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A fuzzy multi-objective approach for sustainable investments

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ABSTRACT

This paper develops models for selecting portfolios for conventional and socially responsible investment (SRI) mutual funds according to the preferences of the SRI investor. This involves constructing an investment portfolio that takes into account both financial and social, environmental and ethical (SEE) criteria. The optimal portfolio selection problem is solved when the expected returns of the assets as well as the periodic returns are not precisely known. Instead, incomplete information on the parameters of the model is modeled by fuzzy numbers, which include the 'true' values and are consistent with the Decision Maker's beliefs on assets' performance.

In this paper, the financial criteria taken into account are the expected return and the difference between the returns of the portfolio and a pre-specified benchmark index i.e. a strategy of tracking error (TE) is followed. Moreover, we assume that the investor's preferences about SEE features of the portfolio are imprecisely known. In order to model these flexible preferences we propose to use fuzzy decision making. The multidimensional nature of the problem leads us to work with techniques of multiple criteria decision making (MCDM), namely goal programming (GP), and the incomplete information is handled by a fuzzy robust approach. The proposed fuzzy goal programming (FGP) model is applied to a database of UK mutual funds.

1. Introduction

Over the last few decades, socially responsible investments (SRIs) (also known as ethical investments) have experienced significant worldwide growth (see McCann et al., 2003; Sparkes, 2002). SRI can be broadly defined as 'an investment process that considers the social and environmental consequences of investments, both positive and negative, within the context of rigorous financial analysis' (Social Investment Forum (SIF), 2001). Unlike conventional forms of investment, SRI funds apply a set of screens to select assets from a universe of investment possibilities based on social, environmental or ethical (SEE) criteria (Renneboog et al., 2007). Investments commonly avoided by SRI funds include the securities of firms involved in armaments, alcohol, tobacco and gambling. SRI funds might also avoid investments in companies that are perceived as having poor records in terms of environmental issues, labor rights, animal abuse or corporate governance.

The investment screens used in SRI have evolved over time. For example Statman (2005) pointed out that in 1999, gambling was the second most likely industry to be rejected by SRI funds, whereas by 2003 it had fallen to fifth. Usually, SRI mutual funds apply a combination of sustainable screens. These screens are often classified into either negative or positive ones.

The oldest and most typical SRI approach is based on negative screens. Negative ethical screening eliminates ethically undesirable companies from SRI portfolios. Common negative screens are alcohol, gambling, tobacco, firearms, environmental damage. A second approach is positive screening, which concentrates on firms with positive records of social performance. Positive screeners tend to look for companies with records of good labor relations, community involvement and superior environmental performance. Recently, socially responsible mutual funds have begun screening according to a 'best-in-class' approach. Rather than excluding sectors, best-in-class funds usually rely on the conjecture that firms within a sector face the same social and environmental challenges and that positive screening within every sector is the most effective approach to identifying firms with a competitive edge. Most SRI funds in the US and the UK employ a combination of negative and positive screens (Social Investment Forum (SIF), 2003).

Negative and positive screens are often respectively referred to as the first and second generation of SRI screens. As mentioned by Renneboog, Ter Horst and Zhang (2007, p. 7) the so-called third generation of screens refers to an integrated approach of selecting companies based on the economic, environmental and social criteria comprised by both negative and positive screens. This approach is often called 'sustainability' or 'triple bottom line' (People, Planet and Profit). The fourth generation combines sustainable investing with shareholder activism and commitment.

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There are some investments that are regarded as being desirable by some SRI funds and as undesirable by others. In this sense Statman (2005, 2006) points out that organizations that construct SRI indexes agree on some SRI characteristics but disagree about others. Kinder, Lydenberg, and Domini (KLD) exclude from the Domini 400 Social Index companies that derive any revenue from the manufacture of alcohol or tobacco products, companies that derive any revenue from the provision of gambling products or services, electric utilities with interests in nuclear power plants, and companies that derive 2% or more of their sales from military weapon systems. Then KLD evaluate companies with respect to areas such as the environment, diversity and employee relations. Problems in one area do not necessarily prompt the elimination of a company from the Domini 400 Social Index, but companies whose records are, on balance, negative are excluded. Calvert, which compiles the Calvert Index, evaluates company performance with respect to the environment, the workplace, product safety and impact, international operations and human rights, community relations, and the rights of indigenous peoples. It excludes companies with interests in gambling, tobacco and military weapons, but includes companies with interests in alcohol, firearms and nuclear power, unless these interests are substantial.

The historical background of modern Socially Responsible Investment (SRI) dates from the early 20th century in the USA, where it originated within religious entities (Domini, 2001). For instance, Anabaptists and Quakers avoided investing in the alcohol and tobacco industries in the belief that it was not ethical to profit from activities that damaged the moral fiber of society. The '1999 Trends Report' from the Social Investment Forum (1999) exposed the fact that while the origins of the social responsibility movement are religious, much of today's social responsibility movement was born in the 1960s, when struggles for civil rights and the rights of women, and for anti-war and pro-environmental policies served to escalate the awareness of social responsibility. Opposition to South African apartheid was a rallying cry that brought many into the SRI movement in the late 1970s. Today the movement continues to grow as emphasis shifts towards corporate governance.

The interest of individual and institutional investors in these types of investment has led to an increasing volume of academic literature on this sector. Several SRI topics have been extensively investigated, with most of the existing empirical studies focusing on the financial performance of SRI funds (see for example: Barnett and Salomon, 2006; Basso and Funari, 2005, 2007; Bauer et al., 2005; Fernández and Matallín, 2008; Galema et al., 2008; Hong and Kacperczyk, 2009; Kempf and Osthoff, 2007; Renneboog et al., 2007, 2008; Statman, 2000), the theoretical background of SRI (see for example: Barnea et al., 2005; Bénabou and Tirole, 2010; Dam and Heijdra, 2011; Heinkel et al., 2001; Jensen, 2001; Tirole, 2001) and flow-performance relation for SRI funds (see for example: Benson and Humphrey, 2008; Bollen, 2007; Renneboog et al., 2006). However there is a dearth of academic literature on the development of methodology based on mathematical programming in order to construct portfolios tailored to the tastes and concerns of SRI investors. In response, the purpose of this paper is to fill in some of the gaps in knowledge in order to cater for this potentially very important (and growing) class of investors.

A pioneering work on this area was conducted by Hallerbach et al. (2004). In addition, there are several other papers on portfolio selection which take into account ethical, social and environmental factors highlighted by SRI. Some academics have tried to extend or complement the classic model of portfolio selection initially proposed by Markowitz (1952) (for example: Drut (2010) and Cañal (2011)), whilst other papers are based on multi-criteria decision making (for example, the articles by Hallerbach et al. (2004) and Bilbao-Terol et al. (2012)).

In this paper, we solve the problem of optimal portfolio selection when neither the expected returns of the assets nor the periodic returns are precisely known. We assume that these parameters 'belong to' a fuzzy number, i.e., a closed interval on the real line where each element has an associated grade of possibility. Set inclusion in order to model incomplete information is closely linked to the classical modeling of uncertainty using probabilities: the fuzzy number of possible parameters corresponds to the state space in probability theory, but no stance is taken on the probability distribution of the 'true' parameter value. Instead, the possibility distribution is known.

As an application of possibility theory to portfolio analysis, possibility portfolio selection models were initially proposed by Tanaka and Guo (1999) who respectively replaced the mean vector and covariance matrix in Markowitz's model with the fuzzy weighted average vector and covariance matrix, extending probability into fuzzy probability which allowed the incorporation of expert knowledge by means of a possibility degree to reflect the degree of similarity between the future state of asset markets and the state of previous periods (Zhang and Nie, 2004). Examples of other papers that use the theory of fuzzy sets in portfolio analysis are Bilbao et al. (2006), Bilbao-Terol et al. (2006), Gupta et al. (2008) and Huang (2007). The main goal of our work is to use fuzzy set theory and multi-objective programming for portfolio selection under the more realistic assumption that the returns are not exactly known or cannot be confidently estimated.

Our proposal to handle the fuzziness of the model is close to robust optimization, i.e., we optimize the objective function in the worst-case realization of the input parameters (see, for instance, Costa and Paiva, 2002; Garlappi et al., 2007; Goldfarb and Iyengar, 2003; Kawas and Thiele, 2011; Lu, 2009, 2011; Tütüncü and Koenig, 2004). A robust portfolio is therefore one designed to optimize the worst-case performance with respect to all the possible values parameters may take within their corresponding uncertainty sets.

Providers of socially responsible investment strategies respond to the desires of institutional investors by introducing investment products that offer not only the promise of high returns, but also low tracking error. In 2005, for example, Barclays Global Investors teamed up with KLD to offer the iShares KLD Social Index fund. Companies with higher social and environmental scores are allotted a higher weighting in the KLD fund than in the Russell 1000 Index. However, the KLD fund also promises to minimize its tracking error away from the Russell 1000 Index (Statman, 2005, p.19). In our paper, the tracking error approach is carried out using a multi-objective method, namely goal programming (GP), which was introduced by Charnes and Cooper (1961) and is one of the most widely used methods due to its applicability to real problems (see Büyüközkan and Berkol, 2011; Chen and Xu, 2012; Jinturkar and Deshmukh, 2011; Lee et al., 2009; Liang, 2010; Romero, 1991; Tsai et al., 2008). Many decision makers prefer to seek solutions that are 'good enough' or 'close enough', rather than optimal. Goal programming allows multiple objectives to be handled under a 'satisfiying philosophy', developed by Nobel Prize winner Herbert Simon (1956) and expressed by means of the concept of 'goal'. Designing a GP model involves the Decision Maker setting a priori 'aspiration levels' for his objectives. Such targets, together with the objectives, constitute the 'goals' of the problem.

There are some works that use GP to obtain a portfolio which as far as possible satisfies investors' preferences (see Arenas-Parra et al., 2001; Ben Abdelaziz et al., 2007; Bilbao et al., 2007; Hsin-Hung, 2008; Kaminski et al., 2009; Lee and Chesser, 1980; Lee and Lerro, 1973). Our work is a novel contribution to portfolio selection when an SRI investor addresses financial and ethical criteria in an imprecise framework.

This paper is organized as follows: In Section 2, some fuzzy theory definitions and theorems are highlighted. In Section 3, the proposed model for portfolio selection of SRI and non-SRI assets is presented. Section 4 is dedicated to presenting a case study using UK mutual funds. Finally, conclusions are drawn in Section 5.

2. The fuzzy approach: previous concepts

In order to make up a portfolio, financial analysts are asked to give subjective evaluations about the market environment because of their technical and expert knowledge. They establish financial parameters for the portfolio, so some degree of subjectivity or imprecision is included in the description of the financial market process. In this kind of situation, fuzzy sets theory and possibility theory may both be helpful, as they allow the quantifying of imprecise information and facilitate reasoning and decision-making based on vague and/or incomplete data (Bilbao-Terol et al., 2012).

First let us introduce some concepts and results of fuzzy technology that will be used throughout this paper.

From now, all symbols with a tilde ($\sim$) above them represent fuzzy parameters.

Definition 1. Letting X denote a universal set, a fuzzy set $\widetilde{A}$ of X can be characterized as a set of ordered pairs of element x and the grade of membership of x in $\widetilde{A}$, $\mu_{\widetilde{A}}(x)$, which is a value between 0 and 1, often written as: $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))/x \in X\}$.

Possibility theory was proposed by Zadeh (1978) and advanced by Dubois and Prade (1985); since then, in many cases the imprecision on decision-making problems have been handled by possibility distributions.

Definition 2. A possibility distribution can be defined in the following way. Let $\widetilde{A}$ be a fuzzy set of a universe of discourse X characterized by its membership function $\mu_{\widetilde{A}}$. Let x be a variable of X. Then $\mu_{\widetilde{A}}(x)$ can be interpreted as the degree of possibility of the proposition 'x is $\widetilde{A}$'. Thus, the possibility distribution (π_x) of x is defined as numerically equal to the membership function of $\widetilde{A}$, i.e. $\pi_x = \mu_{\widetilde{A}}$.

A fuzzy number is one of the most common forms of fuzzy set application (Kaufmann and Gupta, 1988).

Definition 3. A real fuzzy number is a fuzzy set $\widetilde{N}$ on IR, convex and normal, whose membership function $\mu_{\widetilde{N}} : IR \to [0,1]$ is upper semi-continuous and there exist three intervals $[n_1,n_2]$, $[n_2,n_3]$, $[n_3,n_4]$ so that $\mu_{\widetilde{N}}$ is increasing on $[n_1,n_2]$, equal to 1 on $[n_2,n_3]$, decreasing on $[n_3,n_4]$ and equal to 0 elsewhere.

Let $Supp(\widetilde{N})$ be the support of $\widetilde{N}$ denoting the closure of the set where $\mu_{\widetilde{N}}$ is non-null, and let the mode of $\widetilde{N}, Mod(\widetilde{N})$, denote the set where $\mu_{\widetilde{N}}$ is equal to one.

Fuzzy numbers can also be considered as possibility distributions (Dubois and Prade, 1985).

Definition 4. Letting $\alpha \in [0,1]$, the α-cuts – or α-level sets – of $\widetilde{N}$ are then ordinary sets defined by $N_\alpha = \{x \in IR/\mu_{\widetilde{N}}(x) \geqslant \alpha\}$. Observe that the α-cuts of $\widetilde{N}$ are closed and bounded intervals, which are represented by $[n^L_\alpha, n^R_\alpha]$.

Among the various types of fuzzy numbers, the trapezoidal fuzzy numbers are the most used, especially for solving fuzzy mathematical programming problems. A fuzzy number is trapezoidal if its membership function is piecewise linear. From now on, we will denote by $\widetilde{N} = (n_1, n_2, n_3, n_4)$ a trapezoidal fuzzy number where $Supp(\widetilde{N}) = [n_1, n_4]$ and $Mod(\widetilde{N}) = [n_2, n_3]$.

Theorem 1 (*Representation*). *A fuzzy number $\widetilde{N}$ can be represented by a complete ranking family of ordinary sets of IR: its α-cuts $\{\widetilde{N}_\alpha\}, \alpha \in [0,1]$ by which $\widetilde{N}$ can be reconstructed.*

3. Portfolio selection model

The proposed portfolio selection model corresponds to the following multi-objective linear programming problem:

$$\left.\begin{array}{ll} \min & z(x) = (z_1(x), z_2(x), \ldots, z_q(x))^T = (c_1x, c_2x, \ldots, c_qx)^T \\ \text{s.t.} & x \in X = \{x \in R^n / Ax \leqslant b, x \geqslant 0\} \end{array}\right\} \quad (MOLP)$$

where $x = (x_1, x_2, \ldots, x_n)'$ is a n-dimensional vector of decision variables, in our case the portfolio solution and$z_1(x) = c_1x, \ldots, z_q(x) = c_qx$ are q conflicting objectives. Our approach for solving the MOLP-problem is based on goal programming. For goal programming, for each objective function $z_i\,(x)$, a goal is specified by the Decision Maker and therefore the MOLP is converted into the problem of coming 'as close as' to the set of specified goals.

3.1. Definition of model inputs

Let us consider a portfolio, P, made up of a set of $n$assets $\{s_1, s_2, \ldots, s_n\}$. $P = (x_1, x_2, \ldots, x_n)$ where x_i represents the proportion of the total budget to be invested in asset s_i. It is supposed here that short sales are not allowed (therefore a non-negativity condition should be satisfied for x_i) and the entire budget is invested, i.e., $\sum_{i=1}^n x_i = 1$.

Let Y be the vector of historical returns on a benchmark index and R the matrix of historical returns on n assets over T periods. In this work, we consider that the matrix of future returns on assets, denoted by $\widetilde{R}$, is composed of fuzzy numbers, $\widetilde{R}_{ti}$, that $t \in \{T+1, T+2, \ldots, T+f\}$. Each $\widetilde{R}_{ti}$ represents the return on asset s_i in period t and the rows of $\widetilde{R}$ are denoted by the fuzzy vector $\widetilde{R}_t$. The future returns on the benchmark in each period are represented by fuzzy numbers $\widetilde{Y}_t$ which make up the fuzzy vector $\widetilde{Y}$.

3.2. Formulation of the fuzzy goal programming model

We can consider the criteria of evaluation which should be kept in mind for the portfolio optimization process as being split into three groups: the first group contains one criterion, namely the portfolio's expected return; the second is composed of the return of the portfolio in each past and future period t, that $t \in \{1, \ldots, T, T+1, \ldots, T+f\}$, and the third comprises the criteria concerning the SRI features of the portfolio.

3.2.1. Criteria and goals

3.2.1.1. G1: Fuzzy expected return of portfolio. In this paper, the expected return of the portfolio P is an imprecise quantity handled by fuzzy theory that we will call the 'fuzzy expected return'. It is calculated as follows:

$$\widetilde{E}_P(x) = \sum \widetilde{E}(R_i)x_i \tag{1}$$

where $\widetilde{E}(R_i)$ is a fuzzy number that represents the fuzzy expected return of asset s_i. $\widetilde{E}(R_i)$ is obtained by the matrix product M_iR_i where M_i is the matrix composed of fuzzy weights provided by the financial expert, where the values of each row add up to one, and R_i is a column vector composed of the historical returns on asset s_i. $\widetilde{E}_P(x)$ is therefore a fuzzy number that can be obtained by applying fuzzy arithmetic (Kaufmann and Gupta, 1985). Indeed, this criterion involves a 'more better' direction and the fuzzy goal concerned can therefore be expressed as:

$$\widetilde{E}_P = \widetilde{E}x \succsim \widetilde{E}^* \tag{2}$$

where $\widetilde{E}^*$ denotes the target-expected performance, $\widetilde{E}$ is an n-dimensional fuzzy vector composed of the fuzzy expected return of each asset s_i, $\widetilde{E}(R_i)$. The symbol $\succsim$ denotes a fuzzy version of the ordinary inequality $\geqslant$.

We propose the ideal value with respect to the fuzzy expected return of the portfolio as the target for the fuzzy expected return criterion. Therefore, this target is determined by maximizing $\widetilde{E}_P$, subject to the budget constraint and the non-negativity conditions for the decision variables, x_i (these constraints determine the feasible set X). Therefore, in order to obtain $\widetilde{E}^*$, it is necessary to solve the following fuzzy mono-objective problem:

$$\begin{array}{ll} \max & \widetilde{E}_P(x) = \sum \widetilde{E}(R_i)x_i = \widetilde{E}x \\ \text{s.t.} & x \in X \end{array} \tag{3}$$

The solution to (3) will be accomplished using the method developed by Arenas-Parra et al. (1999) for solving fuzzy mono-objective problems which allows one to obtain a fuzzy solution in the objective space defined by its α-cuts. Left and right extremes of these α-cuts are respectively obtained by solving the following problems:

$$\left.\begin{array}{ll} \min & -E^R_\alpha x \\ \text{s.t.} & x \in X \end{array}\right\}(P - cL_\alpha) \tag{4}$$

and

$$\left.\begin{array}{ll} \min & -E^L_\alpha x \\ \text{s.t.} & x \in X \end{array}\right\}(P - cR_\alpha) \tag{5}$$

In the work by Arenas-Parra et al. (1999), it has been proven that $\widetilde{E}^*$ is a fuzzy number, defined through its α-cuts, $\left[E^{*L}_\alpha, E^{*R}_\alpha\right], \alpha \in [0,1]$. E^{*L}_α and E^{*R}_α are the respective solutions of the problems $(P - cL_\alpha)$ and $(P - cR_\alpha)$.

For the treatment of the fuzzy goal (2), we propose an approach closely related to a programming robust (as mentioned above) and we will call this the fuzzy 'robust approach' (FR) in the sense that it involves finding a portfolio that robustly satisfies the fuzzy target-expected performance. We are concerned about the worst-case realization of the data from the uncertainty set. A robust portfolio is one that is designed to optimize the worst-case performance with respect to all the possible values that may be taken by the parameters within their corresponding uncertainty sets.

So considering as 'uncertainty sets' the fuzzy vector $\widetilde{E}$ for the expected return and the fuzzy number $\widetilde{E}^*$ for the target-expected performance, we will say that:

Definition 5. x is a robust portfolio with respect to the pair $(\widetilde{E}, \widetilde{E}^*)$ if x satisfies the following relationships: $E^C_\alpha x \geqslant E^{*C'}_\alpha$ for all $\alpha \in [0,1]$ and where $C, C' \in \{L,R\}$.

We can now show the following result:

Proposition 1. *Let x be a feasible portfolio such that x satisfies the following inequality: $E^L_0 x \geqslant E^{*R}_0$, then x is a robust portfolio.*

The proof follows directly from the relationships of the α-cuts of a fuzzy number.

Hence we propose to convert the fuzzy goal (2) to the following ordinary one:

$$E^L_0 x \geqslant E^{*R}_0 \tag{6}$$

that, by introducing the deviational variables, becomes:

$$E^L_0 x + n_E - p_E = E^{*R}_0 \tag{7}$$

being in this case the undesirable under-achievement.

3.2.2. G2: The fuzzy return of the portfolio in each period

In our approach based on Goal Programming, a portfolio $P = (x_1, x_2, \ldots, x_n)$ is selected such that its return for each of the several periods under consideration exceeds the return of a certain reference point, a benchmark index. Due to the linear performance fees paid to fund managers, we can argue that linear deviations give a more accurate description of the investor's attitude to risk than squared deviations (Rudolf et al., 1999). Therefore the goals concerning these criteria are:

$$R_t x \geqslant Y_t, \quad t = 1, \ldots, T \tag{8}$$

$$\widetilde{R}_t x \gtrsim \widetilde{Y}_t, \quad t = T+1, \ldots, T+f \tag{9}$$

where the fuzzy vector $\widetilde{R}_t$ is composed of the fuzzy future returns on n assets, $\widetilde{R}_{it}$. $\widetilde{R}_{it}$ is obtained in a similar way as the fuzzy expected return $\widetilde{E}(R_i)$, i.e., by the matrix product $M^f_i R^f_i$ where M^f_i is the matrix composed of fuzzy weights which are provided by the financial expert and the values of each row add up to one. R^i_f is a column vector composed of forecasted future returns on asset s_i. Similarly, we obtain $\widetilde{Y}_t$.

The treatment of the fuzzy goals (9) is carried out by a fuzzy robust approach. We will denote as $R^L_{t\alpha}$ and $R^R_{t\alpha}$ the vectors containing the lower and upper extremes of each $\widetilde{R}_t\alpha$-cut, that is:

$$R^L_{t\alpha} = \left(R^L_{t1\alpha}, R^L_{t2\alpha}, \ldots, R^L_{tn\alpha}\right) \tag{10}$$

$$R^R_{t\alpha} = \left(R^R_{t1\alpha}, R^R_{t2\alpha}, \ldots, R^R_{tn\alpha}\right) \tag{11}$$

$R^L_{ti\alpha}$ and $R^R_{ti\alpha}$ respectively being the lower and upper extremes of the α-cut of the fuzzy number $\widetilde{R}_{ti}$, with $Y^L_{t\alpha}$ and $Y^R_{t\alpha}$ respectively denoting the lower and upper extremes of the α-cut of the fuzzy number $\widetilde{Y}_t$. The goal in each future period is then expressed as:

$$R^L_{t0} x \geqslant Y^R_{t0}, \quad t = T+1, \ldots, T+f \tag{12}$$

Introducing the deviational variables, the goals (8) and (9) are expressed as:

$$\begin{array}{l} R_t x + n_t - p_t = Y_t, \quad t = 1, \ldots, T \\ R^L_{t0} x + n_t - p_t = Y^R_{t0}, \quad t = T+1, \ldots, T+f \end{array} \tag{13}$$

in this case being the undesirable under-achievement.

3.2.3. G3: Socially responsible investment

Lower bound for investment in ethical mutual funds: The investor can classify assets according to their social responsibility and decide that an acceptable portfolio should contain a fraction of the total budget 'substantially greater than or equal to j%' of such assets they have classified as being ethical. We will denote these SRI assets by the label EF. Thus, this fuzzy goal of the investor is expressed as:

$$\sum_{s_i \in EF} x_i \gtrsim \frac{j}{100} \tag{14}$$

the flexible treatment of the relationship ($\gtrsim$) could be expressed by eliciting a membership function μ_{EF}, the general form of which is as follows:

$$\mu_{EF}(x) = \begin{cases} 0 & \text{if } \sum\limits_{s_i \in EF} x_i < j/100 - h \\ \text{strictly monotone increasing} & \text{if } j/100 - h \leqslant \sum\limits_{s_i \in EF} x_i \leqslant j/100 \\ 1 & \text{if } \sum\limits_{s_i \in EF} x_i > j/100 \end{cases} \tag{15}$$

It is therefore assumed that the membership function should be 1 if the goal is fully satisfied, 0 if the goal is violated beyond its limit h, and strictly monotone increasing from 0 to 1. With these membership functions, the investor 'softly' delimits goal non-fulfilment, refusing to accept goal violation beyond the h tolerance threshold, with she/he being more satisfied the closer the tolerance threshold is to 0. These membership functions can be interpreted as satisfaction functions relating to the fulfilment of the goal.

Lower bound for investment in assets with social characteristics SC_k: The investor also requires that a proportion of the total budget 'substantially greater than or equal to l_k%' is invested in assets that fulfil the social characteristic $SC_k(k \in \{1,\ldots,K\})$. These assets are a subset of the ethical assets that we shall denote by the label SC_k. This fuzzy goal can thus be expressed as:

$$\sum_{s_i \in SC_k} x_i \gtrsim \frac{l_k}{100} \tag{16}$$

In order to quantify this fuzzy goal, we propose using a membership function, μ_{CS_k}, which can, in general, be expressed as (15). In this case, there are similar considerations to those mentioned above.

Aggregation of the degrees of satisfaction $(\mu_{EF}, \mu_{SC_k}, k \in \{1,\ldots,K\})$ relating to the achievement of each goal will be carried out using the Zimmermann approach (1978). This means that the investor will behave cautiously and that the aggregate degree of satisfaction is the minimum. This will give rise to the following max–min problem:

$$\begin{aligned} &\max \ \lambda \\ &\text{s.t.} \quad \lambda \leqslant \mu_{EF}(x) \\ &\qquad \lambda \leqslant \mu_{SC_k}(x), \ k \in \{1,2,\ldots,K\} \end{aligned} \tag{17}$$

where λ measures the aggregated degree of satisfaction with the ethical profile of the portfolio.

The above problem can be formulated as a goal for the variable λ with target a value of λ^*, less than 1: $\lambda \geqslant \lambda^*$.

$$\lambda + n_{SRI} - p_{SRI} = \lambda^* \tag{18}$$

3.2.4. Our goal programming model

From these objectives, targets and constraints, we propose the following weighted GP model (Romero, 1991) for selecting SRI portfolios:

$$\begin{aligned} \min \quad & a = \omega_E n_E + \sum_{t=1}^{T+f} \omega_t n_t + \omega_{SRI} n_{SRI} \\ \text{s.t.} \quad & E_0^L x + n_E - p_E = E_0^{*R} \\ & R_t^x + n_t - p_t = Y_t, \quad t = 1,\ldots,T \\ & R_{t0}^L x + n_t - p_t = Y_{t0}^R, \quad t = T+1,\ldots,T+f \\ & \lambda \leqslant \mu_{EF}(x) \\ & \lambda \leqslant \mu_{SC_k}(x), \quad k \in \{1,\ldots,K\} \\ & \lambda + n_{SRI} - p_{SRI} = \lambda^* \\ & 0 \leqslant \lambda \leqslant 1 \\ & n_E, n_t, n_{SRI}, p_E, p_t, p_{SRI} \geqslant 0 \\ & x \in X \end{aligned} \tag{19}$$

where ω_E, ω_t and ω_{SRI} are the weights for the deviation in the achievement function and X is the feasible set with the budget constraint and the non-negativity conditions for the decision variables.

In the next section we apply the proposed model to a set of UK-domiciled mutual funds.

4. Case study

The data set used for this research consists of 44 ethical UK-domiciled mutual funds (E) and 81 conventional UK-domiciled mutual funds (C) (see Tables A.1 and A.2 in the appendix for more details). The selection of mutual funds was carried out taking into account the availability of data – these have kindly provided by *Morningstar Inc.*, a leading provider of independent investment research whose mission is to create products that help investors reach their financial goals (see www.morningstar.com).

Once mutual funds had been selected, 10-day overlapped returns from January 2002 to January 2007 were calculated for 125 mutual funds (both ethical and conventional). This period was divided into two parts. The period from January 11th, 2002 to November 4th, 2005 (i.e. the first 200 observations) was used with the aim of forecasting, while the data from following period was used for the optimization and analysis of the method. We take the benchmark index to be the FTSE100 (see Fig. 1). The FTSE 100 is a broad-based financial index comprised of a select group of one hundred publicly traded stocks listed on the London Stock Exchange. The basis for a company's inclusion within the index is market capitalization. These one hundred companies have the highest market capitalization of all stocks listed on the exchange. Stocks within the index include such firms as British Petroleum, Rolls Royce and the Royal Bank of Scotland. The index is seen as a leading indicator for the health of the UK economy, as these one hundred stocks account for approximately 80% of the London Stock Exchange's full market capitalization. The FTSE Group does not use ethical screens for determining the composition of the stocks listed in the index and we can therefore say that the FTSE 100 is a conventional financial index. This is one of the reasons for using it in this study: to answer the question can an ethical investment overcome a conventional financial index?

In our simulation, the optimization problems (19) are performed by varying the final observation (T) from November 4th, 2005 through to January 12th, 2007. Therefore, 63 optimizations have been run and for all the initial observation is set to January 11th, 2002. In each new run, one more observation is added and in all cases we consider one-period-ahead, i.e, $f = 1$ in (9).

The fuzzy expected return on fund $i, \widetilde{E}(R_i)$, is obtained from three previous historical mutual fund returns with weights represented by triangular fuzzy numbers collected in the matrix M_i. The modes of the triangular fuzzy numbers (m_i^1, m_i^3 and m_i^3) and the maximal left and right spread (Ls_i and Rs_i respectively) are provided by a financial expert. Therefore, R_i being the column vector composed of historical returns on asset s_i, $\widetilde{E}(R_i)$ is obtained by the matrix product M_iR_i. That is:

$$E(R_i)_0^L = w_1 \begin{pmatrix} R_{it-1} \\ R_{it-2} \\ R_{it-3} \end{pmatrix} \ E(R_i)_{\alpha=1} = w_2 \begin{pmatrix} R_{it-1} \\ R_{it-2} \\ R_{it-3} \end{pmatrix} \ E(R_i)_0^R = w_3 \begin{pmatrix} R_{it-1} \\ R_{it-2} \\ R_{it-3} \end{pmatrix}$$

where r_j is the j-th row of matrix M_i. The construction of the matrix M_i is based on the relationship between R_{it-1}, R_{it-2} and R_{it-3} in order to guarantee that $\widetilde{E}(R_i)$ is a fuzzy number. So:

- If max $(R_{it-1}, R_{it-2}, R_{it-3}) = R_{it-1}$, then

$$M_i = \begin{pmatrix} m_i^1 - Ls_i & m_i^2 + Ls_i/2 & m_i^3 + Ls_i/2 \\ m_i^1 & m_i^2 & m_i^3 \\ m_i^1 + Rs_i & m_i^2 - Rs_i/2 & m_i^3 - Rs_i/2 \end{pmatrix}$$

- If max $(R_{it-1}, R_{it-2}, R_{it-3}) = R_{it-2}$, then

$$M_i = \begin{pmatrix} m_i^1 + Ls_i/2 & m_i^2 - Ls_i & m_i^3 + Ls_i/2 \\ m_i^1 & m_i^2 & m_i^3 \\ m_i^1 - Rs_i/2 & m_i^2 + Rs_i & m_i^3 - Rs_i/2 \end{pmatrix}$$

- If max $(R_{it-1}, R_{it-2}, R_{it-3}) = R_{it-3}$, then

$$M_i = \begin{pmatrix} m_i^1 + Ls_i/2 & m_i^2 + Ls_i/2 & m_i^3 - Ls_i \\ m_i^1 & m_i^2 & m_i^3 \\ m_i^1 - Rs_i/2 & m_i^2 - Rs_i/2 & m_i^3 + Rs_i \end{pmatrix}$$

For instance, assuming that $m_1^i = 0.6, m_2^i = m_3^i = 0.2, Ls_i = 0.15$, $Rs_i = 0.4$ and max $(R_{it-1}, R_{it-2}, R_{it-3}) = R_{it-1}$, M_i is obtained as:

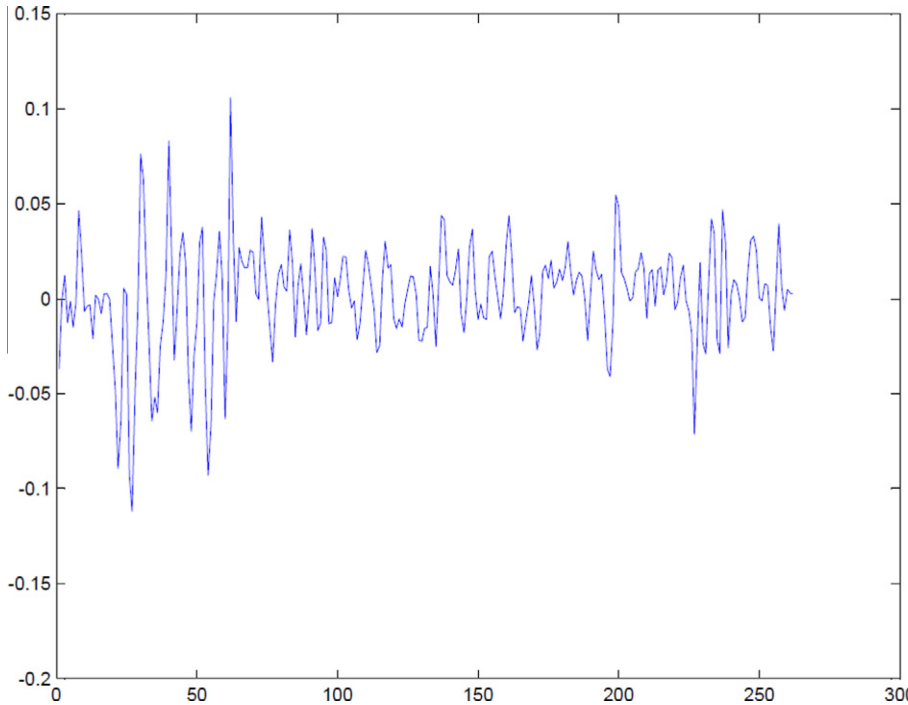

Fig. 1. FTSE 100 returns from 01/11/2002 to 01/19/2007.

$$M_i = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.275 & 0.275 \\ 0.6 & 0.2 & 0.2 \\ 1 & 0 & 0 \end{pmatrix}$$

For the first simulation, in which the 198, 199 and 200 historical returns are used, the fuzzy expected return for our 125 mutual funds (with the fuzzy weights provided by the financial expert, assuming $m_1^i = 0.6, m_2^i = m_3^i = 0.2, Ls_i = 0.15, Rs_i = 0.4$ for all mutual funds) are shown in Table 1.

As we have previously pointed out, the target for the fuzzy expected return of the portfolio is its ideal value, i.e., the solution of problem (3) (see Table 2 and Fig. 2).

The fuzzy vector of future returns, $\widetilde{R}_t$, is obtained from 3 forecasted returns that have been obtained by an autoregressive model using two lags and financial variables (DJ Eurostoxx50, UK t-bills, UK Govern Bonds and FTSE100) on the previous 199 + h historical returns in the h-th optimization (h varying from 1 to 63) (see Table A.3 in the appendix). In this case, we have also fixed $m_1 = 0.6$, $m_2 = m_3 = 0.2$, $Ls = 0.15$ and $Rs = 0.4$. Similarly, we have obtained $\widetilde{Y}_t$ with $m_1 = 0.6$, $m_2 = 0.3$, $m_3 = 0.1$, $Ls = 0.1$ and $Rs = 0.4$ (see Table A.4 in the appendix). We note that the main aim of the case study presented is to demonstrate the novel approach. This approach can be combined with appropriate scenario generation procedures utilizing advanced statistical forecasting techniques. However the search for suitable alternative forecasting techniques for use in this model is beyond the scope of this paper and we consider the autoregressive method useful for the purpose of the illustration.

We have classified ethical criteria into three groups in order to define the socially responsible investment profiles: social, environmental and ethical.

With respect to the SEE characteristics, the goal of the investor is to put at least the thirty percent (with a threshold equal to 6%) of his budget into ethical mutual funds that meet the social and the environmental characteristics in the Table 3. In addition, the investor wishes to achieve a degree of satisfaction with this goal equal to 1. Our weighting goal programming model is therefore:

Table 1
Fuzzy expected return $\widetilde{E}(R_i)$.

Fund	E_0^L	E_1	E_0^R	Fund	E_0^L	E_1	E_0^R
E1	0, 0273	0, 036	0, 0593	E17	0, 0377	0, 0453	0, 0655
E2	0, 0277	0, 0364	0, 0596	E18	0, 0302	0, 0377	0, 0579
E3	0, 0281	0, 0367	0, 0596	E19	0, 0306	0, 0382	0, 0585
E4	0, 0277	0, 0364	0, 0597	E20	0, 0358	0, 0426	0, 0609
E5	−0, 0009	−0, 0003	0, 0016	E21	0, 0362	0, 043	0, 0613
E6	−0, 0093	−0, 0073	−0, 0019	E22	0, 0276	0, 0354	0, 0562
E7	0, 0365	0, 0425	0, 0583	E23	0, 028	0, 0359	0, 0569
E8	0, 0362	0, 0422	0, 058	E24	0, 0274	0, 0353	0, 0564
E9	0, 0265	0, 0324	0, 0482	E25	0, 0277	0, 0357	0, 057
E10	0, 0264	0, 0325	0, 0487	E26	0, 0283	0, 0357	0, 0552
E11	0, 0264	0, 0324	0, 0484	E27	0, 0286	0, 036	0, 0557
E12	0, 0447	0, 052	0, 0713	E28	0, 0313	0, 0377	0, 0548
E13	0, 0445	0, 0518	0, 071	E29	0, 0315	0, 0379	0, 0549
E14	0, 0379	0, 0435	0, 0586	E30	0, 0512	0, 0594	0, 0812
E15	0, 0379	0, 0454	0, 0657	...	...	...	...
E16	0, 0379	0, 0455	0, 0658	C125	0, 0197	0, 0288	0, 0529

Table 2
Ideal fuzzy expected return.

α-cut	$\widetilde{E}_\alpha^{*L}$	$\widetilde{E}_\alpha^{*R}$
0	0.03129145	0.08339616
0.1	0.03227177	0.07933554
0.2	0.03325209	0.07527493
0.3	0.0342324	0.07121431
0.4	0.03521272	0.06715369
0.5	0.03619304	0.06309308
0.6	0.03717336	0.05903246
0.7	0.03822181	0.05497185
0.8	0.03974454	0.05091123
0.9	0.04126727	0.04685062
1	0.04278999	0.04278999

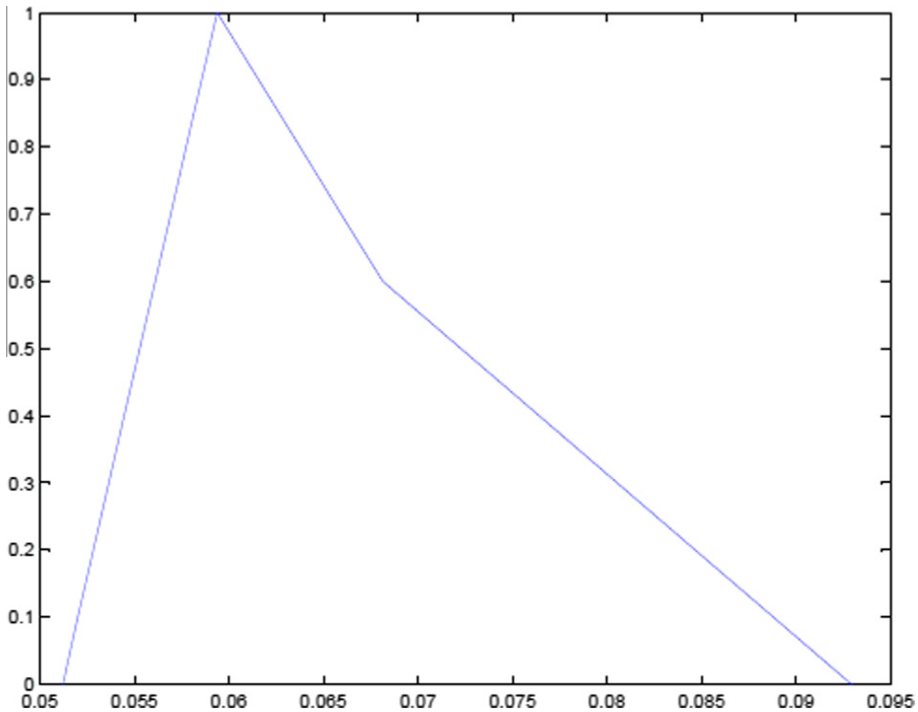

Fig. 2. Ideal fuzzy expected return.

Table 3
Group screening for investor profiles. (Based on the EIRIS Green and Ethical funds directory (2008)).

I. GROUP: Social	II. GROUP: Environmental	III. GROUP: Ethical
1.Access to Medicines	13. Air & Water Pollution	27. Alcohol
2. Bribery & Corruption	14. Biodiversity	28. Animal Testing & Fur
3. Child Labor	15. Climate Change	29. Gambling
4. Community Giving	16. Energy	30. Genetic Engineering
5. Community Initiatives	17. Environmental Management	31. Military
6. Conflict	18. Mining & Quarrying	32. Nuclear Power
7. Corporate Governance	19. Nuclear power	33. Pornography & Adult Entertainment
8. Equal Opportunities	20. Ozone-Depleting Chemicals	34. Repressive Regimes
9. Health & Safety	21. Pesticides	35. Tobacco
10. Human Rights	22. Resource Productivity	36. Firearms
11. Labor Standards	23. Transport	37. Animal-derived Products
12. Supply Chain Management	24. Tropical Hardwood	38. Issues Related to Animal Food
	25. Waste & Toxic Chemical Management	
	26. Water Management	

$$
\begin{aligned}
\min \quad & a = \omega_E n_E + \sum_{t=1}^{T+1} \omega_t n_t + \omega_{SRI} n_{SRI} \\
\text{s.t.} \quad & E_0^L x + n_E - p_E = E_0^{*R} \\
& R_t^x + n_t - p_t = Y_t, \quad t = 1, \ldots, T \\
& R_{t0}^L x + n_t - p_t = Y_{t0}^R, \quad t = T+1 \\
& \sum_{i \in SC_k} x_i - 0.06\lambda \geqslant 0.24, \quad k = 1, 2, \ldots, 26 \\
& \lambda + n_{SRI} - p_{SRI} = 1 \\
& 0 \leqslant \lambda \leqslant 1 \\
& n_E, p_E, n_t, p_t, n_{SRI}, p_{SRI} \geqslant 0, \quad x \in X
\end{aligned}
$$

where $\omega_E = \frac{1}{E_0^{*R}}, \omega_t = \frac{1}{T}, \ t = 1, \ldots, T, \ \omega_{T+1} = \omega_{SRI} = 1$ and Y denotes the return on the chosen benchmark, i.e., the FTSE100. The results for all optimizations are presented in Table 4 (below).

The real returns of the optimized portfolios were obtained from the known mutual fund returns at the moment $T + 1$. The real deviation is calculated as the difference between the real return of the portfolio and the real return of the benchmark in the future period. In this simulation, the portfolios were found to exceed the benchmark 45 times (out of 63 optimizations) – see Fig. 3. These results can be positively evaluated, assuming that the forecasting method has not been refined.

For the most part (in 94% of cases), the portfolios obtained contain ethical mutual funds and only one non-ethical mutual fund. With regards to the amount invested in ethical assets, in 38% of cases at least 60% of the budget is invested in this kind of mutual fund (the investor is fully satisfied with an investment of 30% in ethical funds). Furthermore, all the portfolios obtained meet the requirement that at least 30% of the budget has been invested in mutual funds that fulfill the social and environmental criteria.

A sensitivity analysis of the investor's ethical preferences has been conducted for degrees of satisfaction from 0.1 to 1.

As expected, in Table 5 one can observe that a greater degree of satisfaction with regards to the joint fulfillment (λ) of the SEE preferences produces a greater proportion of the budget invested in ethical mutual funds. However, the results of the real return were

Table 4
Description of the optimal portfolios[a].

Historical returns (from 1/11/2002)	Portfolio	Future period	
		Real return	Real deviation
to 11/04/2005 (200 observations)	E19 = 0.3; E30 = 0.07; C101 = 0.63	0.0292	0.0153
to 11/11/2005 (201 observations)	E19 = 0.3; C67 = 0.7	0.0515	0.0408
to 11/18/2005 (202 observations)	E19 = 0.3; C67 = 0.7	0.0183	0.013
to 11/25/2005 (203 observations)	E19 = 0.3; C65 = 0.7	0.0328	0.034
to 12/02/2005 (204 observations)	E19 = 0.3; C65 = 0.7	0.0321	0.0315
to 12/09/2005 (205 observations)	E19 = 0.3; C65 = 0.7	0.076	0.0618
...	...	...	...
to 01/13/2006 (210 observations)	E19 = 0.3; C99 = 0.7	0.0382	0.0249
to 01/20/2006 (211 observations)	E19 = 0.3; C99 = 0.7		0.0017
to 01/27/2006 (212 observations)	E7 = 0.3; E21 = 0.3; C92 = 0.4	0.0277	0.0316
to 02/03/2006 (213 observations)	E21 = 0.3; E44 = 0.3; C92 = 0.4	0.0183	0.0032
to 02/10/2006 (214 observations)	E19 = 0.3; C122 = 0.7	0.0258	0.0091
to 02/17/2006 (215 observations)	E19 = 0.3; C121 = 0.7	0.0211	0.019
...	...	...	...
to 04/28/2006 (225 observations)	E7 = 0.3; E21 = 0.3; C98 = 0.4	0.0051	0.0235
to 05/05/2006 (226 observations)	E20 = 0.3; E32 = 0.3; C98 = 0.4	−0.0613	0.0101
to 05/12/2006 (227 observations)	E5 = 0.76; E20 = 0.24	0.0008	0.0213
to 05/19/2006 (228 observations)	E5 = 0.76; E21 = 0.24	0.0042	−0.0147
to 05/26/2006 (229 observations)	E5 = 0.3; E21 = 0.3; C62 = 0.4	−0.0002	0.0233
to 06/02/2006 (230 observations)	E21 = 0.3; E36 = 0.3; C62 = 0.4	−0.0056	0.0234
...	...	...	...
to 09/15/2006 (245 observations)	E12 = 0.3; E21 = 0.3; C81 = 0.4	0.0224	0.0081
...	...	...	...
to 12/29/2006 (260 observations)	E19 = 0.3; C67 = 0.7	−0.0169	−0.0198
to 01/05/2007 (261 observations)	E19 = 0.3; C47 = 0.7	−0.0251	−0.0224
to 01/12/2007 (262 observations)	E19 = 0.3; C71 = 0.7	0.0222	0.0204

[a] Assuming $m_1^i = 0.6, m_2^i = m_3^i = 0.2, Ls_i = 0.15, Rs_i = 0.4$ for the expected return, $\widetilde{E}(R_i)$, of all mutual funds, $m_1 = 0.6$, $m_2 = m_3 = 0.2$, $Ls = 0.15$, $Rs = 0.4$ for the fuzzy vector of future returns, $\widetilde{R}_t$, and $m_1 = 0.6$, $m_2 = 0.3$, $m_3 = 0.1$, $Ls = 0.1$, $Rs = 0.4$ for $\widetilde{Y}_t$.

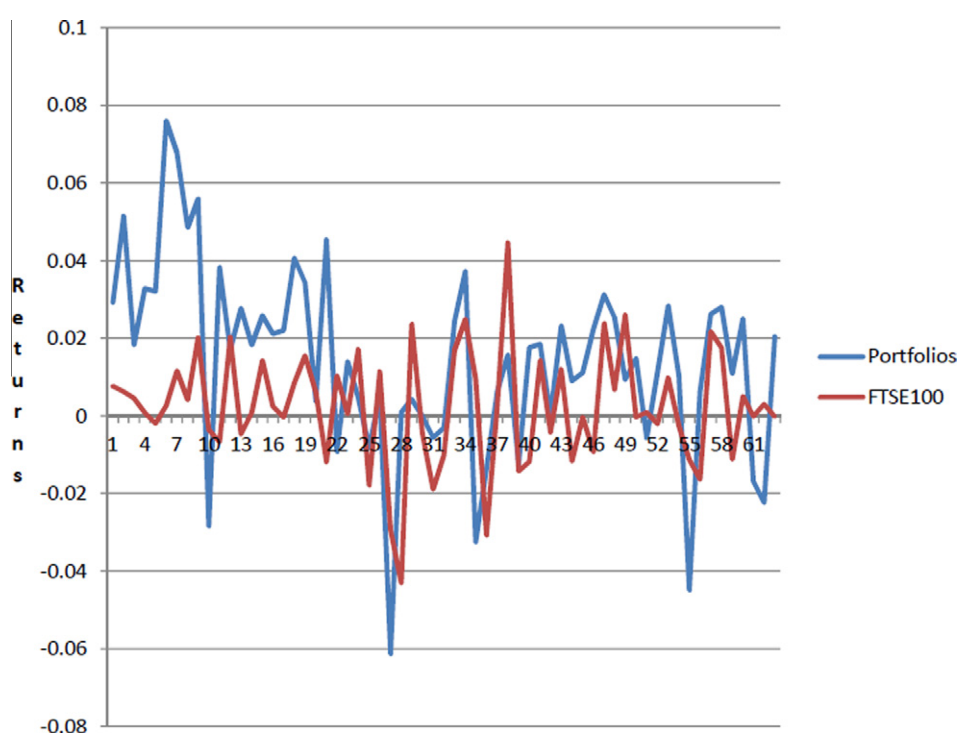

Fig. 3. Portfolio and FTSE 100 returns from 01/11/2002 to 01/19/2007.

somewhat surprisingly: a greater social responsibility on the part of the investor resulted in a greater real return on the portfolio. These results must be considered cautiously and they may not be generalizable.

Finally, a comparison between the portfolios obtained using fuzzy weights and those using crisp weights (we use the most possible value provided by the financial expert) is shown below (Table 6):

Table 5
Portfolios ($T = 200$).

λ	Portfolio	Real return
0.1	E19 = 0.246; C101 = 0.754	0.0277
0.2	E19 = 0.252; C101 = 0.748	0.0278
0.3	E19 = 0.258; C101 = 0.742	0.028
0.4	E19 = 0.264; C101 = 0.736	0.0282
0.5	E19 = 0.27; C101 = 0.73	0.0284
0.6	E19 = 0.276; C101 = 0.724	0.0286
0.7	E19 = 0.282; C101 = 0.718	0.0287
0.8	E19 = 0.288; C101 = 0.712	0.0289
0.9	E19 = 0.294; C101 = 0.706	0.0291
1	E19 = 0.3; E30 = 0.07; C101 = 0.63	0.0292

Table 6
Comparison between portfolios with fuzzy weights and crisp weights (*ceterisparibus*for 63 optimizations).

Weights (m_1, m_2, m_3)	Number of times fuzzy-weights portfolio overcomes crisp-weights portfolio	Number of times crisp-weights portfolio overcomes fuzzy-weights portfolio
(0.6,0.2,0.2)	11	11
(0.2,0.2,0.6)	18	6
(0.1,0.8,0.1)	9	4
(0.3,0.3,0.4)	13	6
(0.1,0.2,0.7)	11	5
(0.1,0.1,0.8)	8	6
(0.8,0.1,0.1)	7	7
(0.25,0.5,0.25)	24	8
(0.1,0.4,0.5)	10	3

Empirical results indicate that, for almost all weights examined, the fuzzy approach is better than the crisp one.

5. Conclusions

In this paper, a new methodology is proposed for constructing portfolios for investors who take into account ethical, social and environmental criteria when making investment decisions.

The interest of individual and institutional investors in these types of investment has led to increasing academic literature relating to this sector. Several SRI topics have been extensively studied, with most of the existing empirical studies on SRIs focusing on the financial performance of SRI funds. However, there is a dearth of academic literature on the development of methods, based on mathematical programming for constructing portfolios tailored to the tastes and concerns of SRI investors. In response, the purpose of this paper has been to fill in some of the knowledge gaps for this potentially very important and growing class of investors.

The suggested methodology is based on multi-criteria decision making and fuzzy set theory. Fuzzy set theory has been applied in several features of the model formulation:

- Firstly, the setting of weights to calculate the uncertain parameters (expected and future returns) was carried out in a soft way by applying fuzzy technology. Thus, it is possible to obtain a fuzzy expected return from historical data and fuzzy weights that are provided by a financial expert. In this paper, the expected return is represented by a fuzzy number. To do this, a new algorithm is presented.
- The future returns are modeled by fuzzy numbers in a similar way.
- The target for the expected return is obtained as the ideal value of a fuzzy programming problem.
- The portfolio risk has been measured by the underperformance on a benchmark index, i.e., a tracking error strategy has been followed.
- The investor preferences relating to social, ethical and environmental characteristics are based on imprecise preferences, which in this paper have been modeled by means of degrees of satisfaction obtained as degrees of membership of a fuzzy set.
- Lastly, a fuzzy goal programming model has been built and a closely robust approach has been proposed for solving it.

With the modeling presented herein, the investor and financial expert can be more comfortable with the specification of the parameters of the model because the requisite information can contain some grade of ambiguity. In addition, this study allows one to combine a set of financial analysts' forecasts if it is possible to assign fuzzy weights to assess the reliability of each individual analyst. The empirical results could indicate that the performance of the proposed approach exceeds that of the corresponding crisp approach.

Finally, it should be pointed out that a major feature of this model is its sensitivity to the opinion of the analyst as well as to the preferences of the investor. This allows interaction between both parties when it comes to designing the best portfolio.

Acknowledgements

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Appendix A

Tables A.1, A.2, A.3, A.4

Table A.1
Ethical mutual funds.

E1	Aberdeen Ethical World Fd A Acc	E23	Henderson Ind of the Ftre I Inc
E2	Aberdeen Ethical World Fd A Inc	E24	Henderson Ind of the Ftre X Inc
E3	Aberdeen Ethical World Fd C Acc	E25	Henderson Ind of the Ftre
E4	Aberdeen Ethical World Fd C Inc	E26	Insight Inv European Ethical A Acc
E5	Aegon Ethical Corp Bd Fund A Acc	E27	Insight Inv European Ethical I Acc
E6	Aegon Ethical Corp Bd Fund A Inc	E28	Norwich UK Ethical Fd 1 Acc
E7	Aegon Ethical Eq B Inc Inst Nav B Acc	E29	Norwich UK Ethical Fd 2 Acc
E8	Aegon Ethical Equity	E30	Old Mutual Ethical Fd A Acc
E9	Allchurches Amity Fund A Inc	E31	Old Mutual Ethical Fd A Inc
E10	Allchurches Amity Fund B Inc	E32	Standard Life UK Ethical Fd Instl Acc
E11	Allchurches Amity Fund C Acc	E33	Standard Life UK Ethical R Acc
E12	AXA Ethical I Acc	E34	Sustainable Fut Abslte Gr 1 Acc
E13	AXA Ethical R Acc	E35	Sustainable Fut Corp Bond 1 Inc
E14	Banner Real Fd A Acc	E36	Sustainable Fut Corp Bond 2 Inc
E15	Credit Suisse Fellowship A Inc	E37	Sustainable Fut Europn Gr 1 Acc
E16	Credit Suisse Fellowship I Inc	E38	Sustainable Fut Europn Gr 2 Acc
E17	Credit Suisse Fellowship R Inc	E39	Sustainable Fut Global Gr 1 Acc
E18	Henderson Gib Care Growth Inc	E40	Sustainable Fut Global Gr 2 Acc
E19	Henderson Gib Care Growth Instl Inc	E41	Sustainable Fut Managed 1 Inc
E20	Henderson Gib CareIncome Inc	E42	Sustainable Fut Managed 2 Inc
E21	Henderson Gib CareIncome Inst Inc	E43	Sustainable Fut UK Gr 1 Acc
E22	Henderson Ind of the Ftre I Acc	E44	Sustainable Fut UK Gr 2 Acc

Table A.2
Conventional mutual funds.

C45	Aberdeen European Growth A Acc	C86	Insight Inv Evergreen Retail Acc
C46	Aberdeen UK Growth Fund Inc	C87	Insight Inv Gib Eq A Acc
C47	AEGON Technology B Acc	C88	Insight Inv Japan Eq A Acc
C48	AEGON UK Equity Fund B Acc	C89	Insight Inv Monthly Inc A Acc
C49	Cazenove European Fund B Acc	C90	JP Morgan UK Equity Inc Class A Acc
C50	Cazenove UK Opportunities Fd B Acc	C91	Lazard European Alpha R Inc
C51	Credit S Wldwd Gr A Inc	C92	Lazard UK Sm Cos R Inc
C52	Credit Suisse Small Companies I Inc	C93	Lincoln North America Trust Inc
C53	F& C FTSE All-Share Tracker Fd 1 Acc	C94	M& G American Fd A GBP Acc
C54	F& C Japan Gr Fd 1 Acc	C95	M& G American Fd A Inc
C55	F& C Stewardship Gr Fd 1 Acc	C96	M& G European Fd A GBP Acc
C56	Fidelity Amer Spec Situations AC Acc	C97	M& G European Fd A Inc
C57	Fidelity American	C98	M& G Intl Growth A EUR Acc
C58	Fidelity Euro Opps GBP Acc	C99	M& G Pan European Fd A GBP Acc
C59	Fidelity European	C100	M& G Pan European Fd A Inc
C60	Fidelity Growth & Income Inc	C101	M& G UK Select A Acc
C61	Fidelity Income Plus Inc	C102	M& G UK Select A Inc
C62	Fidelity Inst OEIC IntlBd A Inc	C103	New Star Balanced Portfolio B Acc
C63	Fidelity International Acc	C104	New Star Balanced Portf Class A Acc
C64	Fidelity Inv Fds (OEIC) Mngd Inte Acc	C105	Newton Balanced Inc
C65	Fidelity Jap Spcl Sit Acc	C106	Newton Continental European Inc
C66	Fidelity Japan Acc	C107	Newton Higher Income Inc
C67	Fidelity South East Asia Acc	C108	Newton Income Inc
C68	Fidelity Special Situations Acc	C109	Newton International Bond Inc
C69	Fidelity UK Aggressive GBP Acc	C110	Newton International Growth Inc
C70	Fidelity UK Growth Acc	C111	Newton Japan Fd Inc
C71	First State Asia Pac A EUR Acc	C112	Newton Managed
C72	First State Asia Pacific	C113	Newton Oriental Inc
C73	First State British Opps A Acc	C114	Norwich Distribution-Fund 1 Inc
C74	First State British Opps A GBP Inc	C115	Schroder Instl Pacific Inst
C75	GAM Global Diversified Acc	C116	Scottish Widows Global Sel Gr B Acc
C76	GAM Global Diversified Inc	C117	Singer & Friedlander Euro Gr Inst I Inc
C77	GAM North American Growth Acc	C118	Singer & Friedlander Euro Gr In R Inc
C78	GAM North American Growth Inc	C119	The Schroder Instl Eur Fd Acc
C79	GAM UK Diversified Acc	C120	The Schroder Instl Eur Fd Inc
C80	GAM UK Diversified Inc	C121	The Schroder Inst Eur Sm Com Fd Acc
C81	Henderson UK Cap Gr A Acc	C122	The Schroder Instl Eur Sm Com Fd Inc
C82	HSBC European Gwth Fd Retail Inc	C123	The Schroder Instl Gib Eq Inc
C83	Insight Inv Asia Pacific Eq A Acc	C124	The Schroder Instl Jap Sm Cos Instl Inc
C84	Insight Inv Eq Hi Income Retail Inc	C125	The Schroder Instl Pacific Fd Inc
C85	Insight Inv European Eq A Acc		

Table A.3
Fuzzy returns one-period-ahead: $\widetilde{R}_{T+1}$ for $T = 200$.

E1	(0.0537, 0.0574, 0.0675)	C1	(0.0412, 0.0463, 0.0602)	C45	(0.043, 0.0455, 0.052)
E2	(0.0538, 0.0576, 0.0677)	C2	(0.0573, 0.0622, 0.0752)	C46	(0.0489, 0.0529, 0.0636)
E3	(0.0538, 0.0577, 0.0678)	C3	(0.0733, 0.0806, 0.0999)	C47	(0.0392, 0.0439, 0.0565)
E4	(0.0539, 0.0577, 0.0678)	C4	(0.0522, 0.0577, 0.0724)	C48	(0.049, 0.0516, 0.0585)
E5	(0.0024, 0.0037, 0.0071)	C5	(0.0369, 0.043, 0.0593)	C49	(0.0633, 0.0698, 0.0872)
E6	(−0.004, −0.0027, 0.001)	C6	(0.0509, 0.0565, 0.0714)	C50	(0.0623, 0.0679, 0.0828)
E7	(0.0525, 0.0541, 0.0582)	C7	(0.0524, 0.0568, 0.0684)	C51	(0.0623, 0.0679, 0.0828)
E8	(0.0522, 0.0538, 0.0579)	C8	(0.044, 0.0475, 0.0568)	C52	(0.0449, 0.0507, 0.066)
E9	(0.0455, 0.0468, 0.0502)	C9	(0.0488, 0.0539, 0.0676)	C53	(0.0449, 0.0507, 0.066)
E10	(0.046, 0.0472, 0.0505)	C10	(0.0444, 0.0552, 0.0838)	C54	(0.0384, 0.0423, 0.0526)
E11	(0.0453, 0.0466, 0.0501)	C11	(0.0551, 0.0565, 0.0601)	C55	(0.0389, 0.0439, 0.0572)
E12	(0.0652, 0.0691, 0.0797)	C12	(0.0617, 0.068, 0.0848)	C56	(0.0388, 0.0438, 0.0571)
E13	(0.0648, 0.0688, 0.0794)	C13	(0.0704, 0.0778, 0.0976)	C57	(0.0615, 0.0662, 0.0788)
E14	(0.0513, 0.0532, 0.0584)	C14	(0.0413, 0.0494, 0.071)	C58	(0.0622, 0.0667, 0.0788)
E15	(0.0597, 0.0625, 0.07)	C15	(0.033, 0.036, 0.044)	C59	(0.044, 0.0456, 0.0501)
E16	(0.0598, 0.0626, 0.0701)	C16	(0.0519, 0.0552, 0.064)	C60	(0.0437, 0.0454, 0.0499)
E17	(0.0596, 0.0624, 0.0698)	C17	(0.0467, 0.0501, 0.0592)	C61	(0.0331, 0.0359, 0.0435)
E18	(0.052, 0.055, 0.063)	C18	(0.0031, 0.0048, 0.0094)	C62	(0.0384, 0.0431, 0.0557)
E19	(0.0525, 0.0556, 0.0638)	C19	(0.055, 0.0584, 0.0677)	C63	(0.0505, 0.0538, 0.0625)
E20	(0.0547, 0.0574, 0.0645)	C20	(0.0593, 0.0635, 0.0746)	C64	(0.0486, 0.0537, 0.0675)
E21	(0.0551, 0.0578, 0.065)	C21	(0.0465, 0.0534, 0.0721)	C65	(0.0103, 0.0117, 0.0154)
E22	(0.0511, 0.0545, 0.0635)	C22	(0.0445, 0.0604, 0.103)	C66	(0.0484, 0.0523, 0.0626)
E23	(0.0518, 0.0551, 0.064)	C23	(0.0817, 0.0863, 0.0986)	C67	(0.0326, 0.0448, 0.0772)
E24	(0.0513, 0.0547, 0.0638)	C24	(0.0521, 0.0544, 0.0604)	C68	(0.0489, 0.0524, 0.0617)
E25	(0.0518, 0.0551, 0.064)	C25	(0.0453, 0.049, 0.0588)	C69	(0.0583, 0.0629, 0.0752)
E26	(0.0512, 0.0527, 0.0568)	C26	(0.0483, 0.0531, 0.0659)	C70	(0.0167, 0.0185, 0.0234)
E27	(0.0516, 0.0531, 0.0571)	C27	(0.0707, 0.0742, 0.0835)	C71	(0.0496, 0.0527, 0.0609)
E28	(0.0505, 0.053, 0.0597)	C28	(0.0592, 0.0617, 0.0683)	C72	(0.0543, 0.0591, 0.0718)
E29	(0.0506, 0.0531, 0.0597)	C29	(0.0458, 0.0517, 0.0675)	C73	(0.034, 0.0381, 0.0489)
E30	(0.0749, 0.0767, 0.0812)	C30	(0.0461, 0.052, 0.0677)	C74	(0.0335, 0.0377, 0.049)
E31	(0.0747, 0.0765, 0.0812)	C31	(0.0547, 0.0571, 0.0634)	C75	(0.0417, 0.0461, 0.0578)
E32	(0.056, 0.0585, 0.0652)	C32	(0.0548, 0.0571, 0.0634)	C76	(0.0415, 0.0459, 0.0578)
E33	(0.0557, 0.0582, 0.0649)	C33	(0.0505, 0.0548, 0.0663)	C77	(0.0117, 0.0178, 0.0341)
E34	(0.0345, 0.038, 0.0474)	C34	(0.0505, 0.0548, 0.0663)	C78	(0.0114, 0.0176, 0.0341)
E35	(−0.0055, −0.0041, −0.0006)	C35	(0.0639, 0.0661, 0.0719)	C79	(0.0534, 0.0577, 0.0694)
E36	(−0.0063, −0.0049, −0.0012)	C36	(0.064, 0.0662 , 0.0719)	C80	(0.0413, 0.0484, 0.0673)
E37	(0.0338, 0.0362, 0.0427)	C37	(0.0588, 0.0624, 0.0721)	C81	(0.0491, 0.0518, 0.0589)
E38	(0.0345, 0.0364, 0.0415)	C38	(0.0413, 0.0467, 0.061)		
E39	(0.0422, 0.0458, 0.0555)	C39	(0.0626, 0.0669, 0.0783)		
E40	(0.0459, 0.0531, 0.0724)	C40	(0.0388, 0.0427, 0.0532)		
E41	(0.0398, 0.042, 0.0477)	C41	(0.0388, 0.0434, 0.0557)		
E42	(0.0404, 0.0425, 0.0481)	C42	(0.0628, 0.0649, 0.0707)		
E43	(0.0478, 0.051, 0.0593)	C43	(0.0534, 0.058, 0.0703)		
E44	(0.0483, 0.0514, 0.0595)	C44	(0.0469, 0.0584, 0.0891)		

Table A.4
$\widetilde{Y}_{T+1}$ with $m_1 = 0.6$, $m_2 = 0.3$, $m_3 = 0.1$, $Ls = 0.1$, $Rs = 0.4$ (one-period-ahead).

$T = 200$	(0.0379, 0.0422, 0.051)	$T = 221$	(−0.0005, 0.0047, 0.0151)	$T = 242$	(0.0044, 0.0098, 0.0205)
$T = 201$	(0.0052, 0.0111, 0.0227)	$T = 222$	(0.0123, 0.0168, 0.0259)	$T = 243$	(−0.0005, 0.0049, 0.0156)
$T = 202$	(0.0072, 0.0125, 0.023)	$T = 223$	(0.0165, 0.0211, 0.0302)	$T = 244$	(−0.01, −0.0044, 0.0068)
$T = 203$	(0.0023, 0.0078, 0.0187)	$T = 224$	(−0.0039, 0.0019, 0.0134)	$T = 245$	(−0.0078, −0.0023, 0.0087)
$T = 204$	(−0.0034, 0.0023, 0.0136)	$T = 225$	(−0.0064, −0.0008, 0.0104)	$T = 246$	(0.0125, 0.0173, 0.027)
$T = 205$	(−0.0001, 0.0053, 0.016)	$T = 226$	(−0.0155, −0.0097, 0.0019)	$T = 247$	(0.0251, 0.0297, 0.0388)
$T = 206$	(0.012, 0.0169, 0.0267)	$T = 227$	(−0.0625, −0.0549, −0.0397)	$T = 248$	(0.0255, 0.0302, 0.0396)
$T = 207$	(0.0119, 0.017, 0.0271)	$T = 228$	(−0.0084, −0.0038, 0.0054)	$T = 249$	(0.0195, 0.0244, 0.0342)
$T = 208$	(0.0209, 0.0254, 0.0346)	$T = 229$	(0.0206, 0.0247, 0.0329)	$T = 250$	(−0.0022, 0.0035, 0.0148)
$T = 209$	(0.012, 0.0171, 0.0273)	$T = 230$	(−0.0224, −0.0161, −0.0035)	$T = 251$	(−0.001, 0.0043, 0.015)
$T = 210$	(−0.0109, −0.005, 0.0067)	$T = 231$	(−0.0211, −0.0154, −0.0042)	$T = 252$	(0.0076, 0.0125, 0.0223)
$T = 211$	(0.015, 0.0194, 0.0282)	$T = 232$	(0.0109, 0.0152, 0.0239)	$T = 253$	(0.0053, 0.0105, 0.0209)
$T = 212$	(0.0114, 0.0165, 0.0267)	$T = 233$	(0.0374, 0.0413, 0.049)	$T = 254$	(−0.0143, −0.0083, 0.0038)
$T = 213$	(−0.007, −0.0011, 0.0108)	$T = 234$	(0.0256, 0.0304, 0.0401)	$T = 255$	(−0.0229, −0.0169, −0.0049)
$T = 214$	(0.014, 0.0187, 0.0281)	$T = 235$	(−0.0236, −0.017, −0.0036)	$T = 256$	(0.0097, 0.0141, 0.0228)
$T = 215$	(0.0122, 0.0173, 0.0275)	$T = 236$	(−0.0217, −0.016, −0.0045)	$T = 257$	(0.0359, 0.0397, 0.0474)
$T = 216$	(−0.0008, 0.0048, 0.016)	$T = 237$	(0.0464, 0.0494, 0.0554)	$T = 258$	(−0.001, 0.005, 0.017)
$T = 217$	(0.0086, 0.0134, 0.023)	$T = 238$	(0.0191, 0.0244, 0.0351)	$T = 259$	(−0.0066, −0.001, 0.0104)
$T = 218$	(0.0218, 0.0262, 0.035)	$T = 239$	(−0.0292, −0.0221, 0.008)	$T = 260$	(0.0057, 0.0106, 0.0205)
$T = 219$	(0.0167, 0.0216, 0.0315)	$T = 240$	(0.0052, 0.010, 0.0195)	$T = 261$	(0.0018, 0.0071, 0.0177)
$T = 220$	−0.0085, −0.0025, 0.0094	$T = 241$	(0.0074, 0.0126, 0.0229)	$T = 262$	(0.0017, 0.007, 0.0176)

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