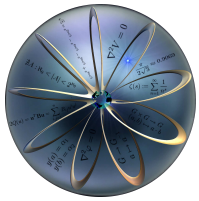

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even-even-odd rule

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The is a mnemonic that is helpful for students for simplifying radical expressions. The phrase even-even-odd stands for the rule: If a real variable to an even http://planetmath.org/Exponent2exponent is under a with an even http://planetmath.org/Radical6index and, when the is eliminated, the resulting on the variable is odd, then absolute value signs must be placed around the variable. (All numbers to which "" and "" refer are natural numbers.) This rule is justified by the following:

Recall that, for any positive integer n, b is the http://planetmath.org/NthRoot$n$th root of a if and only if $b^n = a$ and $\text{sign}(b) = \text{sign}(a)$. Thus, for any positive integer n and $x \in \mathbb{R}$,

$$\sqrt[n]{x^n} = \begin{cases} |x| & \text{if } n \text{ is even} \\ x & \text{if } n \text{ is odd.} \end{cases}$$

The following are some examples of how to use the even-even-odd rule.

Problem. *Let x, and y be real variables. Simplify the expression $\sqrt[4]{x^{12}y^8}$.*

Solution: The on the x is even (12), the of the is even (4), and the that will occur on the x once the is eliminated will be odd (3). Thus, absolute values are necessary on the x.

The on the y is even (8), the of the is even (4), and the that will occur on the y once the is eliminated will be even (2). Thus, according to the rule, absolute values are not necessary on the y. (Note, though, that it would not be incorrect to have them.) The reason that the absolute values are not necessary is that y^2 is nonnegative regardless of the value of y.

Thus, we have $\sqrt[4]{x^{12}y^8} = |x|^3y^2$. (The answer $|x^3|y^2$ is also acceptable.)

Some care is needed in applying the even-even-odd rule, as the next problem shows.

Problem. *Let x be a real variable. Simplify the expression $\sqrt[4]{x^2}$.*

Note that, as stated, the even-even-odd rule does not apply here, since, if the were eliminated, the resulting on the x will be $\frac{1}{2}$. On the other hand, it can still be used to provide a correct answer for this particular problem.

Solution:

$$\sqrt[4]{x^2} = \sqrt{\sqrt{x^2}} = \sqrt{|x|}$$

The good news is that, for square roots, this issue discussed above does not arise: If the even-even-odd rule does not apply, then absolute values are not necessary. That is because, if $n \in \mathbb{N}$ is odd, the expression $\sqrt{x^n}$ only makes sense in the real numbers when x is nonnegative.

I would like to thank Mrs. Sue Millikin, who taught me how to simplify expressions in this manner.