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A group decision making framework based on fuzzy VIKOR approach for machine tool selection with linguistic information

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ABSTRACT

Computer numerical control (CNC) machines are used for repetitive, difficult and unsafe manufacturing tasks that require a high degree of accuracy. However, when selecting an appropriate CNC machine, multiple criteria need to be considered by multiple decision makers. In this study, a multi-criteria group decision making (MCGDM) technique based on the fuzzy VIKOR method is developed to solve a CNC machine tool selection problem. Linguistic variables represented by triangular fuzzy numbers are used to reflect decision maker preferences for the criteria importance weights and the performance ratings. After the individual preferences are aggregated or after the separation values are computed, they are then defuzzified. In this paper, two algorithms based on a fuzzy linguistic approach are developed. Based on these two algorithms and the VIKOR method, a general MCGDM framework is proposed. A CNC machine tool selection example illustrates the application of the proposed approach. A comparative study of the two algorithms using the above case study information highlighted the need to combine the ranking results, as both algorithms have distinct characteristics.

1. Motivation and background

Manufacturing industries today face multiple challenges such as globalization and rapidly changing market requirements because of modern lifestyle trends [18]. Therefore, during the early stages of operation, initial machinery and equipment capital investment decisions are critical to future profitability. Computer numerical control (CNC) machines are considered highly efficient cost effective system centered equipment for repetitious, difficult or dangerous manufacturing tasks. The critical links in CNC machines turn raw materials into finished parts and components for final product assembly. Therefore, the selection of the correct CNC machine tool can decrease production delivery times, improve product flexibility and product quality, and enhance overall productivity [1]. However, selecting the appropriate stand-alone machine tool from the various market alternatives has been a major difficulty for manufacturing companies, as improper machine tool selection could negatively affect the overall manufacturing system performance [6].

Machine tool selection is often a difficult task for engineers and managers as there are many qualitative and quantitative attributes that need to be considered in the selection of the appropriate machine tool [39]. Because of diverse conflicting machine tool selection criteria, quantifying the uncertain qualitative attribute information can be extremely complex. For example, a machine tool that is able to manufacture several different parts could result in increased set-up/changeover times [48]. Effective machine tool selection, therefore, needs to compromise over conflicting tangible and intangible factors, and, to deal with these, multiple criteria decision making (MCDM) has been found to be useful. However, human assessment of qualitative attributes is always subjective and therefore imprecise due to the decision-makers vagueness and uncertainty [5]. Because evaluation data such as alternative ratings or criteria weights are usually expressed in linguistic terms, the fuzzy linguistic approach has been found to be a suitable natural approach to handle such problems.

Against this background, MCDM methods under a fuzzy environment have received significant research attention. As seen in the literature review (see Section 2), fuzzy AHP has been one of the most utilized techniques for this kind of problem. However, fuzzy AHP have often been used to calculate the criteria weights, so an MCDM method that directly considers the conflicting factors is needed. The VIKOR method, which stands for VIse Kriterijumska Optimizacija i Kompromisno Resenje, was first proposed by Opricovic as an efficient tool to handle conflicting criteria [40–42]. This method ranks alternatives and determines a

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compromise solution that is the closest to the ideal. Opricovic and Tzeng conducted a detailed comparative analysis of the VIKOR method and other MCDM methods [42,43], and various extensions of the VIKOR method can be found in the literature. Machine tool selection is an MCDM problem, so the VIKOR method is a suitable method for solving MCDM problems with conflicting criteria. To date, however, there have been few studies which have used the VIKOR method for machine tool selection problems.

Even though research has shown that using fuzzy logic for machine tool selection can achieve good results, few have considered group decision making (GDM), which can consolidate different individual preferences into a single collective preference. The selection of a CNC machine tool generally involves at a minimum the production department, the finance department and the maintenance department. Therefore, to ensure accurate, effective selection decisions, experts from the internal and external environments need to be involved. However, few studies have discussed GDM applications for CNC machine tool selection. In addition, most fuzzy approaches have required some defuzzification process to obtain crisp results, so a further concern has been at which point the defuzzification process should be conducted when using the fuzzy VIKOR method, an issue which has not yet been fully explored in the literature.

Therefore, based on the above motivations, the objective of this study is to present a GDM framework based on a fuzzy VIKOR method to solve a CNC machine tool selection problem. The remainder of this paper is arranged as follows. Related research is reviewed in Section 2. Some preliminaries for the fuzzy set and GDM are introduced in Section 3. In Section 4, two different algorithms are proposed to extend the traditional VIKOR method for a fuzzy GDM case. A general framework for an MCGDM based on the two algorithms is also presented. An application example for a CNC machine tool selection at the Pakistan Machine Tool Factory Private Limited is provided in Section 5 to demonstrate the computational procedure of the proposed approach. In the final section, some conclusions and future research directions are given.

2. Literature review

This section briefly reviews the related literature on machine tool selection and the VIKOR method. Some limitations of the current methods are discussed.

2.1. Machine tool selection

In previous studies, several different approaches have been developed for the selection of the most suitable candidate machines. Approaches based on AHP have been popular in previous research [4,5,15,17,19,63]. For example, Ayağ [4] suggested a fuzzy AHP approach to evaluate machine tool alternatives. Ayağ and Özdemir [4] further utilized a fuzzy analytic network process (ANP)for machine tool selection. Ic et al. [19] presented an AHP method for the development of a component-based machine center selection model. Samvedi et al. [52] used fuzzy AHP and grey relational analysis to develop an integrated machine tool selection approach. Nguyen et al. [38] developed a hybrid fuzzy ANP and COPRAS-G (COmplex PRoportional Assessment of alternatives with Grey relations) approach for machine tool selection. In most of these papers, AHP or fuzzy AHP were used to derive the criteria weights as they are easy to understand and are able to handle both qualitative and quantitative data. However, in order to deal with the assessment values under the criteria, other MCDM methods are required. Önüt et al. [39] described a fuzzy TOPSIS based methodology for the evaluation and selection of vertical CNC machining centers, for which the criteria weights were calculated using fuzzy AHP. Dağdeviren [16] proposed an integrated AHP and PROMETHEE approach for an equipment selection problem. Özgen et al. [45] pointed out that the vagueness of the decision environment was not taken into account in [16], so utilized a new approach which combined a modified DELPHI, a fuzzy AHP and a fuzzy PROMETHEE technique.

Besides MCDM methods, when continuous variables have been involved in machine tool selection problems, optimization-based approaches have been proposed [27]. Rai et al. [50] used fuzzy goal programming to address a complex machine tool selection and operation allocation problem, in which a genetic algorithm (GA) was adopted for model optimization. Mishra et al. [37] developed a fuzzy goal programming technique to solve a machine tool selection and operation allocation problem. Jahromi and Tavakkoli-Moghaddam [21] proposed a new 0-1 linear integer programming model for a dynamic machine-tool selection problem and designed a heuristic method, named the five simple procedures (FSP) to solve the programming. Perçin and Mina [48] suggested a hybrid approach which combined the strengths of quality function deployment (QFD), fuzzy linear regression, and 0-1 goal programming. More recently, He et al. [18] developed an energy-saving optimization method for machine tool selection, in which the machining operation energy consumption and the machine tool idle energy consumption were minimized. Further to these studies, there have been other studies that have investigated CNC efficiencies, all of which are useful when considering CNC selection [9,32].

As seen from the literature review, while there have been a wide range of machine tool selection approaches, MCDM methods have generally been more popular than optimization-based methods. Optimal mathematical models are able to deal well with objective data, but often ignore the qualitative and subjective considerations. When decision makers' preferences for the criteria weights and/or the alternative ratings are involved, MCDM methods are more appropriate. However, there remain some limitations in the current machine tool selection methods. Previous research has focused on the fuzzy AHP in combination with other MCDM methods, yet as the fuzzy AHP method has been found to have some flaws [67], this method should be used with caution. In addition, as mentioned in the previous section, the increasing decision environment complexity makes it less possible for an individual decision maker to simultaneously consider all factors. Therefore, it is more logical to consider machine tool selection problems in group settings. Most machine tool selection approaches have only assumed a single decision maker, so to increase accuracy and efficiency, MCGDM approaches should be used to solve machine tool selection problems.

2.2. VIKOR method

The VIKOR method was developed to solve MCDM problems that had conflicting and non-commensurable criteria. Many researchers have extended this method to a variety of fuzzy environments. Opricovic [43,44] proposed a fuzzy VIKOR method in which both the attribute values and weights could be triangular fuzzy numbers. Sayadi et al. [53] extended the VIKOR method to a decision making problem with interval numbers. Ju and Wang [23] presented an extension of the VIKOR method for a multiple criteria group decision making problem based on the 2-tuple linguistic model. Wan et al. [58] developed an extended VIKOR method for multi-attribute group decision making using triangular intuitionistic fuzzy numbers. Jiang and Shang [22] extended the VIKOR method to group decision making by employing an optimization based method to integrate the decision makers' judgments. Liao et al. [31] developed a hesitant fuzzy linguistic VIKOR

(HFL-VIKOR) method. Qin et al. [49] presented a novel extension of the VIKOR method under an interval type-2 fuzzy environment.

Over the last decade, the VIKOR method has been extensively researched and applied to a wide range of problems [61]. Bazzazi et al. [8] proposed a VIKOR method to derive a preference order for open pit mining equipment. Jahan et al. [20] presented a comprehensive VIKOR method for materials selection. Shemshadi et al. [56] developed a fuzzy VIKOR method with entropy measures for the objective weighting in a best supplier selection problem. Liou et al. [33] utilized a modified VIKOR method to improve domestic airline service quality. Yüenur and Demirel [62] proposed an extended VIKOR method to solve an insurance company problem under a fuzzy environment. Chang [12] developed a fuzzy VIKOR method to evaluate hospital service quality in Taiwan. Kim and Chung [29] proposed a fuzzy VIKOR model to assess the vulnerability of water supply to climate change and variability in South Korea. Safari et al. [54] identified and evaluated enterprise architecture risks using a failure mode and effects analysis (FMEA) and a fuzzy VIKOR method. Liu et al. [34] used the VIKOR method to solve a site selection problem in waste management. Liu et al. [35] further proposed an approach for FMEA based on combination weighting and a fuzzy VIKOR method.

The above studies demonstrate the rapid development of the VIKOR method and its successful application to diverse MCDM problems. Interestingly, however, although defuzzification is important in the fuzzy VIKOR method, it has seldom been taken into account in previous research. For this reason, the defuzzification step should be explicitly considered in the fuzzy VIKOR method framework. As the VIKOR method can efficiently determine compromise solutions to problems with conflicting criteria, it is suitable for machine tool selection problems. Therefore, in this paper, we develop a new MCGDM framework for machine tool selection based on the fuzzy VIKOR method.

3. Preliminaries

In this section, the fuzzy linguistic approach and the GDM aggregation are introduced.

3.1. Fuzzy linguistic approach

A fuzzy set is a set of any objects for which there is no predefined boundary. The fuzzy set concept was first proposed by Zadeh [64] to address uncertainty in human judgment preferences. Many MCDM methods have been developed under fuzzy settings [47,57]. The extensions of fuzzy sets such as intuitionistic fuzzy sets and type-2 fuzzy sets also provide powerful tools for dealing with MCDM problems involving linguistic information [14,28,36]. In general, decision makers are able to express their opinions and preferences using linguistic variables [65], which are treated as fuzzy sets with specified membership functions. Recently, Zadeh [66] underlined three principal rationales for the use of precisiated words instead of numbers.

Definition 1. [26] A fuzzy set $\tilde{A}$ in a universe of discourse X is described using a membership function $\mu_{\tilde{A}}(X)$ which associates a real number in the interval [0,1] with each element x in X. The function value $\mu_{\tilde{A}}(X)$ is called the grade of membership of x in $\tilde{A}$.

Trapezoidal and triangular fuzzy numbers are special fuzzy sets commonly used in different types of decision making problems. In this study, to symbolize the linguistic variables triangular fuzzy numbers are used. A triangular fuzzy number can be denoted $\tilde{A} = (a_l, a_m, a_u)$ and its membership function $\mu_{\tilde{a}}$ is defined as follows:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, & x < a_l, \\ \dfrac{x - a_l}{a_m - a_l}, & a_l \le x \le a_m, \\ \dfrac{a_u - x}{a_u - a_m}, & a_m \le x \le a_u, \\ 0, & x > a_u. \end{cases} \tag{1}$$

The set of all triangular fuzzy number is denoted H_f.

Definition 2. [13] Let $\tilde{A}, \tilde{B} \in H_f$, where $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ are positive triangular fuzzy numbers. Some of the algebraic operations are expressed as follows

$$\begin{aligned} &\textit{Scalar} \quad \textit{summation}: \tilde{A} \oplus \tilde{B} = [a_l + b_l, a_m + b_m, a_u + b_u], \\ &\textit{Scalar} \quad \textit{subtraction}: \tilde{A} \ominus \tilde{B} = [a_l - b_h, a_m - b_m, a_u - b_l], \\ &\textit{Scalar} \quad \textit{multiplication}: \tilde{A} \otimes \tilde{B} = [a_l b_l, a_m b_m, a_u b_u], \\ &\textit{Scalar} \quad \textit{division}: \tilde{A}^{-1} = [1/a_l, 1/a_m, 1/a_u]. \end{aligned} \tag{2}$$

Note, either $\tilde{A}$ or $\tilde{B}$ can be reduced to crisp numbers. For example, for $\lambda > 0$ to be a real number, we have

$$\lambda\tilde{A} = (\lambda a_l, \lambda a_m, \lambda a_u).$$

Definition 3. [55] The operation $\wedge$ for minimum and $\vee$ for maximum are computed by

$$\begin{aligned} &\textit{Operator} \quad \textit{MIN}: \tilde{A} \wedge \tilde{B} = [a_l \wedge b_l, a_m \wedge b_m, a_u \wedge b_u], \\ &\textit{Operator} \quad \textit{MAX}: \tilde{A} \vee \tilde{B} = [a_l \vee b_l, a_m \vee b_m, a_u \vee b_u]. \end{aligned} \tag{3}$$

Linguistic variable values are defined with words or sentences in a natural or artificial language. For example, weight is a linguistic variable and its possible values can be expressed as follows: very low, low, medium, high and very high. Triangular fuzzy numbers are then used to represent these linguistic values.

For example, the following semantics can be assigned to a set of seven terms using triangular fuzzy numbers (see Fig. 1):

$$\begin{aligned} L = \{&\text{None} = (0, 0, 0.17), \text{VeryLow} \\ &= (0, 0.17, 0.33), \text{Low} = (0.17, 0.33, 0.5), \text{Medium} \\ &= (0.33, 0.5, 0.67), \text{High} = (0.5, 0.67, 0.83), \text{VeryHigh} \\ &= (0.67, 0.83, 1), \text{Perfect} = (0.83, 1, 1)\}. \end{aligned}$$

3.2. Aggregation in GDM

GDM (also known as multi-person or collaborative decision making) is a situation in which different stakeholders are collectively included in the decision-making process [11]. One basic problem in GDM is the extraction of the experts' preferences. Once the preference structures are determined, there are generally two lines of reasoning that can be followed: a direct approach and an indirect approach [24]. Assuming there are T decision makers with their preference structures $\{X_1, X_2, \ldots, X_T\}$, the direct approach is denoted as

$$\{X_1, X_2, \ldots, X_T\} \to \textit{solution}.$$

That is, a solution is derived directly (without any intermediate steps) from the set of individual preference structures. The indirect approach is denoted as

$$\{X_1, X_2, \ldots, X_T\} \to X_G \to \textit{solution}.$$

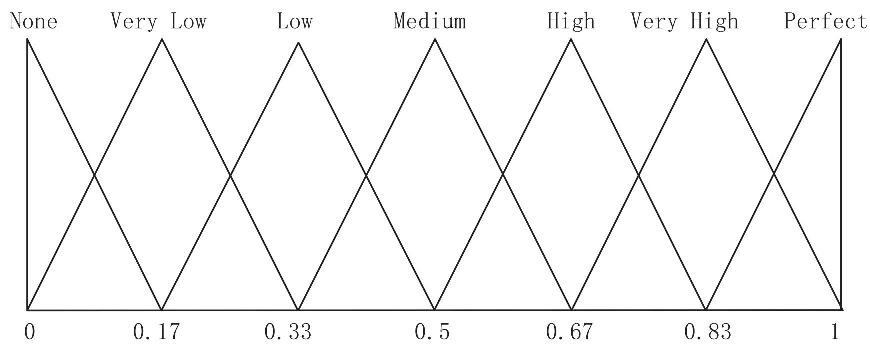

Fig. 1. Set of seven linguistic terms with associated semantics.

That is, from the set of individual preference structures, a social (also called group) preference structure, X_G, is first developed. which is then used to find the solution. Most research has preferred the indirect approach as it involves interaction, which means the final solution is more acceptable to the group members [25].

In the indirect approach, aggregation operators are required to aggregate the individual preferences into group preferences. In this paper, weighted average operators are used. For this kind of operator, the weighting vector reflects the reliability of the information sources.

Definition 4. Let $f: H_f^n \to H_f$, if

$$F(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = w_1\tilde{a}_1 \oplus w_2\tilde{a}_2 \oplus \cdots w_n\tilde{a}_n \oplus = \sum_{i=1}^{n}{}_{\oplus} w_i\tilde{a}_i \tag{4}$$

where $w = (w_1, w_2, \ldots, w_n)$ is the weighting vector such that $0 \le w_i \le 1$, $\sum_{i=1}^{n} w_i = 1$, then F is called the fuzzy weighted arithmetic averaging ($FWAA$) operator. If $w = (1/n, 1/n, \ldots, 1/n)$, then F is called the fuzzy arithmetic averaging (FAA) operator.

4. New fuzzy linguistic VIKOR framework for MCGDM

Machine tool selection is a complex multi-criteria group decision making (MCGDM) problem. Therefore, the fuzzy VIKOR method, as a persuasive decision approach, can be used to solve this MCGDM problem. The fuzzy VIKOR method is used to develop a ranking list, allocate weights and provide a compromise solution, which is the achievable solution closest to the ideal. This compromise solution means that an agreement is established through mutual adjustment [43]. Although the defuzzification step is essential to the fuzzy VIKOR, there has been no agreement as to the best point at which this process should be conducted [12,62]. In the following, two algorithms based on the fuzzy linguistic approach are developed and a general framework proposed to address MCGDM problems.

In the problem, let D_k, $k = 1, 2, 3, \ldots, T$ be the committee of various decision makers. C_j ($j = 1, 2, 3, \ldots, N$) and A_i ($i = 1, 2, 3, \ldots, M$) represent the different criteria and alternatives respectively. Further, let I_b denote the set of benefit criteria and I_c denote the set of cost criteria. Suppose $\tilde{x}_{ij}^{(k)} = (l_{ij}^{(k)}, m_{ij}^{(k)}, u_{ij}^{(k)})$ is a fuzzy value transformed from the linguistic preference value given by decision maker D_k for the ith alternative over the jth criterion and $\tilde{w}_j^{(k)} = (w_{lj}^{(k)}, w_{mj}^{(k)}, w_{uj}^{(k)})$ is the fuzzy importance weight for criterion C_j given by decision maker D_k.

4.1. Early defuzzification GDM VIKOR method

The early defuzzification GDM VIKOR method is described in Algorithm 1, the formation of which has the following steps.

Step 1: Arrange the decision making group committee and define a finite set of criteria and alternatives.

Step 2: Describe the appropriate linguistic variables. These linguistic variables are expressed using triangular fuzzy numbers. In this step, suitable linguistic variables for the criteria importance weights and the alternative fuzzy ratings for each criterion are also expressed.

Step 3: The decision makers aggregate the fuzzy ratings for M alternatives and the fuzzy weights for N criteria.

The fuzzy weight of each criterion is calculated using the following equation.

$$\tilde{w}_j = \frac{1}{T}[\tilde{w}_j^{(1)} \oplus \tilde{w}_j^{(2)} \oplus \tilde{w}_j^{(3)} \oplus \cdots \oplus \tilde{w}_j^{(t)}] \tag{5}$$

The aggregated fuzzy ratings for each alternative are calculated using the following equation.

$$\tilde{x}_{ij} = \frac{1}{T}[\tilde{x}_{ij}^{(1)} \oplus \tilde{x}_{ij}^{(2)} \oplus \tilde{x}_{ij}^{(3)} \oplus \cdots \oplus \tilde{x}_{ij}^{(t)}] \tag{6}$$

Without a loss of generality, equal weights for each decision maker are assumed. However, different weights are allowed for the aggregation of individual information preferences

Remark 1 In [22], an optimization model was given to derive the group judgments to avoid the effect of extreme opinions. In the notations in this paper, the optimization model is as follows:

$$\begin{aligned} &\min \quad z = \sum_{k=1}^{T}(l_{ij} - l_{ij}^k)^2 + \sum_{k=1}^{T}(m_{ij} - m_{ij}^k)^2 + \sum_{k=1}^{T}(u_{ij} - u_{ij}^k)^2 \\ &s.t. \begin{cases} \min\limits_k l_{ij}^k \le l_{ij} \le \max\limits_k l_{ij}^k, \\ \min\limits_k m_{ij}^k \le m_{ij} \le \max\limits_k m_{ij}^k, \\ \min\limits_k u_{ij}^k \le u_{ij} \le \max\limits_k u_{ij}^k, \\ l_{ij} \le m_{ij} \le u_{ij}. \end{cases} \end{aligned} \tag{M-1}$$

We can prove that the optimal solution to M-1 is equivalent to the FAA operator in (5) and (6). In fact, to obtain an analytical solution to M-1, first the unconstrained optimization problem is solved. We have

$$\frac{\partial z}{\partial l_{ij}} = 2\sum_{k=1}^{T}(l_{ij} - l_{ij}^k) \times 1 = 0,$$

and

$$l_{ij} = \frac{\sum_{k=1}^{T} l_{ij}^k}{T}.$$

Similarly, we can get $m_{ij} = (\sum_{k=1}^{T} m_{ij}^k)/T$ and $u_{ij} = (\sum_{k=1}^{T} u_{ij}^k)/T$. It can be verified that the solution also meets the constraints and thus this is the optimal solution to M-1. Therefore, model M-1 actually provides the meaning for the FAA operator in the aggregation. If the weights for T decision makers are available, the *FWAA* operator defined in Definition 4 can be used.

Step 4: Defuzzification. Defuzzification approaches focus on converting the fuzzy quantity into a crisp number to determine the best non-fuzzy performance (BNP) value. Another function of defuzzification is the ranking of the fuzzy numbers, which is a required step in many fuzzy models. There has been significant research focused on the defuzzification and ranking of fuzzy numbers [2,51,59,60]. Some studies addressed the classification and comparison of different defuzzification methods [10,30]. And several defuzzification strategies have been proposed based on specific characteristics, such as the centroid, mode, and median. The centroid method defines the centroid coordinate for the fuzzy numbers on a horizontal axis. The mode rule consists of choosing the support value at which the membership function reaches maximum. The median rule determines the median that divides the area under the membership function into two equal parts [51].

The centroid method (also called the center of gravity in some papers) is computationally simple and its underlying concepts are logically sound. Therefore, in this paper, the centroid method is selected as the defuzzification method. The centroid method can be considered as a special case ($k = 1$) of the kth weighted mean method [44]. Using the centroid method, for a triangular fuzzy number $\tilde{A} = (a_l, a_m, a_u)$, we have

$$CoG(\tilde{A}) = \frac{a_l + a_m + a_u}{3}. \tag{7}$$

After defuzzification, $\tilde{x}_{ij}$ and $\tilde{w}_j$ are denoted x_{ij} and w_j respectively. Formula (7) has been one of the most commonly used defuzzification methods in various applications [35,46].

Remark 2 It is also worth mentioning that different defuzzification methods can produce different BNP values. If another method (say the mode) is chosen, this could affect the final rankings for the VIKOR method. There is no universal acceptance as to which defuzzification method should be used; therefore, the choice of a defuzzification method needs to be considered in the algorithm design (see Fig. 2). To the best of our knowledge, there has been no criterion identified so far for choosing defuzzification methods for group decision making.

Step 5: Calculate the best values f_j^* and the worst values f_j^- for all criteria. When j is associated with a benefit criterion, it follows that

$$f_j^* = \max x_{ij}, \quad f_j^- = \min x_{ij}, j \in I_b. \tag{8}$$

When j is associated with a cost criterion, it follows that

$$f_j^* = \min x_{ij}, \quad f_j^- = \max x_{ij}, j \in I_c. \tag{9}$$

Step 6: Calculate the index S_i, which refers to the separation measure of the ith alternative with the best value. Also, calculate the index R_i, which refers to the separation measure for the ith alternative to the worst value. w_j is the weight of the jth criteria.

$$S_i = \sum_{j=1}^{n} w_j[(f_j^* - x_{ij})/f_j^* - f_j^-)], \tag{10}$$

$$R_i = \max_j[w_j(f_j^* - x_{ij})/f_j^* - f_j^-)]. \tag{11}$$

The solution obtained using R_i has a minimum individual regret function while the solution obtained using S_i has a maximum group utility.

Step 7: Calculate the values Q_i, using the following equation.

$$Q_i = v\frac{S_i - S_{\min}}{S_{\max} - S_{\min}} + (1 - v)\frac{R_i - R_{\min}}{R_{\max} - R_{\min}}, \tag{12}$$

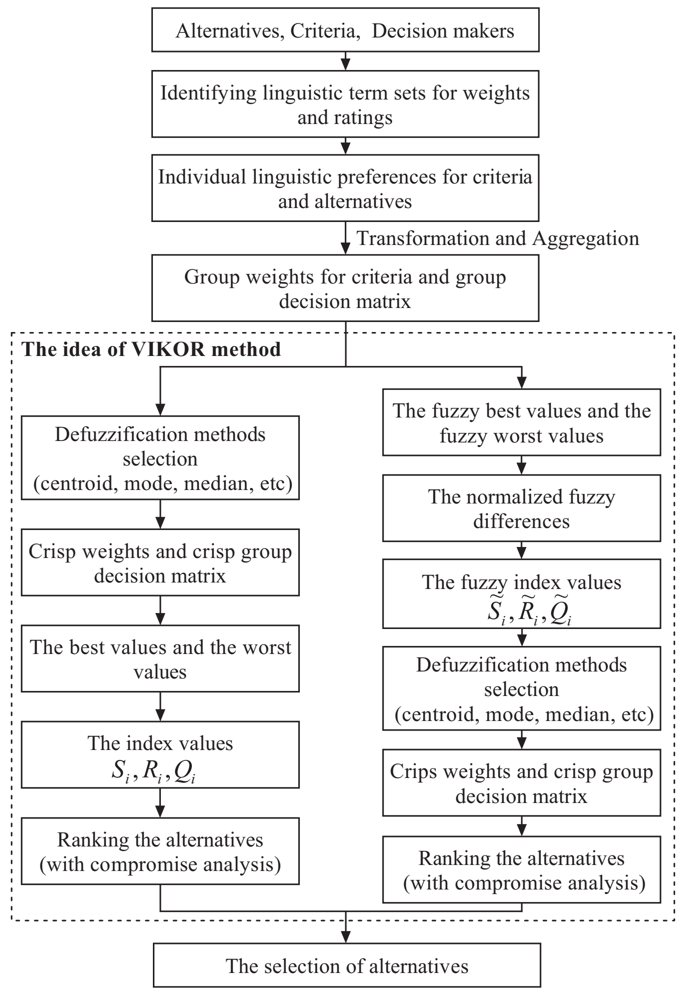

Fig. 2. An MCGDM framework based on the VIKOR method.

where

$$S_{\max} = \max_i S_i, \quad S_{\min} = \min_i S_i,$$
$$R_{\max} = \max_i R_i, \quad R_{\min} = \min_i R_i,$$

and v is the strategy weight for the maximum group utility, while $1 - v$ indicates the individual weight regret function.

Step 8: Rank the alternatives. Rank the alternatives by sorting the crisp values for S, R and Q in ascending order, which results in three ranking lists $\{A\}_S$, $\{A\}_R$, $\{A\}_Q$. The index Q_i indicates the separation measures for the ith alternative A_i from the best alternative, that is, the smaller the value of Q, the better the alternative.

Step 9: Propose a compromise solution. The alternative denoted $A^{(1)}$ is the best ranked using the measure Q (minimum), so is considered a promising solution if the following two conditions are satisfied:

$Cond_1$: Acceptable Advantage: $Adv \geq DQ$

$$Adv = Q(A^{(2)}) - Q(A^{(1)}) \geq 1/(m - 1) \tag{13}$$

where Adv is the advantage of alternative $A^{(1)}$ ranked first, $A^{(2)}$ is the alternative in the second position in $\{A\}_Q$, and $DQ = 1/(m - 1)$ is the threshold.

$Cond_2$: Acceptable Stability in decision making: Alternative $A^{(1)}$ must also be the best when ranked using S and/or R.

If one of the two conditions is not satisfied, then a set of compromise solution is proposed, which consists of:

a) Alternative $A^{(1)}$ and $A^{(2)}$ if only condition $Cond_2$ is not satisfied;
b) Alternative $A^{(1)}, A^{(2)}, \ldots, A^{(M)}$ if condition $Cond_1$ is not satisfied.

$A^{(M_1)}$ is determined from the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for a maximum M_1. This implies that the positions of these alternatives are 'in closeness' and therefore $A^{(1)}, A^{(2)}, \ldots, A^{(M_1)}$ are the set of alternatives to be further considered.

4.2. Late defuzzification GDM VIKOR method

To conduct a comparative study, in the following, another fuzzy VIKOR version, the late defuzzification GDM VIKOR method, is described. The steps for this method are summarized in Algorithm 2.

Algorithm 2

Step 1: The same as Step 1 in Algorithm 1.

Step 2: The same as Step 2 in Algorithm 1.

Step 3: The same as Step 3 in Algorithm 1.

Step 4: Identify the fuzzy best value $\tilde{f}_j^* = (l_j^*, m_j^*, u_j^*)$ and the fuzzy worst values $\tilde{f}_j^- = (l_j^-, m_j^-, u_j^-)$ for all criteria. When j is associated with a benefit criterion, it follows that

$$\begin{aligned} \tilde{f}_j^* &= *MAX_j \tilde{x}_{ij} = (\max_j l_{ij}, \max_j m_{ij}, \max_j u_{ij}), \\ \tilde{f}_j^- &= *MIN_j \tilde{x}_{ij} = (\min_j l_{ij}, \min_j m_{ij}, \min_j u_{ij}). \end{aligned} \tag{14}$$

When j is associated with a cost criterion, it follows that

$$\begin{aligned} \tilde{f}_j^* &= *MIN_j \tilde{x}_{ij} = (\min_j l_{ij}, \min_j m_{ij}, \min_j u_{ij}), \\ \tilde{f}_j^- &= *MAX_j \tilde{x}_{ij} = (\max_j l_{ij}, \max_j m_{ij}, \max_j u_{ij}). \end{aligned} \tag{15}$$

Step 5: Compute the normalized fuzzy difference $\tilde{d}_{ij}$, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$. For the benefit criteria, $\tilde{d}_{ij}$ is calculated as

$$\tilde{d}_{ij} = \frac{\tilde{f}_j^* \ominus \tilde{x}_{ij}}{u_j^* - l_j^-}, \quad j \in I_b \tag{16}$$

For the cost criteria, $\tilde{d}_{ij}$ is calculated as

$$\tilde{d}_{ij} = \frac{\tilde{x}_{ij} \ominus \tilde{f}_j^*}{u_j^- - l_j^*}, \quad j \in I_c. \tag{17}$$

Step 6: Calculate the index values $\tilde{S}_i$ and $\tilde{R}_i$, $i = 1, 2, \ldots, M$, which are measured by

$$\tilde{S}_i = \sum_{j=1}^{N}{}_{\oplus} \tilde{w}_j \otimes \tilde{d}_{ij}, \tag{18}$$

$$\tilde{R}_i = *MAX_j \tilde{w}_j \otimes \tilde{d}_{ij}, \tag{19}$$

where $\tilde{S}_i = (S_i^l, S_i^m, S_i^u)$ is a fuzzy weighted sum referring to the separation measure for alternative A_i from the best values and $\tilde{R}_i = (R_i^l, R_i^m, R_i^u)$ is the separation measure for alternative A_i from the worst values.

Step 7: Calculate the values $\tilde{Q}_i = (Q_i^l, Q_i^m, Q_i^u)$, using the following equation:

$$\tilde{Q}_i = v \frac{\tilde{S}_i \ominus \tilde{S}_{\min}}{S_{\max}^u - S_{\min}^l} \oplus (1 - v) \frac{\tilde{R}_i \ominus \tilde{R}_{\min}}{R_{\max}^u - R_{\min}^l}. \tag{20}$$

where

$$\tilde{S}_{\min} = *MIN_i \tilde{S}_i, \quad S_{\max}^u = \max_i S_i^u, \quad S_{\min}^l = \min_i S_i^l,$$
$$\tilde{R}_{\min} = *MIN_i \tilde{R}_i, \quad R_{\max}^u = \max_i R_i^u, \quad R_{\min}^l = \min_i R_i^l,$$

and v is introduced as a weight for the maximum group utility strategy and $1 - v$ is the weight of the individual regret function.

Step 8: Defuzzify the values $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$. After defuzzification, $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$ are converted into the crisp numbers S_i, R_i and Q_i. Here, as in Algorithm 1, the centroid method is used, and we have

$$S_i = \frac{S_i^l + S_i^m + S_i^u}{3}, \quad R_i = \frac{R_i^l + R_i^m + R_i^u}{3}, \quad Q_i = \frac{Q_i^l + Q_i^m + Q_i^u}{3}. \tag{21}$$

As stated in Step 4 of Algorithm 1, the choice of the defuzzification method should be considered in the algorithm design (see Fig. 2).

Step 9: The same as Step 8 in Algorithm 1.

Step 10: The same as Step 9 in Algorithm 1. However, the Acceptable Advantage is calculated as

$$Adv = \frac{Q(A^{(2)}) - Q(A^{(1)})}{Q(A^{(M)}) - Q(A^{(1)})}. \tag{22}$$

In both these methods, the ranking indexes are determined by considering the maximum group utility and the minimum individual regret function for an opponent. This step to derive the compromise solution is a form of robust analysis. In extreme cases, the compromise solution is composed of all alternatives, although the order can still be clarified using $\{A\}_Q$.

Algorithm 1 adopts an early defuzzification strategy, making the computational calculations much simpler. Algorithm 2, however, adopts a late defuzzification strategy, which means there is more uncertain information in the ranking orders. This is because Algorithm 2 has more steps which involve fuzzy numbers, so a greater amount of uncertain information is propagated. If the decision makers are more deterministic, then Algorithm 1 is more suitable; otherwise Algorithm 2 is more suitable. However, both algorithms should be analyzed to comprehensively understand the results derived using the VIKOR method.

Based on a combination of Algorithms 1 and 2, a general VIKOR method based framework is developed to deal with MCGDM problems, as shown in Fig. 2.

5. Application example

Proper equipment selection, which is a complex, time consuming problem for manufacturing companies, is very important in the global market place. When selecting a machine tool, decision makers need to have adequate relevant criteria to make a proper analysis from the large amount of data. When assessing CNC machines, the main input and output measures are purchase costs and technical specifications.

In this section, the model described above was used to evaluate and select the most suitable candidate CNC machine tool for the Pakistan Machine Tool Factory (PMTF) Private Limited. This company has a technical collaboration with M/s. Oerlikon Buhrle and Co. of Switzerland, a world renowned machine tool manufacturer. The PMTF, which started production in 1971, is located about 35 km from Karachi near the Landhi Industrial Estate. It covers 226 acres, of which 17 are occupied by the manufacturing plant. This factory is a unit of the State Engineering Corporation of Pakistan and is engaged in the production of machine tools. Consequently, the factory management wanted to select the best machine tools to guarantee maximum output at minimum cost. In general, people who are particularly relevant to the machine tool selection should be invited to assist the decision, so the manager responsible for the project needed to carefully select participants. A small group of 3 to 5 decision makers has been suggested as ideal for this kind of problem [17,39]. In this study, two engineers from the factory and one outside expert were invited and agreed to provide advice, so, including the manager, a committee of four decision makers D_1, D_2, D_3, D_4 was organized for the CNC machine tool selection.

In previous research, the criteria used for machine tool selection have varied due to the differences in manufacturing facilities and the different perspectives explored by the group leader. In [3], nine criteria (flexibility, productivity, adaptability, cost, reliability, precision, space, safety and environment, service and maintenance) were presented. Based on the criteria in [3], Ayağ and Özdemir [4] and Ayağ [5] proposed eight main and nineteen subcriteria in their AHP hierarchy. And in another paper, Ayağ and Özdemir [7] selected twelve attributes. Nguyen et al. [38] adopted the same attributes as in [4]. Çimren et al. [15] applied four main criteria and twenty-two subcriteria, with the main criteria being productivity, flexibility, safety and the environment, and adaptability. In Önüt et al.'s study [39], eight criteria (cost, operative flexibility, installation easiness, maintainability and serviceability, productivity, compatibility, safety and user friendliness) were determined, which were also selected in [52]. Özgen et al. [45] identified four main criteria and fifteen subcriteria in their case study at the Inka Fixing Corporation in Istanbul. Five criteria namely stiffness, damping capacity, thermal stability, speed capacity, and accuracy, were considered in Nguyen et al.'s evaluation problem [38]. The above studies indicate that although different criteria have been selected, there have been some commonalities such as cost, capacity and flexibility.

In this paper, from the research analysis and a review of the actual situation, the four decision makers agreed upon the following five alternatives and six criteria. The five alternatives were Mazak (A_1), Romi (A_2), Dossan (A_3), Nakamura (A_4) and Saim (A_5), and the six main criteria were capital costs (C_1), spindle speed (C_2), bar capacity (C_3), horse power (C_4) flexibility (C_5) and turning meter (C_6) (see Fig. 3). The six main criteria were judged important as each could affect the ability of a CNC machine to perform the required manufacturing operations. The proposed fuzzy linguitic VIKOR framework was utilized to solve this MCDGM problem using the following steps:

Step 1: The decision makers' committee was formed, which then described the finite set of criteria and alternatives. In the specified selection problem, there were six criteria, $C_1, C_2, \ldots, C_6$, five alternatives, $A_1, A_2, \ldots, A_5$ and four decision makers D_1, D_2, D_3, D_4.

Step 2: In this step, the appropriate linguistic variables for the criteria importance weights were as follows

$$\begin{aligned} L_1 = \{&\mathrm{VeryLow(VL)} = (0, 0, 0.25), \mathrm{Low(L)} = (0, 0.25, 0.5),\\ &\mathrm{Medium(M)} = (0.25, 0.5, 0.75), \mathrm{High(H)} = (0.5, 0.75, 1),\\ &\mathrm{VeryHigh(VH)} = (0.75, 1, 1)\}. \end{aligned}$$

The linguistic variables for rating the five alternatives with regard to the six criteria were as follows

$$\begin{aligned} L_2 = \{&\mathrm{VeryPoor(VP)} = (0, 0, 0.17), \mathrm{Poor(P)} = (0, 0.17, 0.33),\\ &\mathrm{MediumPoor(MP)} = (0.17, 0.33, 0.5), \mathrm{Fair(F)}\\ = &(0.33, 0.5, 0.67), \mathrm{MediumGood(MG)} = (0.5, 0.67, 0.83),\\ &\mathrm{Good(G)} = (0.67, 0.83, 1), \mathrm{VeryGood(VG)} = (0.83, 1, 1)\}. \end{aligned}$$

The decision makers used the linguistic variables shown above to express their preferences. The evaluations for these criteria importance weights are given in Table 1 and the fuzzy decision matrices are given in Table 2. Note that in Table 2, all values under the cost criterion were transformed using the negative operator *Neg*. Therefore, all values were normalized, and the larger the value, the better the performance.

Table 1
Weights for each criterion.

	C_1	C_2	C_3	C_4	C_5	C_6
D_1	VH	H	M	VH	M	VH
D_2	VH	M	M	VH	H	VH
D_3	VH	H	VH	M	H	H
D_4	H	M	VH	H	H	H

Table 2
Fuzzy ratings for the alternatives over each criterion.

Decision makers	Alternatives	Criteria					
		C_1	C_2	C_3	C_4	C_5	C_6
D_1	A_1	F	VG	F	P	MP	MG
	A_2	G	VG	MP	VG	MG	G
	A_3	VG	MP	G	F	MP	G
	A_4	G	F	MG	MG	G	G
	A_5	VG	G	VG	MG	MG	G
D_2	A_1	G	F	MP	F	P	G
	A_2	G	G	G	MG	MG	VG
	A_3	VG	MP	MG	F	F	G
	A_4	G	MP	MG	G	MG	MG
	A_5	G	VG	G	VG	G	MG
D_3	A_1	MG	VG	MG	F	F	G
	A_2	VG	G	MP	G	MG	G
	A_3	VG	P	VG	MP	F	G
	A_4	VG	F	MG	MG	F	MG
	A_5	G	VG	VG	MG	F	G
D_4	A_1	MP	G	F	F	MP	G
	A_2	G	VG	F	G	G	MG
	A_3	VG	MP	MG	F	F	MG
	A_4	G	F	MG	G	MG	VG
	A_5	VG	G	VG	G	MG	G

Step 3: Individual preferences were aggregated into group preferences, and the linguistic variables were converted into triangular fuzzy numbers. The group decision matrix shown in Table 3 was calculated according to Eq. (6). The fuzzy weights $\tilde{w}_j$ of C_j, j = , 1, 2, . . ., 6 were computed using Eq. (5), as follows

$$\begin{aligned} &\tilde{w}_1 = (0.6875, 0.9375, 1.0000), \quad \tilde{w}_2 = (0.3750, 0.6250, 0.8750),\\ &\quad \tilde{w}_3 = (0.5000, 0.7500, 0.8750),\\ &\tilde{w}_4 = (0.5625, 0.8125, 0.9375), \quad \tilde{w}_5 = (0.4375, 0.6875, 0.9375),\\ &\quad \tilde{w}_6 = (0.6250, 0.8750, 1.0000). \end{aligned}$$

Step 4: The fuzzy numbers were defuzzified into crisp values according to Equation (7) (see Table 4)

Step 5: From Table 4, the best value and the worst value for each criterion were calculated using Eqs. (8) and (9). The results were as follows:

$$\begin{aligned} &f_1^* = 0.9433, \quad f_2^* = 0.8883, \quad f_3^* = 0.9158, \quad f_4^* = 0.8192,\\ &\quad f_5^* = 0.7083, \quad f_6^* = 0.8192,\\ &f_1^- = 0.5833, \quad f_2^- = 0.2917, \quad f_3^- = 0.5000, \quad f_4^- = 0.4167,\\ &\quad f_5^- = 0.3333, \quad f_6^- = 0.7775. \end{aligned}$$

Step 6: The values S_i and R_i were calculated using Eqs. (10) and (11), respectively, and the results were as follows:

$$\begin{aligned} &S_1 = 3.6790, \quad S_2 = 0.9089, \quad S_3 = 2.5600,\\ &\quad S_4 = 2.1176, \quad S_5 = 0.8399,\\ &R_1 = 0.8750, \quad R_2 = 0.7083, \quad R_3 = 0.6910,\\ &\quad R_4 = 0.8333, \quad R_5 = 0.5500. \end{aligned}$$

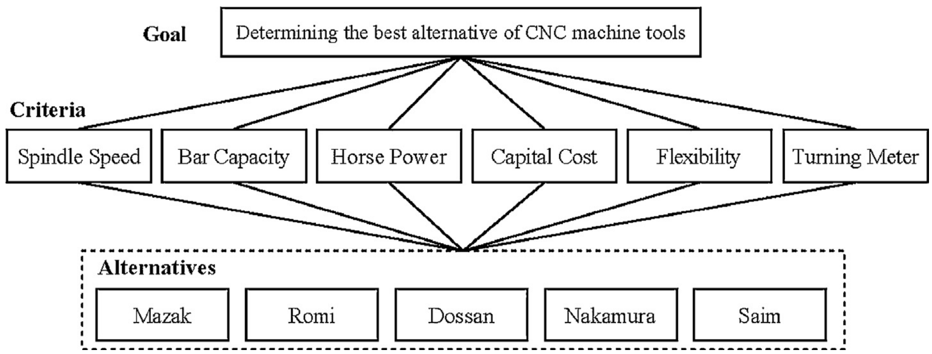

Fig. 3. Hierarchy for the evaluation of the best alternative for machine tool selection.

Table 3
Aggregated fuzzy number decision matrix.

	C_1	C_2	C_3
A_1	(0.4175, 0.5825, 0.7500)	(0.6650, 0.8325, 0.9175)	(0.3325, 0.5000, 0.6675)
A_2	(0.7100, 0.8725, 1.0000)	(0.7500, 0.9150, 1.0000)	(0.3350, 0.4975, 0.6675)
A_3	(0.8300, 1.0000, 1.0000)	(0.1275, 0.2900, 0.4575)	(0.6250, 0.7925, 0.9150)
A_4	(0.7100, 0.8725, 1.0000)	(0.2900, 0.4575, 0.6275)	(0.5000, 0.6700, 0.8300)
A_5	(0.7500, 0.9150, 1.0000)	(0.7500, 0.9150, 1.0000)	(0.7900, 0.9575, 1.0000)
	C_4	C_2	C_6
A_1	(0.2475, 0.4175, 0.5850)	(0.1675, 0.3325, 0.5000)	(0.6275, 0.7900, 0.9575)
A_2	(0.6675, 0.8325, 0.9575)	(0.5425, 0.7100, 0.8725)	(0.6675, 0.8325, 0.9575)
A_3	(0.2900, 0.4575, 0.6275)	(0.2900, 0.4575, 0.6275)	(0.6275, 0.7900, 0.9575)
A_4	(0.5850, 0.7500, 0.9150)	(0.5000, 0.6675, 0.8325)	(0.6250, 0.7925, 0.9150)
A_5	(0.6250, 0.7925, 0.9150)	(0.5000, 0.6675, 0.8325)	(0.6275, 0.7900, 0.9575)

Table 4
Crisp values for decision matrix and each criterion weight.

	C_1	C_2	C_3	C_4	C_5	C_6
Weight	0.8750	0.6250	0.7083	0.7708	0.6875	0.8333
A_1	0.5833	0.8050	0.5000	0.4167	0.3333	0.7917
A_2	0.8608	0.8883	0.5000	0.8192	0.7083	0.8192
A_3	0.9433	0.2917	0.7775	0.4583	0.4583	0.7917
A_4	0.8608	0.4583	0.6667	0.7500	0.6667	0.7775
A_5	0.8883	0.8883	0.9158	0.7775	0.6667	0.7917

Table 5
Ranking of alternatives using *S*, *R* and *Q* in ascending order.

	The ranking order
By *S*	$A_5 \succ A_2 \succ A_4 \succ A_3 \succ A_1$
By *R*	$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$
By *Q*	$A_5 \succ A_2 \succ A_3 \succ A_4 \succ A_1$

Step 7: The values Q_i for each alternative were computed using Eq. (12), and the results were as follows:

$$Q_1 = 1.0000, \quad Q_2 = 0.2557, \quad Q_3 = 0.5199,$$
$$Q_4 = 0.6609, \quad Q_5 = 0.$$

Step 8: Using Table 5, the alternatives were ranked and sorted using the crisp values *S*, *R* and *Q* in ascending order, the results for which were shown in Table 5.

Step 9: Since

$$Q(A^{(2)}) - Q(A^{(1)}) = 0.2557 \geq \frac{1}{5-1} = 0.25,$$

$Cond_1$ was satisfied and alternative A_5 was also the best ranked by *S* and/or *R*, so A_5 was considered to have acceptable stability. Since both $Cond_1$ and $Cond_2$ were verified, A_5, which was the machine from Saim, was recommended as having superior performance to the Dossan, Mazak, Nakamura or Romi.

To provide a comparative study, Algorithm 2 was utilized to tackle the same problem and the steps were as follows. The first three steps were the same as for Algorithm 1.

Step 4: The fuzzy best value $\tilde{f}_j^*$ and the worst values $\tilde{f}_j^-$ were identified for all criteria. From Table 4, the results were as follows

$$\tilde{f}_1^* = (0.8300, 1.0000, 1.0000), \quad \tilde{f}_1^- = (0.4175, 0.5825, 0.7500),$$
$$\tilde{f}_2^* = (0.7500, 0.9150, 1.0000),$$
$$\tilde{f}_2^- = (0.1275, 0.2900, 0.4575), \quad \tilde{f}_3^* = (0.7900, 0.9575, 1.0000),$$
$$\tilde{f}_3^- = (0.3325, 0.4975, 0.6675),$$
$$\tilde{f}_4^* = (0.6675, 0.8325, 0.9575), \quad \tilde{f}_4^- = (0.2475, 0.4175, 0.5850),$$
$$\tilde{f}_5^* = (0.5425, 0.7100, 0.8725),$$
$$\tilde{f}_5^- = (0.1675, 0.3325, 0.5000), \quad \tilde{f}_6^* = (0.6675, 0.8325, 0.9575),$$
$$\tilde{f}_6^- = (0.6250, 0.7900, 0.9150).$$

Step 5: The normalized fuzzy differences $\tilde{d}_{ij}$, $i = 1, 2, \ldots, 5$, $j = 1, 2, \ldots, 6$ were computed. The results were in Table 6.

Table 6
Normalized fuzzy difference values.

	C_1	C_2	C_3
A_1	(0.1373, 0.7167, 1.0000)	(−0.1920, 0.0946, 0.3840)	(0.1835, 0.6854, 1.0000)
A_2	(−0.2918, 0.2189, 0.4979)	(−0.2865, 0, 0.2865)	(0.1835, 0.6891, 0.9963)
A_3	(−0.2918, 0, 0.2918)	(0.3352, 0.7163, 1.0000)	(−0.1873, 0.2472, 0.5618)
A_4	(−0.2918, 0.2189, 0.4979)	(0.1404, 0.5244, 0.8138)	(−0.0599, 0.4307, 0.7491)
A_5	(−0.2918, 0.1459, 0.4292)	(−0.2865, 0, 0.2865)	(−0.3146, 0, 0.3146)
	C_4	C_2	C_6
A_1	(0.1162, 0.5845, 1.0000)	(0.0603, 0.5355, 1.0000)	(−0.8722, 0.1278, 0.9925)
A_2	(−0.4085, 0, 0.4085)	(−0.4681, 0, 0.4681)	(−0.8722, 0, 0.8722)
A_3	(0.0563, 0.5282, 0.9401)	(−0.1206, 0.3582, 0.8262)	(−0.8722, 0.1278, 0.9925)
A_4	(−0.3486, 0.1162, 0.5246)	(−0.4113, 0.0603, 0.5284)	(−0.7444, 0.1203, 1.0000)
A_5	(−0.3486, 0.0563, 0.4683)	(−0.4113, 0.0603, 0.5284)	(−0.8722, 0.1278, 0.9925)

Step 6: The index values $\tilde{S}_i$ and $\tilde{R}_i$, $i = 1, 2, \ldots, 5$ were calculated. The measures were obtained as in the following.

$$\tilde{S}_1 = (-0.3392, 2.2000, 5.0784),\ \tilde{S}_2 = (-1.1960, 0.7221, 3.3142),$$
$$\tilde{S}_3 = (-0.7347, 1.4203, 4.3069),\ \tilde{S}_4 = (-1.0192, 1.0971, 3.8525),$$
$$\tilde{S}_5 = (-1.3866, 0.3359, 2.8820),\quad \tilde{R}_1 = (0.0944, 0.6719, 1.0000),$$
$$\tilde{R}_2 = (0.0918, 0.5169, 0.8722),\quad \tilde{R}_3 = (0.1257, 0.4477, 0.9925),$$
$$\tilde{R}_4 = (0.0527, 0.3277, 1.0000),\quad \tilde{R}_5 = (-0.1074, 0.1368, 0.9925).$$

Step 7: The values for $\tilde{Q}_i$, $i = 1, 2, \ldots, 5$, were calculated, as in the following

$$\tilde{Q}_1 = (-0.6546, 0.3858, 1.0000),$$
$$\tilde{Q}_2 = (-0.7221, 0.2015, 0.8058),$$
$$\tilde{Q}_3 = (-0.6711, 0.2242, 0.9369),$$
$$\tilde{Q}_4 = (-0.7261, 0.1451, 0.9052),$$
$$\tilde{Q}_5 = (-0.8268, -0.0000, 0.8267).$$

Step 8: The values $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$, $i = 1, 2, \ldots, 5$ were defuzzified using the centroid method. The transformed crisp values were presented in Table 7.

Step 9: According to the defuzzified values in Table 7, the alternatives were ranked by sorting the crisp values S, R and Q in ascending order (Table 7).

Step 10: Whether the two conditions were satisfied was then verified. Since

$$\frac{Q(A^{(2)}) - Q(A^{(1)})}{Q(A^{(5)}) - Q(A^{(1)})} \Big/ = \frac{0.0951 - 0}{0.2437 - 0} = 0.3902 \geq \frac{1}{5-1} = 0.25,$$

$Cond_1$ was satisfied and alternative A_5 was also the best ranked by S and/or R.

This comparative study demonstrated that the early defuzzification VIKOR method was simpler, but that both algorithms recognized alternative A_5 as the best option and A_1 as the worst option. From Algorithm 1, the orders obtained using $\{A\}_Q$ and $\{A\}_S$ were similar, whereas when using Algorithm 2, the orders obtained using $\{A\}_Q$ and $\{A\}_R$ were similar. It is also of interest to note that in general the ranking orders between $\{A\}_S$ and $\{A\}_R$ were different as they had different ideas and used different separation measures. The index values showed that the distances between the alternatives in Algorithm 1 were greater than in Algorithm 2. Note that in Algorithm 2, the Acceptable Advantage was calculated based on $Q(A^{(5)}) - Q(A^{(1)})$. The order obtained using $\{A\}_Q$ in both algorithms met the Acceptable Advantage ($Cond_1$) and the Acceptable Stability ($Cond_2$).

Remark 3 In a study in which fuzzy logic is involved, it is meaningful to design a benchmark to compare the usefulness of the various fuzzy algorithms. In this case study, the benchmark problem was to consider a corresponding non-fuzzy information based decision. Because the linguistic information was described using triangular fuzzy numbers and no non-fuzzy information was unavailable, the benchmark problem was to find the corresponding crisp numbers at the very start of the process. In the proposed framework, because the two algorithms had the same first three steps, the benchmark problem was to find the corresponding non-fuzzy information based on Tables 3 and 4, which is somewhat equivalent to the defuzzification step (Step 4) in Algorithm 1. However, in the benchmark analysis, the mode of the triangular fuzzy number was thought to be a better candidate. The mode or core allows for the identification of the set of elements which have the largest degree of membership in a fuzzy number. For a triangular fuzzy number $\tilde{A} = (a_l, a_m, a_u)$, we have $mode(\tilde{A}) = a_m$. An interpretation of the mode is that an element with a higher degree of membership is considered to have more of the property related to the fuzzy set. If we use the mode as the benchmark analysis, the indexes for S, R and Q ($v = 0.5$) were as follows:

$$S_1 = 4.1409,\quad S_2 = 1.0363,\quad S_3 = 2.9631,$$
$$S_4 = 2.2750,\quad S_5 = 1.2216,$$
$$R_1 = 0.9375,\quad R_2 = 0.7500,\quad R_3 = 0.8750,$$
$$R_4 = 0.8235,\quad R_5 = 0.8750,$$
$$Q_1 = 1.0000,\quad Q_2 = 0,\quad Q_3 = 0.6436,$$
$$Q_4 = 0.3956,\quad Q_5 = 0.3632.$$

Now the ranking from S is $A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$, the ranking from R is $A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$, and the ranking from Q is $A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$. The final rank from Q is more similar to the result of Algorithm 2 than that of Algorithm 1. In this sense, Algorithm 2 was shown to be superior to Algorithm 1. This additional comparison also demonstrates that different defuzzification methods can affect the final results. Choosing the right method is a matter of preference and also depends on the context. Therefore, designing appropriate benchmarks to rank the proposed algorithms requires further study.

Table 7
Ranking of alternatives by S, R and Q in ascending order.

	A_1	A_2	A_3	A_4	A_5	The ranking order
By S	2.3131	0.9468	1.6642	1.3101	0.6105	$A_5 \succ A_2 \succ A_4 \succ A_3 \succ A_1$
By R	0.5888	0.4936	0.5220	0.4601	0.3406	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
By Q	0.2437	0.0951	0.1634	0.1081	0.0000	$A_5 \succ A_2 \succ A_4 \succ A_3 \succ A_1$

6. Conclusions

Machine tool evaluation and selection for manufacturing companies is a time-consuming, complex decision-making problem as incorrect selection can have a negative effect on the overall performance of a manufacturing system and eventually lead to quality and productivity losses. From the discussion in this paper, some managerial implications can be derived from the proposed approaches. First, it is flexible, so decision makers are able to express their ratings over the alternatives and the criteria weights in linguistic terms. Fuzzy logic is a natural approach to explicitly handle the decision makers' judgments. Second, in the application of the fuzzy model, to ensure better informed decisions, it is ideal that the decision information be provided by a group of experts rather than a single decision-maker. GDM allows engineers and managers to assess the potential strengths and weaknesses of a problem and apply their expert knowledge and technological understanding. In addition, VIKOR is an appropriate MCDM method for dealing with machine tool selection problems with multiple conflicting criteria. Further, as identified in [33], managers can select suitable weights (v) to highlight different aspects according to their own priorities. If a manager wants to highlight maximum group utility, then $v = 1$ would be used. In contrast, if the manager is concerned about individual regret, then $v = 0$ would be used.

In this study, an integration of fuzzy set theory and a GDM approach was developed to extend the VIKOR method to the selection of suitable machine tool alternatives under a linguistic environment. The main contributions of this paper are summarized as follows

1) Two algorithms, which differed in the defuzzification process, were presented to solve MCGDM problems using the VIKOR method. The first algorithm was simple and carried out the defuzzification process immediately after the fuzzy aggregation. The second algorithm implemented the defuzzification process after the calculation of the fuzzy index values and therefore involved more uncertainty.
2) A general MCGDM framework for machine tool selection based on the two algorithms was presented. As mentioned, this framework allowed for the incorporation of qualitative information expressed in linguistic terms and considered the aggregation of individual preferences.
3) An application example for CNC machine selection at a Pakistani machine tool factory was given to verify the proposed approach. A comparative study demonstrated the distinct characteristics of the two algorithms. It was shown that both algorithms could be used to obtain a better understanding of the solution.

Although the proposed fuzzy VIKOR based framework has some advantages when seeking to solve machine tool selection problems, there are still some limitations left for further research. In this paper, the criteria weights were supplied directly by the decision makers using the linguistic variables. This was simple and easy, but the interdependence and feedback relationships between the criteria were not considered. When such relationships are explicitly involved, some further methods such as DEMATEL and ANP may need to be used. Second, the defuzzification and ranking methods have been very important issues in fuzzy VIKOR methods and other fuzzy MCDM approaches. Since various approaches can be applied, the design of a suitable benchmark for a given decision problem has yet to be addressed. In addition, no criterion has been identified for choosing group decision making defuzzification methods. This issue and the benchmark issue of the framework (the overall algorithm) are the two remaining areas to be resolved in the future. Third, the differing knowledge backgrounds, experience and potentially conflicting interests of the group members led to the necessity for a consensus reaching process before the individual preferences aggregation. Therefore, under a GDM setting, a consensus reaching process could be incorporated into the fuzzy VIKOR method to develop a more robust version.

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