# Mathematical Explanation of the Dynamic Team Selection Program

## Introduction

This document provides a mathematical explanation of the dynamic team selection program. The program selects a team of 11 players from a dataset to maximize a user-selected performance metric while satisfying user-defined constraints on other metrics. The selection is formulated as a linear programming (LP) problem and solved using the simplex method.

## Problem Formulation

### Decision Variables

Let:

$$x_i \in [0, 1] \quad \text{for } i = 1, 2, \ldots, n$$

where:

- $x_i$ represents the fraction of player $i$ selected.
- $n$ is the total number of players in the dataset.

### Parameters

- $c_i$: Value of the performance metric to maximize for player $i$.
- For each constraint $k$:
  - $d_{ik}$: Value of the constrained metric $k$ for player $i$.
  - $b_k$: Right-hand side value for constraint $k$.
  - $\text{op}_k$: Operator for constraint $k$ ($\leq, \geq, =$).

### Objective Function

Maximize the total performance score based on the selected metric:

$$\text{Maximize } Z = \sum_{i=1}^{n} c_i x_i$$

### Constraints

1. Total number of players selected equals 11:

$$\sum_{i=1}^{n} x_i = 11$$

2. User-defined constraints on other metrics:

   For each constraint $k$:

$$\sum_{i=1}^{n} d_{ik} x_i \operatorname{op}_k b_k$$

   where $\operatorname{op}_k$ is one of $\leq, \geq, =$.

3. Selection variables are between 0 and 1:

$$0 \leq x_i \leq 1 \quad \text{for all } i$$

# Explanation

## Decision Variables

The decision variables $x_i$ represent the fraction of each player $i$ selected for the team. Since we are selecting whole players, in practice, we interpret $x_i$ values close to 1 as indicating selection and values close to 0 as non-selection. The LP relaxation allows $x_i$ to be fractional to enable the use of the simplex method.

## Objective Function

The objective function aims to maximize the total sum of the selected performance metric over all players. Each player's contribution to the objective is proportional to their value of the performance metric ($c_i$) and their selection variable ($x_i$).

## Constraints

- **Total Players Constraint**: Ensures exactly 11 players are selected.
- **User-Defined Constraints**: These constraints allow the user to limit or enforce certain team characteristics based on other performance metrics.

  For example, if the user specifies a constraint like:

$$\text{Total fouls} \leq 20$$

  This translates to:

$$\sum_{i=1}^{n} \text{fouls}_i x_i \leq 20$$

- **Variable Bounds**: The selection variables $x_i$ are bounded between 0 and 1.

# Example

Suppose we have a dataset with 5 players and the user wants to maximize goals scored while setting a constraint on fouls committed.

## Data

| Player $i$ | Goals ($c_i$) | Fouls ($d_{i1}$) |
|---|---|---|
| 1 | 10 | 2 |
| 2 | 8 | 1 |
| 3 | 6 | 3 |
| 4 | 5 | 2 |
| 5 | 7 | 1 |

## User Input

- Maximize: Goals - Constraint: Total fouls $\leq 5$ - Total players to select: 3

## Model Formulation

**Decision Variables**:

$$x_i \in [0, 1] \quad \text{for } i = 1, 2, 3, 4, 5$$

**Objective Function**:

$$\text{Maximize } Z = 10x_1 + 8x_2 + 6x_3 + 5x_4 + 7x_5$$

**Constraints**:

1. Total players selected:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3$$

2. Total fouls constraint:

$$2x_1 + 1x_2 + 3x_3 + 2x_4 + 1x_5 \leq 5$$

3. Variable bounds:

$$0 \leq x_i \leq 1 \quad \text{for all } i$$

## Solution

Using the simplex method, we solve the LP problem to find the optimal values of $x_i$ that maximize $Z$ while satisfying the constraints.

Assuming the solution is:

$$x_1 = 1, \quad x_2 = 1, \quad x_5 = 1, \quad x_3 = 0, \quad x_4 = 0$$

## Interpretation

- Selected players: Players 1, 2, and 5. - Total goals:

$$Z = 10(1) + 8(1) + 7(1) = 25$$

- Total fouls:

$$2(1) + 1(1) + 1(1) = 4 \leq 5$$

## Conclusion

The dynamic team selection program uses linear programming to select a team that maximizes a user-selected performance metric while satisfying constraints on other metrics. The decision variables represent the fraction of each player selected, and the simplex method efficiently finds the optimal solution within the feasible region defined by the constraints.