Date:

Growth Model

Electrical Circuit :

$$\rightarrow \frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}$$

Temperature :

$$\rightarrow \frac{d\bar{T}}{dt} + k\bar{T} = k\bar{T}_m$$

Growth & Decay :

$$\rightarrow \frac{dN}{dt} - kN = 0$$

7.1

$N(0) = 20000$

$k = 0.05$

$\therefore \frac{dN}{dt} - kN = 0$

$\frac{dN}{dt} = kN$

$\int \frac{dN}{N} = \int k\,dt$

$\ln(N) = kt + C$

$N(t) = Ce^{0.05t}$

$N(0) = Ce^{0.05(0)}$

$20000 = Ce^{0}$

$N(t) = 20000 \times e^{0.05t}$

$N(3) = 20000 \times e^{0.05(3)}$

$N(3) = 23286.69$

$2N_0 = 20k \times e^{0.05t}$

$40000 = e^{0.05t}$

20000

$2 = e^{0.05t}$

$0.6931 = 0.05t$

$t = 13.86$

Date:

7.2

$N(0) = 5000$

$N(7) = ?$

$k_1 = 0.085 \quad [1^{st} \rightarrow 4^{th}]$

$k_2 = 0.0092 \quad [5^{th} \rightarrow 7^{th}]$

$\therefore \frac{dN}{dt} - kN = 0$

$N(t) = Ce^{0.085t}$

$N(0) = Ce^{0}$

$5000 = C$

$N(t) = 5000\, e^{0.085t}$

$N(4) = 7024.74$

$N(t) = Ce^{0.0092t}$

$N(4) = Ce^{0.0092(4)}$

$7024.74 = C\, e^{0.0092(4)}$

$C = 4852.23$

$N(t) = 4852.23\, e^{0.0925t}$

$N(7) = 9271.44$

7.3 $k = ?$

$t = 6$

$\hookrightarrow 2N_0$

$\frac{dN}{dt} - kN = 0$

$N(t) = Ce^{+kt}$

$N(0) = Ce^{0}$

$C = N_0$

$N(t) = N_0 e^{kt}$

$2N_0 = N_0 e^{k(6)}$

$2 = e^{6k}$

$k = 0.1155$

Date:

7.4 $N(t) = Ce^{kt}$

$1000 = Ce^{k}$

$3000 = Ce^{k(4)}$

~~$\frac{3000 = Ce^{4k}}{1000 = Ce^{4k}}$~~

~~$3 = e^{-3k}$~~

~~$\ln(3) = -3k$~~

~~$\frac{1}{3} = e^{3k}$~~

$\frac{3000}{1000} = \frac{Ce^{4k}}{Ce^{3k}}$

$3 = e^{3k}$

$\ln(3) = 3k$

$k = \frac{1}{3}\ln(3)$

~~$N(t) = 1000e^{-0.366}$~~

$N(t) = Ce^{-0.366}$

$1000 \times e^{0.366} = C$

$N(t) = 694e^{-0.366}$

b) $N(0) = ?$

Date

7.5

At $t=2$, $\rightarrow$ $N(t)$ is double

At $t=3 \rightarrow N(t) = 20{,}000$

$N(t) = Ce^{kt}$

At $t = 0$

$N(0) = Ce^{0}$

$C = N_0$

$N(t) = N_0 e^{kt}$

$2N_0 = N_0 e^{k2}$

$2 = e^{2k}$

$\ln(2) = 2k$

$k = 0.347$

$N(t) = N_0 e^{0.347t}$

$N(3) = N_0 e^{0.347(3)}$

$N_0 = 7062$

1.6

$N(t) = Ce^{kt}$

$N(0) = Ce^{0}$

$50 = C$

$N(t) = 50e^{kt}$ ——— (A)

$N(2) = 50e^{2k}$

$45 = 50e^{2k}$

$k = -0.053$

$N(t) = 50e^{-0.053t}$

$50 - \frac{50}{10}$

Date:

7.8

$$\because \frac{d\bar{T}}{dt} + k\bar{T} = k\bar{T}_m$$

$$\bar{T} = Ce^{-kt}$$

$$\bar{T}(0) = Ce^{0}$$

$$C = 100$$

$$\bar{T} = 100\, e^{-kt}$$

$$\bar{T}(20) = 100\, e^{-k(20)}$$

$$50 = 100\, e^{-20k}$$

$$k = 0.035$$

$$\bar{T} = 100\, e^{-0.035t}$$

$$25 = 100\, e^{-0.035t}$$

$$t = 39.6$$

7.9

$t = 0 \rightarrow \bar{T} = 50$

$$\frac{d\bar{T}}{dt} + k\bar{T} = 100k$$

$$\frac{d\bar{T}}{dt} = 100k - kt$$

$$\int d\bar{T} = \int k(100 - t)\, dt$$

$$\bar{T} = k(100t -$$

$$\frac{d\bar{T}}{dt} = -k(100 - \bar{T})$$

$$\frac{d\bar{T}}{100 - \bar{T}} = -k\, dt$$

$$-\ln(100 - \bar{T}) = -kt + c$$

$$-100 + \bar{T} = Ce^{-kt}$$

$$\bar{T} = Ce^{-kt} + 100$$

$Ce^{-20k} = -15$

$Ce^{-10k} = -30$

$e^{-10k} = \frac{1}{2}$

$-10k = \ln\left(\frac{1}{2}\right)$

$k = \ln\left(\frac{1}{2}\right)\left(\frac{-1}{10}\right)$

$k = 0.069$

7.19

$\because \frac{d\bar{I}}{dt} + \frac{R\bar{I}}{L} = \frac{E}{L}$

$E = 5$

$R = 50$

$L = 1$

$\frac{d\bar{I}}{dt} + 50\bar{I} = 5$

$\frac{d\bar{I}}{dt} = 5 - 50\bar{I}$

$\frac{d\bar{I}}{dt} = 5(1 - 10\bar{I})$

$\frac{d\bar{I}}{5(1-10\bar{I})} = dt$

$\frac{-1}{10}\ln(1-10\bar{I}) = 5t + C$

$-1 + 10\bar{I} = 50t + Ce^{50t}$

$10\bar{I} = Ce^{50t} + 1$

$\bar{I} =$

Date:

$$\bar{I} = \frac{1}{10} + Ce^{-50t}$$

$$\frac{d\bar{I}}{dt} + 20\bar{I} = 6\sin 2t$$

$$IF = e^{20t}$$

$$e^{20t}\bar{I} + 20\bar{I}e^{20t} = 6e^{20t}\sin 2t$$

$$\frac{d}{dt}\left(\bar{I}e^{20t}\right) = 6e^{20t}\sin 2t$$

$$\bar{I}e^{20t} = \int 6e^{20t}\sin 2t\, dt$$

Integration by parts

Let $u = \sin 2t$

$du = 2\cos 2t\, dt$

$dv = e^{20t}\, dt$

$v = \frac{e^{20t}}{20}$

$$\int e^{20t}\sin 2t\, dt = \sin 2t\,\frac{e^{20t}}{20} - \int \frac{e^{20t}}{20}\,2\cos 2t\, dt$$

$$= \sin 2t\,\frac{e^{20t}}{20} - \frac{1}{10}\int \cos 2t\, e^{20t}\, dt$$

Let $u = \cos 2t$

$du = -2\sin 2t\, dt$

$dv = e^{20t}\, dt$

$v = \frac{e^{20t}}{20}$

$$\int e^{20t}\sin 2t = \frac{5e^{20t}\sin 2t}{601}$$

$$= uv - \int v\,du$$

$$\int e^{20t}\sin 2t\,dt = \sin 2t\,\frac{e^{20t}}{20} - \int \frac{e^{20t}}{20}\cdot 2\cos 2t\,dt$$

$$\int e^{20t}\sin 2t\,dt = \frac{e^{20t}}{20}\sin 2t - \frac{1}{10}\int e^{20t}\cos 2t\,dt$$

Let $u = \cos 2t$ $\qquad dv = e^{20t}$

$du = -2\sin 2t$ $\qquad v = e^{20t}/20$

$$\int e^{20t}\cos 2t\,dt = \frac{e^{20t}}{20}\cos 2t + \int \frac{e^{20t}}{20}\cdot\frac{1}{10}(+2\sin 2t)\,dt$$

$$\int e^{20t}\cos 2t\,dt = \frac{e^{20t}}{20}\cos 2t + \frac{1}{10}\int e^{20t}\sin 2t\,dt$$

Date:

$$\int e^{2t}\sin 2t\,dt = \frac{e^{2t}}{20}\sin 2t - \frac{1}{200}e^{2t}\cos 2t - \frac{1}{100}\int e^{2t}\sin 2t\,dt$$

$$\left(\frac{1+1}{100}\right)\int e^{2t}\sin 2t\,dt = \frac{e^{2t}}{20}\sin 2t - \frac{e^{2t}}{200}\cos 2t$$

$$\frac{101}{100}\int e^{2t}\sin 2t\,dt = \text{//}$$

$$\int e^{2t}\sin 2t\,dt = \frac{5e^{2t}}{101}\sin 2t - \frac{e^{2t}}{202}\cos 2t$$

$$I = \frac{30e^{2t}}{101}\sin 2t - \frac{3e^{2t}}{101}\cos 2t + C$$

$$I =$$