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strange root

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In solving certain of equations, one may obtain besides the proper () http://planetmath.org/Equationroots also some *strange roots* which do not satisfy the original equation. Such a thing can happen especially when one has in some stage squared both sides of the treated equation; in this situation one must check all "roots" by substituting them to the original equation.

Example.

$$x-\sqrt{x}=12$$
$$x-12=\sqrt{x}$$
$$(x-12)^2=(\sqrt{x})^2$$
$$x^2-24x+144=x$$
$$x^2-25x+144=0$$
$$x=\frac{25\pm\sqrt{25^2-4\cdot 144}}{2}=\frac{25\pm 7}{2}$$
$$x=16 \quad \vee \quad x=9$$

Substituting these values of x into the left side of the original equation yields

$$16-4=12, \quad 9-3=6.$$

Thus, only $x=16$ is valid, $x=9$ is a strange root. (How $x=9$ is related to the solved equation, is explained by that it may be written $(\sqrt{x})^2-\sqrt{x}-12=0$, from which one would obtain via the quadratic formula that $\sqrt{x}=\frac{1\pm 7}{2}$, i.e. $\sqrt{x}=4$ or $\sqrt{x}=-3$. The latter corresponds the value $x=9$, but it were relevant to the original equation only if we would allow negative values for square roots of positive numbers; the practice excludes them.)

The general explanation of strange roots when squaring an equation is, that the two equations

$$a=b,$$
$$a^2=b^2$$

are not http://planetmath.org/Equivalent3equivalent (but the equations $a=\pm b$ and $a^2=b^2$ would be such ones).