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Quiz-2

Max. Time: 20 min Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [**8**] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting allowed.

i. If $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is $n \times n$, has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$.

(A) True (B) False

ii. A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

(A) True (B) False

iii. If there is a $\mathbf{b}$ in $\mathbb{R}^n$ such that $\mathbf{Ax} = \mathbf{b}$ is inconsistent, then the transformation $T: \mathbf{x} \to \mathbf{Ax}$ is not one-to-one.

(A) True (B) False

iv. The columns of the standard matrix for a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ are not the images of the columns of I_n.

(A) True (B) False

v. Not every elementary matrix is invertible.

(A) True (B) False

vi. Product of invertible matrices is invertible and is given by the product of their inverses in the reverse order.

(A) True (B) False

vii. If an $n\times n$ matrix can be row reduced to some echelon matrix such that all rows have pivot positions, then its inverse does exist.

(A) True (B) False

viii. An $n\times n$ matrix is invertible if it has at most n pivots.

(A) True (B) False

Q.2. [6+6]

a) Determine the standard matrix of linear transformation for the following:

i) $T: \mathbb{R}^2 \to \mathbb{R}^2$ first performs a horizontal shear that maps $\mathbf{e_2}$ to $\mathbf{e_2} - \mathbf{0.5e_1}$ (but leaves $\mathbf{e_1}$ unchanged) and then reflects the result through the **vertical** $\mathbf{x_2}$ axis.

Solution: $T(\mathbf{e_1}) = -\mathbf{e_1}$ and $T(\mathbf{e_2}) = \mathbf{e_2} + \mathbf{0.5e_1} = \begin{bmatrix}0\\1\end{bmatrix} + \mathbf{0.5}\begin{bmatrix}1\\0\end{bmatrix}$

Therefore, standard matrix for linear transformation is: $[T(\mathbf{e_1}) \;\; T(\mathbf{e_2})] = \begin{bmatrix}-1 & 0.5\\0 & 1\end{bmatrix}$

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ii) $T: \mathbb{R}^2 \to \mathbb{R}^2$ that does horizontal contraction by a factor of $\mathbf{0.5}$ followed by a vertical expansion of $\mathbf{1.5}$.

Solution:

$$A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

b) Determine if the following matrix is invertible.

$$A = \begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

Solution: Row reduce A to echelon form to check the number of pivot positions, which should be 4 if the matrix is invertible.

$$A = \begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since A has 4 pivots, it is invertible.

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