



# A fingerprint retrieval system based on level-1 and level-2 features

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## ABSTRACT

This paper proposes a novel fingerprint retrieval system that combines level-1 (local orientation and frequencies) and level-2 (minutiae) features. Various score- and rank-level fusion strategies and a novel hybrid fusion approach are evaluated. Extensive experiments are carried out on six public databases and a systematic comparison is made with eighteen retrieval methods and seventeen exclusive classification techniques published in the literature. The novel approach achieves impressive results: its retrieval accuracy is definitely higher than competing state-of-the-art methods, with error rates that in some cases are even one or two orders of magnitude smaller.

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## 1. Introduction

Automatic fingerprint identification systems are used in forensic and government applications to handle populations of several millions individuals (Maltoni, Maio, Jain, & Prabhakar, 2009; Rath & Bolle, 2003; US-VISIT Program, 2011) or even more than one billion, as in the Indian Unique Identity Project (Unique Identification Authority of India, 2011). When the goal is to accurately and efficiently compare a query fingerprint against a huge database, it is desirable to reduce the number of fingerprints to be considered by using fingerprint retrieval (pre-filtering) techniques (Cappelli, Maio, & Maltoni, 2000; Maltoni et al., 2009). These techniques can be divided into two main categories: (i) exclusive classification and (ii) continuous classification (or fingerprint indexing) (Cappelli & Maio, 2004; Maltoni et al., 2009).

Exclusive classification techniques (Candela, Grother, Watson, Wilkinson, & Wilson, 1995; Cappelli & Maio, 2004; Cappelli, Maio, Maltoni, & Nanni, 2003; Hong & Jain, 1999; Shah & Sastry, 2004) partition the database into a fixed number of classes: during the identification phase, the query fingerprint is compared only to fingerprints belonging to the same class. Techniques belonging to this category are often not able to sufficiently narrow down the search due to (i) the small number of classes and (ii) the uneven distribution of the fingerprints among them. This led the scientific community to investigate retrieval systems that are not based on exclusive classes, but represent each fingerprint in a robust and stable manner so that, in the search phase, it is possible to quickly select a reduced list of the most similar candidates according to an appropriate metric (Cappelli, Maio, & Maltoni, 1999; Cappelli et al.,

2000; Germain, Califano, & Colville, 1997; Jiang, Liu, & Kot, 2006; Lumini, Maio, & Maltoni, 1997). Only such candidates are then compared to the query fingerprint using an automated matching algorithm to produce the identification result (or a shorter candidate list to be finally examined by a human expert).

This paper proposes a novel fingerprint retrieval method that combines level-1 (local ridge-line orientations and frequencies) and level-2 (minutiae positions and angles) features. Extensive experiments on six public databases show that the proposed technique markedly outperforms state-of-the-art methods.

The rest of this paper is organized as follows. Section 2 describes the proposed feature extraction steps; Section 3 introduces the similarity measures and the fusion strategies. In Section 4, experiments on six public datasets compare the novel approach with several state-of-the-art competing techniques. Finally Section 5 draws some conclusions.

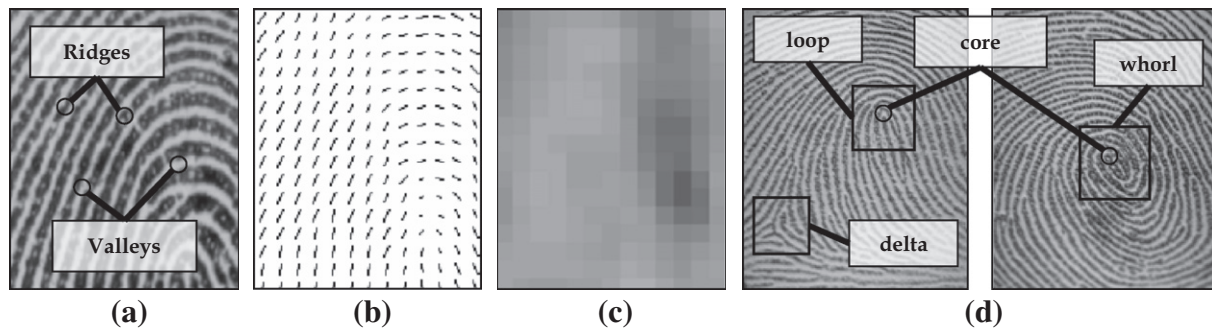
## 2. Feature extraction

A fingerprint is the representation of the epidermis of a finger: it consists of a pattern of interleaved ridges and valleys (Fig. 1(a)) (Maltoni et al., 2009). Ridges and valleys often run in parallel; sometimes they bifurcate and sometimes they terminate. Fingerprint features are generally described in a hierarchical order at three levels: level-1 (pattern), level-2 (minutiae points) and level-3 (pores and ridge shape).

- Level-1: when analyzed at the global level, fingerprint patterns can be characterized by two main features: local orientations (Fig. 1(b)) and local frequencies (Fig. 1(c)). The local orientation at a given position  $(x,y)$  is the angle  $\theta_{xy}$  that the fingerprint ridges forms with the horizontal axis. The local frequency  $f_{xy}$  at point  $(x,y)$  is the number of ridges per unit length along a

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**Fig. 1.** Ridges and valleys in a fingerprint image (a); local orientations (b); local frequencies (darker blocks denote lower frequencies) (c); singular regions and core points (d).

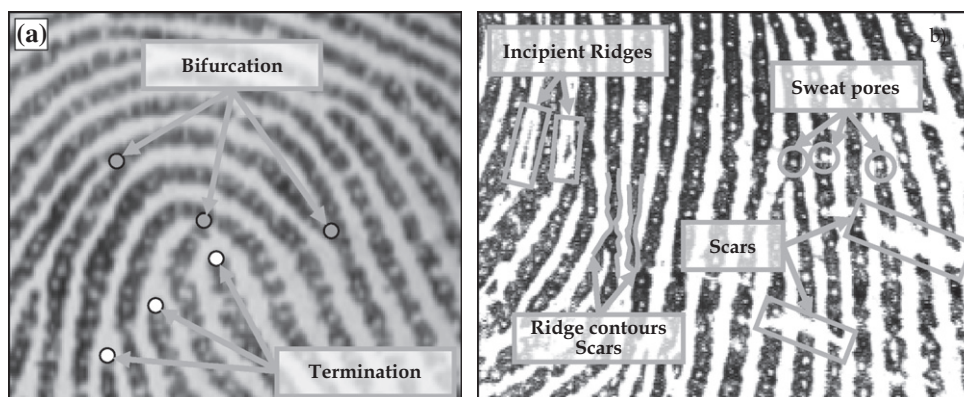
hypothetical segment centered at  $(x,y)$  and orthogonal to the local orientation  $\theta_{xy}$ . Furthermore, at this level, the pattern may exhibit one or more regions where the ridge lines assume distinctive shapes. These regions (called *singularities* or *singular regions*) may be classified into three typologies: *loop*, *delta*, and *whorl* (see Fig. 1(d)). Singular regions belonging to loop, delta, and whorl types are characterized by  $\cup$ ,  $\Delta$ , and  $\bigcirc$  shapes, respectively. The *core point* (used by some algorithms to pre-align fingerprints) corresponds to the center of the north (upper) most loop type singularity.

- **Level-2:** at the local level, other important features, called *minutiae* can be found in fingerprint patterns. Minutiae refer to various ways that the ridges can be discontinuous. For example, a ridge can abruptly come to an end (termination), or can divide into two ridges (bifurcation) (Fig. 2(a)). Although several types of minutiae can be considered, usually only a coarse classification (into these two types) is adopted (Maltoni et al., 2009).
- **Level-3:** at the very local level, further small details can be found in fingerprint patterns. They include sweat pores, dimensional attributes of the ridges (e.g., width, shape, edge contour), incipient ridges, breaks and scars (Fig. 2(b)). Although level-3 features are distinctive and important for latent fingerprint examiners (Maltoni et al., 2009), their reliable detection requires good quality fingerprints and high resolution scanners (at least 1000 dpi).

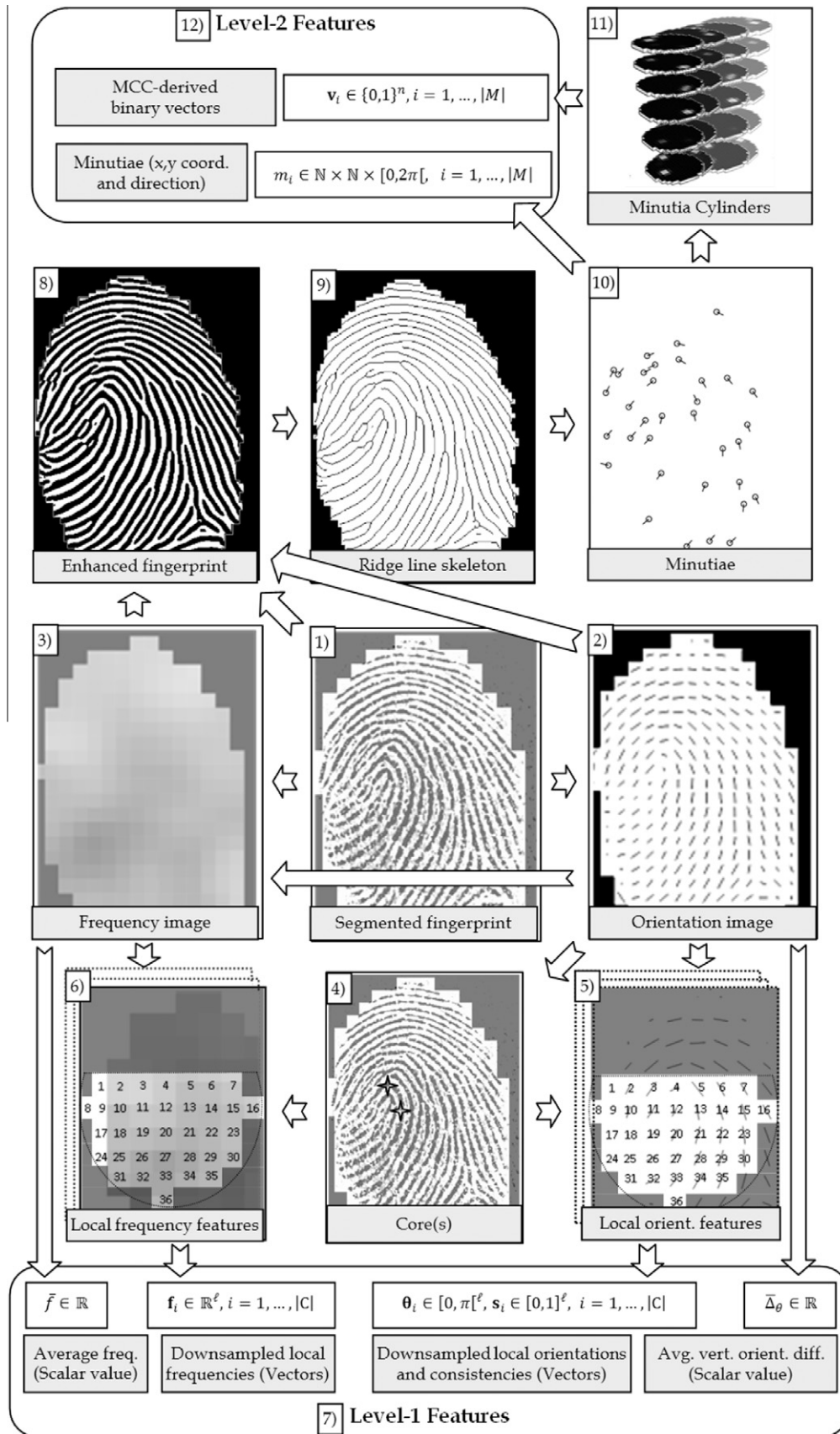
Given a fingerprint image, the proposed approach extracts level-1 and level-2 features by performing the following steps (Fig. 3).

1. The fingerprint pattern is segmented from the background using the approach proposed in Maio and Maltoni (1997), which is based on the block-based average magnitude of the gradient (Fig. 3(1)).

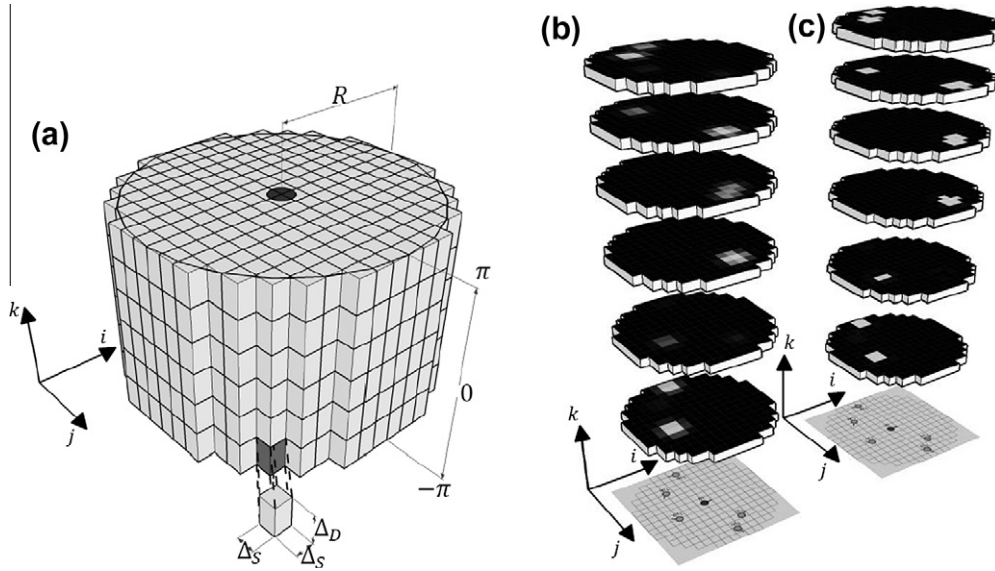
2. The traditional gradient-based technique (Ratha, Chen, & Jain, 1995) is applied to estimate the local orientations every 16 pixels along the horizontal and vertical axes (orientation image), using an averaging window of  $17 \times 17$  pixels. Each estimated orientation element consists of an angle  $\theta \in [0, \pi[$  and a value  $\in [0, 1]$  denoting the reliability (strength) of the estimation (in Fig. 3(2), the strength is represented by the length of the segment denoting each orientation).
3. Once local orientations are available, they are used to estimate local ridge-line frequencies, at the same image locations, as described in Hong, Wan, and Jain (1998) (frequency image). Each frequency element is a value  $f \in \mathbb{R}$ , denoting the inverse of the average ridge-line period estimated in a neighborhood (in Fig. 3(3), light blocks denote higher frequencies).
4. The local orientations are also used to find the fingerprint core (Fig. 3(4)). Since this is a critical step, two techniques are adopted: the iterative singularity detection approach described in Karu and Jain (1996) and the approach based on ridge-line normals described in Rerkrak & Areekul, 2000. In case of ambiguities, both the approaches are allowed to propose more candidates (up to five); in the following,  $C = \{c | c = (x_c, y_c)\}$  denotes the set of candidate cores detected in the current fingerprint ( $|C| \leq 5$ ).
5. For each candidate core  $c = (x_c, y_c)$ , the orientation image is aligned with respect to  $(x_c, y_c)$  and downsampled to one third of its original resolution. As discussed in Cappelli (xxxx), this improves both accuracy and efficiency.
6. Similarly, for each candidate core  $c = (x_c, y_c)$ , the frequency image is aligned with respect to  $(x_c, y_c)$  and downsampled to one third of its original resolution.
7. The level-1 feature vectors are obtained from the downsampled images, by considering only the elements satisfying the following constraint:  $d_x^2 + d_y^2 \leq r^2 \wedge d_y \geq -1$ , where  $d_x$  ( $d_y$ ) is



**Fig. 2.** Termination and bifurcation minutiae in a sample fingerprint (a). Examples of level-3 features in a sample fingerprint acquired at 1000 dpi (b).



**Fig. 3.** A functional schema of the feature extraction approach showing, for a sample fingerprint pattern (1): intermediate steps (2)–(6) and (8)–(11), level-1 features (7), and level-2 features (12).



**Fig. 4.** A graphical representation of the data structure associated to a minutia in the MCC representation (a). A cylinder (b): lighter areas represent higher values; the corresponding minutia and its neighborhood is shown below the base of the cylinder. The same cylinder using a bit-based representation (c).

**Table 1**  
Features extracted by the proposed approach.

Level	Feature	Type	Description
Level-1	$\theta_i$ , $i = 1, \dots,  C $	$ C $ vectors	Core-aligned local orientations
	$s_i$ , $i = 1, \dots,  C $	$ C $ vectors	Core-aligned local orientation consistencies
	$f_i$ , $i = 1, \dots,  C $	$ C $ vectors	Core-aligned local frequencies
	$\bar{f}$	scalar value	Average frequency
	$\bar{\Delta}_\theta$	scalar value	Average vertical orientation difference
Level-2	$m_i$ , $i = 1, \dots,  M $	$ M $ triplets ( $x, y, \theta$ )	Minutiae (pixel coordinates and angles)
	$\mathbf{v}_i$ , $i = 1, \dots,  C $	$ M $ vectors	MCC binary vectors

the horizontal (vertical) displacement of the element with respect to the image center, and  $r = 2$ . In other words, the only elements considered are those inside the region highlighted in Fig. 3(5) and (6). The shape of those regions have been optimized on a training dataset (see Section 4.2). The selected elements are linearized to form the following feature vectors of size  $\ell = 36$ :

- $\theta_i \in [0, \pi]^\ell$  (downsampled local orientations),
- $s_i \in [0, 1]^\ell$  (the corresponding consistencies),
- $f_i \in \mathbb{R}^\ell$  (downsampled local frequencies),

with  $i = 1, \dots, |C|$ .

Furthermore, two additional scalar features are derived from the (non-downsampled) orientation and frequency images: the average frequency  $\bar{f}$  and the average vertical orientation difference  $\bar{\Delta}_\theta$ , which is calculated as:

$$\bar{\Delta}_\theta = \frac{\sum_{y=1}^{h-1} \sum_{x=1}^w (s_{x,y} \cdot s_{x,y+1} \cdot d_\theta(\theta_{x,y}, \theta_{x,y+1}))}{\sum_{y=1}^{h-1} \sum_{x=1}^w (s_{x,y} \cdot s_{x,y+1})} \quad (1)$$

where  $w$  and  $h$  are the width and height of the orientation image, respectively,  $\theta_{x,y}$  and  $s_{x,y}$  are the orientation and strength of the element at position  $(x, y)$ , and  $d_\theta(\theta_a, \theta_b)$  is the angular difference between two orientations.

1. In order to ease the detection of level-2 features, the segmented fingerprint pattern is enhanced by means of contextual filtering with Gabor filters (Hong et al., 1998); the enhancement, which is guided by the orientation and frequency images calculated at steps 2 and 3, produces a nearly-binary image which is then simply binarized using a fixed threshold.
2. The thinning approach introduced in Zhang and Suen (1984) is applied to the binary enhanced pattern to obtain the ridge-line skeleton.
3. Minutiae are extracted from the fingerprint skeleton by using the crossing number (Maltoni et al., 2009); an approach analogous to that proposed in Zhao and Tang (2007) is applied to remove spurious minutiae detected in noisy regions. The result is a set of minutiae  $M = \{m | m = (x_m, y_m, \theta_m)\}$ , where for each minutia  $m$ ,  $x_m$  and  $y_m$  are its location, and  $\theta_m$  is its direction (in the range  $[0, 2\pi]$ ).
4. The Minutia Cylinder-Code representation (MCC Cappelli, Ferrara, & Maltoni, 2010) is adopted to obtain a set of fixed-length invariant features from the minutiae. The MCC representation associates, to each minutia  $m$ , a local structure that encodes spatial and directional relationships between the minutia and its neighborhood, and can be conveniently represented as a cylinder, whose base and height are related to the spatial and directional information, respectively (Fig. 4(a)). The cylinder is discretized into cells and a numerical value is calculated for each cell, by accumulating contributions from minutiae in a neighborhood of the projection of the cell center onto the cylinder base. The contribution of each minutia  $m_t$  to a cell (of the cylinder corresponding to a given minutia  $m$ ), depends both on: spatial information (how much  $m_t$  is close to the center of the cell), and directional information (how much the directional difference between  $m_t$  and  $m$  is similar to the directional difference associated to the cylinder section where the cell lies). In other words, the value of a cell represents the likelihood of finding minutiae that are close to the cell and whose directional difference with respect to  $m$  is similar to a given value. Fig. 4.b shows the cylinder associated to a minutia with five minutiae in its neighborhood. As discussed in Cappelli et al. (2010), the value of each cell can be stored as a bit with a negligible loss of accuracy, hence each cylinder can be treated as a fixed-length bit-vector.



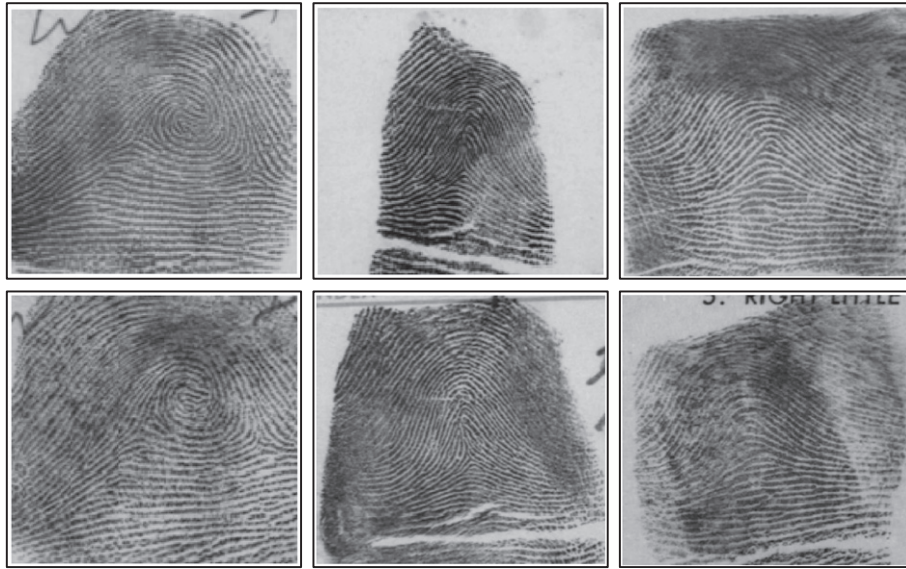


Fig. 5. Sample fingerprints from NIST DB4: 'F' impression (top row) and 'S' impression (bottom row) of three fingers.

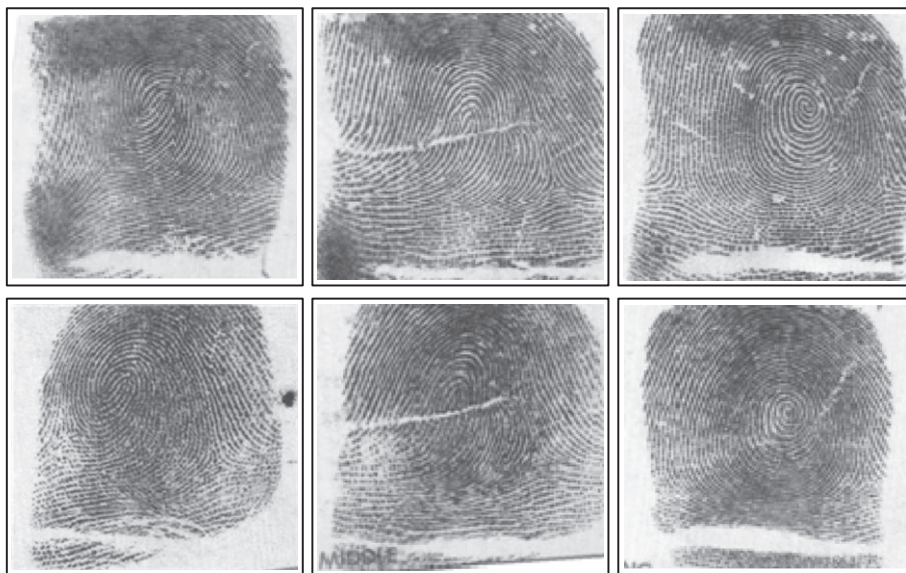


Fig. 6. Sample fingerprints from NIST DB14: 'F' impression (top row) and 'S' impression (bottom row) of three fingers.

5. The following level-2 features are finally obtained:

- the set of minutiae  $M = \{m | m = (x_m, y_m, \theta_m)\}$ , and
- the MCC binary vectors derived from the minutiae.

Table 1 summarizes the features extracted by the proposed approach from a given fingerprint image. This particular set of level-1 features proved to be very effective for fingerprint search even on low-quality fingerprints (Cappelli, xxxx). Another advantage of these level-1 features is that local orientations and frequencies are usually extracted, as an intermediate step, by fingerprint matching algorithms: this means that almost no overhead is required by the retrieval approach for extracting such features. At level-2, MCC binary vectors are very well-suited features for retrieval, as already proved in Cappelli, Ferrara, and Maltoni (2011); in particular: (i) they are invariant for translation and rotation (see Cappelli et al., 2010), (ii) they are robust against skin distortion (which is small at a local level) and against small feature extraction errors (see Cappelli et al., 2010), and (iii) they can be indexed in a very efficient manner (see Cappelli et al., 2011). Note

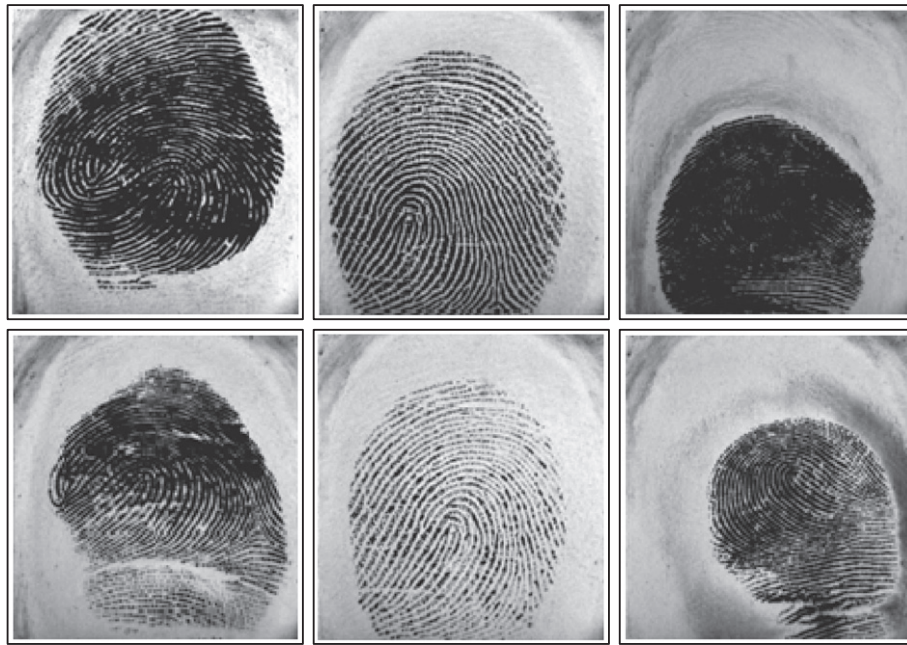
that, although MCC binary vectors are derived from the minutiae, the minutiae themselves are also directly used to apply some constraints (see Section 3.2).

### 3. Similarity measures and fusion approaches

In a single-modality biometric system, multiple features can be integrated at different levels (Rooss & Jain, 2003): feature-level, score-level, rank-level, and decision-level. Although combination at feature level has been shown to be quite effective in various contexts (Maltoni et al., 2009; Rattani, Kisku, Bicego, & Tistarelli, 2006), we decided to focus this study on score level and rank-level fusion. In fact, on the one hand, the particular features used are quite heterogeneous (scalar values and variable number of fixed-length feature vectors for level-1 features, variable number of bit-vectors and minutiae for level-2 features) and it would be difficult to find an effective way to directly integrate them into a single similarity measure; on the other, very effective similarity



**Fig. 7.** Sample fingerprints from FVC2000 DB2: first impression (top row) and one of the other impressions (bottom row) for three fingers.



**Fig. 8.** Sample fingerprints from FVC2000 DB3: first impression (top row) and one of the other impressions (bottom row) for three fingers.

measures are available for the two sets of features and it seems particularly promising to combine them at score- or rank-level. This section introduces the similarity measures used for level-1 and level-2 features and describes the various fusion strategies that have been experimented in this work.

### 3.1. Similarity measure for the level-1 features

An effective similarity measure based on the level-1 features described in Section 2 has been proposed in Cappelli (xxxx): it con-

sists of a weighted sum of four simpler measures. Given the sets of level-1 features extracted from two fingerprints  $\mathfrak{I}_{L1} = (\theta_1, \dots, \theta_{|C|}, \mathbf{s}_1, \dots, \mathbf{s}_{|C|}, \mathbf{f}_1, \dots, \mathbf{f}_{|C|}, \bar{f}, \bar{\Delta}_\theta)$  and  $\mathfrak{I}'_{L1} = (\theta'_1, \dots, \theta'_{|C'|}, \mathbf{s}'_1, \dots, \mathbf{s}'_{|C'|}, \mathbf{f}'_1, \dots, \mathbf{f}'_{|C'|}, \bar{f}', \bar{\Delta}'_\theta)$ , the similarity measure  $S_{L1}(\mathfrak{I}_{L1}, \mathfrak{I}'_{L1})$  is defined as:

$$S_{L1}(\mathfrak{I}_{L1}, \mathfrak{I}'_{L1}) = \omega_1 S_1(\theta_p, \theta'_q, \mathbf{s}'_q) + \omega_2 S_2(\mathbf{f}_p, \mathbf{f}'_q) + \omega_3 S_3(\bar{f}, \bar{f}') + \omega_4 S_4(\bar{\Delta}_\theta, \bar{\Delta}'_\theta) \quad (2)$$

where:





Fig. 9. Sample fingerprints from FVC2002 DB1: first impression (top row) and one of the other impressions (bottom row) for three fingers.

Table 2

Retrieval methods, features used and datasets considered.

Method	Features		NIST			FVC		
	Level	Details	DB4	DB4(Natural)	DB14	2000DB2	2000DB3	2002DB1
<sup>a</sup> Germain et al. (1997)	2	Minutiae triplets	X					X
Lumini et al. (1997)	1	Orientations		X	X			
Cappelli et al. (1999)	1	Orientations		X	X			
<sup>b</sup> De Boer et al. (2001)	1,2	Orientations, FingerCode, minutiae triplets				X		
Cappelli et al. (2002)	1	Orientations			X			
Bhanu and Tan (2003)	2	Minutiae triplets	X					
Lee, Kim, and Park (2005)	1	Orientations, frequencies		X				
Feng and Cai (2006)	2	Minutiae and surrounding ridges						X
Jiang et al. (2006)	1	Orientations, dominant Frequency	X	X		X	X	
Li, Yau, and Wang (2006)	1	Orientations		X				
<sup>b</sup> Liang, Asano, and Bishnu (2006)	2	Minutiae triplets				X		
Liu, Jiang, and Kot (2006)	1	Orientations		X				
<sup>b</sup> Liang, Bishnu, and Asano (2007)	2	Minutiae triplets and surrounding ridges						X
Gyaourova and Ross (2008)	1,2	Match scores of a commercial algorithm (based on level-1 and level-2 features)	X					
Shuai et al. (2008)	c	SIFT features						X
Cappelli et al. (2011)	2	Minutia Cylinder-Code	X	X	X	X	X	X
Cappelli (2011)	1	Orientations, frequencies	X	X	X	X	X	X
Leung and Leung (2011)	1	Orientations	X		X	X	X	X

<sup>a</sup> Results of Germain et al. (1997) on NIST DB4 and FVC2002 DB1 are reported in Bhanu and Tan (2003) and Feng and Cai (2006), respectively.

<sup>b</sup> Results in De Boer et al. (2001) and Liang et al. (2006, 2007) have been obtained using also the 80 fingerprints of training set “B” (see Maio et al., 2002a, 2002b), resulting in 880 fingerprints from 110 fingers.

<sup>c</sup> SIFT are general-purpose image features not directly comparable with fingerprint features.

$$(p, q) = \arg \max_{\substack{p=1, \dots, |C| \\ q=1, \dots, |C'|}} S_1(\theta_p, s_p, \theta'_q, s'_q), \quad \sum_{i=1}^4 \omega_i = 1 \quad (3)$$

and

$$S_1(\theta, s, \theta', s') = \frac{\sqrt{\left(\sum_{j=1}^{\ell} (s_j \cdot \cos(2d_j))\right)^2 + \left(\sum_{j=1}^{\ell} (s_j \cdot \sin(2d_j))\right)^2}}{\sum_{j=1}^{\ell} s_j} \quad \text{with } d_j = d_\theta(\theta[j] - \theta'[j]) \quad \text{and } s_j = s[j] \cdot s'[j] \quad (4)$$

$$S_2(\mathbf{f}, \mathbf{f}') = 1 - \frac{\sum_{j=1}^{\ell} |\frac{1}{f[j]} - \frac{1}{f'[j]}|}{\sum_{j=1}^{\ell} (\frac{1}{f[j]} + \frac{1}{f'[j]})} \quad (5)$$

$$S_3(\bar{f}, \bar{f}') = 1 - \frac{|\bar{f} - \bar{f}'|}{\max(\bar{f}, \bar{f}')} \quad (6)$$

$$S_4(\bar{\Delta}_\theta, \bar{\Delta}'_\theta) = 1 - \frac{|\bar{\Delta}_\theta - \bar{\Delta}'_\theta|}{\pi} \quad (7)$$

$S_1$  (4) measures the homogeneity of the local orientation differences, weighted by their respective consistencies. If the differences between corresponding orientation elements are similar, the score is high, otherwise it is small.  $S_2$  (5) measures the similarity between ridge-line periods.  $S_3$  (6) and  $S_4$  (7) are simply based on normalized differences between the two scalar features. The computation of  $S_{L1}(\mathfrak{T}_{L1}, \mathfrak{T}'_{L1})$  (2) involves finding the pair of candidate cores  $(p, q)$

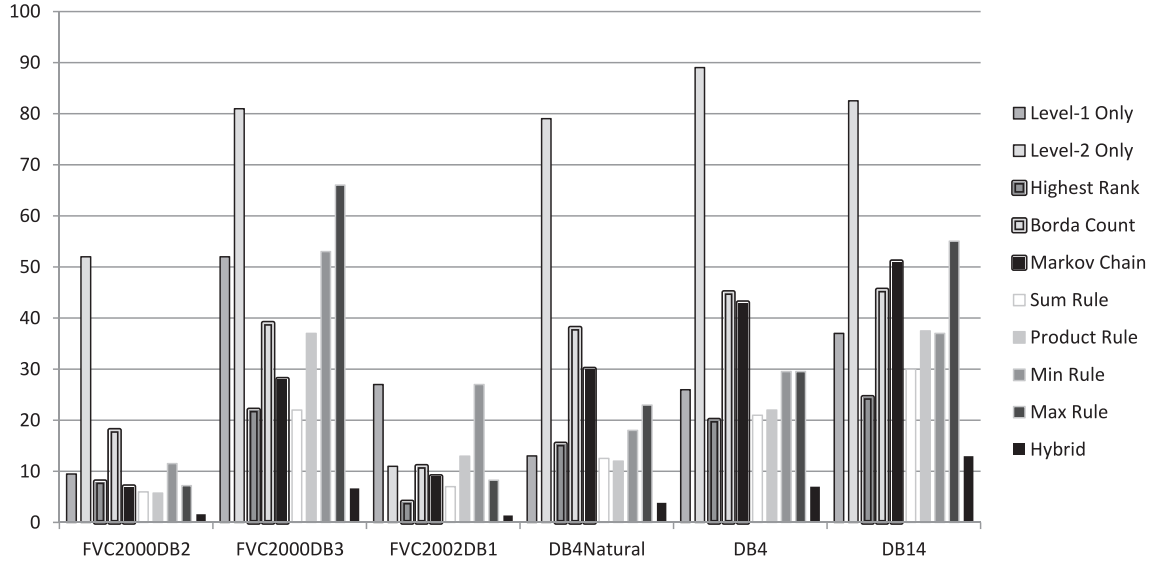


Fig. 10. Average penetration rate at 1% average error rate.

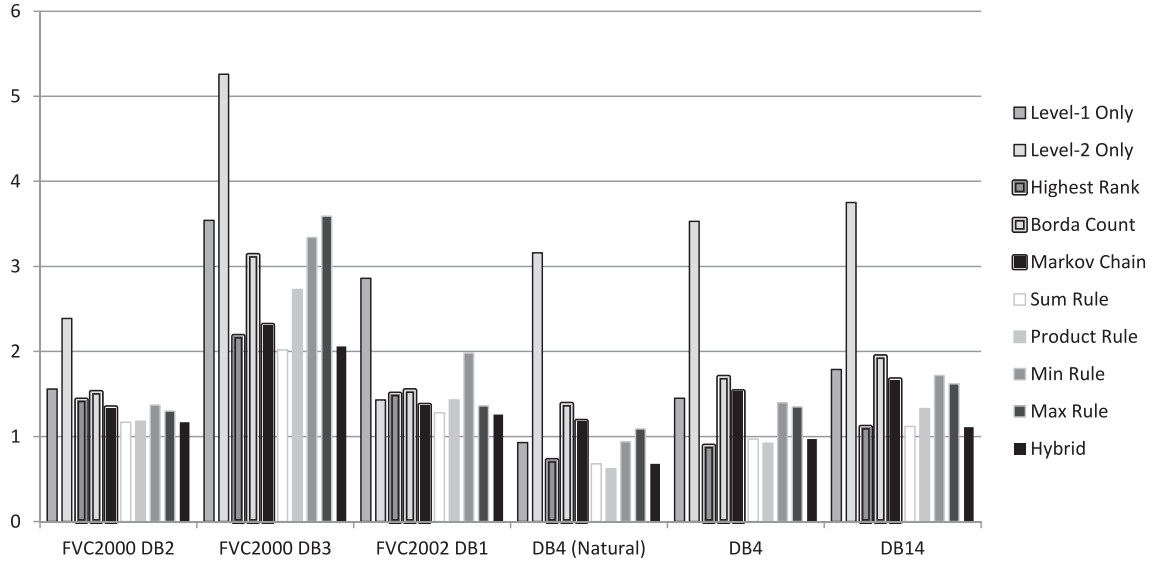


Fig. 11. Average penetration rate for the incremental search scenario.

which maximizes  $S_1$ : the same pair of candidate cores is used to calculate  $S_2$ . The value of  $S_{L1}(\mathfrak{I}_{L1}, \mathfrak{I}'_{L1})$  is always between zero and one (zero means no similarity, one means maximum similarity). The same weights proposed in Cappelli (2011) have been used throughout all the experiments reported in Section 4:  $\omega_1 = 0.16$ ,  $\omega_2 = 0.37$ ,  $\omega_3 = 0.16$ , and  $\omega_4 = 0.31$ .

### 3.2. Similarity measure for the level-2 features

In Cappelli et al. (2011), a similarity measure, based on MCC binary vectors and well suited for fingerprint retrieval, has been introduced, together with a hash-based algorithm to efficiently compute the similarity scores. Given the sets of level-2 features extracted from two fingerprints  $\mathfrak{I}_{L2} = (m_1, \dots, m_{|M|}, \mathbf{v}_1, \dots, \mathbf{v}_{|M|})$  and  $\mathfrak{I}'_{L2} = (m'_1, \dots, m'_{|M'|}, \mathbf{v}'_1, \dots, \mathbf{v}'_{|M'|})$ , the similarity measure  $S_{L2}(\mathfrak{I}_{L2}, \mathfrak{I}'_{L2})$  is defined as:

$$S_{L2}(\mathfrak{I}_{L2}, \mathfrak{I}'_{L2}) = \frac{1}{|M|} \sum_{i=1}^{|M|} \max\{nhs(\mathbf{v}_i, \mathbf{v}'_j) | j = 1, \dots, |M'| \wedge d_\theta(m_i, m_j) \leq \delta_\theta \wedge d_{xy}(m_i, m_j) \leq \delta_{xy}\} \quad (8)$$

where  $d_\theta$  and  $d_{xy}$  are the angular difference and the Euclidean distance between the two minutiae, respectively,

$$nhs(\mathbf{a}, \mathbf{b}) = \left(1 - \frac{d_H(\mathbf{a}, \mathbf{b})}{n}\right)^p \quad (9)$$

$d_H(\mathbf{a}, \mathbf{b})$  is the Hamming distance, and  $n$  is the length of the binary vectors.

$nhs$  is a similarity measure between two binary vectors normalized between zero and one, based on the Hamming distance;  $p$  is a parameter controlling the shape of the similarity function ( $p > 0$ ); in particular, the higher  $p$ , the quicker  $nhs$  drops to zero as  $d_H$  increases.

Eq. (8) defines a similarity score in the range  $[0, 1]$  (zero means no similarity, one means maximum similarity), which is obtained by selecting, for each vector in  $\mathfrak{I}_{L2}$ , the maximum normalized Hamming similarity (9) with those vectors in  $\mathfrak{I}'_{L2}$  corresponding to minutiae that satisfies the constraints on spatial distance and angular difference. Enforcing such constraints results in tolerating a maximum rotation  $\delta_\theta$  and maximum displacement  $\delta_{xy}$  between the query fingerprint and the database fingerprints. The same



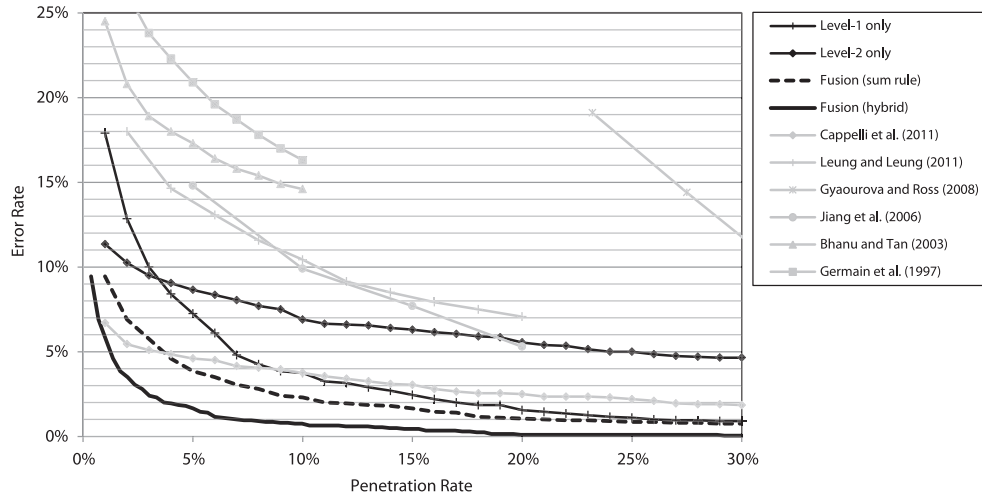


Fig. 12. Penetration rate vs error rate tradeoff on dataset NIST DB4.

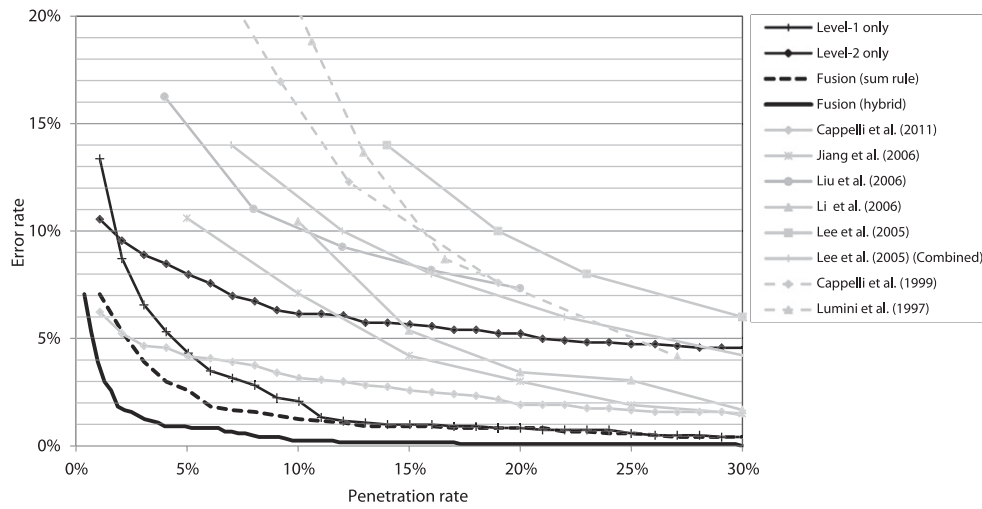


Fig. 13. Penetration rate vs error rate tradeoff on dataset NIST DB4 (Natural).

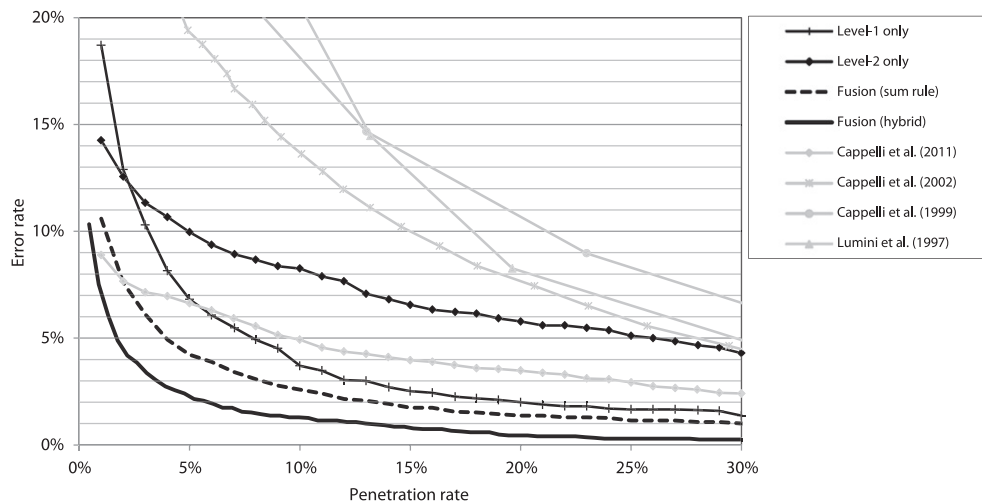


Fig. 14. Penetration rate vs error rate tradeoff on dataset NIST DB14.

parameter values reported in Cappelli et al. (2011) have been used throughout all the experiments of this work:  $p = 30$ ,  $\delta_\theta = \frac{\pi}{4}$ , and  $\delta_{xy} = 256$ . The final similarity score is calculated as the average of

all the maximum normalized Hamming similarities ( $nhs$ ) considered. Although in general Eq. (8) is not the best choice to compare MCC features (the reader should refer to (Cappelli et al., 2010) for a

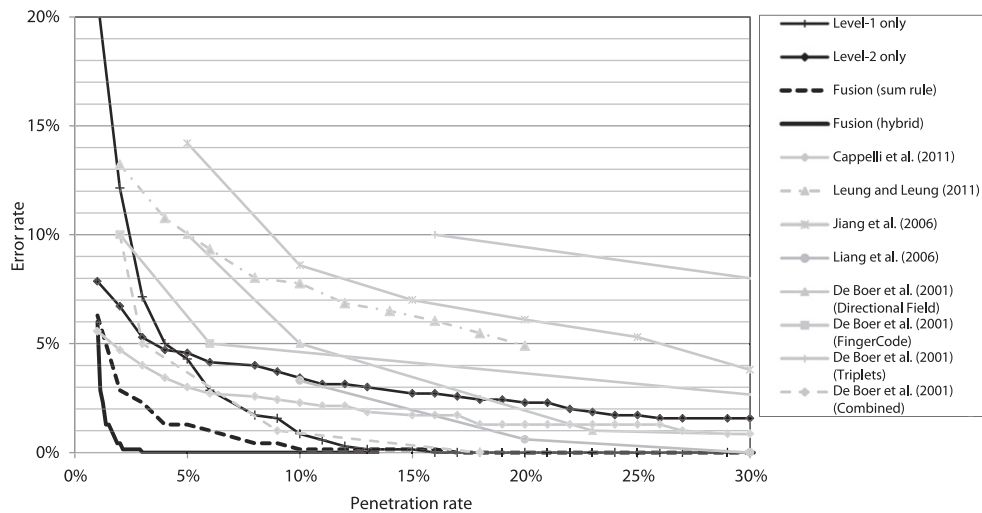


Fig. 15. Penetration rate vs error rate tradeoff on dataset FVC2000 DB2.

Table 3

NIST DB4: error rates of various pre-filtering approaches at 20% penetration rate. The table includes both exclusive classification methods (EC) and other fingerprint retrieval methods not based on fingerprint classes (FR).

Method	Type	Error rate (%)
Level-1 only	FR	1.6
Level-2 only	FR	5.6
Fusion (sum rule)	FR	1.1
Fusion (Hybrid approach)	FR	0.1
Karu and Jain (1996)	EC	14.6
Cappelli et al. (1999)	EC	7.9
Hong and Jain (1999)	EC	12.5
Jain, Prabhakar, and Hong (1999)	EC	10.0
Marcialis, Roli, and Frasconi (2001)	EC	12.1
Yao, Frasconi, and Pontil (2001)	EC	10.7
Cappelli et al. (2003)	EC	4.8
Yao, Marcialis, Pontil, Frasconi, & Roli (2003)	EC	10.0
Cappelli and Maio (2004)	EC	7.0
Zhang and Yan (2004)	EC	15.7
Neuhaus and Bunke (2005)	EC	19.8
Park and Park (2005)	EC	9.3
Tan, Bhanu, and Lin (2005)	EC	8.4
Min, Hong, and Cho (2006)	EC	9.6
Jiang et al. (2006)	FR	5.3
Wang and Dai (2007)	EC	11.5
Hong, Min, Cho, and Cho (2008)	EC	9.2
Li, Yau, and Wang (2008)	EC	6.5
Gyaourova and Ross (2008)	FR	19.1
Cappelli et al. (2011)	FR	2.5
Leung and Leung (2011)	FR	7.1

description of other approaches), as discussed in Cappelli et al. (2011), this particular similarity measure allows a very efficient hash-based implementation.

### 3.3. The fusion approaches evaluated

In this work various score- and rank-level fusion approaches have been experimented to combine the output of the level-1 and level-2 similarity measures described in the previous sections. In this context, the result of a fusion approach is a list of fingerprint candidates sorted, in the descending order, according to their similarity to the query fingerprint.

At score-level, a fused score for each candidate is first obtained by combining the two similarity scores, then the candidate list is generated sorting the database fingerprints

according to their fused score. In order to make level-1 and level-2 scores more homogeneous, the normalization approach proposed in Cappelli, Maio, and Maltoni (2002) is used before applying the fusion rules. The following score-level fusion rules have been evaluated:

- *Sum rule*: the fused score is the sum of the two similarity scores.
- *Product rule*: the fused score is the product of the two similarity scores.
- *Min rule*: the fused score is the minimum of the two similarity scores.
- *Max rule*: the fused score is the maximum of the two similarity scores.

At rank-level, two ordered lists of candidates are created sorting them according to level-1 and level-2 scores; the final rank of each candidate is obtained by combining its ranks in the two lists. The following rank-level fusion approaches have been evaluated:

- *Highest rank*: the final rank of each candidate is the highest rank in the two lists. The final candidate list is then obtained by sorting the fused ranks of all candidates in the ascending order.
- *Borda count*: the Borda count of a candidate is the sum of the number of candidates ranked below it in the two lists. The final candidate list is obtained by sorting in the descending order the Borda counts of all the candidates (Borda, 1781).
- *Markov chain*: a rank-level fusion method based on Markov chain model, recently proposed in Monwar and Gavrilova (2010).

From the result analysis of the various fusion strategies and the score distributions of the two similarity measures over a training dataset (see Section 4.2 for more details), it has been noted that if the difference between the first two candidates according to the level-2 similarity measure is large, then there is a high probability that the top candidate is the true mate. Starting from this observation, an ad-hoc hybrid fusion approach has been defined as follows. Let  $s_1$  and  $s_2$  be the top two level-2 similarity scores, if  $\frac{s_1 - s_2}{s_1} \geq t$  then the final output list will contain only the candidate with the highest level-2 similarity score ( $s_1$ ), otherwise the candidate list will be generated using the Sum rule score fusion method. The value of threshold  $t$  used in the experiments (0.4) has been experimentally determined on a training set (see Section 4.2).

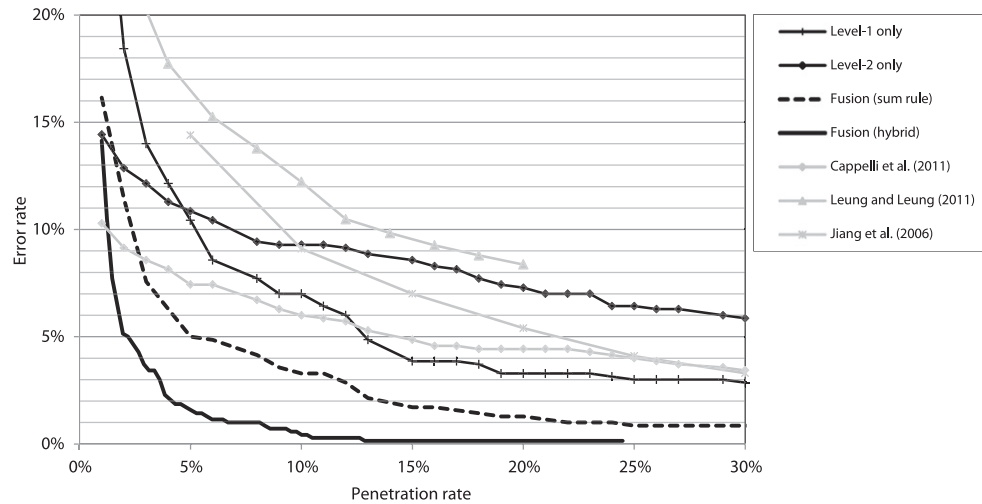


Fig. 16. Penetration rate vs error rate tradeoff on dataset FVC2000 DB3.

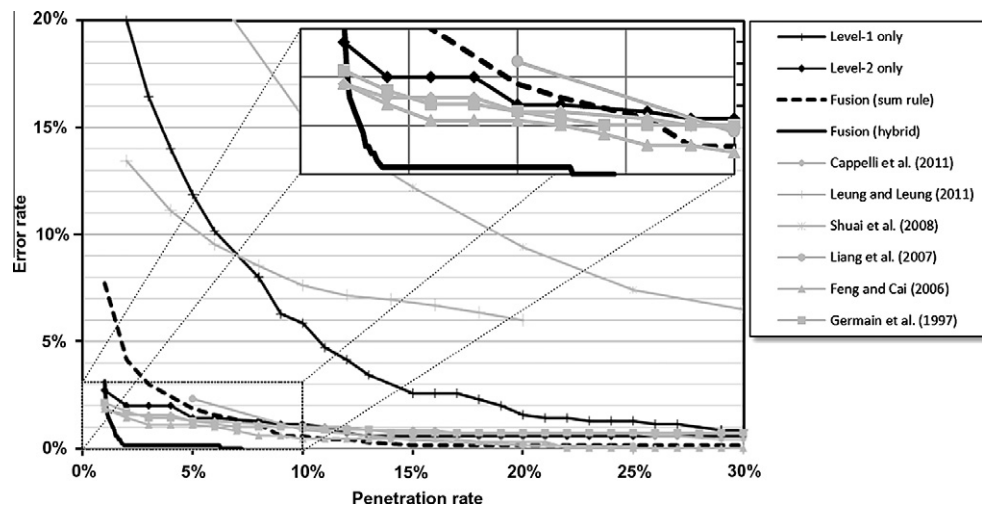


Fig. 17. Penetration rate vs error rate tradeoff on dataset FVC2002 DB1.

Table 4

Average penetration rate for the incremental search scenario: best results are highlighted with bold font.

Method	NIST			FVC		
	DB4	DB4 Nat.	DB14	2000DB2	2000DB3	2002DB1
Level-1 only	1.45	0.93	1.79	1.56	3.54	2.86
Level-2 only	3.53	3.16	3.75	2.39	5.26	1.43
Fusion (sum rule)	<b>0.97</b>	<b>0.68</b>	1.12	<b>1.17</b>	<b>2.02</b>	1.28
Fusion (hybrid)	<b>0.97</b>	<b>0.68</b>	<b>1.11</b>	<b>1.17</b>	2.06	<b>1.26</b>
Lumini et al. (1997)	–	6.90	7.10	–	–	–
Cappelli et al. (1999)	–	5.20	6.40	–	–	–
(DF) De Boer et al. (2001)	–	–	–	2.58	–	–
(FC) De Boer et al. (2001)	–	–	–	2.40	–	–
(Tr.) De Boer et al. (2001)	–	–	–	7.27	–	–
(Comb.) De Boer et al. (2001)	–	–	–	1.34	–	–
Cappelli et al. (2002)	–	–	3.70	–	–	–
Lee et al. (2005)	–	6.14	–	–	–	–
(Combined) Lee et al. (2005)	–	3.85	–	–	–	–
Jiang et al. (2006)	–	2.93	–	–	–	–
Cappelli et al. (2011)	1.59	1.32	2.19	1.72	3.63	1.37

## 4. Experiments and results

This section describes experiments carried out to evaluate the proposed system and to compare it with the state of the art.

### 4.1. Datasets, published results and performance indicators

Most of the published methods for fingerprint pre-filtering have been evaluated on the following datasets.



- NIST DB4: the NIST Special Database 4 (Watson & Wilson, 1992), which contains images of rolled fingerprint impressions scanned from cards; the fingerprints are taken from 2000 fingers uniformly distributed in the five fingerprint classes (Cappelli & Maio, 2004): two different fingerprint instances (F and S) are present for each finger (Fig. 5).
- NIST DB4 (Natural): a subset of NIST DB4 obtained by reducing the cardinality of the less frequent classes in nature, to resemble a natural distribution; the dataset contains 1204 fingerprint pairs.
- NIST DB14: the last 2700 fingerprint pairs from NIST Special Database 14 (Watson, 1993). This dataset contains images of rolled fingerprint impressions scanned from cards; the class distribution of DB14 resembles the fingerprint distribution in nature. Two different fingerprint instances (F and S) are present for each finger (Fig. 6).
- FVC2000 DB2: the second FVC2000 database (Maio, Maltoni, Cappelli, Wayman, & Jain, 2002a), which contains 800 fingerprints from 100 fingers (8 impressions per finger) captured using a capacitive fingerprint scanner (Fig. 7).
- FVC2000 DB3: the third FVC2000 database (Maio et al., 2002a), which contains 800 fingerprints from 100 fingers (8 impressions per finger) captured using an optical fingerprint scanner (Fig. 8).
- FVC2002 DB1: the first FVC2002 database (Maio et al., 2002b), containing 800 fingerprints from 100 fingers (8 impressions per finger) captured using an optical fingerprint scanner (Fig. 9).

Table 2 reports fingerprint retrieval methods for which published results on one or more of the above described datasets are available.

The performance of fingerprint retrieval approaches is typically evaluated by reporting the tradeoff between *error rate* and *penetration rate*. The error rate is defined as the percentage of searched fingerprints whose mate is not present in the candidate list; the penetration rate is the portion of database that the system has to search on the average. In the proposed approach, similarly to other published methods (e.g. De Boer, Bazan, & Gerez, 2001; Feng & Cai, 2006; Shuai, Zhang, & Hao, 2008), the tradeoff between error rate and penetration rate depends on a parameter that controls the maximum number of candidates to be considered. In other words, during the retrieval, if the candidate list is larger than such parameter, it is truncated and only the most similar fingerprints are retrieved.

Some authors, besides the above performance indicators, also consider a retrieval scenario (*incremental search* Cappelli, Lumini, Maio, & Maltoni, 1999), where an ideal matching algorithm is used to stop the search as soon as the true mate is retrieved. In such a scenario there are no retrieval errors, since in the worst case the search can be extended to the whole database, and the average penetration rate is reported as a single performance indicator. An advantage of this approach is that the performance of an algorithm over a dataset can be summarized with just a single value.

#### 4.2. Experimental results

The proposed system has been evaluated over the six datasets described in the previous section. All the parameters (either related to feature extraction, similarity measure calculation, or fusion strategies) have been optimized on a separate dataset (the first 3,000 fingerprint pairs of NIST Special Database 14 (Watson, 1993), to avoid any over-fitting effect.

All the experiments on NIST datasets have been carried out by searching each 'S' impression over a database containing the 'F' impression of each finger. Experiments on the three FVC datasets

have been performed by searching impressions 2–8 over a database containing the first impression of each finger.

The two separate similarity measures and their combinations have been experimented over the six datasets. Fig. 10 reports the average penetration rate at 1% average error rate; Fig. 11 reports the average penetration rate for the incremental search scenario. Observing the two figures, the following remarks can be made:

- Although level-1 similarity measure is much more accurate than level-2 similarity measure (except on FVC2002 DB1), their combination always improves the results.
- Usually Highest rank is the best among the rank-level fusion approaches, while Markov chain approach tends to outperform the Borda count strategy, which sounds reasonable, since, as explained in Monwar and Gavrilova (2010), the Markov chain approach can be seen as a generalization of Borda count.
- Usually the Sum rule is the best score-level fusion approach, except in a few cases where the Product rule leads to similar or slightly better results.
- In general, the Hybrid approach is the best choice except in the incremental search scenario on NIST DB4 (Natural) and NIST DB4, where the Sum and Product rule obtain slightly better performance.

Figures from 12 to 17 report the trade-off between penetration rate and error rate on the six datasets. Although all the fusion approaches introduced in Section 3.3 have been tested over all the datasets, for graph readability, only combined results for the Sum rule and the Hybrid approach are reported. The following observations can be drawn from the graphs:

- The level-2 similarity measure obtains better results than the level-1 similarity measure for small penetration rates (e.g., less than 5%); at larger penetration rates, level-1 similarity measure becomes much more accurate. Probably, this complementarity between the two measures is the main reason of the remarkable performance obtained by their fusion over all the datasets.
- Although the individual performance of the two similarity measures is not always competitive, their score-level combination with the simple Sum rule overcomes almost all state-of-the-art methods.
- The Hybrid approach (introduced in Section 3.3) obtains impressive results over all the six datasets. For instance, on NIST datasets (Figs. 12–14) it achieves an 1% average error rate with an average penetration rate between 5% and 10%, while, to the best of our knowledge, any other published method requires a penetration rate larger than 30%.
- The graph reported in Fig. 15 shows that both fusion approaches reported (Sum rule and Hybrid approach) clearly outperform the method by De Boer et al. (2001), which combines similar level-1 and level-2 features (see Table 2). It is also worth noting that results by De Boer et al. were obtained by manually

**Table 5**

Average search time on NIST DB4 for the proposed system and two published methods.

Method	Search time (ms)	Hardware
Bhanu and Tan (2003)	~1000	Sun Ultra 2
Jiang et al. (2006)	67	Intel Pentium 4 @ 2.26 Ghz
Level-1 only Cappelli (2011)	1.6	Intel Core 2 Quad @ 2.66 Ghz
Level-2 only Cappelli et al. (2011)	14	Intel Core 2 Quad @ 2.66 Ghz
Proposed system	16	Intel Core 2 Quad @ 2.66 Ghz

correcting the core point in 13% fingerprints and discarding 1% fingerprints because no core point was found (see De Boer et al., 2001), while neither manual corrections nor rejections have been made for the proposed approach.

To better understand the current state of the art in this field, it is interesting to compare the proposed system not only with other similar retrieval methods, but also with pre-filtering techniques based on exclusive classification (Cappelli & Maio, 2004). Such a comparison can be performed on NIST DB4 dataset by comparing the average error rates at a 20% penetration rate. In fact, NIST DB4 contains fingerprints uniformly distributed among the five main fingerprint classes, hence any exclusive classification approach has a fixed penetration rate of 20% on that dataset. Table 3 reports the error rates of 17 published exclusive classification methods, four published fingerprint retrieval techniques, and the proposed approach. From the table it is well evident how the Hybrid approach achieves significantly better performance than all other methods reported: it exhibits 0.1% error rate, which is one or two orders of magnitude lower than that of any other pre-filtering method we are aware of. Figs. 16 and 17.

Table 4 reports results for the incremental search scenario: the proposed approach is compared to other published methods for which results on this scenario are available. The novel approach always achieves the best performance over all the datasets.

Finally, Table 5 reports the average search time on NIST DB4 for the proposed system and for two published methods, as indicated in their respective papers. Unfortunately, search times are not reported for other published indexing approaches, and it is not possible to make a systematic comparison like the one done for accuracy in Tables 3 and 4. Although, the search times cannot be directly compared since they have been obtained on different hardware platforms, it can be concluded that the proposed approach exhibits a remarkable efficiency.

## 5. Conclusion

This work proposes a novel fingerprint retrieval system that combines i) local ridge-line orientations and frequencies (level-1 features) and ii) minutiae (level-2 features). Traditional rank-level (e.g., Highest rank) and score-level (e.g., Sum rule) fusion approaches have been evaluated and compared to a new hybrid combination approach. Systematic experiments have been carried out over six publicly available datasets, comparing the new retrieval approach with eighteen published retrieval methods and seventeen exclusive classification techniques. The results are even better than we expected: the proposed Hybrid approach largely outperforms state-of-the-art methods on all the datasets considered, including methods based on the same set of features (e.g., De Boer et al., 2001). The search speed is also very high and compares favorably with other published methods. It is also worth noting that, in order to avoid any data over-fitting, all parameters of the proposed approach have been optimized on a separate dataset and then used on all the test datasets without any specific adjustment, even if such datasets are very heterogeneous (they include rolled, flat, offline-scanned and live-scan fingerprints).

In the future it may be interesting to incorporate level-3 features into the proposed system, by designing appropriate similarity measures and ad-hoc fusion strategies: this may lead to a further improvement of the retrieval accuracy.

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