Project 1 Report

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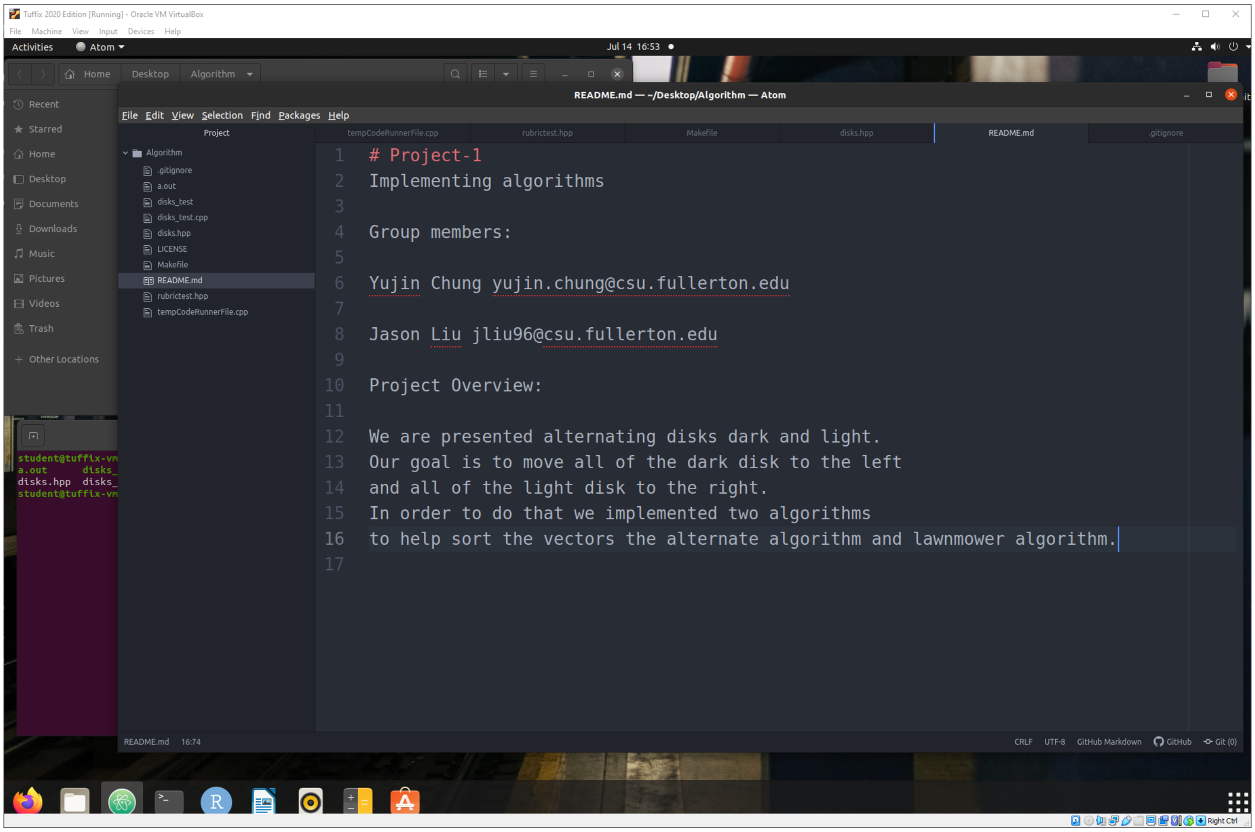
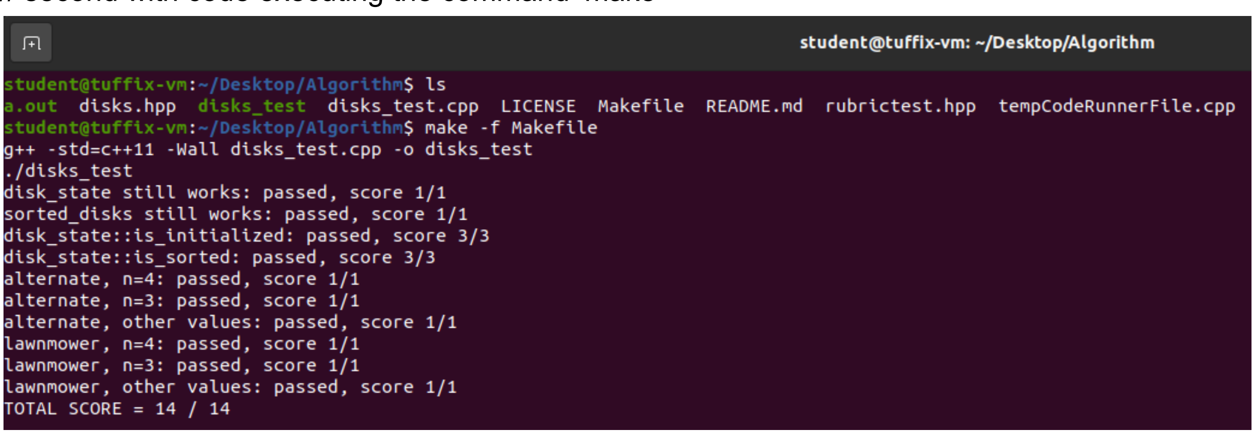
SCREENSHOTS

// one inside Tuffix showing the Atom editor w/ group member names inside Atom (one way to make names appear is to simply open README.md



// second with code executing the command 'make'



ALGORITHM

Lawnmower

```
for t=0 to ceil(n/2)
    for i=1 to 2*n-2 skip 2 do
        if (color[i] == LIGHT && color[i+1] == DARK) do
            swap(i)
        else
            do nothing
    endfor
    for j=2*n-2 down to 1 skip 2 do
        if (color[j]==LIGHT && color[j+1] == DARK) do
            swap(j)
        else
            do nothing
    endfor
endfor
```

Alternate

```
for t=0 to n
      for i=(t%2) to 2*n-2 skip 2 do
             if (color[i] == LIGHT && color[i+1] == DARK) do
             swap(i)
      endfor
endfor
```

TIME COMPLEXITY

First, we have confirmed that the swap() function's complexity is constant when swapping two values. Therefore, while calculating the step count of our algorithms, *we decided to give swap() a time unit of 3*, based on a simple swap function one would write using a temp variable.
For the alternate algorithm given that t%2 can only be either 0 or 1, below is the total step count for both situations.

<u>Lawnmower</u>
SC of if-else block = 3 + max(3, 0) = 6
SC of 1st inner for-loop = [((2*n-2) - 1) / 2 + 1] * 6 = 6n-3
SC of 2nd inner for-loop = [(1 - (2*n-2)) / (-2) + 1] * 6 = 6n-3
Total SC = [(n/2-0) / 1 + 1] * (6n-3 + 6n-3) = $6n^2 + 9n - 6$

Complexity = $O(n^2)$

Proof 1: $6n^2 + 9n - 6 \in O(n^2)$
Choose c = |6| + |9| + |6| = 21
So $6n^2 + 9n - 6 \leq 21n^2 \;\; \forall\; n \geq n_0$
$\Leftrightarrow 21n^2 - 6n^2 - 9n + 6 \geq 0$
$\Leftrightarrow 6n^2 + 6 + 9n^2 - 9n \geq 0 \;\; \forall\; n \geq 0$
$n_0 = 0$

Proof 2: $6n^2 + 9n - 6 \in O(n^2)$ using limit theorem

$$\lim_{n\to\infty} \frac{6n^2 + 9n - 6}{n^2} = \lim_{n\to\infty} \frac{(6n^2 + 9n - 6)'}{(n^2)'} = \lim_{n\to\infty} \frac{12n + 9}{2n} = \lim_{n\to\infty} \frac{(12n + 9)'}{(2n)'} = \lim_{n\to\infty} \frac{12}{2} = 6 \geq 0 \; and\; defined$$

<u>Alternate</u>
If t%2 = 0
SC of if-else block = 3 + max(3,0) = 6
SC of inner for-loop = $\sum_{i=0}^{2n-2} 6$ = 6(2*n-2+1) = 12n-6
Total SC = $\sum_{t=0}^{n}$(12n-6) = 12$\sum_{t=0}^{n} n$ - $\sum_{t=0}^{2n-2}$ 6 = 12n(n+1) - 6(n+1) = $12n^2 + 6n - 6$

If t%2 = 1
SC of if-else block = 3 + max(3,0) = 6

SC of inner for-loop = $\sum_{i=1}^{2n-2} 6$ = 6(2*n-2) = 12n-12

Total SC = $\sum_{t=0}^{n}$ (12n-12) = 12 $\sum_{t=0}^{n} n$ - $\sum_{t=0}^{n} 12$ = 12n(n+1) - 12(n+1) = 12n^2 - 12

Complexity = O(n^2)

Proof 1 if *t%2 = 0*: 12n^2 + 6n - 6 $\in$ O(n^2)
Choose c = |12| + |6| + |6| = 24
So 12n^2 + 6n - 6 ≤ 24n^2 $\forall$ n ≥ n_0
⇔ 24n^2 - 12n^2 - 6n + 6 ≥ 0
⇔ 6n^2 - 6n + 6n^2 + 6 ≥ 0 $\forall$ n ≥ 0
n_0 = 0

Proof 2 if *t%2 = 0*: 12n^2 + 6n - 6 $\in$ O(n^2) using limit theorem

$$\lim_{n\to\infty} \frac{12n^2+6n-6}{n^2} = \lim_{n\to\infty} \frac{(12n^2+6n-6)'}{(n^2)'} = \lim_{n\to\infty} \frac{24n+6}{2n} = \lim_{n\to\infty} \frac{(24n+6)'}{(2n)'} = \lim_{n\to\infty} \frac{24}{2} = 12 \geq 0 \; and \; defined$$

Proof 1 if *t%2 = 1*: 12n^2 - 12 $\in$ O(n^2)
Choose c = |12| + |12| = 24
So 12n^2 - 12 ≤ 24n^2 $\forall$ n ≥ n_0
⇔ 24n^2 - 12n^2 + 12 ≥ 0
⇔ 12n^2 + 12 ≥ 0 $\forall$ n ≥ 0
n_0 = 0

Proof 2 if *t%2 = 1*: 12n^2 - 12 $\in$ O(n^2) using limit theorem

$$\lim_{n\to\infty} \frac{12n^2-12}{n^2} = \lim_{n\to\infty} \frac{(12n^2-12)'}{(n^2)'} = \lim_{n\to\infty} \frac{24n}{2n} = \lim_{n\to\infty} \frac{(24n)'}{(2n)'} = \lim_{n\to\infty} \frac{24}{2} = 12 \geq 0 \; and \; defined$$