proportionality of numbers

Canonical name	ProportionalityOfNumbers
Date of creation	2014-02-23 21:26:01
Last modified on	2014-02-23 21:26:01
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	11
Author	pahio (2872)
Entry type	Definition
Classification	msc 97U99
Classification	msc 12D99
Synonym	proportionality
Related topic	Variation
Related topic	KalleVaisala
Defines	proportional
Defines	directly proportional
Defines	inversely proportional

The nonzero numbers $a_1,\,a_2,\,\ldots,\,a_n$ are (*directly*) *proportional* to the nonzero numbers $b_1,\,b_2,\,\ldots,\,b_n$ if

$$a_1\!:\!a_2\!:\ldots:\!a_n \;=\; b_1\!:\!b_2\!:\ldots:\!b_n, \tag{1}$$

which special notation means the simultaneous validity of the proportion equations

$$a_1\!:\!a_2 \;=\; b_1\!:\!b_2, \quad a_2\!:\!a_3 \;=\; b_2\!:\!b_3, \quad \ldots, \quad a_{n-1}\!:\!a_n \;=\; b_{n-1}\!:\!b_n. \tag{2}$$

It follows however that

$$a_i\!:\!a_j \;=\; b_i\!:\!b_j \quad \text{for all } \ i,\ j. \tag{3}$$

In fact, if one multiplies the left hand sides of e.g. two first equations (2) and similarly their right hand sides, then one obtains $a_1\!:\!a_3 \;=\; b_1\!:\!b_3$.

Swapping the middle members of the proportions (2), which by the http://planetmath.org/Pr entry is allowable, one gets the system of equations

$$\frac{a_1}{b_1} \;=\; \frac{a_2}{b_2} \;=\; \ldots \;=\; \frac{a_n}{b_n} \tag{4}$$

which is http://planetmath.org/Equivalent3equivalent with (1) and (2).

The numbers $a_1,\,a_2,\,\ldots,\,a_n$ are *inversely proportional* to the numbers $b_1,\,b_2,\,\ldots,\,b_n$ if

$$a_1\!:\!a_2\!:\ldots:\!a_n \;=\; \frac{1}{b_1}\!:\!\frac{1}{b_2}\!:\ldots:\!\frac{1}{b_n}.$$

Then we have

$$a_i\!:\!a_j \;=\; b_j\!:\!b_i \quad \text{for all } \ i,\ j.$$

Note. The notation $a_1:a_2:\ldots:a_n$ expressing the "ratio of several numbers" is, of course, , but it behaves as the ratio (= the quotient) of two numbers in the sense that all of its members a_i may be multiplied by a nonzero number without injuring the validity of (1).

Example. Let $a:b = 2:3$ and $b:c = 4:5$. Determine the least positive integers to which the numbers $a,\,b,\,c$ are a) directly, b) inversely proportional.

a) The least common multiple of 3 and 4, the members corresponding the members b in the given proportions, is 12. Thus we must multiply the right hand sides of these proportions respectively by $\frac{12}{3} = 4$ and $\frac{12}{4} = 3$:

$$a:b \;=\; 2:3 \;=\; 8:12, \quad b:c \;=\; 4:5 \;=\; 12:15.$$

Accordingly,

$$a:b:c \;=\; 8:12:15.$$

b) We may write

$$a:b \;=\; \frac{1}{3}:\frac{1}{2} \;=\; \frac{1}{15}:\frac{1}{10}, \quad b:c \;=\; \frac{1}{5}:\frac{1}{4} \;=\; \frac{1}{10}:\frac{1}{8},$$

where the denominators of the right hand sides have been multiplied by 5 and 2, respectively. Consequently,

$$a:b:c \;=\; \frac{1}{15}:\frac{1}{10}:\frac{1}{8},$$

i.e. the required integers are 15, 10, 8.

References

[1] K. Väisälä: *Geometria*. Reprint of the tenth edition. Werner Söderström Osakeyhtiö, Porvoo & Helsinki (1971).