# An Option Pricing Framework for Valuation of Football Players


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*Abstract:* - In this paper we develop a contingent claims framework for determining the financial value of professional football players. The football players are seen as real assets and this methodology can be used in the football industry, insurance industry and the investment world and it is the first of this kind in this area. The mathematical stochastic models are based on the geometric Brownian motion, Poisson processes and jump-diffusion process. We show how player value varies from club to club, depending on club turnover and the total number of performance points generated by the entire team.



*Keywords:* asset pricing, geometric Brownian motion; jump-diffusion, Ito's lemma, Poisson process


## 1 Introduction

In this paper we use contingent claims methodology and standard techniques in stochastic calculus to develop a framework for determining the financial value of professional football players. The question is of vital interest to a sport that has become an industry in its own right ([15]). As the amount of money generated by the football industry grows exponentially year by year, it is punctuated by record financial deals negotiated between the football clubs and the players they hire. Witness, for example, Real Madrid's £37 million bid for Figo in July 2000, its £47 million for Zidane in July 2001 and Manchester United's £46.8 million for three players in the summer of 2001. In fact, Deloitte Touche ([6]) reports that in the year 1999-2000 clubs in the English Premier League spent £255 million on player transfers. This is over 33% of the £772 million of total turnover within the league over the period. Combined with players' wages of £487 million, 96% of the league's 1999-2000 income was consumed by expenses directly associated with players.

Unsurprisingly, for the whole of 1999-2000 only 7 of the 20 Premier League clubs made a profit and the league as a whole traded at a deficit. Similar situations apply across the whole of Europe and no end is in sight as player wages are growing at an annual clip of 25.8%. Given the worldwide popularity of football and the key financial role of the players themselves, surprisingly little academic literature is available on how to determine a player's financial value to a club. In fact, the literature ([5], [14], [15]) on football major determinants from a financial economics point of view is very sparse, the majority of studies concentrating on a statistical modeling approach for score forecasting as in [8], [11],[13] or prediction of the players or team performance [1],[2], [4]. The novelty of this paper, then, is that it is the first published, rigorous framework for valuing football players based on their public performance. We explicitly model the uncertainty surrounding a player's performance, including injury events. We also consider the uncertainty surrounding club income, including the effects of player performance as well as those due to player image, club image, fan loyalty, the economy at large, etc. A major feature of the framework is that it is quite flexible in that it can accommodate synergies among players and coaching expertise. It can also accommodate managerial flexibility associated with real situations such as the possibility of lending or trading a player or terminating his contract.

The rest of the paper is organized as follows. In the next section we outline how a player's performance can be measured with a performance index such as the Carling Opta Index. In sections 3 and 4 we develop the models and in section 5 we conclude.

What we can see very clearly in the modern game, is that players are extremely powerful, more so than ever before. Consequently, to remain in the European elite, you must employ the best, and inevitably, the most expensive players. However, instability has been introduced into the football industry as a consequence of recent European Union (EU) legislation associated with the cases of John Bosman and Valery Karpin. The outcomes of these cases have changed the nature of the relationship between player and club by extending the right of free movement of labour enjoyed by other EU and potentially non-EU citizens to footballers.

Overall, it looks like increasingly complex solutions are necessary for the future of this fast growing industry. In particular, the question arises of how clubs can continue to meet rising wage demands and, in an increasing number of cases where clubs are now plc's, satisfy shareholder aspirations, while maintaining their performance on the pitch? A major part of the answer is an accurate assessment of a player's financial value to the club. In fact, market values for football players are already a reality. Buying shares in footballers is common in some Latin American countries. A recent example is presented by the Argentinean player Esteban Fuentes, whose proposed transfer to English Premier League club Derby County was held up when three different clubs claimed partial ownership of the player, leading to protracted negotiation. Already one firm of football agents in the UK has been exploring the potential of mortgaging player transfers to facilitate transfer deals that otherwise may have been abandoned for lack of financial resources. Ultimately, if EU transfer regulations develop further in favor of freedom of employment mobility, players will be able to establish themselves as corporations and to issue shares in themselves in exchange for a proportion of their earnings, possibly derived from licensed products, archive footage of past performances, appearances, and so on. This may only be a possibility among a very small number of players, but it may increasingly provide a solution to some of the problems referred to above.

## 2 A Measure of Player Performance

The potential for other financial instruments to develop in football would appear to be limited only by the ingenuity of the product. However, there are associated difficulties. Football is both quantitative and qualitative, and subjectivity in evaluation is a major issue in all proposed transfers and salary negotiations. From a subjective point of view, a footballer's value is in the eye of the beholder. Some form of statistical analysis of value could offer the potential for a solution. Whether transfer fees remain in European football or not, the ultimate question facing an investor (who may be the chairman of a football club or an investment company) is assessing the value held by the football player that they own or pay the salary of. Thus, there is now a very real need to quantitatively evaluate a footballer, to understand his value adding capability, and to put a price upon that capacity. The first step in that direction is an objective performance index.

One of the pioneers in this field of research and applications are Carling Opta ([3]), the official player performance statisticians to the Premier League. The company was set up at the end of the 1990's and their system of monitoring players was developed in conjunction with former England coach Don Howe. They report an index of each player based on the analysis of all matches played in the Premiership, recording more than 90 distinct actions and outcomes for players, including kinds of shots, passes, tackles, saves, etc. The players are categorised by the position they play: goalkeepers, defenders, midfielders and attackers, and various utilities can be attached to each category. The Carling Opta database system is comprehensive and is based on the video evidence given by an independent assessor following each match. The Index is a sort of quantitative indicator of the form of the player and it is already used by the betting industry, the media, and in fantasy games, as well as by the clubs themselves. Their system is expanding to a wider audience in countries like Italy, France and Holland, Austria, Australia, Singapore and Estonia. The index is calculated as follows. Through his play on the pitch in a game, a player earns a total number of points called the Game Score. The Index Score is then simply calculated as the total number of points from the last six Game Scores, taking into account the number of minutes played as an exposure factor. The Index Score is a moving average-type of statistical measure, and with any new match the previous (6th) match is removed from the calculation. The Opta database has been used in [4] to investigate the team performance in a statistical manner. In this paper however, the Carling Opta Index serves as the "underlying" in the contingent claims, asset pricing literature in finance.

## 3 The Modelling Approach

Let $T$ be the turnover for the club, $N$ be the number of Opta Index points for the individual player under evaluation, $S$ be the sum of Opta Index points for all players playing for the club. Then it follows that

$$X = \frac{T}{S} \tag{1}$$

is the value in money for the club of a single Opta Index point and $Y = NX$ is the Opta value of the player. Denote by $V$ the value of the player to the club. This is a function of $Y$ and of a Poisson process modelling the occurrence of injuries.

We assume that the turnover $T$ and the Opta value of the whole club (team) are correlated and they follow a geometric Brownian motion, see [9] and [12]

$$\begin{aligned} dT(t) &= \alpha T(t)dt + \sigma T dz(t) \\ dS(t) &= \gamma S(t)dt + \delta S(t)dw(t) \\ E(dS(t)dT(t)) &= \rho dt \end{aligned} \tag{2}$$

For an individual player, the number of Opta index points again can be assumed to follow a geometric Brownian motion described by

$$dN(t) = aN(t)dt + bN(t)dH(t) \tag{3}$$

We assume that all technical conditions, as described in [10], regarding the existence of the mathematical quantities involved are realized.

### 3.1 The money value of one Opta Index point

The first thing that needs to be calculated is how much is the value in cash of one point of the Opta Index. This calculation is done relative to the club that is evaluating the player. For example, if Manchester United is looking to calculate the value of an Opta Index point for David Beckham then the Opta point is calculated in sterling; if A.C. Milan is looking to buy David Beckham, they will use for calculations euro's.

The value of one point is $X$ from equation (1) it can be seen that it depends on two stochastic elements, the turnover $T$ of the club and the sum $S$ of Opta points for the whole team playing for that club. Hence, applying a multivariate Ito's lemma ([9], [10], [12]) we get

$$dX(t) = \left[\frac{\partial X}{\partial t} + (\alpha + \delta^2 - \gamma - \rho\,\delta\sigma)X\right]dt$$
$$+ Xdz(t) - \delta X dw(t)$$

$$dX(t) = \left[\frac{\partial X}{\partial t} + AX\right]dt + X\sqrt{\sigma^2 + \delta^2 - 2\sigma\,\delta\rho}\,d\Omega(t)$$

where $A = \alpha + \delta^2 - \gamma - \rho\,\delta\sigma$ and

$$d\Omega(t) = \frac{\sigma - \delta dw(t)}{\sqrt{\sigma^2 + \delta^2 - 2\sigma\,\delta\rho}} \tag{4}$$

A player can be evaluated by the club for which he is already playing for or by a club manifesting an interest in him. This dichotomy is important because in the former case the processes $N$ and $X$ are correlated whereas for the latter case they are not.

### 3.2 The evaluation by an outside club

In this case $Y = NX$ and $N$ and $X$ are uncorrelated. Applying a multivariate version of Ito's lemma using equations (3) and (4) we get the stochastic differential equation describing the evolution of the Carling Opta value of the player

$$dY(t) = (A + a)Ydt + BYd\Delta(t) \tag{5}$$

where

$$B = \sqrt{b^2 + \sigma^2 + \delta^2 - 2\sigma\,\delta\rho}$$

$$d\Delta(t) = \frac{bdH(t) + \sqrt{\sigma^2 + \delta^2 - 2\sigma\,\delta\rho}\,d\Omega(t)}{\sqrt{b^2 + \sigma^2 + \delta^2 - 2\sigma\,\delta\rho}}$$

### 3.3 The effect of injuries

It is of paramount importance to consider the effect of injuries incurred by the player on his value. The best way to model the arrival of injuries for a football player is via a Poisson process. This type of stochastic process is appropriate for rare events and it has been used successfully in the insurance industry. The injuries are explicit events arriving at discrete times. The intensity parameter of the Poisson process is a parameter $\lambda > 0$ that is characteristic to each player. The estimate of this parameter will depend on the position of player, age, previous records, medical examinations and so on.

Injuries have a one-to-one association with jumps in the value of the player. This discontinuous character needs to be dealt with when calculating the value of the player. There are two ways of taking this into consideration. First we may assume that the Poisson process affects the Opta value of the player $Y$. This will be translated automatically for the money value of the player, which is $V$. The second approach would be to consider that the Poisson process representing the injuries affects only the money value $V$ of the player.

The latter alternative is considered next. If $l(Y,t)$ denotes the loss per interval of time $dt$ due to injuries then the expected amount of losses per interval of time $dt$ is \lambda $l(Y,t)dt$. Then the return on the value of the player per interval of time $dt$, using a riskless interest rate, is equal to the expected change in the money value of the player minus the expected losses due to injuries. Thus

$$rVdt = E_{\Delta}(dV(t)) - \lambda l(Y,t)dt \qquad (6)$$

The first term on the right side is calculated by again applying Ito's lemma and take the expectation with respect to the Wiener process $\{\Delta(t)\}$. Therefore

$$E_{\Delta}(dV(t)) = \left[\frac{\partial V}{\partial t} + (A+a)Y\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2}\right]dt$$

$$rV = \frac{\partial V}{\partial t} + (A+a)Y\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2} - \lambda l(Y,t) \qquad (7)$$

After the specification of the loss function $l(Y,t)$ the equation (7) can be solved either analytically or by resorting to numerical methods.

### 3.4 The evaluation by the player's club

If the evaluation is made by the club where the player is playing the only difference from the above outlined calculations is that $N$ and $X$ are correlated. Let the correlation between $N$ and $X$ be given by the correlation coefficient $\psi$ so

$$E(dH(t)d\Omega(t) = \psi dt \qquad (8)$$

Then the application of Ito's lemma gives

$$dY(t) = \frac{\partial Y}{\partial t}dt + \frac{\partial Y}{\partial N}dN(t) + \frac{\partial Y}{\partial X}dX(t) + \psi X\sqrt{\sigma^2+\delta^2-2\sigma\delta\rho}\, N\frac{\partial^2 Y}{\partial N \partial X}dt \qquad (9)$$

Following the same steps as in the uncorrelated case we get $dY(t) = DYdt + BYd\Delta(t)$ (10)

where $D = A + a + \psi\sqrt{\sigma^2+\delta^2-2\sigma\delta\rho}$

This leads to the following equation

$$rV = \frac{\partial V}{\partial t} + DY\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2} - \lambda l(Y,t) \qquad (11)$$

that has the same form with equation (7), the only difference being in the coefficient of $\partial V/\partial Y$. Therefore, from the mathematical point of view the same equation is solved.

## 4 Modelling the Value of the Player with a Jump-Diffusion process

Financial modelling can be adapted to deal with the assumption that the jumps affect directly the evolution of the underlying and following that the evolution of the financial product depending on that underlying. A similar approach can be pursued here. One can argue that $N$, that is the number of Opta index points for the player under evaluation, is a time series that experiences downward jumps when that player has injuries. It is not realistic however to assume that the injuries of an individual player affect the variable $X$, that is the value in money of a single Opta index point. The change that is made then is on equation (3) which under the new assumption is expanded to

$$dN(t) = aN(t)dt + bN(t)dH(t) + K(N)dJ(t) \qquad (12)$$

where $K(N)$ is the amplitude of the jump in $N$ and $J(t)$ is a counting process representing the number of jumps occurring in the interval of time [0,t]. In mainstream financial modelling (see [9], [12]) this process is usually a Poisson process with intensity parameter $\lambda$, which means that the probability to have a jump in the next interval of time $dt$ is $\lambda dt$.

### 4.1 The amplitude of the jump

Suppose that the amplitude of the jumps is given by $K(N) = N(t)(k-1)$. Therefore

$N_{after} - N_{before} = N_{before} + K(N) - N_{before}$ and so $N_{after} = kN_{before}$ (13)

It is obvious that $k$ cannot be negative and we get downward jumps if and only if $k < 1$. It can be showed that

$$N(t) = N(0)k^{J(t)}\exp\left[a - \frac{1}{2}b^2 + \lambda\ln(kt) + bH(t)\right] \qquad (14)$$

which, upon specification of a probability distribution for $k$ gives the possibility of simulating a path of the variable $N$ from 0 to $t$.

### 4.2 The pricing equation for the value of the player

In order to determine the value of the player, as before, we need to dichotomize between the two possible situations: evaluation for the club the player is playing for and for an outside club. In the following only the latter case is considered, the former case being similar. We need to find out the equation of $Y = NX$, which is

$$dY = (A + a)Ydt + BYd\Delta(t) + \Delta_K(Y)dJ(t) \quad (15)$$

where

$$B = \sqrt{b^2 + \sigma^2 + \delta^2 - 2\sigma\ \delta\rho}$$

$$d\Delta(t) = \frac{bdH(t) + \sqrt{\sigma^2 + \delta^2 - 2\sigma\ \delta\rho}d\Delta(t)}{\sqrt{b^2 + \sigma^2 + \delta^2 - 2\sigma\ \delta\rho}}$$

$$\Delta_K(Y) = Y(N(t) + K(N(t)), X, t) - Y(N(t), X, t)$$

It can be shown that this leads to

$$dY(t) = (A + a)Ydt + BYd\Delta(t) + [\Delta_K(Y)]dJ(t) \quad (16)$$

The last step is to find the pricing equation for $V=V(Y,t)$. Applying the same methodology as for $Y$ we get that

$$E_\Delta(dV(t)) = G(V)dM(t) + \left[\frac{\partial V}{\partial t} + (A + a)Y\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2} + \lambda G(V)\right]dt \quad (17)$$

where

$$dM(t) = dJ(t) - \lambda dt$$

$$E[dM(t)] = 0 \quad (18)$$

$$G(V) = V(Y + \Delta_K(Y), t) - V(Y, t)$$

Applying again the expectation with respect to the martingale $\{M(t)\}$ we get

$$E_\Delta(dV(t)) = [\frac{\partial V}{\partial t} + (A+a)Y\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2}]dt + \lambda G(V)dt \quad (19)$$

$$rV = E[dV(t)] = \lambda G(V)dt + [\frac{\partial V}{\partial t} + (A + a)Y\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial^2 V}{\partial Y^2}]dt \quad (20)$$

The last partial differential equation can be solved by numerical methods under some constraints like $V(Y,T^*)=0$ for all $Y$ where $T^*$ is the maturity of the player.

# 5 Conclusion and Further Developments

## 5.1 The evaluation of football clubs

A football player is an element of a large organization that is a football club. Having established a way to evaluate football players individually it seems natural to extend the analysis by evaluating the football club as a whole based on some sort of aggregation process for all players in the squad. Big clubs increasingly have big squads in order to maintain their positions in the elite group and it is not unheard of clubs with 30-35 players. If we considers players owned by the club and lent to other clubs this number may increase. Therefore, because of the natural correlations (or synergies as some managers prefer to call them) between players performances the financial modelling process becomes multidimensional and perhaps the problem is of a computational nature.

At a first glance it looks like we need only $S$, the sum of Opta index points for all players hired by the club, and then multiply this quantity by $X$ the value in money of one Opta index point for that club. This gives $T$ and the whole story seems to fall down. However, the value of the club is not given by the simple sum $S$ because the synergy (or the correlations between the players) effects must be taken into consideration.

## 5.2 Extended use of real options for management of football players

The two models described above are better than any other decision tree or discount cash flow techniques because they recognize the potential of upside movements in the value of the player while limiting the downside movements ([7]). Several types of real options, already investigated in other context, may prove fruitful for the financial management of football players and will be the subject of future research.

The investor in a player may at any time cancel his investment under the contractual obligations agreed. Therefore, this option limits the downside that a player may lose a considerable proportion of his value and the investor then tries to salvage some of the initial investment. This type of option is discussed for other types of project in [16].

The investor may also invest in a player but not use him or leave him to develop latently by lending the player to a club in a lower division. He may recall the player if needed and this extra option provided can be also evaluated. Also of interest is the dual problem of waiting to invest. This type of option may interact with interest rates financial modelling and, if the player bought is in a different country than the club manifesting an interest, it may interact with exchange rates modelling.

It is very common in the football world for one club to change one of its players for another player from another club, usually playing in a different position. Most of the time there is an additional balancing sum of cash involved. Therefore, even if a player does not play extremely well or does not play very often, he may have an extra value component if his club can offset part of the cash needed to buy a new player .

Major football clubs, the majority being already listed on stock exchanges, have become in recent years potentially significant actors in the financial investment world. The transactions are evaluated almost always in millions of dollars and players are generally viewed as the most important assets for the clubs. This market is growing at a phenomenal pace and if it is to follow what is already evident in USA for games like basketball, baseball and American football, we are going to see dramatic changes in the financial investments in football in Europe and world-wide. Although a lot of statistical information is ready available from reliable sources there is no theoretical financial methodology for the big question "How much is this player worth at this moment in time".

In this paper, we set up a general theoretical framework for this question. Our approach is based on real options like models, one of the best theoretical tools for decision making. We emphasize that although the players are sold and bought in a sort of market this does not correspond to the financial meaning of market. In addition, the transfer of the players from one club to another in exchange for a sum of money is likely to disappear in the immediate future. Whether that is the case or not does not affect the methodology proposed in this paper. One way or another, the real options models shown here can be applied in the same manner. It is just that their results might need to be used slightly different.

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