

Name: _____

Roll Number: _____

Quiz-1

Max. Time: 20 min

Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [8] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting allowed.

i. In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

(A) True (B) **False**

ii. The equation $Ax = b$ is referred to as vector equation.

(A) True (B) **False**

iii. The equation $Ax = b$ is consistent if the augmented matrix $[A \ b]$ has a pivot position in every row.

(A) **True** (B) False

iv. If A is an $m \times n$ matrix whose columns do not span \mathbb{R}^m , then the equation $Ax = b$ is inconsistent for some b in \mathbb{R}^m .

(A) **True** (B) False

v. A homogeneous system is always consistent.

(A) **True** (B) False

vi. If \mathbf{x} is a nontrivial solution of $Ax = 0$, then every entry in \mathbf{x} is nonzero.

(A) True (B) **False**

vii. The effect of adding \mathbf{p} to a vector is to move the vector in a direction parallel to \mathbf{p} .

(A) **True** (B) False

viii. The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through \mathbf{v} parallel to \mathbf{p} .

(A) True (B) **False**

Q.2.

a) Row reduce the following matrix to reduced echelon form.

Solution:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Name: _____

Roll Number: _____

b) Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is \mathbf{b} in the plane spanned by \mathbf{a}_1 and \mathbf{a}_2 ?

Solution:

Row reduce the system $[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{b}]$ to echelon form.

$$[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{b}] = \begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{bmatrix}$$

The vector \mathbf{b} is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2\}$ when the above system is consistent, i.e. $h + 17 = 0$, which implies $h = -17$

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