Section 11 Propellers

11.1 Abbreviations and Terminology

Abbreviations

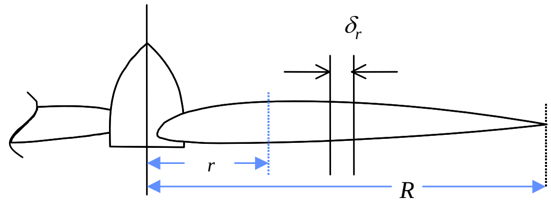
AF	activity factor = $\frac{100,000}{16}\int_{.15}^{1.0}\left(\frac{b}{D}\right)x^3dx$
B	number of blades
b	blade section width (feet)
BHP	brake horsepower (measured at engine crankshaft)
C_{LD}	blade section design lift coefficient
C_{Li}	integrated design lift coefficient = $4\int_{.15}^{1.0}(C_{LD})x^3dx$
C_P	power (absorbed) coefficient
C_T	thrust coefficient
D	propeller diameter (feet)
f_c	ratio of speed of sound at standard day sea level to speed of sound at operating condition $f_c = \frac{1}{\sqrt{\theta}}$
HP	horsepower (1 HP = 550 ft-lb/sec)
G.R.	gear ratio, propeller speed/engine speed
J	Propeller advance ratio $=V_T/nD$ (nondimensional)
M	aircraft Mach number
N	propeller speed, revolutions per minute (*RPM*)
n	propeller speed, revolutions per second
N_e	engine speed, *RPM*
P_a	ambient pressure
P	power output (ft-lb/sec)
Q	torque (ft-lb)
q	dynamic pressure
T	thrust
T_a	absolute ambient temperature
R	blade radius at propeller tip (feet)
r	radius at blade element (feet)
SHP	shaft horsepower (measured at propeller shaft)
T	propeller thrust (pounds)
V_T	freestream velocity (ft/sec)
V_K	freestream velocity (knots)
x	fraction of propeller tip radius, r/R
V_{tan}	tangential velocity
V_R	resultant velocity
V_{tip}	tip speed
α	local angle of attack
β	local blade twist angle, measured between chord and plane of rotation, same as θ (degrees).
ΔM	Mach number adjustment for effect of blade camber
ϕ	propeller disk angle of attack
η	isolated propeller efficiency.
η_{comp}	composite prop efficiency (includes tip and blockage corrections)
$\theta^{3/4}$	propeller blade twist angle at x =3/4 (degrees), same as $\beta^{3/4}$
σ	ratio of operating density to sea level standard density = ρ_a/ρ_o.
ω	propeller rotation speed (radians/second)

Terminology

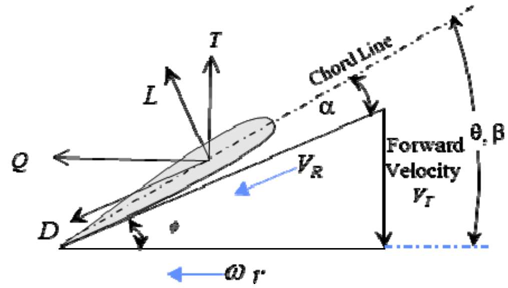
blade aspect ratio	measured as [R / max blade width].
effective pitch	actual advance per revolution.
experimental pitch	necessary advance to generate zero thrust.
geometric pitch (p)	advance per revolution if blade element moves according to β (i.e., with no slip).
reduction gear	gearing between the engine crankshaft and prop shaft that reduces the propeller rotation speed .
right-handed	moves clockwise (viewed from the slipstream).
solidity	fraction of prop disk covered by blade area = $2\pi R/Bb$.
total width ratio (TWR)	measured as [WR x B]
thickness ratio (TR)	blade thicknessmeasured locally or at .75R to represent entire prop.
width ratio (WR)	calculated as {b @ .75R } / D

11.2 Propeller Geometry

- δ_r is the width of any element along blade radius.



- $x = r/R$, the local fraction of prop tip radius



- Prop blade chord extends from leading edge to trailing edge.
- Blade twist angle θ, measured between rotation plane and local chord.
- Relative wind is the resultant velocity (V_R), comprised of aircraft forward speed and tangential speed at radial location along blade.

$$V_R = \sqrt{V_T^2 + (2\pi rn)^2}$$

$$\phi = \tan^{-1}\frac{V_T}{r\omega} = \tan^{-1}\frac{V_T}{r2\pi n} = \tan^{-1}\frac{V_T}{xD\pi n}: \qquad \phi_{tip} = \tan^{-1}\frac{1}{\pi}\frac{V_T}{nD}$$

- Angle ϕ is measured between plane of rotation and local V_R

$$\alpha^x = \theta^x - \phi = \theta^x - \tan^{-1}\frac{1}{\pi}\frac{V_T}{r2n} = \theta^x - \tan^{-1}\frac{1}{\pi}\frac{V_T}{xDn} = \theta^x - \tan^{-1}\frac{J}{\pi x}$$

- Advance ratio (J) is defined as $J = V_T/nD$.
- Local angle of attack at any fraction of radius(α^x) is measured between the local chord line and relative wind
- Lift and drag are perpendicular and parallel to V_R, respectively
- Thrust (T) and torque (Q) are perpendicular and parallel to the plane of rotation, respectively.

11.3 Propeller Coefficients

Integrating lift and drag along a blade gives the thrust (*T*) and torque (*Q*). Multiply by number of blades (*B*) to determine total *T* and *Q*.

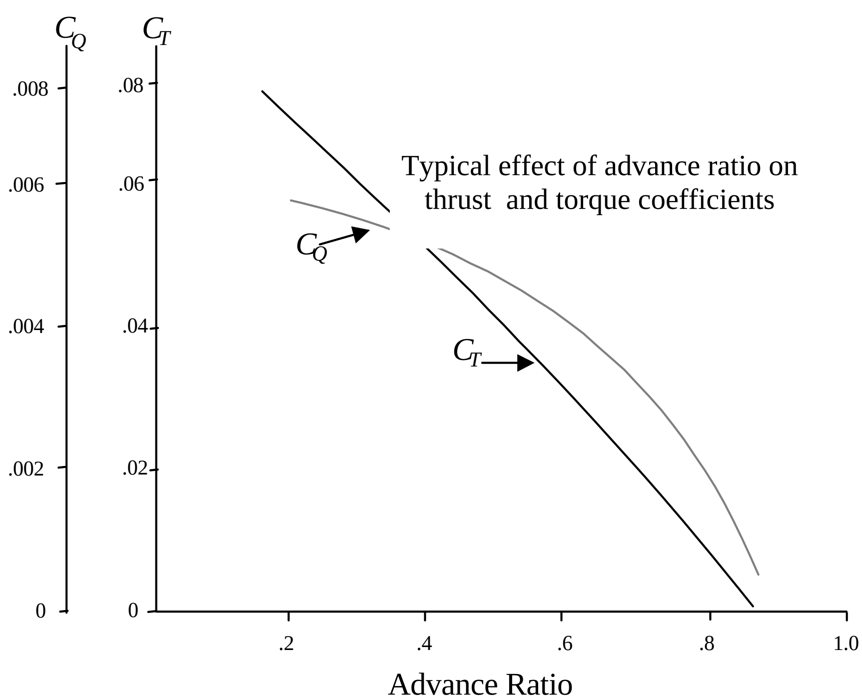
$$T = qB\int_{R_1}^{R_2} \frac{c}{\sin^2\phi}\left(C_L \cos\phi - C_D \sin\phi\right)dr$$

$$Q = qB\int_{R_1}^{R_2} \frac{cr}{\sin^2\phi}\left(C_L \sin\phi + C_D \cos\phi\right)dr$$

Thrust Coefficient, C_T $\equiv \dfrac{T}{\rho n^2 D^4}$

Torque Coefficient, C_Q $\equiv \dfrac{Q}{\rho n^2 D^5}$

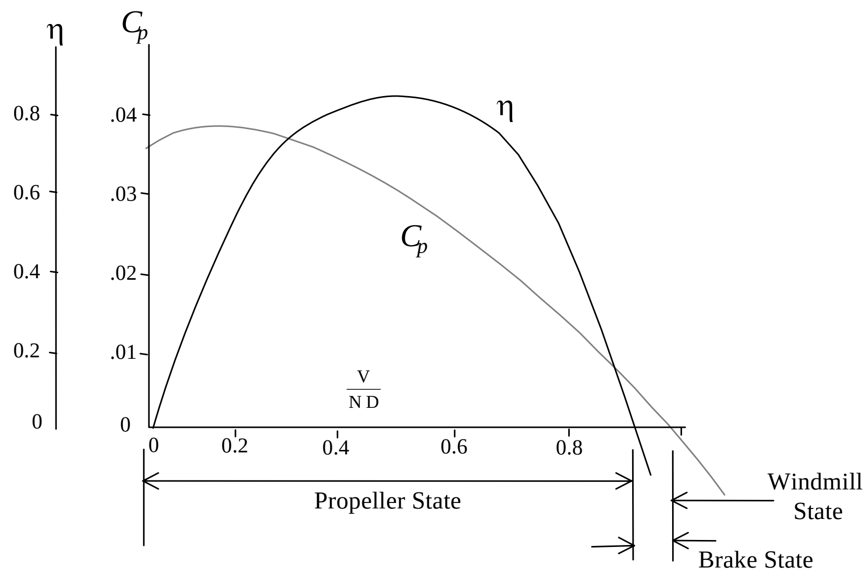
Power Coefficient, C_P $\equiv \dfrac{P}{\rho n^3 D^5} = \dfrac{Q\times\omega}{\rho n^3 D^5} = \dfrac{Q\times 2\pi n}{\rho n^3 D^5} = 2\pi\dfrac{Q}{\rho n^2 D^5} = 2\pi C_Q$



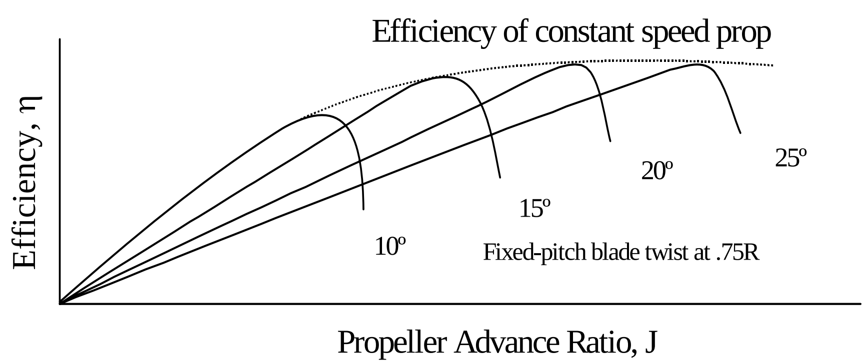
11.4 Propeller Efficiency and States

Propeller efficiency (η)

$$\eta \equiv \frac{P_{out}}{P_{in}} = \frac{Thrust \times V_T}{Q \times \omega} = \frac{C_T \rho n^2 D^4 \times V_T}{C_Q \rho n^2 D^5 \times 2\pi n} = \frac{1}{2\pi}\frac{C_T}{C_Q}\frac{V_T}{nD} = \frac{C_T}{C_P} J$$



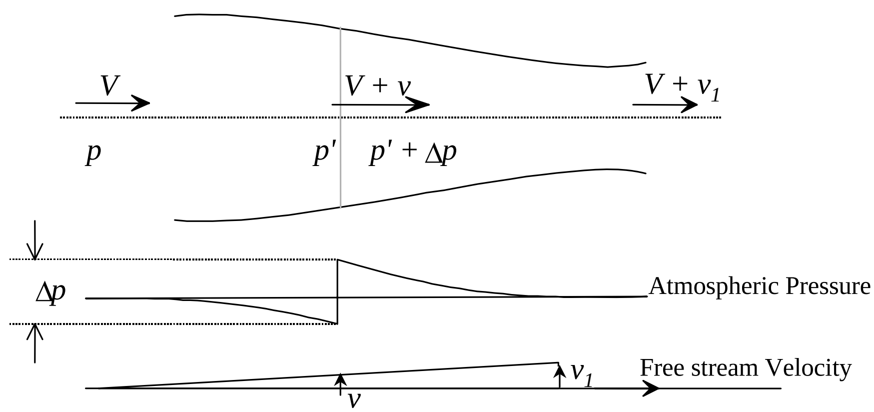
Propeller state: positive thrust & efficiency, power supplied by engine.
Brake state: negative thrust & efficiency, power supplied by engine.
Windmill state: negative thrust & η, power supplied by freestream.



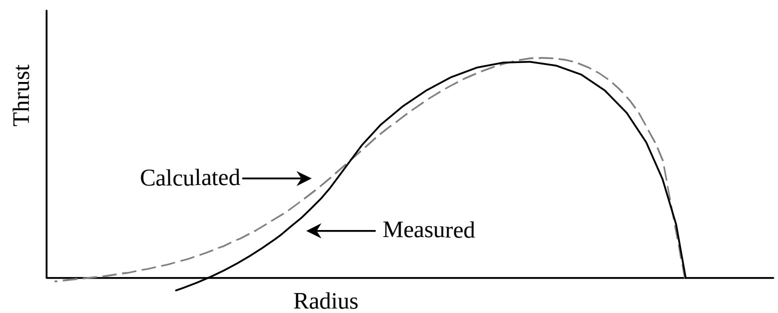
11.5 Propeller Theory

Simple momentum theory describes pressure jump (ΔP) across propeller disk .

- The downstream velocity increment (v_1) is twice the velocity increment at the disk (v) .
- Thrust (ΔP) = ΔP x disk area
- Froude's momentum theory: efficiency = $\eta \equiv \dfrac{TV_T}{T(V_T + v)} = \dfrac{V_T}{V_T + v}$



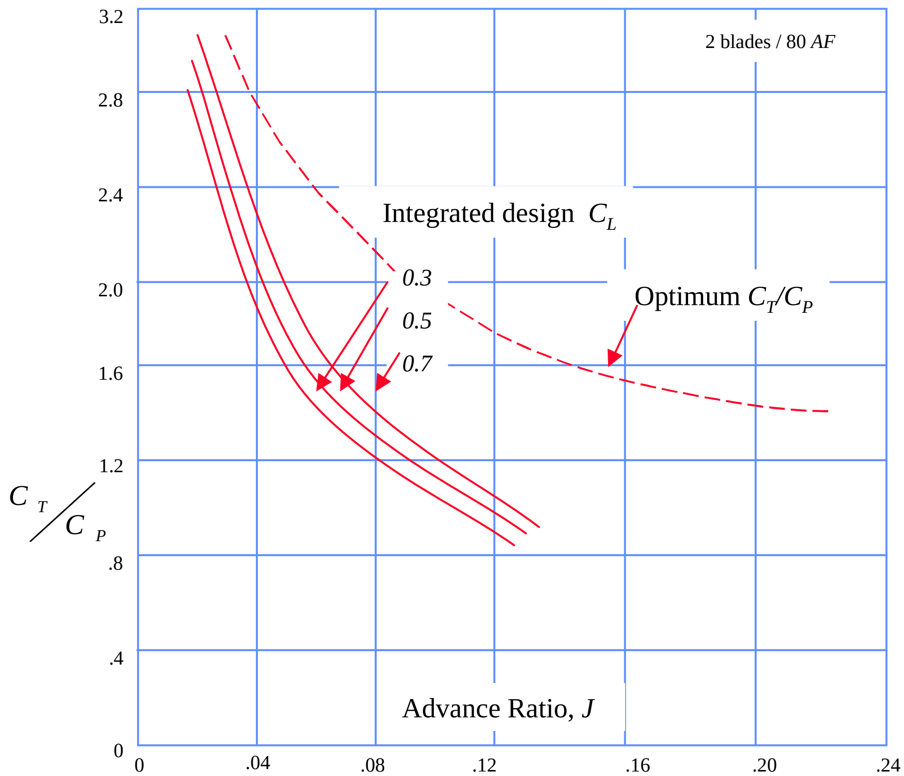
Blade element theory tends to be more complex and may include corrections for tip swirl losses, Mach effects, nacelle blockage, etc. Below is a comparison of typical calculated and measured thrust distribution.



11.6 Propeller Modeling

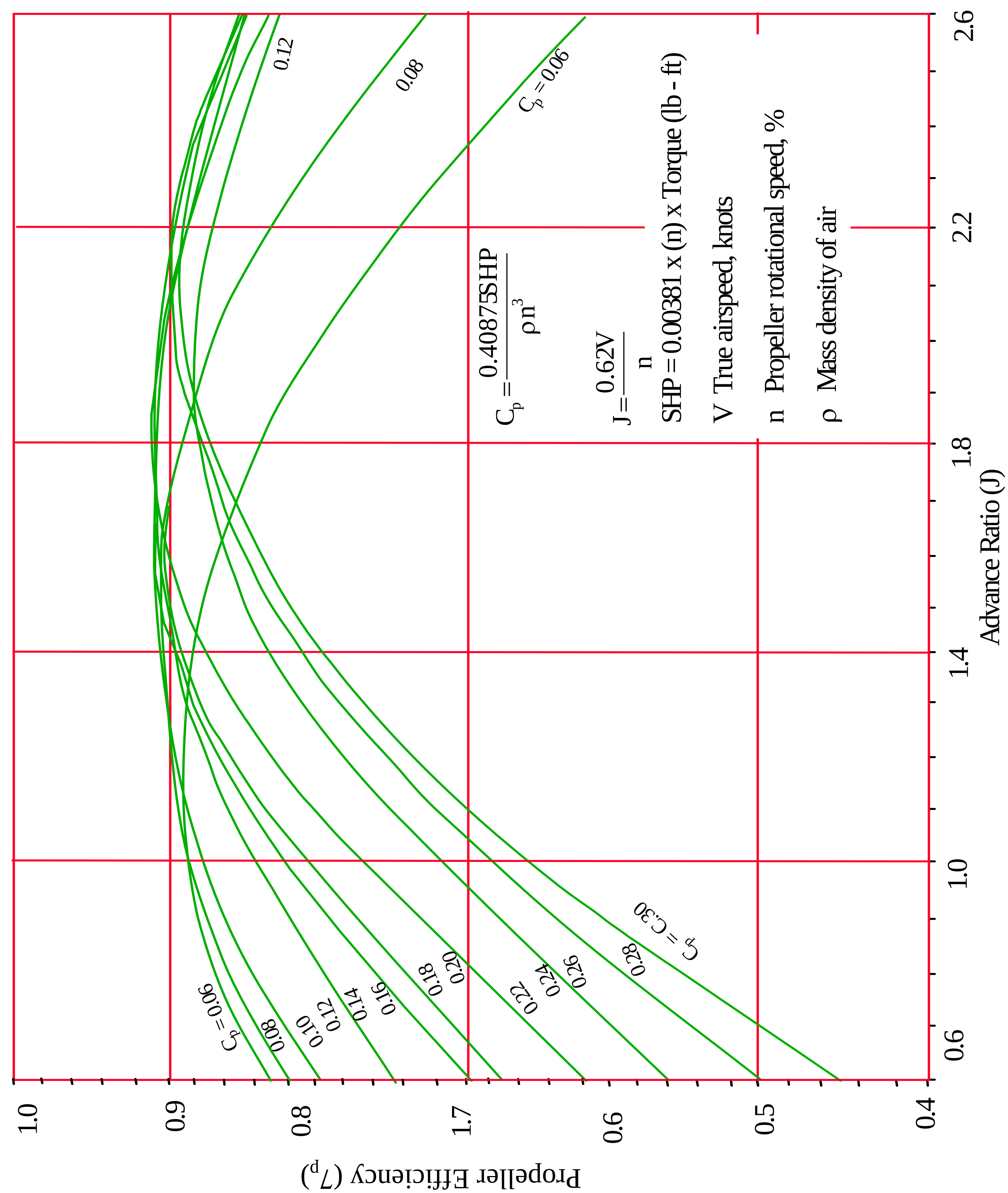
- For a specified propeller geometry; C_T, C_P, J, and blade angle (θ) are interrelated such that knowledge of any two defines the other two.
- Calculate propeller efficiency as $\eta = JC_T / C_P$.
- Models assume isolated conditions, i.e., without nacelle blockage.
- Models assume negligible Mach effects at propeller tips.
- Different models required for static and "in-flight" conditions.

Determine static C_T and C_P using "**Static Thrust Chart**" (ref 11.2)



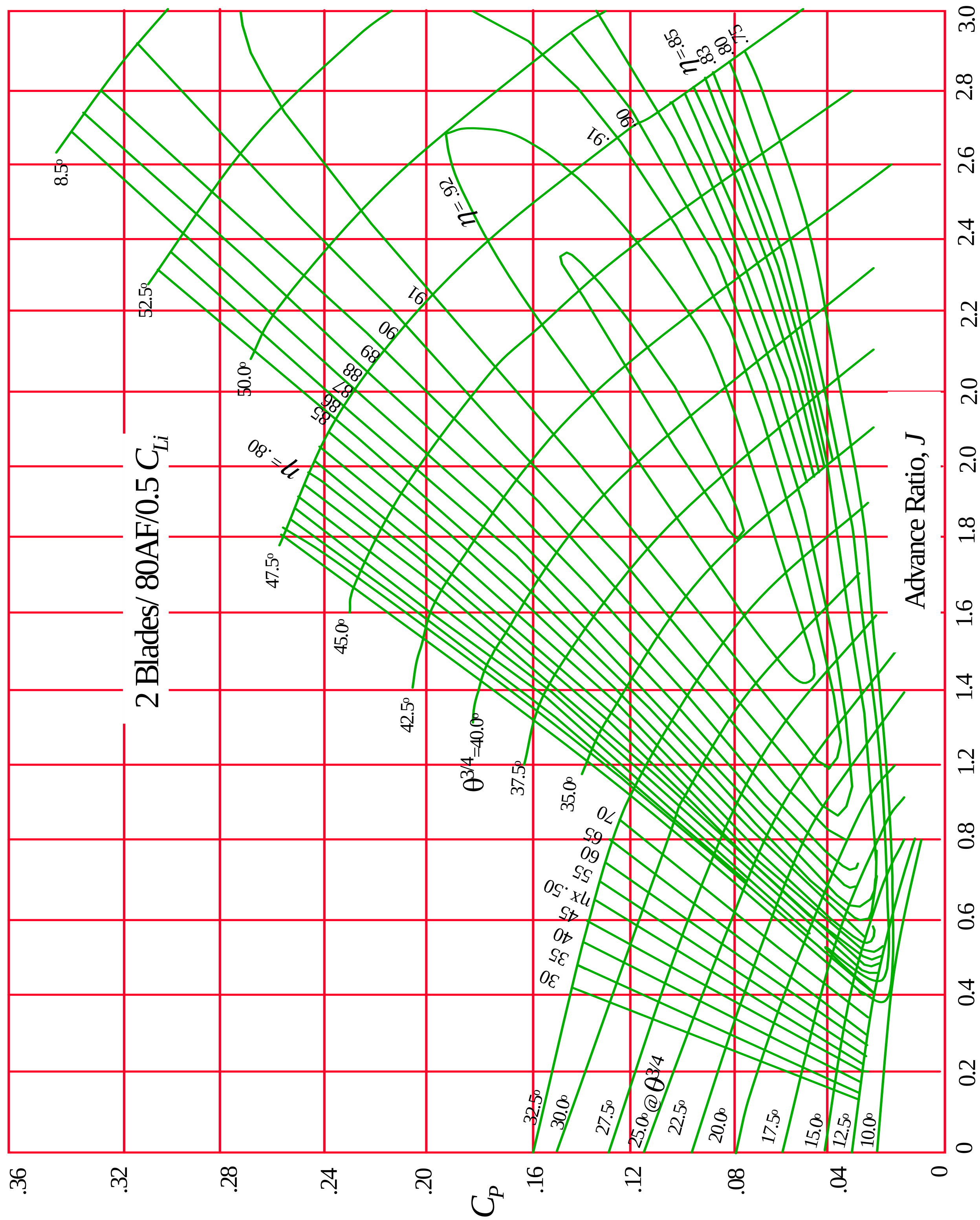
~ Separate charts exist for each combination of AF and # of blades (B).
~ Enter chart at appropriate J & C_{li}

Static Thrust = $T_{static} = \frac{C_T}{C_P}\frac{SHP}{ND}33{,}000$ where N = Propeller RPM Determine isolated propeller in-flight efficiency (η) from the appropriate **"Flight Charts."** They are typically presented in one of two forms.

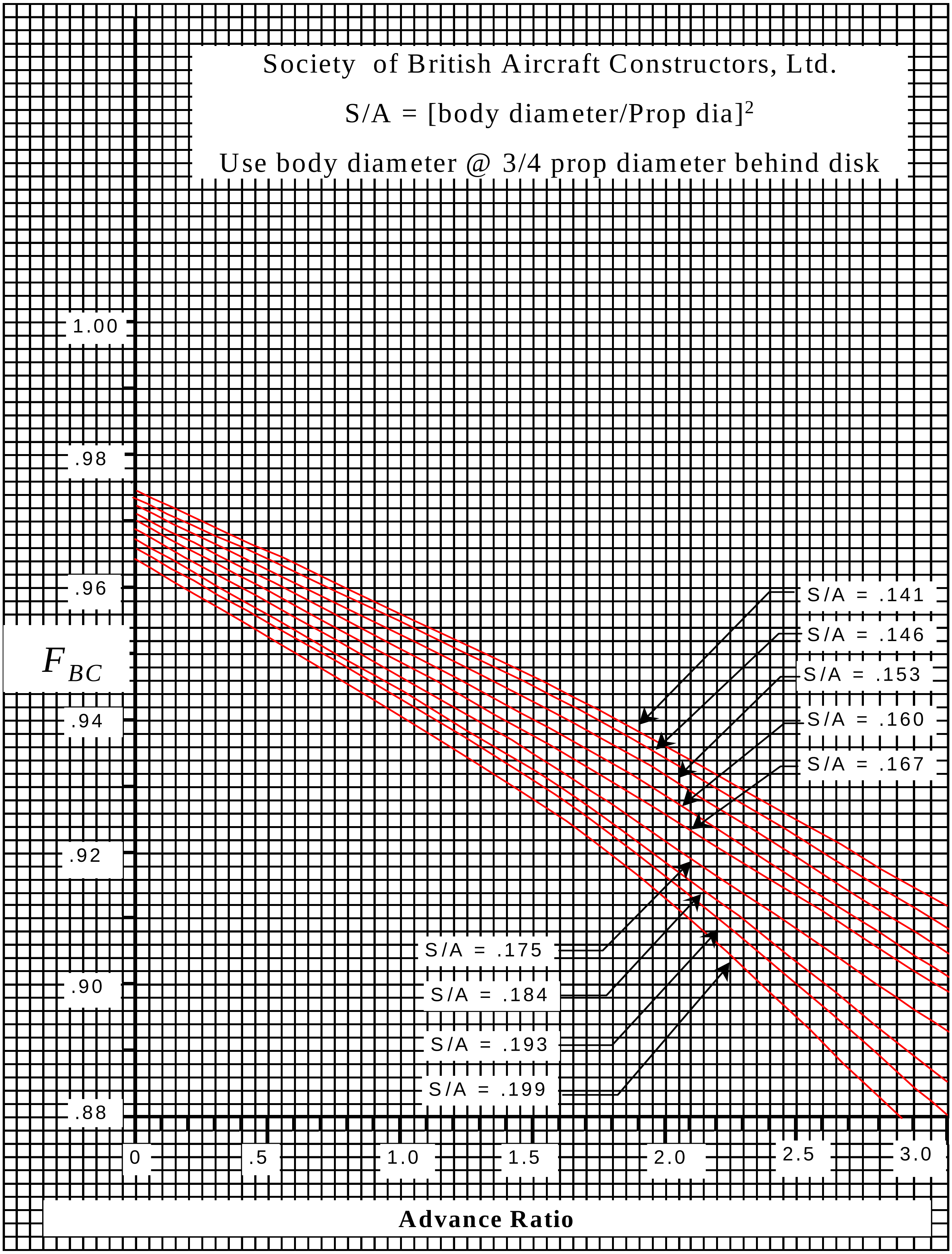


Above example for AiResearch TPE331-3U-303G engines and Hartzell T10282HDB-4R 3-blade, constant speed, feathering propellers.

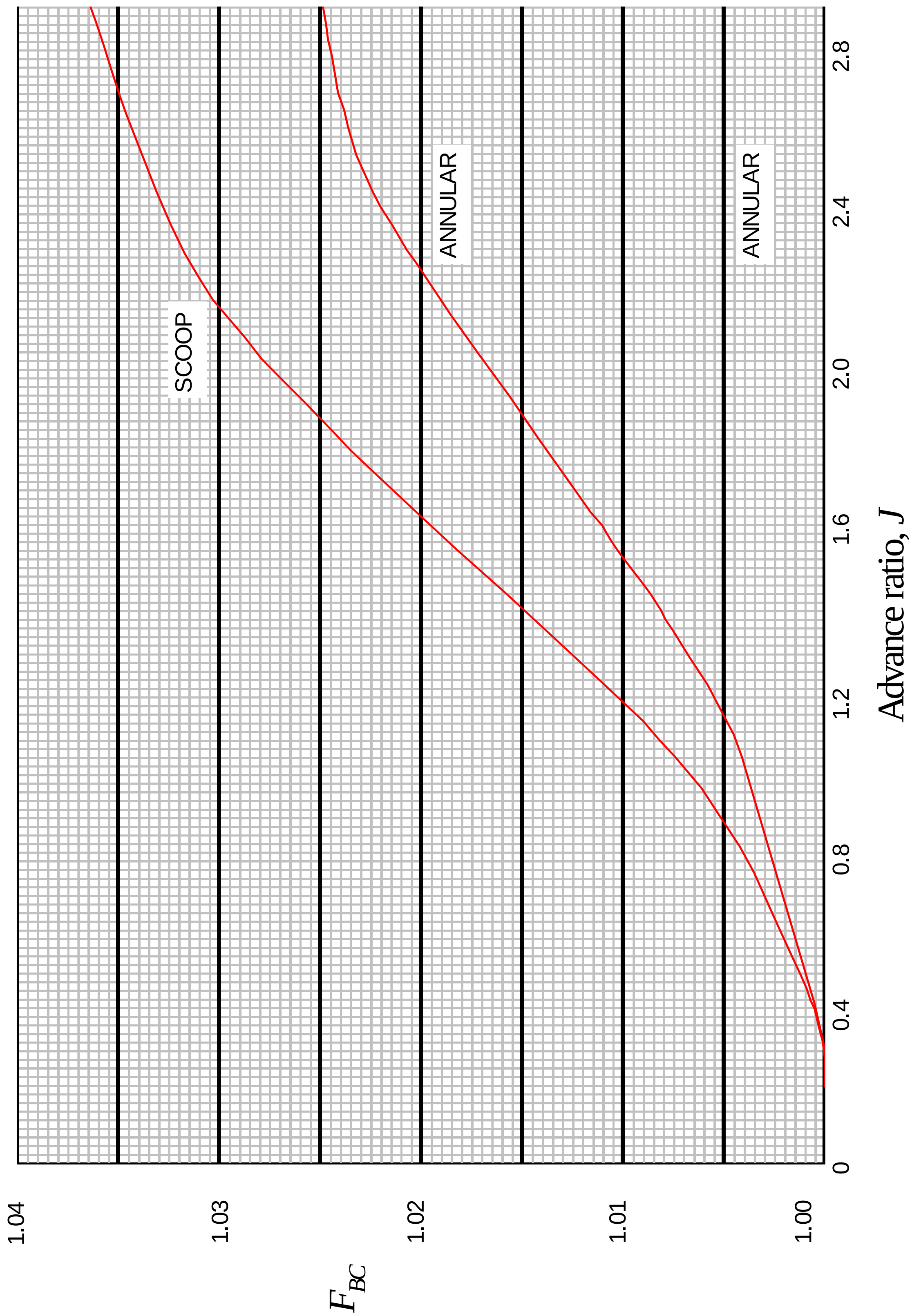
The other "in-flight η" format also requires calculation of C_P and J. Below is a typical flight chart published by Hamilton Standard (Ref 11.2). This applies to a propeller with 2 blades, AF= 80, and C_{Li} =0.5



A **body correction factor** (F_{BC}) should be applied to account for reduced efficiency due to body flow blockage immediately behind the propeller. Two examples follow.



Hamilton Standard also publishes a generalized nacelle blocking correction for typical scoop and annual inlet nacelles used on typical turboprops.

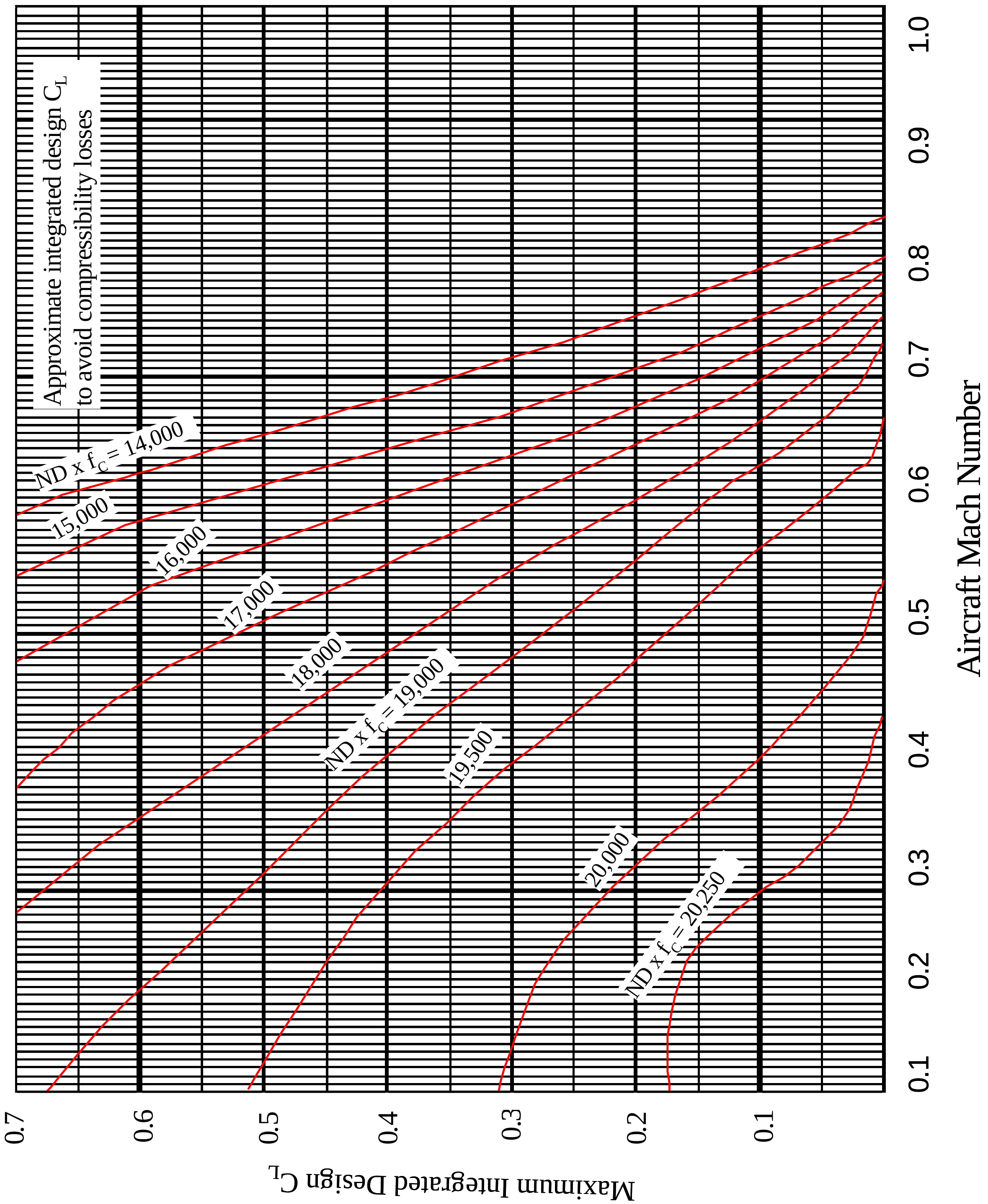


To determine if tip compressibility corrections are appropriate, find the maximum integrated design lift coefficient, C_{Limax} from the graph below.

~ Enter at flight Mach number, and move across at appropriate NDf_c.

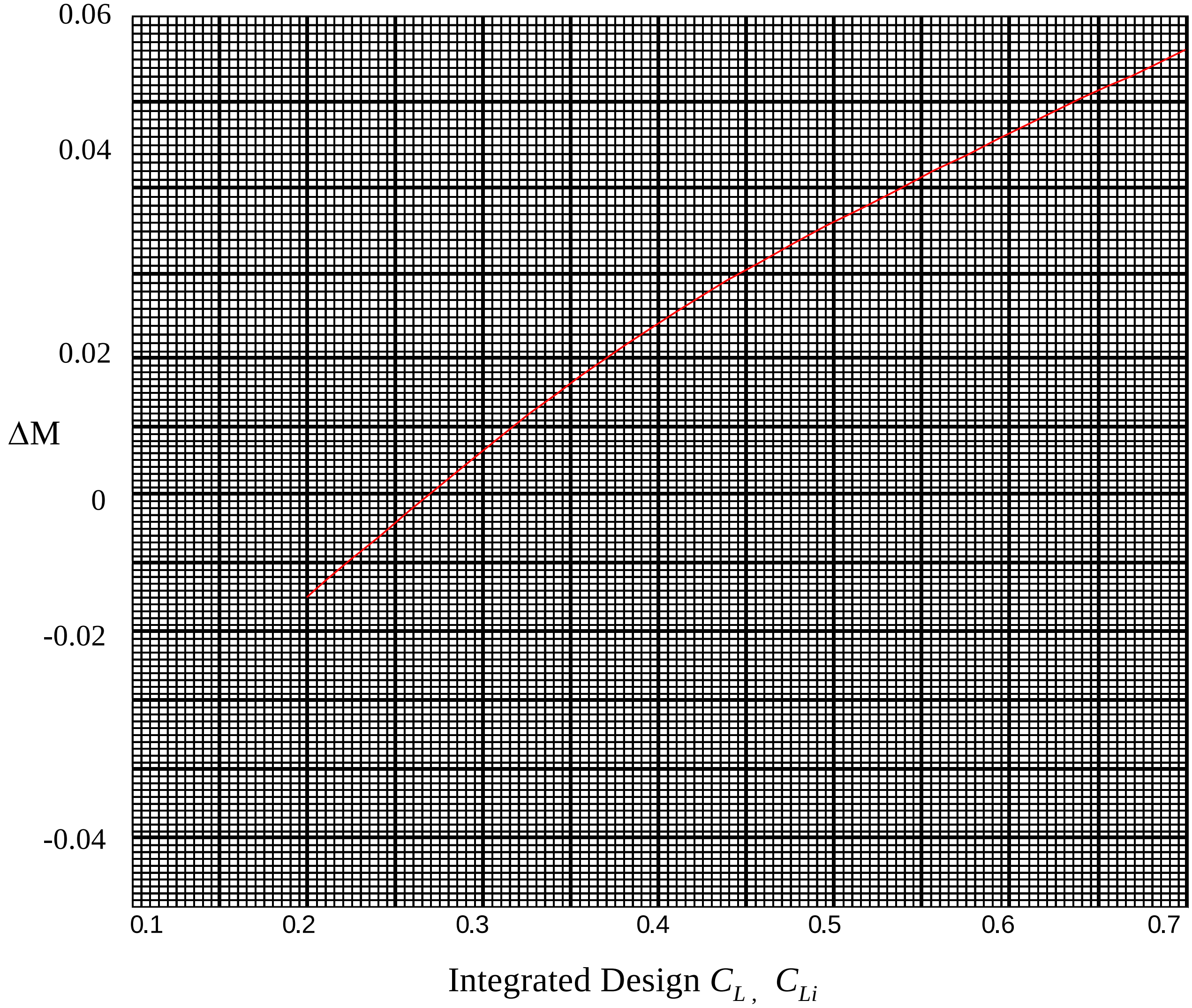
$$f_c = \frac{1}{\sqrt{\theta}}$$

~ If C_{Limax} is below calculated C_{li} , then corrections are required.



If tip compressibility corrections are necessary, then the first step is to

- Determine the **Mach number adjustment for the effect of blade camber** (ΔM) from the figure below.

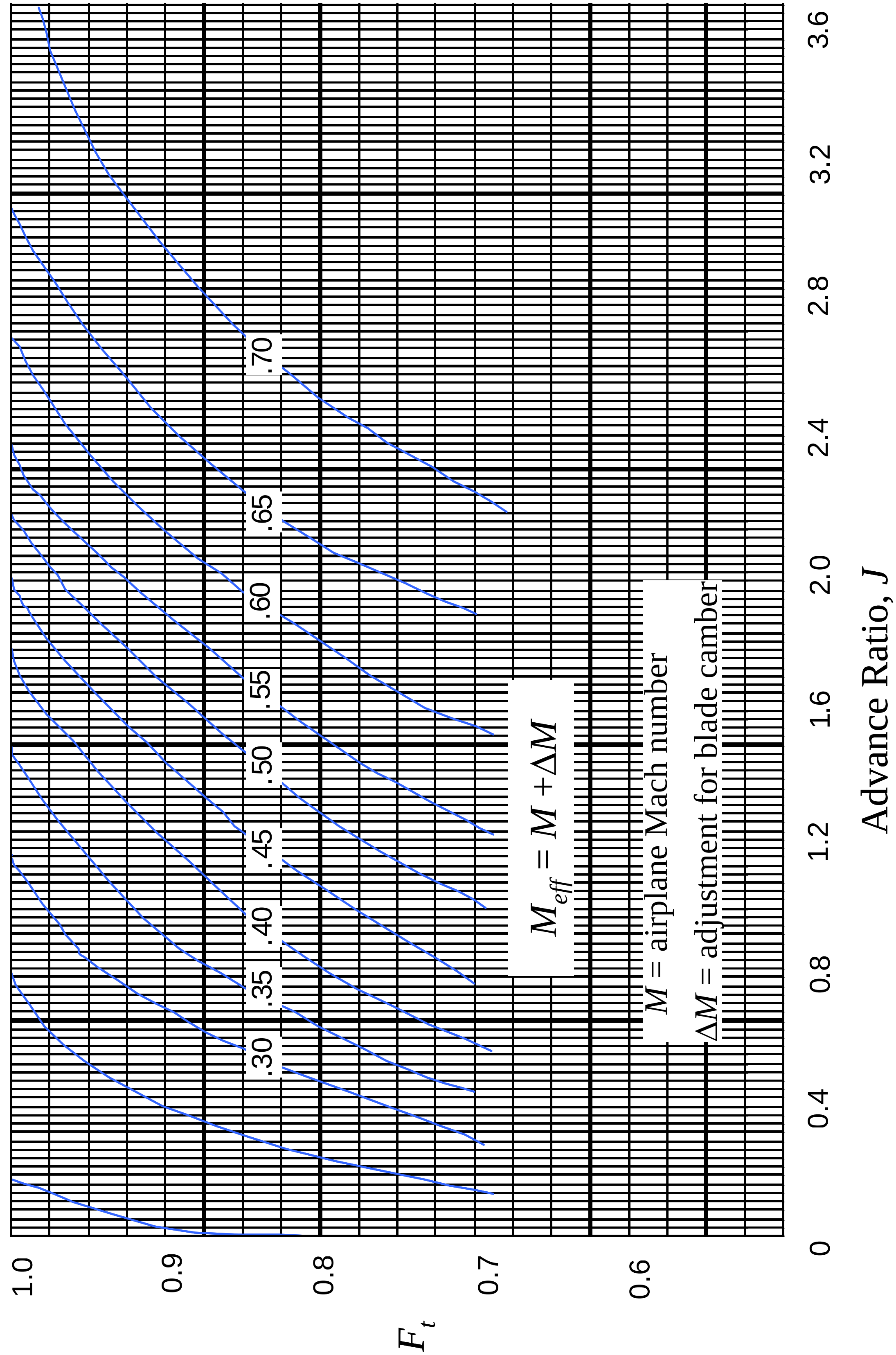


- Next, add ΔM from above to flight Mach number to get M_{eff}.
- Enter adjacent **generalized compressibility correction chart** to determine propeller efficiency tip factor (F_t)
- Calculate composite propeller efficiency as

$$\eta_{\text{comp}} = \eta \text{ x } F_t \text{ x } F_{BC}$$

- Calculate in-flight thrust as

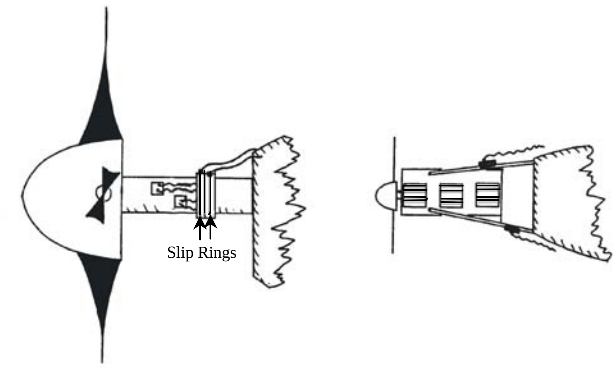
$$T = \frac{\eta_{comp} SHP}{V_T} = \frac{326\,\eta_{comp} SHP}{KTAS}$$



1.0
0.9
0.8
0.7
0.6
F_t
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
Advance Ratio, J
.30
.35
.40
.45
.50
.55
.60
.65
.70
$M_{eff} = M + \Delta M$
M = airplane Mach number
ΔM = adjustment for blade camber

11.7 Propeller Flight Test

The best method for determining η_{comp} is to instrument the prop shaft and/or engine mounts to measure thrust and torque.



Calculate efficiency as

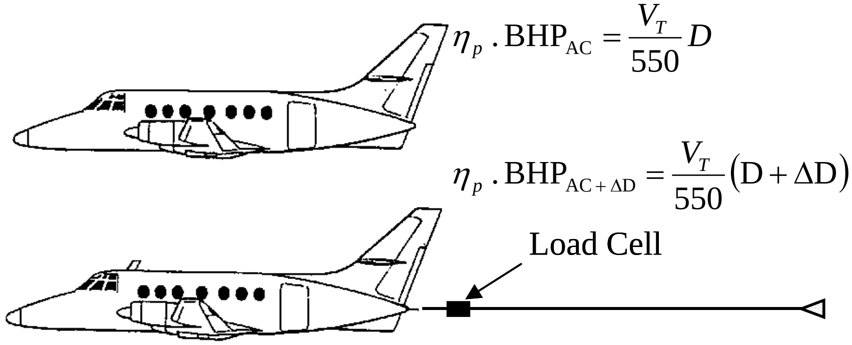
$$\eta_{comp} = \frac{T\,V_T}{Q\ \omega}$$

As an alternate, the **incremental drag method** requires an accurate engine power model, a load cell and a small drag device.

- Trim the aircraft at test *RPM*, V_T , & altitude. Note *SHP* required.
- Repeat above test with drag device and load cell attached. Note the power requirement change (ΔSHP) and load cell drag (ΔD).

$$\eta_p \,.\, \mathrm{BHP_{AC}} = \frac{V_T}{550} D$$

$$\eta_p \,.\, \mathrm{BHP_{AC+\Delta D}} = \frac{V_T}{550}\left(\mathrm{D} + \Delta\mathrm{D}\right)$$



- Calculate aircraft drag and prop efficiency as

$$D = \frac{\Delta D(SHP)}{\Delta SHP} \qquad \qquad \eta = \frac{V_T\, D}{550\ (SHP\ \ )}$$

- This technique assumes the same η for both tests and is valid if J is constant and the C_P change is small. The drag device must therefore be small enough to not violate this assumption, yet large enough for the change in *SHP* to be measurable on engine instruments.

11.8 References

11.8.1 Roberts, Sean, “Light Aircraft Performance for Test Pilots and Flight Test Engineers,” Flight Re search Inc., Mojave CA, 1982.

11.8.2 anon., Hamilton Standard Propeller Efficiency Charts (a.k.a.Redbook), PDB 6101.

11.8.3 Von Mises, Richard, “Theory of Flight,” McGraw-Hill, 1945.

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