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Quiz-2

Max. Time: 20 min Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [**8**] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting allowed.

i. When two linear transformations are performed one after another, the combined effect may not always be linear.

(A) True (B) False

ii. A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector **x** in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

(A) True (B) False

iii. If **A** is 3×2 matrix, then the transformation from **x** to **Ax** cannot be one-to-one.

(A) True (B) False

iv. The columns of the standard matrix for a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ are the images of the columns of I_n.

(A) True (B) False

v. Every elementary matrix is not invertible.

(A) True (B) False

vi. Product of invertible matrices is invertible and is given by the product of their inverses in the same order.

(A) True (B) False

vii. If a matrix cannot be row reduced to identity matrix, then its inverse does not exist.

(A) True (B) False

viii. An $n \times n$ matrix is invertible if it has at most n pivots.

(A) True (B) False

Q.2. [6+6]

a) Determine the standard matrix of linear transformation for the following:

i) $T: \mathbb{R}^2 \to \mathbb{R}^2$first reflects points through the horizontal $x_1$axis and then reflects points through the line $x_2 = x_1$.

Solution: $T(\mathbf{e_1}) = \mathbf{e_2}, T(\mathbf{e_2}) = -\mathbf{e_1}$. Therefore, $[T(\mathbf{e_1}) \;\; T(\mathbf{e_2})] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

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ii) $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a horizontal shear transformation that leaves the unit vector $\mathbf{e_1}$ for horizontal axis unchanged and maps the unit vector for the vertical axis $\mathbf{e_2}$ to $\mathbf{e_2} + \mathbf{3e_1}$

Solution:

$$T(\mathbf{e_1}) = \mathbf{e_1};\ ,T(\mathbf{e_2}) = \begin{bmatrix}0\\1\end{bmatrix} + \mathbf{3}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$$

$$[T(\mathbf{e_1})\ \ T(\mathbf{e_2})] = \begin{bmatrix}1 & 3\\0 & 0\end{bmatrix}$$

b) Determine if the following matrix is invertible.

$$A = \begin{bmatrix}-1 & -3 & 0 & 1\\3 & 5 & 8 & -3\\-2 & -6 & 3 & 2\\0 & -1 & 2 & 1\end{bmatrix}$$

Solution: Row reduce A to echelon form to check the number of pivot positions, which should be 4 if the matrix is invertible.

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$$A = \begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since A has 4 pivots, it is invertible.