product of negative numbers

Canonical name	ProductOfNegativeNumbers
Date of creation	2013-03-22 17:35:41
Last modified on	2013-03-22 17:35:41
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	13
Author	pahio (2872)
Entry type	Derivation
Classification	msc 97D40
Classification	msc 13A99
Synonym	product of two negative numbers
Related topic	OppositeNumber
Related topic	PlusSign
Related topic	Multiplication
Related topic	KalleVaisala

<div align="center">Why is $(-a)(-b) = ab$?</div>

Negative numbers are less than zero, positive ones are more than zero. One can study the former, concrete question in fact from a more general of view, where the used letter variables in the latter question may be thought equally well positive as negative (but it may be simpler and better to think that a and b there mean positive numbers). We use the notation convention, usual in mathematics, that $+a$ and $+b$ mean the same as a and b (see plus sign), when it is easy to speak of changing the sign.

The following three properties of multiplication are well known:

- The product of two numbers equals to zero always when one of the numbers equals to zero.

- Commutative law: $ab = ba$

- Distributive law: $a(b+c) = ab+ac$

It is natural to require these properties regardless of whether the numbers are positive or negative.

We need in the following calculation only the first and the last property:

$$0 = (+a)\cdot 0 = (+a)[(+b)+(-b)] = (+a)(+b)+(+a)(-b) = ab+(+a)(-b)$$

Because the value of the sum in the end is zero, the latter summand $(+a)(-b)$ must be the opposite number of the former addend ab. Accordingly we may write:

$$(+a)(-b) = -(ab) \tag{1}$$

This result means that as the sign of the second `http://planetmath.org/Product`factor of the product $(+a)(+b)$ is changed, the sign of the whole product changes. The same concerns of course also the first factor of the product, since by the commutative law, the order of the factors can be changed.

But if one changes in the product $(+a)(+b)$ the signs of both factors, first one and then the other, the sign of the product changes twice, i.e. it remains unchanged. Thus we obtain the final result

$$(-a)(-b) = ab. \tag{2}$$

<div align="center">1</div>

If we think that a and b are positive numbers, the result (2) must be understood that *the product of two negative numbers is necessarily positive*, in order to keep the three properties of multiplication in for all numbers.

Remark. The justification of (1) and (2) makes apparent, that these formulae are in in every ring (cf. the `http://planetmath.org/XcdotYXcdotY2`parent entry where one assumes the ring unity).

References

[1] K. Väisälä: *Algebran oppi- ja esimerkkikirja I*. Fifth edition. Werner Söderström osakeyhtiö, Porvoo & Helsinki (1952).