bullet v\bullet w$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	$\Leftarrow u\bullet v\bullet w$

[3.13] $f(a,b,c,d)=(a+\bar{b})\cdot c+b\cdot\bar{c}\cdot d+b\cdot c$

$=a\cdot c+\bar{b}\cdot c+b\cdot\bar{c}\cdot d+b\cdot c$

$=a\cdot c+(\bar{b}+b)\cdot c+b\cdot\bar{c}\cdot d$

$=a\cdot c+c+b\cdot\bar{c}\cdot d=c+b\cdot\bar{c}\cdot d$

a	b	c	d	f(a,b,c,d)	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	1	$\Leftarrow$ c
0	0	1	1	1	$\Leftarrow$ c
0	1	0	0	0	
0	1	0	1	1	$\Leftarrow b\cdot\bar{c}\cdot d$
0	1	1	0	1	$\Leftarrow$ c
0	1	1	1	1	$\Leftarrow$ c
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	1	$\Leftarrow$ c
1	0	1	1	1	$\Leftarrow$ c
1	1	0	0	0	
1	1	0	1	1	$\Leftarrow b\cdot\bar{c}\cdot d$
1	1	1	0	1	$\Leftarrow$ c
1	1	1	1	1	$\Leftarrow$ c

[3.14] $g(a,b)=m_0+m_2=\bar{a}\cdot\bar{b}+a\cdot\bar{b}$

[3.15] $f(x,y,z)=m_0+m_1+m_5+m_6$

$f(x,y,z)=\bar{x}\cdot\bar{y}\cdot\bar{z}+\bar{x}\cdot\bar{y}\cdot z+x\cdot\bar{y}\cdot z+x\cdot y\cdot\bar{z}$

[3.16]

(a) $f(x,y,z)=m_0+m_2+m_4+m_5$

$f(x,y,z)=\bar{x}\cdot\bar{y}\cdot\bar{z}+\bar{x}\cdot y\cdot\bar{z}+x\cdot\bar{y}\cdot\bar{z}+x\cdot\bar{y}\cdot z$

(b) $f(x,y,z)=\bar{x}\cdot(\bar{y}+y)\cdot\bar{z}+x\cdot\bar{y}\cdot(\bar{z}+z)=\bar{x}\cdot\bar{z}+x\cdot\bar{y}$

[3.17] $h(a,b,c)=\Sigma m(0, 2, 4, 5, 6)$

$h(a,b,c)=\bar{a}\cdot\bar{b}\cdot\bar{c}+\bar{a}\cdot b\cdot\bar{c}+a\cdot\bar{b}\cdot\bar{c}+a\cdot\bar{b}\cdot c+a\cdot b\cdot\bar{c}$

$=\bar{a}\cdot(\bar{b}+b)\cdot\bar{c}+a\cdot\bar{b}\cdot(\bar{c}+c)+a\cdot b\cdot\bar{c}$

$=\bar{a}\cdot\bar{c}+a\cdot\bar{b}+a\cdot b\cdot\bar{c}=\bar{a}\cdot\bar{c}+a\cdot(\bar{b}+b\cdot\bar{c})$

$=\bar{a}\cdot\bar{c}+a\cdot(\bar{b}+\bar{c})=\bar{a}\cdot\bar{c}+a\cdot\bar{b}+a\cdot\bar{c}$

$=(\bar{a}+a)\cdot\bar{c}+a\cdot\bar{b}=\bar{c}+a\cdot\bar{b}$

c \ ab	00	01	11	10
0	1	1	1	1
1	0	0	0	1

$h(a,b,c)=\bar{c}+a\cdot\bar{b}$

[3.18] $h(x,y,z)=\Sigma m(1, 2, 4)$

$=\bar{x}\cdot\bar{y}\cdot z+\bar{x}\cdot y\cdot\bar{z}+x\cdot\bar{y}\cdot\bar{z}$

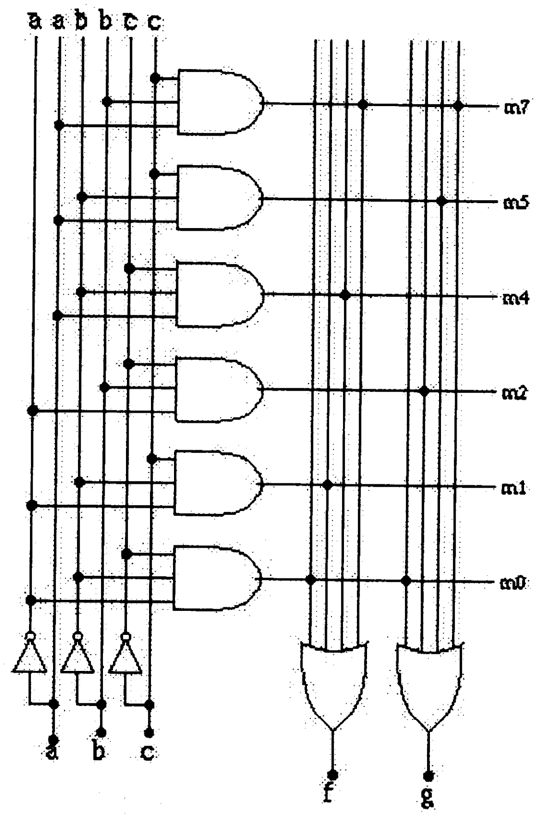
[3.19] $G(a,b)=M_0\cdot M_2=(a+b)\cdot(\bar{a}+b)$

[3.20] $G(A,B,C)=\prod M(1, 3, 4, 7)$

$G(A,B,C)=(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+\bar{C})$

[3.21] $f(a,b,c)=\Sigma m(0, 1, 4, 7)$

$g(a,b,c)=\Sigma m(0, 2, 5, 7)$

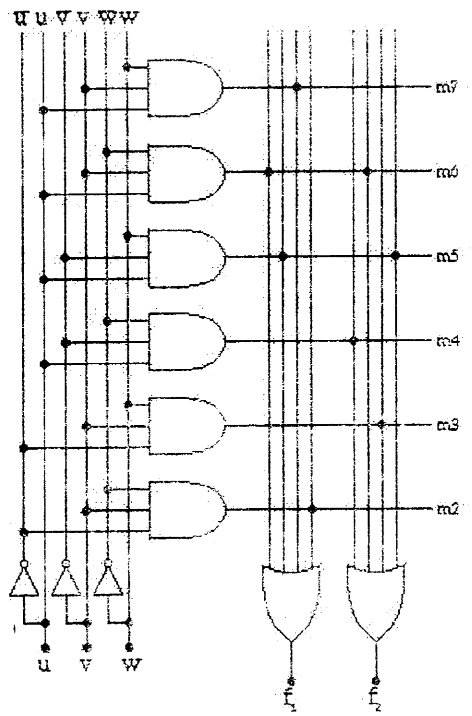


[3.22] $f_1=u\cdot v\cdot\bar{w}+u\cdot\bar{v}\cdot w+u\cdot v\cdot w+\bar{u}\cdot v\cdot\bar{w}$

$=\Sigma m(6, 5, 7, 2)$

$f_2=u\cdot\bar{v}\cdot\bar{w}+u\cdot v\cdot\bar{w}+\bar{u}\cdot v\cdot w+u\cdot\bar{v}\cdot w$

$=\Sigma m(4, 6, 3, 5)$



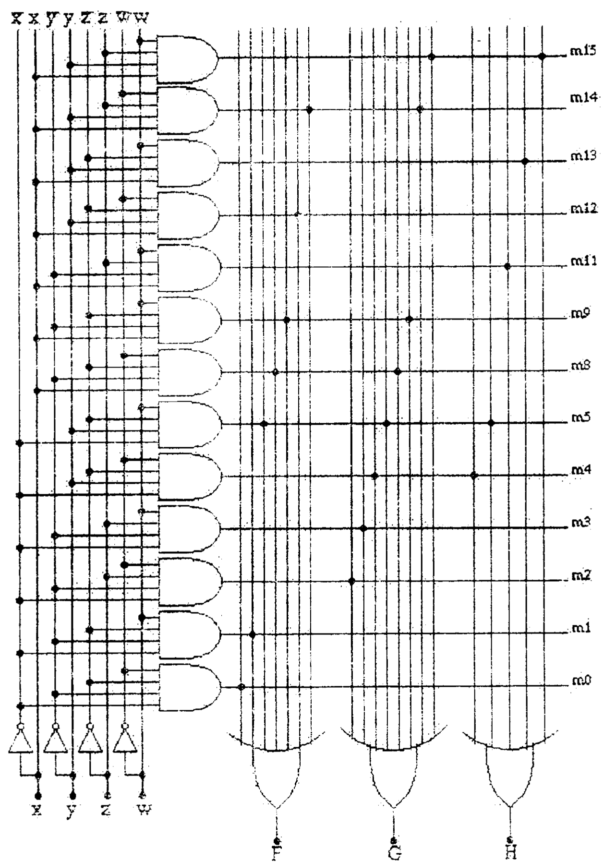
There are more than one ways of designing AND-OR PLA for these equations. The method shown here is to extend the G(x,y,z) into G(x,y,z,w) using the following fact that $(\overline{w}+w)=1$. Thus,

$G(x,y,z)=G(x,y,z,w)=\sum m(2,3,4,5,8,9,14,15)$

[3.23] $F(x,y,z,w)=\sum m(0, 1, 5, 8, 9, 12, 14)$

$H(x,y,z,w)=\sum m(4, 5, 11, 13, 15)$

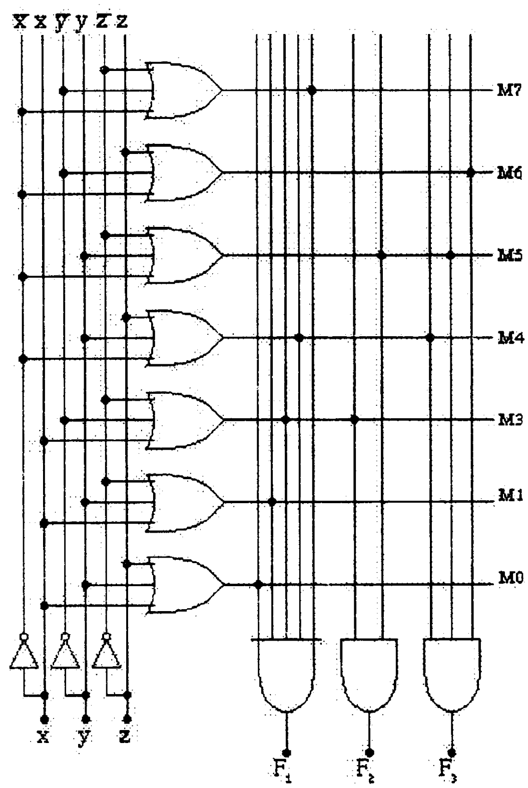
$G(x,y,z)=\sum m(1, 2, 4, 7)$



[3.24] $F_1=\prod M(0, 1, 3, 4, 7)$

$F_2=\prod M(3, 5)$

$F_3=\prod M(4, 5, 6)$



[3.25]

$g_0=(x+\overline{y}+z+\overline{w})\bullet(x+\overline{y}+z+w)\bullet(\overline{x}+y+z+w)$

$=\prod M(5, 4, 8)$

$g_1=(\overline{x}+y+z+w)\bullet(\overline{x}+\overline{y}+\overline{z}+w)\bullet(x+\overline{y}+z+w)$

$=\prod M(8, 14, 4)$

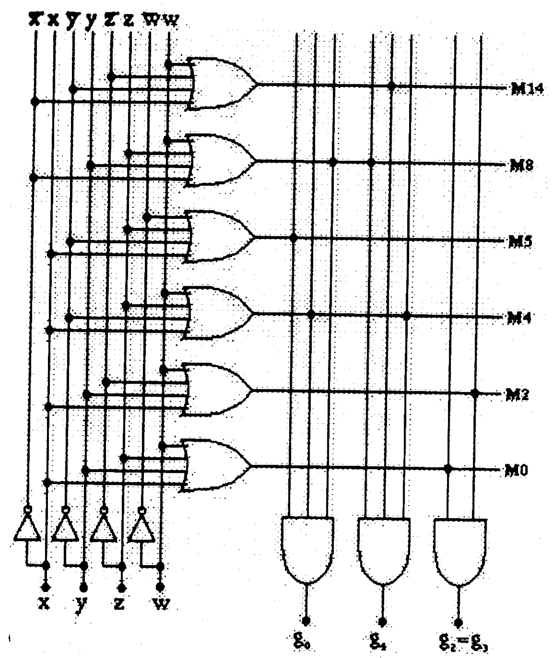
$g_2=(x+y+z+w)\bullet(x+y+z+w)\bullet(x+y+\overline{z}+w)$

$=(x+y+z+w)\bullet(x+y+\overline{z}+w)$

$=\prod M(0, 2)$

$g_3=(x+y+\overline{z}+w)\bullet(x+y+\overline{z}+w)\bullet(x+y+z+w)$

$=(x+y+\overline{z}+w)\bullet(x+y+z+w)=g_2$



[3.26]

$f=a\cdot b\cdot c+a\cdot c+a\cdot\bar{b}+\bar{a}\cdot b$

$=a\cdot b\cdot c+a\cdot(b+\bar{b})\cdot c+a\cdot\bar{b}\cdot(c+\bar{c})$
$+\bar{a}\cdot b\cdot(c+\bar{c})$

$=a\cdot b\cdot c+a\cdot b\cdot c+a\cdot\bar{b}\cdot c+a\cdot\bar{b}\cdot c$
$+a\cdot\bar{b}\cdot\bar{c}+\bar{a}\cdot b\cdot c+\bar{a}\cdot b\cdot\bar{c}$

$=a\cdot b\cdot c+a\cdot\bar{b}\cdot c+a\cdot\bar{b}\cdot\bar{c}+\bar{a}\cdot b\cdot c+\bar{a}\cdot b\cdot\bar{c}$

c \ ab	00	01	11	10
0	0	1	0	1
1	0	1	1	1

$f=\bar{a}\cdot b+a\cdot c+a\cdot\bar{b}$

$=\bar{a}\cdot b+a\cdot(c+\bar{b})$

[3.27] $h(x,y,z)=x\cdot y+x\cdot(\bar{y}+z)+(x+y)\cdot\bar{z}$

z \ xy	00	01	11	10
0	0	1	1	1
1	0	0	1	1

$h(x,y,z)=y\cdot\bar{z}+x$

[3.28] $g(a,b,c)=\sum m(0, 1, 2, 3, 6)$

c \ ab	00	01	11	10
0	1	1	1	0
1	1	1	0	0

$g(a,b,c)=\bar{a}+b\cdot\bar{c}$

[3.29]

(a) Problem [3.4]

C \ AB	00	01	11	10
0	0	1	1	1
1	0	1	1	0

$g(A,B,C)=B+A\cdot\bar{C}$

(b) Problem [3.9]

z \ xy	00	01	11	10
0	1	1	0	0
1	1	0	1	0

$f=\bar{x}\cdot\bar{y}+\bar{x}\cdot\bar{z}+x\cdot y\cdot z$

$=\bar{x}\cdot(\bar{y}+\bar{z})+x\cdot y\cdot z$

(c) Problem [3.10]

c \ ab	00	01	11	10
0	0	1	0	1
1	1	0	1	1

$G=\bar{b}\cdot c+\bar{a}\cdot b\cdot\bar{c}+a\cdot c+a\cdot\bar{b}$

$=\bar{b}\cdot c+\bar{a}\cdot b\cdot\bar{c}+a\cdot(c+\bar{b})$

(d) Problem [3.11] (a)

w \ uv	00	01	11	10
0	1	0	0	1
1	1	0	1	1

$B=\bar{v}+u\cdot w$

Problem [3.11] (b)

c \ ab	00	01	11	10
0	1	0	1	1
1	0	1	1	1

$$R = b \cdot c + \overline{b} \cdot \overline{c} + a$$

[3.30]

$$F = A \cdot B \cdot C + \overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot C + A \cdot B \cdot C + \overline{A} \cdot \overline{B} \cdot C$$

C \ AB	00	01	11	10
0	0	1	0	0
1	1	0	1	1

$$F = \overline{B} \cdot C + \overline{A} \cdot B \cdot \overline{C} + A \cdot C$$
$$= \overline{A} \cdot B \cdot \overline{C} + (A + \overline{B}) \cdot C$$

[3.31] $f(x,y,z) = \sum m(0, 1, 3, 4, 7)$

z \ xy	00	01	11	10
0	1	0	0	1
1	1	1	1	0

$$f = \overline{y} \cdot \overline{z} + \overline{x} \cdot z + y \cdot z$$
$$= \overline{y} \cdot \overline{z} + (\overline{x} + y) \cdot z$$

[3.32] $F(x,y,z,w) = \sum m(2, 3, 4, 7, 8, 14, 15)$

zw \ xy	00	01	11	10
00	0	1	0	1
01	0	0	0	0
11	1	1	1	0
10	1	0	1	0

$$F = \overline{x} \cdot \overline{y} \cdot z + \overline{x} \cdot y \cdot \overline{z} \cdot \overline{w} + y \cdot z \cdot w + x \cdot y \cdot z + x \cdot \overline{y} \cdot \overline{z} \cdot \overline{w}$$
$$= \overline{x} \cdot \overline{y} \cdot z + (\overline{x} \cdot y + x \cdot \overline{y}) \cdot \overline{z} \cdot \overline{w} + (x + w) \cdot y \cdot z$$

[3.33]

(a)

x \ yz	00	01	11	10
0	0	1	0	0
1	1	1	1	1

$$f = x + \overline{y} \cdot z$$

(b)

x \ yz	00	01	11	10
0	1	0	0	1
1	1	0	1	1

$$f = \overline{z} + x \cdot y$$

[3.34]

(a)

u \ vw	01	11	10	00
0	0	1	1	0
1	1	0	0	1

$$f = u \cdot \overline{v} + \overline{u} \cdot v$$

(b)

u \ vw	11	10	00	01
0	0	1	1	0
1	0	1	1	1

$$f = \overline{w} + u \cdot \overline{v}$$

[3.35]

(a)

u \ vw	00	01	11	10
0	0	1	1	0
1	1	0	1	1

$$f = u \cdot \overline{w} + \overline{u} \cdot w + v \cdot w$$
$$= u \cdot \overline{w} + (\overline{u} + v) \cdot w$$

(b)

u \ vw	00	01	11	10
0	1	1	1	0
1	1	0	0	0

$$f=\bar{v}\bullet\bar{w}+\bar{u}\bullet w$$

(c)

u \ vw	10	11	01	00
0	0	1	1	0
1	1	1	0	1

$$f=u\bullet\bar{w}+v\bullet w+\bar{u}\bullet w$$
$$=u\bullet\bar{w}+(v+\bar{u})\bullet w$$

[3.36]

xy \ zw	00	01	11	10
00	0	1	1	1
01	1	0	1	1
11	1	1	0	0
10	0	1	0	0

$$f=y\bullet\bar{z}\bullet\bar{w}+x\bullet\bar{z}\bullet w+\bar{x}\bullet\bar{y}\bullet w+\bar{x}\bullet z$$
$$=y\bullet\bar{z}\bullet\bar{w}+(x\bullet\bar{z}+\bar{x}\bullet\bar{y})\bullet w+\bar{x}\bullet z$$

[3.37]

$$h=x\bullet y\bullet(z+\bar{w})+(\bar{x}+y)\bullet z$$

x	y	z	w	h(x,y,z,w)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

The canonical SOP form of h(x,y,z,w) is

$$h=\bar{x}\bullet\bar{y}\bullet z\bullet\bar{w}+\bar{x}\bullet\bar{y}\bullet z\bullet w+\bar{x}\bullet y\bullet z\bullet\bar{w}+\bar{x}\bullet y\bullet z\bullet w$$
$$+x\bullet y\bullet\bar{z}\bullet\bar{w}+x\bullet y\bullet z\bullet\bar{w}+x\bullet y\bullet z\bullet w$$

[3.38]

xy \ zw	00	01	11	10
00	1	1	1	0
01	1	0	1	1
11	1	1	0	1
10	0	0	0	0

$$F=y\cdot\bar{w}+x\cdot y\cdot\bar{z}+\bar{x}\cdot\bar{y}\cdot\bar{z}+\bar{x}\cdot z\cdot w$$
$$=y\cdot\bar{w}+(x\cdot y+\bar{x}\cdot\bar{y})\cdot\bar{z}+\bar{x}\cdot z\cdot w$$

[3.39]

xy \ zw	11	10	00	01
01	1	1	0	1
11	1	1	0	0
10	1	0	0	0
00	0	1	0	1

$$F = y\cdot z + x\cdot z\cdot w + \bar{x}\cdot z\cdot \bar{w} + \bar{x}\cdot \bar{z}\cdot w$$
$$= (y + x\cdot w + \bar{x}\cdot \bar{w})\cdot z + \bar{x}\cdot \bar{z}\cdot w$$

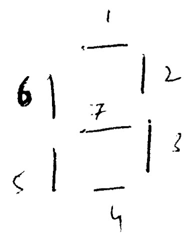
[3.40]

uv \ de	00	01	11	10
11	0	1	1	1
10	1	0	0	0
00	1	0	0	0
01	1	1	1	1

$$g = \bar{v}\cdot \bar{d}\cdot \bar{e} + \bar{u}\cdot v + v\cdot e + v\cdot d$$
$$= \bar{v}\cdot \bar{d}\cdot \bar{e} + v\cdot(\bar{u} + e + d)$$

ADDITIONAL PROBLEMS

ABCD	F1	F2	F3
0000	0	X	X
0001	1	X	X
0010	1	X	X
0011	0	1	0
0100	1	0	0
0101	0	0	1
0110	0	1	1
0111	1	0	1
1000	1	0	1
1001	0	1	1
1010	0	0	0
1011	1	0	1
1100	0	1	1
1101	1	X	X
1110	1	X	X
1111	0	X	X



1)

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$$F1 = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + AB\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD$$

$$= (AB + \bar{A}\bar{B})\bar{C}D + C\bar{D}(AB + \bar{A}\bar{B}) + \bar{C}\bar{D}(A\bar{B} + \bar{A}B) + CD(A\bar{B} + \bar{A}B)$$

$$= (\overline{A \oplus B})\bar{C}D + C\bar{D}(\overline{A \oplus B}) + \bar{C}\bar{D}(A \oplus B) + CD(A \oplus B)$$

$$= (\overline{A \oplus B})(C\bar{D} + \bar{C}D) + A \oplus B(\bar{C}\bar{D} + CD)$$

$$= (\overline{A \oplus B})(C \oplus D) + A \oplus B(\overline{C \oplus D})$$

[The rest I leave it up to you.]

$$= A \oplus B \oplus C \oplus D$$

2)

AB \ CD	00	01	11	10
00	X	X	1	X
01	0	0	0	1
11	1	X	X	X
10	0	1	0	0

F2:- $A\bar{C}D + AB + \bar{A}\bar{B} + BC\bar{D}$ [This is one possible solution].

$\Rightarrow A\bar{C}D + BC\bar{D} + \overline{A \oplus B}$

3)

AB \ CD	00	01	11	10
00	X	X	0	X
01	0	1	1	1
11	1	X	X	X
10	1	1	1	0

F3:→ $BC + AD + A\bar{C} + BD$ [This is one possible Solution]

1 NOT GATE
4 AND2 GATES
1 OR4 GATE

→ $B(C+D) + A(\bar{C}+D)$ → 1 NOT GATE, 2 AND2 GATES, 3 OR2 GATES } A BIT CHEAPER MODEL