CS 228 - 1st Test - WS 1997 - Dr. Zobrist
100 pts Total - Question Points Noted on Exam
Closed Book - 8 1/2 Crib Sheet Allowed

(25) 1. Use the iterative method for finding a root to $f(x) = x^{**}2 - 1$; i.e., $x = g(x)$. Answer the following questions.

a. Where are the roots (find by solving equation)

roots = ______________________________

b. Choose a starting point x0 = 0.95. Do three iterations.

iteration	x	g(x)
1		
2		
3		

c. Does it converge? Yes ______ No ______

d. Will any starting point converge to the positive root? Yes ______ No ______

e. Demonstrate analytically whether it will, or will not converge.

(20) 2. Use the Biscetion Method on $f(x) = x^{**}2 -1$ to find the positive root; start at x0 = -0.5 and proceed in increments of 1.0.

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(25) 3. From the table below find the representation for
x = (1/3 + 1/2) and determine the error.

mantissa	exponent n = -1	n = 0	n = +1
.100	.25	.5	1.
.101	.3125	.625	1.25
.110	.375	.75	1.5
.111	.4375	.875	1.75

(10) 4. Define the following:
a. Absolute error

b. Relative error

(10) 5. Determine the maximum absolute error for
f(x) = x**2 - 1, when x = 5.1 +/- 0.05

(10) 6. Convert the following binary representation to decimal form,

101.1101 ----> ________________

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C.Sc. 228 **2nd Test** **Dr. Zobrist** **WS 97**

100 pts. total - All questions equally weighted - Closed book - 8 ½ x 11 crib sheet allowed.

1.

X	$\sqrt{X}$	f[,]	f[,,]	f[,,,]
1.05	1.02470			
1.10	1.04881			
1.15	1.07238			
1.20	1.09544			

Complete the divided difference table.

Note: f[,]; f[,,]; f[,,,] are the first, second, third divided difference, respectively.

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2. Apply Lagranges formula (first order) to obtain $\sqrt{1.12}$ from the Table in question #1. Choose $X_0 = 1.10$, $X_1 = 1.15$.

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3. Obtain a formula of the type $p(x) = Ae^{MX}$ from the following data (in the least squares sense):

xi	1	2	3	4
Pi	7	11	17	27

Use the data linearization method, i.e., logarithmic transformations.

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4. Solve by Gaussian Elimination

$$X_1 + \tfrac{1}{2} X_2 + \tfrac{1}{3} X_3 = 1$$

$$\tfrac{1}{2} X_1 + \tfrac{1}{3} X_2 + \tfrac{1}{4} X_3 = 0$$

$$\tfrac{1}{3} X_1 + \tfrac{1}{4} X_2 + \tfrac{1}{5} X_3 = 0$$

Utilize $AX = B \overset{\text{trf}}{\Rightarrow} UX = Y$, i.e., upper triangularization followed by back substitution.

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5. Show that the following equations satisfy property III for a cubic spline, i.e., $S_k\ (X_{k+1}) = S_{k+1}(X_{k+1})$

and

x	0	1	2	3
y	0	.5	2.0	1.5

$$S_0 = .4X^3 + .1X \quad ; \quad 0 \le x \le 1$$

$$S_1 = -(x-1)^3 + 1.2(x-1)^2 + 1.3(x-1) + .5 \quad ; \quad 1 \le x \le 2$$

$$S_2 = .6(x-2)^3 - 1.8(x-2)^2 + .7(x-2) + 2 \quad ; \quad 2 \le x \le 3$$

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CS 228 - 2nd Test - WS 1997 - Dr. Zobrist
100 pts Total - Question Points Noted on Exam
Closed Book - 8 1/2 Crib Sheet Allowed

(30) 1. For $f(x) = \cos x : 0 \leq x \leq \pi/2$; calculate the upper bound on the error in integration using the Trapezoidal Rule, when using four intervals. Also, determine the value of $\int_0^{\pi/2} f(x)\,dx$ using the Trapezoidal Rule, using the same four intervals, and

compare with the actual value (ie, determine error).

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(30) 2. For $f(x) = \cos x$: $x = 0$, $\pi/2$; construct an interpolating linear polynomial $p(x)$ passing through the given points, and then differentiate to obtain $p'(x)$. Compare the result with the true value at $x = \pi/4$.

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(20) 3. Estimate the round off error behavior for the formula

$$(y_1 - y_{-1}) / 2h .$$

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(20) 4. Define <u>Adaptive Quadrature</u>.

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CS 228 - Final Exam - WS 1997 - Dr. Zobrist
100 pts Total - Question Points Noted on Exam
Closed Book - 8 1/2 Crib Sheet Allowed

(30) 1. Apply the Euler Method to determine the solution of:

$y' = -2xy$; $y(0) = 1$; Step Size = .2;

determine y for x = 0., .2, .4, .6, .8, 1.

Compare to the exact solution ($y_{exact} = e^{-x^2}$).

Also, determine the single step truncation error and the global truncation error.

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(30) 2. Apply the Fourth Order Runge-Kutta Method to determine the solution of:

$y' = -2xy$; $y(0) = 1$; Step Size = .2;

determine y for x = 0., .2, .4, .6, .8, 1.

Compare to the exact solution ($y_{exact} = e^{-x^2}$).

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(40) 3. Apply the Method described below to determine the solution of:

$$y' = -2xy; \; y(0) = 1., \text{ Step Size } = .5;$$

determine y for x = 0., .5, 1..

Compare to the exact solution ($y_{exact} = e^{-x^2}$)

METHOD

$$y_{k+1} = y_k + (h/2)[\, f(x_k, y_k) + f(x_{k+1}, p_{k+1})]$$

$$p_{k+1} = y_k + h\, f(x_k, y_k)$$

$$x_{k+1} = x_k + h$$

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