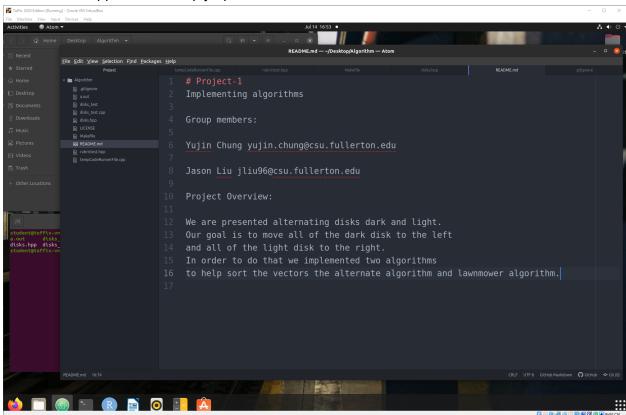
Project 1 Report

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SCREENSHOTS

// one inside Tuffix showing the Atom editor w/ group member names inside Atom (one way to make names appear is to simply open README.md



// second with code executing the command 'make'

```
student@tuffix-vm:~/Desktop/Algorithm$ ls
a.out disks.hpp disks_test disks_test.cpp LICENSE Makefile README.md rubrictest.hpp tempCodeRunnerFile.cpp
student@tuffix-vm:~/Desktop/Algorithm$ make -f Makefile
g++ -std=c++11 -Wall disks_test.cpp -o disks_test
./disks_test
disk_state still works: passed, score 1/1
sorted_disks still works: passed, score 1/1
disk_state::is_initialized: passed, score 3/3
disk_state::is_sorted: passed, score 3/3
alternate, n=4: passed, score 1/1
alternate, n=3: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, other values: passed, score 1/1
TOTAL SCORE = 14 / 14
```

ALGORITHM

```
Lawnmower
for t=0 to ceil(n/2)
  for i=1 to 2*n-2 skip 2 do
     if (color[i] == LIGHT && color[i+1] == DARK) do
       swap(i)
     else
       do nothing
  endfor
  for j=2*n-2 down to 1 skip 2 do
     if (color[j]==LIGHT && color[j+1] == DARK) do
       swap(j)
     else
       do nothing
  endfor
endfor
<u>Alternate</u>
for t=0 to n
   for i=(t%2) to 2*n-2 skip 2 do
        if (color[i] == LIGHT && color[i+1] == DARK) do
        swap(i)
   endfor
endfor
```

TIME COMPLEXITY

First, we have confirmed that the swap() function's complexity is constant when swapping two values. Therefore, while calculating the step count of our algorithms, we decided to give swap() a time unit of 3, based on a simple swap function one would write using a temp variable. For the alternate algorithm given that t%2 can only be either 0 or 1, below is the total step count for both situations.

Lawnmower

SC of if-else block =
$$3 + max(3, 0) = 6$$

SC of 1st inner for-loop = $[((2*n-2) - 1) / 2 + 1] * 6 = 6n-3$
SC of 2nd inner for-loop = $[(1 - (2*n-2)) / (-2) + 1] * 6 = 6n-3$
Total SC = $[(n/2-0) / 1 + 1] * (6n-3 + 6n-3) = 6n^2 + 9n - 6$

Complexity = $O(n^2)$

Proof 1:
$$6n^2 + 9n - 6 \subseteq O(n^2)$$

Choose $c = |6| + |9| + |6| = 21$
So $6n^2 + 9n - 6 \le 21n^2 \quad \forall \quad n \ge n_0$
 $\Leftrightarrow 21n^2 - 6n^2 - 9n + 6 \ge 0$
 $\Leftrightarrow 6n^2 + 6 + 9n^2 - 9n \ge 0 \quad \forall \quad n \ge 0$
 $n_0 = 0$

Proof 2: $6n^2 + 9n - 6 \in O(n^2)$ using limit theorem

$$\lim_{n \to \infty} \frac{6n^2 + 9n - 6}{n^2} = \lim_{n \to \infty} \frac{(6n^2 + 9n - 6)'}{(n^2)'} = \lim_{n \to \infty} \frac{12n + 9}{2n} = \lim_{n \to \infty} \frac{(12n + 9)'}{(2n)'} = \lim_{n \to \infty} \frac{12}{2} = 6 \ge 0 \text{ and defined}$$

Alternate

If
$$t\%2 = 0$$

SC of if-else block =
$$3 + max(3,0) = 6$$

SC of inner for-loop =
$$\sum_{i=0}^{2n-2} 6 = 6(2*n-2+1) = 12n-6$$

Total SC =
$$\sum_{t=0}^{n} (12n-6) = 12 \sum_{t=0}^{n} n - \sum_{t=0}^{2n-2} 6 = 12n(n+1) - 6(n+1) = 12n^2 + 6n - 6$$

If
$$t\%2 = 1$$

SC of if-else block =
$$3 + max(3,0) = 6$$

SC of inner for-loop =
$$\sum_{i=1}^{2n-2} 6 = 6(2*n-2) = 12n-12$$

Total SC =
$$\sum_{t=0}^{n} (12n-12) = 12 \sum_{t=0}^{n} n - \sum_{t=0}^{n} 12 = 12n(n+1) - 12(n+1) = 12n^2 - 12$$

Complexity = $O(n^2)$

Proof 1 if
$$t\%2 = 0$$
: $12n^2 + 6n - 6 \in O(n^2)$
Choose $c = |12| + |6| + |6| = 24$
So $12n^2 + 6n - 6 \le 24n^2 \quad \forall \quad n \ge n_0$
 $\Leftrightarrow 24n^2 - 12n^2 - 6n + 6 \ge 0$
 $\Leftrightarrow 6n^2 - 6n + 6n^2 + 6 \ge 0 \quad \forall \quad n \ge 0$

Proof 2 if t%2 = 0: $12n^2 + 6n - 6 \in O(n^2)$ using limit theorem

 $n_0 = 0$

$$\lim_{n \to \infty} \frac{12n^2 + 6n - 6}{n^2} = \lim_{n \to \infty} \frac{(12n^2 + 6n - 6)'}{(n^2)'} = \lim_{n \to \infty} \frac{24n + 6}{2n} = \lim_{n \to \infty} \frac{(24n + 6)'}{(2n)'} = \lim_{n \to \infty} \frac{24}{2} = 12 \ge 0 \text{ and defined}$$

Proof 1 if
$$t\%2 = 1$$
: $12n^2 - 12 \in O(n^2)$
Choose $c = |12| + |12| = 24$
So $12n^2 - 12 \le 24n^2 \quad \forall \quad n \ge n_0$
 $\Leftrightarrow 24n^2 - 12n^2 + 12 \ge 0$
 $\Leftrightarrow 12n^2 + 12 \ge 0 \quad \forall \quad n \ge 0$
 $n_0 = 0$

Proof 2 if t%2 = 1: $12n^2 - 12 \in O(n^2)$ using limit theorem

$$\lim_{n \to \infty} \frac{12n^2 - 12}{n^2} = \lim_{n \to \infty} \frac{(12n^2 - 12)'}{(n^2)'} = \lim_{n \to \infty} \frac{24n}{2n} = \lim_{n \to \infty} \frac{(24n)'}{(2n)'} = \lim_{n \to \infty} \frac{24}{2} = 12 \ge 0 \text{ and defined}$$