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freshman's dream error

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The name "freshman's dream theorem" comes from the fact that people who are unfamiliar with mathematics commonly make the error of distributing exponents over addition and/or subtraction, typically when working in fields of characteristic zero. An example is the equation $(x + y)^2 = x^2 + y^2$ for $x, y \in \mathbb{R}$. The equation is incorrect unless $x = 0$ or $y = 0$. By no means does the exponent need to be a natural number or an integer for this error to occur. An example of this is the equation $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ for $x, y \in \mathbb{R}$ with $x \geq 0$ and $y \geq 0$. This equation can be rewritten using the exponent $\frac{1}{2}$, and again, the equation is incorrect unless $x = 0$ or $y = 0$.

An easy way to explain to someone who is under the impression that exponents distribute over addition and/or subtraction is to provide a counterexample. For instance, when $x = 3$ and $y = 4$, we have:

$$(x + y)^2 \quad = \quad (3 + 4)^2 \quad = \quad 7^2 \quad = \quad 49$$

$$x^2 + y^2 \quad = \quad 3^2 + 4^2 \quad = \quad 9 + 16 \quad = \quad 25$$

On the other hand, the freshman's dream theorem yields some instances in which exponents can be distributed over addition and/or subtraction.