Test of Hypothesis using SPSS

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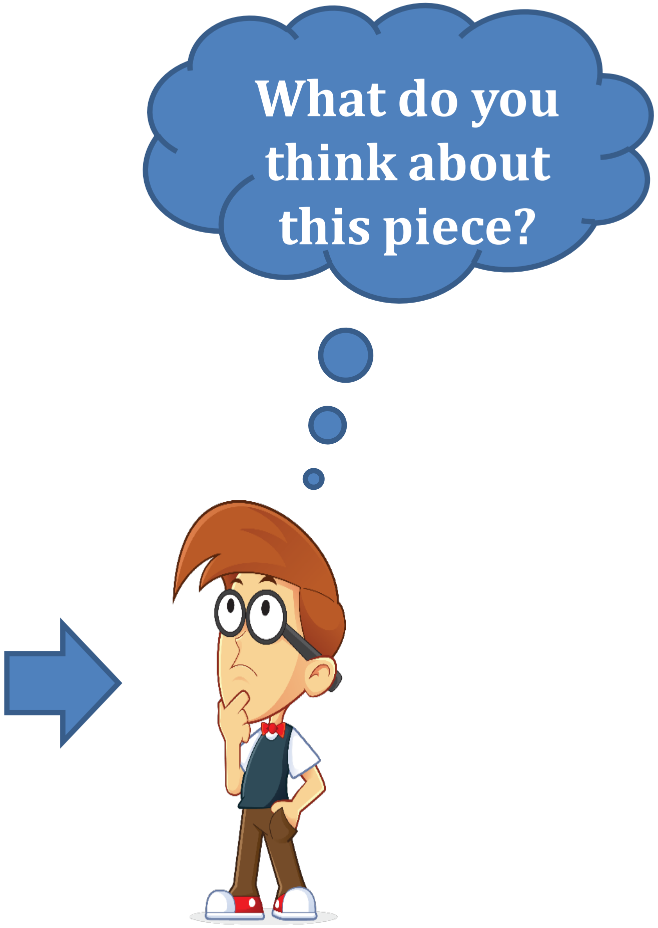
Concept Covered

- Hypothesis testing
- Errors in Hypothesis testing
- Hypothesis testing procedure
- Hypothesis testing – case studies
- t-test
- Nonparametric Test

Introduction



Statistical Hypothesis

- **Hypothesis:** Any statement regarding the parameter of a random variable is called hypothesis.

- If the hypothesis is stated in terms of population parameters (such as mean, variance etc.), the hypothesis is called **statistical hypothesis**.

- Data from a sample (which may be an experiment) are used to test the validity of the hypothesis.

- A procedure that enables us to agree (or disagree) with the statistical hypothesis is called a **test of the hypothesis**.

Hypothesis Testing

- ▷ The main purpose of statistical hypothesis testing is to choose between two competing hypotheses.

Example

- ▷ One hypothesis might claim that wages of men and women are equal, while the **alternative** might claim that men make more than women.

- ▷ Hypothesis testing start by making a set of two statements about the parameter(s) in question.
- ▷ **Null Hypothesis:** The hypothesis which is usually tested under rejection, is call null hypothesis. It is denoted by $\boldsymbol{H_0}$.
- ▷ **Alternative Hypothesis:** The alternative hypothesis is the hypothesis used in hypothesis testing that is contrary to the null hypothesis. It is denoted by $\boldsymbol{H_1}$ or $\boldsymbol{H_A}$.

Hypothesis Testing

Statistical inference

Null hypothesis

Sample

Alternative hypothesis

Assumptions

- Independence
- Normality
- Homogeneity (variances are equal)

Conceptions in Hypothesis Test

- One-sided and Two-sided tests
- Type-I error and Type-II error
- Level of Significance
- Conclusion with probability
- Means of P-value
- Significance

The Hypotheses

- **One-sided test:** A statistical test in which the alternative hypothesis specifies that the population parameter lies entirely above or below the value specified in H_0 is called a one-sided (or one-tailed) test. e.g.,

$$H_0: \mu \leq 100 \quad vs. \quad H_1 > 100$$

$$\text{or, } H_0: \mu \geq 0 \quad vs. \quad H_1 < 100$$

- **Two-sided test:** An alternative hypothesis that specifies that the parameter can lie either sides of the value specified by H_0 is called a two-sided (or two-tailed) test. e.g.,

$$H_0: \mu = 100 \quad vs. \quad H_1 \neq 100$$

Errors in Hypothesis Testing

Type I error:	A type I error occurs when we incorrectly reject H_0 (i.e. we reject the null hypothesis, when H_0 is true).
Type II error:	A type II error occurs when we incorrectly fail to reject H_0 (i.e. we accept H_0 when it is not true).

Observation	**Decision**	
	Reject H_0	Do not reject H_0
H_0 is true	Type I error (α)	Decision is Correct
H_0 is false or H_1 is true	Decision is Correct Power $(1-\beta)$	Type II error (β)

Probability of Making Errors

- **Level of Significance:** The probability of rejecting the null hypothesis when it is true is called level of significance. It is the probability of Type-I error. It is denoted by α.

- **Power of the Test:** The probability of rejecting the null hypothesis when it is false is called power of the test. It is denoted by $(1-\beta)$.

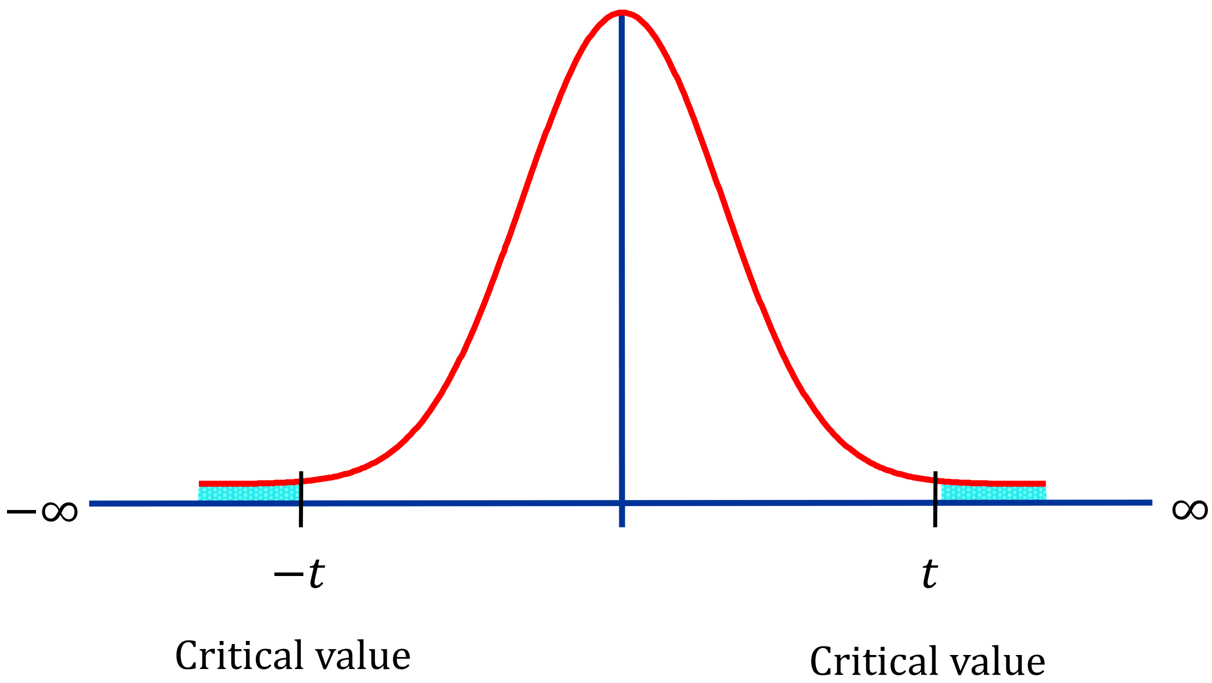
$$\therefore Power = 1 - (\text{probability of Type} - \text{II error}) = 1 - \beta$$

- ***p*-value:** A p *-value* (probability value) describes the exact significance level associated with a particular result.

  - For example: The p-value of 0.01 indicates that the test is statistically significant at 1% level.

p-value

- The p-value of a test is the smallest value of α (level of significance) for which null hypothesis can be rejected.



What does mean by *P*-value?

- If $P \leq \alpha$, H_0 is statistically significant.
  - If the data are not consistent with the null hypothesis, the difference is said to be statistically significant.
- If $P > \alpha$, No statistical significance.
  - No sufficient evidence.

5 Steps of Hypothesis test

1. Specify H_0 and H_1, the null and alternate hypothesis, and an acceptable level of $\boldsymbol{\alpha}$ ($\boldsymbol{significance\ level}$).
2. Determine an appropriate sample-based test statistics and the rejection region for the specified H_0.
3. Collect the sample data and calculate the test statistics.
4. Make a decision to either reject or fail to reject H_0.
5. Interpret the result in common language suitable for practitioners.

T-test: One sample

- Example: An outbreak of Salmonella-related illness was attributed to ice cream produced at a certain factory. Scientists measured the level of Salmonella in 9 randomly sampled batches of ice cream. The levels (in MPN/g) were: 0.593, 0.142, 0.329, 0.691, 0.231, 0.793, 0.519, 0.392, 0.418

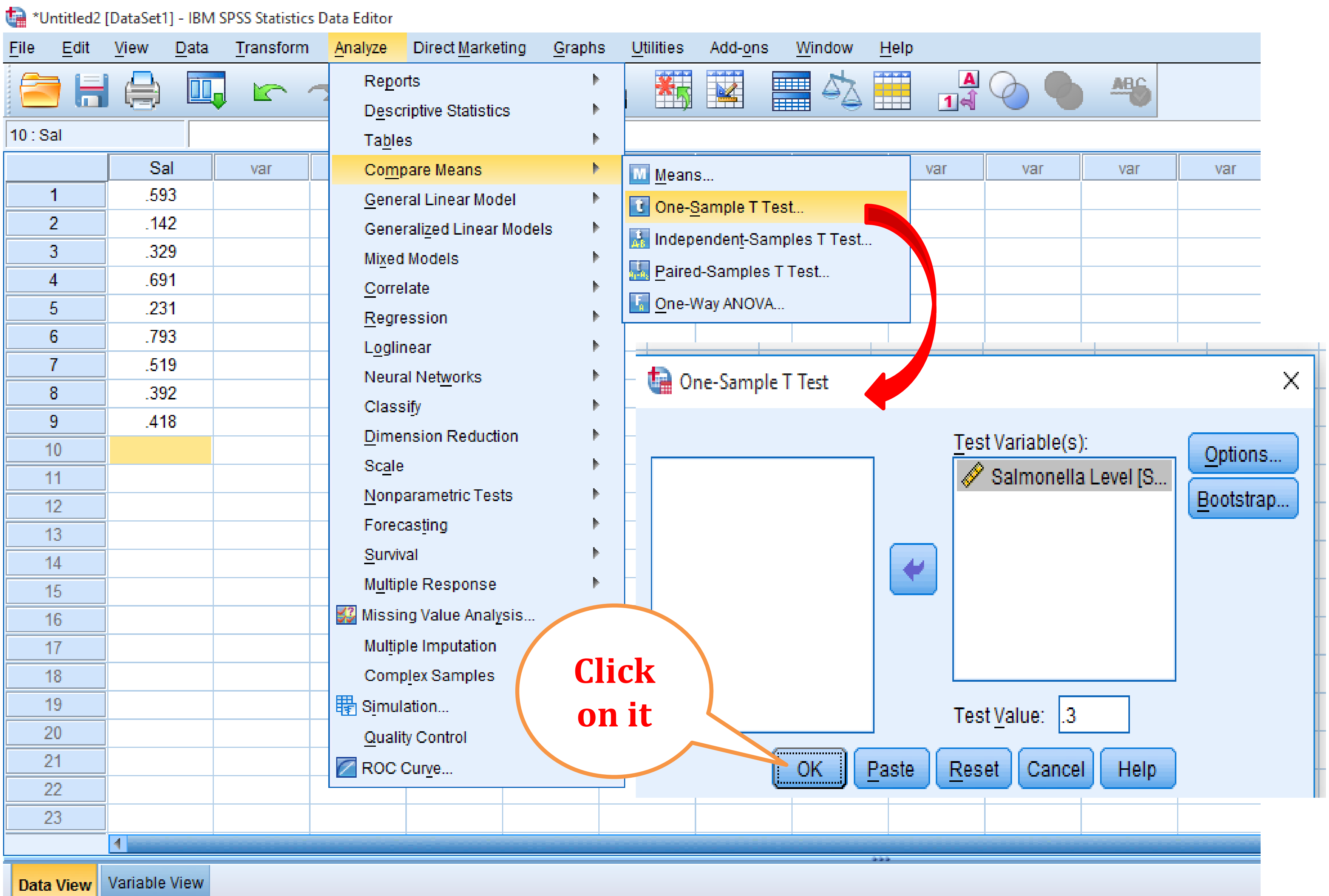
- Question: Is there evidence that the mean level of Salmonella in the ice cream is greater than 0.3 MPN/g?

- **Solution:** The null and alternative hypotheses are

$$H_0: \mu \leq 0.3 \ vs. H_1: \mu > 0.3$$

  – Let's input the above data in SPSS

T-test: One sample



T-test: One sample

$$\boldsymbol{H_0: \mu \leq 0.3} \quad vs. \quad \boldsymbol{H_1: \mu > 0.3}$$

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Salmonella Level	9	.45644	.212844	.070948

One-Sample Test

	Test Value = .3					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Salmonella Level	2.205	8	.059	.156444	-.00716	.32005

$$\boldsymbol{P = 0.059/2}$$
$$\boldsymbol{= 0.0295 < 0.05}$$

- **Conclusion**: Since the $p - value$ (0.0295) $< \alpha$ (0.05), we may reject the H_0. Therefore, we may conclude that the mean level of Salmonella in the ice cream is greater than 0.3 MPN/g.

T-test: Two independent samples

- **Body Temperature data description (.sav)**: The data represent the 100 people with heart rate and body temperature.

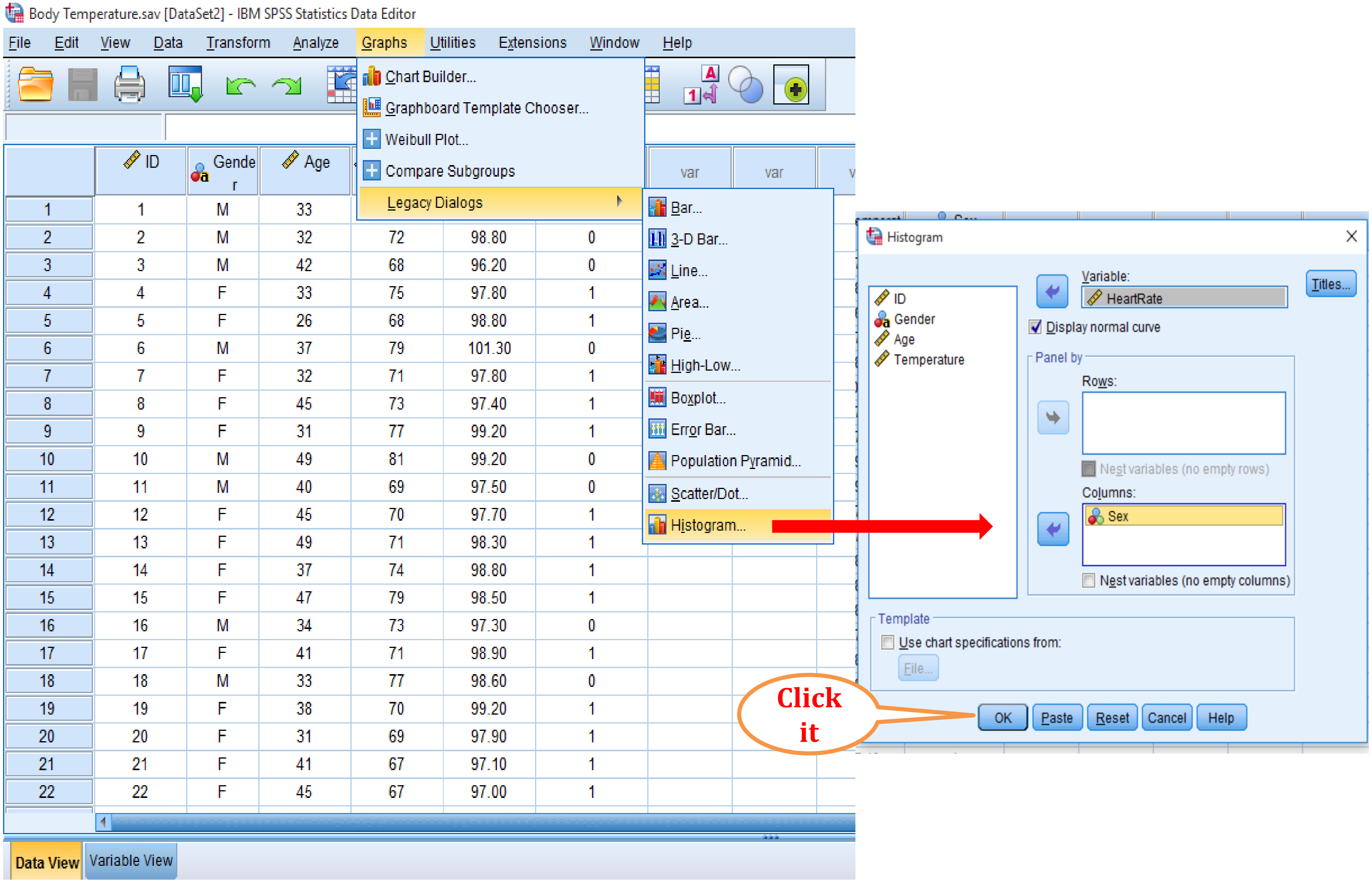
Let's import the data first!

- Question 1: Is there any significant changes in the heart rate of male and female?
- Recode the Gender variable into numerical coding ($0 = Male$, $1 = Female$)
- Check the distribution of heart rate variable!
- **Solution:** The null and alternative hypotheses are

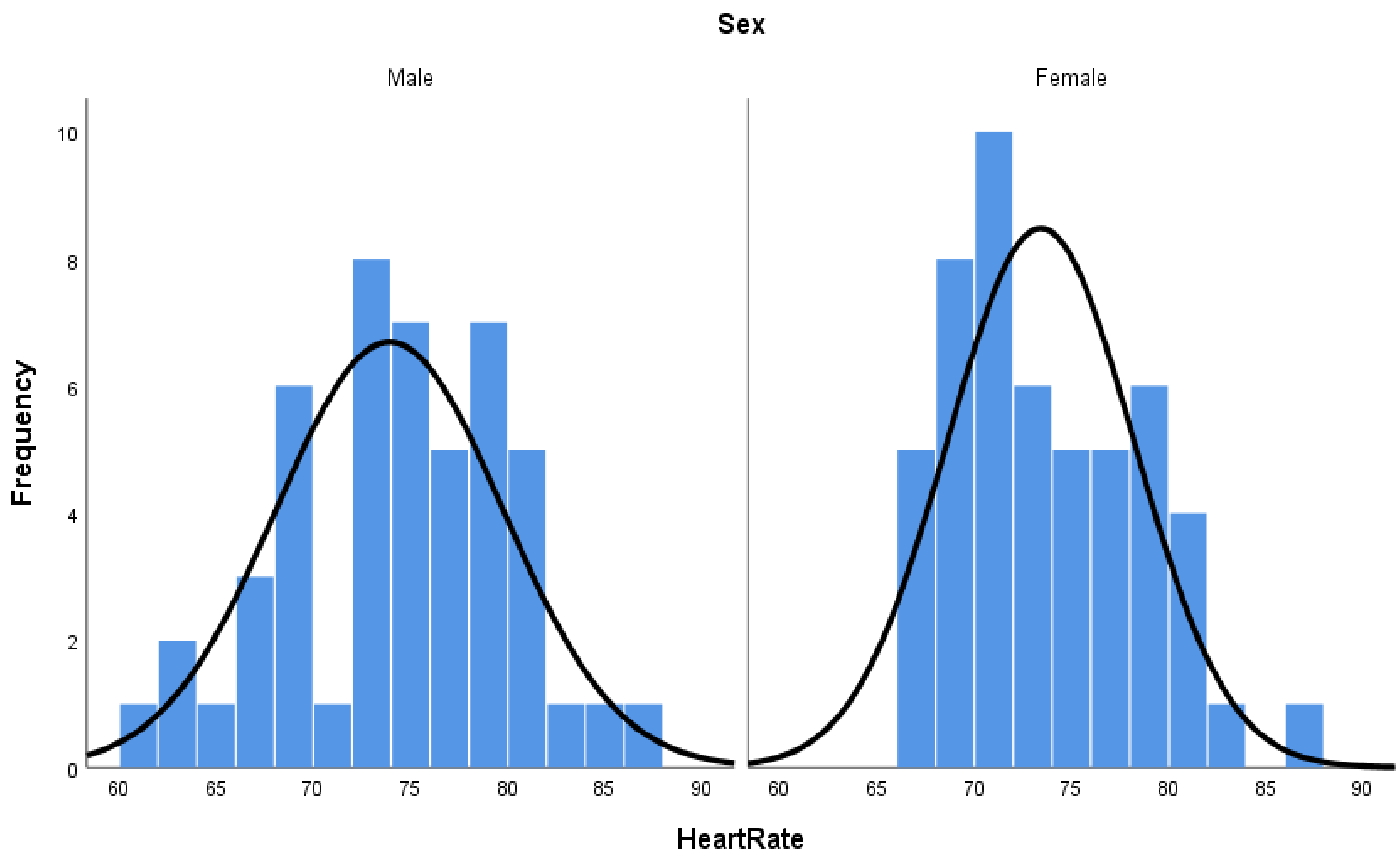
  $\boldsymbol{H_0: \mu_1 = \mu_2}$ (mean HR of male and female are same)

  $\boldsymbol{H_1: \mu_1 \neq \mu_2}$ (mean HR of male and female are different)

Checking the distribution of heart rate variable

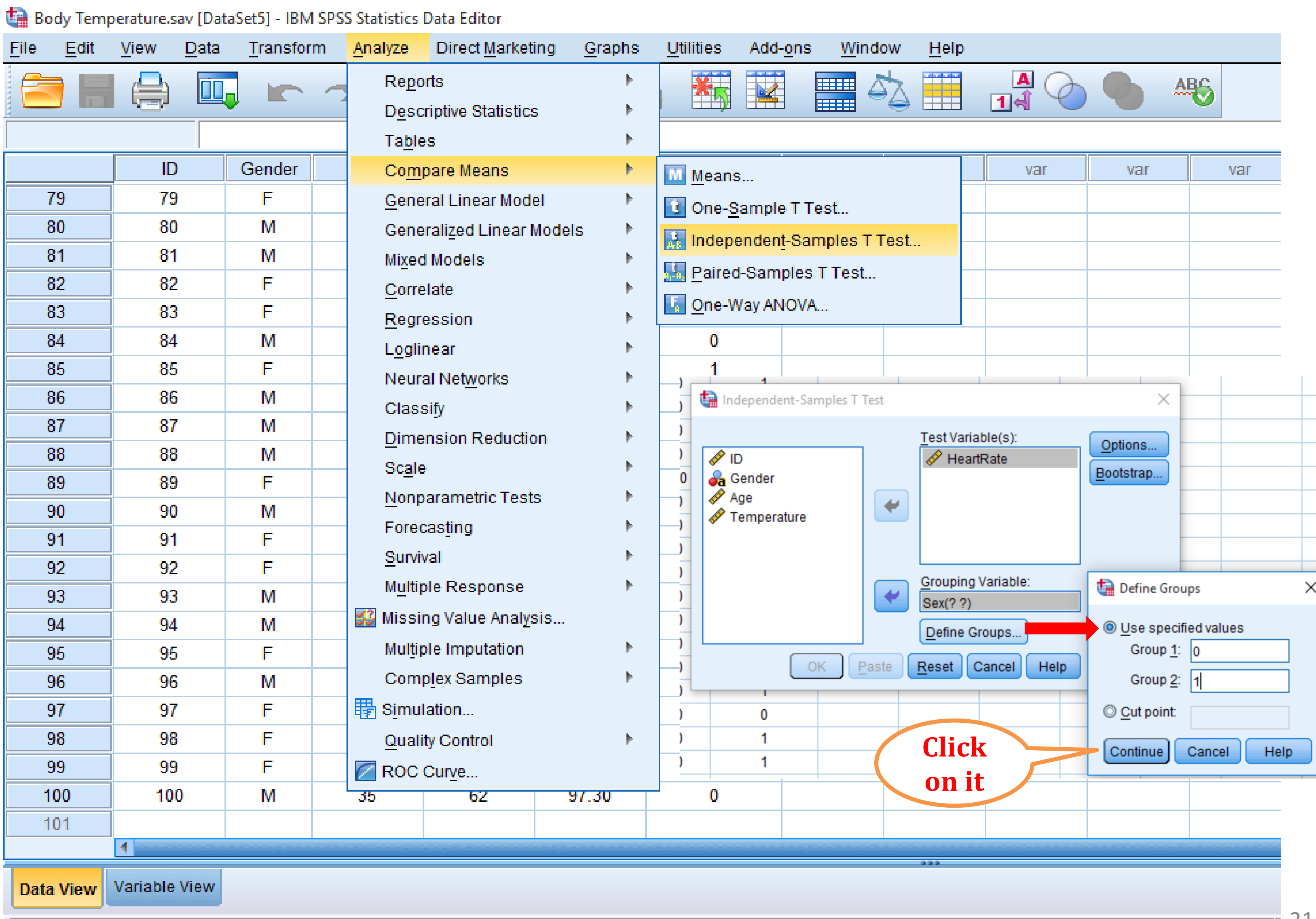


Checking the distribution of heart rate variable



▷ **Summary**: The distribution of HR for male seems normal but the distribution of HR for female looks a bit skewed to the right. For both gender, the HR is approximately normally distributed.

T-test: Two independent samples



T-test: Two independent samples

Group Statistics

	Sex	N	Mean	Std. Deviation	Std. Error Mean
HeartRate	Female	51	73.41	4.797	.672
	Male	49	73.92	5.841	.834

$$H_0:\ var(HR_{male}) = var(HR_{female})$$
$$H_1:\ var(HR_{male}) \neq var(HR_{female})$$

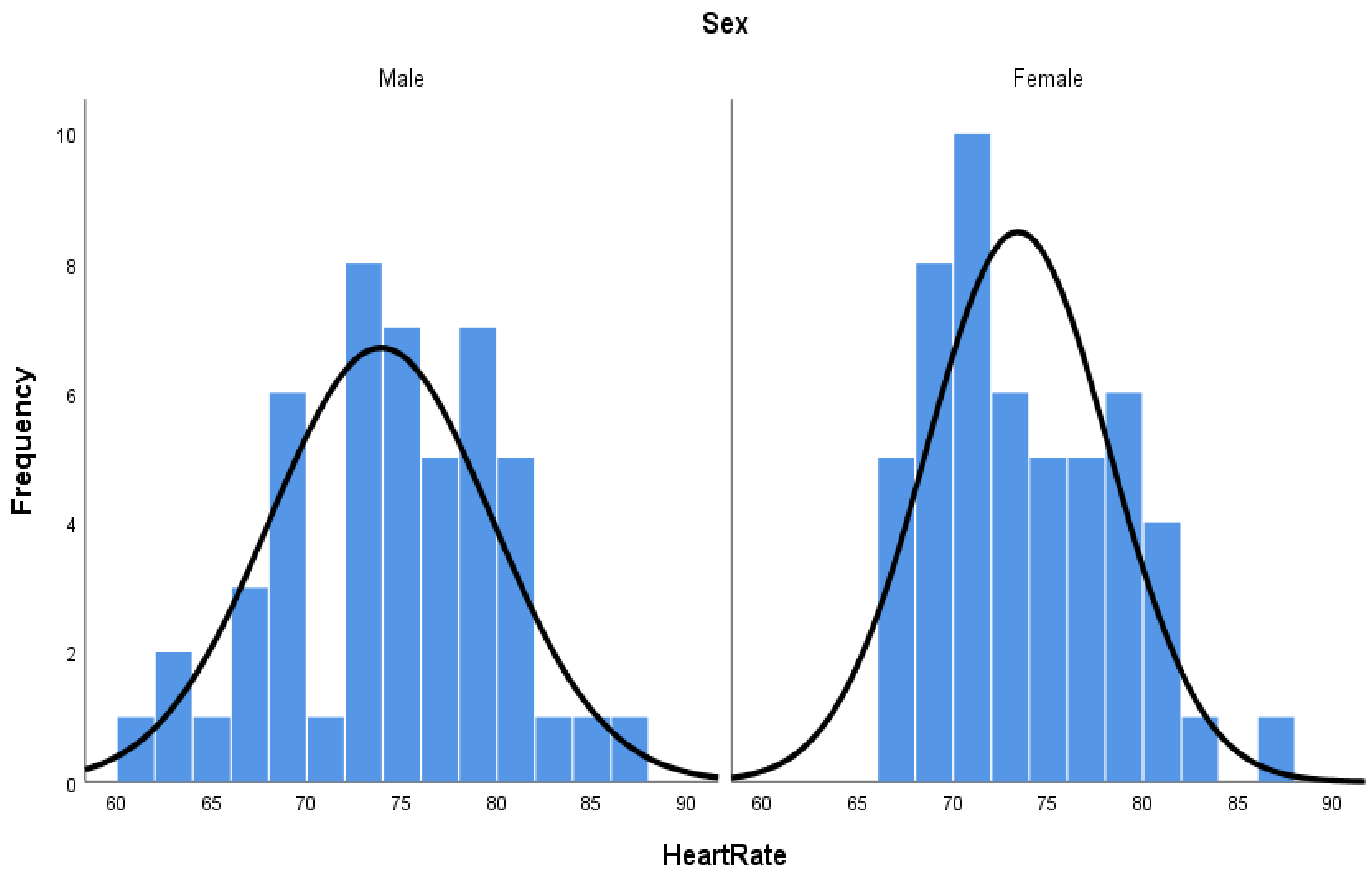
Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
HeartRate	Equal variances assumed	1.366	.245	-.475	98	.636	-.507	1.067	-2.624	1.611
	Equal variances not assumed			-.473	92.907	.637	-.507	1.071	-2.634	1.621

Which one will you choose??

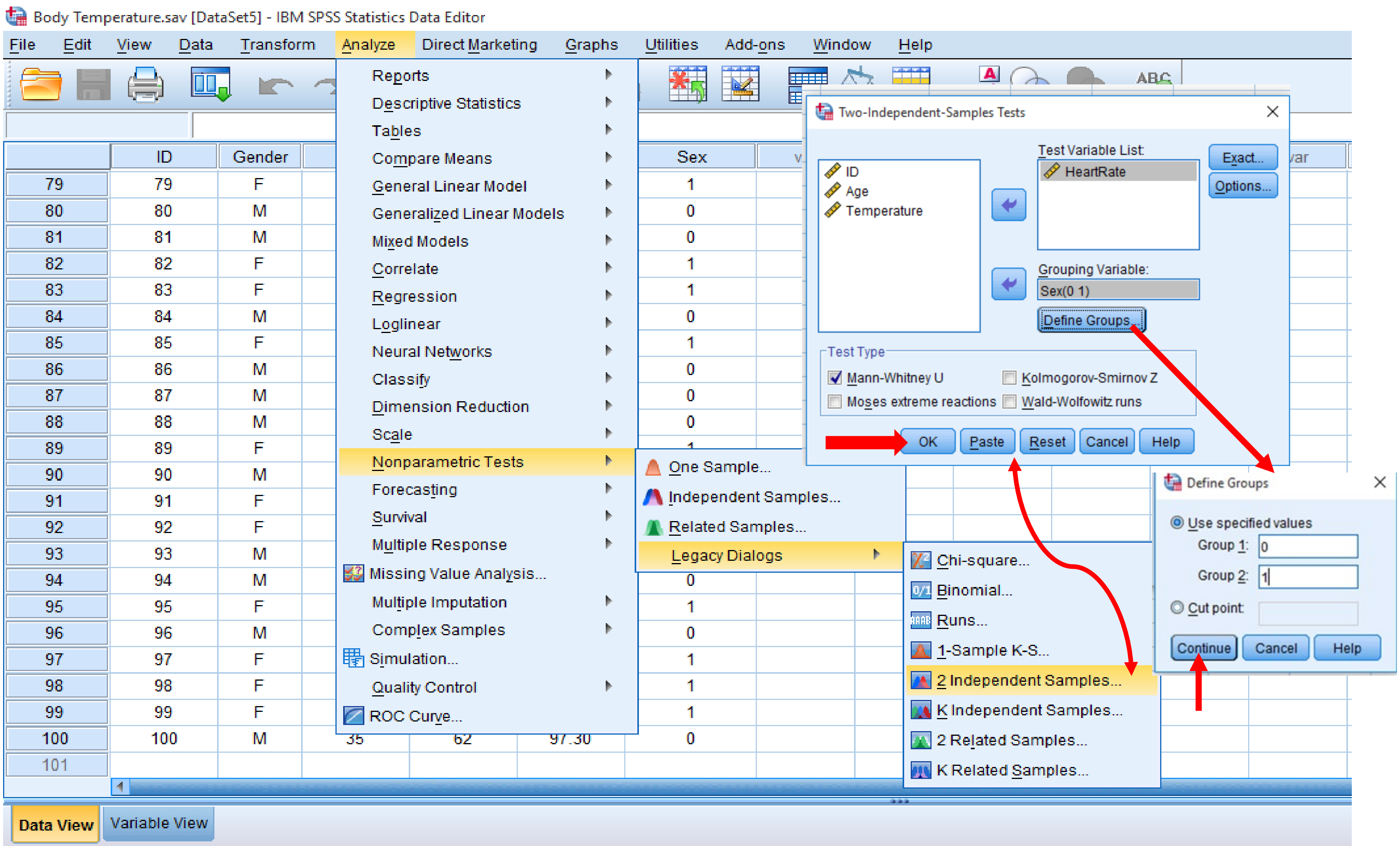
$\boldsymbol{P > 0.05}$

Checking the distribution of heart rate variable



▷ **Summary**: The distribution of HR for male seems normal but the distribution of HR for female looks skewed to the right (positively skewed). Overall, the HR is not normally distributed.

Non-parametric: Mann-Whitney U test



Mann-Whitney U test

Ranks

	Sex	N	Mean Rank	Sum of Ranks
HeartRate	Male	49	52.76	2585.00
	Female	51	48.33	2465.00
	Total	100		

Test Statistics[a]

	HeartRate
Mann-Whitney U	1139.000
Wilcoxon W	2465.000
Z	-.763
Asymp. Sig. (2-tailed)	.445

a. Grouping Variable: Sex

$P > 0.05$

Paired t-test

- Weight of the mice before treatment
- 200.1, 190.9, 192.7, 213, 241.4, 196.9, 172.2, 185.5, 205.2, 193.7
- Weight of the mice after treatment
- 392.9, 393.2, 345.1, 393, 434, 427.9, 422, 383.9, 392.3, 352.2
- **Question:** Is there any significant changes in the weights of mice before and after treatment?

Let’s Import the data in SPSS

Paired t-test

- The null hypothesis (two-sided) is:

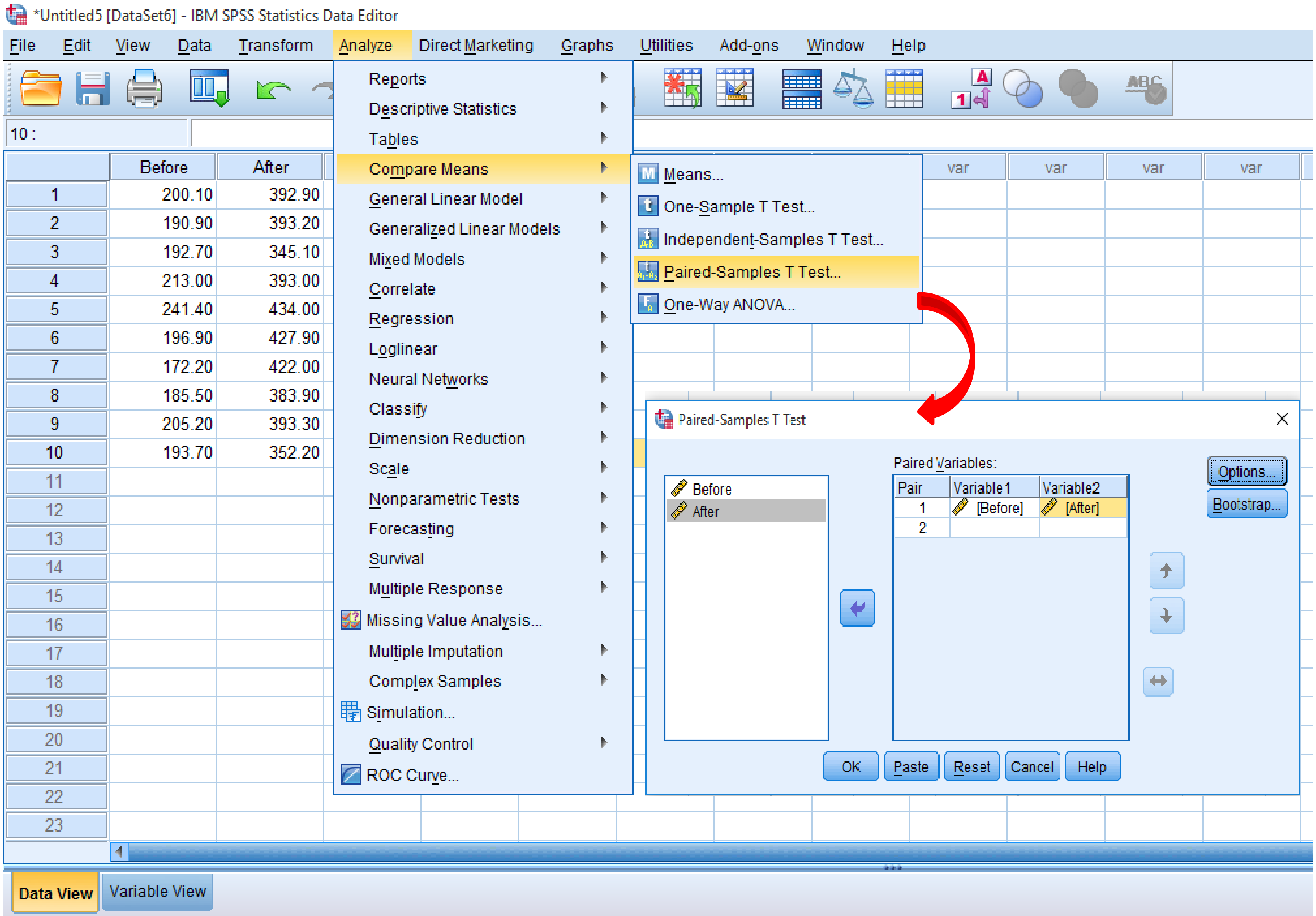
$$H_0 : d_{difference} = 0$$

(The population average weight loss is zero)

$$H_1 : d_{difference} \neq 0$$

(The population average weight loss is not zero)

Paired t-test



Paired t-test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before	199.1600	10	18.47354	5.84185
	After	393.7500	10	29.39460	9.29539

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Before & After	10	.313	.379

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	Before - After	-194.59000	29.42491	9.30497	-215.63931	-173.54069	-20.912	9	.000

Thank You!