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Modelling of a'fortiori reasoning

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ABSTRACT

The paper presents the model of two variants of a'fortiori reasoning applicable in the case of statutory law as well the example of the genuine law case, which has been modeled with use of established methodology. The model of reasoning assumes the existence of "less–more" relation between the analyzed actions, which has been expressed by means of strict partial order and some additional assumptions. The paper also contains the implementation of the analyzed example.

1. Introduction

IT systems employed to support legal interpretation are still the domain of research laboratories rather than that of the common legal practice. The immediate reason behind such a state of affairs lies not only because lawyers approach such tools with circumspection but also because common sense plays an important role, legal acts lack precision, and it is essential to analyse the context of a situation under analysis. At the core if it all is the inability to foresee a myriad of situations which occur in real life.

Law makers are not able to predict all such situations, and, therefore, legal provisions never (or nearly never) regulate the entire scope of events. Naturally, there is nothing wrong in this, particularly if one realises that there are proven ways to handle such cases both in theory and in practice.

Legal acts demonstrate a different level of generality, but even the most precise and coherent ones are not deterministic and complete enough for lawyers who, during argumentation, are bound to seek various rules of interpretation, conflict of law rules or other instruments which help them deal with the imperfections of statutory laws.

These methods, which go beyond formal principles of logic, are largely based on intuitive and informal reasoning. Lack of a formal model for such interpretation makes it impossible to use these methods in IT assisted legal counselling.

This paper demonstrates an attempt to formalise a'fortiori interpretation for the provisions of codified law. To begin with, an attempt to generalise the establish model for two versions of this method is made. This is followed by an attempt to show how a'fortiori interpretation can be applied to a specific example from real life.

A'fortiori interpretation hinges on axiological evaluation, which leads to a great deal of discretionary power that can, albeit not necessarily, apply. This paper is an attempt to formalise interpretation with a view to applying the same in advisory computer systems (particularly for resolving cases that are not expressly regulated by law). Nevertheless, a fully automatic application of such interpretation in an IT-assisted interpretation system remains to be seen. There is no doubt that such interpretation may be of significance in argumentation modelling systems and in selecting proper tools which help determine the right argument.

2. Related works

This study is an expansion of the research results presented during the ICAIL 2011 conference and further discussed in Zurek (2011).

Although, generally speaking, the issues of formal modelling for a'fortiori inference is not discussed in detail in the literature, some authors have made certain formalisation attempts. Hage (2005b) treats a'fortiori inference as a kind of inference by analogy. In general, the analogy-based inference model, as presented by Hage, is as follows:

A rule is applied analogously if

1. It is applied to a case that does not satisfy its conditions (otherwise it would be normal application).
2. Because the case is sufficiently similar to cases that do satisfy the rule conditions.

According to the model, a'fortiori inference relies on comparing two cases, i.e. the Old Case (OC) and the New Case (NC). At first, the author defines the premises for a'fortiori inference to occur Hage (2005b):

- NC provides better numerical support for *C* than OC, and equal or better weight-support, or
- NC provides equal or better numerical support for *C* than OC, and it gives better weight-support.

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Later in his study, the author specifies the conditions for better numerical and weight support:

Better numerical support is when the facts of NC that plead for legal consequence C are a proper superset of the facts of OC that plead for legal consequence *C* and the facts of NC that plead against legal consequence *C* are a (not necessarily proper) subset of the facts of OC that plead against legal consequence *C* or when the facts of NC that plead against legal consequence *C* are a proper subset of the facts of OC that plead against legal consequence *C* and the facts of OC that plead for legal consequence *C* are a (not necessarily proper) superset of the facts of NC that plead for legal consequence.

Better weight support is when the facts of NC that plead for legal consequence *C* provide as a group more weight-support for *C* than the facts of OC that plead for *C*, and the facts of NC that plead against legal consequence *C* provide as a group equal or less weight-support against *C* than the facts of OC that plead against *C*, or the facts of NC that plead against legal consequence *C* provide as a group less weight-support against *C* than the facts of OC that plead against *C*, and the facts of OC that plead for legal consequence *C* provide as a group more or equal weight-support for *C* than the facts of NC that plead for C set of reasons S1 provide as a group more weight support for *C* than reasons in set S2 if both sets support conclusion *C*, S2 is a subset of S1, at least one reason has more weight in S1 than in S2 and there is no reason which has more weight in S2 than in S1.

In his model, Hage treats a'fortiori inference as a kind of inference by analogy, referring more to the case law (that compares specific cases, rather than rules) than to the statutory law. Contrary to the model described in this study, the model presented in Hage (2005b) does not refer to any specific deontic states or legal rules, but it constitutes a much more general tool that allows us to structure the argumentation for a wide variety of "for" and "against" factors, and to accurately match a new case analysed to a previously-encountered one.

A kind of a'fortiori reasoning was also presented in very detailed way in Abraham, Gabbay, and Schild (2009). The authors of the paper called it "matrix abduction", and discussed it in the context of traditional talmudic law and some real life examples. The model can be employed in reasoning with incomplete information. By way of simplification, this model can be described as follows: it is assumed that specific cases under analysis exhibit certain properties (the matrix columns feature specific cases in the "property" lines). Value 1 in a matrix field means that the feature occurs in the case given whereas value 0 implies that it does not. Symbol "?" in one of the fields means that the value of this feature for this instant case is unknown. According to the model, one case is major over the other when it bears all features of the minor case plus at least one more. Further in the article, the authors present the label-assigning mechanism that allows for creating a graph which represents the order of analysed cases. In order to determine the missing value, two graphs are created by substituting "?" with 0 and 1. Then, in line with the simplicity criterion, as defined by the authors, the best matching value is selected. In general, the model described in Abraham et al. (2009) allows us to represent the mutual major–minor relations between analysed cases (which do not necessarily refer to legal problems) and to possibly infer the missing value of one case feature in the matrix. However, this does not directly refer to any deontic notions.

How does the aforementioned model relate to the one presented in this study? Firstly, the model in Abraham et al. (2009) is oriented towards exploring and seeking the major–minor relations in analysed cases (contrary to the model described here, which assumes a certain predefined order), rather than towards the process of legal inference. It is more general so it can be used in more diversified contexts, but it provides a less accurate reflection on a'fortiori inference, especially on the deontic notions of statutory laws. Inter alia, it makes no argumentative distinction between a minori ad maius and a maiori ad minus, nor does it refer to the values arising from the orders analysed, which are crucial to the reasoning process. The model in Abraham et al. (2009) admits several major–minor relations, though the shield et.cons. fail to analyse their source or quality, focusing on the identification of such orders and at predicting whether a given case "matches" a certain order (the issue of predicting the missing value in the matrix). Additionally, the model in Abraham et al. (2009) does not allow for the occurrence of two independent orders in two or more identical cases.

Different types of a'fortiori interpretation were also shortly discussed in Sion (1995) as well as in the other works of Hage (2001) and Roth (2001) who seek similarities between cases in case based reasoning.

The model presented in this paper focuses on a somewhat different variant of a'fortiori rule, whose scope is narrowed down to statutory law and deontic notions. This is the reason why most of the above mentioned models (including the most developed Abraham et al. (2009)) are not useful for representing such specific way of inferencing. However some solutions mentioned in Abraham et al. (2009) may be useful e.g. partial order as a representation of "better – worse" relation or way of ordering of groups of entities, as well as utilization of inclusion as a source of "better – worse" relation Hage (2005a).

Problem of open nature of the law, modeling of the rules and their application was widely described in Hage (2005b).

A'fortiori reasoning modeled in this article may be also treated as a way of increasing of applicability of the rules and contribution into better representation of open nature of the law. The issue of representing the character (whether positive or negative) of the inference value is yet another problem dealt with in this study. Various studies devoted to value-based inference Araszkiewicz (2010), Grabmair and Ashley (2011) tend to declare a certain value V (usually defined as a legal concept abstracting a set of one or more interests of an individual or a group (Grabmair & Ashley, 2011)) that is strengthened or weakened in various circumstances. The solution presented here is slightly different. Namely, an order that represents the major–minor relation can be negative, positive or neutral, i.e. one object may differ from another one in a positive way (it may be better), in a negative way (it may be worse) or in a neutral way (where the relation character cannot be expressly declared).

3. A'fortiori interpretation

A'fortiori interpretation is based on the following scheme: "if yes ... then the more ...," or: "if *A* then the more *B*". Such interpretation occurs in two variants Leszczynski (2001):

- Argumentum a maiori ad minus (from more to less), based on the following scheme: "If norm N1 obliging to do more is binding, then norm N2 obliging to do less is binding all the more". In a somewhat different construction interpretation from more to less is based on the following scheme: "if norm N1 permitting one to do more is binding, then norm N2 permitting one to do less is binding all the more."
- Argumentum a minori ad maius (from less to more), based on the following scheme: "if norm N1 imposing a ban on doing less is binding, then norm N2 imposing a ban on doing more is binding all the more."

Both variants of a'fortiori interpretation are particularly useful where a given situation or action is not expressly regulated by law, which allows one to interpret the deontic functor of a given action on the basis of the existence of an obligation or admission to do more or a ban to do less. It should be noted that the a'fortiori argumentation involves the axiological assessment which allows us to define to do "more" of what and "more" than what. Furthermore, one has to decide upon the area of the axiological assessment, namely, in what sense there is "more" of something than of something else. Also, the question of whether we can treat it as the right argument has to be answered.

4. General model

An attempt was made to define a general model of a'fortiori interpretation on the basis of the afore mentioned definitions. For the purpose of this study, it is assumed that the analysed set of classes contains action A of a certain rigid partial order. This order is represented by the "more-and-less" relation between the analysed actions. It follows that $Y < X$ relations means the occurrence of action of X class is something more than the occurrence of action of Y class. The analysed set of class actions may contain more orders and each of them may be determined on the basis of a different criterion.

It is assumed that the actions of a given order meet, besides the conditions determined by axiology, the conditions of a rigid partial order, i.e. they are counter-directional and transitive.

Given the specific nature of a'fortiori interpretation, it is worth considering the matter of defeasibility of rules. In Horty and Jan (2001) and Prakken and Sartor (1997) it was suggested that two types of implications should be distinguished: those that can be defeasible and those that cannot. Defeasible rules are non-monotonic in nature – the appearance of new conditions may undermine the operation of a rule. In Horty and Jan (2001) and Prakken and Sartor (1997) a defeasible implication was marked with "⇒". A classic implication that is not defeasible was marked with "→". The models of two variants of a'fortiori interpretation are presented below:

4.1. A maiori ad minus

It is assumed that a set of A class actions exists. The relation of partial order in $A : R = (A, <)$. This relation maps the "less-and-more" relation among the elements of the set.

Action classes of X, Y are elements of set A, i.e.:

$X, Y \in A,$

X represents more according to a set order than Y, i.e.:

$Y < X$

From more-to-less principle: If norm N1 obliging to do more is binding, then norm N2 obliging to do less is binding all the more. Model:

$$Obl(does(X)) \Rightarrow Obl(does(Y)) \tag{1}$$

Another variant of this principle: if norm N1 allowing to do more is binding, then norm N2 permitting one to do less is binding all the more.

Model:

$$Perm(does(X)) \Rightarrow Perm(does(Y)) \tag{2}$$

It is worth considering a particular case where Y is a part of X. If the analysed relation of partial order $Y < X$ results from the relation of parthood, the a'fortiori principle becomes easier to implement, since it is possible to define relevant ontology which describes classes of actions and relations of inclusion among them.

4.2. A minori ad maius

Similar to the previous case, it is initially assumed that there is a set of A class actions. It is also assumed that there exist a relation of partial order in the set of A actions $R = (A, <)$. Similar to the previous case, the determined order is based on the axiological evaluation of actions from set A. Classes of actions X, Y belong to set A:

$X, Y \in A,$

We assume that:

$X < Y$

Then, in accordance with the a'fortiori principle

$$Forb(does(X)) \Rightarrow Forb(does(Y)) \tag{3}$$

Similar to the previous model, in a specific case the ordering relation may come from the fact that one action is a part of another one (X is part of Y). Implementation in such a case is easier, since it is relatively simple to map the relations of inclusion by means of relevant ontology.

5. Many orders

In practice, it is not uncommon to find a situation where there are more orders than one in a given set of class actions, and where each order may result from a different criterion. These order may (and usually are) completely unrelated with one another. The application of a given order in specific a'fortiori interpretation depends on the specific character of the case under analysis, and it is not always reasonable to apply it in the context of an analysed situation.

In such a case one needs to consider a set of relations of partial order O in a set of A class actions. The set of pairs $R_n = (A, <_n)$, where $<_n$ is an N-relation of a rigid partial order in set A, sets of actions X, $Y \in A$ and where $Y <_n X$ may support the a'fortiori principle of a maiori ad minus formulated as follows:

$$Obl(does(X)) \Rightarrow Obl(does(Y)) \tag{4}$$

or

$$Perm(does(X)) \Rightarrow Perm(does(Y)) \tag{5}$$

or for a minori ad maius:

$$Forb(does(Y)) \Rightarrow Forb(does(X)) \tag{6}$$

There may exist more orders than one which support a specific a'fortiori principle, and without an in-depth analysis of a specific problem it is impossible to determine whether a given order actually supports the interpretation suggested. Nevertheless, such orders can be treated as potentially supportive of the a'fortiori principle.

Even where action X has a major status to $Y : X >_1 Y$ and $X >_2 Y$ in two different orders, deciding which of them should be used in the argumentation process is still not negligible. Therefore, it needs to be decided which order is adequate to the problem under analysis. This considerably restricts the possibility to pass "automatic judgements" but it does not make it impossible to implement this mechanism as an advisory tool for the argumentation process.

One way of dealing with the order selection problem may be to put certain labels to various orders and rules, describing specific comparison criteria and interpretation contexts, in order to allow for selecting the order corresponding to the purpose of a given inference process. This is shown on the following example:

Let us take two actions:

- travel_by_plane
- travel_by_train

two orders:

- $travel\_by\_plane >_{speed} travel\_by\_train$ (plane is faster than train)
- $travel\_by\_plane >_{cost} travel\_by\_train$ (plane is more expensive than train)

and a rule:

$$r1 : important\_bussiness\_travel \Rightarrow_{cost} Perm(does(travel\_by\_plane))$$

then the a'fortiori principle leads to the inference that:

$$r2 : important\_bussiness\_travel \Rightarrow_{cost} Perm(does(travel\_by\_train))$$

The model presented above can be generalised by assuming that when the following elements are given: a set of actions A, a set of partial order relations O, and a set of pairs $R_n = (A, <_n)$, where $<_n$ is a relation of sharp partial order on the set of actions A, and n is a relations label, and the rules take the following form:

$$condition \Rightarrow_m Perm(doesX) \tag{7}$$

or

$$condition \Rightarrow_m Forb(doesY) \tag{8}$$

or

$$condition \Rightarrow_m Obl(doesX) \tag{9}$$

where m is the rule label, whereas X and Y are sets of actions, and where: $X, Y \in A$ and $Y <_n X$ and $m = n$, then the a'fortiori interpretation, in the form of a maiori ad minus, can be presented as follows:

$$Obl(does(X)) \Rightarrow Obl(does(Y)) \tag{10}$$

or

$$Perm(does(X)) \Rightarrow Perm(does(Y)) \tag{11}$$

In the form of minori ad maius, it can be expressed as:

$$Forb(does(Y)) \Rightarrow Forb(does(X)) \tag{12}$$

The actual notion of the rule label seems worth deliberating. According to the author, the label should describe either the objective or the idea behind the rule. While creating a conjunction, or action alternatives (based on the formulas provided in previous chapters), the orders with different labels should be treated as incomparable.

It is worth noting that the problem of defeasibility Horty and Jan (2001), Prakken and Sartor (1997) has been discussed in various studies, which suggest that rules may be ordered and prioritised. In contrast, the model presented here entails ordering actions, not rules.

Unfortunately, even the labelling of the inference context may not be sufficient to make the analysed order match the argumentation process. In many cases, while defining the orders, one may face the problem of interpreting the notion of major–minor. Sometimes, apart from the occurrence of the relation itself, the values specific to it are also significant. Analysing two different orders ($battery < aggravated\_battery$) and ($wounds\_dressing < wounds\_dressing\_and\_hiring\_the\_doctor$), it can be noted that the first order is negative since aggravated battery is major and worse than battery whereas the second one is positive since wounds dressing and hiring the doctor is not only major, but also better than wounds dressing.

Coming back to our previous example:

When:

$$r1 : important\_bussiness\_travel \Rightarrow_{cost} Perm(does(travel\_by\_plane))$$
$$travel\_by\_plane >_{cost} travel\_by\_train$$

Then it can be inferred that:

$$r2 : important\_bussiness\_travel \Rightarrow_{cost} Perm(does(travel\_by\_train))$$

It can be intuitively assumed that $travel\_by\_plane >_{cost} travel\_by\_train$ means that the former is more expensive than the latter, that is if travel by plane is permitted, the more so is travel by train.

However, sometimes two different orders, describing opposing relations, can be assigned to some contexts. For instance, the order labelled "cost" can be viewed from different angles, i.e. $A >_{cost} B$ may imply that A is more expensive than B (as in the example quoted) or $A >_{cost} B$ may mean that A is cheaper than B (major, that is more favourable). Theoretically speaking, the order illustrated by our example can be reversed by assuming that travel by train, in terms of cost, is major (better) than travel by plane, as it is cheaper (such order can be labelled as "$cost'$"):

$$travel\_by\_train >_{cost'} travel\_by\_plane$$

Obviously, such an order can no longer be used in a'fortiori inference, described above.

Therefore, it is worth exploring the possible reasons behind this situation:

Firstly, it can easily be noted that the former order is negative ($A >_{cost} B$ implies that A is more expensive, i.e. worse than B) whereas the latter is positive ($A >_{cost'} B$ implies that A is more favourable than B). Secondly, it can be assumed that if there is a rule of law that permits a given action (referred to as express permission) then, intuitively speaking, it must refer to a negative action (our example deals with spending a large sum of money). On these grounds, one could make a generalisation: if the action permitted is negative then, based on the a'fortiori principle, something stronger, in a negative sense, will be even more so. A similar situation applies to the other type of a'fortiori inference. If an action is banned, then probably it is of a negative character. As a result, based on argumentum a minori ad maius, it can be assumed that something major (in a negative sense) will be banned as well. This can be illustrated on a simple example: Given the rule which says that travelling by train on a less important business trip is banned by reason of costs:

$$not\_important\_bussiness\_travel \Rightarrow_{cost} Forb(does(travel\_by\_train))$$

and given the order:

$$travel\_by\_plane >_{cost} travel\_by\_train$$

then, referring to the a'fortiori principle, it can be inferred that travelling by plane is banned as well.

$$not\_important\_bussiness\_travel \Rightarrow_{cost} Forb(does(travel\_by\_plane))$$

Reversing the order (as shown in the previous example) also fails to allow for such inference. Things look slightly different for the command (or order) related a'fortiori a maiori ad minus inference, as in this event the minor–major relation usually refers to something positive (it can aim at maximising the positive value), though it does not have to. Essentially, the command refers to a specific action and something minor, whether positive or negative, which does not form part of this command, does not refer to it. The command to go on an important business trip is, on no account, tantamount to performing any other minor action, such as travelling by train (irrespective of how the order between them is defined). In this case, the a'fortiori principle may be limited only to the situation where the minor–major relation results from the inclusion relation of one action within the other (as mentioned in the previous chapters). If a given action is ordered, then its part is even more so. For instance, if a person is ordered to pay taxes on the entire income he or she earns, then this also applies to taxes on salary. Based on the formula included in the previous chapter, it can be assumed that:

$$paying\_taxes\_on\_salary\ \boldsymbol{partOf}\ paying\_taxes\_on\_the\_entire\_income \Rightarrow (paying\_taxes\_on\_salary <_{partOf} paying\_taxes\_on\_the\_entire\_income)$$

The aforementioned examples show that not only the major–minor order, but also its character (positive, negative or neutral) is crucial to at least two kinds of a'fortiori inference. Therefore, with a view to developing a more thorough model of such inference, this needs to be considered as well. The idea is to add more information, concerning the character of the analysed order, to the model described. Summing up, given a set of action A, a set of relations of partial order O and a set of pairs $R_n = (A, C<_n)$, where $C<_n$ is the relation of a sharp partial positive order on the set of action A, n is the label and C represents the character of this relation and takes the value + (for the positive character), – (for the negative one) or empty (for the neutral one).

Yet another issue of the major–minor relation, resulting from the fact of being a part of the whole (partOf), is worth discussing. Such a relation is more general than, and fully independent of, any specific contexts indicated by the label. Therefore, one may reasonably assume that the order resulting from being "a part" is universal and can be used in the inference process, regardless of the context. However, where there is no predefined character, it remains neutral and can be considered from both a positive and negative angle.

If the "partOf" order has a declared character, then its negative or neutral feature can be used with argumentum a minori ad maius and a maiori whereas with the command-related argumentum a maiori ad minus any relation can be used, whether positive, negative or neutral.

Based on the above, a'fortiori inference in the permission-related kind of argumentum a maiori ad minus (If norm N1 permitting one to do more is binding, then norm N2 permitting one to do less is binding all the more) may be defined in a such way:

$$X, Y \in A \wedge ((Y - <_n X) \vee (Y - <_{partOf} X) \vee (Y <_{partOf} X))$$

May support the a'fortiori principle of a maiori ad minus formulated as follows:

$$(condition \Rightarrow_n Perm(does(X))) \Rightarrow (condition \Rightarrow_n Perm\ (does(Y))) \tag{13}$$

Analogically for argumentum a minori ad maius:

When:

$$X, Y \in A \wedge ((Y - <_n X) \vee (Y - <_{partOf} X) \vee (Y <_{partOf} X))$$

Then:

$$(condition \Rightarrow_n Forb(does(Y))) \Rightarrow (condition \Rightarrow_n Forb(does(X))) \tag{14}$$

In a second variant of a maiori ad minus (If norm N1 obliging to do more is binding, then norm N2 obliging to do less is binding all the more):

When:

$$X, Y \in A \wedge ((X + <_{partOf} Y \vee X - <_{partOf} Y \vee X <_{partOf} Y))$$

Then:

$$(condition \Rightarrow_{partOf} Obl(does(Y))) \Rightarrow (condition \Rightarrow_{partOf} Obl(does(X))) \tag{15}$$

In the event where order $A+<_{label} B$ is given and the inference requires a negative order, the latter can be inferred through:

$$A + <_{label} B \iff B - <_{label} A$$

Undoubtedly, a careful observer may note that the character of inter-action relations can be viewed in a different way from various perspectives. For the buyer, $A-<_{cost} B$ may imply that B is more expensive (i.e. worse because the order is negative) whereas for the seller, $A-<_{cost} B$ may imply that B is cheaper (i.e. worse because it brings less profits). In this event, the safest solution will be to distinguish between the buyers order: $A-<_{cost} B$ and the sellers order: $B-<_{profit} A$.

6. Relations between orders

The relations between different action classes can be varied. Even the simplest major–minor relations, as discussed above, can be interrelated in various ways. It may be worth exploring the inclusion relations ofone context, described by a certain order, within the other. This can be illustrated by these two orders:

$$travel\_by\_train\ - <_{cost}\ travel\_by\_plane$$
$$travel\_by\_train\ - <_{ticketPrice}\ travel\_by\_plane$$

As the ticket price is a component of the overall travel cost, it can be assumed that the context labelled "ticketPrice" is included in the context labelled "cost." At this point, we may ask whether a certain order referring to a more specific context can be inferred from a more general order (and vice versa). We can easily find some examples showing that, in some cases, such inference could be wrong: if travelling by train is known to be cheaper that travelling by plane, we may intuitively assume that a train ticket is cheaper than a plane ticket. However, it may occur that, due to a hugely discounted price, the latter will be very cheap (and cheaper than the former), though the overall cost of getting to and from the airport, as well as paying additional charges will still exceed the overall cost of travelling by train.

What would happen if only the a general order was given and the legal rule label referred to a more specific order? If no specific order is determined, that would directly refer to the rule, with respect to which the argumentation process would require applying the a'fortiori inference, then the system itself could suggest referring to the general order. Obviously, such a solution would not always guarantee the accurate result of the argumentation process, but it would create an opportunity, in case of missing data, to construct the argumentation process that would be at least potentially correct.

How would this process work?

It would be based on the previously discussed a'fortiori inference mechanism, provided that the necessary order was not defined, the major order was given, and the inclusion relation between these two was known. For instance, for the permission-related argumentum a maiori ad minus.

Assumptions:

$$X, Y, Z \in A$$

and

$$Y - <_m X$$

and

$$n \in m$$

and

$$condition \Rightarrow_n Perm(does(X))$$

may support the a'fortiori principle of a maiori ad minus formulated as follows:

$$Perm(does(X)) \Rightarrow_n Perm(does(Y)) \tag{16}$$

This can be illustrated as follows:

If it is known that:

$$r1 : important\_bussiness\_travel \Rightarrow_{ticketPrice} Perm(does(travel\_by\_plane))$$
$$travel\_by\_train \ -<_{cost} travel\_by\_plane$$

There is no declared relation of the plane ticket price to the train ticket price. However, it is known that the context labelled "cost" is a set of contexts and context labelled "ticketPrice" is included in the context labelled "cost" label. It can be, therefore, assumed that:

$$(travel\_by\_plane >-_{cost} travel\_by\_train) \wedge (ticketPrice \subset cost) \wedge \sim (travel\_by\_train >-_{ticketPrice} travel\_by\_plane) \Rightarrow (travel\_by\_plane >-_{ticketPrice} travel\_by\_train)$$

Considering the above, one could apply a'fortiori inference, where necessary, i.e. based on the rule r1 and the order inferred:

$$travel\_by\_plane \ >-_{ticketPrice} travel\_by\_train$$

then, it can be inferred that:

$$r2 : important\_bussiness\_travel \Rightarrow_{ticketPrice} Perm(does(travel\_by\_train))$$

As has already been proven, such a solution does not guarantee that the order labelled "ticketPrice" actually exists but the occurrence of the more general order, in the absence of contradictory evidence, allows us to assume the existence of the "ticketPrice" order. The inference mechanism described is clearly defeasible and the appearance of any undermining evidence, such as that $travel\_by\_plane >-_{ticketPrice} travel\_by\_train$ may exclude it from argumentation.

7. Sample judicial decision

The application of the methodology presented allows for modelling a specific application of a'fortiori interpretation Court (2009):

Case summary: In this instant case Mr. G.D., legal adviser and power of attorney of Mr. W.J.M., filed a complaint against the refusal of the Director of the Treasury Office to forward a copy to Mr. D.B. of a letter from the Fiscal Inspection Office sent to the Statistical Office in L on the grounds that the letter in question was excluded from the inspection in view of public interest. The complaint was dismissed.

Justification

It can be inferred from the provisions of Article 179, paragraph 1 of the Tax Ordinance Act that the tax authority is authorised to exclude a document from the case records on the grounds of public interest. Since the competences of the tax authority extend that far, and an entire document can be excluded from case records, it is only legitimate to accept a motion concerning deprivation of access of a party exclusively to a part of the document. This conclusion can be drawn from the type of reasoning called argumentum a'fortiori (from the stronger) and its variant a maiori ad maius (from more to less). If someone is authorised to do more, he/she is also authorised to do less. Doing more is connected in this instant case with a bigger restriction of rights of the party to said proceedings. It follows that if a tax authority can restrict the rights of a party to a wider extent, it can also restrict such rights to a narrower extent.

Case model

- X – class of action: exclusion of a document from case records
- Y – class of action: deprivation of one party of access to parts of the document
- x – specific action: exclusion of the document from case records
- y – specific action: deprivation of one party of access to parts of the document
- x instanceOf X – action x is an instance of class X
- y instanceOf Y – action y is an instance of class Y
- cond (X) – condition which action X must meet: in this case the aim of action X is public interest (whether this is really in public interest or not is not determined in this example).

Assumptions resulting from the provisions of Article 179, paragraph 1:

$$cond(X) \Rightarrow_{excludingDocument} Perm(does(X)) \tag{17}$$

Under the necessary assumption in a'fortiori interpretation in the a maiori ad maius variant, action y must be less than action x. A question thus arises about the nature of the distinction between more and less. The nature of this distinction is not specified precisely in the definition of a'fortiori interpretation. In the general model of a'fortiori interpretation, as shown above, it was assumed that the "more–less" dependence between the actions is reflected in the relation of rigid partial order between the classes of actions. In some cases the ordering relation may result from the relation of being a *part of* with respect to individual action classes. Indeed, in the analysed case the relation of parthood occurs and it has negative character. Action x consists in an exclusion of the document from case records while action y consists in depriving one party of access to parts of the document. This relation meets the requirements of a rigid partial order whereas other, previously specified conditions which a given order must meet do not refer to this case, since there are only two classes of action involved.

Therefore, the following can be stated for the analysed case:

$$\begin{aligned}&(cond(X) \Rightarrow_{excludingDocument} Perm\ (does(X))) \wedge (xinstanceOf\ X)\\ &\wedge (y\ instanceOf\ Y) \wedge (Y\ -<_{excludingDocument} X)\\ &\Rightarrow (Cond(y)\ \Rightarrow\ Perm\ (does(y)))\end{aligned} \tag{18}$$

or

$$\begin{aligned}&(cond(X) \Rightarrow_{excludingDocument} Perm\ (does(X))) \wedge (x\ instanceOf\ X)\\ &\wedge (y\ instanceOf\ Y) \wedge (Y\ -<_{partOf} X)\\ &\Rightarrow (Cond(y)\ \Rightarrow Perm\ (does(y)))\end{aligned} \tag{19}$$

After the transformation, one can examine whether action y is permissible:

$$\begin{aligned}&(cond(X) \Rightarrow_{excludingDocument} Perm\ (does(X))) \wedge (x\ instanceOf\ X)\\ &\wedge (y\ instanceOf\ Y) \wedge Cond(y)\ \wedge (Y\ -<_{partOf} X)\\ &\Rightarrow\ Perm\ (does(y))\end{aligned} \tag{20}$$

8. Implementation

In order to visualise the model described above, the PROLOG language was employed to implement a'fortiori interpretation in the example provided. The choice of the language was determined by its relatively easy and concise manner of representing logical dependencies. In the first instance, a declaration of actions x and y as well as types thereof is effected:

```
action(x, exclusion_document_from_case_records).
action(y, deprivation_of_one_party_of_right_to
_access_part_document).
```

Afterwards, the property of action y is defined, i.e. its aim or public interest in this instant case:

```
action_property(y, aim, public_interest).
```

The next step is to define the more–less relation and its character:

```
more(exclusion_of_document_from
_case_records,                  deprivation_of_one-
party_of_access_to_
parts_of_document, excludingDocument, negative).
```

The definition of the rule of permitting whose arguments include type of condition, condition, action permitted, and legal basis:

```
permitting_rule(aim, public_interest, exclusion_of
_document_from_case_records, Article179, excluding
Document).
```

Definition of express permission under the rule of permitting:

```
perm(X):- permitting_rule(Aim, Y,Z,Basis,_),
target_action (Z,Basis), action(X, Z),
action_property (X, Aim, Y).
```

Definition of permission under the a'fortiori principle:

```
perm(Y):- permitting_rule(Aim, Condition, Action,
Basis, Context), target_action (Action, Basis),
action_property(Y,Aim,Condition), action(Y,Name),
more(Action, Name, negative, Context).
```

or

```
perm(Y):- permitting_rule(Aim, Condition, Action,
Basis, partOf), target_action (Action, Basis),
action_property(Y,Aim,Condition), action(Y,Name),
more(Action, Name, negative, partOf).
```

or

```
perm(Y):- permitting_rule(Aim, Condition, Action,
Basis, partOf), target_action (Action, Basis),
action_property(Y,Aim,Condition), action(Y,Name),
more(Action, Name, neutral, partOf).
```

One of the variants of the a'fortiori principle defined in such a way allows for drawing correct conclusions for situations which are not expressly defined in law. Expressly defined legal rules are – in the example specified above – represented by the permitting_rule(...) and executed by the first definition of the perm(...) predicate. Despite its clarity and conciseness in representing logical dependencies, the PROLOG language has some drawbacks which are described in Gordon (1987) and which could prove insufficient in the implementation of a larger system. The methodology employed here can, however, be applied in using other tools, including those available from commonly available rule engines or by combining the PROLOG language with other tools.

9. Conclusions

This study presents two a'fortiori interpretation models, used in the statutory law. This kind of interpretation is based on the minor–major relation, which stipulates that whenever a certain obligation (or admission) refers to the major issue, then it much more refers to the minor issue, or – in the opposite case – if a certain ban refers to the minor issue, then it much more refers to the major one. Representing the minor-major relation is the crucial aspect of such reasoning. In the model analysed, this relation is presented as the relation of partial order on a set of actions. The source of a certain minor-major relation, which is often based on the common sense criteria, and as such is not easily represented on a formal model, creates another challenging problem.

A model situation comprising several orders representing the minor-major relation was also presented later in the study. The multiplicity of orders representing such relations makes the original, and relatively simple, assumptions behind the a'fortiori interpretation model considerably more complex. First of all, it may occur that action A has a major status to action B in one order, whereas a different order may create an inverted effect. Furthermore, even if *A* has a major status to B in both orders, we still cannot make a random choice of order, since the order representing the improper relation may be meaningless in the context of certain argumentation, making it entirely senseless. To make a choice of order in line with the interpretation context, the author suggests attaching labels the orders which represent the minor-major relation, and the applicable legal rules. The labels should either refer to the context or to the interpretation objective, thereby making it possible to relate a given order to the right argumentation process. This study also casts some light on the character of the minor-major relations, considering that it is not without essence to the a'fortiori inference process whether the major–minor relation is positive, negative or neutral. The author suggests that only negative orders should be used for the a'fortiori inference in the kind of permission-related argumentum a maiori ad minus and argumentum a minori ad maius whereas in the case of command-related argumentum a maiori ad minus only such orders should be applied that arise from the inclusion relation of one action within the other. The minor-major relations can be defined at various generalisation levels. This study proposes a mechanism that allows for a more specific order to be inferred from the more general one. It can be useful in the event where the a'fortiori inference in the argument constructed requires a representation of the minor-major relation which is not declared in the knowledge base. It is worth noting that in such a model of a'fortiori inference, the positive and neutral relations, other than of the partOf kind, cannot be used directly. Positive relations may, nevertheless, be used to infer negative ones (based the formula presented at the end of Chapter X). One of the most serious challenges posed by the model described stems from the disputability of assessing the relation character, i.e. whether something is major in a positive or negative sense. Such an assessment is rather subjective and often disputable, and it may vary according to specific interests. This problem can be partly solved by attaching various labels to the description of the relations assessed from different angles. However, as regards disputable relations, the problem unfortunately stems not from the model itself, but from the commonsense knowledge and the subjective assessment by users. A'fortiori inference, which is defeasible, can be used in the argumentation process but we should be mindful that other arguments may defend it. In general, legal argumentation is of a discretionary nature: the potentially correct line of reasoning may not always be considered reasonable (e.g., by the court). This may also be the case with the model presented herein. As a result, it should not be treated as a mechanism that ensures the right conclusion, but rather as a mechanism proposing a potentially correct one.

The a'fortiori interpretation models presented in this study by no means exhaust the possible applications of this reasoning method, and they are limited to two specific types, related to deontic notions.

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