Matching Methods for Causal Inference with Time-Series Cross-Sectional Data

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    - *any* binary treatment pattern

Outline of talk

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1. Motivation
2. The Proposed Matching Methodology
3. Simulation Studies
4. Empirical Analysis
5. Conclusion

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  - Find **all** two-period DiD cases in the data with a matching algorithm
- Empirical applications:
  - Democracy and economic growth (Acemoglu et al.)
  - Interstate war and inheritance tax (Scheve & Stasavage)

Democracy and Economic Growth

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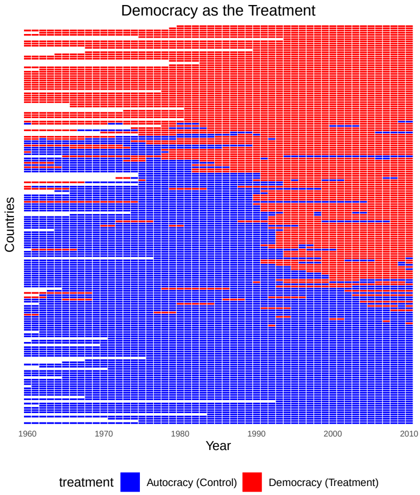
- Two-way FE: Country and year fixed effects
- Binary variable for democracy or not as the treatment

Treatment Variation Plot

- Treatment introduced at different times for different countries



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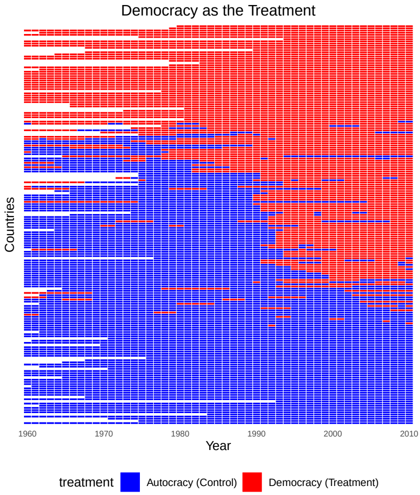

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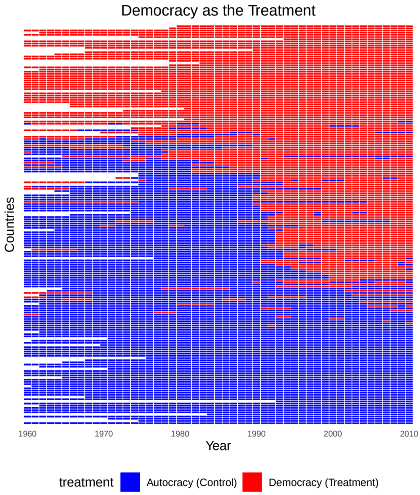

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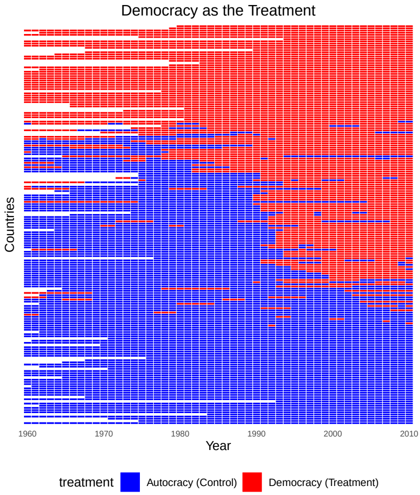
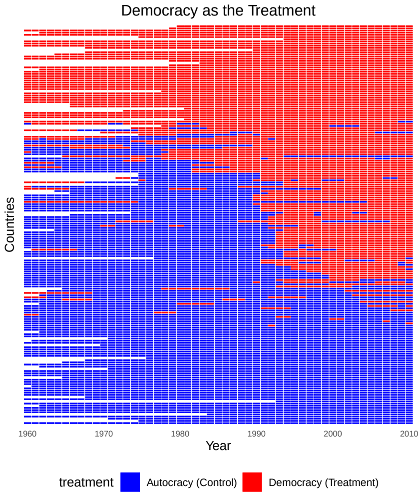


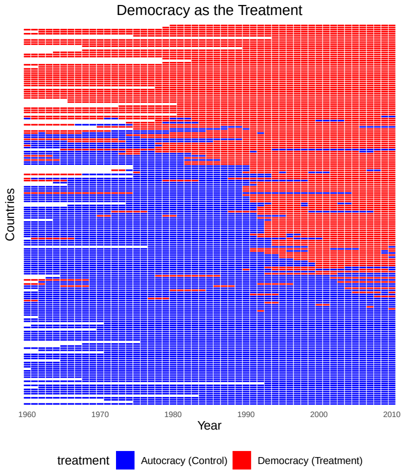
Treatment Variation Plot

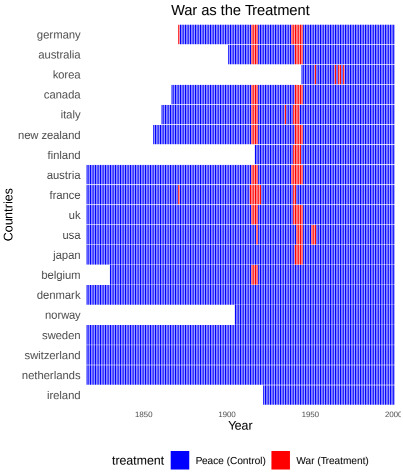
- Treatment introduced at different times for different countries
- Beyond staggered adoption:
  - democratic transition and authoritarian reversal
- Identification strategy:
  - within-unit over-time variation
  - within-time across-units variation
  - variation in the timing of treatment



Treatment Variation Plot

Democracy and Growth by Acemoglu et al.(Left)
War and Inheritance Taxation by Scheve and Stasavage (Right)





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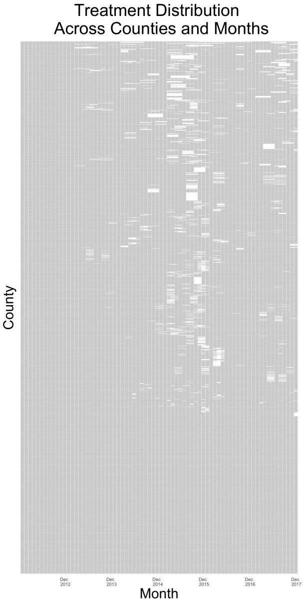

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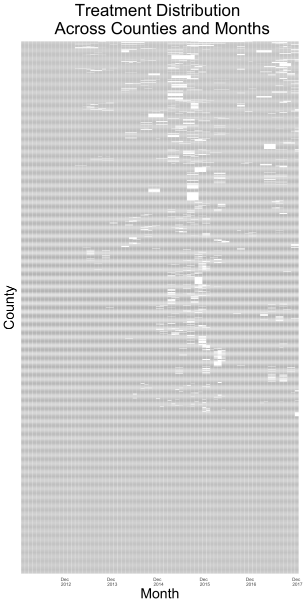

A micro data example: anti-corruption inspection and bureaucrats' productivity (Wang 2022)

72 months and $2,698$ counties

Each cell a "county-month"

Treated (under inspection) cells in white

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  - Treatment timing differs across units
  - Once treated, will remain treated in the following periods

DiD with Staggered Adoption

The $N \times T$ matrix $\boldsymbol{W}$ with typical element W_{it} has the form:

$$\boldsymbol{W}_{N\times T} = \left(\begin{array}{cccccccr}
1 & 2 & 3 & 4 & \dots & T & & \text{(time period)} \\
 & & & & & & & \\
0 & 0 & 0 & 0 & \dots & 0 & & \text{(never adopter)} \\
0 & 0 & 0 & 0 & \dots & 1 & & \text{(late adopter)} \\
0 & 0 & 0 & 0 & \dots & 1 & & \\
0 & 0 & 1 & 1 & \dots & 1 & & \\
0 & 0 & 1 & 1 & \dots & 1 & & \text{(medium adopter)} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & & \\
0 & 1 & 1 & 1 & \dots & 1 & & \text{(early adopter)}
\end{array} \right).$$

Source: Athey & Imbens (2021)

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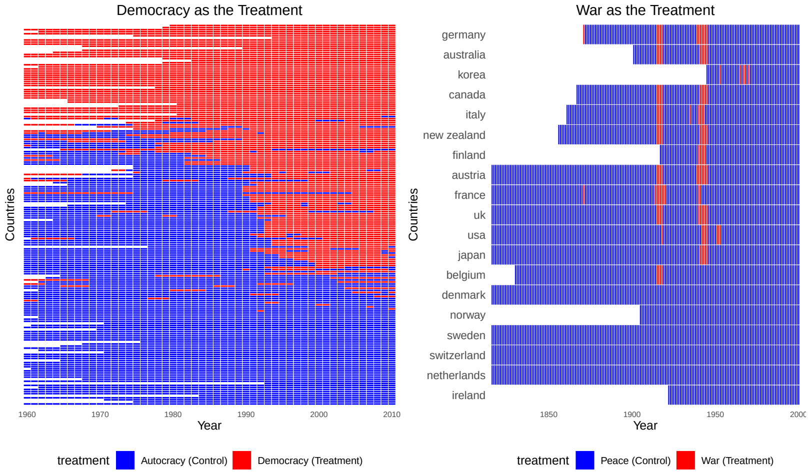

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- Average Treatment Effect of Policy Change for the Treated (ATT):

$$\delta(F, L) = \mathbb{E}\left\{ Y_{i,t+F}\left(X_{it} = 1, X_{i,t-1} = 0, \{X_{i,t-\ell}\}_{\ell=2}^{L}\right) - Y_{i,t+F}\left(X_{it} = 0, X_{i,t-1} = 0, \{X_{i,t-\ell}\}_{\ell=2}^{L}\right) \mid X_{it} = 1, X_{i,t-1} = 0 \right\}$$

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  3. Parallel trend after conditioning:

$$\begin{aligned} &\mathbb{E}[Y_{i,t+F}\left(X_{it} = X_{i,t-1} = 0, \{X_{i,t-\ell}\}_{\ell=2}^{L}\right) - Y_{i,t-1} \\ &\qquad \mid X_{it} = 1, X_{i,t-1} = 0, \{X_{i,t-\ell}, Y_{i,t-\ell}\}_{\ell=2}^{L}, \{\mathbf{Z}_{i,t-\ell}\}_{\ell=0}^{L}] \\ = \; &\mathbb{E}[Y_{i,t+F}\left(X_{it} = X_{i,t-1} = 0, \{X_{i,t-\ell}\}_{\ell=2}^{L}\right) - Y_{i,t-1} \\ &\qquad \mid X_{it} = 0, X_{i,t-1} = 0, \{X_{i,t-\ell}, Y_{i,t-\ell}\}_{\ell=2}^{L}, \{\mathbf{Z}_{i,t-\ell}\}_{\ell=0}^{L}] \end{aligned}$$

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Average Treatment Effect of Policy Reversal for the Treated (ART):

$$\xi(F,L) = \mathbb{E}\left\{ Y_{i,t+F}\left(X_{it}=0, X_{i,t-1}=1, \{X_{i,t-\ell}\}_{\ell=2}^{L}\right) - \right.$$
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Assumption:

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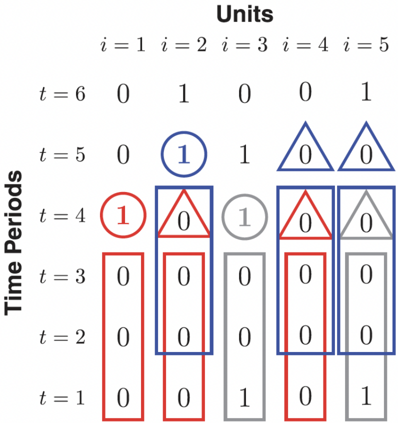
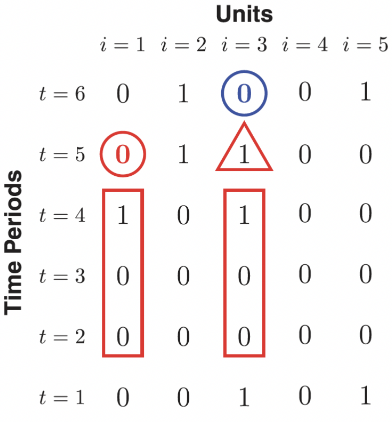
  $\rightsquigarrow$ change the quantity of interest by dropping them
- Similar to the risk set of Li et al. (2001) but we do not exclude those who already receive the treatment

Matched Sets for ATT and ART



(a) Matched Sets for ATT



(b) Matched Sets for ART

An Example of Matched Set

	Country	Year	Democracy	logGDP	Population	Trade
1	Argentina	1974	**1**	888.20	29.11	14.45
2	Argentina	1975	**1**	886.53	29.11	12.61
3	Argentina	1976	**0**	882.91	29.15	12.11
4	Argentina	1977	**0**	888.09	29.32	15.15
5	**Argentina**	**1978**	**0**	**881.99**	**29.57**	**15.54**
6	Argentina	1979	0	890.24	29.85	15.93
7	Argentina	1980	0	892.81	30.12	12.23
8	Argentina	1981	0	885.43	30.33	11.39
9	Argentina	1982	0	878.82	30.62	13.40
10	Thailand	1974	**1**	637.24	43.32	37.76
11	Thailand	1975	**1**	639.51	42.90	41.63
12	Thailand	1976	**0**	645.97	42.44	42.33
13	Thailand	1977	**0**	653.02	41.92	43.21
14	**Thailand**	**1978**	**1**	**660.57**	**41.39**	**42.66**
15	Thailand	1979	1	663.64	40.82	45.27
16	Thailand	1980	1	666.57	40.18	46.69
17	Thailand	1981	1	670.27	39.44	53.40
18	Thailand	1982	1	673.52	38.59	54.22

Another Example of Matched Set (Wang 2022)

	County	Time	Inspection
1	Youxi	Sep 2014	**0**
2	Youxi	Oct 2014	**1**
3	Youxi	Nov 2014	**1**
4	Youxi	Dec 2014	**0**
5	Youxi	Jan 2015	**1**
6	Guangze	Sep 2014	**0**
7	Guangze	Oct 2014	**1**
8	Guangze	Nov 2014	**1**
9	Guangze	Dec 2014	**0**
10	Guangze	Jan 2015	**0**
11	Mengla	Sep 2014	**0**
12	Mengla	Oct 2014	**1**
13	Mengla	Nov 2014	**1**
14	Mengla	Dec 2014	**0**
15	Mengla	Jan 2015	**0**

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  1. Compute the distance between treated and matched control obs.

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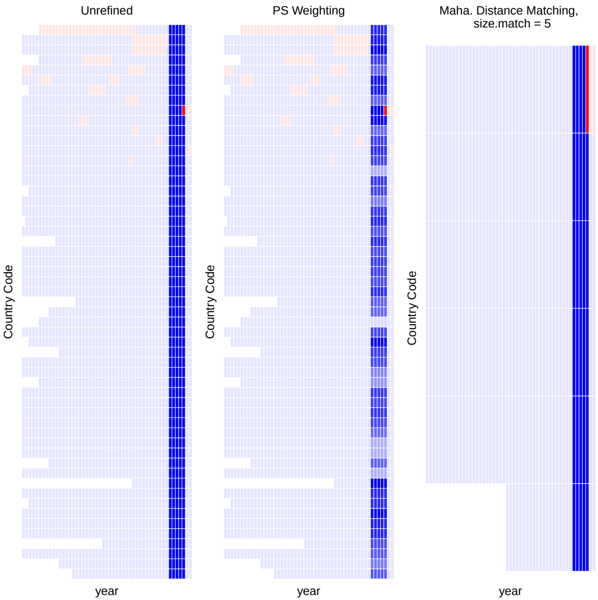
  2. Weight each matched control observation by its inverse
     - could then use scores as weights or pick the closest J, each of which receives equal weight

An Example of Refinement: Bhutan in 2008



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- Same with ART
- Block bootstrap for standard error calculation

An Example of Refinement

	Country	Year	Democracy	logGDP	Population	Trade	**Weight**
1	Argentina	1979	**0**	890.24	29.85	15.93	1.00
2	Argentina	1980	**0**	892.81	30.12	12.23	1.00
3	Argentina	1981	**0**	885.43	30.33	11.39	1.00
4	Argentina	1982	**0**	878.82	30.62	13.40	1.00
5	**Argentina**	**1983**	**1**	**881.09**	**30.75**	**16.46**	**1.00**
6	Argentina	1984	1	881.76	30.77	15.67	1.00
7	Mali	1979	**0**	542.02	43.80	31.18	0.26
8	Mali	1980	**0**	535.65	43.96	41.82	0.26
9	Mali	1981	**0**	529.10	44.07	41.92	0.26
10	Mali	1982	**0**	522.25	44.45	42.53	0.26
11	**Mali**	**1983**	**0**	**524.84**	**44.74**	**43.65**	**0.26**
12	Mali	1984	0	527.13	44.95	45.92	0.26
13	Chad	1979	**0**	506.71	44.61	44.80	0.27
14	Chad	1980	**0**	498.36	44.84	45.75	0.27
15	Chad	1981	**0**	497.18	45.07	51.58	0.27
16	Chad	1982	**0**	500.07	45.44	43.97	0.27
17	**Chad**	**1983**	**0**	**512.20**	**45.76**	**29.22**	**0.27**
18	Chad	1984	0	511.63	46.04	29.91	0.27
19	Uruguay	1979	**0**	858.39	27.23	41.51	0.47
20	Uruguay	1980	**0**	863.39	27.04	37.99	0.47
21	Uruguay	1981	**0**	864.28	26.93	36.20	0.47
22	Uruguay	1982	**0**	853.36	26.86	35.84	0.47
23	**Uruguay**	**1983**	**0**	**841.87**	**26.83**	**33.36**	**0.47**
24	Uruguay	1984	0	840.08	26.82	42.98	0.47

Checking Covariate Balance

- Balance for covariate j at time $t - \ell$ in each matched set:

$$B_{it}(j, \ell) \; = \; \frac{V_{i,t-\ell,j} - \sum_{i' \in \mathcal{M}_{it}} w_{it}^{i'} \; V_{i',t-\ell,j}}{\sqrt{\frac{1}{N_1 - 1} \sum_{i'=1}^{N} \sum_{t'=L+1}^{T-F} D_{i't'}(V_{i',t'-\ell,j} - \overline{V}_{t'-\ell,j})^2}}$$

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- Average this measure across all treated observations:

$$\overline{B}(j,\ell) \;=\; \frac{1}{N_1}\sum_{i=1}^{N}\sum_{t=L+1}^{T-F} D_{it}B_{it}(j,\ell)$$

Outline of the Talk

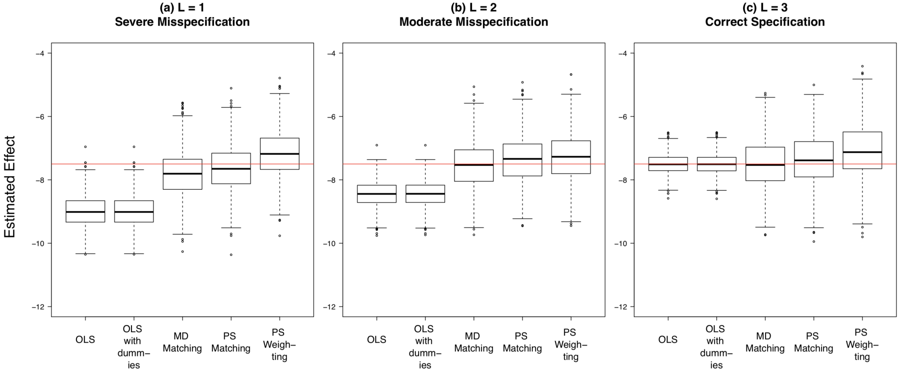
Simulation Studies

- Make simulations realistic by using the Acemoglu et al. data
- Balanced TSCS data with $N = 162$ and $T = 51$
- Impute missing values in the original data set
- Treatment and outcome variables:

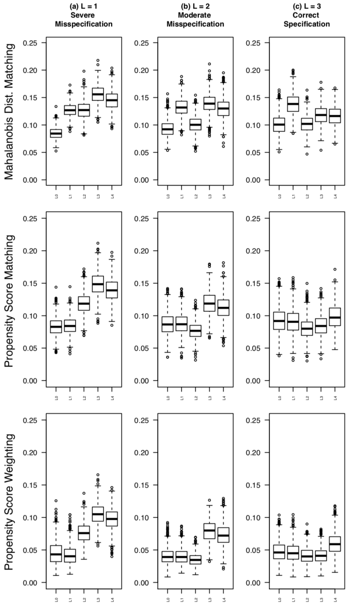
$$
\begin{aligned}
X_{it} &\sim \text{Benoulli}\left(\text{logit}^{-1}\left\{\tilde{\alpha}_i + \tilde{\gamma}_t + \sum_{\ell=1}^{L} \tilde{\beta}_\ell^\top X_{i,t-\ell} + \right.\right. \\
&\qquad\qquad \left.\left. \sum_{\ell=0}^{L}\left(\tilde{\zeta}_\ell^\top \mathbf{Z}_{i,t-\ell} + \tilde{\phi}_\ell^\top [\mathbf{Z}_{i,t-\ell}^{(1)} : \mathbf{Z}_{i,t-\ell}^{(3)}]\right)\right\}\right) \\
Y_{it} &= \alpha_i + \gamma_t + \sum_{\ell=0}^{L} \beta_\ell^\top X_{i,t-\ell} + \sum_{\ell=0}^{L}\left(\zeta_\ell^\top \mathbf{Z}_{i,t-\ell} + \phi_\ell^\top [\mathbf{Z}_{i,t-\ell}^{(1)} : \mathbf{Z}_{i,t-\ell}^{(3)}]\right) + \epsilon_{it}
\end{aligned}
$$

where $\epsilon \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2)$, and $\mathbf{Z}_{i,t-\ell} = (\mathbf{Z}_{i,t-\ell}^{\top(1)}, \mathbf{Z}_{i,t-\ell}^{\top(2)}, \mathbf{Z}_{i,t-\ell}^{\top(3)})^\top$

Robustness to Model Misspecification



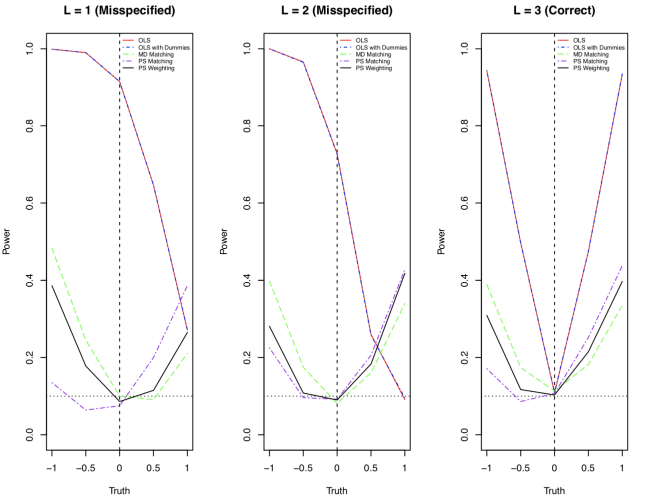
Balance Metrics in the Simulations



Average absolute standardized mean difference across covariates for each L

Desirable balance for moderate misspecification and correct specification

Bias-variance Tradeoff



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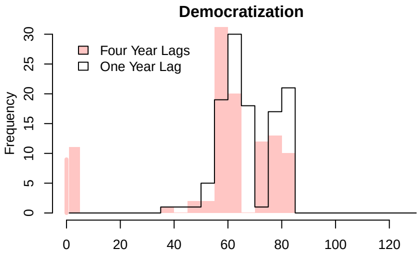
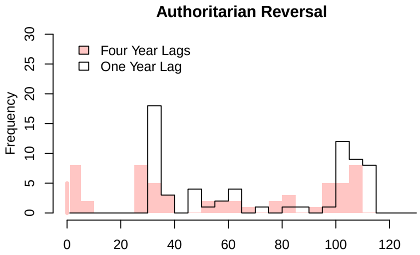
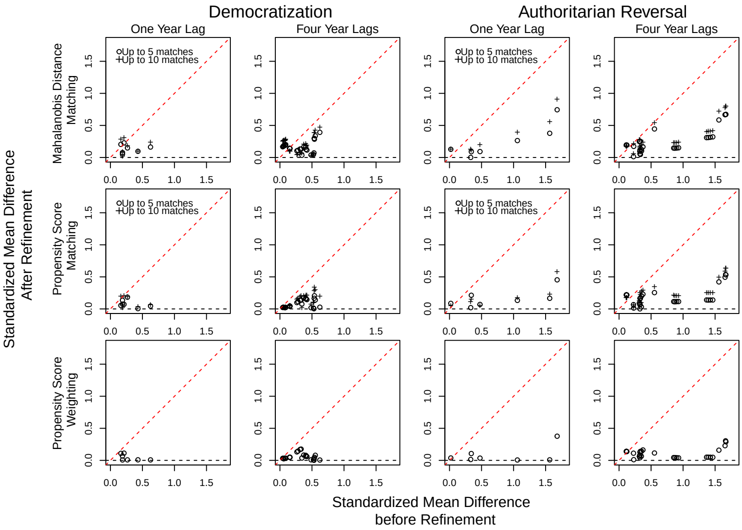
- 9 (5) treated observations have no matched control





Improved Covariate Balance



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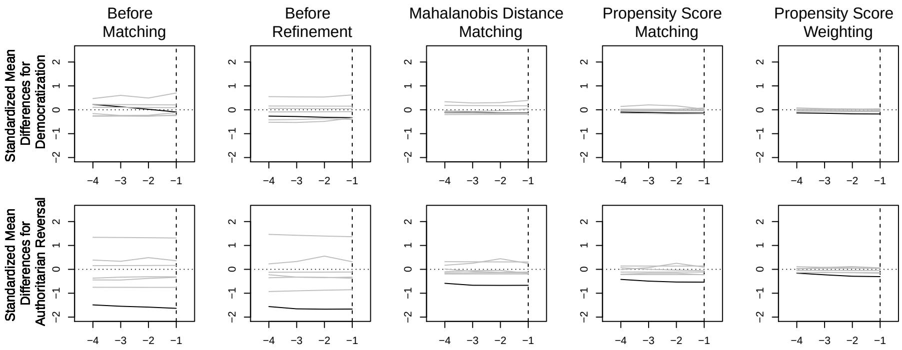
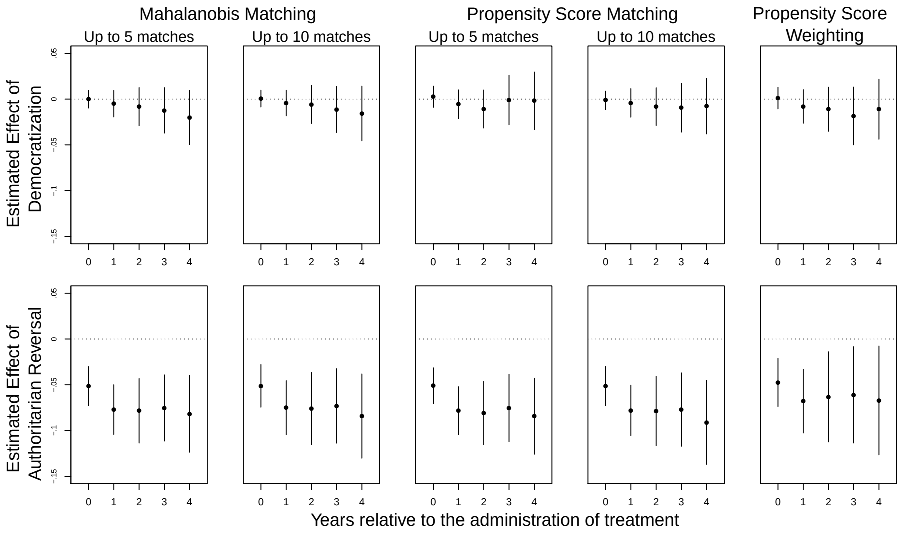


Estimated Causal Effects



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- Ongoing research: possible spillover effects and continuous treatment

References

- Imai, Kim, and Wang. "Matching Methods for Causal Inference with Time-Series Cross-Sectional Data." *American Journal of Political Science*, Forthcoming

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- Send comments and suggestions to:

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