Causal Graphs

Julian Schuessler

Presentation for MZES Methods Bites

November 5, 2019

Today

Love & Mercy

Survey and Overview

Graph Basics and d-separation

Definition and Identification of Causal Effects

Post-Treatment Bias

Causal Mediation

Section 1

Love & Mercy

Love & Mercy

Love & Mercy

- “The ’insights’ offered by the graphical approach to causal inference are generally not helpful” (Donald Rubin)

Love & Mercy

- “The 'insights' offered by the graphical approach to causal inference are generally not helpful” (Donald Rubin)
- “There are a lot of people who never use graphs, who are well-known in causal inference, who use this to the extent that they say we shouldn't use graphs. A famous saying in science is: Science progresses on the deaths of the older scientists. And I don't think graphs will have any trouble in the future.” (James Robins)

Love & Mercy

- "The 'insights' offered by the graphical approach to causal inference are generally not helpful" (Donald Rubin)
- "There are a lot of people who never use graphs, who are well-known in causal inference, who use this to the extent that they say we shouldn't use graphs. A famous saying in science is: Science progresses on the deaths of the older scientists. And I don't think graphs will have any trouble in the future." (James Robins)
- "The DAG approach fully deserves the attention of all researchers and users of causal inference as one of its leading methodologies." (Guido Imbens)

Love & Mercy

- "The 'insights' offered by the graphical approach to causal inference are generally not helpful" (Donald Rubin)
- "There are a lot of people who never use graphs, who are well-known in causal inference, who use this to the extent that they say we shouldn't use graphs. A famous saying in science is: Science progresses on the deaths of the older scientists. And I don't think graphs will have any trouble in the future." (James Robins)
- "The DAG approach fully deserves the attention of all researchers and users of causal inference as one of its leading methodologies." (Guido Imbens)
- "Knowledge of causal graphical models is a plus" (job ad by Facebook)

Section 2

Survey and Overview

Survey

- How would you rate your knowledge in potential outcomes / counterfactuals?

Survey

- How would you rate your knowledge in potential outcomes / counterfactuals?
  - Very good
  - Good
  - Ok
  - A bit
  - Nada
- How would you rate your knowledge in causal graphs?

Survey

- How would you rate your knowledge in potential outcomes / counterfactuals?
  - Very good
  - Good
  - Ok
  - A bit
  - Nada
- How would you rate your knowledge in causal graphs?
  - Very good
  - Good
  - Ok
  - A bit
  - Nada

Survey: Structural Equations

- $Y = X\beta + \epsilon$. When does β stand for the causal effect of X on Y?

Survey: Structural Equations

- $Y = X\beta + \epsilon$. When does β stand for the causal effect of X on Y?
  - Never

Survey: Structural Equations

- $Y = X\beta + \epsilon$. When does β stand for the causal effect of X on Y?
  - Never
  - If a researcher says so, and then w/o further assumptions

Survey: Structural Equations

- $Y = X\beta + \epsilon$. When does β stand for the causal effect of X on Y?
  - Never
  - If a researcher says so, and then w/o further assumptions
  - If $E[\epsilon|X] = 0$ ("exogeneity")

Choosing Control Variables

- Suppose you are interested in effect of X on Y

Choosing Control Variables

- Suppose you are interested in effect of X on Y
- You ponder statistical control for other variable Z

Choosing Control Variables

- Suppose you are interested in effect of X on Y
- You ponder statistical control for other variable Z
- A good rule for choosing whether to include Z would have two properties:

Choosing Control Variables

- Suppose you are interested in effect of X on Y
- You ponder statistical control for other variable Z
- A good rule for choosing whether to include Z would have two properties:
  - It tells you which Z you must control for

Choosing Control Variables

- Suppose you are interested in effect of X on Y
- You ponder statistical control for other variable Z
- A good rule for choosing whether to include Z would have two properties:
  - It tells you which Z you must control for
  - It never tells you to control for a Z which actually introduces bias

Choosing Control Variables

- What do you think are good rules for choosing control variables?

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y
  - Z associated with X

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y
  - Z associated with X
  - Unaffected by X and associated with X and Y (K. Imai)

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y
  - Z associated with X
  - Unaffected by X and associated with X and Y (K. Imai)
  - affects Y

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y
  - Z associated with X
  - Unaffected by X and associated with X and Y (K. Imai)
  - affects Y
  - $X \perp\!\!\!\perp Y | Z$

Choosing Control Variables

- What do you think are good rules for choosing control variables?
- Control for Z if...
  - Z associated with Y
  - Z associated with X
  - Unaffected by X and associated with X and Y (K. Imai)
  - affects Y
  - $X \perp\!\!\!\perp Y | Z$
  - $X \perp\!\!\!\perp Y(x) | Z$

Choosing Control Variables

- *All* of these rules violate the two requirements

Choosing Control Variables

- *All* of these rules violate the two requirements
- Except for $X \perp\!\!\!\perp Y(x) | Z$

Choosing Control Variables

- *All* of these rules violate the two requirements
- Except for $X \perp\!\!\!\perp Y(x) | Z$
- "Conditional ignorability"

Choosing Control Variables

- *All* of these rules violate the two requirements
- Except for $X \perp\!\!\!\perp Y(x) | Z$
- "Conditional ignorability"
- "Conditional on Z, the potential outcome of Y when X is set to x is independent of X"

Choosing Control Variables

- *All* of these rules violate the two requirements
- Except for $X \perp\!\!\!\perp Y(x) | Z$
- "Conditional ignorability"
- "Conditional on Z, the potential outcome of Y when X is set to x is independent of X"
- Central assumption in potential outcomes framework

Choosing Control Variables

- *All* of these rules violate the two requirements
- Except for $X \perp\!\!\!\perp Y(x) | Z$
- "Conditional ignorability"
- "Conditional on Z, the potential outcome of Y when X is set to x is independent of X"
- Central assumption in potential outcomes framework
- Who is confident in understanding this assumption?

Structural Causal Models

- SCM: Graphs are understood as structural equations, and define potential outcomes
- Potential outcomes are an important, indispensable part of "the DAG approach"
- Every assumption in potential outcomes can be depicted using a graph

Structural Causal Models

- SCM: Graphs are understood as structural equations, and define potential outcomes
- Potential outcomes are an important, indispensable part of "the DAG approach"
- Every assumption in potential outcomes can be depicted using a graph
- Contra Twitter, *there is no formal difference. It is only about representation of the math, and how easy to understand it is*

Structural Causal Models

- Three main problems and tools:

Structural Causal Models

- Three main problems and tools:
  - Understanding independencies implied by graph

Structural Causal Models

- Three main problems and tools:
  - Understanding independencies implied by graph
  - Definition and identification of causal effects using graphical rules

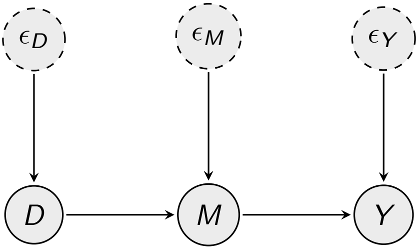
Structural Causal Models

- Three main problems and tools:
  - Understanding independencies implied by graph
  - Definition and identification of causal effects using graphical rules
  - Structural Definition of Potential Outcomes / Counterfactuals

Section 3

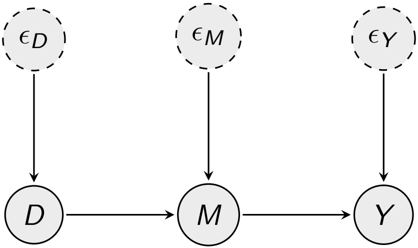
Graph Basics and d-separation

What are the Assumptions?



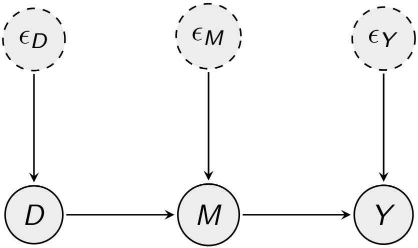
- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?

What are the Assumptions?



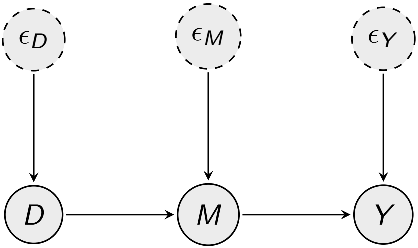
- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?
- *Absent* arrows:

What are the Assumptions?



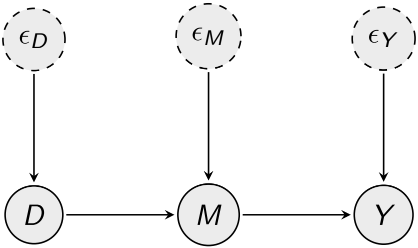
- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?
- *Absent* arrows:
  - From D to Y

What are the Assumptions?



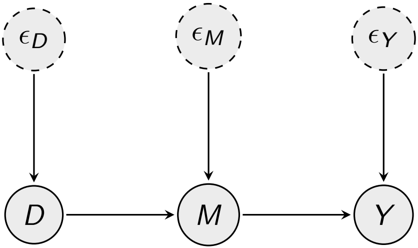
- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?
- *Absent* arrows:
  - From D to Y
  - from ϵ_D to M and Y, etc.

What are the Assumptions?



- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?
- *Absent* arrows:
  - From D to Y
  - from ϵ_D to M and Y, etc.
  - from M to D and Y to M or D (would make graph cyclic)

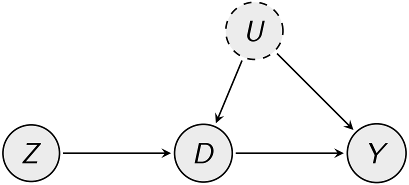
What are the Assumptions?



- This is a directed acyclic graph (DAG)
- Which assumptions are depicted here?
- *Absent* arrows:
  - From D to Y
  - from ϵ_D to M and Y, etc.
  - from M to D and Y to M or D (would make graph cyclic)
- Sth. you have to learn: Look for arrows that aren't there

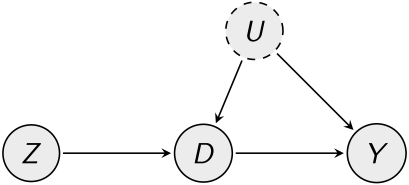
Exercise:
Draw "the" instrumental variables
graph with
instrument Z
treatment X
outcome Y
unobserved confounder U

The IV Graph



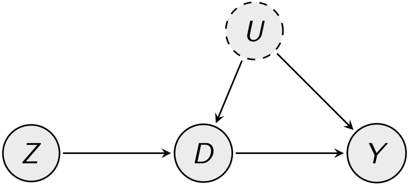
- Suppose you run a regression of Y on Z and D. Will the coefficient on Z be zero?

The IV Graph



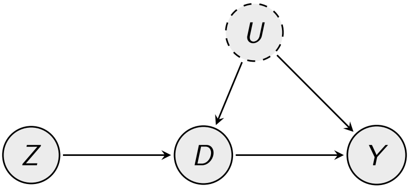
- Suppose you run a regression of Y on Z and D. Will the coefficient on Z be zero?
- Discuss with your neighbor!

The IV Graph



- Suppose you run a regression of Y on Z and D. Will the coefficient on Z be zero?
- Discuss with your neighbor!
- We want to know whether a graph implies independencies between variables

The IV Graph



- Suppose you run a regression of Y on Z and D. Will the coefficient on Z be zero?
- Discuss with your neighbor!
- We want to know whether a graph implies independencies between variables
- $\implies$ d-separation

d-separation I

- In this graph, do D and Y correlate?

d-separation I

- In this graph, do D and Y correlate?
  - Yes

d-separation I

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?

d-separation I

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?
  - No

d-separation I

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?
  - No
- The path is *open*. Conditional on M, it is *blocked*

d-separation I

$$D \longrightarrow M \longrightarrow Y$$

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?
  - No
- The path is *open*. Conditional on M, it is *blocked*
- Simulation in R:

```
D <- rnorm(1000)
M <- 0.4*D + rnorm(1000)
Y <- -0.6*M rnorm(1000)
lm(Y ~ D)
lm(Y ~ D + M)
```

d-separation II

- In this graph, do D and Y correlate?

d-separation II

$$D \leftarrow M \rightarrow Y$$

- In this graph, do D and Y correlate?
  - Yes

d-separation II

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?

d-separation II

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?
  - No

d-separation II

- In this graph, do D and Y correlate?
  - Yes
- Do D and Y correlate when I control for / condition on M?
  - No
- The path is *open*. Conditional on M, it is *blocked*

d-separation III

- In this graph, do D and Y correlate?

d-separation III

- In this graph, do D and Y correlate?
  - No

d-separation III

- In this graph, do D and Y correlate?
  - No
- Do D and Y correlate when I control for / condition on M?

d-separation III

- In this graph, do D and Y correlate?
  - No
- Do D and Y correlate when I control for / condition on M?
  - Yes

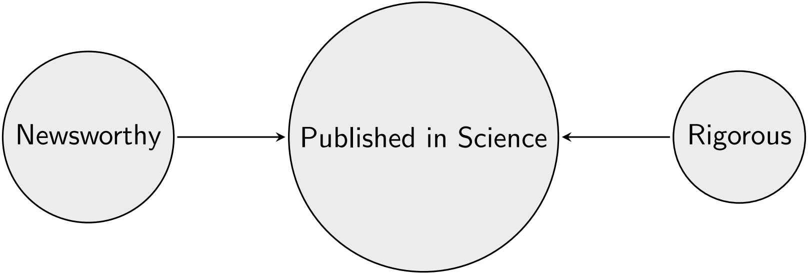
d-separation III

- In this graph, do D and Y correlate?
  - No
- Do D and Y correlate when I control for / condition on M?
  - Yes
- The path is *blocked*. Conditional on M, it is *open*

d-separation III

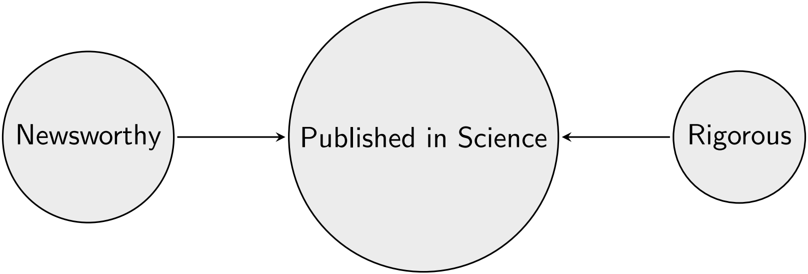
- In this graph, do D and Y correlate?
  - No
- Do D and Y correlate when I control for / condition on M?
  - Yes
- The path is *blocked*. Conditional on M, it is *open*
- M acts as a *collider*

Collider: Example



- If study is newsworthy and published in science...

Collider: Example



- If study is newsworthy and published in science...
- ... it is probably less rigorous

d-separation: Summary

- Chain of mediation: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y | M$.

d-separation: Summary

- Chain of mediation: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y | M$.
- Common cause/fork: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y | M$.

d-separation: Summary

- Chain of mediation: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y | M$.
- Common cause/fork: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y | M$.
- Collider: Path is blocked unconditionally, but open conditional on the middle node or one of its descendants. $D \perp\!\!\!\perp Y$ but $D \not\perp\!\!\!\perp Y | M$.

d-separation: Summary

- Chain of mediation: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y|M$.
- Common cause/fork: Path is open unconditionally, but blocked conditional on the middle node. $D \not\perp\!\!\!\perp Y$ but $D \perp\!\!\!\perp Y|M$.
- Collider: Path is blocked unconditionally, but open conditional on the middle node or one of its descendants. $D \perp\!\!\!\perp Y$ but $D \not\perp\!\!\!\perp Y|M$.
- What if there are multiple, longer paths between D and Y? Will D and Y be (conditionally) independent? **d-separation** gives the answer

d-separation: Definition

- A path p is blocked by a set of nodes Z if and only if
  1. p contains a chain of nodes $X \rightarrow M \rightarrow Y$ or a fork $X \leftarrow M \rightarrow Y$ such that the middle node M is in Z (i.e., M is conditioned on), or
  2. p contains a collider $X \rightarrow M \leftarrow Y$ such that the collision node M is not in Z, and no descendant of B is in Z

d-separation: Definition

- A path p is blocked by a set of nodes Z if and only if
  1. p contains a chain of nodes $X \rightarrow M \rightarrow Y$ or a fork $X \leftarrow M \rightarrow Y$ such that the middle node M is in Z (i.e., M is conditioned on), or
  2. p contains a collider $X \rightarrow M \leftarrow Y$ such that the collision node M is not in Z, and no descendant of B is in Z
- If Z blocks every path between two nodes X and Y, then X and Y are **d-separated**, conditional on Z, and thus are independent conditional on Z

d-separation: Definition

- A path p is blocked by a set of nodes Z if and only if
  1. p contains a chain of nodes $X \rightarrow M \rightarrow Y$ or a fork $X \leftarrow M \rightarrow Y$ such that the middle node M is in Z (i.e., M is conditioned on), or
  2. p contains a collider $X \rightarrow M \leftarrow Y$ such that the collision node M is not in Z, and no descendant of B is in Z
- If Z blocks every path between two nodes X and Y, then X and Y are **d-separated**, conditional on Z, and thus are independent conditional on Z
- **testable implication** of the graph

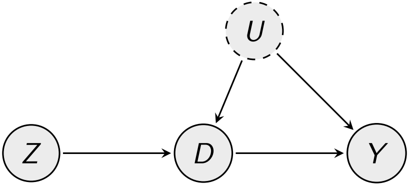
d-separation: Definition

- A path p is blocked by a set of nodes Z if and only if
  1. p contains a chain of nodes $X \to M \to Y$ or a fork $X \leftarrow M \to Y$ such that the middle node M is in Z (i.e., M is conditioned on), or
  2. p contains a collider $X \to M \leftarrow Y$ such that the collision node M is not in Z, and no descendant of B is in Z
- If Z blocks every path between two nodes X and Y, then X and Y are **d-separated**, conditional on Z, and thus are independent conditional on Z
- **testable implication** of the graph
- "d-separation" = "directional separation" (in directed graphs)

d-separation: Definition

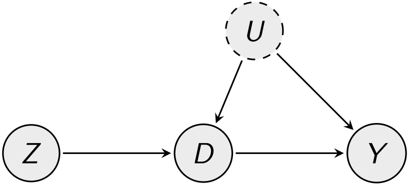
- A path p is blocked by a set of nodes Z if and only if
  1. p contains a chain of nodes $X \to M \to Y$ or a fork $X \leftarrow M \to Y$ such that the middle node M is in Z (i.e., M is conditioned on), or
  2. p contains a collider $X \to M \leftarrow Y$ such that the collision node M is not in Z, and no descendant of B is in Z
- If Z blocks every path between two nodes X and Y, then X and Y are **d-separated**, conditional on Z, and thus are independent conditional on Z
- **testable implication** of the graph
- "d-separation" = "directional separation" (in directed graphs)
- Path p may be very long, but as long as you block sub-path, you block the whole path

The IV Graph



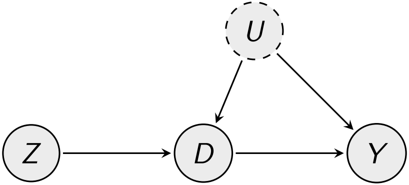
- Suppose you run a regression of Y on Z and D. Will the coefficient on Z be zero?
- Enumerate all paths between Z and Y. Check whether there are any open paths, conditional on D
- Then check back with your neighbor

The IV Graph



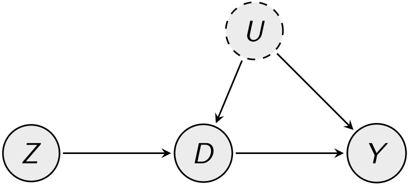
- $Z \to D \to Y$

The IV Graph



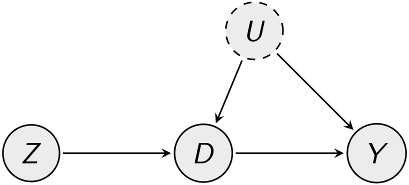
- $Z \to D \to Y$
- $Z \to D \leftarrow U \to Y$

The IV Graph



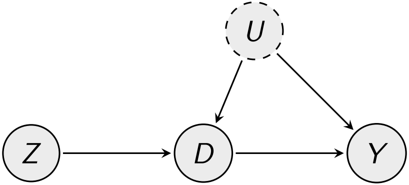
- $Z \rightarrow D \rightarrow Y$
- $Z \rightarrow D \leftarrow U \rightarrow Y$
- First path is "mediation", blocked conditional on D

The IV Graph



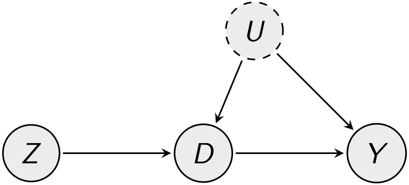
- $Z \rightarrow D \rightarrow Y$
- $Z \rightarrow D \leftarrow U \rightarrow Y$
- First path is "mediation", blocked conditional on D
- Second path: D is collider, open conditional on D

The IV Graph



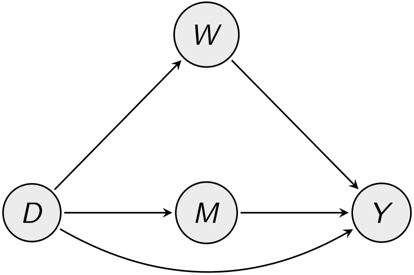
- $Z \to D \to Y$
- $Z \to D \leftarrow U \to Y$
- First path is "mediation", blocked conditional on D
- Second path: D is collider, open conditional on D
- $\implies$ In regression of Y on Z and D, coefficient of Z will be non-zero

The IV Graph



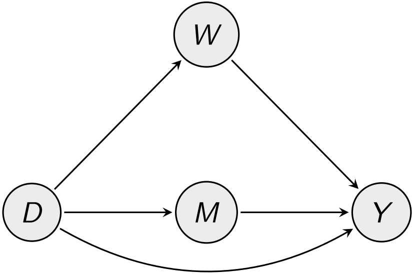
- $Z \to D \to Y$
- $Z \to D \leftarrow U \to Y$
- First path is "mediation", blocked conditional on D
- Second path: D is collider, open conditional on D
- $\implies$ In regression of Y on Z and D, coefficient of Z will be non-zero
- Even though Z is a valid IV, no direct effect

d-separation in Mediation Model



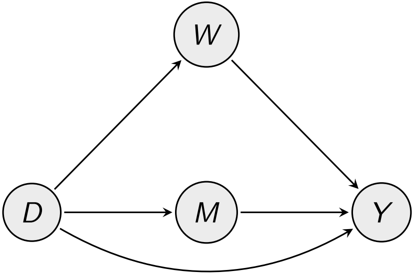
- Does this graph have any testable implications? Check by inspecting whether any pair of variables are d-separated

d-separation in Mediation Model



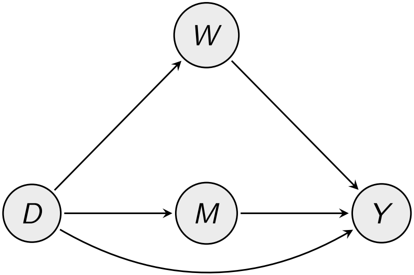
- Does this graph have any testable implications? Check by inspecting whether any pair of variables are d-separated
- Hint: Variables that are directly connected can never be d-separated

d-separation in Mediation Model



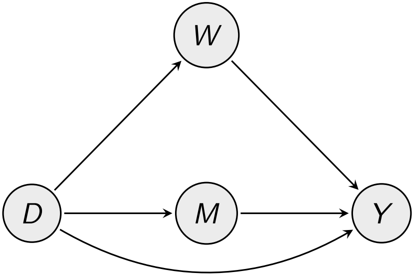
- Does this graph have any testable implications? Check by inspecting whether any pair of variables are d-separated
- Hint: Variables that are directly connected can never be d-separated
- Work with your neighbor

d-separation in Mediation Model



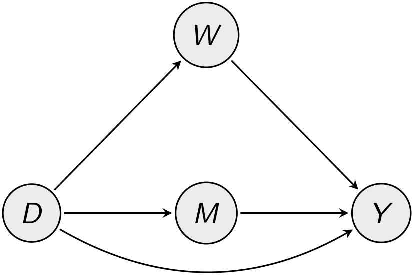
- Only candidate pair: M and W

d-separation in Mediation Model



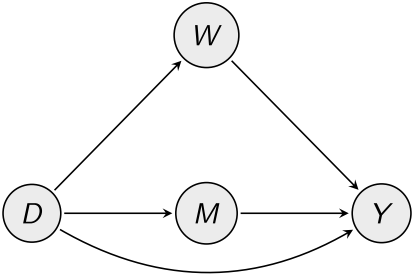
- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$

d-separation in Mediation Model



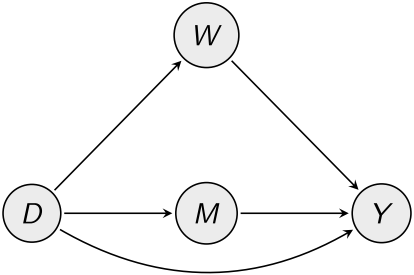
- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$

d-separation in Mediation Model



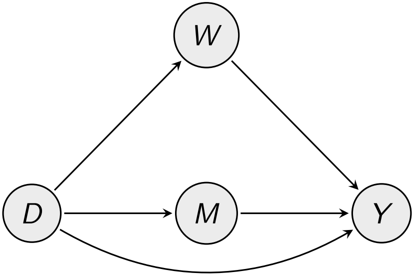
- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$
- $W \leftarrow D \rightarrow Y \leftarrow M$

d-separation in Mediation Model



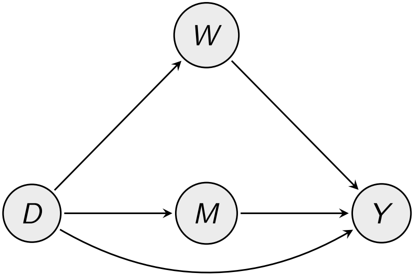
- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$
- $W \leftarrow D \rightarrow Y \leftarrow M$
- $\implies$ W and M d-separated conditional on D

d-separation in Mediation Model



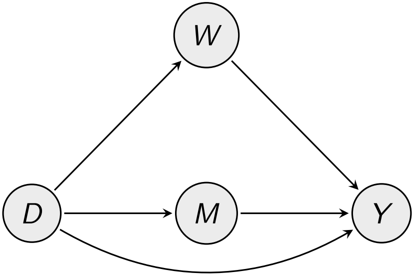
- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$
- $W \leftarrow D \rightarrow Y \leftarrow M$
- $\implies$ W and M d-separated conditional on D
- Test, e.g.: Regress W on M and D. Which coefficient should be zero?

d-separation in Mediation Model



- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$
- $W \leftarrow D \rightarrow Y \leftarrow M$
- $\implies$ W and M d-separated conditional on D
- Test, e.g.: Regress W on M and D. Which coefficient should be zero?
- Coefficient on M

d-separation in Mediation Model

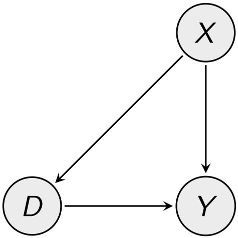


- Only candidate pair: M and W
- $W \leftarrow D \rightarrow M$
- $W \rightarrow Y \leftarrow M$
- $W \leftarrow D \rightarrow Y \leftarrow M$
- $\implies W$ and M d-separated conditional on D
- Test, e.g.: Regress W on M and D. Which coefficient should be zero?
- Coefficient on M
- If not: misspecified regression / Type 1 error / graph wrong

Section 4

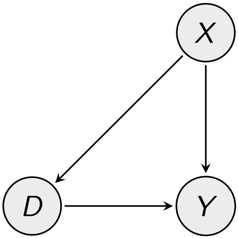
Definition and Identification of Causal Effects

Definition of Causal Effects



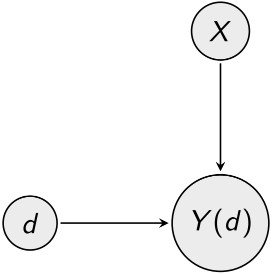
- Hypothetical control of D used to define causal effects

Definition of Causal Effects



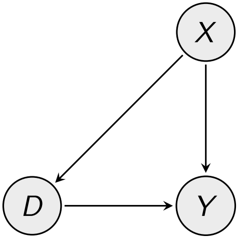
- Hypothetical control of D used to define causal effects
- How does the graph change if D is set to d externally?

Definition of Causal Effects

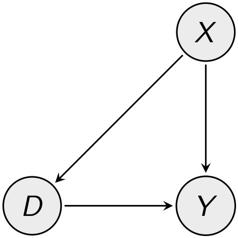


- $E[Y|do(D = d)] = E[Y(d)]$

Intuition

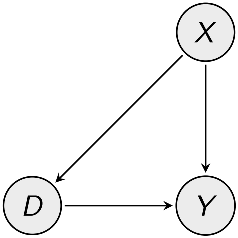


Intuition



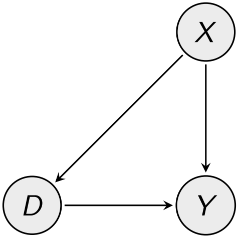
- Which paths does the association between D and Y consist of?

Intuition



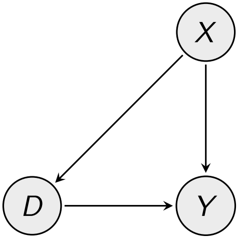
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and

Intuition



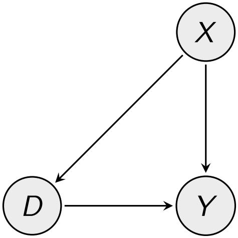
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and 2) confounding due to X

Intuition



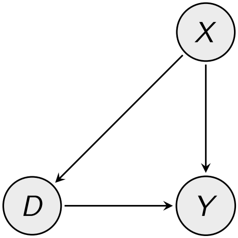
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and 2) confounding due to X
- We want to estimate $E[Y|do(D = d)]$

Intuition



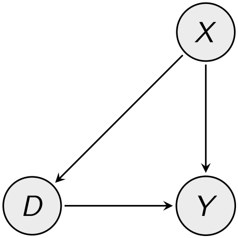
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and 2) confounding due to X
- We want to estimate $E[Y|do(D = d)]$
- If you cannot $do(d)$ in reality, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked

Intuition



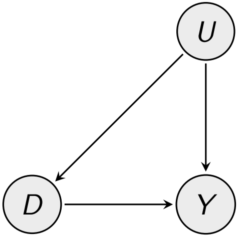
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and 2) confounding due to X
- We want to estimate $E[Y|do(D = d)]$
- If you cannot $do(d)$ in reality, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked
  - All "causal" paths are left open

Intuition



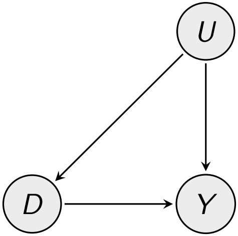
- Which paths does the association between D and Y consist of?
- 1) causal effect of D on Y and 2) confounding due to X
- We want to estimate $E[Y|do(D = d)]$
- If you cannot $do(d)$ in reality, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked
  - All "causal" paths are left open
  - No new "non-causal" paths are opened up (colliders...)

Intuition



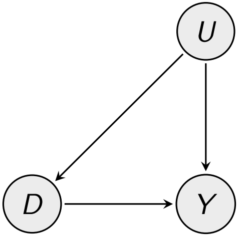
- If you cannot intervene, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked
  - All "causal" paths are left open
  - No new "non-causal" paths are opened up (colliders...)

Intuition



- If you cannot intervene, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked
  - All "causal" paths are left open
  - No new "non-causal" paths are opened up (colliders...)
- This is UNRELATED to d-separation: d-separation is for testing graphs; and if two variables are d-separated, by definition all paths between them are blocked

Intuition



- If you cannot intervene, find control variables such that
  - "Bad", "spurious", "non-causal" paths between D and Y are blocked
  - All "causal" paths are left open
  - No new "non-causal" paths are opened up (colliders...)
- This is UNRELATED to d-separation: d-separation is for testing graphs; and if two variables are d-separated, by definition all paths between them are blocked
- But for identifying causal effects, we certainly want to leave certain paths open (although we also want to block *some*)

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and

  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and

  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

- *Ordered* pair because D is cause, Y is effect

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and
  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

- *Ordered* pair because D is cause, Y is effect
- A path that starts with an arrow into D is called a **back-door path**

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and

  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

- *Ordered* pair because D is cause, Y is effect
- A path that starts with an arrow into D is called a **back-door path**
- Blocking back-door paths makes sure we block "bad" paths

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and
  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

- *Ordered* pair because D is cause, Y is effect
- A path that starts with an arrow into D is called a **back-door path**
- Blocking back-door paths makes sure we block "bad" paths
- Not conditioning on descendants of D makes sure we leave all "good" causal paths open and that we do not open up new bad paths

The Back-Door Criterion

- Given an ordered pair of variables (D, Y) in a DAG G, a set of variables X satisfies the backdoor criterion relative to (D, Y) if

  1) no node in X is a descendant of D, and

  2) X blocks every path between D and Y that contains an arrow into D, or: X d-separates D from Y in $G_{\underline{D}}$

- *Ordered* pair because D is cause, Y is effect
- A path that starts with an arrow into D is called a **back-door path**
- Blocking back-door paths makes sure we block "bad" paths
- Not conditioning on descendants of D makes sure we leave all "good" causal paths open and that we do not open up new bad paths
- Holds for any DAG $\implies$ non-parametric, distribution-free

Section 5

Post-Treatment Bias

Post-Treatment Variables: Problem 1

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?

Post-Treatment Variables: Problem 1

$$D \longrightarrow M \longrightarrow Y$$

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary

Post-Treatment Variables: Problem 1

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)

Post-Treatment Variables: Problem 1

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)
- No paths into D - just like we intervened on it

Post-Treatment Variables: Problem 1

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)
- No paths into D - just like we intervened on it
- Does M correlate with D and Y?

Post-Treatment Variables: Problem 1

$$D \longrightarrow M \longrightarrow Y$$

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)
- No paths into D - just like we intervened on it
- Does M correlate with D and Y?
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"

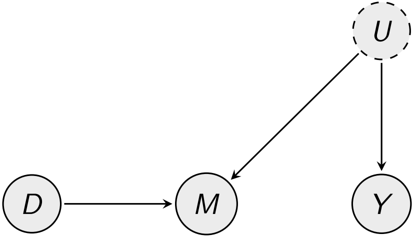
Post-Treatment Variables: Problem 1

- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)
- No paths into D - just like we intervened on it
- Does M correlate with D and Y?
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"
- Bad idea: Conditional on M, D and Y are d-separated! Even though D may have an effect on Y

Post-Treatment Variables: Problem 1

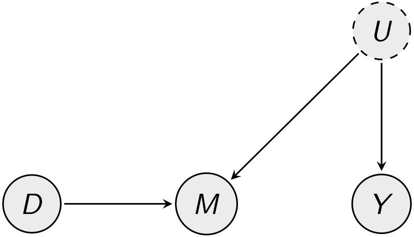
- Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set $\emptyset$ - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0) = E[Y|D=1] - E[Y|D=0]$ (correlation is causation)
- No paths into D - just like we intervened on it
- Does M correlate with D and Y?
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"
- Bad idea: Conditional on M, D and Y are d-separated! Even though D may have an effect on Y
- Montgomery et al. 2018 AJPS estimate that 50 % of political science experiments do this. Huge problem.

Post-Treatment Variables: Problem 2



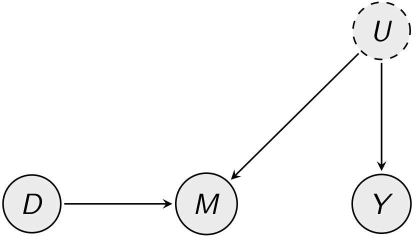
- It gets worse.

Post-Treatment Variables: Problem 2



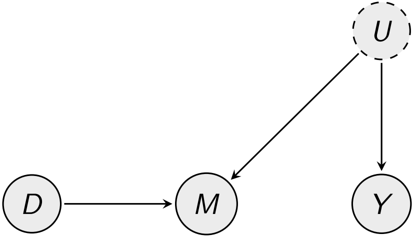
- It gets worse. Which set of variables in this graph satisfy the BDC wrt effect of D on Y?

Post-Treatment Variables: Problem 2



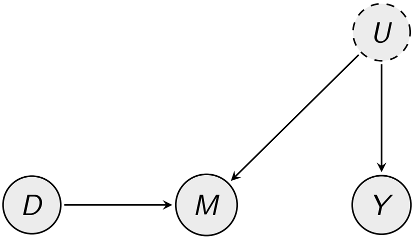
- It gets worse. Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set - no controls necessary

Post-Treatment Variables: Problem 2



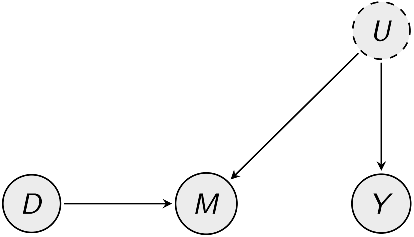
- It gets worse. Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0)] = E[Y|D=1] - E[Y|D=0]$. What is $E[Y|D]$?

Post-Treatment Variables: Problem 2



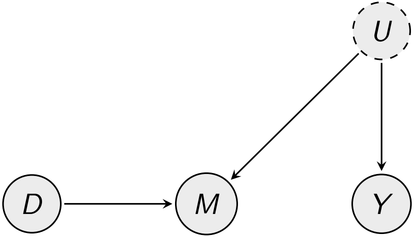
- It gets worse. Which set of variables in this graph satisfy the BDC wrt effect of D on Y?
- The empty set - no controls necessary
- $E[Y|do(D=1)] - E[Y|do(D=0)] = E[Y|D=1] - E[Y|D=0]$. What is $E[Y|D]$?
- $E[Y|D] = E[Y]$ by d-separation. Correct estimator equals $E[Y] - E[Y] = 0$. Which is also clear from the graph.

Post-Treatment Variables: Problem 2



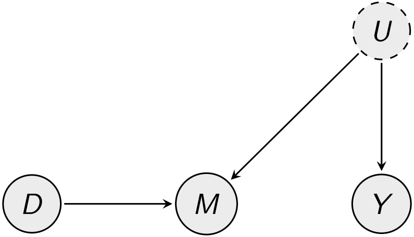
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"

Post-Treatment Variables: Problem 2



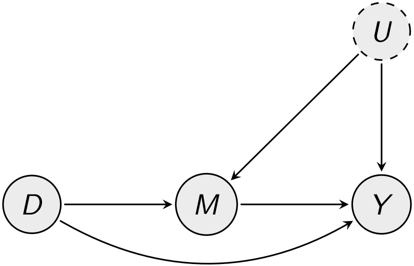
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"
- Bad idea: Conditional on M, D and Y are d-connected! Collider!

Post-Treatment Variables: Problem 2



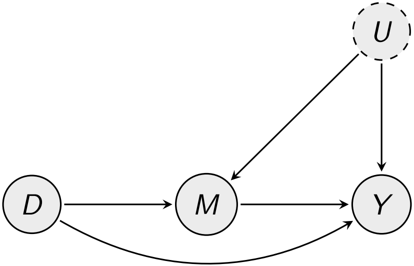
- "M correlates with D and Y. I've learned in stats that I need to control for it. Otherwise, I have omitted-variable bias"
- Bad idea: Conditional on M, D and Y are d-connected! Collider!
- $E[Y|D=1, M=m] \neq E[Y|D=1]$

Post-Treatment Variables: General Case



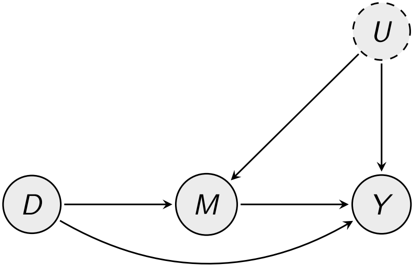
- This graph applies to situations where there are no back-door paths into D. Perhaps via randomization, or you block them by conditioning on X (not shown).

Post-Treatment Variables: General Case



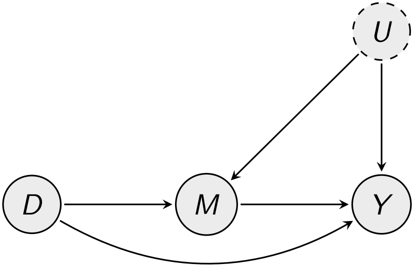
- This graph applies to situations where there are no back-door paths into D. Perhaps via randomization, or you block them by conditioning on X (not shown).
- Conditioning on M is forbidden by the BDC and will have two consequences:

Post-Treatment Variables: General Case



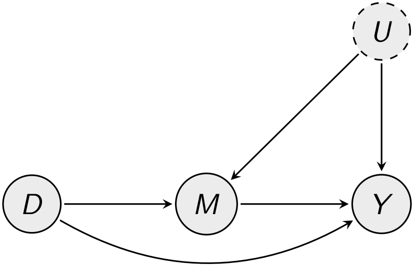
- This graph applies to situations where there are no back-door paths into D. Perhaps via randomization, or you block them by conditioning on X (not shown).
- Conditioning on M is forbidden by the BDC and will have two consequences:
- 1. You block a causal path, which you do not want

Post-Treatment Variables: General Case



- This graph applies to situations where there are no back-door paths into D. Perhaps via randomization, or you block them by conditioning on X (not shown).
- Conditioning on M is forbidden by the BDC and will have two consequences:
- 1. You block a causal path, which you do not want
- 2. You open up a non-causal path, which you do not want

Post-Treatment Variables: General Case



- This graph applies to situations where there are no back-door paths into D. Perhaps via randomization, or you block them by conditioning on X (not shown).
- Conditioning on M is forbidden by the BDC and will have two consequences:
- 1. You block a causal path, which you do not want
- 2. You open up a non-causal path, which you do not want
- This introduces bias, and it can go in any direction

Post-Treatment Variables: Remarks

- Although it is intuitively clear using causal graphs, the fact that conditioning on the descendants of the treatment may actually introduce bias is not well-known

Post-Treatment Variables: Remarks

- Although it is intuitively clear using causal graphs, the fact that conditioning on the descendants of the treatment may actually introduce bias is not well-known
- Usually not mentioned in textbooks that do not use causal graphs

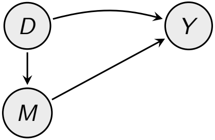
Post-Treatment Variables: Remarks

- Although it is intuitively clear using causal graphs, the fact that conditioning on the descendants of the treatment may actually introduce bias is not well-known
- Usually not mentioned in textbooks that do not use causal graphs
- Even if mentioned, not really explained (see for example "Mostly Harmless Econometrics", section on "Bad Control")

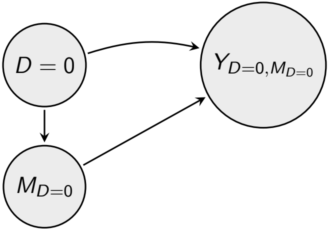
Section 6

Causal Mediation

Intuition for Natural Direct Effects

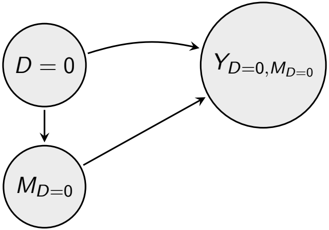


Intuition for Natural Direct Effects

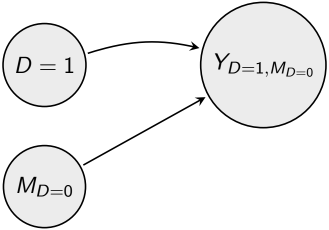


(a) Set D = 0

Intuition for Natural Direct Effects

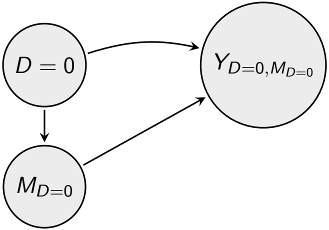


(a) Set D = 0

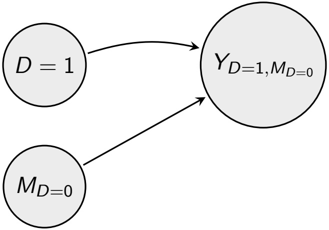


(b) Set D = 1, but disable D $\rightarrow$ M path

Intuition for Natural Direct Effects



(a) Set D = 0

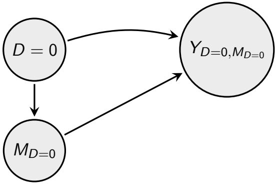


(b) Set D = 1, but disable D $\rightarrow$ M path

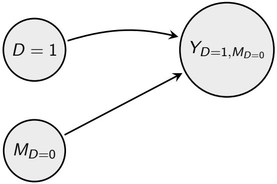
- Note that D is fixed while $M_{D=d}$ is random throughout!

Definition of Natural Direct Effects

- First intervention gives $E[Y_{D=0,M_{D=0}}] = E[Y_{D=0}]$



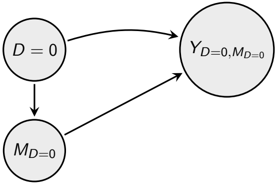
(a) Set D = 0



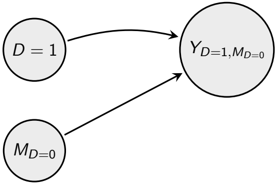
(b) Set D = 1, but disable D $\rightarrow$ M path

Definition of Natural Direct Effects

- First intervention gives $E[Y_{D=0,M_{D=0}}] = E[Y_{D=0}]$
- Second intervention gives $E[Y_{D=1,M_{D=0}}]$



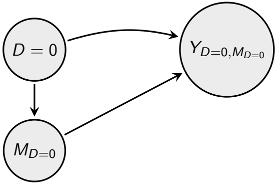
(a) Set D $=$ 0



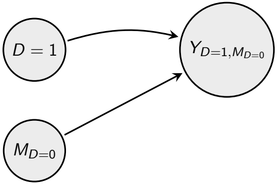
(b) Set D $=$ 1, but disable D $\rightarrow$ M path

Definition of Natural Direct Effects

- First intervention gives $E[Y_{D=0,M_{D=0}}] = E[Y_{D=0}]$
- Second intervention gives $E[Y_{D=1,M_{D=0}}]$
- Difference is $E[Y_{D=1,M_{D=0}}] - E[Y_{D=0}] = NDE(0,1)$



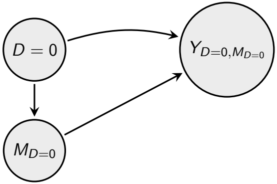
(a) Set D = 0



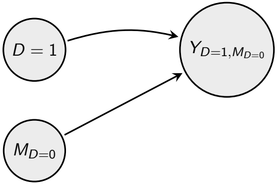
(b) Set D = 1, but disable D → M path

Definition of Natural Direct Effects

- First intervention gives $E[Y_{D=0,M_{D=0}}] = E[Y_{D=0}]$
- Second intervention gives $E[Y_{D=1,M_{D=0}}]$
- Difference is $E[Y_{D=1,M_{D=0}}] - E[Y_{D=0}] = NDE(0,1)$
- The **natural direct effect** of changing D from 0 to 1 while leaving mediator(s) M as if $D = 0$



(a) Set D = 0



(b) Set D = 1, but disable D → M path

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it
- $D \perp\!\!\!\perp M(d), Y(d', m) | X$

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it
- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | D, X$

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it
- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | D, X$
- or was it

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it
- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | D, X$
- or was it
- $D, M(d) \perp\!\!\!\perp Y(d', m) | X$

Sequential Ignorability: Imai et al. 2010

- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | X$
- or was it
- $D \perp\!\!\!\perp M(d), Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | D, X$
- or was it
- $D, M(d) \perp\!\!\!\perp Y(d', m) | X$
- $Y(d', m) \perp\!\!\!\perp M(d') | D, X$?

Sequential Ignorability: Graphical Version

- Graphical version of Sequential Ignorability (Imai et al. 2010) due to Pearl 2014:
- There are covariates X such that

Sequential Ignorability: Graphical Version

- Graphical version of Sequential Ignorability (Imai et al. 2010) due to Pearl 2014:
- There are covariates X such that
- 1. X and D block all D-avoiding back-door paths from M to Y
- 2. X blocks all back-door paths from D to M and from D to Y, and no member of of X is descendant of D

Comparison

- D is jointly independent from potential outcome of M when D is set to d and potential outcome of Y when D is set to d' and M is set to m, conditional on X
- The potential outcome of Y when D is set to d and M is set to m is independent form the potential outcome of M when D is set to d'

Comparison

- D is jointly independent from potential outcome of M when D is set to d and potential outcome of Y when D is set to d' and M is set to m, conditional on X
- The potential outcome of Y when D is set to d and M is set to m is independent form the potential outcome of M when D is set to d'
- There are covariates X such that
- 1. X and D block all D-avoiding back-door paths from M to Y
- 2. X blocks all back-door paths from D to M and from D to Y, and no member of of X is descendant of D

Section 7

Wrap-Up

Wrap-Up

- Everyone interested in causal inference should learn about causal graphs
- Ideally, about structural causal models: Graphs plus structural equations plus potential outcomes
- Best textbook on the market: Pearl/Jewell/Glymour: Causal Inference. A Primer.
- Teaching material at julianschuessler.net

More stuff?

- Sample selection bias / generalizability
- Multiple interventions / controlled direct effects
- Panel Data
- More on IV
- Sensitivity Analysis