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absolute value inequalities

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Let $a, b, c \in \mathbb{R}$ and $f(x) \in \mathbb{R}[x]$. There is a mnemonic device that is useful for solving inequalities of the following forms:

$$\begin{array}{ll} a|f(x)| + b \leq c & c \geq a|f(x)| + b \\ a|f(x)| + b < c & c > a|f(x)| + b \\ a|f(x)| + b \geq c & c \leq a|f(x)| + b \\ a|f(x)| + b > c & c < a|f(x)| + b \end{array}$$

Before using the mnemonic device, the expression $|f(x)|$ should be on the left hand of the inequality. Once this is accomplished, the absolute value must be dealt with: One statement should look to the previous one, the only being that the absolute value are dropped. The other statement should also have the absolute value dropped, but the inequality needs reversed and the number (on the) needs to be negated.

The two statements as described above should be using either or or and. The mnemonic that aids in remembering which one to use is *greater less than*. That is, when the inequality before splitting up has $>$ or \geq , the connector or should be used; when the inequality before splitting up has $<$ or \leq , the connector and should be used.

Here is an example:

$$\begin{array}{l} 8 > 3 + |2x - 7| \\ 5 > |2x - 7| \\ |2x - 7| < 5 \end{array}$$

Since the inequality is $<$, and should be used.

$$\begin{array}{lll} 2x - 7 < 5 & \text{and} & 2x - 7 > -5 \\ 2x < 12 & \text{and} & 2x > 2 \\ x < 6 & \text{and} & x > 1 \end{array}$$

$$1 < x < 6$$

I would like to thank Mrs. Sue Millikin, who taught me absolute value inequalities in this manner.