

# Kalman filter

Motion model:

$$\begin{cases}
 x_t = x_{t-1} + V_t \cos(\theta_{t-1})dt \\
 y_t = y_{t-1} + V_t \sin(\theta_{t-1})dt \\
 \theta_t = \theta_{t-1} + \omega_t dt \\
 V_t^x = V_t \cos(\theta_{t-1} + \omega_t dt) \\
 V_t^y = V_t \sin(\theta_{t-1} + \omega_t dt) \\
 \omega_t^\theta = \omega_t
 \end{cases}$$

$$G = \begin{bmatrix}
 1 & 0 & -V_t \sin(\theta_{t-1})dt & 0 & 0 & 0 \\
 0 & 1 & V_t \cos(\theta_{t-1})dt & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & -V_t \sin(\theta_{t-1}) & 0 & 0 & 0 \\
 0 & 0 & V_t \cos(\theta_{t-1}) & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0
 \end{bmatrix}$$

$$\begin{cases}
 x_t = x_{t-1} + \frac{V_t}{\omega_t} (\sin(\theta_{t-1} + \omega_t dt) - \sin(\theta_{t-1})) \\
 y_t = y_{t-1} + \frac{V_t}{\omega_t} (-\cos(\theta_{t-1} + \omega_t dt) + \cos(\theta_{t-1})) \\
 \theta_t = \theta_{t-1} + \omega_t dt \\
 V_t^x = V_t \cos(\theta_{t-1} + \omega_t dt) \\
 V_t^y = V_t \sin(\theta_{t-1} + \omega_t dt) \\
 \omega_t^\theta = \omega_t
 \end{cases}$$

$$G = \begin{bmatrix}
 1 & 0 & \frac{V_t}{\omega_t} (\cos(\theta_{t-1} + \omega_t dt) - \cos(\theta_{t-1})) & 0 & 0 & 0 \\
 0 & 1 & \frac{V_t}{\omega_t} (\sin(\theta_{t-1} + \omega_t dt) - \sin(\theta_{t-1})) & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & -V_t \sin(\theta_{t-1}) & 0 & 0 & 0 \\
 0 & 0 & V_t \cos(\theta_{t-1}) & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0
 \end{bmatrix}$$

Sensor model:

$$h = \begin{bmatrix}
 \theta_t \\
 \frac{V_t^x}{\cos(\theta_t)} \\
 \omega_t^\theta
 \end{bmatrix}$$

$$H = \begin{bmatrix}
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & \frac{V_t^x \sin(\theta_t)}{\cos(\theta_t)^2} & \frac{1}{\cos(\theta_t)} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}$$