General Solution of Differential Equations

Example 1: Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} = 30$$

Further, calculate integration constants with initial conditions x(0)=x'(0)=0. Give specific solution as well.

Solution

Step 1: First Integration (with respect to t)

Start with the second derivative equation:

$$\frac{d^2x}{dt^2} = 30$$

Now, integrate it once with respect to t to find the first derivative:

$$\frac{dx}{dt} = \int 30\,dt$$

$$\frac{dx}{dt} = 30t + C_1$$

Here, C_1 is the constant of integration.

Step 2: Second Integration (with respect to t)

Next, integrate the first derivative with respect to t to find the original function $x(t)$:

$$x(t) = \int (30t + C_1)\,dt$$

$$x(t) = \int 30t\,dt + \int C_1\,dt$$

Now, integrate each term separately:

$$x(t) = 30\int t\,dt + C_1\int 1\,dt$$

$$x(t) = 30\left(\frac{t^2}{2}\right) + C_1 t + C_2$$

In this step, we introduced another constant of integration, C_2.

So, the general solution to the differential equation is:

$$x(t) = 15t^2 + C_1 t + C_2$$

Here, C_1 and C_2 are arbitrary constants.

To find out specific solution:

We will find the values of C_1 and C_2 based on the initial conditions:

$$\begin{aligned} \text{Initial Condition 1: } x(0) &= 0 \\ x(0) &= 15(0)^2 + C_1(0) + C_2 \\ 0 &= 0 + 0 + C_2 \\ C_2 &= 0 \end{aligned}$$

$$\begin{aligned} \text{Initial Condition 2: } x'(0) &= 0 \\ x'(t) &= 30t + C_1 \\ 0 &= 30(0) + C_1 \\ C_1 &= 0 \end{aligned}$$

So, based on the initial conditions $x(0) = 0$ and $x'(0) = 0$, we have found that $C_1 = 0$ and $C_2 = 0$. Therefore, the particular solution to the differential equation is:

$$x(t) = 15t^2$$

Example: Calculate the general solution for the distance x(t) of a bicycle that accelerates at 1m/sec/sec from an initial velocity of 4 m/s for 10 seconds.

Step 1: First Integration (with respect to t)

Given $x'' = 1$, we integrate once to find the velocity function:

$$x' = \int 1\, dt = t + C_1$$

Using the initial velocity condition $x'(0) = 4\,\text{m/s}$:

$$4 = 0 + C_1$$

So, $C_1 = 4$.

Step 2: Second Integration (with respect to t)

Now, we integrate the velocity function to find the displacement function:

$$x = \int (t + C_1)\, dt = \frac{t^2}{2} + C_1 t + C_2$$

Using the initial condition $x(0) = 0$ (assuming the bicycle starts from rest at $t = 0$):

$$0 = 0 + 0 + C_2$$

So, $C_2 = 0$.

Therefore, the general solution for the distance $x(t)$ is:

$$x(t) = \frac{t^2}{2} + 4t$$

Here, $C_1 = 4$ and $C_2 = 0$ are the integration constants, and this equation represents the position of the bicycle as a function of time t.

Verification of solution of the ODE:

Example 2:

We know the general solution of the equation $\frac{d^2x}{dt^2} = 40$ is $x(t) = 20t^2 + 10t + 20$.

To verify that it is the solution of the given differential equation, we can differentiate $x(t)$ twice with respect to t:

$$\frac{dx}{dt} = \frac{d}{dt}(20t^2 + 10t + 20)$$

$$\frac{dx}{dt} = 20\frac{d}{dt}(t^2) + 10\frac{d}{dt}(t) + \frac{d}{dt}(20)$$

Evaluating the derivatives on the right-hand side:

$$\frac{dx}{dt} = 20(2t) + 10(1) + 0$$

Simplifying further:

$$\frac{dx}{dt} = 40t + 10$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}(40t + 10)$$

$$\frac{d^2x}{dt^2} = 40$$

Example 3:

We know the general solution of the equation $\frac{dy}{dx} = xy^{1/2}$ is $y = \frac{1}{16}x^4$.

To verify that it is the solution of the given differential equation, we can differentiate $y(x)$ with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{16}x^4\right)$$

Using the linearity of differentiation, we have:

$$\frac{dy}{dx} = \frac{1}{16}\frac{d}{dx}(x^4)$$

Evaluating the derivative on the right-hand side:

$$\frac{dy}{dx} = \frac{1}{16} \cdot 4x^3$$

Simplifying further:

$$\frac{dy}{dx} = \frac{1}{4}x^3$$

Now, we will compare this result with the given equation $\frac{dy}{dx} = xy^{1/2}$.

LHS: $\frac{dy}{dx} = \frac{1}{4}x^3$

RHS: $xy^{1/2} = x\left(\frac{1}{16}x^4\right)^{1/2} = x\left(\frac{1}{4}x^2\right) = \frac{1}{4}x^3$

Since the LHS is equal to the RHS, we have successfully verified that $y(x) = \frac{1}{16}x^4$ is a solution of the given differential equation.

Example 4 :

Verify that $x(t) = e^{3t}$ is a solution of the equation $\frac{d^3x}{dt^3} - 9\frac{d^2x}{dt^2} = 0$ or not?

Let's calculate the derivatives:
First derivative:

$$\frac{dx}{dt} = \frac{d}{dt}(e^{3t}) = 3e^{3t}$$

Second derivative:

$$\frac{d^2x}{dt^2} = \frac{d}{dt}(3e^{3t}) = 9e^{3t}$$

Third derivative:

$$\frac{d^3x}{dt^3} = \frac{d}{dt}(9e^{3t}) = 27e^{3t}$$

Now, we'll evaluate the LHS of the equation:

$$\frac{d^3x}{dt^3} - 9\frac{d^2x}{dt^2} = 27e^{3t} - 9 \cdot 9e^{3t}$$

Simplifying:

$$27e^{3t} - 81e^{3t} = -54e^{3t}$$

So, the LHS of the equation is $-54e^{3t}$.

$LHS \neq RHS$

Example 5:

Verify that $x(t) = \ln(t)$ is a solution of the equation $t\frac{dx}{dt} = 1$ or not?.

Let's calculate the first derivative:

First derivative:

$$\frac{dx}{dt} = \frac{d}{dt}(\ln(t)) = \frac{1}{t}$$

Now, we'll evaluate the LHS of the equation:

$$t\frac{dx}{dt} = t\left(\frac{1}{t}\right) = 1$$

So, the LHS of the equation is 1.

Example 6

To check whether $y = 3e^{-2x}$ is a solution of the differential equation $y' + 2y = 0$, we follow these steps:

1. Start with the function $y = 3e^{-2x}$.

2. Find the derivative y' of y with respect to x:

$$y' = \frac{d}{dx}(3e^{-2x}) = -6e^{-2x}$$

3. Now, apply the derivative y' to the left-hand side of the differential equation $y' + 2y = 0$:

$$(y' + 2y) = (-6e^{-2x}) + 2(3e^{-2x})$$

4. Simplify the expression:

$$(-6e^{-2x}) + 2(3e^{-2x}) = -6e^{-2x} + 6e^{-2x} = 0$$

The left-hand side of the differential equation simplifies to zero.

Since the left-hand side equals zero, which matches the right-hand side of the differential equation $y' + 2y = 0$, we can conclude that $y = 3e^{-2x}$ is indeed a solution to the given differential equation.

The solution $y = 3e^{-2x}$ satisfies the differential equation $y' + 2y = 0$.

Example 7

To check whether $y = e^{3x}$ is a solution of the differential equation $y'' - 9y = 0$, we'll follow these steps:

1. Start with the function $y = e^{3x}$.

2. Find the first and second derivatives y' and y'' of y with respect to x:

$$y' = \frac{d}{dx}(e^{3x}) = 3e^{3x}$$

$$y'' = \frac{d^2}{dx^2}(e^{3x}) = 9e^{3x}$$

3. Now, apply the derivatives y'' and y to the left-hand side of the differential equation $y'' - 9y = 0$:

$$(y'' - 9y) = (9e^{3x} - 9e^{3x})$$

4. Simplify the expression:

$$(9e^{3x} - 9e^{3x}) = 0$$

The left-hand side of the differential equation simplifies to zero.

Since the left-hand side equals zero, which matches the right-hand side of the differential equation $y'' - 9y = 0$, we can conclude that $y = e^{3x}$ is indeed a solution to the given differential equation.

The solution $y = e^{3x}$ satisfies the differential equation $y'' - 9y = 0$.

Example 8

To check whether $y = e^{-2x}$ is a solution of the differential equation $y'' + 4y' + 4y = 0$, we'll follow these steps:

1. Start with the function $y = e^{-2x}$.

2. Find the first and second derivatives y' and y'' of y with respect to x:

$$y' = \frac{d}{dx}(e^{-2x}) = -2e^{-2x}$$

$$y'' = \frac{d^2}{dx^2}(e^{-2x}) = 4e^{-2x}$$

3. Now, apply the derivatives y'' and y' to the left-hand side of the differential equation $y'' + 4y' + 4y = 0$:

$$(y'' + 4y' + 4y) = (4e^{-2x} + 4(-2e^{-2x}) + 4e^{-2x})$$

4. Simplify the expression:

$$(4e^{-2x} + 4(-2e^{-2x}) + 4e^{-2x}) = 4e^{-2x} - 8e^{-2x} + 4e^{-2x} = 0$$

The left-hand side of the differential equation simplifies to zero.

Since the left-hand side equals zero, which matches the right-hand side of the differential equation $y'' + 4y' + 4y = 0$, we can conclude that $y = e^{-2x}$ is indeed a solution to the given differential equation.

The solution $y = e^{-2x}$ satisfies the differential equation $y'' + 4y' + 4y = 0$.

Example 9

To solve the third-order differential equation $\frac{d^3x}{dt^3} = 6$ by integrating it thrice step by step and finding the general solution for $x(t)$ using integration constants like C_1, C_2, etc., you can follow these steps:

1. Start with the third-order differential equation:

$$\frac{d^3x}{dt^3} = 6$$

2. Integrate it once with respect to t to find the second derivative $x''(t)$:

$$x''(t) = \int 6\,dt$$

Integrating with respect to t:

$$x''(t) = 6t + C_1$$

Here, C_1 is the first integration constant.

3. Integrate $x''(t)$ once more to find the first derivative $x'(t)$:

$$x'(t) = \int (6t + C_1)\,dt$$

Integrating with respect to t:

$$x'(t) = 3t^2 + C_1 t + C_2$$

Here, C_2 is the second integration constant.

4. Finally, integrate $x'(t)$ to find $x(t)$:

$$x(t) = \int (3t^2 + C_1 t + C_2)\,dt$$

Integrating with respect to t:

$$x(t) = t^3 + \frac{C_1}{2}t^2 + C_2 t + C_3$$

Here, C_3 is the third integration constant.
So, the general solution for $x(t)$ is:

$$x(t) = t^3 + \frac{C_1}{2}t^2 + C_2t + C_3$$

Example 10

To find the specific solution for the differential equation $\frac{d^3x}{dt^3} = 6$ with initial conditions $x(0) = x'(0) = x''(0) = 0$, we start from the general solution:

$$x(t) = t^3 + \frac{C_1}{2}t^2 + C_2 t + C_3$$

We apply the initial conditions as follows:

1. $x(0) = 0$:

$$0 = 0^3 + \frac{C_1}{2} \cdot 0^2 + C_2 \cdot 0 + C_3$$

This simplifies to:

$$C_3 = 0$$

2. $x'(0) = 0$:

$$0 = 3 \cdot 0^2 + C_1 \cdot 0 + C_2$$

This simplifies to:

$$C_2 = 0$$

3. $x''(0) = 0$:

$$0 = 6 \cdot 0 + C_1$$

This simplifies to:

$$C_1 = 0$$

So, all three integration constants C_1, C_2, and C_3 are determined to be zero based on the given initial conditions.

Therefore, the specific solution for the differential equation with the initial conditions $x(0) = x'(0) = x''(0) = 0$ is:

$$x(t) = t^3$$

This is the unique solution that satisfies the given initial conditions.

Example 11

To find the specific solution for the differential equation $\frac{d^3x}{dt^3} = 6$ with new initial conditions $x(0) = 10$, $x'(0) = 20$, and $x''(0) = 30$, we start from the general solution:

$$x(t) = t^3 + \frac{C_1}{2}t^2 + C_2 t + C_3$$

We apply the new initial conditions as follows:

1. $x(0) = 10$:

$$10 = 0^3 + \frac{C_1}{2} \cdot 0^2 + C_2 \cdot 0 + C_3$$

This simplifies to:

$$C_3 = 10$$

2. $x'(0) = 20$:

$$20 = 3 \cdot 0^2 + C_1 \cdot 0 + C_2$$

This simplifies to:

$$C_2 = 20$$

3. $x''(0) = 30$:

$$30 = 6 \cdot 0 + C_1$$

This simplifies to:

$$C_1 = 30$$

So, we have determined the values of the integration constants based on the new initial conditions:

$C_1 = 30$, $C_2 = 20$, and $C_3 = 10$.

Therefore, the specific solution for the differential equation with the new initial conditions $x(0) = 10$, $x'(0) = 20$, and $x''(0) = 30$ is:

$$x(t) = t^3 + 15t^2 + 20t + 10$$

Example 12

To find the specific solution for the differential equation $\frac{d^3x}{dt^3} = 6$ with new initial conditions $x(0) = 10$, $x'(0) = 20$, and $x''(0) = 0$, we start from the general solution:

$$x(t) = t^3 + \frac{C_1}{2}t^2 + C_2t + C_3$$

We apply the new initial conditions as follows:

1. $x(0) = 10$:

$$10 = 0^3 + \frac{C_1}{2} \cdot 0^2 + C_2 \cdot 0 + C_3$$

This simplifies to:

$$C_3 = 10$$

2. $x'(0) = 20$:

$$20 = 3 \cdot 0^2 + C_1 \cdot 0 + C_2$$

This simplifies to:

$$C_2 = 20$$

3. $x''(0) = 0$:

$$0 = 6 \cdot 0 + C_1$$

This simplifies to:

$$C_1 = 0$$

So, we have determined the values of the integration constants based on the new initial conditions:

$C_1 = 0$, $C_2 = 20$, and $C_3 = 10$.

Therefore, the specific solution for the differential equation with the new initial conditions $x(0) = 10$, $x'(0) =$

20, and $x''(0) = 0$ is:

$$x(t) = t^3 + 10t + 10$$

Example 13

To solve the second-order differential equation $\frac{d^2x}{dt^2} = te^t$ and find $x(t)$, we integrate it step by step:

1. Start with the differential equation:

$$\frac{d^2x}{dt^2} = te^t$$

2. Integrate once to find $x'(t)$:

$$x'(t) = \int te^t \, dt$$

Using integration by parts:

$$x'(t) = te^t - e^t + C_1$$

Here, C_1 is the integration constant.

3. Integrate $x'(t)$ once more to find $x(t)$:

$$x(t) = \int (te^t - e^t + C_1) \, dt$$

Simplifying:

$$x(t) = \frac{1}{2}t^2e^t - e^t + (C_1t + C_2)$$

Here, C_2 is another integration constant.

So, the solution for $x(t)$ is:

$$x(t) = \frac{1}{2}t^2e^t - e^t + C_1t + C_2$$

This is the general solution with C_1 and C_2 as arbitrary constants.

Example 14: In the above example, apply the given initial conditions and find out specific solution: x(0)=10; x'(0)=0

Assume the general solution of some differential equation is:

$$x(t) = \frac{1}{2}t^2e^t - e^t + C_1t + C_2$$

Given that $x'(t) = te^t - e^t + C_1$.

Now, apply the initial conditions:

1. $x(0) = 10$:

$$\frac{1}{2}(0)^2e^0 - e^0 + C_1(0) + C_2 = -1 + C_2 = 10$$

Solving for C_2:

$$C_2 = 10 + 1 = 11$$

2. $x'(0) = 0$:

$$(0)e^0 - e^0 + C_1 = -1 + C_1 = 0$$

Solving for C_1:

$$C_1 = 1$$

The specific solution of the differential equation, with the given initial conditions, is:

$$x(t) = \frac{1}{2}t^2e^t - e^t + t + 11$$

Problems related to Velocity and Acceleration

Example 15

A car travelling at 10 m/s accelerates uniformly at 2m/s/s.

(a) Set up the differential equation for the problem.

(b) Write down the initial conditions.

(c) Find out the expression for x'(t)

(d) Evaluate x'(t) at t= 5 sec.

Solution: To solve the differential equation

$$\frac{d^2x}{dt^2} = 2$$

for $x'(t)$:

Step 1: Integrate the equation with respect to t once to obtain the first derivative $x'(t)$. Remember that when you integrate, you introduce an integration constant, which we'll call C_1:

$$\frac{dx}{dt} = \int 2\,dt + C_1$$

This simplifies to:

$$x'(t) = 2t + C_1$$

Step 2: To find the particular solution, you'll need initial conditions. You're given $x'(0) = 10$, which allows you to determine the value of C_1. Substitute $t = 0$ and $x'(t) = 10$ into the equation:

$$10 = 2(0) + C_1$$

Solving for C_1:

$$C_1 = 10$$

Step 3: Now that you know the value of C_1, you can write the specific solution for $x'(t)$:

$$x'(t) = 2t + 10$$

So, the particular solution for $x'(t)$ is $x'(t) = 2t + 10$, assuming the initial condition $x'(0) = 10$

Example 16: A train slows down from 22.2 m/s with a uniform retardation of 2 m/s/s. How long will it take to attain a speed of 5.6m/s.

(a) Set up the differential equation for the problem.

(b) Write down the initial conditions.

(c) Find out the expression for x'(t)

(d) How long will it take to attain a speed of 5.6m/s?

To solve the differential equation

$$\frac{d^2x}{dt^2} = -2$$

for $x'(t)$:

Step 1: Integrate the equation with respect to t once to obtain the first derivative $x'(t)$. Remember that when you integrate, you introduce an integration constant, which we'll call C_1:

$$\frac{dx}{dt} = \int (-2)\, dt + C_1$$

This simplifies to:

$$x'(t) = -2t + C_1$$

Step 2: To find the particular solution, you'll need initial conditions. You're given $x'(0) = 22.2$, which allows you to determine the value of C_1. Substitute $t = 0$ and $x'(t) = 22.2$ into the equation:

$$22.2 = -2(0) + C_1$$

Solving for C_1:

$$C_1 = 22.2$$

Step 3: Now that you know the value of C_1, you can write the specific solution for $x'(t)$:

$$x'(t) = -2t + 22.2$$

So, the particular solution for $x'(t)$ is $x'(t) = -2t + 22.2$, assuming the initial condition $x'(0) = 22.2$.

Calculate t.....

Example 17

A stone is dropped the top of the tower. The stone hits the ground after 5 seconds. Do the following:

(a) Set up the differential equation for the problem.

(b) Write down the initial conditions.

(c) Find out the expression for x'(t) and x(t)

(d) Find height of the tower [Hint: find x(5)]

(e) Find the velocity with which the stone hits the ground. [Hint: find x'(5)]

To solve the differential equation

$$\frac{d^2x}{dt^2} = 10$$

for $x'(t)$:

Step 1: Integrate the equation with respect to t once to obtain the first derivative $x'(t)$. Remember that when you integrate, you introduce an integration constant, which we'll call C_1:

$$\frac{dx}{dt} = \int 10\,dt + C_1$$

This simplifies to:

$$x'(t) = 10t + C_1$$

Next, to find the general solution for $x(t)$ with C_2 as the second integration constant, we integrate $x'(t)$ with respect to t:

$$x(t) = \int (10t + C_1)\,dt + C_2$$

This simplifies to:

$$x(t) = 5t^2 + C_1 t + C_2$$

Step 2: To find the particular solution, you'll need initial conditions. You're given $x(0) = x'(0) = 0$, which allows you to determine the values of C_1 and C_2.

Substitute $t = 0$ into the equations:

$$x(0) = 5(0)^2 + C_1(0) + C_2 = 0$$

This implies $C_2 = 0$.

$$x'(0) = 10(0) + C_1 = 0$$

Solving for C_1:

$$C_1 = 0$$

Step 3: Now that you know the values of C_1 and C_2, you can write the specific solution for $x(t)$:

$$x(t) = 5t^2$$

So, the particular solution for $x(t)$ is $x(t) = 5t^2$, assuming the initial conditions $x(0) = x'(0) = 0$.

Example: A boy throws a ball vertically up. It returns to the ground after 5 sec. Find the maximum height reached by the ball.

To solve the differential equation

$$\frac{d^2x}{dt^2} = 10$$

for $x'(t)$:

Step 1: Integrate the equation with respect to t once to obtain the first derivative $x'(t)$. Remember that when you integrate, you introduce an integration constant, which we'll call C_1:

$$\frac{dx}{dt} = \int 10\, dt + C_1$$

This simplifies to:

$$x'(t) = 10t + C_1$$

Next, to find the general solution for $x(t)$ with C_2 as the second integration constant, we integrate $x'(t)$ with respect to t:

$$x(t) = \int (10t + C_1)\, dt + C_2$$

This simplifies to:

$$x(t) = 5t^2 + C_1 t + C_2$$

Step 2: To find the particular solution, you'll need initial conditions. You're given $x(0) = x'(0) = 0$, which allows you to determine the values of C_1 and C_2.

Substitute $t = 0$ into the equations:

$$x(0) = 5(0)^2 + C_1(0) + C_2 = 0$$

This implies $C_2 = 0$.

$$x'(0) = 10(0) + C_1 = 0$$

Solving for C_1:

$$C_1 = 0$$

Step 3: Now that you know the values of C_1 and C_2, you can write the specific solution for $x(t)$:

$$x(t) = 5t^2$$

So, the particular solution for $x(t)$ is $x(t) = 5t^2$, assuming the initial conditions $x(0) = x'(0) = 0$.

Separation of variables technique

Example

To solve the differential equation $\frac{dx}{dt} = x^2 t$ using separation of variables, we have:

$$\frac{1}{x^2} dx = t dt$$

Now, integrating both sides:

$$\int \frac{1}{x^2} dx = \int t dt$$

Solving the left integral:

$$-\frac{1}{x} = \frac{1}{2} t^2 + C$$

Solving for x:

$$x(t) = -\frac{1}{\frac{1}{2} t^2 + C}$$

This is the general solution to the differential equation.

Example

To solve the differential equation $\frac{dx}{dt} = x^2 t$ with the initial condition $x(0) = 1$, we find the value of C as follows:

Given the general solution: $x(t) = -\frac{1}{\frac{1}{2}t^2 + C}$

1. Apply the initial condition $x(0) = 1$:

$$x(0) = -\frac{1}{\frac{1}{2}(0)^2 + C} = -\frac{1}{C} = 1$$

2. Solve for C:

$$-\frac{1}{C} = 1$$

Multiplying both sides by -1:

$$C = -1$$

Now, we can write the specific solution:

Specific solution: $x(t) = -\frac{1}{\frac{1}{2}t^2 - 1}$

Example

To solve the differential equation $\frac{dx}{dt} = \frac{2}{3}x$ using separation of variables, we have:

$$\frac{1}{x}dx = \frac{2}{3}dt$$

Now, integrating both sides:

$$\int \frac{1}{x}dx = \int \frac{2}{3}dt$$

Solving the left integral:

$$\ln x = \frac{2}{3}t + C$$

Solving for x by taking the exponential of both sides:

$$x = e^{\frac{2}{3}t}e^{C}$$

$$x = C_1 e^{\frac{2}{3}t]}$$

This is the general solution to the differential equation.

Exapme

To solve the differential equation $dx = \frac{1-e^t}{e^{2t}}dt$ by keeping only dx on the right side and taking the integration of both sides:

$$
\begin{aligned}
dx &= \frac{1-e^t}{e^{2t}}dt \\
x &= \int \frac{1}{e^{2t}}dt - \int \frac{e^t}{e^{2t}}dt \\
x &= \int e^{-2t}dt - \int e^{-t}dt \\
x &= -\frac{1}{2}e^{-2t} + e^{-t} + C_1 \\
x &= -\frac{1}{2}e^{-2t} + e^{-t} + C_1
\end{aligned}
$$

Example

To solve the differential equation $\frac{dy}{dx} + 2xy = 0$ using the variable separable technique, we'll rearrange the equation to separate variables and then integrate both sides.

The differential equation is: $\frac{dy}{dx} + 2xy = 0$

First, we'll move dy to the left side and dx to the right side to separate variables: $\frac{dy}{y} = -2xdx$

Next, we'll integrate both sides :

$\ln y = -x^2 + C_1$

Where C_1 is the constant of integration.

$y = e^{-x^2+C_1}$

$y = \; e^{C_1}e^{-x^2}$

$$y(x) = Ce^{-x^2}$$

Example:

Solve the differential equation $\frac{dy}{dx} + 2xy^2 = 0$ using the variable separable technique

Start with the differential equation:

$$\frac{dy}{dx} + 2xy^2 = 0$$

Rearrange the terms to separate variables:

$$\frac{dy}{y^2} = -2xdx$$

Now, let's integrate both sides simultaneously while keeping one integration constant C_1 with x:

$$-y^{-1} = -x^2 - C_1$$

$$y^{-1} = x^2 - C_1$$

$$y = \frac{1}{x^2 - C_1}$$

$$y(x) = \frac{1}{x^2 - C_1}$$

Example

To solve the differential equation $\frac{dy}{dx} = y\sin(x)$ using the variable separable technique and keeping one integration constant C_1 on the right side with the x variable, follow these steps:

Start with the differential equation:

$$\frac{dy}{dx} = y\sin(x)$$

Rearrange the terms to separate variables:

$$\frac{dy}{y} = \sin(x)dx$$

Now, let's integrate both sides

$$\ln y = -\cos(x) + C_1$$

$$y = e^{-\cos(x)+C_1}$$
$$y = e^{-\cos(x)}e_1^C$$
$$y = Ce^{-\cos(x)}$$

Example