# Section 5 Aerodynamics

## 5.0 Recurring Terminology

$a$ slope of lift curve, $dC_L/d\alpha$
$a.c.$ aerodynamic center, location along the chord where pitching moments about this center do not change with angle of attack (25% MAC for airfoils in subsonic flow, 50% MAC for airfoils in supersonic flow)
$AOA$ angle of attack
AR aspect ratio = [wing span]$^2$/ [reference wing area] = $b^2/S$
B wing span
$b_t$ horizontal tail span
$C$ coefficient, a non-dimensional representation of an aerodynamic property
$c$ wing chord length Camber maximum curvature of an airfoil, measured at maximum distance between chord line and amber line, expressed in % of MAC.Camber line theoretical line extending from an air foil's leading edge to the trailing edge, located halfway between the upper and lower surfaces.
$C_D$ drag coefficient
$C_{Di}$ induced drag coefficient
$C_{Do}, C_{Dpe}$ parasitic drag coefficient
$c_f$ friction coefficient
Chord straight-line distance from an airfoil's leading edge to its trailing edge.
$C_L$ lift coefficient
$Cp$ pressure coefficient = $\Delta p/q$
$e$ Oswald efficiency factor
$l$ distance traveled by flow, or characteristic length of surface
$M$ Mach number
MAC mean aerodynamic chord, chord length of location on wing where total aerodynamic forces can be concentrated.
MGC mean geometric chord, the average chord length, derived only from a plan form view of a wing (similar to MAC if wing has no twist and constant cross section & thickness-to-chord ratio).
$P$ pressure
$P_{req'd}$ power required
$q$ dynamic pressure = $\frac{1}{2} \rho_a V_T^2 = \frac{1}{2} \rho_o V_T^2$
$R$ gas constant
$R_n, R_e$ Reynolds number
$S$ reference wing area, includes extension of wing to fuselage centerline.
$S_t$ horizontal tail surface area
$S_W$ wetted area of surface
$T$ temperature
$V$ true velocity
$V_e$ equivalent velocity
$\alpha$ angle of attack
$\alpha_i$ induced angle of attack
$\delta$ depth of boundary layer, or surface wedge angle
$\mu$ viscosity, or wave angle
$\nu$ flow turning angle
$\theta$ shock wave angle
$\rho$ density

•Perfect Fluid

~ incompressible, inelastic, and non-viscous

~ used in flow outside of boundary layers at $M < .7$

•Incompressible, inelastic, viscous

~ used for boundary layer studies at $M < .7$

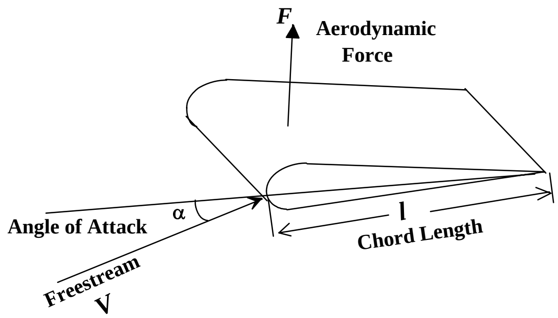
•Compressible, non-viscous, elastic fluid

~ used outside boundary layers up to $M = 5$

## 5.1 Dimensional Analysis Interpretations (ref 5.2)

Aerodynamic force = $F$

- $F = f(\rho, \mu, T, V,$ shape, orientation, size, roughness, gravity)
- For aircraft ignore $R$, $K$ & hypersonic effects




- Initially assume similar body orientations, shapes & roughness.

- Dimensional Analysis reveals four non-dimensional (π) parameters:

Force Coefficient $\pi_1 = \dfrac{F}{\rho V^2 l^2}$

Reynolds Number $\pi_2 = \dfrac{\rho V l}{\mu}$

Mach Number $\pi_3 = \dfrac{V}{a}$

Froude Number $\pi_4 = \dfrac{V}{\sqrt{l\,g}}$

A closer look at the force coefficient:

$$C_F = \frac{F}{\rho V^2 l^2} \Rightarrow \frac{F}{\frac{1}{2}\rho V^2 S}$$

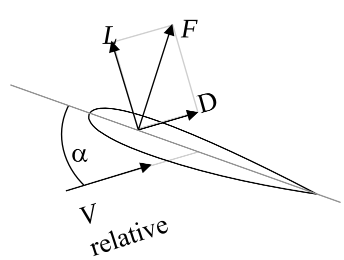
where $^1/_2\ \rho_a V_T^2 = {}^1/_2\ \rho_o V_e^2$ =dynamic pressure, $q$
dimensions of reference wing area, $S$ are the same

**A feel for *q***

- Kinetic energy of a moving object = ½ $mV_T^2$
- Block of moving air kinetic energy = ½ ρ (volume) $V_T^2$
- Dividing through by volume yields KE per volume of moving air = ½ ρ $V_T^2$
- "Dynamic pressure" or "*q*" = potential for converting each cubic foot of the airflow's kinetic energy into frontal stagnation pressure
- Feel *q* by extending your hand out the window of a moving car

**A feel for coefficients**

- $C_F = (F/S)/q$ = the ratio between the total force pressure and the flow 's dynamic pressure
- Lift is the component of the total force perpendicular to the free stream flow
- Drag is the component along the flow
- Break total into lift and drag coefficients:

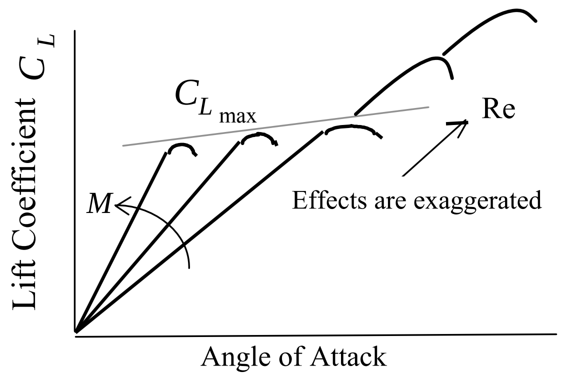


- Froude number is not significant in aerodynamic phenomena
- Recall that forces are aslo a function of angle of attack, shape & surface roughness, therefore

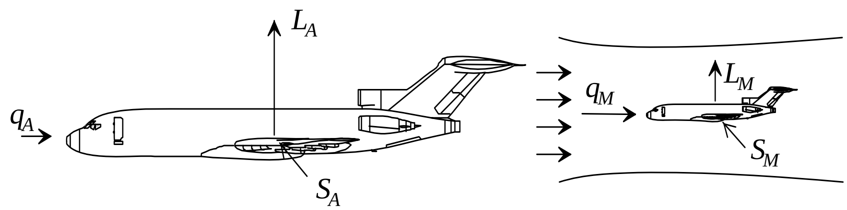
Froude number is not significant in aerodynamic phenomena

Recall that forces are also a function of angle of attack, shape & surface roughness

$C_L, C_D = f\,[M, \mathrm{R}e, \alpha]$ for a given shape, roughness




To compare test day and standard day aircraft or to match wind tunnel $C_F$ data to actual aircraft; the shape, roughness, $M$, $R_n$ and $\alpha$ must be equal for both aircraft




$$\frac{L_A}{q_A S_A} = C_L = \frac{L_M}{q_M S_M}$$

## 5.2 General Aerodynamic Relations (refs 5.1, 5.2, 5.10)

Lift & Drag forces can be described using two approaches:

1) Change in momentum of airstream, $F = d\{mv\}/dt$
2) "Bernoulli" approach which requires the continuity and conservation of energy equations

**Continuity Equation**

Fluid M ass in = Fluid Mass out

$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

For subsonic (incompressible) flow $\rho_1 = \rho_2$

$V_1 A_1 = V_2 A_2$

**Conservation of Energy** (Bernoulli) **Equation:**

Potential + Kinetic + Pressure = constant

(changes in Potential energy are negligible)

Energy per unit volume is pressure then

Dynamic Pressure + Static Pressure = Total Pressure

$$\tfrac{1}{2}\rho V^2 + p_s = \text{constant}$$

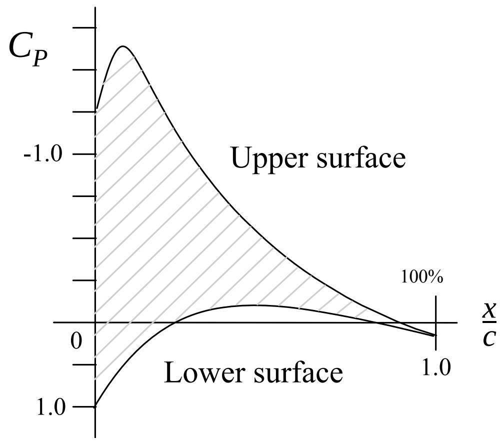
$$\tfrac{1}{2}\rho V^2 + p_s = p_t$$

- This classic approach only applies in the "potential flow" region and not in the boundary layer where energy losses occur

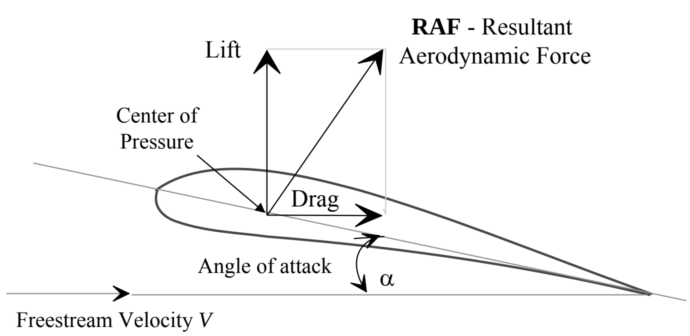
- Pressures around a surface can be calculated or measured from tests and converted into pressure coefficients,

$$c_p = (p_{\text{local}} - p_{\text{ambient}})/\text{dynamic pressure} = \Delta p/q$$

- $c_p$ values can be mapped out for all surfaces




- Summation of all pressures perpendicular to surface yield the pitching moments and the "**Resultant Aerodynamic Force**" which is broken into lift and drag components

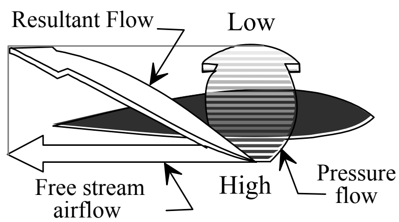


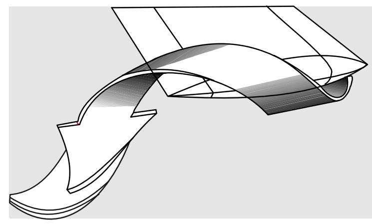

- Lift & drag forces are referred to the aerodynamic center (*ac*) where the pitching moment is constant for reasonable angles of attack.
- Pitching moments increase with airfoil camber, are zero if symmetric.
- Aerodynamic center is located at 25% MAC for fully subsonic flow and at 50% MAC for fully supersonic flow.

## 5.3 Wing Design Effects on Lift Curve Slope (refs 5.1, 5.2, 5.10)

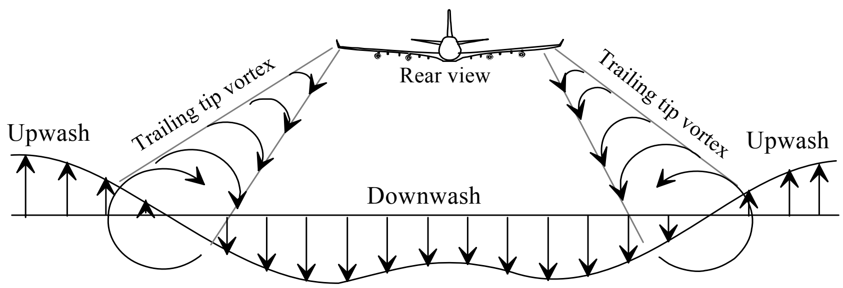
**Aspect Ratio Effect**

- Pressure differential at wingtip causes tip vortex

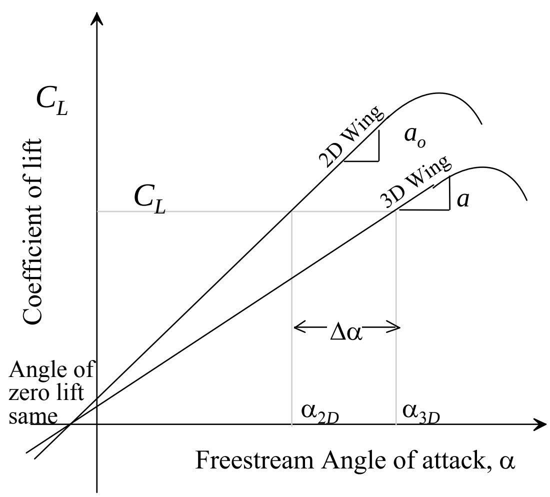


- Vortex creates flow field that reduces AOA across wingspan




- Local AOA reductions decrease average lift curve slope



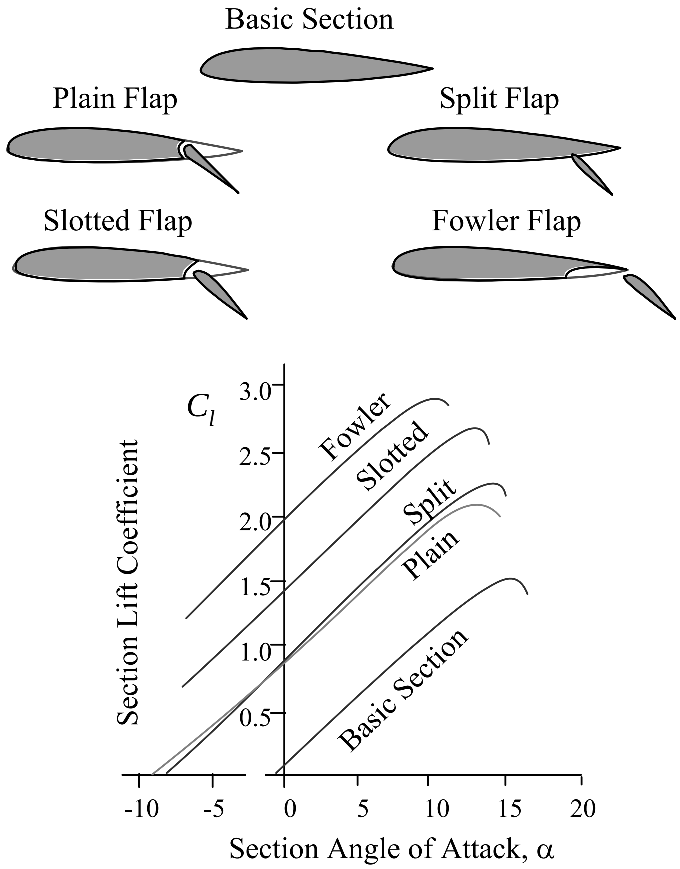

2D wing = wind tunnel airfoil extending to walls (infinite aspect ratio).

$a_o$ = Lift curve slope for an infinite wing

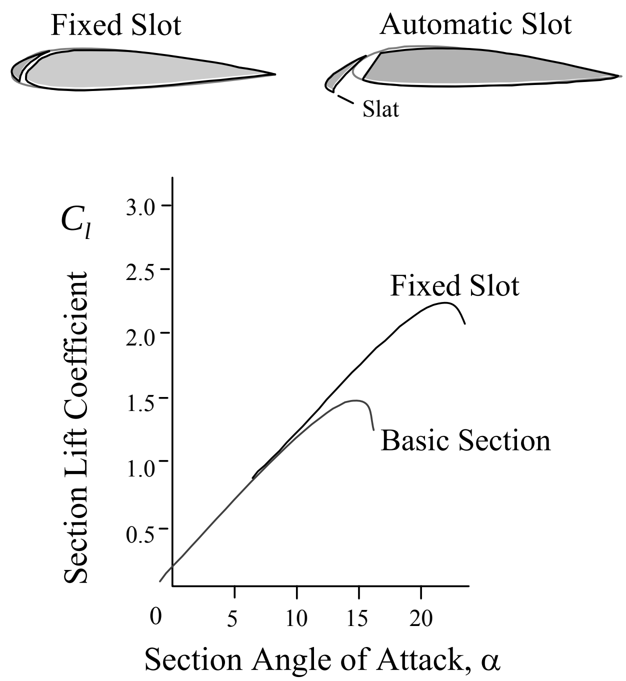
$a$ = Lift curve slope for a finite wing

- Above relationship estimated as $$a = \frac{dC_L}{d\alpha} = \frac{a_o}{1 + \frac{57.3 a_o}{\pi AR}}$$

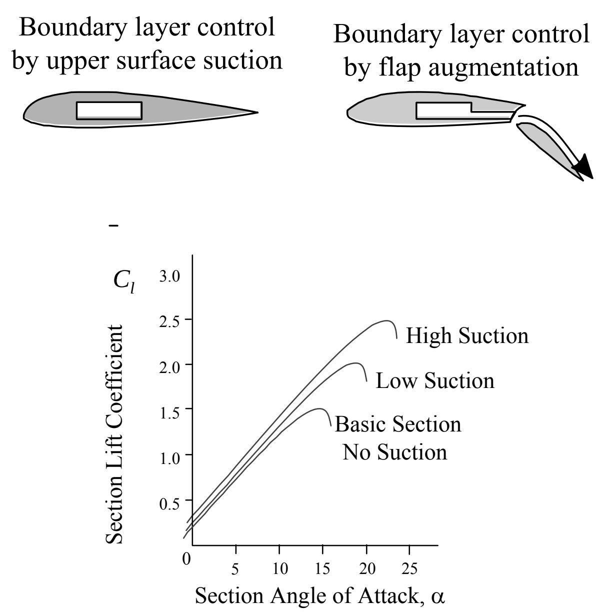
**Trailing Edge Flap Effects**



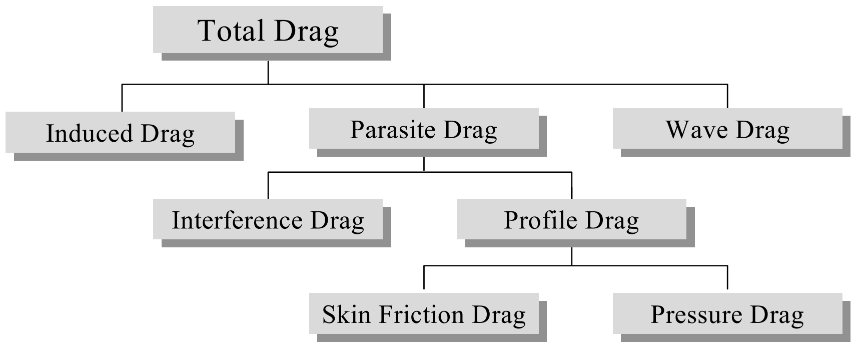
**Leading Edge Flap Effects**



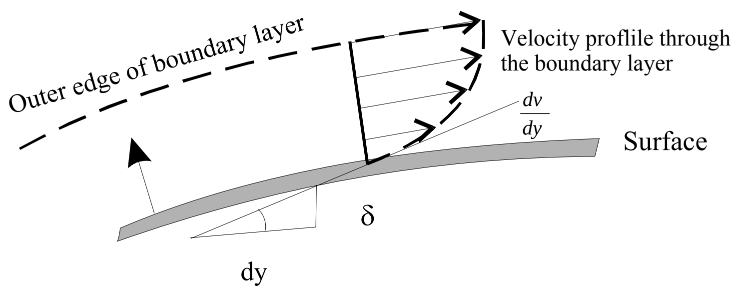

**Boundary Layer Control Effects**



## 5.4 Elements of Drag (refs 5.1, 5.2, 5.10)




- Skin friction shear stress is a function of velocity profile at surface




$$\text{Shear stress } \Im_w = \mu\left(\frac{dv}{dy}\right)_{y=0}$$

- **Viscosity** (μ) increases with temperature (ref 5.9)

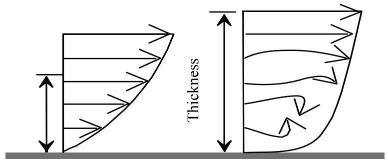
Sutherland law: $$\mu = \mu_o \frac{\left(\frac{T}{T_o}\right)^{1.5}(T_o + S)}{(T + S)}$$ Power law: $$\mu = \mu_o\left(\frac{T}{T_o}\right)^n$$

Where $T_o$ = 273.15 $K$ = 518.67 $R$.
For air: $S$ =110.4 $K$ = 199 $R$; $n$=.67
For air at 273 K: $\mu_o$= 1.717x10$^{-5}$ [kg/m s] = 3.59x10$^{-7}$ [slug/ft s]
Inserting air values ($T_K$=Kelvin and $T_R$=Rankin) into Sutherland law gives

$$\mu = 1.458x10^{-6}\frac{T_K^{1.5}}{T_K + 110.4}\left[\frac{kg}{s\cdot m}\right] = 2.2x10^{-8}\frac{T_R^{1.5}}{T_R + 199}\left[\frac{s\lg}{s\cdot ft}\right]$$

**Reynolds Number Effects** (ref 5.10)

- Laminar boundary layers have more gradual change in velocity near surface than turbulent boundary layers.
- High Reynolds numbers help propagate turbulent flow.



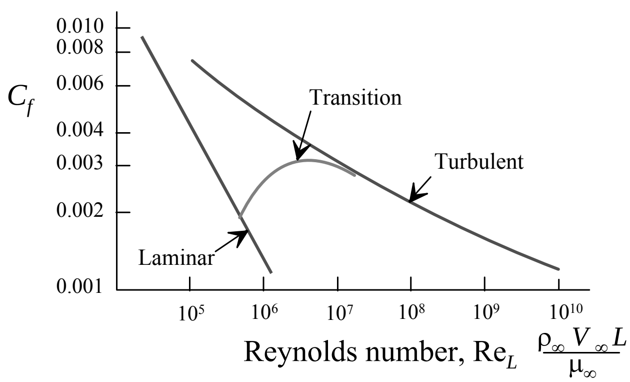

Laminar Turbulent

Shearing stress $\Im_w = \mu \left( \frac{dv}{dy} \right)_{y=0}$

Skin friction coefficient $C_f = \frac{\Im_w}{\frac{1}{2}\rho_\infty V_\infty^{\,2}} = \frac{\Im_w}{q_\infty}$

Laminar boundary layer Total $C_f = \frac{1.328}{(\mathrm{Re}_L)^{1/2}}$

Turbulent boundary layer Total $C_f = \frac{.455}{(\log \mathrm{Re}_L)^{2.58}} \approx \frac{0.074}{(\mathrm{Re}_L)^{1/5}}$




$Re_L$ based on total length of flat plate

- Depth of boundary layer (δ) depends on local Reynolds number ($\mathrm{Re}_x$) and whether the flow is turbulent or laminar.

$$\mathrm{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} \equiv \frac{\text{Inertia Forces}}{\text{Viscous Forces}}$$

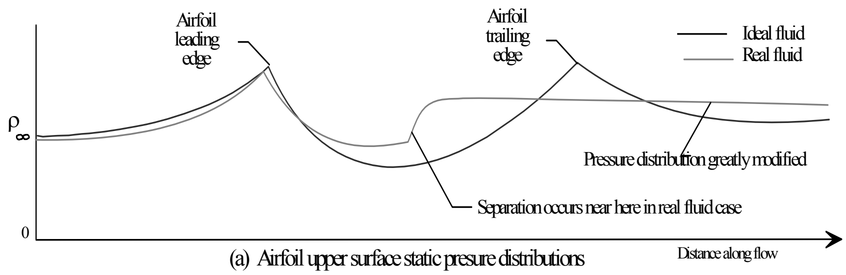
$x$= distance traveled to point in question

$$\delta_{lam} = \frac{5.2\,x}{\sqrt{\mathrm{Re}_x}}$$

$$\delta_{turb} = \frac{.37\,x}{\mathrm{Re}_x^{\,.2}}$$

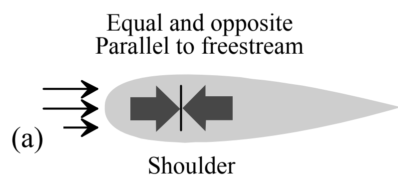
### 5.4.2 Pressure Drag

- Ideal frictionless flow has no losses and leads to zero pressure drag
- Real fluids have friction and energy losses along surface
- Energy losses negate total pressure recovery, lead to decreasing total pressure along surface

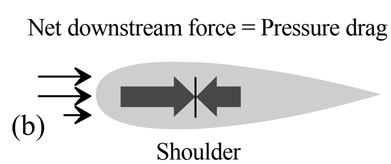



(a) Airfoil upper surface static presure distributions

- Imbalance of pressures on surfaces causes pressure drag

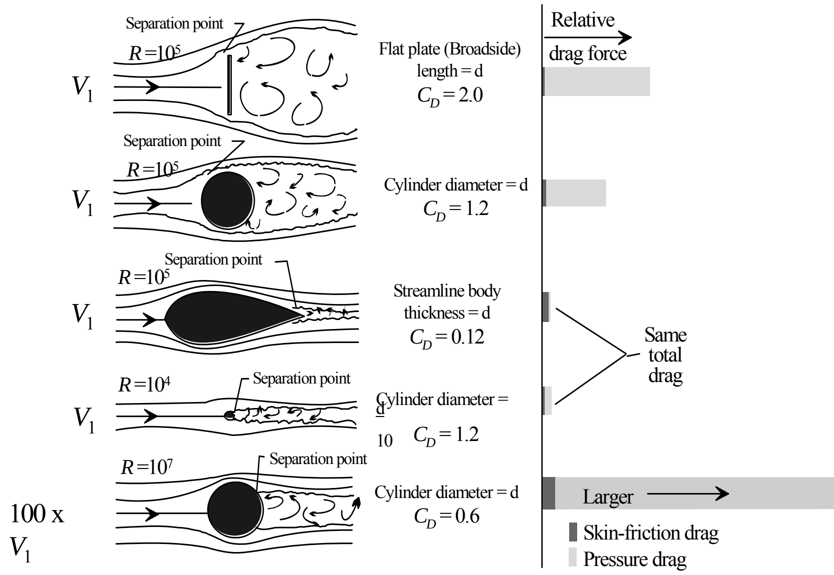



(b) Ideal fluid air foil (no pressure drag)
Sum of horizontal pressures = 0



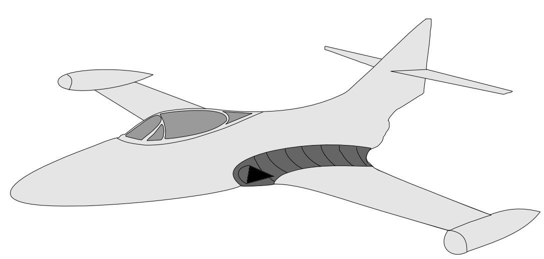

(c) Real fluid airfoil (net pressure drag) more drag pressure than thrust pressure

- Profile streamlining reduces pressure drag



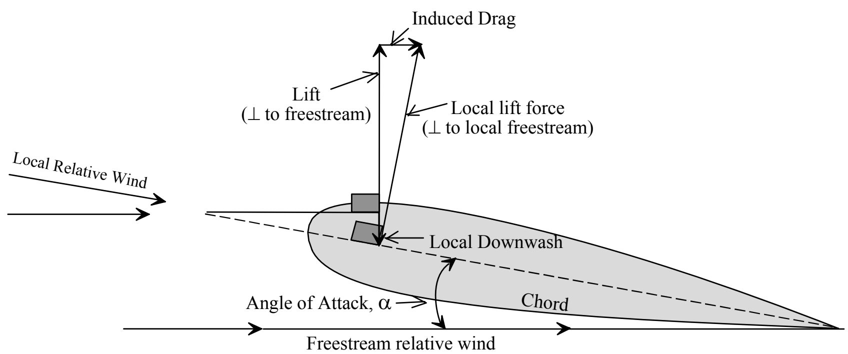

### 5.4.3 Interference Drag

- Occurs with multiple surfaces approximately parallel to flow
- Caused by flow's interference with itself or by excessive adverse pressure gradient due to rapidly decreasing vehicle cross section
- Most severe with surfaces at acute angles to each other
- Effects often reduced by fillets around contracting surfaces



### 5.4.4 Induced Drag

- Wingtip vortex reduces local AOA at each station along wing
- Local lift vector is perpendicular to local AOA
- Local lift vector is therefore tilted back relative to freestream lift
- Induced drag defined as rearward component of local lift vector




$$\text{Induced Drag } (D_i) = \text{Lift}(L)\sin\alpha_i = L\alpha_i$$

$$\text{Induced Drag } (D_i) = L(\alpha_i)$$

$$\text{For elliptical lift distributions} \quad \alpha_i = \frac{C_L}{\pi \text{AR}}$$

$$\therefore D_i = L\left(\frac{C_L}{\pi \text{AR}}\right) \quad \text{but} \quad L = qSC_L$$

$$C_{D_i} = \frac{D_i}{qS} = \frac{C_L{}^2}{\pi \text{AR}} = \text{induced drag coefficient}$$

Oswald efficiency factor, *e*, accounts for losses in excess of those predicted above (due to uneven downwash and changing interference drag effects).

$$\therefore C_{D_i} = \frac{C_L{}^2}{\pi \text{AR} e}$$

## 5.5 Aerodynamic Compressibility Relations (reference 5.8)

**Prandtl/Glauert Approximation**
Approximates Mach effects on aerodynamics below critical Mach

$$C_{P_{compressible}} = \frac{1}{\sqrt{1-M^2}} C_{P_{incompressible}}$$

**Total vs Ambient Property Relations for Adiabatic Flow**

$$\frac{T_T}{T} = 1 + \frac{\gamma - 1}{2} M^2$$ Isentropic flow not required

$$\frac{P_T}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$ Isentropic (shockless) flow required

$$\frac{\rho_T}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma - 1}}$$ Isentropic flow required

**Normal Shock Relations**

Assumes isentropic flow on each side of the shock
Assumes flow across shock is adiabatic
Property changes occur in a constant area (throat)

$$\frac{P_2}{P_1} = \frac{1 - \gamma + 2\gamma M_1^2}{1 + \gamma}$$

$$\frac{\rho_2}{\rho_1} = \left[\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}\right]^{-1}$$

$$\frac{T_2}{T_1} = \left[\frac{1 - \gamma + 2\gamma M_1^2}{1 + \gamma}\right]\left[\frac{2 + (\gamma - 1)M_1^2}{(1 + \gamma)M_1^2}\right]$$

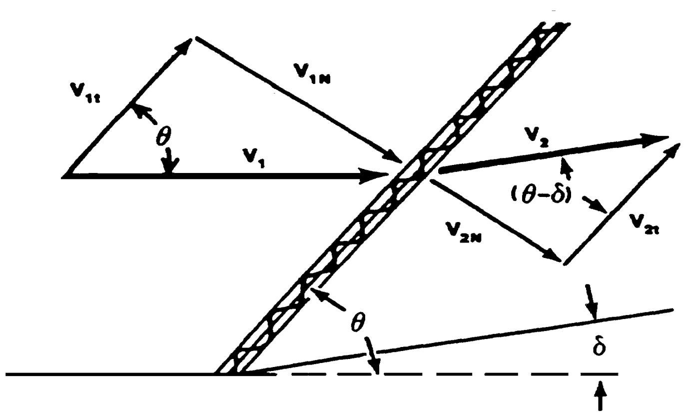
$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Normal shock summary

$$P_{T_1} > P_{T_2} \quad \rho_{T_1} > \rho_{T_2} \quad T_{T_1} = T_{T_2} \quad M_1 > M_2$$
$$P_1 < P_2 \quad \rho_1 < \rho_2 \quad T_1 < T_2 \quad s_1 < s_2$$

### 5.5.1 Oblique Shocks

**Oblique Shock Description**




$\delta$ = surface turning angle

$\theta$ = shock wave angle

Subscript 1 denotes upstream conditions

Subscript 2 denotes downstream conditions

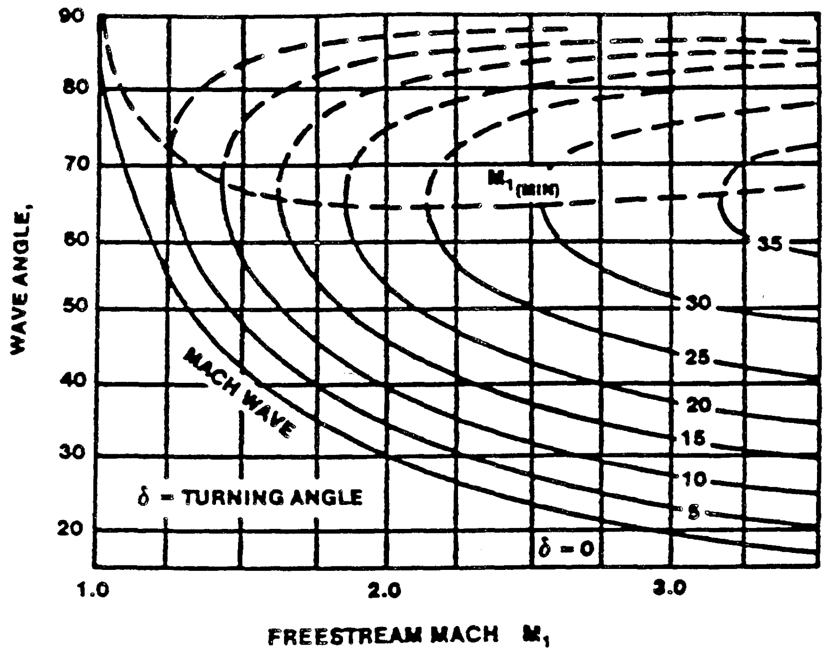
**Oblique Shock Relations**

- Calculate $P_2/P_1$, $T_2/T_1$, and $\rho_2/\rho_1$ across oblique shocks by using normal shock equations and substituting $M_1 \sin\theta$ in place of $M_1$
- Calculate total pressure loss across oblique shock as
- Calculate relation between Mach number and angles as

$$\frac{P_{T_2}}{P_{T_2}} = \left\{ \left[ \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2 \sin^2\theta} \right]^{\gamma} \left[ \frac{2\gamma}{\gamma+1} M_1^2 \sin^2\theta - \frac{\gamma-1}{\gamma+1} \right] \right\}^{\frac{1}{1-\gamma}}$$

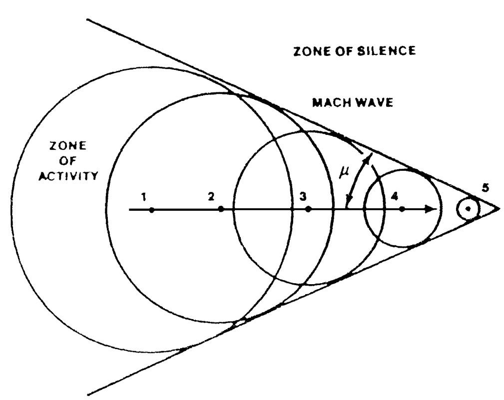
$$M_2^2 \sin^2(\delta-\theta) = \frac{M_1^2 \sin^2\theta + \dfrac{2}{\gamma-1}}{\dfrac{2\gamma}{\gamma-1} M_1^2 \sin^2\theta - 1}$$

**Oblique Shock Turning Angle as a Function of Wave Angle**



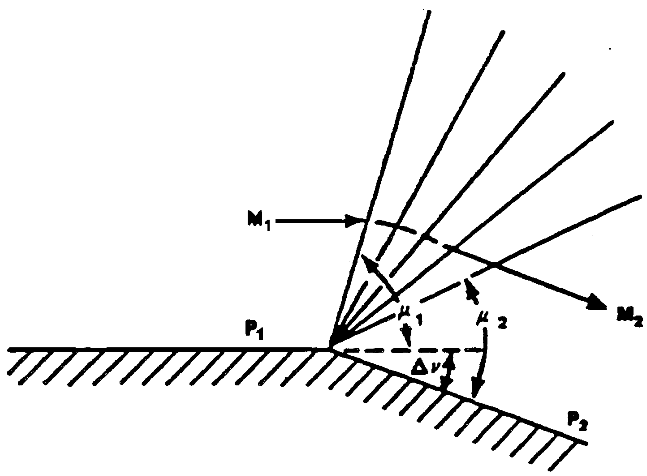

- Two θ solutions exist for every $M_1$ & $\delta$ combination
  These represent the strong and weak shock solutions
  Weak shocks normally occur in nature
- There is a minimum Mach number for each turning angle
- The wave angle of a weak shock decreases with increased Mach
- For a given Mach number, $\theta$ approaches $\mu$ as $\delta$ decreases

**Mach Cone Angle**




Minimum Wave Angle
$\mu = \sin^{-1}(1/M)$

### 5.5.2 Supersonic Isentropic Expansion Relations



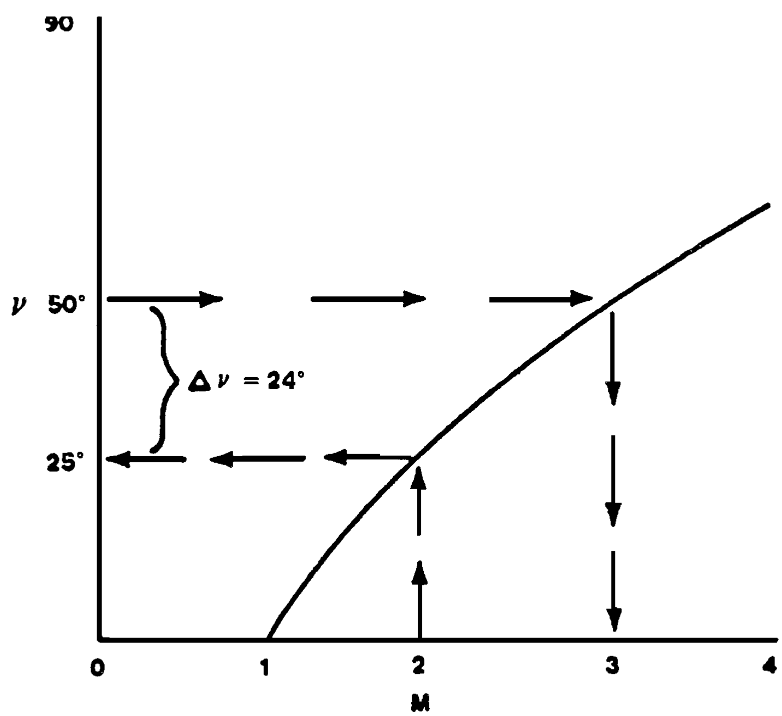

- The wave angle $\mu$ determines where the lower pressure can be felt and thus where the flow can be accelerated
- As the flow accelerates, a new wave angle forms and the subsequent lower pressure further accelerates the flow
- Results in a series of Mach waves forming a “fan” until the flow turns and accelerates so that it is parallel to the new boundary

**Prandtl-Meyer Function**

Shows flow's required turning angle ($\nu$) to accelerate from one Mach number to another

$$\nu_M = \sqrt{\frac{\gamma+1}{\gamma-1}}\left[\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)}\right] - \tan^{-1}\sqrt{M^2-1}$$

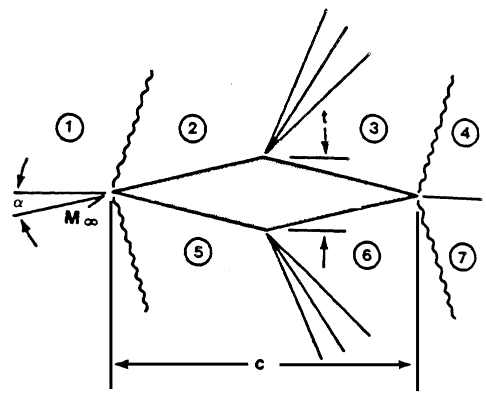
- If upstream Mach ($M_1$) =1, then $\nu_1 = 0$, and equation directly relates downstream Mach ($M_2$) to surface turning angle ($\Delta\nu$)
- If $M_1$>1, determine $M_2$ as follows:
  - Calculate upstream $\nu_1$ from above equation
  - Calculate $\nu_2 = \nu_1 + \Delta\nu$
  - Reverse above equation to obtain corresponding $M_2$
- Above equation is tabulated in NACA TR 1135 and is plotted below




Example: Flow initially at $M_1$ = 2.0 accelerates through an expansion corner of 24 *deg*. Exit Mach number is 3.0

### 5.5.3 Two-Dimensional Supersonic Airfoil Approximations

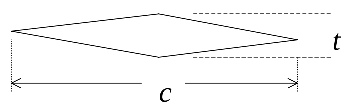
- Determine surface static pressures by calculating changes through obliques shocks and expansion fans




- Ackert approximations for thin wings are based on

$$C_p = \frac{\Delta P}{q} \cong \pm \frac{2\delta}{\sqrt{M^2 - 1}}$$

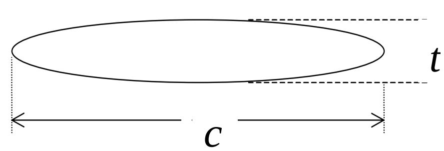
- Double wedge airfoil approximations




$$C_L \cong \frac{4\alpha}{\sqrt{M^2 - 1}}$$

$$C_D \cong \frac{4\alpha^2}{\sqrt{M^2 - 1}} + \frac{4}{\sqrt{M^2 - 1}}\left(\frac{t}{c}\right)^2$$

- Biconvex wing approximations




$$C_L \cong \frac{4\alpha}{\sqrt{M^2 - 1}}$$

$$C_D \cong \frac{4\alpha^2}{\sqrt{M^2 - 1}} + \frac{5.33}{\sqrt{M^2 - 1}}\left(\frac{t}{c}\right)^2$$

## 5.6 Drag Polars (ref 5.2)

### 5.6.1 Drag Polar Construction and Terminology

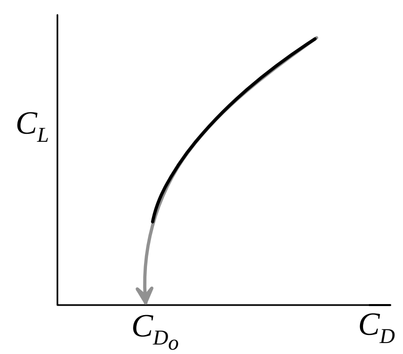
$C_L$ = lift coefficient
$C_D$ = drag coefficient
$C_{Di}$ = induced drag coefficient
$C_{Do}$ = parasitic drag coefficient
AR = aspect ratio
$e$ = Oswald efficiency factor
$l$ = length flow has traveled
$S_{wet}$ = wetted area of surface
$S$ = reference wing area

**Simple Drag Polar Equation Limitations**

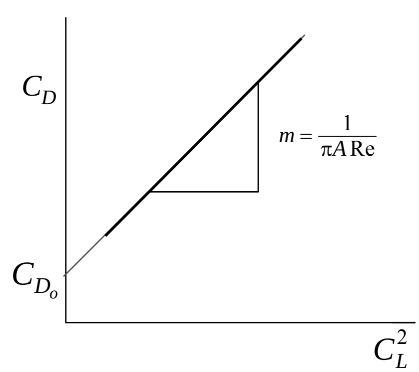
- No separated flow losses
- Symmetric Camber
- Applies at one Mach, Altitude, *cg*

$$C_D = C_{D_o} + \frac{C_L^2}{\pi A \mathrm{Re}} = C_{D_o} + C_{D_i}$$

"Polar" form of simple drag polar

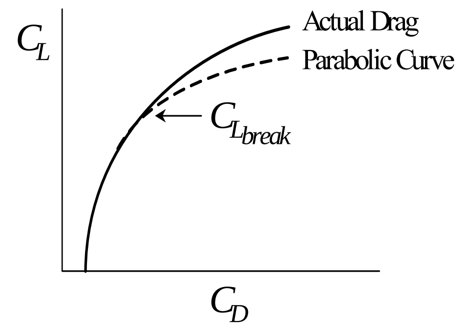



Linearized form of simple drag polar



### 5.6.2 Complicating Factors

**Airflow Separation Effects**




Drag Polar Equation Accounting for Flow Separation:

$$C_D = C_{D_{\min}} + \frac{\left(C_L - C_{L_{\min}}\right)^2}{\pi A \mathrm{Re}} + k_2\left(C_L - C_{L_{break}}\right)$$

- Delete last term if $C_L < C_{lbreak}$
- Determine $k_2$ from flight test

**Reynolds Number Effects** (refs 5.4, 5.11)

- Calculate length $Re_L$ and friction coefficient ($c_f$) for each surface as

$$\mathrm{Re}_L = \frac{\rho V l}{\mu} = 7.101\times10^6 M\left[\frac{\delta}{\theta^2}\right]\left[\frac{T_K + 110}{398}\right] l$$

($T_K$ = Kelvin, $l$= total length, ft)

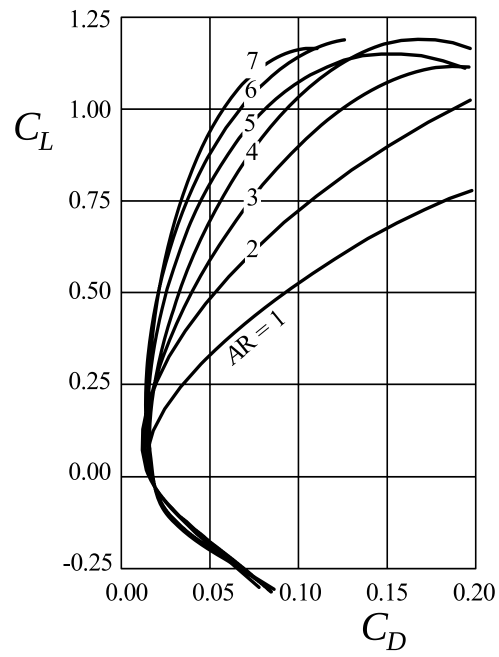
$$c_f = \left\{\frac{1.328}{\sqrt{\mathrm{Re}_L}}\right\}\left[1 + 0.1305 M^2\right]^{-0.12} \text{ laminar,or} = \left[\frac{.074}{\left(\mathrm{Re_L}\right)^2} - \frac{1700}{\mathrm{Re}_L}\right]\text{transition}$$

or $$c_f = 0.455\left\{\log \mathrm{Re}_L\right\}^{-258}\left\{1 + 0.144 M^2\right\}^{-0.65} \text{ turbulent}$$

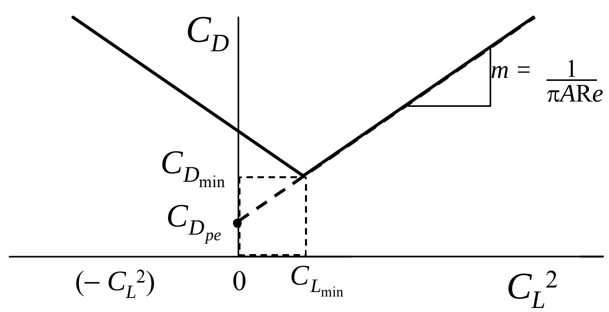
- In general, $c_f$ decreases as $R_n$ increases (unless transitioning from laminar to turbulent flow)
- Friction drag = $c_f\, q\, S_{wet}$ for each component ($S_{wet}$ = wetted area)
- Correct from test day to standard day aircraft drag coefficient by summing differences of each component's drag change

$$\Delta C_D = \frac{\Sigma\left(c_{f_s} - c_{f_t}\right) S_{wet}}{S}$$

**Wing Camber or Incidence Angle Effects**




Note slight increase in drag as lift decreases towards zero




Linearized drag po-lafor aircraft with wing camber and/or incidence

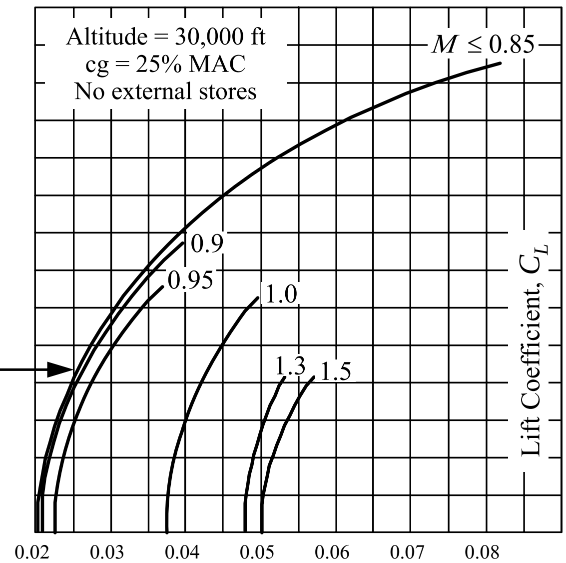
Revised drag polar equation accounting for wing camber or incidence

$$C_D = C_{D_{min}} + \frac{\left(C_L - C_{L_{min}}\right)^2}{\pi A \mathrm{Re}}$$

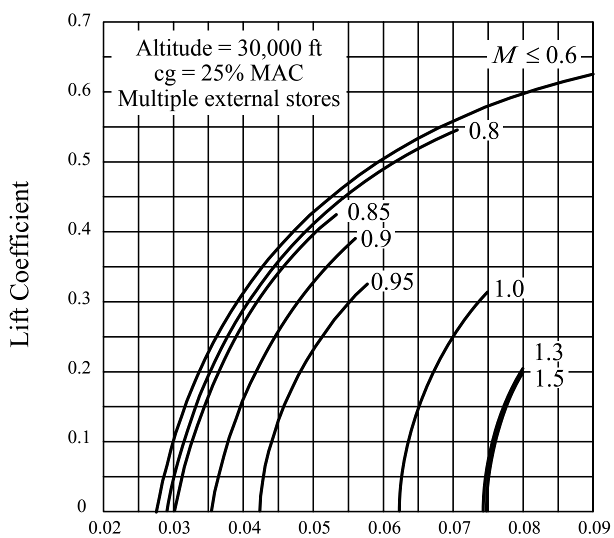
- Generally not necessary since most flight occurs above $C_{Lmin}$

**Mach Number Effects**

- Aircraft with low parasitic drag coefficients and high fineness ratios pay a relatively small "wave drag" penalty.




- With external stores, same aircraft pays larger Mach penalty

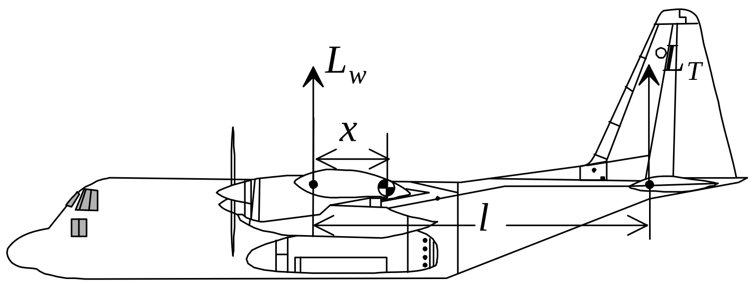



**Propeller Slipstream Effects**

- a.k.a "scrubbing" drag
- Propwash increases flow speed over surface within slipstream
- More drag is created by higher *q* and vorticity.
- Function of prop speed and power absorbed ($C_p$)or thrust ($C_T$)
- Problem should be addressed in airframe or propeller models

**Trim Drag Effects** (reference 5.4)

e = wing Oswald efficiency factor
$e_t$ = tail Oswald efficiency factor
b = span, $b_t$ = tail span
*x* = wing *ac*-to-*cg* distance
*l*= wing *ac*-to tail *ac* dist.

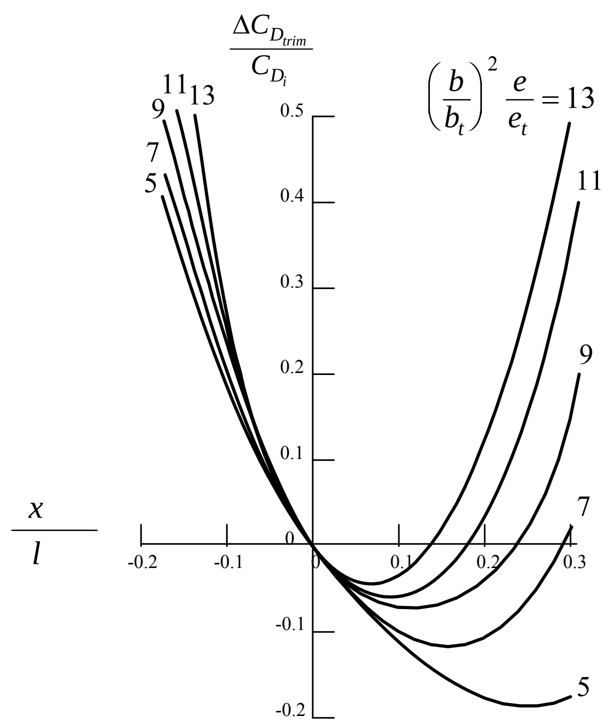



$$\Delta C_{D_{trim}} = \frac{W^2}{\pi q^2 S b^2 e}\left\{\frac{2}{lW}\left[x_0 - x_1\right] + \frac{1}{l^2}\left[1 + \frac{S}{S_t}\frac{e}{e_t}\left(\frac{b}{b_t}\right)^2\right]\left[x_0^2 - x_1^2\right]\right\}$$

Trim drag change relative to total induced drag:

$$\frac{\Delta C_{D\,trim}}{\Delta C_{D_i}} = \frac{x}{l}\left[\frac{x}{l}\left(\frac{b}{b_t}\right)^2 \frac{e}{e_t} - 2\right]$$

Plot of above equation



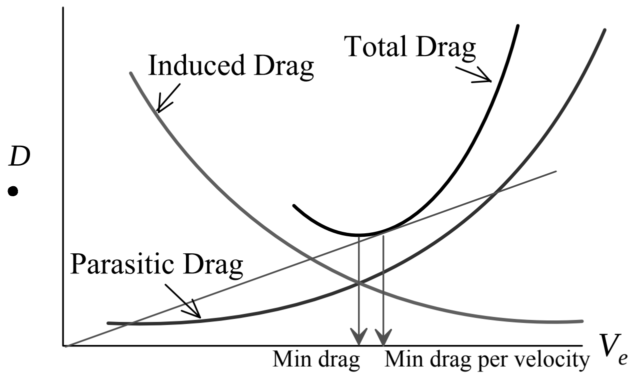
### 5.6.3 Drag Polar Analysis

$$D = \bar{q}SC_D = \bar{q}S\left[C_{D_o} + \frac{C_L^2}{\pi A \,\mathrm{Re}}\right] = \tfrac{1}{2}\rho_o V_e^{\,2} S\left[C_{D_o} + \frac{W^2}{\pi A \,\mathrm{Re}\left(\tfrac{1}{2}\rho_o V_e^{\,2} S\right)^2}\right]$$

- For a given configuration ($C_{Do}$, $S$, $AR$, $e$)

$$D = k_1 V_e^2 + k_2 \frac{W^2}{V_e^2}$$ first term = parasitic drag, second term = induced drag

- For any given weight, $D = f$(equivalent airspeed) only




- Minimum total drag occurs when $D_{induced} = D_{parasitic}$
  - same as speed where $C_{Di} = C_{Do}$
  - occurs at max $C_L/C_D$ ratio (same as max $L/D$ ratio)
- Minimum drag/velocity occurs at min slope of Drag vs V curve
  - same as speed where $3C_{Di} = C_{Do}$
  - occurs at max $C_L^{1/2}/C_D$ ratio

Power required = drag x true airspeed

$$P_{req} = DV_T = D\frac{V_e}{\sqrt{\sigma}} = k_1\frac{V_e^3}{\sqrt{\sigma}} + k_2\frac{W^2}{\sqrt{\sigma}V_e}$$

Minimum total $P_{req'd}$ occurs when $P_{induced} = P_{parasitic}$
- same as speed where $C_{Di} = 3C_{Do}$
- occurs at max $C_L^{3/2}/C_D$ ratio

Minimum power/velocity occurs at min slope of $P_{req'd}$ vs $V$ curve
- same as speed where $C_{Di} = C_{Do}$
- occurs at max $C_L/C_D$ ratio

**Optimum Aerodynamic Flight Conditions**

*Gliders/ Engine-Out Flight*

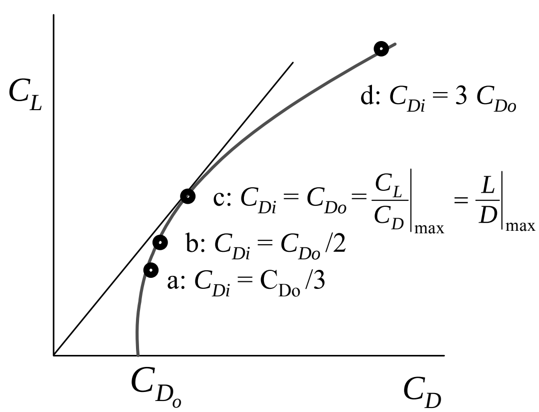
- Max range (minimum glide slope) occurs at max $C_L/C_D$
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic
- Min sink rate (minimum power req'd) occurs at max $C_L^{3/2}/C_D$ ratio same as condition where $3C_{Do} = C_{Di}$ *if* drag polar is parabolic

*Reciprocating Engine Aircraft (assuming constant BSFC & prop $\eta$)*

- Max range (minimum power/velocity) occurs at max $C_L/C_D$ ratio
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic
- Max endurance (minimum power req'd) occurs at max $C_L^{3/2}/C_D$
  same as condition where $3C_{Do} = C_{Di}$ *if* drag polar is parabolic

*Turbine Jet Engine Aircraft (assuming constant TSFC)*

- Max range at constant altitude (minimum drag/velocity)
  occurs at max $C_L^{1/2}/C_D$ ratio
  same as condition where $C_{Do} = 3C_{Di}$ *if* drag polar is parabolic
- Best cruise/climb range (maximum [$M$ x $L/D$] ratio)
  occurs at max $C_L/C_D^{3/2}$ ratio
  same as condition where $C_{Do} = 2C_{Di}$ *if* drag polar is parabolic
- Best endurance (minimum drag)
  occurs at max $C_L/C_D$ ratio
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic




To calculate optimum speed $V_2$ for configuration$_2$ & weight$_2$ based on optimum speed $V_1$ at configuration$_1$ & weight$_1$

$$V_2 = \left(\frac{C_{D_{o1}}}{C_{D_{o2}}}\right)^{\frac{1}{4}} \left(\frac{W_2}{W_1}\right)^{\frac{1}{2}} V_1$$

## 5.7 References

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