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A random fuzzy minimum spanning tree problem through a possibility-based value at risk model

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ABSTRACT

This paper considers a minimum spanning tree problem under the situation where costs for constructing edges in a network include both fuzziness and randomness. In particular, this article focuses on the case that the edge costs are expressed by random fuzzy variables. A new decision making model based on a possibility measure and a value at risk measure is proposed in order to find a solution which fully reflects random and fuzzy information. It is shown that an optimal solution of the proposed model is obtained by a polynomial-time algorithm.

1. Introduction

A minimum spanning tree problem is one of the most important combinatorial optimization problems, which is seen as real-world decision making problems (Ferreira, Ochi, Parada, & Uchoa, 2012; Kim, Park, Kwon, & Chang, 2012). It is well known that when edge costs are given as constants, minimum spanning tree problems can be solved by polynomial-time algorithms (Cherition & Tarjan, 1976; Gabow, Galil, Spencer, & Tarjan, 1986; Kruskal, 1956; Prim, 1957).

Since costs for constructing edges are often uncertain in the real world, some articles dealt with the cases where the costs are expressed as random variables or fuzzy numbers. Ishii, Shiode, Nishida, and Namasuya (1981) firstly considered a minimum spanning tree problem with random edge costs and constructed a polynomial-time algorithm, which was revised by Geetha and Nair (1993) as a more efficient one. Considering the ambiguity of experts' knowledge or judgments, fuzzy mathematical programming approaches have been developed in order to handle various decision making problems such as supply chain network problem (Bilgen, 2010), project management decision (Liang, 2010), pricing and marking planning model (Sadjadi, Ghazanfari, & Yousefli, 2010) and so on. As for minimum spanning tree problems under fuzziness, Itoh and Ishii (1996) coped with minimum spanning tree problems with fuzzy edge costs. These studies focus on environments in which either randomness or fuzziness exists.

However, in real-world problems, decision makers are often faced with situation where not only randomness related to uncertain factors involved in stochastic phenomena but also fuzziness related to human's judgments are to be considered (Luhandjula, 2006). Kato et al. discussed a fuzzy programming approach to stochastic programming problems (Kato, Sakawa, Katagiri, & Wasada, 2004; Kato, Sakawa, Katagiri, & Perkgoz, 2010) and bi-level (two-level) programming (Kato, Katagiri, Sakawa, & Wang, 2006). Applying fuzzy random variables (Kwakernaak, 1978; Puri & Ralescu, 1985), some researchers considered single/multiobjective linear programming problems with fuzzy random variables (Katagiri & Sakawa, 2011; Katagiri, Sakawa, & Ishii, 2005; Katagiri, Sakawa, Kato, & Ohsaki, 2005; Katagiri, Sakawa, Kato, & Nishizaki, 2008), fuzzy random 0–1 programming problems (Katagiri & Sakawa, 2004) and fuzzy random inventory models (Wang, 2011).

In particular, as for minimum spanning tree problems closely related to the topic of this paper, Katagiri et al. considered a fuzzy random minimum spanning tree problems (Katagiri, Mermri, Sakawa, Kato, & Nishizaki, 2005) and a bottleneck type of fuzzy random minimum spanning tree problems (Katagiri & Ishii, 2000; Katagiri, Sakawa, & Ishii, 2004). Although a fuzzy random variable is a useful mathematical tool for handling uncertain parameters with both fuzziness and randomness, it has a limitation in the range of applications because fuzzy random variables can be mainly applied to the case where the realized values of random variables becomes fuzzy sets.

In order to deal with fuzzy stochastic parameters that cannot be represented with fuzzy random variables, recently, a random fuzzy variable (Liu, 2002, 2004) draws attention as a new tool for decision making problems under random fuzzy environments, such as random fuzzy programming problems (Liu, 2002), portfolio

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selection problems (Hasuike, Katagiri, & Ishii, 2009), project selection problems (Huang, 2007), investment decision problems (Sakalli & Baykoc, 2010) and facility location problems (Wen & Iwamura, 2008). For example, when edge costs are expressed by random variables in formulating a minimum spanning tree problem, there is a situation where the mean of a random variable is estimated not as a constant but as a fuzzy set because of a lack of information. In this case, the edge costs can be expressed not by fuzzy random variables but by random fuzzy variables.

Under these circumstances, this article firstly tackles a new minimum spanning tree problem where edge costs are represented by random fuzzy variables, especially by Gaussian random variables with fuzzy mean. It should be noted here that it is generally difficult to construct reasonable decision making models for problems involving random fuzzy variables. Furthermore, even if a certain decision making model is constructed, it is often difficult to obtain an optimal solution of the decision making model because of nonlinearity of the formulated problem. In order to overcome the difficulty, we shall propose a new decision making model using both a possibility measure and show that a deterministic equivalent minimum spanning tree problem can be exactly solved by a polynomial-time algorithm.

This paper is organized as follows. Section 2 introduces a definition of random fuzzy variables and provides a fuzzy set-based simple definition. In Section 3, we formulate a minimum spanning tree problem with random fuzzy edge costs. Section 4 proposes a decision making model based both on stochastic and possibilistic programming, especially, using a possibility measure and a value at risk measure. Furthermore, we show the problem can be transformed into the deterministic nonlinear minimum spanning tree problem. Section 5 constructs a polynomial-time algorithm to solve the deterministic problem obtained in Section 4. After providing a numerical example in Section 6, we conclude this paper and discuss future studies in Section 7.

2. Random fuzzy variables

When one uses a random variable in order to express an uncertain parameter related to a stochastic factor of real systems, it is implicitly assumed that there exists a single random variable as a proper representation of the uncertain parameter. However, in some cases, experts may think of several possible random variables, namely a set of random variables, rather than a single one, in order to express the uncertain parameter as straightly as possible. In this case, it would be quite natural to assign the different degrees of possibility to elements in a set of random variables, reflecting the experts' different conviction degrees that these random variables properly represent the uncertain parameter. For handling such real-world decision making situations, a random fuzzy variable was introduced by Liu (2002) and explicitly defined (Liu, 2004) as a function from a possibility space to a collection of random variables.

In this section, we firstly introduce the Liu's original definition of random fuzzy variables, which is considered to be an extended concept of fuzzy variable (Nahmias, 1978). Next, we provide a simple definition of random fuzzy variables as a natural extension of fuzzy sets. Our purpose is to provide just another mathematical way of dealing with random fuzzy variables from the viewpoint of fuzzy set theory, and our definition does not contradict the Liu's original one.

2.1. Liu's original definition of a random fuzzy variable

In preparation for introducing a random fuzzy variable, the definition of possibility space is given as follows:

Definition 1 (*Possibility space (Liu, 2004)*). Let Θ be a nonempty set, and $P(\Theta)$ be the power set of Θ. For each $A \in P(\Theta)$, there is a nonnegative number *Pos{A}*, called its possibility, such that

1. $Pos\{\emptyset\} = 0$, $Pos\{\Theta\} = 1$; and
2. $Pos\{\cup_k A_k\} = \sup_k Pos\{A_k\}$ for any arbitrary collection $\{A_k\}$ in $P(\Theta)$.

The triplet (Θ, $P(\Theta)$, *Pos*) is called a possibility space, and the function *Pos* is referred to as a possibility measure.

Then, a random fuzzy variable is firstly defined by Liu (2004) as a function from a possibility space to a collection of random variables.

Definition 2 (Random fuzzy variable (Liu, 2004)). A random fuzzy variable is defined as a function from the possibility space (Θ, $P(\Theta)$, *Pos*) to the set of random variables.

An example of random fuzzy variables are given by Liu (2004) as follows:

Example 1 (*Liu, 2004*). Assume that $\bar{\eta}_1, \bar{\eta}_2, \ldots, \bar{\eta}_m$ are random variables and $u_1, u_2, \ldots, u_m$ are real numbers in [0,1] such that $u_1 \vee u_2 \vee \cdots \vee u_m = 1$. Then $\bar{\tilde{\xi}}$ is a random fuzzy variable expressed as

$$\bar{\tilde{\xi}} = \begin{cases} \bar{\eta}_1 & \text{with possibility} \quad u_1 \\ \bar{\eta}_2 & \text{with possibility} \quad u_2 \\ & \cdots \\ \bar{\eta}_m & \text{with possibility} \quad u_m. \end{cases} \tag{1}$$

It should be note here that $\bar{\tilde{\xi}}(i) = \bar{\eta}_i,\ i = 1, 2, \ldots, m$ are can be regarded as functions from a possibility space (Θ, $P(\Theta)$, *Pos*) to a collection of random variables Γ if we define $\Theta = \{1, 2, \ldots, m\}$, $Pos\{i\} = u_i$, $i = 1, 2, \ldots, m$ and $\Gamma = \{\bar{\eta}_1, \bar{\eta}_2, \ldots, \bar{\eta}_m\}$.

Similar to the Nahmias's approach (Nahmias, 1978), the membership function of a random fuzzy variable is defined as follows:

Definition 3 (*Membership function of a random fuzzy variable* (Liu, 2004)). Let ξ be a random fuzzy variable on the possibility space (Θ, $P(\Theta)$, *Pos*). Then its membership function is derived from the possibility measure *Pos* by

$$\mu(\bar{\eta}) = Pos\{\theta \in \Theta | \xi(\theta) = \bar{\eta}\}, \quad \bar{\eta} \in \Gamma. \tag{2}$$

2.2. Fuzzy set-based definition of a random fuzzy variable

Liu's definition of random fuzzy variables is based on fuzzy variables that were defined by Nahmias (1978). As Nahmias mentioned in his article (Nahmias, 1978), the fuzzy variables can be viewed as another way of dealing with the imprecision which was originally represented by fuzzy sets. In this section, we introduce a simple definition of random fuzzy variables as an extended concept of fuzzy sets. Note that our definition does not either contradict or reach beyond Liu's definition, and that it just provides another way of viewing random fuzzy variables from a slightly different direction.

Definition 4 (*Random fuzzy variable*). Let Γ be a collection of random variables. Then, a random fuzzy variable $\bar{\tilde{C}}$ is defined by its membership function

$$\mu_{\bar{\tilde{C}}} : \Gamma \to [0, 1]. \tag{3}$$

Remark 1. In Definition 4, the membership function $\mu_{\bar{\tilde{C}}}$ assigns each random variable $\bar{\gamma} \in \Gamma$ to a real number $\mu_{\bar{\tilde{C}}}(\bar{\gamma})$. If Γ is defined as $\mathbb{R}$, then (3) becomes equivalent to the membership function of an ordinary fuzzy set. In this sense, a random fuzzy variable can be regarded as an extended concept of fuzzy sets.

Remark 2. In Definition 4, if Γ is defined as a singleton $\Gamma = \{\bar{\gamma}\}$ of which membership function is given as $\mu_{\bar{\tilde{C}}}(\bar{\gamma}) = 1$, then the corresponding random fuzzy variable $\bar{\tilde{C}}$ can be viewed as an ordinary random variable. Hence, a random fuzzy variable can be viewed as an extended concept of random variables.

Using the above definition, we express the membership function of the random fuzzy variable shown in Example 1.

Example 2. Assume that $\bar{\eta}_1, \bar{\eta}_2, \ldots, \bar{\eta}_m$ are random variables and $u_1, u_2, \ldots, u_m$ are real numbers in [0,1] such that $\max\{u_1, u_2, \ldots, u_m\} = 1$. Then $\bar{\tilde{\eta}}$ is a random fuzzy variable and its membership function is expressed as

$$\mu_{\bar{\tilde{\xi}}}(\bar{\gamma}) = \begin{cases} u_1 & \text{if} \quad \bar{\gamma} = \bar{\eta}_1 \\ u_2 & \text{if} \quad \bar{\gamma} = \bar{\eta}_2 \\ & \ldots \\ u_m & \text{if} \quad \bar{\gamma} = \bar{\eta}_m. \end{cases} \tag{4}$$

As another simple example of random fuzzy variables, we consider a Gaussian random variable of which mean is not definitely specified as a constant but as a fuzzy set.

Example 3. Let $\bar{\gamma}$ be a Gaussian random variable $N(s, 10^2)$ where an expert identifies possible mean values s as a set $(s_1, s_2, s_3) = (100, 200, 300)$. If the membership function $\mu_{\bar{\tilde{C}}}$ is defined by

$$\mu_{\bar{\tilde{C}}}(\bar{\gamma}) = \begin{cases} 0.6 & \text{if } \bar{\gamma} \sim N(100, 10^2) \\ 0.8 & \text{if } \bar{\gamma} \sim N(200, 10^2) \\ 0.4 & \text{if } \bar{\gamma} \sim N(300, 10^2) \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

then $\bar{\tilde{C}}$ is a random fuzzy variable.

Example 3 corresponds to the case where the mean of Gaussian random variable is expressed by fuzzy set $\tilde{S} = \{(100, 0.6), (200, 0.8), (300, 0.4)\}$ as a set of pairs of each element and its membership function value. More generally, we can consider the case where $\tilde{S}$ is a fuzzy number, which will be later discussed in the main theme of this paper.

3. Problem formulation

3.1. Definition of spanning tree

Let $G = (N, E)$ denote an undirected graph consisting of a vertex set $N = \{v_1, v_2, \ldots, v_k\}$ and an edge set $E = \{e_1, e_2, \ldots, e_l\} \subset N \times N$. Let c_j be the cost attached to an edge e_j, $j = 1, 2, \ldots, l$. Then a spanning tree $T = T(N, E_S)$ of G is a partial graph satisfying the following four conditions: (1) T has the same vertex set as G; (2) $|E_S| = k - 1$, where $|E_S|$ denotes the cardinality of set E_S; (3) T is connected; (4) T includes no cycle. T can be denoted with 0–1 variables $x_1, x_2, \ldots, x_l$; each x_j is 1 if $e_j \in E_S$ and otherwise, x_j is 0.

Conversely, if the edge set $\{e_j | x_j = 1\}$ constructs a spanning tree of G with a vertex set N, then $\boldsymbol{x} = (x_1, x_2, \ldots, x_l)$ is also called a spanning tree hereafter in this paper.

3.2. Minimum spanning tree problem with random edge costs

In this paper, we deal with the following minimum spanning tree problem:

$$\left.\begin{array}{ll} \text{minimize} & \bar{\tilde{\boldsymbol{C}}}\boldsymbol{x} \\ \text{subject to} & \boldsymbol{x} \in X \end{array}\right\} \tag{6}$$

where $\bar{\tilde{\boldsymbol{C}}} = (\bar{\tilde{C}}_1, \bar{\tilde{C}}_2, \ldots, \bar{\tilde{C}}_l)$ is a coefficient vector, $\boldsymbol{x}$ is a $1 \times l$ column decision vector, and X is a set of all spanning trees in a given graph. Assume that each $\bar{\tilde{C}}_j$ is a random fuzzy variable. In this article, we focus on the case where each $\bar{\tilde{C}}_j$ is a Gaussian random variable with fuzzy mean, namely, $\bar{\tilde{C}}_j \sim N\left(\tilde{M}_j, \sigma_j^2\right)$. To be more explicit, we assume that the probability density function f_j of $\bar{\tilde{C}}_j$ is formally represented with

$$f_j(z) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(z - \tilde{M}_j)^2}{2\sigma_j^2}\right), \quad j = 1, 2, \ldots, l, \tag{7}$$

where $\tilde{M}_j$ is an L–R fuzzy number characterized by the following membership function:

$$\mu_{\tilde{M}_j}(t) = \begin{cases} L\left(\frac{m_j - t}{\alpha_j}\right) & (m_j \geqslant t) \\ R\left(\frac{t - m_j}{\beta_j}\right) & (m_j < t). \end{cases} \tag{8}$$

Functions L and R are called reference functions or shape functions which are nonincreasing upper semi-continuous functions $[0, \infty) \to [0, 1]$ satisfying the following conditions:

(a) $L(0) = R(0) = 1$.
(b) There exists a t_0^L such that $L(t) = 0$ for any t larger than t_0^L.
(c) There exists a t_0^R such that $R(t) = 0$ for any t larger than t_0^R.

When $\bar{\tilde{C}}_j$ is a random fuzzy variable characterized by (7) and (8), the membership function $\mu_{\bar{\tilde{C}}_j}$ of $\bar{\tilde{C}}_j$ is expressed as

$$\mu_{\bar{\tilde{C}}_j}(\bar{\gamma}_j) = \sup_{s_j} \left\{ \mu_{\tilde{M}_j}(s_j) | \bar{\gamma}_j \sim N\left(s_j, \sigma_j^2\right) \right\}, \quad \forall \bar{\gamma}_j \in \Gamma, \tag{9}$$

where Γ is a universal set of all possible Gaussian random variables as follows:

$$\Gamma \triangleq \{\bar{u} \mid \bar{u} \sim N(s, \sigma^2), \quad s \in (-\infty, \infty), \quad \sigma^2 \in (0, \infty)\}. \tag{10}$$

Each membership function value $\mu_{\bar{\tilde{C}}_j}(\bar{\gamma}_j)$ is interpreted as a degree of possibility or compatibility that $\bar{\tilde{C}}_j$ is equal to $\bar{\gamma}_j$. Then, we define the objective function $\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}$ based on Zadeh's extension principle.

Definition 5. The objective function $\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}$ is defined as a random fuzzy variable characterized by the following membership function:

$$\mu_{\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}}(\bar{u}) \triangleq \sup_{\bar{\boldsymbol{\gamma}}} \left\{ \min_{1 \leqslant j \leqslant l} \mu_{\bar{\tilde{C}}_j}(\bar{\gamma}_j) \middle| \bar{u} = \sum_{j=1}^{l} \bar{\gamma}_j x_j \right\}, \quad \forall \bar{u} \in Y, \tag{11}$$

where $\bar{\boldsymbol{\gamma}} = (\bar{\gamma}_1, \ldots, \bar{\gamma}_l)$ and

$$Y \triangleq \left\{ \sum_{j=1}^{l} \bar{\gamma}_j x_j \middle| \bar{\gamma}_j \in \Gamma, \quad j = 1, \ldots, l \right\}.$$

Then the following theorem holds:

Lemma 1. *For any $\bar{u} \in Y$, it holds that*

$$\mu_{\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}}(\bar{u}) = \sup_{\boldsymbol{s}} \left\{ \min_{1 \leqslant j \leqslant l} \mu_{\tilde{M}_j}(s_j) \middle| \bar{u} \sim N\left(\sum_{j=1}^{l} s_j x_j, \sum_{j=1}^{l} \sigma_j^2 x_j^2 \right) \right\}, \tag{12}$$

where $s = (s_1, s_2, \ldots, s_l)$.

Proof. From (9) and (11), we obtain

$$\begin{aligned} \mu_{\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}}(\bar{u}) &= \sup_{\bar{\boldsymbol{\gamma}}} \left\{ \min_{1 \leqslant j \leqslant l} \mu_{\bar{\tilde{C}}_j}(\bar{\gamma}_j) \middle| \bar{u} = \sum_{j=1}^{l} \bar{\gamma}_j x_j \right\} \\ &= \sup_{\boldsymbol{s}} \left\{ \min_{1 \leqslant j \leqslant l} \mu_{\tilde{M}_j}(s_j) \middle| \bar{\gamma}_j \sim N\left(s_j, \sigma_j^2\right), \bar{u} = \sum_{j=1}^{l} \bar{\gamma}_j x_j \right\} \\ &= \sup_{\boldsymbol{s}} \left\{ \min_{1 \leqslant j \leqslant l} \mu_{\tilde{M}_j}(s_j) \middle| \bar{u} \sim N\left(\sum_{j=1}^{l} s_j x_j, \sum_{j=1}^{l} \sigma_j^2 x_j^2 \right) \right\}, \end{aligned} \tag{13}$$

where $s = (s_1, s_2, \ldots, s_l)$. □

From the above theorem, problem (6) is reformulated as the following problem:

$$\left.\begin{array}{ll}\text{minimize} & \bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}\\ \text{subject to} & \boldsymbol{x}\in X,\end{array}\right\}\tag{14}$$

where

$$\mu_{\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}}(\bar{u})=\sup_{\boldsymbol{s}}\left\{\min_{1\leqslant j\leqslant l}\mu_{\widetilde{M}_j}(s_j)\,\middle|\,\bar{u}\sim N\left(\sum_{j=1}^{l}s_jx_j,\sum_{j=1}^{l}\sigma_j^2x_j^2\right)\right\}.$$

4. Possibility-based value-at-risk model for random fuzzy minimum spanning tree problems

Since problem (14) includes both randomness and fuzziness in the objective function, the objective function value does not become a deterministic value even when a decision variable vector $\boldsymbol{x}$ is determined. This means that unlike deterministic problems, problem (14) is an ill-defined problem and cannot be directly optimized as it is.

Therefore, a new optimization model is needed in order to interpret problem (14) as a well-defined problem. As a first step to construct a new model, instead of minimizing the objective function $\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}$, we focus on a probability that the objective function $\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}$ is smaller than or equal to a target level f, which is denoted by $Prob(\omega\mid\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)$. Similar to an optimization criterion in stochastic programming, especially, using fractile criterion (Geoffrion, 1967; Kataoka, 1963) or value-at-risk criterion (Jorion, 1996; Pritsker, 1997), we consider the following problem:

$$\left.\begin{array}{ll}\text{minimize} & f\\ \text{subject to} & Prob\left(\omega\,\middle|\,\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f\right)\gtrsim p_1\\ & \boldsymbol{x}\in X,\end{array}\right\}\tag{15}$$

where $\gtrsim$ means "substantially greater than or equal to." It should be noted here that this constraint is still an ill-defined constraint, and that it is necessary to introduce a certain interpretation so that the constraint is transformed into a well-defined one. In the next subsection, we shall consider a possibilistic constraint as one of proper interpretations of the ill-defined constraint in problem (15).

4.1. Possibility-based value-at-risk model

This section devotes to constructing a new random fuzzy optimization model by incorporating a possibilistic programming approach into a value-at-risk model and discussing how to transform problem (15) into a deterministic equivalent problem which can be solved by a polynomial-time algorithm.

In preparation for constructing a new model, we define the probability $Prob(\omega\mid\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)$ as a fuzzy set.

Definition 6. Based on Zadeh's extension principle, the probability $Prob(\omega\mid\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)$ can be defined as a fuzzy set $\widetilde{P}$ characterized by the following membership function:

$$\mu_{\widetilde{P}}(p)=\sup_{\bar{u}}\left\{\mu_{\bar{\tilde{\boldsymbol{C}}}\boldsymbol{x}}(\bar{u})\mid p=Prob(\omega\mid\bar{u}(\omega)\leqslant f)\right\}.\tag{16}$$

From (13) and (16), we obtain

$$\begin{aligned}\mu_{\widetilde{P}}(p)=\sup_{\boldsymbol{s}}\min_{1\leqslant j\leqslant l}&\left\{\mu_{\widetilde{M}_j}(s_j)\mid p=Prob(\omega|\bar{u}(\omega)\leqslant f),\right.\\ &\left.\bar{u}\sim N\left(\sum_{j=1}^{l}s_jx_j^2,\sum_{j=1}^{l}\sigma_j^2x_j^2\right)\right\}.\end{aligned}\tag{17}$$

Next, we transform the fuzzy constraint $Prob(\omega\mid\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)\gtrsim p_1$ in (15) into a deterministic constraint, using possibilistic programming. Possibilistic programming was originally developed by Dubois and Prade (1980), which is one of methodologies for decision making under existence of ambiguity. In possibilistic programming, a degree of possibility or necessity (Zadeh, 1978) is maximized. In order to interpret $Prob(\omega\mid\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)\gtrsim p_1$ using a possibility measure, we focus on a degree of possibility that fuzzy probability $\widetilde{P}$ is substantially greater than or equal to fuzzy goal $\widetilde{G}$:

$$\Pi_{\widetilde{P}}(\widetilde{G})\triangleq\sup_{p}\min\{\mu_{\widetilde{P}}(p),\ \mu_{\widetilde{G}}(p)\},\tag{18}$$

where $\mu_{\widetilde{G}}(p)$ is a membership function of fuzzy goal $\widetilde{G}$ for probability. We define $\mu_{\widetilde{G}}(p)$ as the following nonincreasing upper semi-continuous function:

$$\mu_{\widetilde{G}}(p)=\begin{cases}0, & p\leqslant p_0\\ g(p), & p_0\leqslant p\leqslant p_1\\ 1, & p\geqslant p_1,\end{cases}\tag{19}$$

where p_0 and p_1 are constants, and $g(p)$ is a nondecreasing upper semi-continuous function. Here, we assume it holds that

$$p_0>1/2,\tag{20}$$

which is considered to be a natural assumption in view of real decision making situations because a decision maker is not usually satisfied with the probability $p\leqslant 1/2$ at all.

Figs. 1 and 2 illustrate a possible shape of the membership function for the fuzzy goal $\widetilde{G}$ and the degree of possibility $\Pi_{\widetilde{P}}(\widetilde{G})$, respectively.

As a result of the above discussions, the constraint $Prob(\omega|\bar{\tilde{\boldsymbol{C}}}(\omega)\boldsymbol{x}\leqslant f)\gtrsim p_1$ is interpreted as $\Pi_{\widetilde{P}}(\widetilde{G})\geqslant h$ where h is given by a decision maker as a constant satisfying $0\leqslant h\leqslant 1$. Consequently, we consider the following problem as an interpretation of (15) from the view point of possibilistic programming:

$$\left.\begin{array}{ll}\text{minimize} & f\\ \text{subject to} & \Pi_{\widetilde{P}}(\widetilde{G})\geqslant h\\ & \boldsymbol{x}\in X.\end{array}\right\}\tag{21}$$

Although the above problem is a well-defined problem, it is necessary to transform the problem into an explicit form of the mathematical constraint. As a preliminary preparation, we show the following lemma:

Lemma 2. *The condition*

$$\mu_{\widetilde{P}}(p)\geqslant h\tag{22}$$

is equivalently transformed into the condition

$$Prob(\omega|\bar{u}^R(\omega)\leqslant f)\leqslant p\leqslant Prob(\omega|\bar{u}^L(\omega)\leqslant f),\tag{23}$$

where $\bar{u}^L$ and $\bar{u}^U$ are defined by

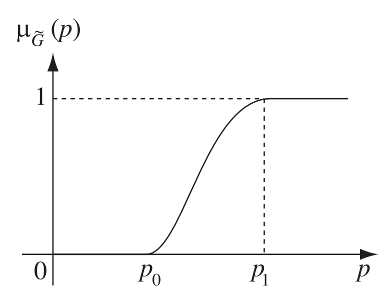

Fig. 1. Membership function $\mu_{\widetilde{G}}(y)$ of fuzzy goal $\widetilde{G}$.

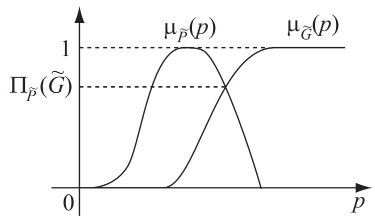

Fig. 2. Degree of possibility $\Pi_{\widetilde{P}}(\widetilde{G})$.

$$\bar{u}^L \sim N\left(\sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right),$$

$$\bar{u}^R \sim N\left(\sum_{j=1}^{l}\{m_j + R^\star(h)\beta_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right),$$

$$L^\star(h) \triangleq \begin{cases} \sup\{t \mid L(t) \geqslant h\} & (0 < h \leqslant 1) \\ \infty & (h = 0), \end{cases}$$

$$\text{and } R^\star(h) \triangleq \begin{cases} \sup\{t \mid R(t) \geqslant h\} & (0 < h \leqslant 1) \\ \infty & (h = 0). \end{cases}$$

Proof. In view of (17), the condition $\mu_{\widetilde{P}}(p)$ is equivalently replaced with

$$\sup_{\boldsymbol{s}} \min_{1\leqslant j\leqslant l}\left\{\mu_{\widetilde{M}_j}(s_j) \,\middle|\, p = Prob(\omega|\bar{u}(\omega) \leqslant f),\ \bar{u} \sim N\left(\sum_{j=1}^{l} s_j x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right)\right\} \geqslant h \tag{24}$$

This implies

$$\exists \boldsymbol{s} : \min_{1\leqslant j\leqslant l}\mu_{\widetilde{M}_j}(s_j) \geqslant h, \quad \bar{u} \sim N\left(\sum_{j=1}^{l} s_j x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right),$$
$$p = Prob(\omega|\bar{u}(\omega) \leqslant f). \tag{25}$$

This is equivalent to calculating the possible value of $p = Prob(\omega \mid \bar{u}(\omega) \leqslant f)$ under the condition of

$$\min_{1\leqslant j\leqslant l}\mu_{\widetilde{M}_j}(s_j) \geqslant h,$$
$$\bar{u} \sim N\left(\sum_{j=1}^{l} s_j x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right).$$

It follows that

$$\min_{1\leqslant j\leqslant l}\mu_{\widetilde{M}_j}(s_j) \geqslant h \Longleftrightarrow \mu_{\widetilde{M}_j}(s_j) \geqslant h, \quad j = 1, 2, \ldots, l.$$

From (8), it holds that

$$\mu_{\widetilde{M}_j}(s_j) \geqslant h \Longleftrightarrow s_j \in \left[m_j - L^\star(h)\alpha_j, m_j + R^\star(h)\beta_j\right], \tag{26}$$

where $L^\star(h)$ and $R^\star(h)$ are pseudo inverse functions defined as

$$L^\star(h) \triangleq \begin{cases} \sup\{t \mid L(t) \geqslant h\} & (0 < h \leqslant 1) \\ \infty & (h = 0) \end{cases}$$

and

$$R^\star(h) \triangleq \begin{cases} \sup\{t \mid R(t) \geqslant h\} & (0 < h \leqslant 1) \\ \infty & (h = 0). \end{cases}$$

Then, consider the following two random variables corresponding to the left and right end points of the interval (26):

$$\bar{u}^L \sim N\left(\sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right)$$

$$\text{and } \bar{u}^R \sim N\left(\sum_{j=1}^{l}\{m_j + R^\star(h)\beta_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right).$$

Since $Prob(\omega|\bar{u}^R(\omega) \leqslant f) \leqslant Prob(\omega|\bar{u}^L(\omega) \leqslant f)$, the range of probability $p = Prob(\omega|\bar{u}(\omega) \leqslant f)$ is calculated as the closed interval

$$p \in [Prob(\omega|\bar{u}^R(\omega) \leqslant f),\ Prob(\omega|\bar{u}^L(\omega) \leqslant f)]$$
$$\text{or } Prob(\omega \mid \bar{u}^R(\omega) \leqslant f) \leqslant p \leqslant Prob(\omega \mid \bar{u}^L(\omega) \leqslant f). \qquad \square$$

Then, we obtain the following theorem:

Theorem 1. *The following equivalent transformation holds:*

$$\Pi_{\widetilde{P}}(\widetilde{G}) \geqslant h \Longleftrightarrow \sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j + \Phi^{-1}\left(\mu^\star_{\widetilde{G}}(h)\right)\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2} \leqslant f, \tag{27}$$

where Φ denotes the probability distribution function of the standard normal random variable N(0,1) and Φ^{-1} denotes its inverse function. Functions $\mu^\star_{\widetilde{G}}(h)$ and $L^\star(h)$ are pseudo inverse functions defined as

$$\mu^\star_{\widetilde{G}}(h) = \inf\{p \mid \mu_{\widetilde{G}}(p) \geqslant h\}$$

and

$$L^\star(h) = \begin{cases} \sup\{t \mid L(t) \geqslant h\} & (0 < h \leqslant 1) \\ \infty & (h = 0) \end{cases}$$

Proof. From (19), the constraints $\Pi_{\widetilde{P}}(\widetilde{G}) \geqslant h$ in (21) means

$$\sup_p \min\{\mu_{\widetilde{P}}(p), \quad \mu_{\widetilde{G}}(p)\} \geqslant h,$$

which is equivalent to the condition

$$\exists p : \ \mu_{\widetilde{P}}(p) \geqslant h \quad \text{and} \quad \mu_{\widetilde{G}}(p) \geqslant h. \tag{28}$$

From Lemma 2,

$$\mu_{\widetilde{P}}(p) \geqslant h \tag{29}$$

is equivalently transformed into the condition

$$Prob(\omega|\bar{u}^R(\omega) \leqslant f) \leqslant p \leqslant Prob(\omega|\bar{u}^L(\omega) \leqslant f), \tag{30}$$

where $\bar{u}^L$ and $\bar{u}^R$ are defined by

$$\bar{u}^L \sim N\left(\sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right),$$

$$\bar{u}^R \sim N\left(\sum_{j=1}^{l}\{m_j + R^\star(h)\beta_j\}x_j, \sum_{j=1}^{l}\sigma_j^2 x_j^2\right).$$

Hence, (28) is equivalent to

$$\exists p : Prob(\omega \mid \bar{u}^R(\omega) \leqslant f) \leqslant p \leqslant Prob(\omega \mid \bar{u}^L(\omega) \leqslant f),$$
$$\text{and} \quad p \geqslant \mu^\star_{\widetilde{G}}(h).$$

This means that

$$Prob(\omega \mid \bar{u}^L(\omega) \leqslant f) \geqslant \mu^\star_{\widetilde{G}}(h). \tag{32}$$

Since $Prob(\omega \mid \bar{u}^L(\omega) \leqslant f)$ is transformed into

$$Prob\left(\omega \,\middle|\, \frac{\bar{u}^L - \sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j}{\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2}} \leqslant \frac{f - \sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j}{\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2}}\right),$$

in consideration of

$$\frac{\bar{u}^L - \sum_{j=1}^{l}\{m_j - L^\star(h)\alpha_j\}x_j}{\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2}} \sim N(0,1),$$

(32) is equivalently transformed as

$$\Phi\left(\frac{f-\sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j}{\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2}}\right) \geqslant \mu_{\tilde{G}}^{\star}(h), \tag{33}$$

where Φ is a probability distribution function of the standard Gaussian random variable $N(0,1)$.

From the monotonicity of Φ, (33) is rewritten as

$$\sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j+\Phi^{-1}(\mu_{\tilde{G}}^{\star}(h))\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^2} \leqslant f, \tag{34}$$

where Φ^{-1} is the inverse function of Φ. □

Remark 3. The transformation (27) was firstly shown by Katagiri, Ishii, and Sakawa (2002) in 2002, although the validity of the transformation was not definitely proved, unlike Theorem 1 in this article. Later, Hasuike et al. (2009) applied the computational result to a portfolio selection problem with random fuzzy variable returns.

By using the above theorem and the fact that $x_j^2 = x_j$ holds for any $x_j \in \{0,1\}$, problem (21) is transformed into the following problem:

$$\begin{array}{ll} \text{minimize} & f \\ \text{subject to} & \sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j+\Phi^{-1}\left(\mu_{\tilde{G}}^{\star}(h)\right)\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j} \leqslant f \\ & \boldsymbol{x}\in X. \end{array}$$

This is equivalently transformed into the following simpler problem:

$$\left.\begin{array}{ll} \text{minimize} & \sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j+\Phi^{-1}\left(\mu_{\tilde{G}}^{\star}(h)\right)\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j} \\ \text{subject to} & \boldsymbol{x}\in X. \end{array}\right\} \tag{35}$$

In (35), it holds that $\Phi^{-1}\left(\mu_{\tilde{G}}^{\star}(h)\right) \geqslant 0$ because $\mu_{\tilde{G}}^{\star}(h) \geqslant 1/2$ for $h \geqslant 0$. It should be noted here that problem (35) includes neither randomness nor fuzziness, and that it is a deterministic nonlinear minimum spanning tree problem.

Remark 4. Problem (35) can be considered as a generalized version of stochastic minimum spanning tree problems that were previously discussed in existing articles (Geetha & Nair, 1993; Ishii et al., 1981). To be more precise, in problem (35), if $\alpha_j = 0, \forall j$ and if the fuzzy goal $\tilde{G}$ is replaced by crisp goal G such as

$$\mu_G(p) = \begin{cases} 0 & \text{if } p < p_1 \\ 1 & \text{if } p \geqslant p_1, \end{cases}$$

problem (35) is equivalent to the stochastic minimum spanning tree problem discussed in previous studies.

$$\left.\begin{array}{ll} \text{minimize} & \sum_{j=1}^{l} m_j x_j+\Phi^{-1}(p_1)(h)\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j} \\ \text{subject to} & \boldsymbol{x}\in X. \end{array}\right\} \tag{36}$$

5. Polynomial-time algorithm for obtaining an optimal solution

In this section, we definitely provide a polynomial-time algorithm for exactly solving problems (35) which is based on the algorithm proposed by Geetha and Nair as a solution technique for stochastic minimum spanning tree problems (Geetha & Nair, 1993). In addition, we prove that the time-complexity of the algorithm can be improved by using the knowledge of discrete computational geometry (Dey, 1998).

In preparation for solving problem (35), following the Geetha–Nair's approach (Geetha & Nair, 1993), we firstly introduce the following bi-objective minimum spanning tree problem:

$$\begin{array}{ll} \text{minimize} & \sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j \\ \text{minimize} & \sum_{j=1}^{l}\sigma_j^2 x_j \\ \text{subject to} & \boldsymbol{x}\in X. \end{array}$$

Applying Geetha–Nair's idea and algorithm, we provide a polynomial-time algorithm for solving problem (35) as follows:

Step 1 Set $q := 1$. Find a minimum spanning tree $\boldsymbol{x}^{(q)} \in \arg\min_{\boldsymbol{x}\in X}\sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j$. Calculate $z_1^{(q)} = \sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j^{(q)}$ and $z_2^{(q)} = \sum_{j=1}^{l}\sigma_j^2 x_j^{(q)}$.

Step 2 Set $q := 2$ and find a minimum spanning tree $\boldsymbol{x}^{(q)} \in \arg\min_{\boldsymbol{x}\in X}\sum_{j=1}^{l}\sigma_j^2 x_j$. Calculate $z_2^{(q)} = \sum_{j=1}^{l}\sigma_j^2 x_j^{(q)}$ and $z_1^{(q)} = \sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j^{(q)}$. Let $DS = \{(1,2)\}$ and $PS = \{1,2\}$.

Step 3 Set $q := q+1$. Select arbitrarily an element $(s,t) \in DS$ and calculate

$$\eta_1^{(s,t)} := |z_2^{(t)} - z_2^{(s)}|,$$
$$\eta_2^{(s,t)} := |z_1^{(t)} - z_1^{(s)}|.$$

Let $\hat{\boldsymbol{x}}$ be an optimal solution of the following minimum spanning tree problem:

$$\left.\begin{array}{ll} \text{minimize} & \sum_{j=1}^{l}\left[\eta_1^{(s,t)}\{m_j-L^{\star}(h)\alpha_j\}+\eta_2^{(s,t)}\sigma_j^2\right]x_j \\ \text{subject to} & \boldsymbol{x}\in X. \end{array}\right\} \tag{37}$$

If $\hat{\boldsymbol{x}}$ is equivalent to either $\boldsymbol{x}^{(s)}$ or $\boldsymbol{x}^{(t)}$, then set $DS = DS \setminus \{(s,t)\}$ and go to Step 4. Otherwise, set

$$DS := DS \cup \{(s,q),(q,t)\} \setminus \{(s,t)\},$$
$$PS := PS \cup \{q\}$$

and go to Step 4.

Step 4 If $|PS| \geqslant 1$, then go to Step 5. Otherwise, return to Step 3.

Step 5 Find an optimal solution

$$\boldsymbol{x}^{opt} \in \arg\min_{q\in PS}\sum_{j=1}^{l}\{m_j-L^{\star}(h)\alpha_j\}x_j^{(q)}+\Phi^{-1}\left(\mu_{\tilde{G}}^{\star}(h)\right)\sqrt{\sum_{j=1}^{l}\sigma_j^2 x_j^{(q)}}.$$

If the result of Geetha and Nair (1993) is directly applied to the proposed algorithm, using the fact that the number of PS is $O(k^{2.5})$ (Gusfield, 1980), the time-complexity of the above algorithm is $O(k^5 \log k)$. However, if one uses some knowledge of discrete computational geometry and more efficient algorithm for minimum spanning tree problems, it is shown that the time-complexity of the above algorithm can be reduced.

Theorem 2. *The time-complexity of the above algorithm is* $O\left(l^2 k^{\frac{1}{3}} \log \beta(l, k)\right)$, *where*

$$\beta(l, k) = \min\{i \mid \log^{(i)} n \leqslant l/k\}$$

$$\log^{(0)}(w) = w, \ \log^{(i+1)} w = \log\log^{(i)} w$$

Proof. The above algorithm finds a set of nondominated solutions or Pareto optimal solutions (spanning trees) corresponding to the endpoints of convex hull in objective space. Dey (1998) shows that there is a close relation between nondominated spanning trees and k-set problem in discrete computational geometry, and that the number of possible spanning trees, namely, the cardinality $|PS|$, is $O(lk^{1/3})$. Since it takes $O(l \log \beta(l,k))$ to find every element in PS, namely, to solve a minimum spanning tree problem (37) (Gabow et al., 1986), the total time-complexity of the above algorithm is $O\left(l^2 k^{\frac{1}{3}} \log \beta(l, k)\right)$. □

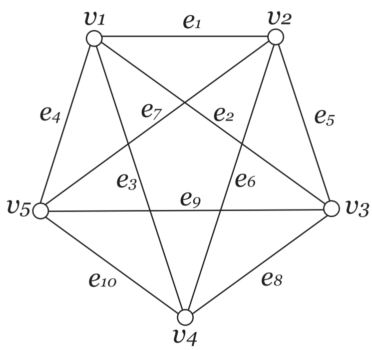

Fig. 3. Complete graph G.

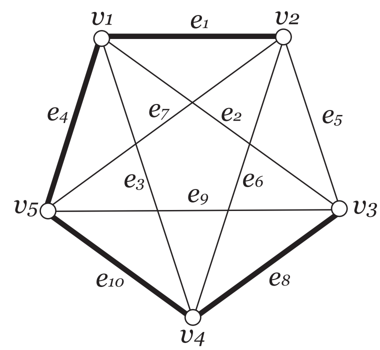

Fig. 5. Optimal solution of the proposed model for $h = 0.8$.

Table 1
Parameters of fuzzy random edge costs.

	m_j	σ_j^2	α_j	β_j
$\tilde{\bar{C}}_1$	3.0	0.30	0.30	0.90
$\tilde{\bar{C}}_2$	5.0	0.60	0.80	0.60
$\tilde{\bar{C}}_3$	6.0	0.50	1.00	0.40
$\tilde{\bar{C}}_4$	4.0	0.50	0.80	0.30
$\tilde{\bar{C}}_5$	5.0	0.70	1.20	0.50
$\tilde{\bar{C}}_6$	7.0	0.80	0.90	1.10
$\tilde{\bar{C}}_7$	4.0	0.30	0.20	0.90
$\tilde{\bar{C}}_8$	3.0	0.20	0.40	1.20
$\tilde{\bar{C}}_9$	3.5	0.20	1.20	0.70
$\tilde{\bar{C}}_{10}$	3.0	0.60	0.20	1.00

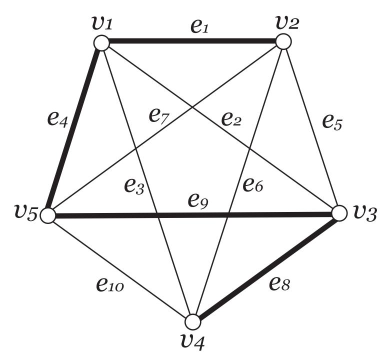

Fig. 6. Optimal solution of the proposed model for $h = 0.5$.

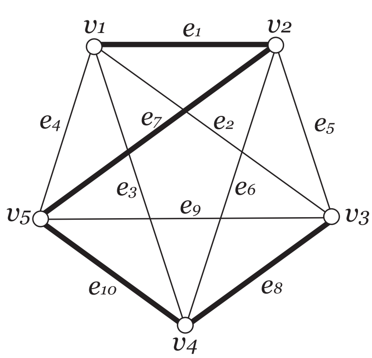

Fig. 4. Optimal solution of the stochastic programming model.

6. Numerical examples

In this section we give a very simple example of problem (6) which is a construction cost minimization problem in telecommunication stations network. Let G be a complete graph with five vertices, $k = 5$ (see Fig. 3).

We consider the situation that each cost attached to an edge varies dependent on random phenomenon such as weather conditions and economics and that mean of random variables are given as fuzzy number. Parameters in this example are given in Table 1.

In order to understand characteristics of the proposed model, we firstly compare an optimal solution of stochastic programming model (36) with that of our model (35). Using the solution algorithm proposed by Geetha and Nair (1993), we obtain the optimal solution $\boldsymbol{x}^* = [1,0,0,0,0,0,1,1,0,1]$ of problem (36) (see Fig. 4). On the other hand, the optimal solution of problem (35) for $h = 0.8$ is $\boldsymbol{x}^* = [1,0,0,1,0,0,0,1,0,1]$ (see Fig. 5).

Comparing the obtained optimal solutions of (35) and (36), it is found that e_4 and e_7 are exchanged. It should be noted here that the spread parameters $\alpha_4 = 0.80$ of $\tilde{\bar{C}}_4$ is fairly larger than $\alpha_7 = 0.20$ of $\tilde{\bar{C}}_7$. From this result, it is understood that the proposed model has a tendency to select an edge in which the attached cost for construction has larger ambiguity than the stochastic programming model. This characteristic is consistent with the fact that our model is developed based on a possibility measure because a possibility measure is useful for decision makers that hold optimistic views of decision making environments.

Next, we compare our models when values of parameter h are different. Fig. 6 depicts the optimal solution of our model for $h = 0.5$. Comparing Fig. 5 with Fig. 6, we observe that e_9 and e_{10} are exchanged. It should be noted here that the spread parameters $\alpha_9 = 1.20$ of $\tilde{\bar{C}}_9$ is fairly larger than $\alpha_{10} = 0.20$ of $\tilde{\bar{C}}_{10}$. From this result, it is observed that our model has a tendency to select an edge in which attached cost for construction has larger ambiguity as the value of h decreases. This characteristic is also consistent with the fact that our model is developed using a possibility measure.

7. Conclusion

In this paper, we have considered a minimum spanning tree problem where costs are random fuzzy variables, especially, random variables with fuzzy mean. We have proposed a random fuzzy programming model using a possibility measure and shown that the problem based on the proposed model is equivalently transformed into a deterministic nonlinear minimum spanning tree problem. The framework of our model and solution techniques has an advantage that an optimal solution of the formulated problem is obtained by a polynomial-time algorithm, whereas, in general, problems under both fuzziness and randomness are exactly difficult to solve.

In the future, we will apply or extend our model to solve other decision making problems, such as k-minimum spanning tree problems (Katagiri, Nishizaki, Hayashida, & Guo, 2012), under random fuzzy environments. Moreover, since we have proposed only a model using a possibility measure, we will propose a model using a necessity measure.

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