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## numerable set

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Let X be a set. An *enumeration* on X is a surjection from the set of natural numbers  $\mathbb{N}$  to X.

A set X is called *numerable* if there is a bijective enumeration on X.

It is easy to show that  $\mathbb{Z}$  and  $\mathbb{Q}$  are numerable.

It is a standard fact that  $\mathbb{R}$  is not numerable. For, if we suppose that the numbers [0,1] were countable, we can arrange them in a list (given by the supposed bijection).

Representing them in a binary form, it is not hard to construct an element in [0,1], which is not in the list.

This contradiction implies that  $[0,1] \subset \mathbb{R}$  is not numerable.

**Remark**. If the enumeration  $\mathbb{N} \to X$  is furthermore a computable function, then we say that X is *enumerable*. There exists numerable sets that are not enumerable.