# Homework 5

STA-360-602

Total points: 3 (reproducibility) + 15 (Q1) + 25 (Q2) = 50 points.

1. (15 points, 5 points each) Hoff, 3.12 (Jeffrey's prior).

   Jeffrey's (1961) suggested a default rule for generating a prior distribution of a parameter $\theta$ in a sampling model $p(y \mid \theta)$. Jeffreys' prior is given by

   $$p_J(\theta) \propto \sqrt{I(\theta)},$$

   where

   $$I(\theta) = -E\left[\frac{\partial^2 \log p(Y \mid \theta)}{\partial \theta^2} \mid \theta\right]$$

   is the Fisher information.

   (a) Let $Y \sim \text{Binomial}(n, \theta)$. Show that $p_J(\theta) \propto Beta(1/2, 1/2)$.

   (b) Re-parameterize the model in part a such that $\psi = \log[\theta/(1-\theta)]$. This implies that $p(y \mid \psi) = \binom{n}{y} e^{\psi y}(1+e^{\psi})^{-n}$. Show that

   $$p_J(\psi) \propto \frac{n^{1/2} e^{\psi/2}}{1+e^{\psi}} \propto \frac{e^{\psi/2}}{1+e^{\psi}}.$$

   (c) Take the prior distribution from (a) and apply the change of variables formula from exercise 3.10 (see Homework 4) to obtain the induced prior density on $\psi$. This density should be the same as the one derived in part b) of this exercise. This consistency under re-parametrization is the defining characteristic of Jeffereys' prior.

2. Lab component (25 points total) Please refer to lab 5 and complete tasks 4—5.

   (a) (10 points) Task 4

   (b) (15 points) Task 5

**You can refer to class notes to help you check your answers. See Module 5, slides 44 - 51**