

# Homework 4

STA-360-602

Total points: 3 (reproducibility) + 6 (Q1) + 5 (Q2) = 14 points.

1. (6 points, 3 points each) Hoff, 3.10 (Change of variables).

Let  $\psi = g(\theta)$ , where  $g$  is a monotone function of  $\theta$ , and  $h$  be the inverse of  $g$  so that  $\theta = h(\psi)$ . If  $p_\theta(\theta)$  is the probability density of  $\theta$ , then the probability density of  $\psi$  induced by  $p_\theta$  is given by

$$p_\psi(\psi) = p_\theta(h(\psi)) \times \left| \frac{dh}{d\psi} \right|.$$

- (a) Let  $\theta \sim \text{beta}(a, b)$  and let  $\psi = \log[\theta/(1 - \theta)]$ . Obtain the form of  $p_\psi$  and plot  $p_\psi$  for the case that  $a = b = 1$ .

**Hint: Show that**

$$p_\psi(\psi) = \frac{1}{B(a, b)} \frac{e^{a\psi}}{(e^\psi + 1)^{(a+b)}} = \frac{e^\psi}{(e^\psi + 1)^2},$$

when  $a = b = 1$ .

- (b) Let  $\theta \sim \text{gamma}(a = \text{shape}, b = \text{rate})$  and let  $\psi = \log(\theta)$ . Obtain the form of  $p_\psi$  and plot  $p_\psi$  for the case that  $a = b = 1$ .

**Hint: Show that**

$$p_\psi(\psi) = \frac{b^a}{\Gamma(a)} e^{a\psi - be^\psi} = e^{\psi - e^\psi},$$

when  $a = b = 1$ .

2. Lab component (5 points total) Please refer to lab 4 and complete tasks 4—5.

- (a) (2 points) Task 4  
(b) (3 points) Task 5