POTENTIAL OUTCOMES

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Logistics

- Workshop materials: `github.com/socialsciencedatalab/potential-outcomes`
- Live stream: `zoom.us/j/386654642`

General information

- Introduction to Potential Outcomes (60 min)
- Simulation Exercise (30 min)
- Prep course for Florian Foos “Randomized Experiments and Randomization Inference”, Sept 16, 15:30-17:00

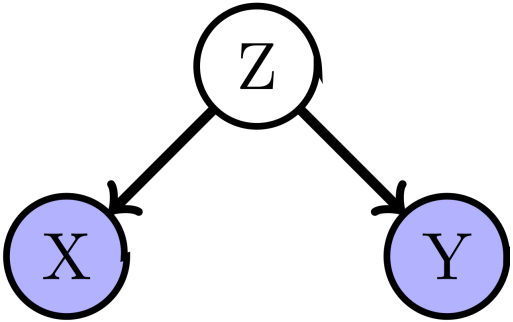
Introduction

USING CORRELATIONS/ASSOCIATIONS FOR CAUSAL INFERENCE

- we nearly always find notable correlations/associations among variables in (observational) data
- correlation/association indicates that there *may* be an effect that *may* have a causal interpretation
- exact identification hinges on the assumption of no unobserved heterogeneity (aka exogeneity, no confounding bias)

CONFOUNDING BIAS (IMAI 2017, P. 58)

A pretreatment variable that is associated with both the treatment and the outcome variables is called a **confounder** and is a source of **confounding bias** in the estimation of the treatment effect.



CAUSAL IDENTIFICATION

- We can try to make this assumption more credible, for instance by
  - subsetting the sample to rather homogeneous subgroups
  - using covariate adjustment to reduce unit heterogeneity
  - matching
  - randomization
- Ultimately, causal identification requires that all observed and unobserved confounders are accounted for – or 'eliminated' by design
- This assumption remains untestable – no matter the research design!
- Causal inference thus requires making untestable assumption
- Causal identification: Development and application of an identification strategy that makes these assumptions as plausible and defensible as possible

Potential Outcomes

POTENTIAL OUTCOMES

- "What if?": Potential outcomes is all about hypothetical counterfactuals
- More specifically: Comparing factual and the counterfactual states
- Core assumption: Every unit in the population has a potential outcome Y_i under each possible treatment condition.
- For a binary treatment D (received=1, not received=0), we speak of treatment and control
- Causal effects can only be identified in comparison of different treatment states at the unit level: $\tau_i = Y_i(D_i = 1) - Y_i(D_i = 0)$

POTENTIAL OUTCOMES: NOTATION

- $i = 1, ..., N$: subscript to denote a unit
- $D_i \in \{0, 1\}$: hypothetical treatment status (here: binary)
- $d_i \in \{0, 1\}$: factual treatment status
- $z_i \in \{0, 1\}$: assignment to treatment/control group (may differ from d_i!)
- $(Y_i(0), Y_i(1))$: potential outcomes of unit i in its untreated and treated state (general: $Y_i(D_i)$)
- $Y_i(d_i)$: factual/observed outcome of unit i
- X_i: covariate(s) (potential confounder(s)) for unit i
- $E[Y_i(1)]$: expectatation of outcome across all units under treatment
- $E[Y_i(1)|D_i = 0]$: expectatation of outcome across all units under treatment conditional on being in the control group – a counterfactual conditional expectation

OUR FICTIONAL EXAMPLE FOR TODAY

Setup

- We have a sample of $N = 1000$ students. Our outcome of interest, Y_i, is knowledge about counterfactual causality measured by test scores in a quiz on causal inference.
- The treatment of interest, D_i, is attending a 90 minute talk titled "Introduction to the Potential Outcomes Framework".
- Assignment of treatment status equals actual receipt of treatment, $z_i = d_i$.
- Students' potential outcomes under the control, $Y_i(0)$, range between 20 and 80.
- Individual treatment effects, τ_i, range between 0 and 20.
- Potential outcomes under the treatment, $Y_i(1)$, range between 20 and 100.

POTENTIAL OUTCOMES, EXAMPLE

i	$Y_i(0)$	$Y_i(1)$	τ_i
1	42.1	49.1	7
2	33.3	46	12.7
3	60.7	71.4	10.8
⋮	⋮	⋮	⋮
998	61.1	73.6	12.5
999	54.8	61.8	7
1000	43.6	52.9	9.3
(mean)	50.5	60.6	10.1

Table 1: What if students took the class? What if they didn't? Potential Outcomes under treatment and control plus idiosyncratic treatment effects

THE FUNDAMENTAL PROBLEM OF CAUSAL INFERENCE (FPCI)

- we never observe more than one potential outcome for a given unit at a time
- if $D_i \in \{0, 1\}$, we observe $Y_i = d_i Y_i(1) + (1 - d_i) Y_i(0)$
  - we observe $Y_i(1)$ if $d_i = 1$
  - we observe $Y_i(0)$ if $d_i = 0$
- if $D_i \in \{0, 1, ..., K\}$, we observe $Y_i = \sum_{k=1}^{K} 1\{d_i = k\} Y_i(k)$
  i.e., we observe $Y_i(k)$ if and only if $d_i = k$

i	$Y_i(0)$	$Y_i(1)$	τ_i
1	42.1	?	?
2	?	46	?
3	?	71.4	?
⋮	⋮	⋮	⋮
998	61.1	?	?
999	?	61.8	?
1000	?	52.9	?
(mean*)	51.2	59.7	8.5

Table 2: The fundamental problem of causal inference illustrated.
* sample means based on observed values.

POSSIBLE SOLUTIONS TO THE FPCI

Remember: Idiosyncratic causal effects can only be identified in comparison of different treatment states at the unit level: $\tau_i = Y_i(1) - Y_i(0)$.

1. Assume temporal stability and causal transience
   - expose each unit i to control, measure $Y_i(0)$; then expose to treatment, measure $Y_i(1)$
   - calculate $\tau_i = Y_i(1) - Y_i(0)$
2. Assume unit homogeneity
   - expose one unit to control, measure $Y_i(0)$; expose other unit to treatment, measure $Y_j(1)$
   - calculate $\tau_{ij} = Y_j(1) - Y_i(0)$
3. Statistical Method
   - shift the focus to expected causal effects
   - randomly split many units into control and treatment groups
   - expose one to control, one to treatment
   - estimate $E[\tau] = E[Y_i(1)] - E[Y_i(0)]$ per $\overline{Y(1)} - \overline{Y(0)}$

AVERAGE TREATMENT EFFECT (MORGAN AND WINSHIP 2015, CH. 2)

- Remember: $Y_i(1)$ and $Y_i(0)$ are hypothetical random variables defined over all units for different treatment conditions
- ATE is the average in potential outcomes:

$$\begin{aligned} E[\tau_i] =& E[Y_i(1) - Y_i(0)] \\ =& E[Y_i(1)] - E[Y_i(0)] \\ =& \{\pi E[Y_i(1)|D_i = 1] + (1-\pi)E[Y_i(1)|D_i = 0]\} \\ & - \{\pi E[Y_i(0)|D_i = 1] + (1-\pi)E[Y_i(0)|D_i = 0]\} \end{aligned}$$

- Which of these can we estimate from observed data? Which are unobservable counterfactual quantities?

AVERAGE TREATMENT EFFECT (ATE)

What we want to know: $E[\tau_i] = E[Y_i(1)] - E[Y_i(0)]$

$$\{\pi \underbrace{E[Y_i(1)|D_i = 1]}_{\text{factual}} + (1-\pi) \underbrace{E[Y_i(1)|D_i = 0]}_{\text{counterfactual}}\}$$
$$- \{\pi \underbrace{E[Y_i(0)|D_i = 1]}_{\text{counterfactual}} + (1-\pi) \underbrace{E[Y_i(0)|D_i = 0]}_{\text{factual}}\}$$

What we do know:

$$\underbrace{\hat{\tau} = E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0]}_{\text{naive estimator}}$$

ESTIMATING THE ATE

- The naive estimator: difference-in-means, $\hat{\tau} = \overline{Y(1)} - \overline{Y(0)}$
- When does this estimator yield an unbiased estimate? What assumption do we make?

$$\{\pi \underbrace{E[Y_i(1)|D_i = 1]}_{\text{as observed}} + (1-\pi) \underbrace{E[Y_i(1)|D_i = 0]}_{\text{assume} = E[Y_i(1)|D_i=1]} \}$$
$$- \{\pi \underbrace{E[Y_i(0)|D_i = 1]}_{\text{assume} = E[Y_i(0)|D_i=0]} \} + (1-\pi) \underbrace{E[Y_i(0)|D_i = 0]}_{\text{as observed}}\}$$

- If these assumptions hold:

$$\underbrace{\{\pi E[Y_i(1)|D_i = 1] + (1-\pi)E[Y_i(1)|D_i = 0]\}}_{=E[Y_i(1)]}$$
$$- \underbrace{\{\pi E[Y_i(0)|D_i = 1]\} + (1-\pi)E[Y_i(0)|D_i = 0]\}}_{=E[Y_i(0)]}$$

- But when are these defensible assumptions to make?

RANDOMIZATION

Imagine you were randomly split into two groups. What would be your expectation regarding the distribution of...

- observable characteristics?
- unobservable characteristics?
- idiosyncracies that result in random or systematic measurement error?
- possible responses to an experimental treatment?

Thus, under randomization:

- $Y_i(0), Y_i(1) \perp D_i$ for all $i = 1, ..., N$
- $X_i \perp D_i$ for all $i = 1, ..., N$

BIAS IN THE ATE IN THE ABSENCE OF RANDOMIZATION

$$E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0] = \underbrace{E[\tau_i]}_{\text{True ATE}} + \underbrace{\{E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0]\}}_{\text{selection bias}} + \underbrace{(1 - \pi)\{E[\tau_i|D_i = 1] - E[\tau_i|D_i = 0]\}}_{\text{differential treatment effect bias}}$$

...so?

BIAS IN THE ATE IN THE ABSENCE OF RANDOMIZATION

$$E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0] = E[\tau_i]$$

only holds if:

1. no selection bias:

$$E[Y_i(0)|D_i = 1] = E[Y_i(0)|D_i = 0]$$

2. no differential treatment effects

$$\begin{aligned} E[\tau_i|D_i = 1] &= E[\tau_i|D_i = 0] \\ E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 1] &= E[Y_i(1)|D_i = 0] - E[Y_i(0)|D_i = 0] \\ E[Y_i(1)|D_i = 1] - E[Y_i(1)|D_i = 0] &= E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0] \end{aligned}$$

TWO CORE ASSUMPTIONS ABOUT POTENTIAL OUTCOMES

1. Excludability (Exclusion Restriction)
   - Potential outcomes only respond to d, the actual receipt of the treatment
   - Over and beyond d, they are unaffected by assignment to the treatment group, z

2. Non-Interference (SUTVA)
   - Potential outcomes are unaffected by the pattern of actual or assigned treatments of other units

Summary

TAKE-AWAY POINTS

- The Fundamental Problem of Causal Inference does not make causal inference impossible. It makes causal inference without making untested assumptions impossible!
- An identification strategy encompasses the clear statement of these assumptions and a research design that makes them as plausible and defensible as possible.
- The most defensible design is the experimental ideal: Randomized control trials.
- The more your design approximates the experimental ideal, the more defensible it is.

WHEN YOU WANT TO MAKE CAUSAL CLAIMS...

- Identify your causal question.
- Think about the ideal experiment to answer your question.
- How does deviation from the ideal affect your results?
- What if you could more strongly resemble an experiment? What could change?
- What assumptions do you have to make in order for your design to approximate the experimental ideal?
- Justify these assumptions!

Simulation Exercise

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- Individual treatment effects, τ_i, range between 0 and 20.
- Potential outcomes under the treatment, $Y_i(1)$, range between 20 and 100.
- Students' probability of selecting into the class is a direct function of their prior ability, i.e., of their potential outcomes under the control, $Y_i(0)$.
- Similarly, prior ability affects how much they learn (i.e., the size of their idiosyncratic treatment effects, τ_i) – students with a higher prior ability tend get more out of the class.

OUR FICTIONAL EXAMPLE FOR TODAY

Two Scenarios

1. Randomization: Pure chance determines whether students attend the talk or not
2. Self-Selection: Students self-select into the class, based on prior ability

We want to quantify the magnitude of selection bias and differential treatment effect bias under randomization and under self-selection.

PROMPT i

1. Before you start, set a seed for reproducibility of your random variable generation (`set.seed()`).
2. Generate an integer, `N <- 1000L`, to be used in determining the length of the variables we create below.
3. Generate a variable of potential outcomes under the control, `Y_0`, containing random draws from a uniform distribution with support $[20, 80]$ (`runif()`).
4. Generate a variable of individual treatment effects, `tau`. As `tau` is a function of the potential outcomes under the control, we have differential treatment effects. We draw these from a normal distribution with some random error (`tau <- 0.2 * rnorm(Y_0, 2.5)`) and then truncate the distribution at 0 and 20.
5. Show a scatter plot with `Y_0` on the *x*-axis and `tau` on the *y*-axis. Briefly interpret the pattern you observe.

PROMPT ii

6. Generate a variable of potential outcomes under the treatment, `Y_1`, using the two variables you previously generated.
7. Generate a randomly assigned binary treatment indicator, `D`, which takes on values of either 0 or 1 from a Bernoulli distribution, which is a special case of the binomial distribution with $n = 1$, meaning we take $n = 1$ draw for each observation i (`rbinom`). Suppose everyone has equal probability of being assigned to either treatment or control group.
8. Show that the potential outcomes are independent of treatment status. Choose an appropriate test or statistic and give a brief interpretation.
9. Generate a variable `Y_obs` that takes the values of `Y_0` if `D==0` and the values of `Y_1` if `D==1`.
10. Run a regression of `Y_obs` on `D` to gauge the average treatment effect. Give a brief interpreation.

PROMPT iii

11. Next, generate a variable `D_sel` that indicates receipt of the treatment in a scenario of self selection. For this, generate a variable `prob_sel` equal to the potential outcomes under control divided by 100. This gives you each student's probability of selecting into the treatment. Use these probabilities to draw `D_sel` from a Bernoulli distribution.
12. Using the same test you chose above, test if the potential outcomes are independent of `D_sel` and give a brief interpretation.
13. Generate a variable `Y_obs_sel` that takes the values of `Y_0` if `D_sel==0` and the values of `Y_1` if `D_sel==1`.
14. Run a regression of `Y_obs_sel` on `D_sel` to gauge the average treatment effect under selection bias.
15. Specify the magnitude of both selection bias and differential treatment effect bias. Explain how the two differ conceptually and interpret the estimates.

References

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