empty sum

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The *empty sum* is such a borderline case of sum where the number of the addends is zero, i.e. the set of the addends is an empty set.

- One may think that the zeroth multiple $0a$ of a ring element a is the empty sum; it can spring up by adding in the ring two multiples whose integer coefficients are opposite numbers:

  $$(-n)a+na \;=\; (-n+n)a = 0a$$

  This empty sum equals the additive identity 0 of the ring, since the multiple $(-n)a$ is defined to be

  $$\underbrace{(-a)+(-a)+\ldots+(-a)}_{n\text{ copies}}$$

- In using the `http://planetmath.org/Summing`sigma notation

  $$\sum_{i=m}^{n} f(i) \tag{1}$$

  one sometimes sees a case

  $$\sum_{i=m}^{m-1} f(i). \tag{2}$$

  It must be an empty sum, because in

  $$\sum_{i=m}^{m} f(i) \tag{3}$$

  the number of addends is clearly one and therefore in (2) the number is zero. Thus the value of (2) may be defined to be 0.

Note. The sum (1) is not defined when n is less than $m-1$, but if one would want that the usual rule

$$\sum_{i=m}^{n} f(i) + \sum_{i=n+1}^{k} f(i) \;=\; \sum_{i=m}^{k} f(i) \tag{4}$$

would be true also in such a cases, then one has to define

$$\sum_{i=m}^{n} f(i) \;=\; -\sum_{i=n+1}^{m-1} f(i) \qquad\qquad (n < m-1),$$

because by (4) one could calculate

$$0 \;=\; -\sum_{i=n+1}^{m-1} f(i) + \sum_{i=n+1}^{m-1} f(i) \;=\; \sum_{i=m}^{n} f(i) + \sum_{i=n+1}^{m-1} f(i) \;=\; \sum_{i=m}^{m-1} f(i).$$