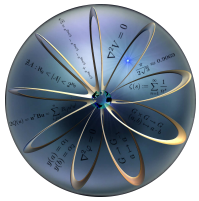

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inequality with absolute values

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Recalling that the `http://planetmath.org/AbsoluteValue`absolute value of a real number means on the *number line* (real axis) the distance of the point from the origin, we have the following three rules which remove the absolute value signs from an inequality (we use the logical symbol "$\vee$" for alternativeness 'or'). Note that the symbols "$\geq$" and "$\leq$" may also be without the equality bar.

1. $|a| \geq b \quad \Leftrightarrow \quad a \leq -b \ \vee \ a \geq b$
2. $|a| \leq b \quad \Leftrightarrow \quad -b \leq a \leq b$
3. $|a| \geq |b| \quad \Leftrightarrow \quad a^2 \geq b^2$

These rules are valid for all real values of a and b. For example, if one has a case

$$|x| < -5$$

corresponding the rule 2, this inequality seems to be impossible since no absolute value is negative; but now also the result $-(-5) < x < -5$ given by the rule 2 is impossible — no real number is simultaneously greater than $+5$ and less than -5.

Examples. We solve some inequalities with absolute values.

a) $|2x+1| > 5x$

$2x+1 < -5x$ or $2x+1 > 5x$ (rule 1)

$7x < -1$ or $-3x > -1$

$x < -1/7$ or $x < 1/3$

$x < 1/3$ (combined)

b) $8|x| + |x-2| > 6$

$|8x| > 6 - |x-2|$

$8x < -6 + |x-2|$ or $8x > 6 - |x-2|$ (rule 1)

$|x-2| > 8x+6$ or $|x-2| > 6-8x$

$x-2 < -8x-6$ or $x-2 > 8x+6$ or $x-2 < -6+8x$ or $x-2 > 6-8x$ (rule 1 twice)

$9x < -4$ or $-7x > 8$ or $-7x < -4$ or $9x > 8$

$x < -4/9$ or $x < -8/7$ or $x > 4/7$ or $x > 8/9$

$x < -4/9$ or $x > 4/7$ (from the number line)

c) $|1-5x| \leq 3$

$-3 \leq 1-5x \leq 3$ (rule 2)

$-4 \leq -5x \leq 2$ (subtracted 1 from all parts)

$4/5 \geq x \geq -2/5$ (divided by -5)
$-2/5 \leq x \leq 4/5$ (rewritten from end to begin)