Section 3 Universe/Earth/Atmospheric Properties

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Section 3 Recurring Nomenclature

H_p = pressure altitude. The pressure associate with geopotential altitude on a standard day.
T = absolute temperature (Rankin or Kelvin)
T_R = absolute temperature, Rankin scale
T_o = standard day seal level absolute temperature
P = ambient pressure
P_o= standard day seal level ambient pressure
ρ = ambient density
ρ_o= standard day seal level ambient density
$\delta = P/P_o$ = atmospheric pressure/std day sea level pressure
$\theta = T/T_o$ = atmospheric absolute temp / std day sea level absolute temp
$\sigma = \rho/\rho_o$ = atmospheric density/std day sea level density
g = acceleration due to gravity
g_o = standard earth acceleration due to gravity
a_o = speed of sound at std day sea level temperature

Section 3.1 Universal Constants (reference 3.1)

Avogadros number, N_o	6.022169×10^{23} molecules/mole	
Boltzmann constant, k	1.380×10^{-23}	J/^{o}K
electron charge, e	$1,602 \times 10^{-19}$	coulomb
electron mass, m_e	9.109×10^{-31}	kg
gas constant, R	8.31434J/^{o}K	mole
gravitational constant, G	6.673×10^{-11}	Nm^2/kg^2
neutron mass, m_n	1.674×10^{-27}	kg
Planck constant, h	6.625×10^{-34}	J sec
proton mass, m_p	1.672×10^{-27}	kg
speed of light in a vacuum, c	2.998×10^{8}	m/sec
unified atomic mass constant, m_u	1.660×10^{-27}	kg
volume of ideal gas (std temp & press)	2.241 x 10	m^3/mol

Newtonian Gravity

The gravitaional field (g) near any mass can be calculated as

$$g = \frac{GM}{(R_A)^2}$$

where G is the universal gravitational constant and R_A is the absolute distance from the center of mass M

Section 3.2 Earth Properties (references 3.9.2, 3.9.3)

Std Earth gravitational acceleration, g_o = 9.8066 m/s^2 = 32.174 ft/s^2
mass = 5.9722 × 10^{24} *kg* = 13.22 x 10^{24} *lb*
rotation rate, ω = 7.292115 × 10^{-5} *rad/sec*
average density = 5.522 g/cm^3 = 344.7 lb/ft^3
radius average, R_{avg} = 6,367,444 *m* = 3956.538 *st. miles* = 20,890,522 *ft*
radius at the equator (R_e) is 6,378,137 *m* (±2)
radius at the poles R_p = 6,356,752 [*m*]
radius as a function of latitude, ϕ (assumes perfect ellipsoid):

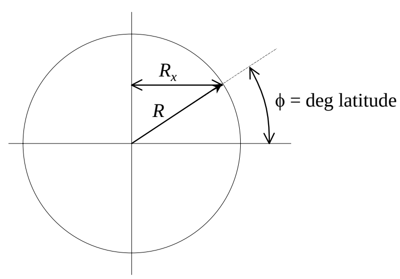
$$R = \left[\left(\frac{\cos\phi}{R_e} \right)^2 + \left(\frac{\sin\phi}{R_p} \right)^2 \right]^{-\frac{1}{2}}$$

Centrifugal Relief from Gravity

The earth's "normal" gravity field includes both the Newtonian Law and a correction for the centrifugal force caused by the earth's rotation. The centrifugal relief correction is

$$\Delta CR = -\frac{V^2}{R_x} = -\frac{(R_x \omega)^2}{R_x} = R_x \omega^2$$

where ω is the earth's rotation rate and R_x is the perpendicular distance from the earth's axis to the surface and can be calculated as $R_x = R \cos\phi$ (see figure below).



For any centrifugal relief calculations associated with aircraft performance, it is sufficiently exact (g ±0.00004 m/s^2) to use the average earth radius. An aircraft flying eastward contributes to centrifugal relief while a west-bound aircraft diminishes it.

The International Association of Geodesy publishes the following equation (accurate to 0.005%) to calculate *local sea level gravity including the effects of centrifugal relief for any point fixed to the earth's surface*

$$g_{lsl} = 9.780327\left(1 + 0.00530224\sin^2\phi - 0.000058\sin^2 2\phi\right)\left[\frac{m}{s^2}\right]$$

The above equation is tabulated below for quick reference.

Latitude (deg)	**Normal g_{local}**	
	(m/s^2)	**(ft/s^2)**
0	9.780327	32.088
15	9.783659	32.098
30	9.792866	32.188
45	9.805689	32.171
60	9.818795	32.214
75	9.828569	32.249
90	9.832185	32.258

The standard acceleration (g_o) corresponds to a latitude of 46.0625°.
g_{lsl} at the equator and the poles varies ±0.27% from g_o.

Altitude Effect on Gravitational Acceleration
R_A is the sum of the earth's local radius and the geometric distance (h_G) above the surface: $R_A = R + h_G$

Gravitational acceleration at any geometric altitude:

h_G (1000 ft)	$\frac{g_{alt}}{g_{lsl}}$
0	1
10	0.99904
20	0.99809
40	0.99618
60	0.99428
80	0.99238
100	0.99049

$$g_{alt} = g_{sls}\left(\frac{R}{R + h_G}\right)^2$$

Actual Gravitational Pull on an Aircraft

Adding a centrifugal relief correction due to the aircraft's velocity, a complete calculation for its gravitational acceleration is

$$g_{A/C} = \left[g_{lsl} + \omega^2 R\cos\phi\right]\left[\frac{R}{R+h_G}\right]^2 - \left(\omega + \frac{V_G \sin\sigma}{R+h_G}\right)^2 (R+h_G)\cos\phi$$

where V_G = ground speed and σ = ground track angle (0° = true North, 90° = East, etc.).

Gravity Influence on Aircraft Cruise Performance

Even at the same altitude, changes in gravity due to latitude or centrifugal relief directly alter the required lift, drag, and fuel flow. For example, with sufficiently precise instrumentation, data collected heading West could show about 0.5% more drag and fuel flow than data collected heading East (centrifugal relief effect). After determining test and standard (or mission) values for g, flight test values for C_L, C_D, drag, and fuel flow can be corrected to standard as follows:

$$C_{L_{std}} = C_{L_t} \frac{N_{z_{\omega_{std}}}}{N_{z_{\omega_t}}}\left[\frac{g_{std}}{g_{A/C}}\right]$$

$$C_{D_{std}} = \frac{\left(C_{L_{std}}\right)^2}{\pi A \mathrm{Re}}$$

$$\Delta D = D_{std} - D_t = qS\left[C_{D_{i_{std}}} - C_{D_{i_t}}\right]$$

$$\dot{W}_{f_{std}} = \dot{W}_{f_t} + \Delta D \cdot TFSC$$

where N_z = normal load factor,
C_L = lift coefficient, C_D = drag coefficient,
AR = aspect ratio, e = Oswald efficiency factor,
ΔD change in drag force,
$TSFC$ = thrust specific fuel consumption, and

$\dot{W}_{f_{std}}$ = standard day fuel flow

Section 3.3 General Properties of Air (reference 3.9.1)

Gas constant, R = 53.35 ft lb/R lbm = 287.074 J/kg K
= 1716 lb(ft)/slgs(*R*) = 3089.7 lb(ft)/slgs(*K*)

Speed of sound = $a_o(\theta)^{½}$
= 49.02 $(T_R)^{1/2}$ *ft/sec*
= 33.42 $(T_R)^{1/2}$ *miles/hr*
= 29.04 $(T_R)^{1/2}$ *knots*
= 20.05 $(T_R)^{1/2}$ *m/sec*

Density, ρ = .0023769 *slug/ft*^{3} = 1.225 *kg/m*^{3} *(at 15^o C)*

Specific weight, g_ρ = .07647 *sec*^{2}*/ft*^{4}

Specific heat capacity at 59^o*F* (=T_o)
at constant pressure, c_p = .240 *BTU/lb R* = 1004.76 *J/kg K*
at constant volume, c_v = .1715 *BTU/lb R* = 717.986 *J/kg K*
specific heat ratio, $\gamma = \{c_p / c_v\}$ =1.4

Normal Composition of clean, dry atmospheric air near sea level

Nitrogen, N_2	78.084 % by volume
Oxygen, O_2	20.948 %
Argon, *A*	0.934 %
Carbon Dioxide, CO_2	0.031 %
Neon, *Ne*	0.002 %
total	99.9988 %

plus traces of helium, kryton, xenon, hydrogen, methane, nitrous oxide, ozone, sulfur dioxide, nitrogen dioxide, ammonia, carbon monoxide, and iodine.

Viscosities of Air

Coefficient of Viscosity, $\mu_c = \dfrac{7.3025x10^{-7}(T_R)^{3/2}}{T_R + 198.72}$ lb/ft sec

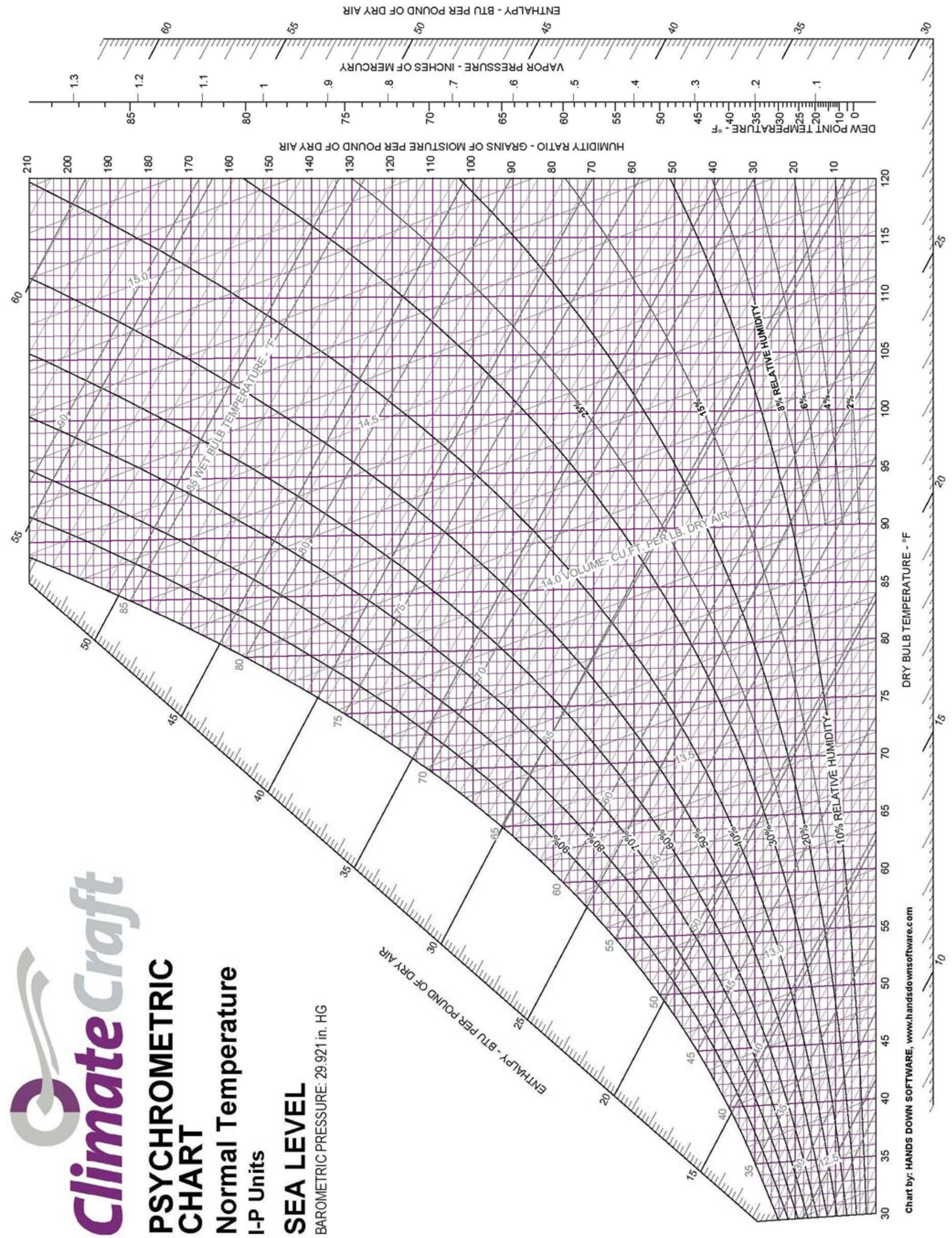
Kinematic viscosity, $\nu = \dfrac{\mu_c}{g\rho}$ ft^2/sec

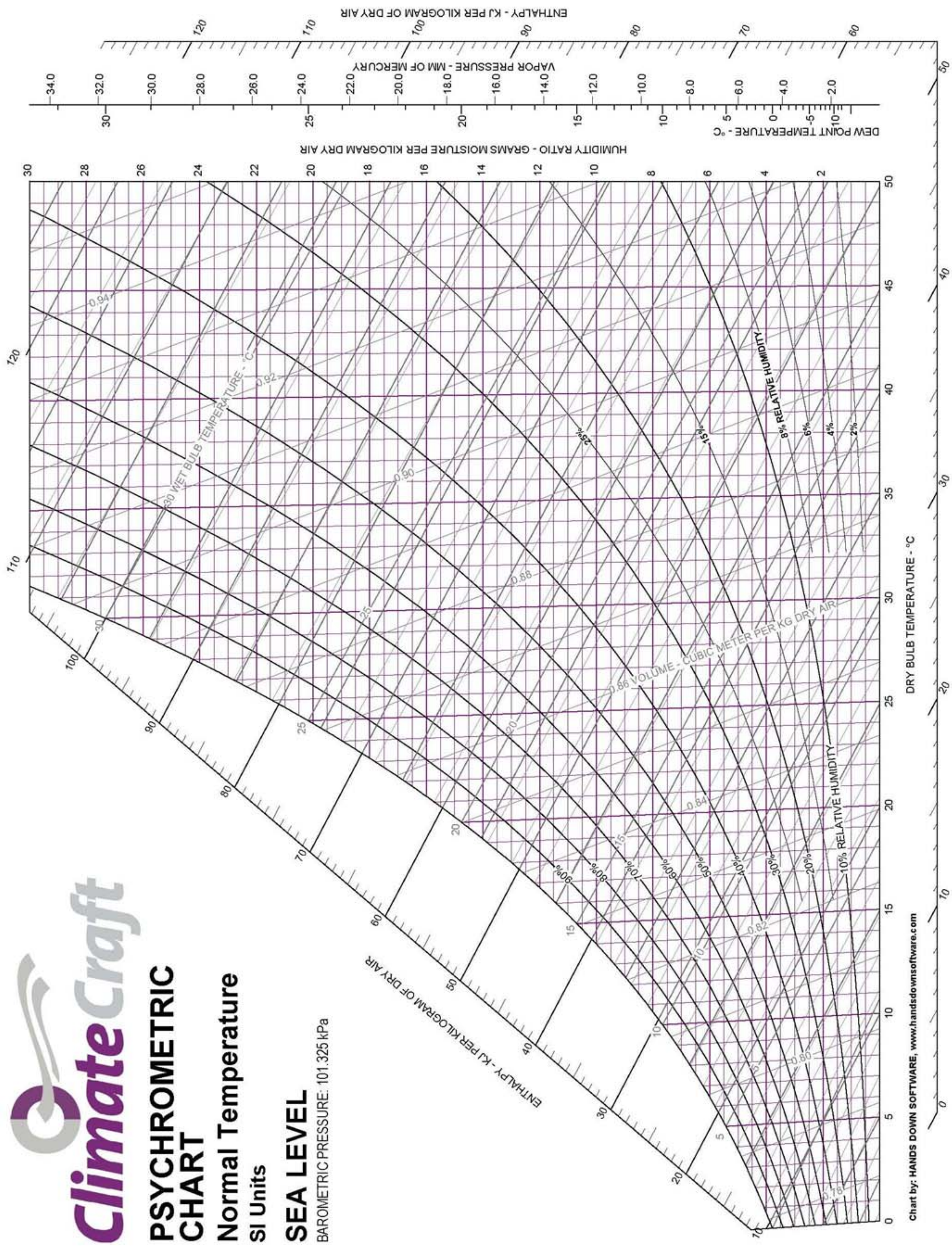
Absolute Viscosity, lb $\mu = \rho\nu = \left[.317(T_R)^{3/2}\left(\dfrac{734.7}{T_R + 216}\right)\right]x10^{-10}$ sec/ft^2

Atmospheric Viscosity (U.S. Standard Atmosphere)

Pressure Altitude *ft*	Kinematic Viscosity υ *(ft^2/sec)*	Absolute Viscosity μ *(lb sec/ft^2)*
0	1.572 x 10^{-4}	3.737 x 10^{-7}
5,000	1.776	3.638
10,000	2.013	3.538
15,000	2.293	3.435
20,000	2.625	3.330
25,000	3.019	3.224
30,000	3.493	3.115
35,000	4.065	3.004
40,000	5.074	2.981
45,000	6.453	2.982
50,000	8.206	2.983
55,000	10.44	2.985
60,000	13.27	2.986
70,000	21.69	3.005
80,000	35.75	3.043
90,000	58.53	3.080
100,000	95.19	3.118
150,000	1066	3.572
200,000	6880	3.435

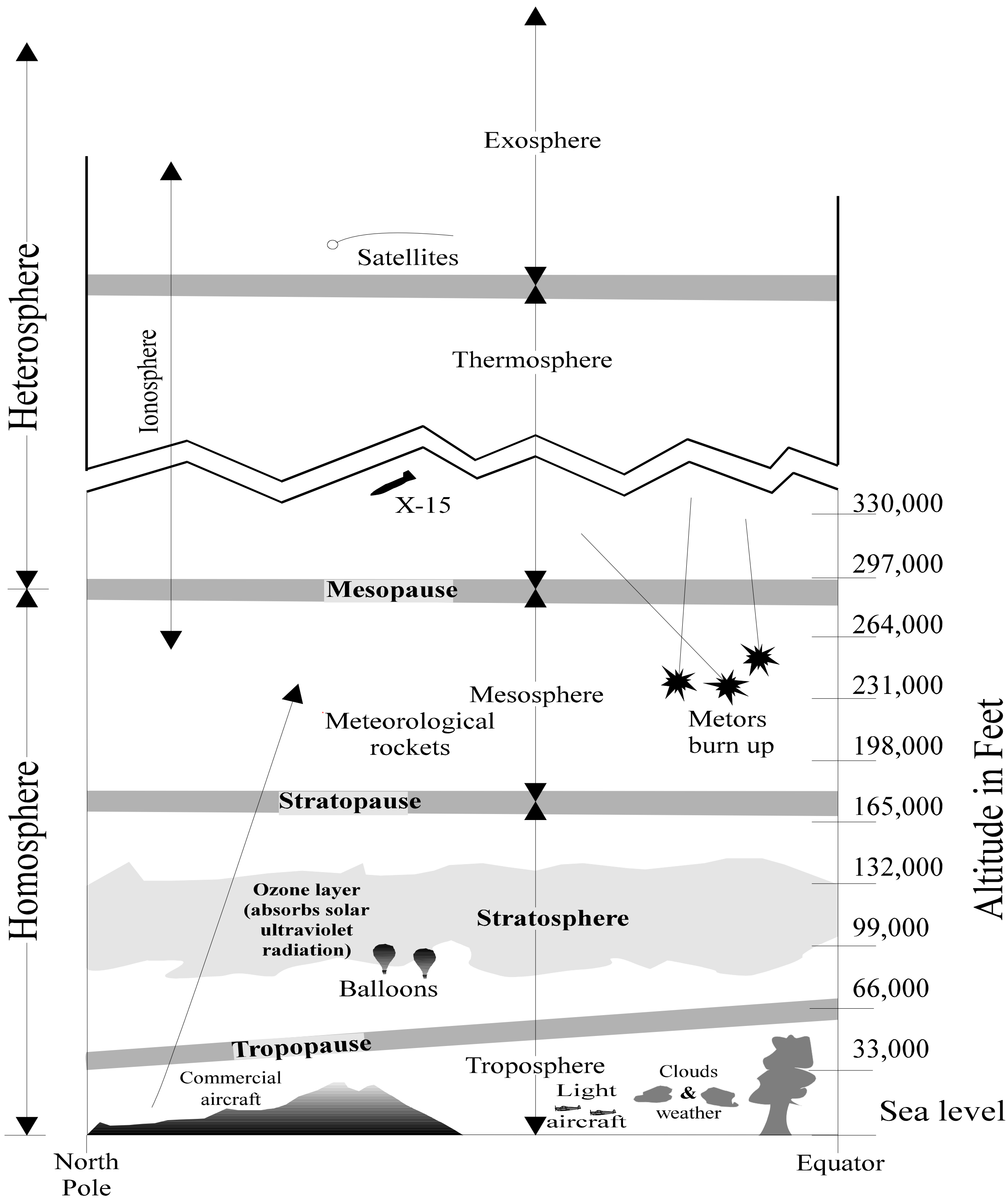
Psychrometric Chart for Seal Level Barometric Pressure




ClimateCraft
PSYCHROMETRIC CHART
Normal Temperature
SI Units
SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
ENTHALPY - KJ PER KILOGRAM OF DRY AIR
VAPOR PRESSURE - MM OF MERCURY
DEW POINT TEMPERATURE - °C
HUMIDITY RATIO - GRAMS MOISTURE PER KILOGRAM DRY AIR
DRY BULB TEMPERATURE - °C
WET BULB TEMPERATURE - °C
VOLUME - CUBIC METER PER KG DRY AIR
RELATIVE HUMIDITY
Chart by: HANDS DOWN SOFTWARE, www.handsdownsoftware.com

Section 3.4 Standard Atmosphere

Divisions of the Atmosphere



Constantly changing atmospheric conditions cannot be duplicated at will to provide the exact environment in which a flight takes place. A standard atmosphere provides a common basis to relate all flight test, wind tunnel results, aircraft design and general performance. Several models of "standard atmosphere" exist with minor differences based on mathematical constants used in the calculations.

Geometric altitude, h_G , is defined as the height of an aircraft above sea level (also called **tapeline** altitude)

Absolute altitude, h_a, is defined as the height of an aircraft above the center of the earth: (geometric altitude + radius of the earth).

Geopotential altitude, h, is required because g changes with height.If potential energy is calculated using sea level weight ($W_{SL} = mg_o$) instead of actual weight ($W = mg$), then the altitude must be lower.

$$W\, h_G = W_{SL}\;\; h$$

Pressure altitude, H_p is the altitude, on a standard day, at which the test day pressure would be found

Density altitude is the altitude, on a standard day, at which the test day density would be found

Temperature altitude is the altitude, on a standard day, at which the test day temperature would be found

Assumptions on which the standard atmosphere is built

1. The air is dry (only 0.4% per volume of water vapor)
2. The air is a perfect gas and obeys the equation of state,

$$P = \rho g R T$$

where $R = 53.35$ ft lb/^{o}R lbm

3. The gravitational field decreases with altitude
4. Hydrostatic equilibrium exists ($\Delta p = -\rho g_o \Delta h$)

Standard Day Sea Level Atmospheric Conditions

P_o = 2116.22 lb/ft^2 = 14.696 lb/in^2 = 29.921 in Hg
= 1013.25 HPa (mb) = 101325 Pa

T_o = 288.15 K = 518.67 R = 59 ^oF = 15 ^oC

ρ_o = 0.0023769 slgs/ft^3 = 0.07647 lbm/in^3 = 1.255 kg/m^3 (at 15^o C)

a_o = 1116.45 ft/sec = 661.478 KTAS = 761.14 mph = 340.294 m/sec

g_o = 32.174 ft/sec^2 = 9.80665 m/sec^2

L = standard temperature lapse rate = 0.0019812 K /ft

1976 U.S Standard Atmosphere Equations

Troposhere - below 36,089 ft (11,000 m) < 22636 Pa

$$\theta = 1 - (L/To)\, h = 1\text{-}(6.8755856 \times 10^{-6})\, h$$

$$\sigma = \theta^{n-1}$$

$$\delta = \theta^{n}$$

where n = 5.255876, h = geopotential altitude (ft)

Stratosphere- between 36,089 ft and 65,616 ft (20,000 m) the standard day temperature is a constant 216.65 K, therefore:

$$\theta = 0.751865$$

$$\sigma = .297076\ e^{-0.000048063\ [h-36,089]}$$

$$\delta = .223361\ e^{-0.000048063\ [h-36,089]}$$

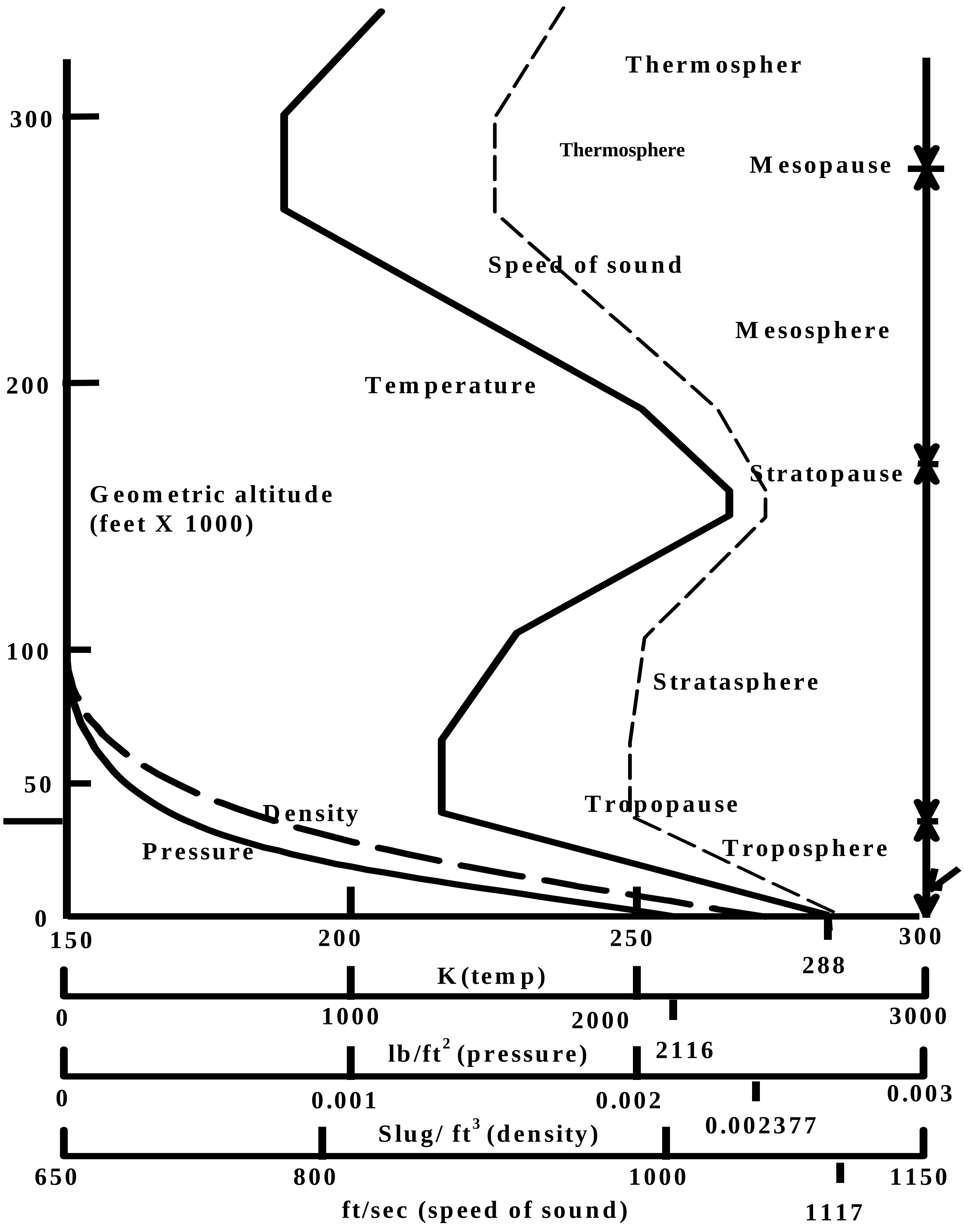
The above relations characterize the standard atmosphere table in this handbook. They may be re-written to solve for pressure altitude (H_p) for any ambient pressure. Below the tropopause (ambient pressure greater than 472.683 psf or 22632 Pa)

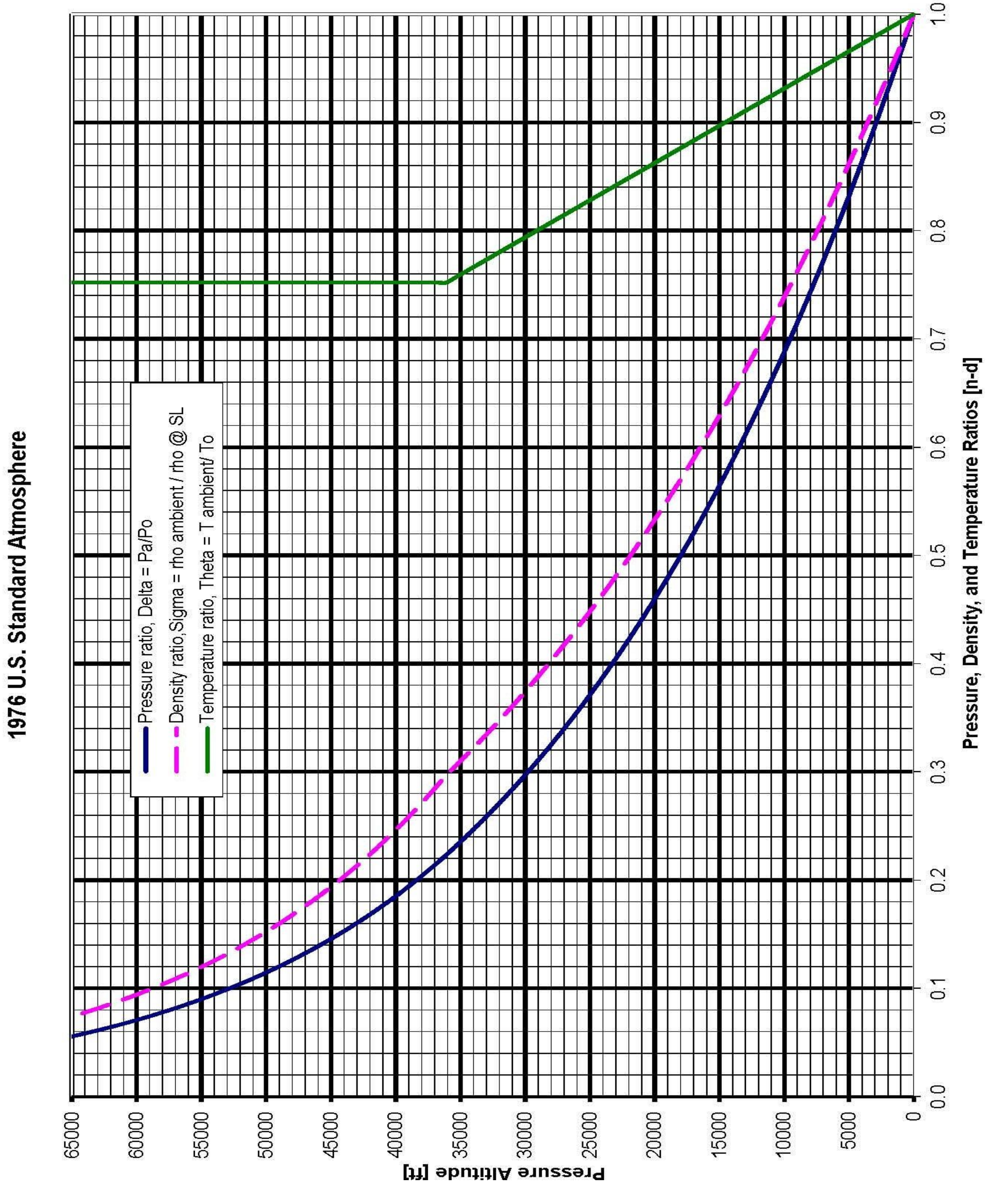
$$H_p\,[\text{ft}] = [1\text{-}(P_a/P_o)^{0.1902632}]/[6.8755856 \times 10^{-6}]$$

In the troposphere (ambient pressure between 114.347 and 472.683 psf or between 5475 and 22632.1 Pascal)

$$H_p\,[\text{ft}] = 36089+[\ln(P_a/P_o)+1.498966]/\ 0.000048063$$

1976 U.S. Standard Atmosphere Graph




1976 U.S. Standard Atmosphere
Pressure ratio, Delta = Pa/Po
Density ratio,Sigma = rho ambient / rho @ SL
Temperature ratio, Theta = T ambient/ To
Pressure, Density, and Temperature Ratios [n-d]
Pressure Altitude [ft]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
50000
55000
60000
65000

1976 U.S. Standard Atmosphere - Below Tropopause [<11 Km]

		Po = 14.696	Po = 2116.22807	Po = 101325	Po = 29.92126		ρo = .0023769	ρo = 1.225		To = 288.15	To = 15	To = 518.67	To = 59
		Ambient Air Pressure (P a)					Amb. Air Density (ρa)			Ambient Air Temperature (T a)			
Hp [ft]	δ (=Pa/Po)	[psi]	[psf]	[Pa]	[in Hg]	σ (= ρa/ρo)	[slg/ft3]	[kg/m3]	θ (=Ta/To)	[K]	[deg C]	[R]	[deg F]
-1000	1.036670	15.23490	2193.8300	105040.58	31.01847	1.029591	0.0024472	1.261249	1.00688	290.131	16.98	522.24	62.57
0	1.000000	14.69600	2116.2281	101325.00	29.92126	1.000000	0.0023769	1.225000	1.00000	288.150	15.00	518.67	59.00
1000	0.964388	14.17264	2040.8640	97716.57	28.85569	0.971064	0.0023081	1.189554	0.99312	286.169	13.02	515.10	55.43
2000	0.929809	13.66447	1967.6881	94212.91	27.82106	0.942773	0.0022409	1.154897	0.98625	284.188	11.04	511.54	51.87
3000	0.896241	13.17116	1896.6514	90811.67	26.81668	0.915117	0.0021751	1.121019	0.97937	282.206	9.06	507.97	48.30
4000	0.863662	12.69238	1827.7057	87510.55	25.84185	0.888086	0.0021109	1.087906	0.97250	280.225	7.08	504.41	44.74
5000	0.832048	12.22778	1760.8036	84307.27	24.89593	0.861671	0.0020481	1.055546	0.96562	278.244	5.09	500.84	41.17
6000	0.801378	11.77705	1695.8984	81199.62	23.97824	0.835860	0.0019868	1.023929	0.95875	276.263	3.11	497.27	37.60
7000	0.771630	11.33987	1632.9442	78185.37	23.08813	0.810645	0.0019268	0.993040	0.95187	274.282	1.13	493.71	34.04
8000	0.742782	10.91592	1571.8959	75262.38	22.22497	0.786016	0.0018683	0.962870	0.94500	272.300	-0.85	490.14	30.47
9000	0.714814	10.50490	1512.7089	72428.50	21.38813	0.761964	0.0018111	0.933406	0.93812	270.319	-2.83	486.57	26.90
10000	0.687705	10.10651	1455.3396	69681.66	20.57699	0.738479	0.0017553	0.904637	0.93124	268.338	-4.81	483.01	23.34
11000	0.661434	9.72043	1399.7449	67019.78	19.79093	0.715552	0.0017008	0.876551	0.92437	266.357	-6.79	479.44	19.77
12000	0.635982	9.34639	1345.8825	64440.85	19.02938	0.693173	0.0016476	0.849137	0.91749	264.376	-8.77	475.88	16.21
13000	0.611329	8.98409	1293.7108	61942.87	18.29172	0.671334	0.0015957	0.822384	0.91062	262.394	-10.76	472.31	12.64
14000	0.587455	8.63324	1243.1889	59523.88	17.57740	0.650025	0.0015450	0.796281	0.90374	260.413	-12.74	468.74	9.07
15000	0.564342	8.29357	1194.2766	57181.96	16.88583	0.629238	0.0014956	0.770816	0.89687	258.432	-14.72	465.18	5.51
16000	0.541971	7.96481	1146.9344	54915.22	16.21646	0.608963	0.0014474	0.745979	0.88999	256.451	-16.70	461.61	1.94
17000	0.520324	7.64668	1101.1234	52721.79	15.56874	0.589191	0.0014004	0.721759	0.88312	254.470	-18.68	458.05	-1.62
18000	0.499382	7.33891	1056.8054	50599.84	14.94213	0.569915	0.0013546	0.698145	0.87624	252.488	-20.66	454.48	-5.19
19000	0.479127	7.04126	1013.9430	48547.59	14.33610	0.551124	0.0013100	0.675127	0.86936	250.507	-22.64	450.91	-8.76
20000	0.459544	6.75345	972.4992	46563.26	13.75013	0.532812	0.0012664	0.652694	0.86249	248.526	-24.62	447.35	-12.32
21000	0.440613	6.47525	932.4379	44645.13	13.18370	0.514968	0.0012240	0.630836	0.85561	246.545	-26.61	443.78	-15.89
22000	0.422319	6.20640	893.7235	42791.48	12.63632	0.497585	0.0011827	0.609542	0.84874	244.564	-28.59	440.21	-19.46
23000	0.404645	5.94666	856.3211	41000.65	12.10749	0.480655	0.0011425	0.588802	0.84186	242.582	-30.57	436.65	-23.02
24000	0.387575	5.69580	820.1964	39271.00	11.59672	0.464169	0.0011033	0.568607	0.83499	240.601	-32.55	433.08	-26.59
25000	0.371092	5.45357	785.3157	37600.92	11.10355	0.448119	0.0010651	0.548946	0.82811	238.620	-34.53	429.52	-30.15
26000	0.355182	5.21975	751.6460	35988.81	10.62749	0.432497	0.0010280	0.529809	0.82123	236.639	-36.51	425.95	-33.72
27000	0.339829	4.99412	719.1548	34433.13	10.16810	0.417296	0.0009919	0.511187	0.81436	234.658	-38.49	422.38	-37.29
28000	0.325017	4.77645	687.8104	32932.36	9.72492	0.402506	0.0009567	0.493070	0.80748	232.676	-40.47	418.82	-40.85
29000	0.310733	4.56653	657.5815	31485.00	9.29752	0.388121	0.0009225	0.475448	0.80061	230.695	-42.45	415.25	-44.42
30000	0.296961	4.36414	628.4375	30089.59	8.88545	0.374133	0.0008893	0.458312	0.79373	228.714	-44.44	411.69	-47.98

1976 U.S. Standard Atmosphere - Below Tropopause [<11 Km]

		Po = 14.696	Po = 2116.2280	Po = 101325	Po = 29.92126		ρo = .0023769	ρo = 1.225		To = 288.15	To = 15	To = 518.67	To = 59
		Ambient Air Pressure (Pa)					Amb. Air Density (ρa)			Ambient Air Temperature (Ta)			
Hp [ft]	δ (=Pa/Po)	[psi]	[psf]	[Pa]	[in Hg]	σ (= ρa/ρo)	[slg/ft3]	[kg/m3]	θ (=Ta/To)	[K]	[deg C]	[R]	[deg F]
31000	0.283688	4.16908	600.3483	28744.68	8.48830	0.360533	0.0008570	0.441653	0.78686	226.733	-46.42	408.12	-51.55
32000	0.270899	3.98113	573.2845	27448.86	8.10565	0.347315	0.0008255	0.425461	0.77998	224.752	-48.40	404.55	-55.12
33000	0.258581	3.80011	547.2172	26200.76	7.73708	0.334471	0.0007950	0.409727	0.77311	222.770	-50.38	400.99	-58.68
34000	0.246721	3.62581	522.1181	24999.01	7.38221	0.321993	0.0007653	0.394442	0.76623	220.789	-52.36	397.42	-62.25
35000	0.235305	3.45804	497.9594	23842.30	7.04063	0.309875	0.0007365	0.379597	0.75935	218.808	-54.34	393.85	-65.82
36000	0.224321	3.29662	474.7139	22729.30	6.71196	0.298109	0.0007086	0.365184	0.75248	216.827	-56.32	390.29	-69.38

Standard Atmosphere Calculator Website Link
http://www.digitaldutch.com/atmoscalc/

1976 U.S. Standard Atmosphere - Troposphere [11-20 Km]

		Ambient Air Pressure (P_a)					Amb. Air Density (ρ_a)			Ambient Air Temperature (T_a)			
Hp [ft]	δ (=Pa/Po)	[psi]	[psf]	[Pa]	[in Hg]	σ (= ρa/ρo)	[slg/ft3]	[kg/m3]	θ (=Ta/To)	[K]	[deg C]	[R]	[deg F]
36089	0.223361	3.28251	472.683	22632.1	6.68324	0.297076	0.0007061	0.363918	0.751865	216.650	-56.50	389.97	-69.70
37000	0.213795	3.14192	452.438	21662.7	6.39700	0.284352	0.0006759	0.348332	0.751865	216.650	-56.50	389.97	-69.70
38000	0.203762	2.99449	431.207	20646.2	6.09681	0.271009	0.0006442	0.331986	0.751865	216.650	-56.50	389.97	-69.70
39000	0.194200	2.85397	410.972	19677.3	5.81071	0.258291	0.0006139	0.316407	0.751865	216.650	-56.50	389.97	-69.70
40000	0.185087	2.72004	391.686	18753.9	5.53804	0.246171	0.0005851	0.301559	0.751865	216.650	-56.50	389.97	-69.70
41000	0.176402	2.59240	373.306	17873.9	5.27816	0.234619	0.0005577	0.287408	0.751865	216.650	-56.50	389.97	-69.70
42000	0.168124	2.47075	355.788	17035.1	5.03048	0.223609	0.0005315	0.273921	0.751865	216.650	-56.50	389.97	-69.70
43000	0.160234	2.35480	339.092	16235.7	4.79441	0.213116	0.0005066	0.261067	0.751865	216.650	-56.50	389.97	-69.70
44000	0.152715	2.24430	323.180	15473.9	4.56943	0.203115	0.0004828	0.248816	0.751865	216.650	-56.50	389.97	-69.70
45000	0.145549	2.13899	308.014	14747.7	4.35500	0.193584	0.0004601	0.237140	0.751865	216.650	-56.50	389.97	-69.70
46000	0.138719	2.03861	293.560	14055.7	4.15064	0.184500	0.0004385	0.226012	0.751865	216.650	-56.50	389.97	-69.70
47000	0.132209	1.94295	279.785	13396.1	3.95587	0.175842	0.0004180	0.215406	0.751865	216.650	-56.50	389.97	-69.70
48000	0.126005	1.85177	266.656	12767.5	3.77023	0.167590	0.0003983	0.205298	0.751865	216.650	-56.50	389.97	-69.70
49000	0.120092	1.76487	254.142	12168.3	3.59331	0.159726	0.0003797	0.195664	0.751865	216.650	-56.50	389.97	-69.70
50000	0.114457	1.68206	242.216	11597.3	3.42469	0.152230	0.0003618	0.186482	0.751865	216.650	-56.50	389.97	-69.70
51000	0.109086	1.60312	230.850	11053.1	3.26398	0.145087	0.0003449	0.177731	0.751865	216.650	-56.50	389.97	-69.70
52000	0.103967	1.52789	220.017	10534.4	3.11081	0.138278	0.0003287	0.169391	0.751865	216.650	-56.50	389.97	-69.70
53000	0.099088	1.45620	209.693	10040.1	2.96484	0.131790	0.0003133	0.161442	0.751865	216.650	-56.50	389.97	-69.70
54000	0.094438	1.38786	199.853	9568.9	2.82571	0.125605	0.0002986	0.153866	0.751865	216.650	-56.50	389.97	-69.70
55000	0.090006	1.32273	190.474	9119.9	2.69311	0.119711	0.0002845	0.146646	0.751865	216.650	-56.50	389.97	-69.70
56000	0.085783	1.26066	181.536	8691.9	2.56673	0.114093	0.0002712	0.139764	0.751865	216.650	-56.50	389.97	-69.70
57000	0.081757	1.20151	173.017	8284.1	2.44628	0.108739	0.0002585	0.133206	0.751865	216.650	-56.50	389.97	-69.70
58000	0.077921	1.14512	164.898	7895.3	2.33149	0.103637	0.0002463	0.126955	0.751865	216.650	-56.50	389.97	-69.70
59000	0.074264	1.09139	157.160	7524.8	2.22208	0.098773	0.0002348	0.120997	0.751865	216.650	-56.50	389.97	-69.70
60000	0.070779	1.04017	149.785	7171.7	2.11781	0.094138	0.0002238	0.115319	0.751865	216.650	-56.50	389.97	-69.70
61000	0.067458	0.99136	142.756	6835.2	2.01842	0.089721	0.0002133	0.109908	0.751865	216.650	-56.50	389.97	-69.70
62000	0.064292	0.94484	136.057	6514.4	1.92371	0.085510	0.0002032	0.104750	0.751865	216.650	-56.50	389.97	-69.70
63000	0.061275	0.90050	129.673	6208.7	1.83344	0.081498	0.0001937	0.099835	0.751865	216.650	-56.50	389.97	-69.70
64000	0.058400	0.85825	123.588	5917.4	1.74740	0.077673	0.0001846	0.095150	0.751865	216.650	-56.50	389.97	-69.70
65000	0.055659	0.81797	117.788	5639.7	1.66540	0.074028	0.0001760	0.090685	0.751865	216.650	-56.50	389.97	-69.70
65617	0.054034	0.79408	114.347	5475.0	1.61675	0.071866	0.0001708	0.088036	0.751865	216.650	-56.50	389.97	-69.70

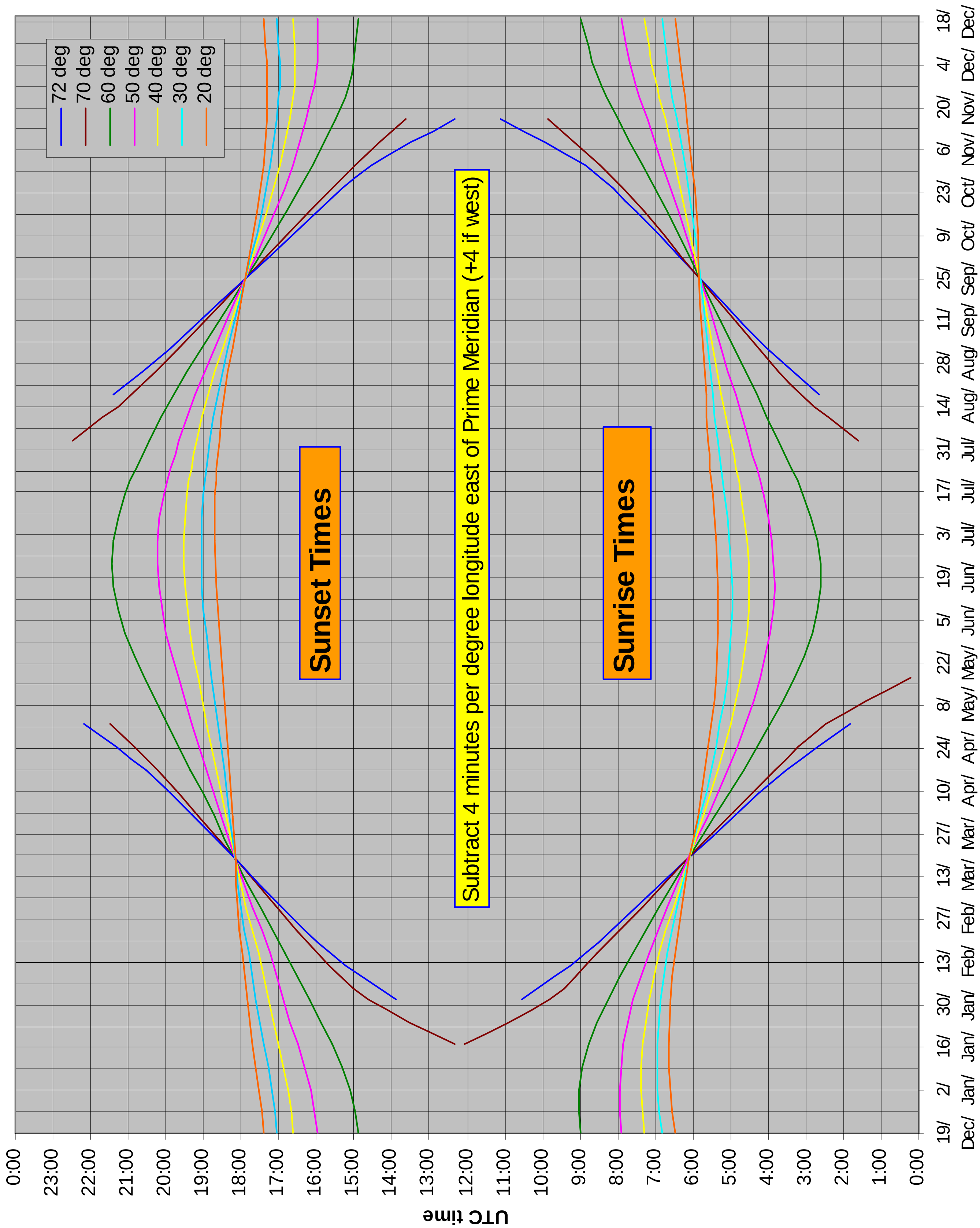
Section 3.5 Sea States

(ref 3.3) Sea Stale
International Swell Scale

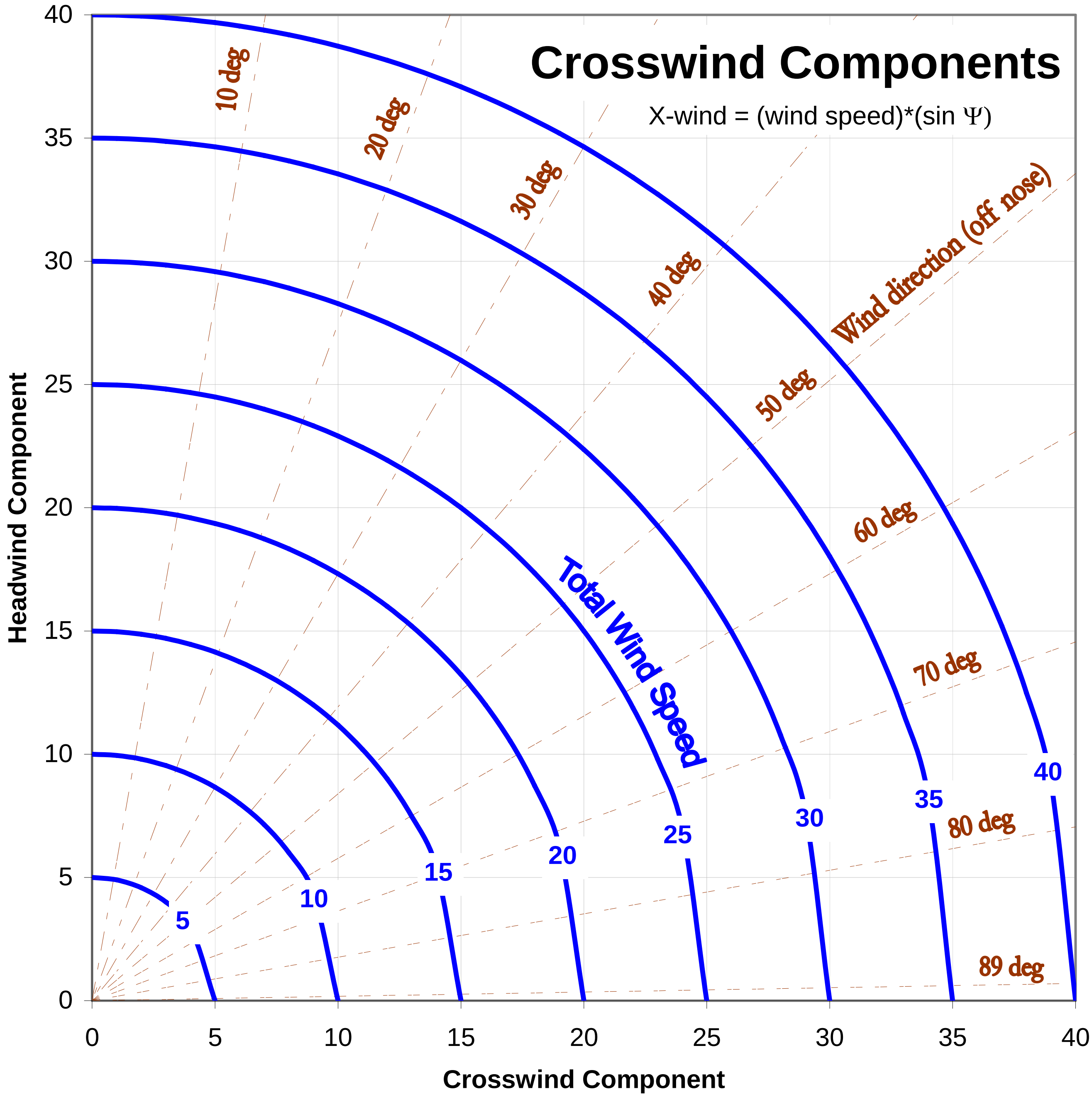
Code	Sea	Wave Height, Crest to Trough (ft)
0	Calm	0
1	Smooth	Less than 1
2	Slight	1-3
3	Moderate	3-5
4	Rough	5-8
5	Very rough	8-12
6	High	12-20
7	Very high	20-40
8	Mountainous	40+
9	Confused, Used as additional description 1-8	

Code	Swell	In Open Sea
0	None	low
1	Short or average	
2	Long	
3	Short	Moderate height
4	Average	
5	Long	
6	Short	heavy
7	Average	
8	Long	
9	Confused, Used as additional description 1-8	

Section 3.6 Sunrise Sunset Times



Section 3.7 Crosswind Components



Section 3.8 Geodetic Measurements

Acronyms, Abbreviations and Symbols

DGPS Differential Global Positioning System

ECEF Earth Centered Earth Fixed coordinate system

GPS Global Positioning System

INS Inertial Navigation System

WGS84 World Geodetic System 1984

a Earth's semi-major axis radius

b Earth's semi-minor axis radius

D Great circle distance between two points

e eccentricity of the Earth square

f Earth's flatness factor

h geodetic height

N radius of curvature in prime vertical

P radius of curvature in prime vertical

$\vec{P}$ Vector from earth center extending to coordinates

r Earth's radius

X ECEF x coordinate

Y ECEF y coordinate

Z ECEF z coordinate

ϕ Geodetic latitude

φ Angle between the two vectors originating at the Earth's center and extending to their respective coordinates at the start and end points.

λ Geodetic longitude

ψ Runway heading with $\vec{P}$ respect to true North.

Earth Modeling

The Geodetic System (i.e. latitude, longitude and height) commonly defines the location of any point relative to the earth (Figure 3.8-1, point P). Longitude and latitude are expressed in degrees, minutes, seconds. Longitude lines extend ± 180 degrees from the Prime Meridian, run north to south, and converge at the poles. Latitude lines are parallel to the equator and extend ± 90°.

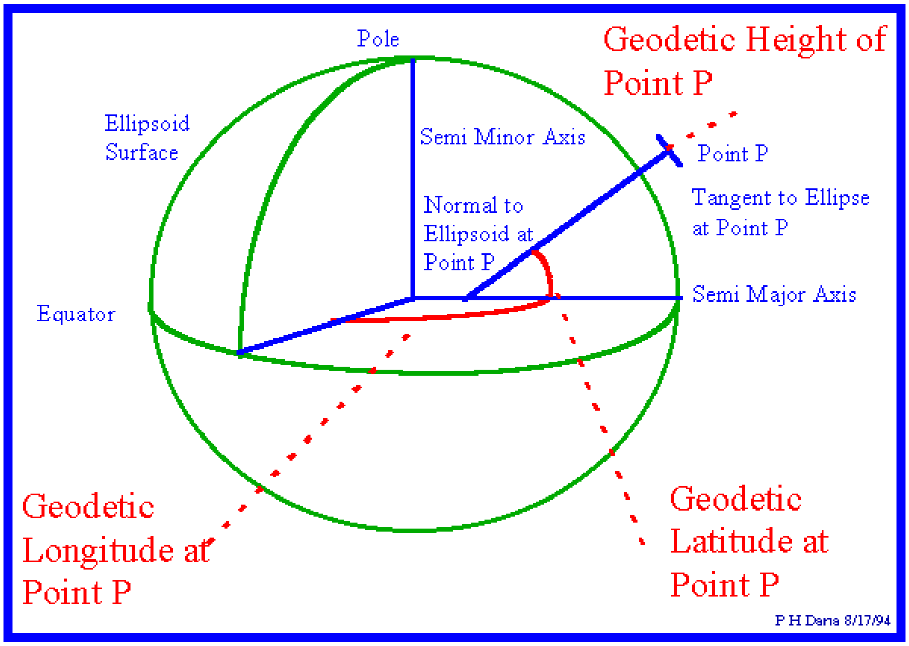


Figure 3.8-1 Geodetic Coordinate System

The 1984 world geodetic system, WGS84, models the earth's surface as an oblate spheroid - an ellipsoid rotated about its semi-minor axis. In this model (used by GPS systems), the earth's semi-major axis, a is 6,378,137.0 meters and the semi-minor axis, b is 6,356,752.314 meters.

The flatness factor (f) is defined as:

$$f = \frac{a - b}{a}$$

For the WGS84 model, f =1/298.257223563

Any plane passing through the center of a spheroid traces a great circle around the perimeter of that spheroid. The shortest distance between two points on the surface is that portion of the great circle arc encompassing both points (Figure 3.8-2).

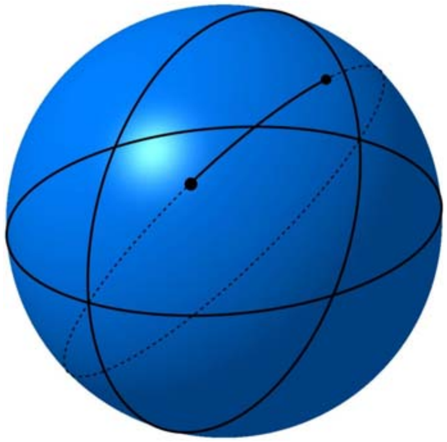


Figure 3.8-2 Great Circle Arc

Note: navigating along a great circle route has the disadvantage of not intercepting longitude lines at the same angle (except when flying along the equator); in other words, the heading changes along the route.

For the purpose of performance, navigation, or noise analysis, flight testers may require distances between two points (the shortest being along the great circle arc) and the average heading of that arc. Calculating these from typical Geodetic System Lat/Long inputs requires conversion to the Earth Centered Earth Fixed (ECEF) coordinate system as shown in Figure 3.8-3.

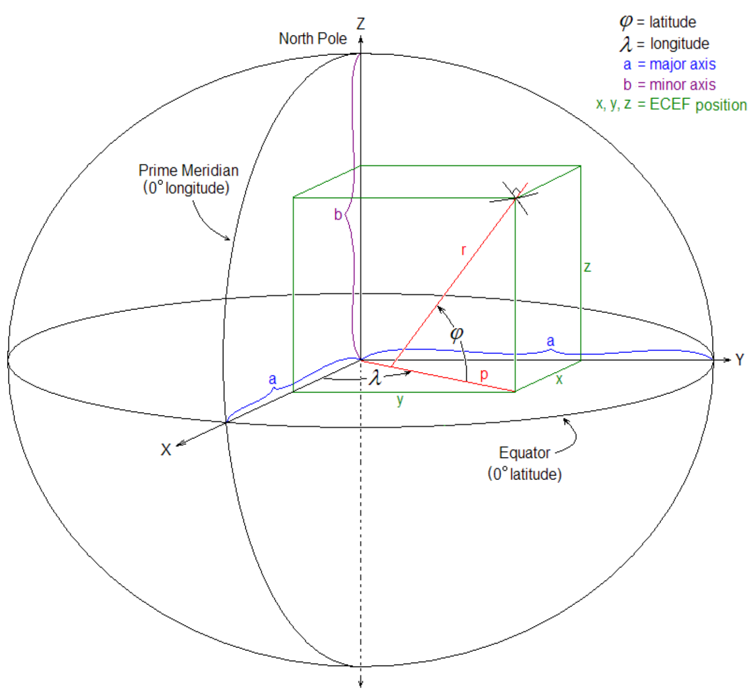


Figure 3.8-3 Earth Centered Earth Fixed Coordinate System

The ECEF coordinate system is a Cartesian system with the origin at the earth's center. In this system, the X-axis is defined by the intersection of the Prime Meridian and equatorial planes. The Z-axis goes through the North Pole. The Y-axis completes a right-handed orthogonal system by a plane 90 degrees east of the X-axis and its intersection with the equator. Geodetic System (lat/long/height) data converts to ECEF as follows:

$$x = (N + h) \bullet \cos(\phi) \bullet \cos(\lambda)$$

$$y = (N + h) \bullet \cos(\phi) \bullet \sin(\lambda)$$

$$z = (N(1-e^2) + h) \bullet \sin(\phi)$$

where,

x = ECEF coordinate parallel to the X-axis

y = ECEF coordinate parallel to the Y-axis

z = ECEF coordinate parallel to the Z-axis

ϕ = geodetic latitude

λ = geodetic longitude

h = height above geodetic surface

N = Normal radius; distance from earth center to any point on the modeled surface at that latitude

$$N = \frac{a}{\sqrt{1 - e^2 \cdot \sin^2(\phi)}}$$

where,

a = semi-major axis radius

e^2 = eccentricity squared; $e^2 = 1 - \left(\frac{a}{b}\right)^2 = 2 \cdot f - f^2$ = 0.00669438002290 (Earth, per WGS84)

Great Circle Distance

The great circle distance (D) between points (subscripts 1 and 2) can be calculated as

$$P_1 = \sqrt{x_1^2 + y_1^2 + z_1^2}\ ,\quad P_2 = \sqrt{x_2^2 + y_2^2 + z_2^2}$$

$$\vec{P}_1 \cdot \vec{P}_2 = P_1 \cdot P_2 \cos\varphi = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

$$\varphi = \arccos\left(\frac{x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot Zz_2}{P_1 \cdot P_2}\right)$$

$$D = N(\phi_{avg}) \cdot \varphi$$

where

P = distance from earth center to any point (including height above the spheroid surface).

$\vec{P}$ = Vector from the Earth's center to point P.

φ = Angle between the two $\vec{P}$ vectors

$N(\phi_{avg})$ = Normal radius using average latitude of points 1 and 2

For shorter distances typical of local flight testing, the Great Circle model approximates a two dimensional model.

Distance North-South (Northing):	$dy = N(\phi) \bullet \sin(\Delta\phi)$
Earth's radius East-West:	$r = N(\phi) \bullet \cos(\phi)$
Distance East-West (Easting):	$dx = r \sin \bullet (\Delta\lambda)$
Distance between two points:	$D = \sqrt{dx^2 + dy^2}$
Heading between two points (relative to true north)	$\psi = \arctan(dy/dx)$

Runway Distance Transformation

Data from a runway survey provides a reference point for determining the distance to key locations (e.g. brake release, liftoff, microphone array). From runway survey data, a local coordinate system can be established that uses any point on the runway as the origin, and can either

a) keep the same X, Y and Z axes or
b) set height as the Z-axis, direction along the runway heading as the X-axis, and direction normal to the runway centerline as the Y-axis (Figure 3.8-4).

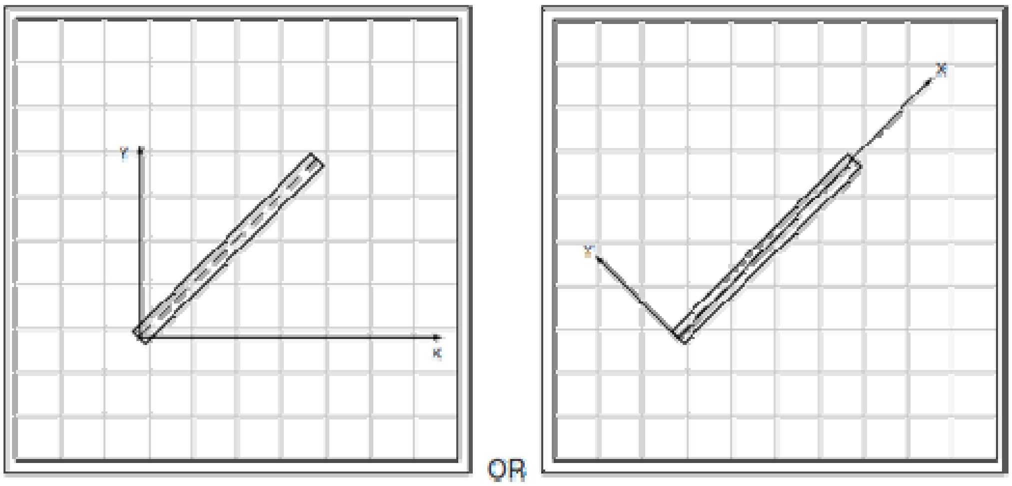


Figure 3.8-4 Local Coordinate System

For the second option above, distance along and normal to the runway can be calculated using the following geometry definitions and equations (Figures 3.8-5 and 3.8-6):

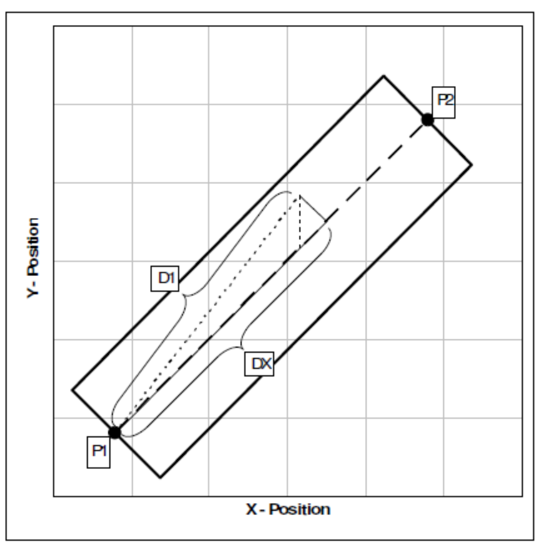


Figure 3.8-5 Local Geometry 1

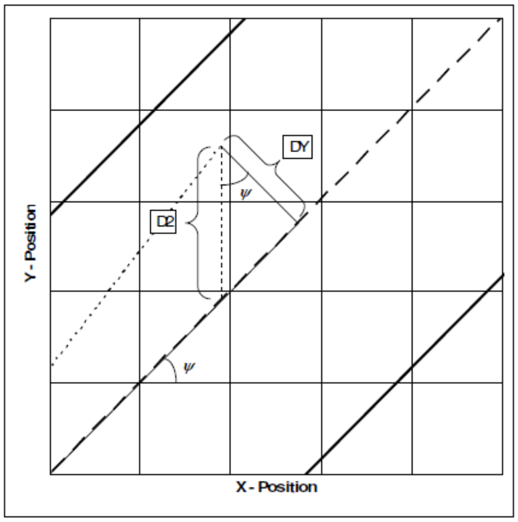


Figure 3.8-6 Local Geometry 2

P1= Main reference point along runway centerline (subscript 1 in the following equations)
P2= 2nd point defining runway centerline (subscript 2 in the following equations)
ψ = runway centerline heading with respect to true north.
*D*1 = total distance from reference point to current position.
*D*2 = distance from current position to runway along the same X position (i.a.w. ECEF coordinates)
DX = distance along runway centerline from reference point to current position.
DY = normal distance between runway centerline and current position.
m = slope of runway centerline equation.
b = intercept of runway centerline equation.

$$\psi = arctan\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$D1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

$$y = mx + b, \quad m = \frac{y_2 - y_1}{x_2 - x_1}, \quad b = \frac{x_2 \cdot y_1 - x_1 \cdot y_2}{x_2 - x_1}$$

$$\cos\psi = \frac{DY}{D2} = \frac{DY}{y - (mx + b)} = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} = \frac{1}{\sqrt{m^2 + 1}}$$

$$DY = \frac{|y - mx - b|}{\sqrt{m^2 + 1}}$$

$$DX = \sqrt{D1^2 - DY^2}$$

Note: the above heading equation applies between any two points and may be useful for navigation analysis in a local environment.

Section 3.9 References

3.1 Anon., “Aeronautical Vestpocket Handbook” ,Part No. P&W 079500, United Technologies Pratt & Whitney, Canada, 1990.

3.2 Lawless, Alan. R. et al, “Aerodynamics for Flight Testers”, National Test Pilot School, P.O. Box 658, Mojave CA, 93501, 1999.

3.3 Denno, Richard R., et al “AIAA Aerospace Design Engineers Guide” ISBN 0-930403-21-5, AIAA, 1987.

3.4 Global Positioning System Overview, Peter H. Dana, Department of Geography, University of Texas at Austin, 1994. (www.colorado.edu/geography/gcraft/notes/gps/gps_f.html)

3.5 Charles D Ghilani, Penn State College of Engineering, 2008 (http://surveying.wb.psu.edu/sur351/georef/Ellip4.htm)

3.6 Standard Atmosphere Calculator Website Link http://www.digitaldutch.com/atmoscalc/.

NOTES