Homework 4

STA-360-602

Total points: 3 (reproducibility) + 6 (Q1) + 5 (Q2) = 14 points.

1. (6 points, 3 points each) Hoff, 3.10 (Change of variables).

   Let $\psi = g(\theta)$, where g is a monotone function of θ, and h be the inverse of g so that $\theta = h(\psi)$. If $p_\theta(\theta)$ is the probability density of θ, then the probability density of ψ induced by p_θ is given by

   $$p_\psi(\psi) = p_\theta(h(\psi)) \times |\frac{dh}{d\psi}|.$$

   (a) Let $\theta \sim$ beta(a, b) and let $\psi = \log[\theta/(1-\theta)]$. Obtain the form of p_ψ and plot is for the case that $a = b = 1$.
   Hint: Show that

   $$p_\psi(\psi) = \frac{1}{B(a,b)} \frac{e^{a\psi}}{(e^\psi + 1)^{(a+b))}} = \frac{e^\psi}{(e^\psi + 1)^2},$$

   when $a = b = 1$.

   (b) Let $\theta \sim$ gamma$(a = shape, b = rate)$ and let $\psi = \log(\theta)$. Obtain the form of p_ψ and plot is for the case that $a = b = 1$.
   Hint: Show that

   $$p_\psi(\psi) = \frac{b^a}{\Gamma(a)} e^{a\psi - be^\psi} = e^{\psi - e^\psi},$$

   when $a = b = 1$.

2. Lab component (5 points total) Please refer to lab 4 and complete tasks 4—5.

   (a) (2 points) Task 4

   (b) (3 points) Task 5