Society of Flight Test Engineers

Reference Handbook

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Society of Flight Test Engineers Reference Handbook

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1.1 Unit Conversions

(references 1.1, 1.2)

Prefix Multipliers

10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

(common FTE conversions in boldface)

Time	days (solar)	24	hours
	days (sidereal)	23.934	hours
	days (solar)	1.0027	days (sidereal)
	hours	60	minutes
	minutes	60	seconds
	months (sdrl)	27d + 7hr +43min +11.47sec	
	months (lunar)	29d +12hr +44min + 2.78sec	
	year	365.24219879	days

Temperature Kelvin = ^{o}*C*+273.15^o

Rankin = ^{o}*F* + 459.67^o

^{o}Centigrade = [$^o\tilde{F}$ 32^o] 5/9

^{o}Fahrenheit = (9/5)^o*C* + 32

	Multiply	***by***	***To Obtain***
Length	angstroms	10^{-10}	meters
	astronmcl units	$1.496x10^{11}$	meters
	cable lengths	120	fathoms
	caliber	0.01	inches
	cubit	0.4572	meters
	fermi	10^{-15}	meters
	fathoms	6	feet
	feet	12	inches
	furlongs	40	rods
	Multiply	***by***	***To Obtain***
	hands	4	inches
	inches	**2.54**	**cm**
	kilometers	**3281**	**feet**
	kilometers	**0.53996**	**nautical miles**
	leagues(U.S.)	3	nautical miles
	light years	$5.88x10^{12}$	statute miles
	links (engnr's)	12	inches
	links (srvyr's)	7.92	inches
	meters	**3.28084**	**feet**
	meters	**39.370079**	**inches**
	microns	0.1^{6}	meters
	mils	0.001	inches
	nautical miles	**1.15078**	**statute miles**
	nautical miles	**1,852**	**meters**
	nautical miles	**6,076.115486**	**feet**
	paces	0.762	meters
	parsec	$1.9163x10^{13}$	statute miles
	perch	5.0292	meters
	pica (printers)	0.0042175176	meters
	point (printers)	0.0003514598	meters
	pole (=rod)	5.0292	meters
	skein	109.728	meters
	statute miles	**5,280**	**feet**
	statute miles	**1.609344**	**kilometers**
	statute miles	8	furlongs
	yards	3	feet

	Multiply	*by*	*To Obtain*
Velocity	inches/sec	0.0254	meters/sec
	knots	**1.68781**	**feet/sec**
	km/hr	**0.621371**	**mph**
	km/hr	**0.9113**	**feet/sec**
	Knots (kts)	**1.15078**	**mph**
	Knots (kts)	**1.852**	**km/hr**
	Knots (kts)	0.51444	meters/sec
	meters/sec	3.281	ft/sec
	meters/sec	3.6	km/hr
	meters/sec	**196,85**	**feet/min**
	mph	1.466667	feet/sec
Linear Acceleration	feet/sec2	1.09728	kilometers/hr/sec
	feet/sec2	0.3048	meters/sec2
	feet/sec2	0.6818	mph/sec
	g	32.174049	feet/sec2
	g	9.80665	meters/sec2
	gals (Galileo)	0.01	meters/sec2
	knots/sec	1.6878	feet/sec2
	meters/sec2	3.6	kilometers/hr/sec
	mph/sec	0.447	meters/sec^2
	mph/sec	1.609	kilometers/hr/sec
Mass	carats	*200*	milligrams
	grams	0.035274	ounces*
	grains	6.479891×10^{-5}	kilograms
	hndrdwght long	50.80 234544	kilograms
	hndrdwght shrt	45.359237	kilograms
	kilograms	**0.06852**	**slugs**
	kilograms	6.024x1026	atomic mass units
	kilograms	**2.2046**	**pounds***
	ounces (avd)*	*28.349523125*	grams
	ounces (troy)*	*31.1034768*	grams
	pounds (mass)	1	pounds (force)*
	pounds (mass)	**0.*45359237***	**kilograms**
	pounds (mass)	**0.031081**	**slugs**
	scuples (apoth)	0.0012959782	kilograms
	slugs	**32.174**	**pounds ***
	slugs	**14.594**	**kilograms**
	tons (long)	1016.047	kilograms
	tons (assay)	0.02916	kilograms
	tons (metric)	*1000*	kilograms
	tons (short)	907.1847	kilograms

	Multiply	by	To Obtain
Area	acres	43,560	ft^2
	ares	100	m^2
	barn	10^{-28}	m^2
	circular mils	7.854×10^{-7}	in^2
	cm^2	100	mm^2
	ft^2	**144**	**in^2**
	ft^2	**0.09290304**	**m^2**
	in^2	6.452	cm^2
	in^2	10^6	$mils^2$
	m^2	10.76	ft^2
	section	2,589,988.1	m^2
	st. $mile^2$	27,780,000	ft^2
	st. $mile^2$	2.590	km^2
	township	93,239,572	m^2
	yd^2	9	ft^2
	yd^2	0.8361	m^2
Volume	acre-feet	43,560	ft^3
	acre-feet	1,233	m^3
	acre-feet	3.259×10^5	gals (U.S.)
	barrels	31.5	gals (U.S.)
	board-feet	144	in^3
	bushels	1.244	ft^3
	bushels	32	quarts (dry)
	bushels	4	pecks
	cm^3	0.001	liters
	cm^3	0.03381	fluid ounces
	cm^3	0.06102	in^3
	cord-feet	4x4x1	ft^3
	cords	128	ft^3
	cups	0.5	pints (liquid)
	dram (fluid)	3.69669×10^{-6}	m^3
	ft^3	**0.0283167**	**m^3**
	ft^3	1728	in^3
	ft^3	28.32	liters
	ft^3	**7.481**	**gals (U.S.)**
	gals (Imperial)	**1.2009**	**gals (U.S.)**
	gals (Imperial)	277.42	in^3
	gals (U.K.)	4546.1	cm^3
	gals (U.S.)	**231**	**in^3**
	gals (U.S.)	**0.003785**	**m^3**

Multiply	***by***	***To Obtain***
gals (U.S.)	**3.785**	**liters**
gals (U.S.)	4	quarts (liquid)
gals (U.S.)	0.0238095	barrels (U.S.)
gils	7.219	in^3
hogshead	2	barrels
in^3	16.39	cm^3
liters	0.02838	bushels
liters	0.9081	quarts (dry)
liters	1.057	quarts (liquid)
liters	1000	cm^3
liters	61.03	in^3
m^3	1.308	yd^3
m^3	1000	liters
m^3	264.2	gals (U.S.)
m^3	**35.314667**	**ft^3**
mil-feet (circ.)	0.0001545	cm^3
ounces (U.K.)	28.413	cm^3
ounces (U.S.)	29.574	cm^3
pecks	8	quarts (dry)
pecks	8.81	liters
perches	0.7008	m^3
perches	24.75	ft^3
pints (dry)	33.60	in^3
pints (liquid)	28.88	in^3
pints (liquid)	4	gals
quarts (dry)	1.164	quarts (liquid)
quarts	2	pints
register tons	100	ft^3
shipping ton (U.S.)	40	ft^3
shipping ton (Br.)	42	ft^3
steres	1000	liters
tablespoons	0.0625	cups
teaspoons	0.3333	tablespoons

	Multiply	***by***	***To Obtain***
Density	grams/cm^3	0.03613	pounds/in^3 *
	grams/cm^3	62.43	pounds/ft^3 *
	kg/m^3	16.02	pounds/ft^3*
	slugs/ft^3	**515.4**	**kg/m^3**
	pounds/in^3 *	**1728**	**pounds/ft^3***
	slugs/ft^3	1.94	grams/cm^3

* conversion from pounds force to pounds mass assumes g = 32.174 ft/s^2

Force	dynes	$3.597\text{x}10^{-5}$	ounces
	kilogram force	**9.80665**	**Newtons**
	kilopond force	9.80665	Newtons
	kip	4,448.221	Newtons
	Newtons	**0.224808931**	**pounds**
	Newtons	100,000	dynes
	ounce	20	pennyweights
	ounces (troy)	480	grains
	pennyweights	24	grains
	pound	12	ounces
	pounds	32.174	poundals
	pounds	**4.4482216**	**Newtons**
	pounds	5760	grains
	quintals (long)	112	pounds
	quintals (met.)	100	kilograms
	stones	14	pounds
	tons (long)	2,240	pounds
	tons (metric)*	1.102	tons (short)
	tons (short)	2000	pounds
Pressure	**atmospheres**	**14.696**	**pounds/in^2**
	atmospheres	**29.92**	**inches of Hg**
	atmospheres	76	cm of Hg
	bars	1,000,000	dynes/cm^2
	bars	**29.52**	**inches of Hg**
	barye	**0.1**	**Newtons/m^2**
	dynes/cm^2	10	Newtons/m^2
	nches of H_2O	**5.20237**	**pound/ft^2**
	inches of Hg	**70.72619**	**pounds/ft^2**
	inches of Hg	0.491154	pounds/in^2
	inches of Hg	13.595	inches of H_2O
	kiloPascals	**100**	**bars**
	pounds/ft^2	**47.88**	**Pa**

	Multiply	by	To Obtain
	pounds/ft^2	**0.3325**	**Pa**
	hectoPascals	**1**	**millibars**
	watt-second	0.73756	foot-pounds
	millibars	**0.02953**	**inches of Hg**
	mm of Hg	0.019337	pounds/in^2
	mm of Hg	133.32	Newtons/m^2
	Pascals	**1**	**Newton/m^2**
	pieze	1000	Newtons/m^2
	pounds/ft^2	**0.01414**	**inches of Hg**
	pounds/ft^2	**47.88**	**Newtons/m^2**
	pounds/in2	**2.036**	**inches of Hg**
	pounds/in2	**27.681**	**inches of H2O**
	torrs	**133.32**	**Newtons/m2**
Energy	Btu	1.055x10^{10}	ergs
Work	Btu	1055.1	Joules (Newton-meters)
	Btu	2.9302x10^{-4}	kilowatt-hours
	Btu	251.99	calories (gram)
	Btu	778.03	foot-pounds
	calories	4.1868	watt-seconds
	calories	3.088	foot-pounds
	electron volt	1.519x10^{-22}	Btu
	ergs	1	dyne-centimeters
	ergs	7.376x10^8	foot-pounds
	foot-pounds	**1.3558**	**Joules (N-m)**
	foot-pounds	3.766x10^{-7}	kilowatt-hours
	foot-pounds	5.051x10^{-7}	horsepower-hours
	hp-hours	0.7457	kilowatt-hours
	hp-hours	2546.1	Btu
	Joules	0.23889	calories
	Joules	1	Newton-meters
	Joules	1	watt-seconds
	Joules	10^7	ergs
	kilowatt-hours	**3.6x10^6**	**Joules**
	thermies	4.1868x10^6	Joules
	watt-second	0.73756	foot-pounds

	Multiply	*by*	*To Obtain*
Torque	**foot-pounds**	**1.3558**	**Newton-meters**
	foot-pounds	0.1383	kilogram-meters*
	ounce-inches	72.008	gram-centimeters*
	pound-inches	1129800	dyne-centimeters

* conversion from kg mass to kg force assumes g = 32.174 ft/s^2

Power	btu/min	0.01758	kilowatts
	calories(kg)/min	3087.46	foot-pounds/min
	ergs/sec	$7.376x10^{-8}$	foot-pounds/sec
	ft(lbs)/min	$2.260x10-^{5}$	kilowatts
	ft(lbs)/sec	0.07712	btu/min
	hp	**550**	**ft(lb)/sec**
	hp	**33,000**	**ft(lbs)/min**
	hp	**10.69**	**calories (kg)/min**
	hp	**745.7**	**watts [J/sec]**
	hp (metric)	**735.5**	**watts**
	hp	**1.1014**	**horsepower (metric)**
	kilowatts	**1.341**	**horsepower**
	watts	**107**	**ergs/sec**
	watts	**1**	**Joules/sec**
Fuel	**gal**	**5.8**	**lbs (U.S. AV gas)**
	gal	**7.5**	**lbs (U.S. oil)**
	Liter (jet A)	**0.804**	**kilograms**
	Liter (jet A)	**1.7725**	**pounds**

Note: Fuel densities are temperature dependent

Angles	circles	1	circumferences
	circles	12	signs
	circles	21,600	minutes
	circles	2	radians
	cirles	360	degrees
	degrees	01111	quadrants
	degrees	3600	seconds
	degrees	60	minutes
	mils (Army)	.05625	degrees
	mils (Navy)	.05729	degrees
	quadrants	90	degrees
	radians	57.2958	degrees
	revolutions	360	degrees
	sphere	4	steradians # #solid angle measurement

	Multiply	***by***	***To Obtain***
Angular	cycles/sec	6.2814	rads/sec
Velocity	rads/sec	0.1592	rev/sec (cycles/sec)
	rads/sec	9.549	rpm
	rad/sec	57.296	deg/sec
	rpm	0.01667	rev/sec
	rpm	0.10472	rad/sec
Angular Acceleration	rev/min2	0.001745	rad/sec2
Electrical	amperes	0.1	abamperes
Quantities	amperes	1.0365x10-5	faradays/sec
	amperes	2.998x109	statamperes
	amperes.cicmil	1.973x105	amperes/cm2
	ampere-hours	3,600	coulombs
	ampere-hours	1.079x1013	statcoulombs
	ampere turn/cm	1.257	gilberts/cm
	ampere turn/cm	1.257	oersteds
	coulombs	0.1	abcoulombs
	coulombs	6.243x1018	electronic charges
	coulombs	1.037x10-5	faradays
	coulombs	2.998x109	statcoulombs
	faradays	26.8	apmere-hours
	farads	10-9	abfarads
	farads	106	microfarads
	farads	8.986x1011	statfarads
	gausses	1	maxwells/cm2
	gausses	6.452	lines/in2
	gilberts	0.7958	ampere turns
	henries	109	abhenries
	henries	1.113x10-12	stathenries
	maxwells	1	lines
	oersteds	2.998x1010	statoersteds
	ohms	109	abohms
	ohms	1.113x1012	statohms
	ohm-cm	6.015x106	circ mil-ohms/ft
	volts	108	abvolts
	volts	0.003336	statvolts

	Multiply	***by***	***To Obtain***
Viscosity	centistokes	10-6	m2/sec
	ft2/sec	0.0929	m2/sec
	pound sec/ ft2	47.880258	Newton secs/ m2
	poise	0.1	Newton secs/ m2
	rhe	10	m2/Newton second
Illumination	candles	1	lumens/steradian
	candles/cm2		lamberts
	candlepower	12.566	lumens
	foot-candles	1	lumens/ft2
	foot-candles	10.764	lux
	foot-lamberts	1	lumen/ft2
	lamberts	295.72	candles/ft2
	lamberts	929.03	lumens/ft2
	lumens	0.001496	watts
	lumens/in2	1	fots
	lumens/m2	1	lux
	lux	1	meter-candles
	lux	0.0001	fots
	meter-candles	1	lumens/m2
	millilamberts	0.2957	candles/ft2
	millilamberts	0.929	foot-lamberts
	milliphots	0.929	foot-candles
	milliphots	0.929	lumens/ft2
	milliphots	10	meter-candles
Moments of Inertia	gram-cm2	0.737x10-7	slug-ft2
	pound-ft2*	0.031081	slug-ft2
	slug-in2	0.0069444	slug-ft2
	slug-ft2	1.3546	kg-m2
	slug-ft2	32.174	pound-ft2 *
	slug-ft2	12.00	pound-inch-sec2 *
	slug-ft2	192.00	ounce-inch-sec2 *

*converting from force to mass assumes g=32.147 ft/sec2

1.2 Greek Alphabet

Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
p	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	ϕ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

1.3 Greek Symbols Used for Aircraft

α	angle of attack (degrees or radians)
τ	tail angle of attack
β	angle of sideslip (degrees)
γ	flight path angle relative to horizontal
γ	specific heat ratio (1.4 for air)
δ	relative pressure ratio (P_a/P_o)
δ_a	aileron deflection angle
δ_r	rudder deflection angle
δ_e	elevator deflection angle
ε	downwash angle at tail (degrees)
ζ	damping ratio
η	efficiency
θ	body axis/pitch angle
θ	relative temperature ratio, T_a/T_o
ι	angle of incidence
ι_F	thrust angle of incidence
ι_T	horizontal tail angle of incidence
λ	pressure lag constant
Λ	wing sweep angle
μ	coefficient of absolute viscosity = $\rho\nu$

μ	Mach cone angle
ν	kinematic viscosity = μ/g
π	nondimensional parameter
ρ	density
ρ_a	ambient air density
ρ_o	standard atmospheric density (slugs/ft3)
σ	air density ratio (ρ_α / ρ_o)
σ_{cr}	critical density
τ	shear stress (pounds per square inch) psi
τ_R	Roll Mode Time Constant (sec)
ϕ	bank angle (degrees)
ψ	aircraft heading (degrees)
ω	frequency
ω	rotational velocity (radians per second)
ω_d	damped natural frequency
ω_n	natural undamped frequency

1.4 Common Subscripts

a	aileron
a	ambient
alt	at test altitude
avg	average
c	calibrated
e	elevator
e	equivalent
E	endurance leg of mission
F	final
I	initial
i	inbound leg of mission
i	indicated
ic	instrument corrected
l	subscript for coefficient of rolling moment
m	mission conditions
m	pitching moment
n	yawing moment
O	outbound leg of mission
o	sea-level standard day
o	sea level
r	reserve leg of mission
r	rudder
S	standard day
s	standard day at altitude
SL	sea level
T	True
t	test day

1.5 Common Abbreviations

a	lift curve slope
a	linear acceleration (ft/sec^2 or m/sec^2)
a	speed of sound
A/A	air-to-air
a/c	aircraft
AAA	anti aircraft artillery
AC	aerodynamic center
ac	alternating current
ACM	air combat maneuvering
A/D	analog to digital
ADC	air data computer
ADC	analog-to-digital converter
ADF	automatic direction finder
ADI	attitude direction indicator
AFMC	Air Force Materiel Command
AFOTEC	Air Force Operational Test and Evaluation Center
A/G	air-to-ground
AGL	above ground level
AHRS	attitude heading reference system
AM	amplitude modulation
AOA	angle of attack
AOED	age of ephemeris data
APU	auxiliary power unit
AR	air refuel (mode of flight)
AR	aspect ratio = b^2 / S
ARDP	advanced radar data processor
ARSP	advanced radar signal processor
ASPJ	airborne self protection jammer
ATC	air traffic control
avg	average
ax	longitudinal acceleration
ay	lateral acceleration
AZ	azimuth
b	span of wing (feet)
B/N	bombadier/navigator
bbl	barrel
BHP	brake horsepower
BICOMS	bistatic coherent measurement system
BID	bus interface device
BIT	built-in test
BSFC	brake specific fuel consumption
Btu	British thermal unit
BW	bandwidth
^oC	degrees centigrade...see *T*
c	aerodynamic chord of a wing
c	brake specific fuel consumption (BSFC)
c	speed of light in a vacuum (186,282 miles/sec = 299,792,500 [m/s])

c	mean aerodynamic chord (MAC) of a wing
C/A	coarse acquisition
C/N_o	carrier to noise ratio
CADC	central air data computer
CARD	cost analysis requirement document
C_D	coefficient of drag
$C_{D\,i}$	induced drag coefficient
$C_{D\,o}$	zero lift drag coefficient (also parasitic drag coefficient for symmetric wing)
CDI	course deviation indicator
CDMA	code division multiplex access
CDR	critical design review
CDRL	contracts data requirement list
CDU	control display unit
CEA	circular error average
CEP	circular error probable
C_f	coefficient of friction
CFE	contractor furnished equipment
CFT	conformal fuel tank
cg	center of gravity (normally in % MAC)
C_H	hinge moment coefficient
cine	cinetheodolite
C_l	rolling moment coefficient, airfoil section lift coefficient
C_L	lift coefficient
CLHQ	closed loop handling qualities
C_{lp}	roll damping coefficient
C_{lr}	roll moment due to yaw rate coefficient
C_m	pitching moment coefficient
C_M	moment coefficient
cm	centimeters
cos	cosine
cot	cotangent
$C_{l\beta}$	(dihedral) rolling moment due to sideslip
$C_{l\delta a}$	aileron power coefficient
Cm_q	pitch damping coefficient
Cm_α	longitudinal static stability coefficient
$Cm_{\delta e}$	elevator power coefficient
Cn	yawing moment coefficient
Cn_r	yaw damping coefficient
cnst	constant
Cn_β	directional stability coefficient
$Cn_{\delta a}$	adverse yaw coefficient
$Cn_{\delta r}$	rudder power coefficient
COTS	commercial, off–the-shelf
CP	center of pressure
C_P	propeller power coefficient
CPU	central processing unit
c_r	wing root chord
CRM	crew resource management

c_t	wing tip chord
CTF	combined test force
CY	calendar year
C_Y	side force coefficient
CY_{β}	side force due to sideslip coefficient
$CY_{\delta r}$	side force due to rudder coefficient
D	diameter
D	drag
D/A	digital/analog
DAC	digital to analog converter
DAPS	data acquisition and processing system
DARPA	Defense Advanced Research Projects Agency
db	decibel
DC	direct current
deg	degrees
DG	directional gyro
DGPS	differential GPS
DMA	Defense Mapping Agency
DME	distance measuring equipment
DoD	Department of Defense
DOP	dilution of precision
DSN	defense switched network
DT	development test
DTC	data transfer cartridge
DTIC	Defense Technical Information Center
e	Oswald efficiency factor
e	natural mathematical constant = 2.718281828459
E	energy
E	lift-to-drag ratio (C_L/C_D, L/D)
EAS	equivalent airspeed
EC	electronic combat
ECCM	electronic counter countermeasures
ECM	electronic countermeasures
ECP	engineering change proposal
ECS	environmental control system
EGT	exhaust gas temperature
EL	elevation
ELINT	electronic intelligence
ELV	expendable launch vehicle
EM	electromagnetic
E_{max}	maximum lift-to-drag ratio
EMC	electromagnetic compatibility
EMI	electromagnetic interference
EMP	electromagnetic pulse
EO	electro optical
EOM	equations of motion
EPR	engine pressure ratio
EPROM	electrically programmable read only memory
Es	specific energy
ESA	European Space Agency

ESD	Electronic Systems Division
ESHP	equivalent shaft horsepower
ETA	estimate time of arrival
ETE	estimate time en-route
EW	early warning
EW	electronic warfare
^{o}F	degrees Fahrenheit
f	frequency...hertz (originally cycles per second)
F.S.	fuselage station
F_a	aileron force
FAA	Federal Aviation Administration
FAR	Federal Aviation Regulation
FCF	functional check flight
FDC	flight data computer
F_e	elevator force
F_{ex}	excess thrust
F_g	gross thrust
FL	flight level
Flip	flight information publication
FLIR	forward-looking infra red
FM	frequency modulation
FMC	fully mission capable
FMS	flight management system
FMS	foreign military sales
Fn	net thrust
Fn/δ	corrected thrust parameter
FOM	figure of merit
FOT&E	follow-on test & evaluation
FOUO	for official use only
FOV	field of view
fpm	feet per minute
fps	feet per second
FQT	formal qualification test
Fr	rudder force
FRD	functional requirements document
FRL	fuselage reference line
FRL	force, rudder, left
FRR	force, rudder, right
FRR	flight readiness review
FSD	full scale development
FSI	full scale integration
ft	feet
ft-lb	English unit of work...foot-pound...
fwd	forward
FY	fiscal year
g	acceleration due to gravity at altitude
G	gravitational constant = $6.6732 x 10^{-11}$ [N m^2/kg^2]
GAO	Government Accounting Office
GCA	ground control approach
GCI	ground controlled intercept

GDOP	geometric dilution of precision
GMT	Greenwich mean time
g_o	standard acceleration due to gravity (sea level, 46 deg latitude)
GPS	global positioning system
GS	ground speed
GSI	glide slope indicator
h	% MAC
H	altitude
HARM	high-speed anti-radiation missile
H_c	calibrated altitude (assumed to be pressure altitude in flight test)
H_D	density altitude
HDDR	high density digital recorder
HDOP	horizontal dilution of precision
HF	high frequency
Hg	mercury
H_i	indicated altitude
h_m	stick-fixed maneuver point (%MAC)
h'_m	stick-free maneuver point (%MAC)
h_n	stick-fixed neutral point (%MAC)
h'_n	stick-free neutral point (%MAC)
hp	horsepower
hr	hour
hrs	hours
HSI	horizontal situation indicator
HUD	head-up display
HV	host vehicle
Hz	hertz
I/O	input/output
IAS	indicated airspeed
IAW	in accordance with
ICAO	International Civilian Aviation Organization
ICU	interface computer unit
ICBM	intercontinental ballistic missile
IFF	identification friend or foe
IFR	instrument flight rules
ILS	instrument landing system
IMC	instrument meteorological conditions
IMN	indicated Mach number
IMU	inertial measuring unit
in	inch
INS	inertial navigation system
INU	inertial navigation unit
IOC	initial operational capability
IOT&E	initial operational test & evaluation
IUGG	International Union of Geodesy and Geographics
I_x, I_x, I_z	moments of inertia
I_{xy}, I_{xz}, I_{yz}	products of inertia
J	joules energy, (Newton-Meter)

J	propeller advance ratio
J&S	jamming and spoofing
JCS	Joint Chiefs of Staff
K	Kelvin (absolute temperature)
K	temperature probe recovery factor
K, $k1$	constants
KCAS	knots calibrated airspeed
KEAS	knots equivalent airspeed
kg	kilogram, metric unit of mass
KIAS	knots indicated airspeed
KISS	keep it simple, stupid
km	kilometer
KTAS	knots true airspeed
kts	knots
L	Lift (lbs)
l	length
L	rolling moment
L/D	Lift-to-drag ratio
L/D	lift-to-drag ratio
LANTIRN	low altitude navigation and targeting IR for night
lat	lateral
lb	pound
lb_f	English unit of force, often just lb (pound)
lb_m	English unit of mass, often just lb (slug)
LCC	life cycle cost
LCD	liquid crystal display
LED	light emitting diode
LLH	latitude, longitude, height
ln	natural log, log to the base e
LO	low observables
Log	common log, to the base 10
LOS	line of sight
l_t	distance from *cg* to tail's aerodynamic cent
$L_{\delta a}$	rolling moment due to aileron deflection
M	moment (ft-lbs)
M	Mach number
m	mass
m	meter (length)
M	pitching moment
MAG	magnetic
MAP	manifold pressure
mb	millibar
MCA	minimum crossing altitude
M_{cr}	critical Mach number
M_d	drag divergence Mach number
M_{ac}	mean aerodynamic cord
M_{GC}	mean geometric chord
MHz	megahertz
mHZ	millihertz
M_{ic}	instrument-corrected Mach number

MilSpec	military specification
MIL-STD	military standard (publication)
min	minute (time)
Mm	millimeters
MOA	memorandum of agreement
MOE	measure of effectiveness
MOP	measures of performance
MOU	memorandum of understanding
MP	manifold pressure
MSL	mean sea level
MTBF	mean time between failures
MTTR	mean time to repair
MX	maintenance
N	newton (force)
N	rotational speed (RPM)
n	load factor (g's)
N	yawing moment
N_1	low pressure compressor speed
N_2	high pressure compressor speed
NACA	National Advisory Committee for Aeronautics
NADC	Naval Air Development Center
NASA	National Aeronautics and Space Administration
NAV	navigation
NED	North, East, Down
NM, nm	nautical mile (6080 feet)
NOE	nap-of-the-earth
NOFORN	not releasable to foreign nationals
NOTAM	notice to airmen
NRC	National Research Council (Canada)
NWC	Naval Weapons Center
N_x	longitudinal load factor (g's)
N_y	lateral load factor (g's)
N_z	normal load factor (g's)
OAT	outside air temperature
OAT	on aircraft test
OEI	One engine inoperative
OPR	Office of Primary Responsibility
OSD	Office of the Secretary of Defense
OT&E	operational test & evaluation
p	aircraft roll rate (degrees/sec)
P	pressure (N/m^2 ,pounds per square inch)
P_a	ambient pressure
PCM	pulse code modulation
P-code	precision code
PD	pulse Doppler
PDM	pulse duration modulation
PGM	precision guided munitions
PIO	pilot induced oscillations
P_{iw}	total thrust horsepower required
Pk	probability of kill

PLF	power for level flight
P_o	standard atmospheric pressure (2116.22 lb/ft^2)
POC	point of contact
P_p	pitot pressure
ppm	parts per million
Prop	propeller
P_s	static pressure
PS	pulse search
psf	pounds per square foot
psi	pounds per square inch
P_T	total pressure
PW	pulse width
Q or q	dynamic pressure = $0.5\rho V^2$
q	aircraft pitch rate
Q	engine torque
q_c	impact pressure ($P_t - P_a$)
^oR	degrees Rankine = $^oF + 459.67$
R	perfect gas constant = 8314.34 [J/kmol K]
r	aircraft yaw rate (degrees/sec)
R	earth radius
R	range
R&D	research and development
R&M	reliability and maintainability
R/C	rate of climb
rad	radians
Radar	radio detection and ranging
RAF	resultant aerodynamic force
RAM	radar absorbing material
RAT	ram air turbine
RCS	radar cross section
Re	Reynolds number (dimensionless)
REP	range error probable
RF	range factor
RLG	ring laser gyro
rms	root mean square
RNG	range
ROC	rate of climb
ROC	required obstacle clearance
RPM	revolutions per minute (a.k.a. N)
R/T	receiver/transmitter
RTO	refused takeoff
RTO	rejected takeoff
RTO	responsible test organization
S	wing area (ft^2 or m^2)
S_a	horizontal distance between liftoff and specified height or between specified height and touch down.
SA	selective availability
SA	situational awareness
SE	specific endurance
sec	seconds (time or angle)

SFC	specific fuel consumption
S_g	ground roll distance
SHP	shaft horsepower
SI	international system of units
SIGINT	signal intelligence
sin	sine
SL	sea level
SLAM	standoff land attack missile
SLR	side-looking radar
S/N	serial number
S/N	signal -to-noise ratio
SOF	special operations forces
SOW	stand-off weapon
SR	specific range
SRB	safety review board
S_T	tail area
std	standard
S_T	total takeoff or landing distance ($S_a + S_g$)
STOL	short takeoff and landing
STOVL	short takeoff and vertical landing
T	period of oscillation
T	temperature
t	thickness
T, t	time (sec)
t/c	thickness-to-chord ratio
T_a	ambient temperature
TACAN	tactical air navigation
tan	tangent
T_{as}	standard temperature at altitude
TAS	true airspeed
TBD	to be determined
TD	touchdown
TED	trailing edge down
TEL	trailing edge left
TEMP	test and evaluation master plan
TER	trailing edge right
TEU	trailing edge up
TF	terrain following
THP	Thrust Horsepower
THP_{alt}	horsepower available at altitude
THP_{max}	maximum horsepower available
THP_{min}	minimum horsepower required
THP_{SL}	horsepower required at sea level
TIT	turbine inlet temperature
TM	telemetry
TMN	true Mach number
T/O	takeoff
T_o	standard sea level temperature (59.0 ^oF, 15 ^oC)
TO	technical order
TRB	technical review board

TRD	technical requirements document
TRP	technical resources plan
TSFC	thrust specific fuel consumption
TSPI	time, space, position information
T_t	total temperature
TV	television
T/W	thrust to weight ratio
TWT	track while scan
TWT	traveling wave tube
u	velocity along aircraft's x-axis
UAV	uninhabited aerial vehicle
UHF	ultra high frequency
UPT	undergraduate pilot training
USA	US Army
USAF	US Air Force
USCG	US Coast Guard
USMC	US Marine Corps
USN	US Navy
UT	universal time
UV	ultraviolet
v	velocity along aircraft's lateral axis
V_H	horizontal tail volume coefficient
V_V	vertical tail volume coefficient
V_1	takeoff decision speed
V_2	takeoff safety speed
V_A	design maneuvering speed
VAC	volts AC
V_b	buffet airspeed
V_B	design speed for max gust intensity
V_{br}	velocity for best range
V_c	calibrated airspeed
V_D	design diving speed
VDC	volts DC
VDOP	vertical dilution of precision
V_e	equivalent velocity
V_{FE}	maximum flap extended speed
V_{FR}	visual flight rules
V_g	ground speed
VHF	very high frequency
V_i	indicated airspeed
V_{ic}	indicated airspeed corrected for instrument error
V_{iw}	velocity at sea level std day and std weight
VLE	max speed with landing gear extended

NOTES

Section 2 Mathematics

Section 2.1 Algebra

(reference 2.1)

LAWS

commutative: $a+b = b+a$

$ab = ba$

associative: $a+(b+c) = (a+b)+c$

distributive: $a(b+c) = ab+ac$

IDENTITIES

exponents: $a^x a^y = a^{x+y}$

$(ab)^x = a^x b^x$

$(a^x)y = a^{xy}$

$a^{mn} = (a^m)^n$

if $a^0 = 1 \qquad a \neq 0$

$$a^{-x} = \frac{1}{a^x} \qquad \left(\frac{1}{a}\right)^x$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\sqrt[x]{ab} = \left[\sqrt[x]{a}\right]\left[\sqrt[x]{b}\right]$$

$$a^{x/y} = \sqrt[y]{a^x} = \left(\sqrt[y]{a}\right)^x$$

$$a^{1/y} = \sqrt[y]{a}$$

$$a^{x/y} = \sqrt[y]{a^x} = \left(\sqrt[y]{a}\right)^x$$

$$\sqrt[x]{a}\sqrt[y]{a} = a^{(1/x)+(1/y)} = \sqrt[xy]{a^{x+y}}$$

$$\sqrt{a} + \sqrt{b} = \sqrt{a + b + 2\sqrt{ab}}$$

logarithms: if M, N,b are positive and

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b MN = \log_b M + \log_b N$

$\log_b [M/N] = \log_b M - \log_b N$

$\log_b M^p = p \log_b M$

$\log_b [1/M] = -\log_b M$

$\log_b \sqrt[q]{M} = \frac{1}{q} \log_b M$

$\log_b M = \log_c M \log_b c = \dfrac{\log_c M}{\log_c b}$

examples: log 6.54 = .8156,

log 6540 = log (6.54 x 10^3) = .8156 +3 = 3.8156

log .654 = log (6.54 x 10^{-1}) = .8156 -1 = 9.8156 -10

log .000654 = log (6.54 x 10^{-4}) = .8156 -4 = 6.8156 -10

calculate 68.31 x .2754:

log 68.31 = 1.8354

log .2754 = -.56

1.8354 +(-.56) = 1.2745

$\log^{-1}$ 1.2745 = 18.81

calculate $[.6831]^{1.53}$:

log .6831 = -.1655

1.53 x (-.1655) = -.253

$\log^{-1}$[-.253] = .5582

calculate $[.6831]^{1/5}$:

log .6831 = -.1655

1/5 x(-.1655) = -.0331

$\log^{-1}$ (-.0331) = .9266

solve for x in $.6931^x = 27.54$:

$\log[.6931^x] = \log 27.54$

$x \log [.6931] = \log 27.54$

$x = \log 27.54 / \log [.6931]$

=1.44/[-.1655] = -8.701

EQUATIONS

Quadratic Equation: for $ax^2 + bx + c = 0$

(has two roots, both real or both complex)

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cubic Equation: for $y^3 + py^2 + qy + r = 0$

(has three roots, all real or one real & two complex)

let $y = x - (p/3)$ to rewrite equation in form of $x^3 + ax + b = 0$

where $a = (3q - p^2)/3$ and $b = (2p^3 - 9pq - 27r)/27$

let $$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

and $$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

then $x_1 = A + B$

$x_2 = -(A + B)/2 + \{[-3]^{1/2}/2\}(A - B)$

$x_3 = -(A + B)/2 - \{[-3]^{1/2}/2\}(A - B)$

special cases…

if $(b^2/4 + a^3/27 < 0)$, then the real roots are

$x_{1,2,3} = 2[-a/3]^{1/2}\cos(\phi/3 + 120^{o}k)$

where $k = 0,1,2$

and $\cos\phi = +[(b^2/4)/(-a^3/27)]^{1/2}$ if $b<0$

or $\cos\phi = -[(b^2/4)/(-a^3/27)]^{1/2}$ if $b>0$

if $(b^2/4 + a^3/27 > 0)$ and $a>0$, the single real root is

$x = 2[a/3]^{1/2}\cot(2\phi)$

where $\tan(\phi) = [\tan(\psi)]^{1/3}$

and $\cot(2\psi) = +[(b^2/4)/(-a^3/27)]^{1/2}$ if $b<0$

or $\cot(2\psi) = -[(b^2/4)/(-a^3/27)]^{1/2}$ if $b>0$

if $(b^2/4 + a^3/27 = 0)$, the three real roots are

$x_1 = -2[-a/3]^{1/2}$, $x_2 = x_3 = +[-a/3]^{/2\ 1}$ if $b>0$

or $x_1 = +2[-a/3]^{1/2}$, $x_2 = x_3 = -[-a/3]^{1/2}$ if $b<0$

Quartic (biquadratic) Equation: for $y^4 + py^3 + qy^2 + ry + s = 0$

let $[y = x - (p/4)]$ to rewrite equation as $x^4 + ax^2 + bx + c = 0$

let *l, m, n* denote roots of the following resolvent cubic…

$t^3 + at^2/2 + (a^2 - 4c)t/16 - b^2/64 = 0$
the roots of the quartic are
$x_1 = +[l]^{1/2} + [m]^{1/2} + [n]^{1/2}$
$x_2 = +[l]^{1/2} - [m]^{1/2} - [n]^{1/2}$
$x_3 = -[l]^{1/2} + [m]^{1/2} - [n]^{1/2}$
$x_4 = -[l]^{1/2} - [m]^{1/2} + [n]^{1/2}$

INTEREST AND ANNUITIES
(reference 2.3)

Amount:

P principal at i interest for n time accumulates to amount A_n:
simple interest: $A_n = P(1 + ni)$
at interest compounded each n interval: $A_n = P(1 + i)^n$
at interest compounded q times per n interval: $A_n = P(1 + r/q)^{nq}$
where r is the nominal (quoted) rate of interest

Effective Interest:

The rate per time period at which interest is earned during each period is called the effective rate i. $i = (1 + r/q)^q - 1$

Solve above equations for P to determine investment required now to accumulate to amount A_n
True discount , $D = A_n - P$

Annuities:

rent R is consistent payment at each period n

let $$s_n \equiv \frac{(1+i)^n - 1}{i}$$

and let $$r_n \equiv \frac{1-(1+i)^{-n}}{i}$$

then $A_n = Rs_n$

or $$n = \frac{\log(A_n + R) - \log R}{\log(1+i)}$$

present value of an annuity, A is the sum of the present values of all the future payments. $A = Rr_n$

Section 2.2 Geometry

(references 2.1, 2.2)

General definitions:

A = area
a = side length
b = base length
C = circumference
D = diameter
h = height
n = number of sides
R = radius
V = volume
x, y, z = distances along orthogonal coordinate system
β = interior vertex angle

triangle: A = bh/2
sum of interior angles = 180^o

rectangle: A = bh
sum of interior angles = 360^o

parallelogram (opposite sides parallel):
A = ah = absin β

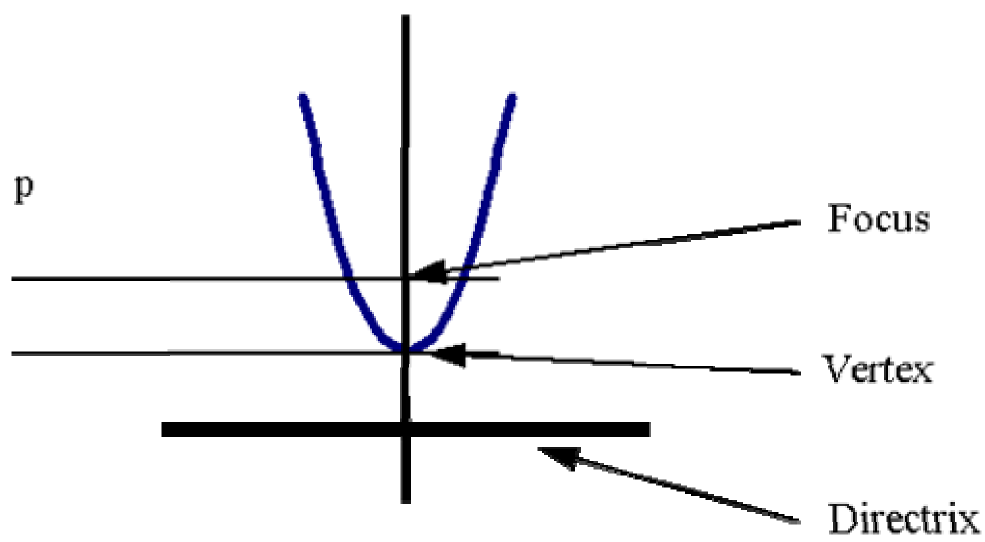
trapezoid (4 sides, 2 parallel):
A = h(a+b)/2

pentagon, hexagon, and other *n*-sided polygons:
$A = \{(na^2)\cot(180^o/n)\}/4$
R = radius of circumscribed circle = $\{a \csc(180^o/n)\}/2$
r = radius of inscribed circle = $\{a \cot(180^o/n)\}/2$
$\beta = 180^o - (360^o/n)$
sum of interior angles = $n180^o - 360^o$

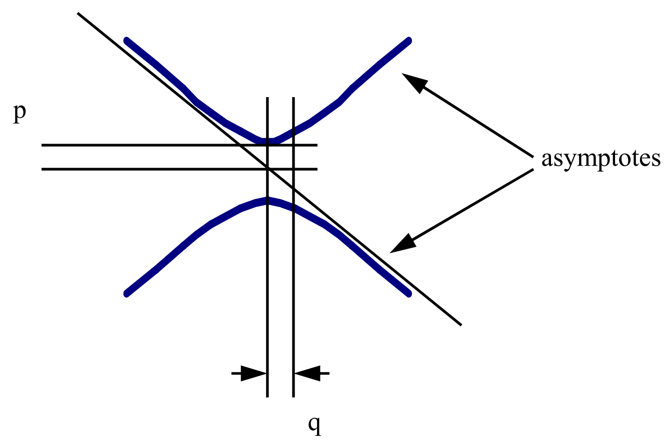
circle: $A = \pi R^2$
$C = 2\pi R = \pi D$
perimeter of n-sided polygon inscribed within a circle
$= 2nR\sin(\pi/n)$
area of circumscribed polygon $= nR^2\tan(\pi/n)$
area of inscribed polygon $= \{nR^2\sin(2\pi/n)\}/2$
equation for a circle with center at (h,k): $R^2 = (x-h)^2 + (y-k)^2$

ellipse: f = semimajor axis
g = semiminor axis
e = eccentricity $= ([f^2-g^2]^{1/2})/f$
$A = \pi ef$
equation for ellipse with center at (h,k):
$(x-h)^2/f^2 + (y-k)^2/g^2 = 1$ if major axis along x-axis
or $(y-k)^2/f^2 + (x-h)^2/g^2 = 1$ if major axis along y-axis
distance from center to either focus $= [f^2-g^2]^{1/2}$
latus rectum $= (2g^2)/a$

parabola: p = distance from vertex to focus
e = eccentricity = 1
equation for parabola with vertex at (h,k), focus at (h+p,k):
$(y-k)^2 = 4j(x-h)$ if $(j>0)$
equation for parabola with vertex at (h,k), focus at (h,k+p):
$(x-h)^2 = 4j(y-k)$ if $(j<0)$



hyperbola: p = distance between center and vertex
q = distance between center and conjugate axis
e = eccentricity = $([p^2+q^2]^{1/2})/p$
equation for hyperbola centered at (h, k):
$(x-h)^2/p^2 - (y-k)^2/q^2 = 1$ if (asymptotes slopes = +/- q/p)
or $(y-k)^2/p^2 - (x-h)^2/q^2 = 1$ if (asymptotes slopes = +/- p/q)



sphere: $A = 4\pi R^2$
$V = 4\pi R^3/3$
equation for sphere centered at origin: $x^2+y^2+z^2 = R^2$

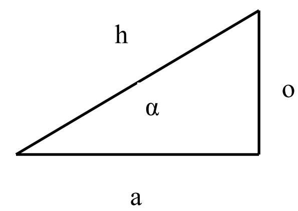
torus: $A= 4\pi^2 R\rho$
$V = 2\pi^2 R\rho^2$
ρ = smaller radius

Section 2.3 Trigonometery

(references 2.1, 2.2)

For any right triangle with hypotenuse h, an acute angle α, side length o opposite from α, and side length a adjacent to α, the following terms are defined:

sine $\alpha = \sin\ \alpha = o/h$
cosine $\alpha = \cos\ \alpha = a/h$
tangent $\alpha = \tan\ \alpha = o/a = \sin\ \alpha / \cos\ \alpha$
cotangent $\alpha = \cot\ \alpha = \mathrm{ctn}\ \alpha = a/o = 1/\tan\ \alpha = \cos\ \alpha / \sin\ \alpha$
secant $\alpha = \sec\ \alpha = h/a = 1/\cos\ \alpha$
cosecant $\alpha = \csc\ \alpha = h/o = 1/\sin\ \alpha$
exsecant $\alpha = \mathrm{exsec}\ \alpha = \sec\ \alpha - 1$
versine $\alpha = \mathrm{vers}\ \alpha = 1 - \cos\ \alpha$
coversine $\alpha = \mathrm{covers}\ \alpha = 1 - \sin\ \alpha$
haversine $\alpha = \mathrm{hav}\ \alpha = (\mathrm{vers}\ \alpha)/2$



also defined are the following…

hyperbolic sine of $x = \sinh x = (e^x - e^{-x})/2$
hyperbolic cosine of $x = \cosh x = (e^x + e^{-x})/2$
hyperbolic tangent of $x = \tanh x = \sinh x / \cosh x$
$\mathrm{csch}\ x = 1/\sinh x$
$\mathrm{sech}\ x = 1/\cosh x$
$\coth x = 1/\tanh x$

Identities

Pythagorean Identities:

$\sin^2 \alpha + \cos^2 \alpha = 1$
$1 + \tan^2 \alpha = \sec^2 \alpha$
$1 + \cot^2 \alpha = \csc^2 \alpha$

Half Angle Identities:

$\sin [\alpha/2] = +/- \ [(1 - \cos \alpha)/2]^{1/2}$
(negative if $[\alpha/2]$ is in quadrant III or IV)
$\cos [\alpha/2] = +/- \ [(1 + \cos \alpha)/2]^{1/2}$

NOTES

Section 3 Universe/Earth/Atmospheric Properties

Section 3 Recurring Nomenclature

T = absolute temperature (Rankin or Kelvin)
T_R = absolute temperature, Rankin scale
T_o = standard day seal level absolute temperature
P = ambient pressure
P_o= standard day seal level ambient pressure
ρ = ambient density
ρ_o= standard day seal level ambient density

$\delta = P/P_o$ = atmospheric pressure/std day sea level pressure
$\theta = T/T_o$ = atmospheric absolute temp / std day sea level absolute temp
$\sigma = \rho/\rho_o$ = atmospheric density/std day sea level density

g = acceleration due to gravity
g_o = standard earth acceleration due to gravity
a_o = speed of sound at std day sea level temperature

Section 3.1 Universal Constants (reference 3.1)

Avogadros number, N_o	6.022169×10^{23} molecules/mole	
Boltzmann constant, k	1.380×10^{-23}	J/^{o}K
electron charge, e	$1,602 \times 10^{-19}$	coulomb
electron mass, m_e	9.109×10^{-31}	kg
gas constant, R	8.31434J/^{o}K	mole
gravitational constant, G	6.673×10^{-11}	Nm^2/kg^2
neutron mass, m_n	1.674×10^{-27}	kg
Planck constant, h	6.625×10^{-34}	J sec
proton mass, m_p	1.672×10^{-27}	kg
speed of light in a vacuum, c	2.998×10^{8}	m/sec
unified atomic mass constant, m_u	1.660×10^{-27}	kg
volume of ideal gas (std temp & press)	2.241 x 10	m^3/mol

Newtonian Gravity

The gravitaional field (g) near any mass can be calculated as

$$g = \frac{GM}{(R_A)^2}$$

where G is the universal gravitational constant and R_A is the absolute distance from the center of mass M

Section 3.2 Earth Properties (references 3.2, 3.3)

Std Earth gravitational acceleration, g_o = 9.8066 m/s^2 = 32.174 ft/s^2
mass = 5.98333 × 10^{24} *kg* = 13.22 x 10^{24} *lb*
rotation rate, ω = 7.292115 × 10^{-5} *rad/sec*
average density = 5.522 g/cm^3 = 344.7 lb/ft^3
radius average, R_{avg} = 6,367,444 *m* = 3956.538 *st. miles* = 20,890,522 *ft*
radius at the equator (R_e) is 6,378,137 *m* (±2)
radius at the poles R_p = 6,356,752 [*m*]
radius as a function of latitude, ϕ (assumes perfect ellipsoid):

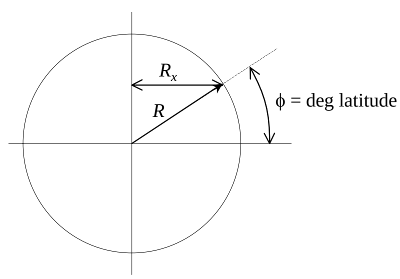
$$R = \left[\left(\frac{\cos\phi}{R_e} \right)^2 + \left(\frac{\sin\phi}{R_p} \right)^2 \right]^{-\frac{1}{2}}$$

Centrifugal Relief from Gravity

The earth's "normal" gravity field includes both the Newtonian Law and a correction for the centrifugal force caused by the earth's rotation. The centrifugal relief correction is

$$\Delta CR = -\frac{V^2}{R_x} = -\frac{(R_x\omega)^2}{R_x} = R_x\omega^2$$

where ω is the earth's rotation rate and R_x is the perpendicular distance from the earth's axis to the surface and can be calculated as $R_x = R \cos\phi$ (see figure below).



For any centrifugal relief calculations associated with aircraft performance, it is sufficiently exact (g ±0.00004 m/s^2) to use the average earth radius. An aircraft flying eastward contributes to centrifugal relief while a west-bound aircraft diminishes it.

The International Association of Geodesy publishes the following equation (accurate to 0.005%) to calculate *local sea level gravity including the effects of centrifugal relief for any point fixed to the earth's surface*

$$g_{lsl} = 9.780327\left(1 + 0.00530224\sin^2\phi - 0.000058\sin^2 2\phi\right)\left[\tfrac{m}{s^2}\right]$$

The above equation is tabulated below for quick reference.

Latitude (deg)	**Normal g_{local}**	
	(m/s^2)	**(ft/s^2)**
0	9.780327	32.088
15	9.783659	32.098
30	9.792866	32.188
45	9.805689	32.171
60	9.818795	32.214
75	9.828569	32.249
90	9.832185	32.258

The standard acceleration (g_o) corresponds to a latitude of 46.0625°.
g_{lsl} at the equator and the poles varies ±0.27% from g_o.

Altitude Effect on Gravitational Acceleration
R_A is the sum of the earth's local radius and the geometric distance (h_G) above the surface: $R_A = R + h_G$

Gravitational acceleration at any geometric altitude:

h_G (1000 ft)	
0	1
10	0.99904
20	0.99809
40	0.99618
60	0.99428
80	0.99238
100	0.99049

$$g_{alt} = g_{sls}\left(\frac{R}{R + h_G}\right)^2$$

$$\frac{g_{alt}}{g_{lsl}}$$

Actual Gravitational Pull on an Aircraft

Adding a centrifugal relief correction due to the aircraft's velocity, a complete calculation for its gravitational acceleration is

$$g_{A/C} = \left[g_{lsl} + \omega^2 R\cos\phi\right]\left[\frac{R}{R+h_G}\right]^2 - \left(\omega + \frac{V_G \sin\sigma}{R+h_G}\right)^2 (R+h_G)\cos\phi$$

where V_G = ground speed and σ = ground track angle (0° = true North, 90° = East, etc.).

Gravity Influence on Aircraft Cruise Performance

Even at the same altitude, changes in gravity due to latitude or centrifugal relief directly alter the required lift, drag, and fuel flow. For example, with sufficiently precise instrumentation, data collected heading West could show about 0.5% more drag and fuel flow than data collected heading East (centrifugal relief effect). After determining test and standard (or mission) values for g, flight test values for C_L, C_D, drag, and fuel flow can be corrected to standard as follows:

$$C_{L_{std}} = C_{L_t}\frac{N_{z_{\omega std}}}{N_{z_{\omega t}}}\left[\frac{g_{std}}{g_{A/C}}\right]$$

$$C_{D_{std}} = \frac{\left(C_{L_{std}}\right)^2}{\pi A\mathrm{Re}}$$

$$\Delta D = D_{std} - D_t = qS\left[C_{D_{i_{std}}} - C_{D_{i_t}}\right]$$

$$\dot{W}_{f_{std}} = \dot{W}_{f_t} + \Delta D \cdot TFSC$$

where N_z = normal load factor,
C_L = lift coefficient, C_D = drag coefficient,
AR = aspect ratio, e = Oswald efficiency factor,
ΔD change in drag force,
$TSFC$ = thrust specific fuel consumption, and

$\dot{W}_{f_{std}}$ = standard day fuel flow

Section 3.3 General Properties of Air (reference 3.1)

Gas constant, R = 53.35 ft lb/^{o}R lbm = 287.074 J/kg ^{o}K
= 1716 lb(ft)/slgs(o*R*) = 3089.7 lb(ft)/slgs(o*K*)

Speed of sound = $a_o(\theta)^{½}$
= 49.02 $(T_R)^{1/2}$ *ft/sec*
= 33.42 $(T_R)^{1/2}$ *miles/hr*
= 29.04 $(T_R)^{1/2}$ *knots*
= 20.05 $(T_R)^{1/2}$ *m/sec*

Density, ρ = .0023689 *slug/ft*^{3} *= 1.225 kg/m*^{3}

Specific weight, g_ρ = .07647 *sec*^{2}*/ft*^{4}

Specific heat capacity at 59^o*F* (=T_o)
at constant pressure, c_p = .240 *BTU/lb* ^{o}*R* = 1004.76 *J/kg* ^{o}*K*
at constant volume, c_v = .1715 *BTU/lb* ^{o}*R* = 717.986 *J/kg* ^{o}*K*
specific heat ratio, $\gamma = \{c_p / c_v\}$ =1.4

Normal Composition of clean, dry atmospheric air near sea level

Nitrogen, N_2	78.084 % by volume
Oxygen, O_2	20.948 %
Argon, *A*	0.934 %
Carbon Dioxide, CO_2	0.031 %
Neon, *Ne*	0.002 %
total	99.9988 %

plus traces of helium, kryton, xenon, hydrogen, methane, nitrous oxide, ozone, sulfur dioxide, nitrogen dioxide, ammonia, carbon monoxide, and iodine.

Viscosities of Air

Coefficient of Viscosity, $\mu_c = \dfrac{7.3025x10^{-7}(T_R)^{3/2}}{T_R + 198.72}$ lb/ft sec

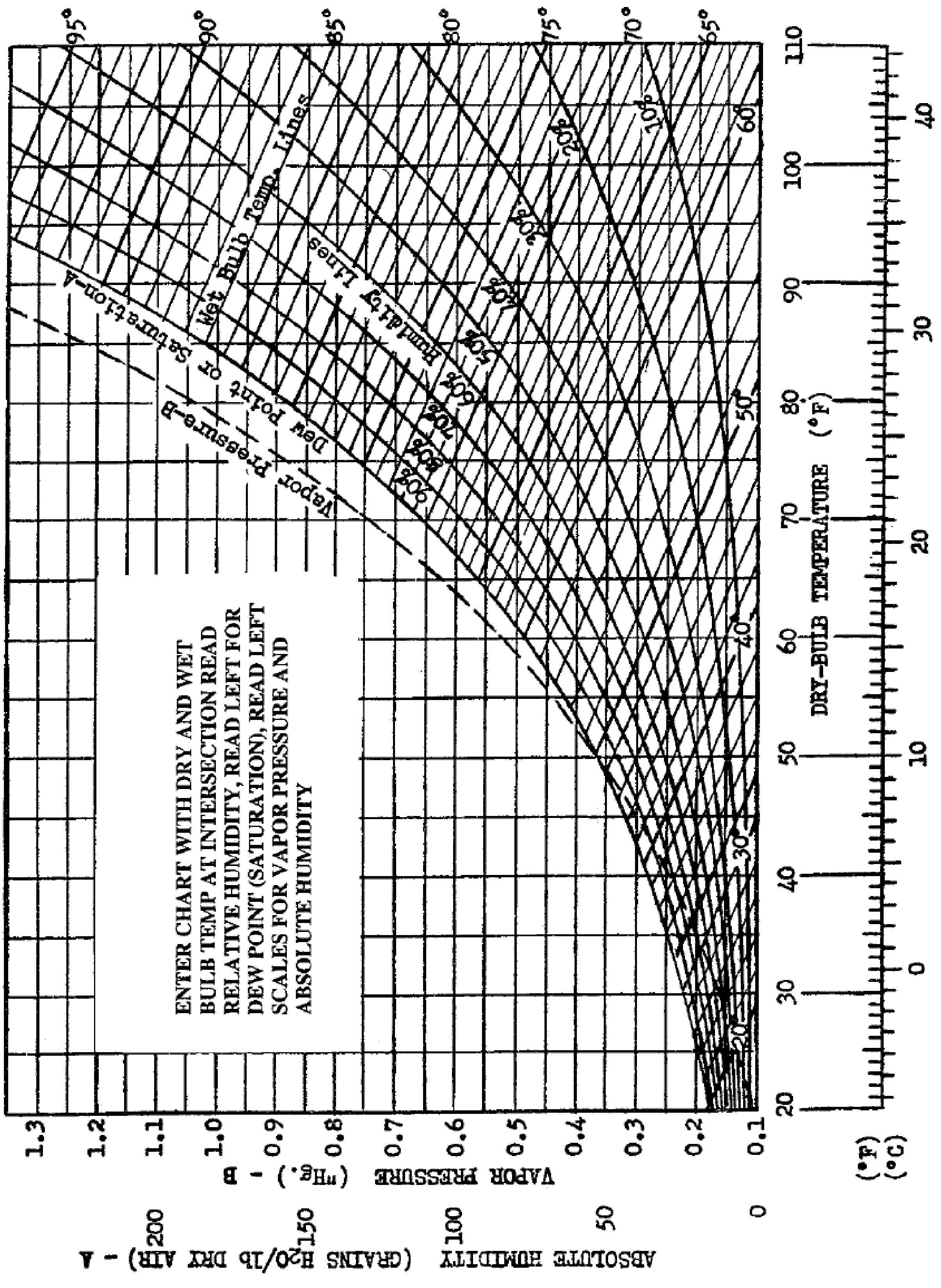
Kinematic viscosity, $\nu = \dfrac{\mu_c}{g\rho}$ ft^2/sec

Absolute Viscosity, lb $\mu = \rho\nu = \left[.317(T_R)^{3/2}\left(\dfrac{734.7}{T_R + 216}\right)\right]x10^{-10}$ sec/ft^2

Atmospheric Viscosity (U.S. Standard Atmosphere)

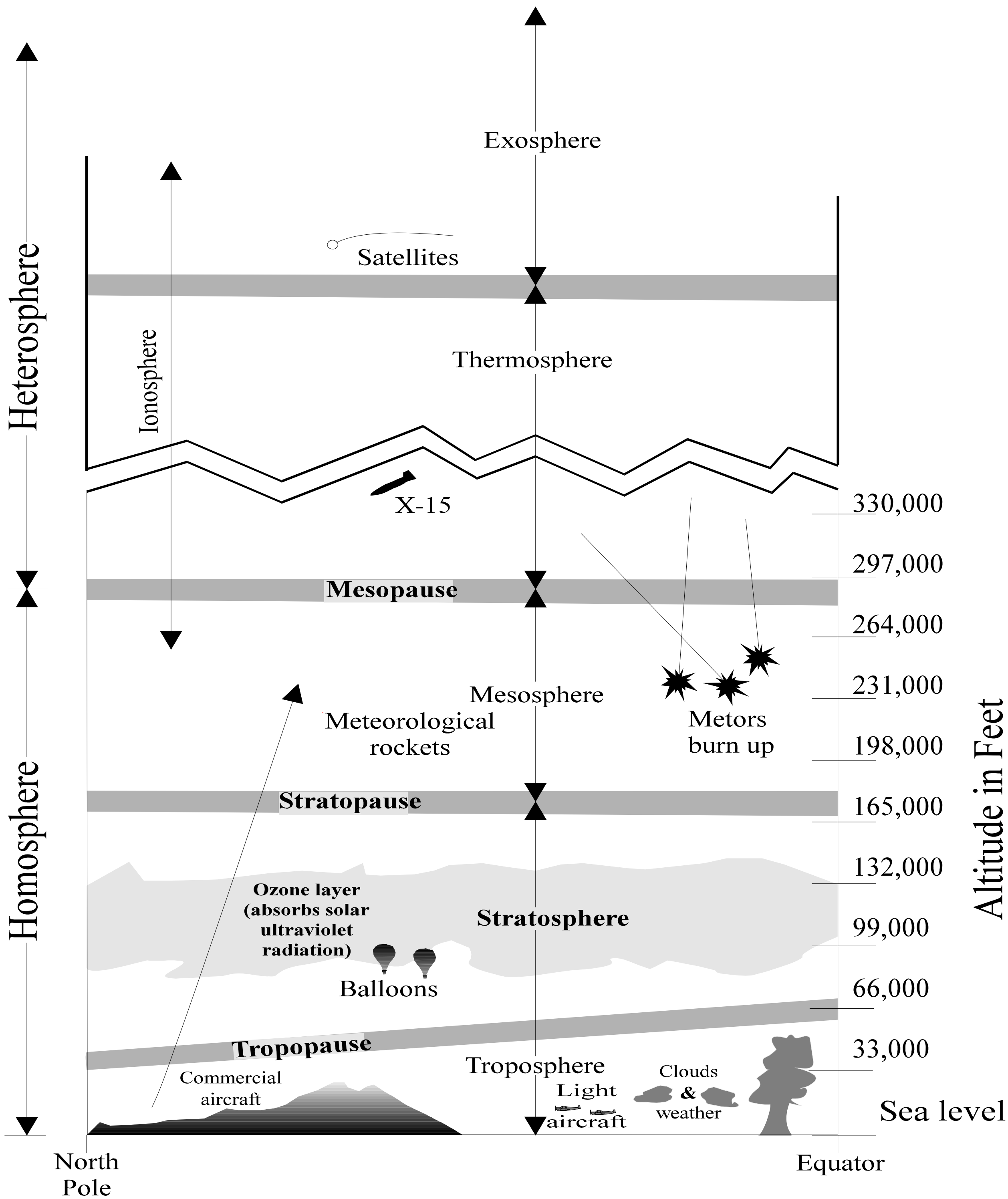
Pressure Altitude *ft*	Kinematic Viscosity υ *(ft^2/sec)*	Absolute Viscosity μ *(lb sec/ft^2)*
0	1.572 x 10^{-4}	3.737 x 10^{-7}
5,000	1.776	3.638
10,000	2.013	3.538
15,000	2.293	3.435
20,000	2.625	3.330
25,000	3.019	3.224
30,000	3.493	3.115
35,000	4.065	3.004
40,000	5.074	2.981
45,000	6.453	2.982
50,000	8.206	2.983
55,000	10.44	2.985
60,000	13.27	2.986
70,000	21.69	3.005
80,000	35.75	3.043
90,000	58.53	3.080
100,000	95.19	3.118
150,000	1066	3.572
200,000	6880	3.435

Psychrometric Chart for Seal Level Barometric Pressure



Section 3.4 Standard Atmosphere

Divisions of the Atmosphere



Constantly changing atmospheric conditions cannot be duplicated at will to provide the exact environment in which a flight takes place. A standard atmosphere provides a common basis to relate all flight test, wind tunnel results, aircraft design and general performance. Several models of "standard atmosphere" exist with minor differences based on mathematical constants used in the calculations.

Geometric altitude, h_G , is defined as the height of an aircraft above sea level (also called **tapeline** altitude)

Absolute altitude, h_a, is defined as the height of an aircraft above the center of the earth: (geometric altitude + radius of the earth).

Geopotential altitude, h, is required because g changes with height.If potential energy is calculated using sea level weight ($W_{SL} = mg_o$) instead of actual weight ($W = mg$), then the altitude must be lower.

$$W\, h_G = W_{SL}\ \ h$$

Pressure altitude is the altitude, on a standard day, at which the test day pressure would be found

Density altitude is the altitude, on a standard day, at which the test day density would be found

Temperature altitude is the altitude, on a standard day, at which the test day temperature would be found

Assumptions on which the standard atmosphere is built

1. The air is dry (only 0.4% per volume of water vapor)

2. The air is a perfect gas and obeys the equation of state,

$$P = \rho g R T$$

where $R = 53.35$ ft lb/^{o}R lbm

3. The gravitational field decreases with altitude

4. Hydrostatic equilibrium exists ($\Delta p = -\rho g_o \Delta h$)

Standard Day Sea Level Atmospheric Conditions

P_o = 2116.22 lb/ft^2 = 14.696 lb/in^2 = 29.921 in *Hg*
= 1013.25 HPa (mb) = 101325 Pa

T_o = 288.15 ^oK = 518.67 ^oR = 59 ^oF = 15 ^oC

ρ_o = 0.0023689 slgs/ft^3 = 0.07647 lbm/in^3 = 1.255 kg/m^3

a_o = 1116.45 ft/sec = 661 KTAS = 761.14 mph = 340.294 m/sec

g_o = 32.174 ft/sec^2 = 9.80665 m/sec^2

L = standard temperature lapse rate = 0.0019812 ^oK /ft

1976 U.S Standard Atmosphere Equations

Troposhere - below 36,089 ft (11,000 m) < *22636 Pa*

$$\theta = 1 - (L/To)\, h = 1 - (6.866 \times 10^{-6})\, h$$

$$\sigma = \theta^{n-1}$$

$$\delta = \theta^{n}$$

where n = 5.25585, h = geopotential altitude

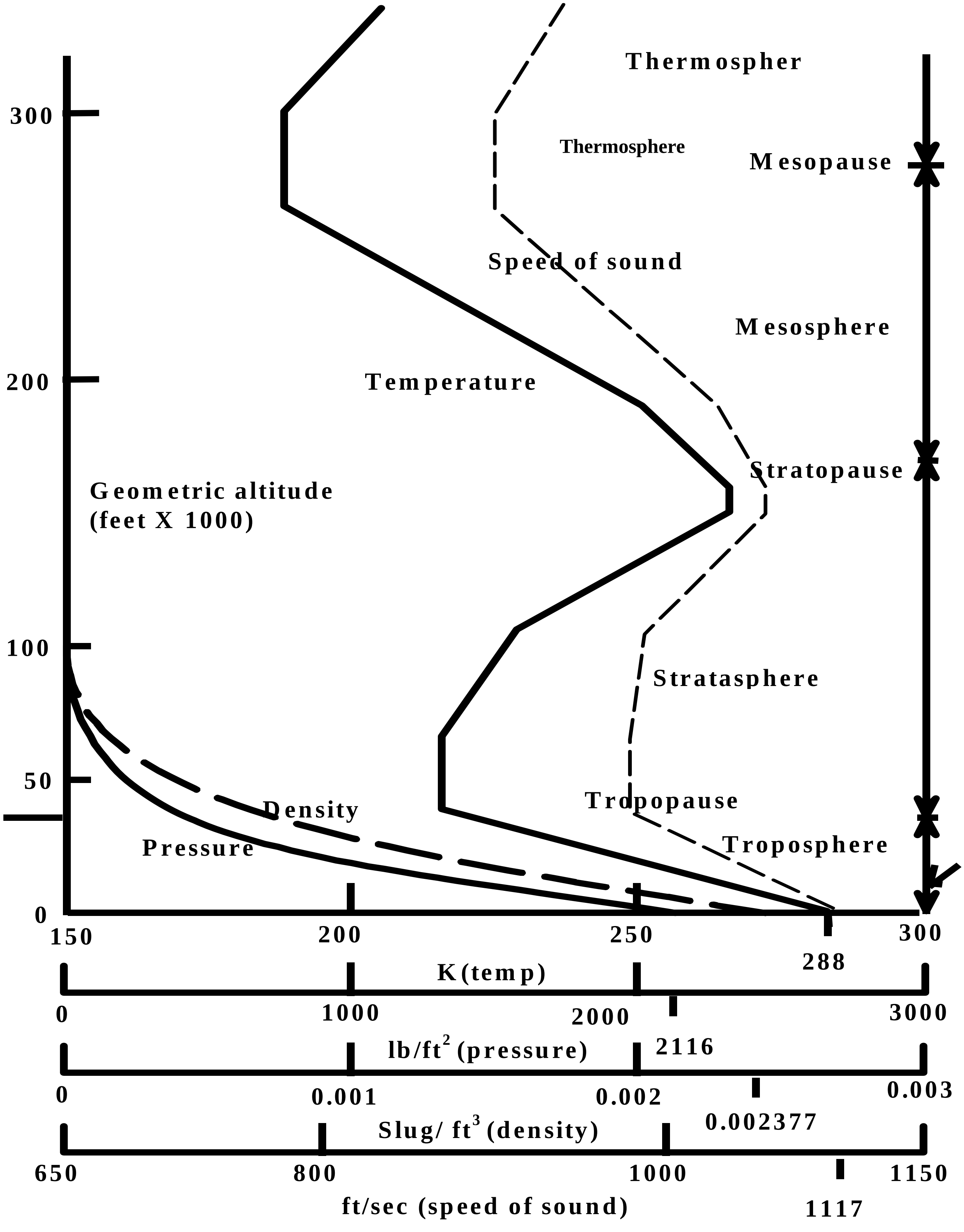
Stratosphere- between 36,089 ft and 65,616 ft (20,000 m) the standard day temperature is a constant 216.66 ^oK, therefore:

$$\theta = 0.7519$$

$$\sigma = .29707\ e^{-0.00004806\ [h - 36,089]}$$

$$\delta = .223358\ e^{-0.00004806\ [h - 36,089]}$$

1976 U.S. Standard Atmosphere Graph



1976 U.S. Standard Atmosphere Tables

H (ft)	δ (=Pa/Po)	θ (=Ta/To)	σ (=ρa/ρo)
-1,000	1.0367	1.0069	1.0296
0	1.0000	1.0000	1.0000
1,000	0.9644	0.9931	0.9711
2,000	0.9298	0.9862	0.9428
3,000	0.8962	0.9794	0.9151
4,000	0.8637	0.9725	0.8881
5,000	0.8320	0.9656	0.8617
6,000	0.8014	0.9587	0.8359
7,000	0.7716	0.9519	0.8106
8,000	0.7428	0.9450	0.7860
9,000	0.7148	0.9381	0.7620
10,000	0.6877	0.9312	0.7385
11,000	0.6614	0.9244	0.7156
12,000	0.6360	0.9175	0.6932
13,000	0.6113	0.9106	0.6713
14,000	0.5875	0.9037	0.6500
15,000	0.5643	0.8969	0.6292
16,000	0.5420	0.8900	0.6090
17,000	0.5203	0.8831	0.5892
18,000	0.4994	0.8762	0.5699
19,000	0.4791	0.8694	0.5511
20,000	0.4595	0.8625	0.5328
21,000	0.4406	0.8556	0.5150
22,000	0.4223	0.8487	0.4976
23,000	0.4046	0.8419	0.4807
24,000	0.3876	0.8350	0.4642
25,000	0.3711	0.8281	0.4481
27,000	0.3398	0.8144	0.4173
28,000	0.3250	0.8075	0.4025
29,000	0.3107	0.8006	0.3881
30,000	0.2970	0.7937	0.3741
31,000	0.2837	0.7869	0.3605
32,000	0.2709	0.7800	0.3473
33,000	0.2586	0.7731	0.3345
34,000	0.2467	0.7662	0.3220
35,000	0.2353	0.7594	0.3099
36,000	0.2243	0.7525	0.2981

1976 U.S. Standard Atmosphere Tables (cont.)

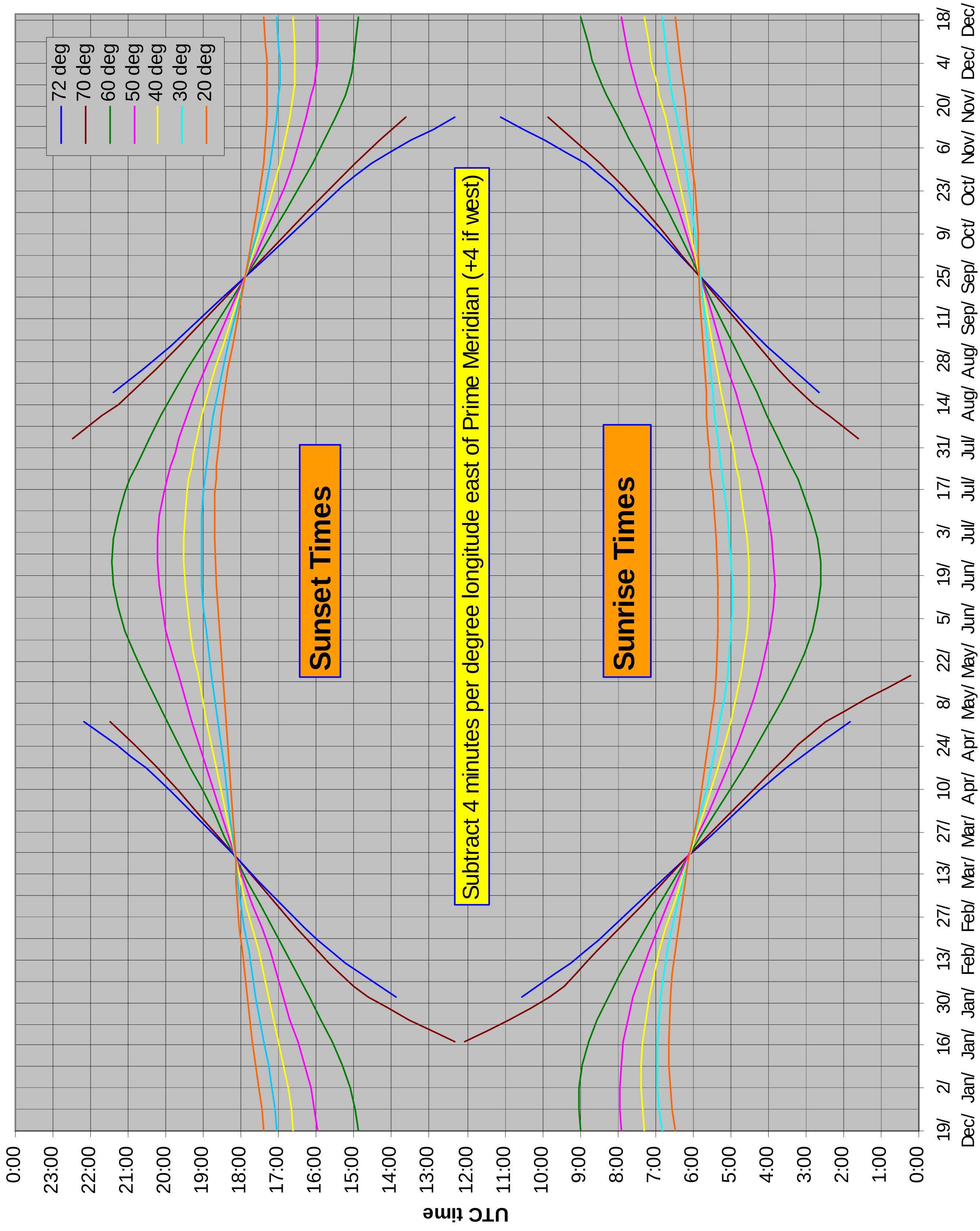
H (ft)	δ (=Pa/Po)	θ (=Ta/To)	σ (=ρa/ρo)
37,000	0.2138	0.7519	0.2843
38,000	0.2038	0.7519	0.2710
39,000	0.1942	0.7519	0.2583
40,000	0.1851	0.7519	0.2462
41,000	0.1764	0.7519	0.2346
42,000	0.1681	0.7519	0.2236
43,000	0.1602	0.7519	0.2131
44,000	0.1527	0.7519	0.2031
45,000	0.1455	0.7519	0.1936
46,000	0.1387	0.7519	0.1845
47,000	0.1322	0.7519	0.1758
48,000	0.1260	0.7519	0.1676
49,000	0.1201	0.7519	0.1597
50,000	0.1145	0.7519	0.1522
51,000	0.1091	0.7519	0.1451
52,000	0.1040	0.7519	0.1383
53,000	0.0991	0.7519	0.1318
54,000	0.0944	0.7519	0.1256
55,000	0.0900	0.7519	0.1197
56,000	0.0858	0.7519	0.1141
57,000	0.0818	0.7519	0.1087
58,000	0.0779	0.7519	0.1036
59,000	0.0743	0.7519	0.0988
60,000	0.0708	0.7519	0.0941
61,000	0.0675	0.7519	0.0897
62,000	0.0643	0.7519	0.0855
63,000	0.0613	0.7519	0.0815
64,000	0.0584	0.7519	0.0777
65,000	0.0557	0.7519	0.0740
70,000	0.0443	0.7563	0.0586
75,000	0.0350	0.7615	0.0459
80,000	0.0276	0.7667	0.0360
85,000	0.0219	0.7712	0.0284
90,000	0.0174	0.7773	0.0224
95,000	0.0138	0.7825	0.0177
100,000	0.0110	0.7877	0.0140
150,000	0.0013	0.9236	0.0015
200,000	0.0002	0.8811	0.0002

Section 3.5 Sea States

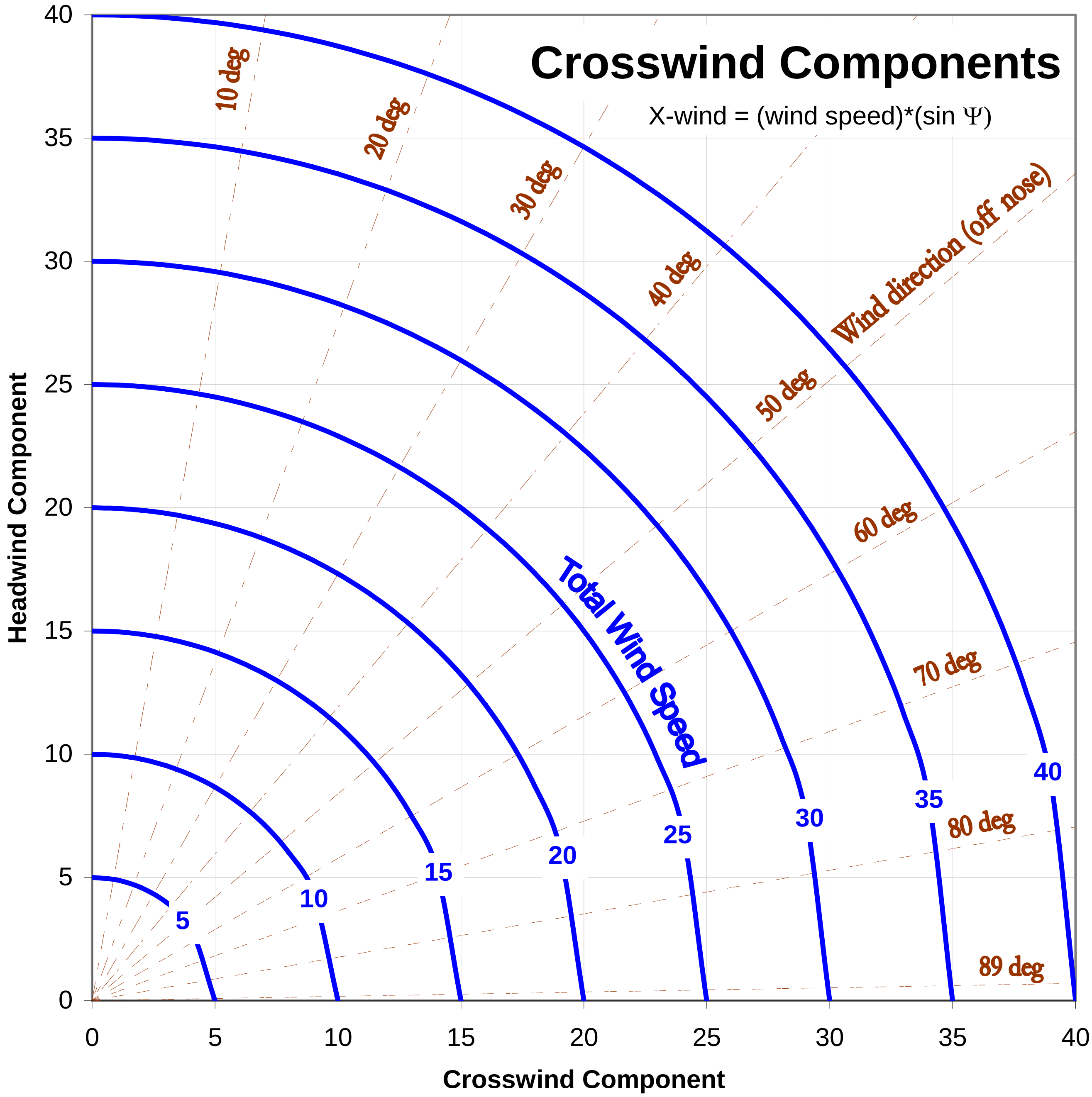
(ref 3.3) Sea Stale
International Swell Scale

Code	Sea	Wave Height, Crest to Trough (ft)
0	Calm	0
1	Smooth	Less than 1
2	Slight	1-3
3	Moderate	3-5
4	Rough	5-8
5	Very rough	8-12
6	High	12-20
7	Very high	20-40
8	Mountainous	40+
9	Confused, Used as additional description 1-8	
Code	**Swell**	**In Open Sea**
0	None	low
1	Short or average	
2	Long	
3	Short	Moderate height
4	Average	
5	Long	
6	Short	heavy
7	Average	
8	Long	
9	Confused, Used as additional description 1-8	

Section 3.6 Sunrise Sunset Times



Section 3.7 Crosswind Components



Section 3.8 References

3.1 Anon., "Aeronautical Vestpocket Handbook" ,Part No. P&W 079500, United Technologies Pratt & Whitney, Canada, 1990.

3.2 Lawless, Alan. R. et al, "Aerodynamics for Flight Testers", National Test Pilot School, P.O. Box 658, Mojave CA, 93501, 1999.

3.3 Denno, Richard R., et al "AIAA Aerospace Design Engineers Guide" ISBN 0-930403-21-5, AIAA, 1987.

NOTES

Section 4 Pitot Statics

Editor's Note

In an effort to reduce confusion and conflict regarding pitot and static pressure nomenclature, SFTE has elected to change two definitions and symbols since the first edition of this handbook was released. Henceforth, ΔPs shall indicate static pressure ERROR ($\Delta P_s = P_s - P_a$) and ΔP_T shall indicate total (i.e. pitot) pressure ERROR ($\Delta P_T = P_p - P_T$). This nomenclature eliminates the ΔP_p symbol and confusion as to whether it indicates position error or pitot error.

Section 4 Common Nomenclature

a = speed of sound
a_o = speed of sound at sea level on a std day
M = Mach number
P_a =ambient pressure
P_o = ambient pressure at sea level on a std day
(=2116.2 lb/ft^2 = 29.92 in Hg)
P_p = pitot pressure corrected for instrument error only
P_s = static pressure (indicated at static port)
P_T = total pressure
q = incompressible dynamic pressure
q_c = compressible differential pressure (P_T-P_a)
q_{cic} = instrument corrected differential pressure (= P_p-P_s)
T_a = ambient temperature (absolute scale)
T_o =ambient temperature at sea level on a std day
(=288.15 ^{o}K = 15 ^{o}C = 518.7 ^{o}R = 59 ^{o}F)
T_T = total temperature (absolute scale)
V_c = calibrated airspeed
V_e = equivlent airspeed
V_g = ground speed
V_i = indicated airspeed
V_{ic} = instrument corrected airspeed
V_T = true airspeed
ΔH_{ic} = altimeter instrument correction
ΔH_{pc} = altimeter position error correction
ΔP_D = dynamic pressure error (= P_T-ΔP_s)
ΔP_T = total (pitot) pressure error (= P_P - P_T)
ΔP_s =static pressure error (= P_s - P_a)
ΔV_{ic} = airspeed instrument correction
ΔV_{pc} =airspeed position error correction
δ= pressure ratio between ambient and sea level std (= P_a / P_o)
θ= temperature ratio between ambient and sea level std (= T_a / T_o)
ρ_o =ambient density at sea level on a std day (=.002377 slg/ft^3)
σ= density ratio between ambient and sea level std (= ρ_a /ρ_o)
γ = ratio of specific heats (= 1.4 for air)

Section 4.1 Subsonic Airspeed and Mach Equations

True Airspeed

$$V_T = \left[\frac{2\gamma}{\gamma - 1} \frac{P_a}{\rho_a} \left(\left[\frac{P_T - P_a}{P_a} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{1}{2}}$$

Equivalent Airspeed

(= V_T equation with assumption of std day sea level density)

$$V_e = \sqrt{7 \frac{P_a}{\rho_o} \left(\left[\frac{P_T - P_a}{P_a} + 1 \right]^{\frac{2}{7}} - 1 \right)} = V_T \sqrt{\frac{\rho_a}{\rho_o}} = V_T \sqrt{\sigma}$$

Calibrated Airspeed

(= V_e equation with assumption of std day sea level pressure)

$$V_c = \left[\frac{2\gamma}{\gamma - 1} \frac{P_o}{\rho_o} \left(\left[\frac{P_T - P_a}{P_o} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{1}{2}}$$

Applying British units (*lb/ft*^{2}) and converting from *ft/sec* to knots yields

$$V_c = 1478 \sqrt{\left[\frac{P_T - P_a}{2116} + 1 \right]^{\frac{2}{7}} - 1} \quad \text{(kts)}$$

Mach Number

$$M = \frac{V_T}{a} = \sqrt{\frac{2}{\gamma - 1} \left[\left(\frac{P_T - P_a}{P_a} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} = \sqrt{5 \left(\left[\frac{q_c}{P_a} + 1 \right]^{\frac{2}{7}} - 1 \right)}$$

Section 4.2 Scale Altitude (Compressibility) Correction

The name comes from the fact that although the equivalent airspeed equation does correct for compressibility, the sea level pressure assumption used for calibrated airspeed makes the compressibility correction valid only for that (sea level) pressure. Above sea level, the calibrated airspeed must be re-scaled for pressure effects on compressibility. The mathematical method for determining V_e from V_c is to first solve the calibrated airspeed equation for q_c

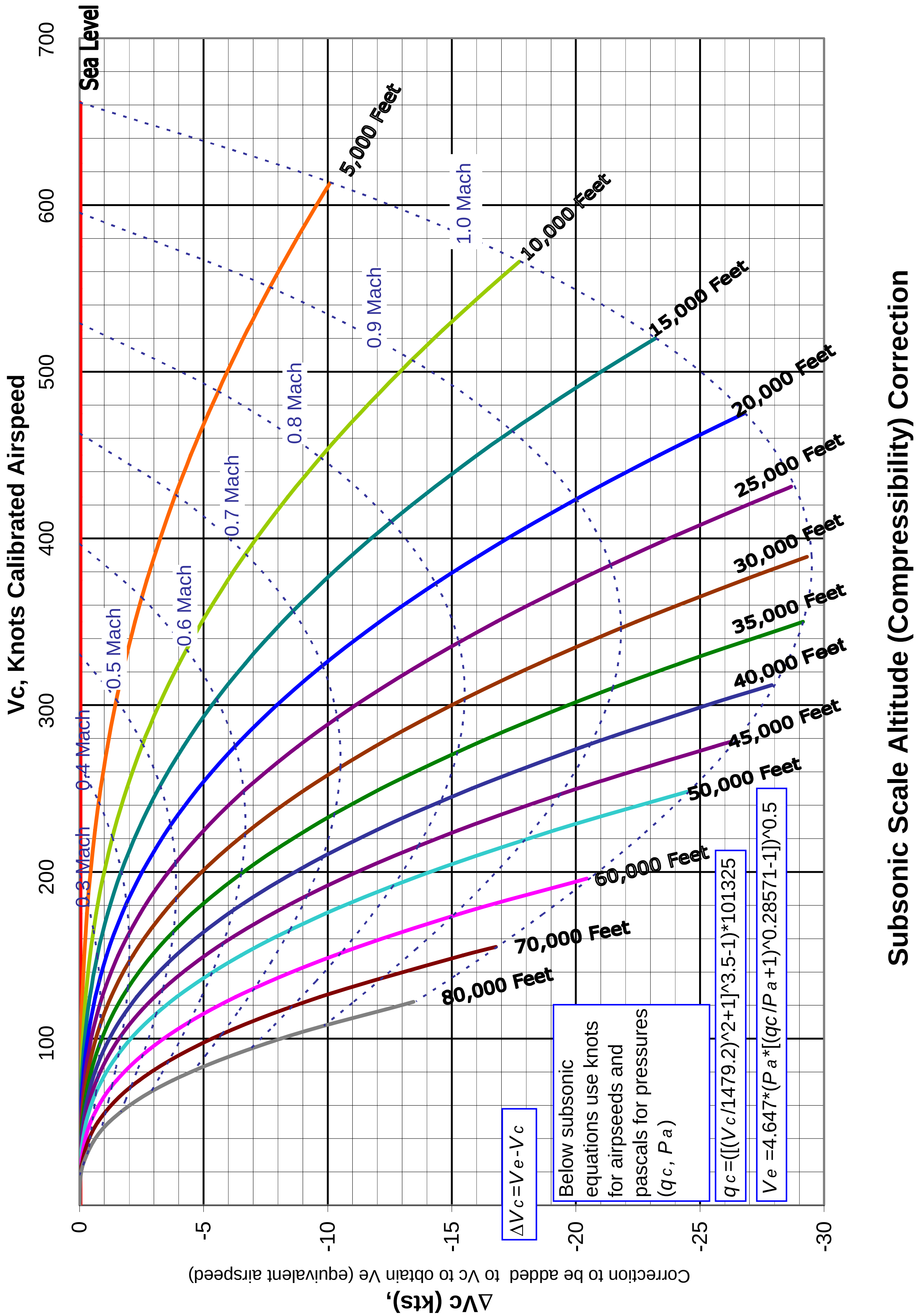
$$q_c = P_o\left[\left(\frac{\rho_o}{P_o}\frac{V_c^2}{7}+1\right)^{3.5}-1\right]$$

Next, substitute this value and the ambient pressure (P_a) into the equivalent airspeed equation. ($q_c = P_T - P_a$)

$$V_e = \sqrt{7\frac{P_a}{\rho_o}\left(\left[\frac{q_c}{P_a}+1\right]^{2/7}-1\right)}$$

The adjacent chart does this graphically for all subsonic airspeeds.

$$V_e = V_c + \Delta V_c$$


Subsonic Scale Altitude (Compressibility) Correction
Vc, Knots Calibrated Airspeed
ΔVc (kts),
Correction to be added to Vc to obtain Ve (equivalent airspeed)
Sea Level
5,000 Feet
10,000 Feet
15,000 Feet
20,000 Feet
25,000 Feet
30,000 Feet
35,000 Feet
40,000 Feet
45,000 Feet
50,000 Feet
60,000 Feet
70,000 Feet
80,000 Feet
0.3 Mach
0.4 Mach
0.5 Mach
0.6 Mach
0.7 Mach
0.8 Mach
0.9 Mach
1.0 Mach
ΔVc=Ve-Vc
Below subsonic equations use knots for airspeeds and pascals for pressures (qc, Pa)
qc=([(Vc/1479.2)^2+1]^3.5-1)*101325
Ve =4.647*(Pa*[(qc/Pa+1)^0.28571-1])^0.5
0
100
200
300
400
500
600
700
-5
-10
-15
-20
-25
-30

Section 4.3 Subsonic Relations Between Compressible and Incompressible Dynamic Pressure

For constant density (incompressible) flow Bernoulli's equation reduces to

$$V_T = \sqrt{\frac{2}{\rho_a}\left(P_T - P_a\right)} = \sqrt{\frac{2q}{\rho_a}}$$

Where incompressible dynamic pressure q is defined as P_T-P_a **.**

As airflow speed increases, its density at the stagnation point increases thereby increasing the sensed pressure. The ratio between compressible & incompressible dynamic pressure can be written as a function of Mach number

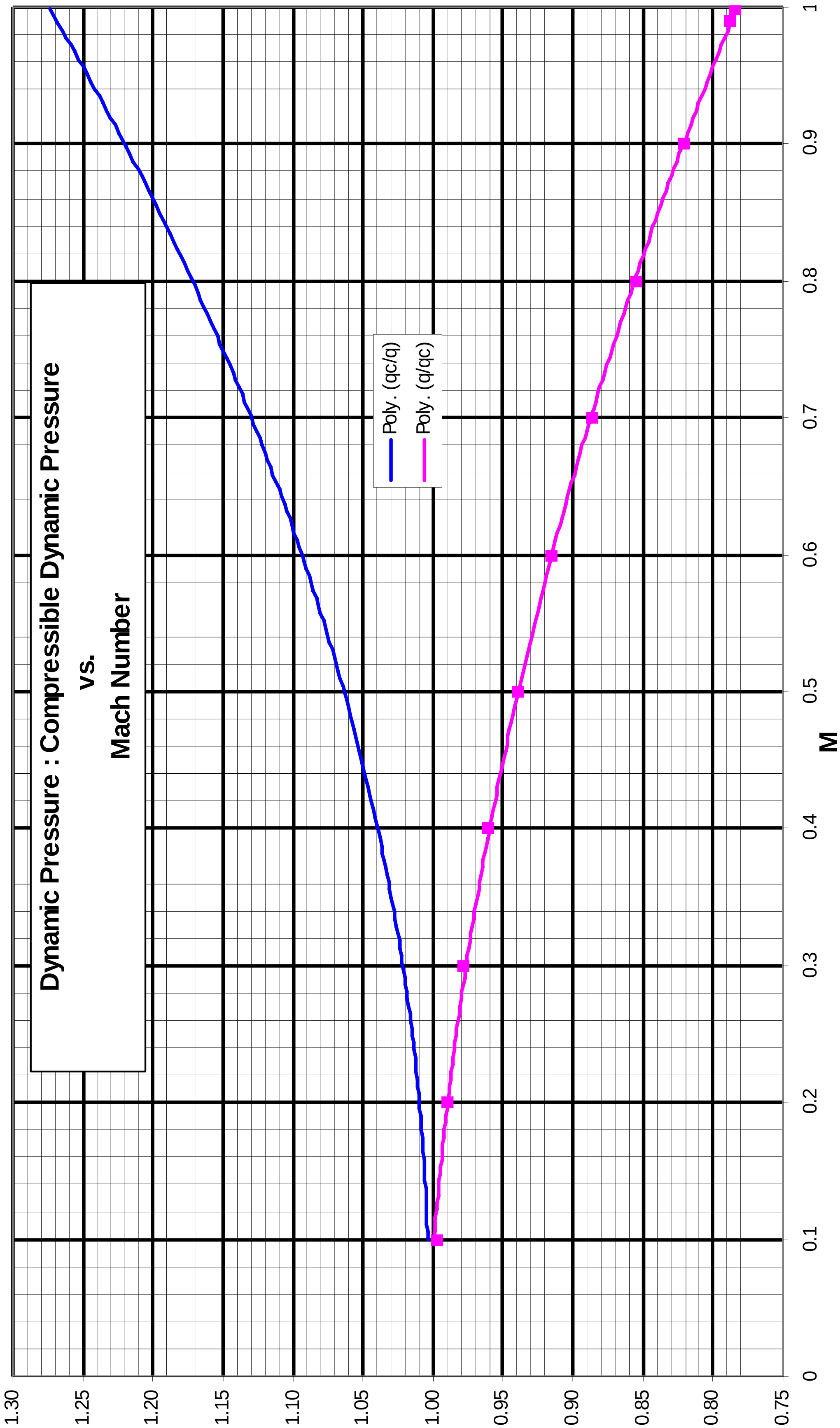
$$q_c = q\left[1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^6}{1600} + ...\right]$$

True dynamic pressure q (as used in modeling) is defined in dimensional analysis as:

$$q = {}^1\!/_2\, \rho_a V_T^{\,2} = {}^1\!/_2\, \rho_o V_e^{\,2}$$

This value for q should not be confused with compressible (a.k.a. impact or differential) pressure, q_c $(= P_T\text{-}P_a)$

$$q = \frac{1}{2}\rho_o 7\frac{P_a}{P_o}\left(\left[\frac{q_c}{P_a} + 1\right]^{2/7} - 1\right)$$


Dynamic Pressure : Compressible Dynamic Pressure
vs.
Mach Number
Poly. (qc/q)
Poly. (q/qc)
M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75

Section 4.4 Supersonic Airspeed and Mach Equations

P_T' denotes pitot pressure behind shock wave

True Airspeed

$$\frac{P'_T - P_a}{P_a} = \frac{q_c}{P_a} = \left[\frac{\gamma+1}{2}\left(\frac{V}{a}\right)^2\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1}{\frac{2\gamma}{\gamma+1}\left(\frac{V}{a}\right)^2 - \frac{\gamma-1}{\gamma+1}}\right]^{\frac{1}{\gamma-1}} - 1$$

Equivalent Airspeed

$$\frac{q_c}{P_a} = \frac{166.92\left[\frac{V_e}{a_o\sqrt{\delta}}\right]^7}{(7\left[\frac{V_e}{a_o\sqrt{\delta}}\right]^2 - 1)^{2.5}} - 1$$

Calibrated Airspeed

$$\frac{q_c}{P_o} = \frac{166.92\left[\frac{V_c}{a_o}\right]^7}{(7\left[\frac{V_c}{a_o}\right]^2 - 1)^{2.5}} - 1$$

Mach Number

$$\frac{q_c}{P_a} = \frac{166.92[M]^7}{(7[M]^2 - 1)^{2.5}} - 1$$

Section 4.5 Total Temperature Equation

Since stagnation exists at the probe, it absorbs the energy of the air

Apply Bernoulli: $\frac{V^2}{2} + \frac{\gamma}{\gamma - 1} \cdot \frac{P_s}{\rho_s} = \frac{\gamma}{\gamma - 1} \cdot \frac{P_p}{\rho_p}$

also $P/_\rho = RT$ and $a^2 = \gamma RT$

$$\therefore \frac{T_T}{T_a} = 1 + \left(\frac{\gamma - 1}{2} \right) M^2$$

Use K (probe recovery factor) to account for heat losses:

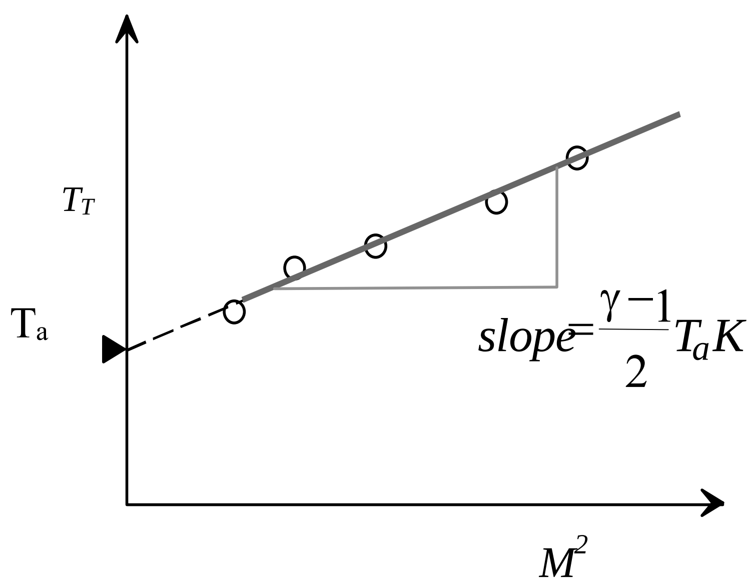
$$\frac{T_T}{T_a} = \left[1 + \frac{K(\gamma - 1)}{2} M^2 \right]$$

During position error flight testing, mea T_{i*} e

From V_c and H_{pc} determine M

$$T_i + \Delta T_{ic} = T_T = T_a + T_a KM^2$$

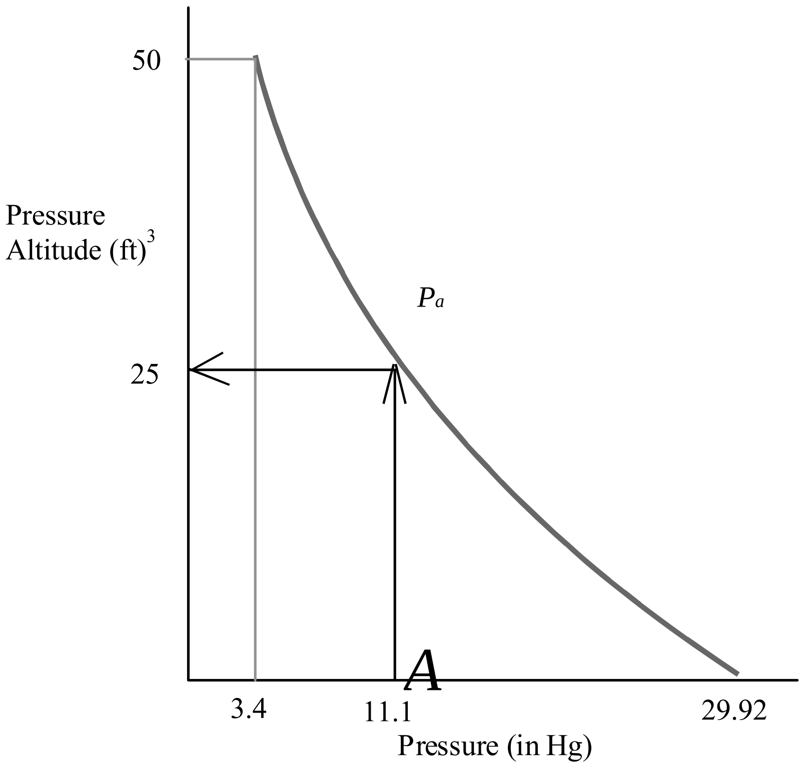
$$\text{plot } T_i \sim M^2$$



Section 4.6 Altimeter Equation

$$P_a = P_o (1\text{-}6.87535 \times 10^{-6}\ H)^{5.256} \quad \text{below 36,089 ft}$$

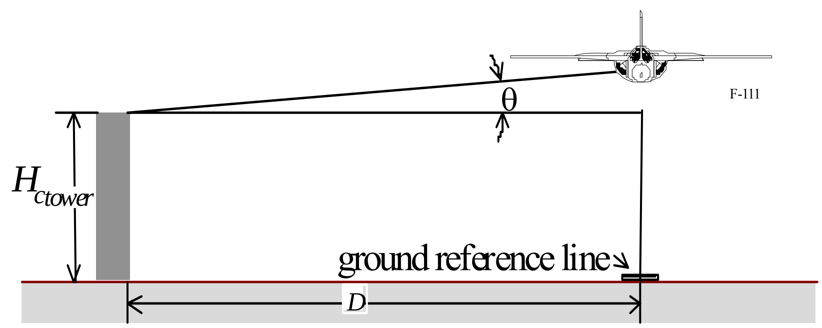
$$P_a = P_o\,(.22335)\ e^{-.00004806[H-36,089]} \quad \text{above 36,089 ft}$$



H_i	Indicated pressure altitude (29.92" Hg)
$+ \Delta H_{ic}$	Instrument error correction
$= H_{ic}$	Altimeter corrected of instrument error
$+ \Delta H_{pc}$	Position error correction
$= H_c$	Calibrated pressure altitude

Section 4.7 Position Error Test Methods

4.7.1 Tower Fly-by



Assumptions

1. No errors in total head.
2. Constant height runs
3. Surveyed course

$$\text{Actual}H_c = H_{c_{tower}} + \left(D \tan\theta \cdot \frac{T_s}{T_t} \right)$$

$$\Delta H_{pc} = \text{Actual}H_c - \left(H_i + \Delta H_{ic} \right)$$

$$\Delta P_S = -\rho g \Delta H_{pc}$$

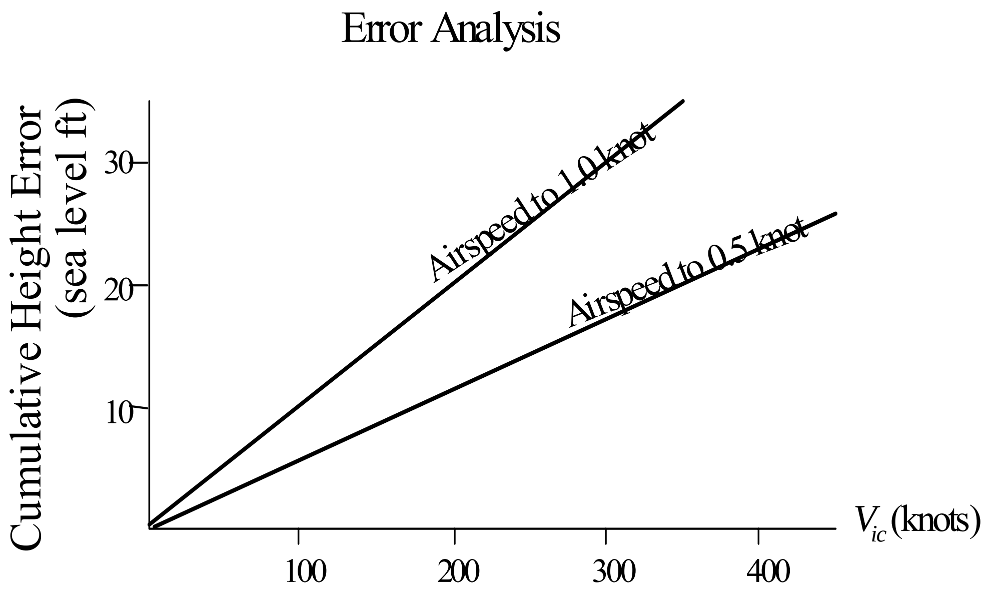
$$\Delta P_S = q_c - q_{cic}$$

$$\Delta P_S = \tfrac{1}{2} \rho_0 V_c^{\,2} - \tfrac{1}{2} \rho_0 V_{ic}^{\,2} \quad \text{(low Mach only)}$$

Solve for V_c

$$\Delta V_{pc} = V_c - V_{ic}$$

See flowchart for high mach or $\Delta P_T \neq 0$ cases.

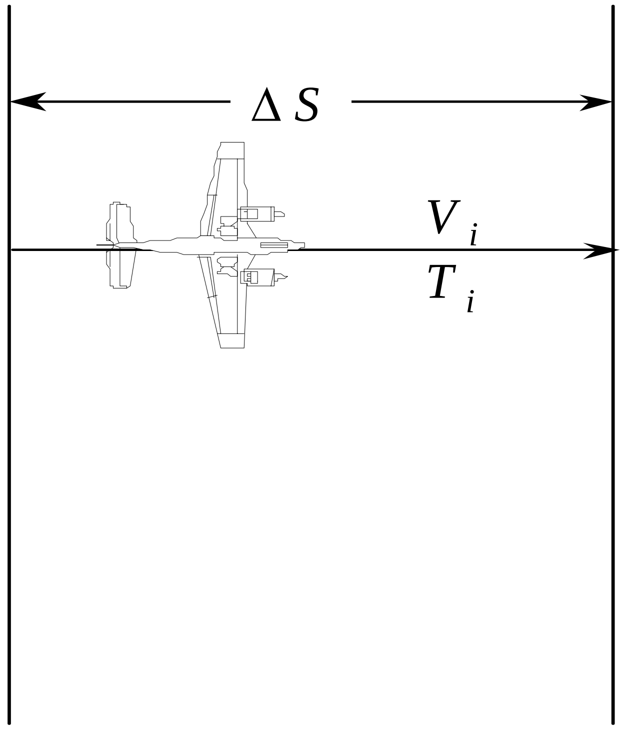


Note: A check on basic instrument calibration is easily accomplished using a "ground block" where a parked test aircraft compares altimeters with tower. Any error can be treated as a bias.

This altitude-based Test method determines altimeter corrections and therefore static error directly. Accurately converting this static source error to an airspeed correction also requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined either through direct comparison with a calibrated noseboom pitot pressure or from one of the air-speed-based methods that directly yield airspeed corrections (pace, ground course, GPS). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated value for airspeed correction.

Section 4.7.2 Ground Course Method

$$V_i$$
$$+\ \Delta V_{ic}$$
$$=\ V_{ic}$$
$$+\ \Delta V_{pc}$$
$$=\ V_c$$
$$\div\ \sqrt{\sigma}$$
$$=\ V_T$$
$$=\ \frac{\Delta S}{\Delta T}$$



Fly known course at constant V_i

Elapsed time $= \Delta T, \therefore V_T = \Delta S / \Delta T$

Use H_i and T_i to compute $V_e = V_T(\sqrt{\sigma}) = V_c$ for low altitude.

Correct V_i for instrument error corrections (ΔV_{ic}) using

$$V_{ic} = V_i + \Delta V_{ic}$$

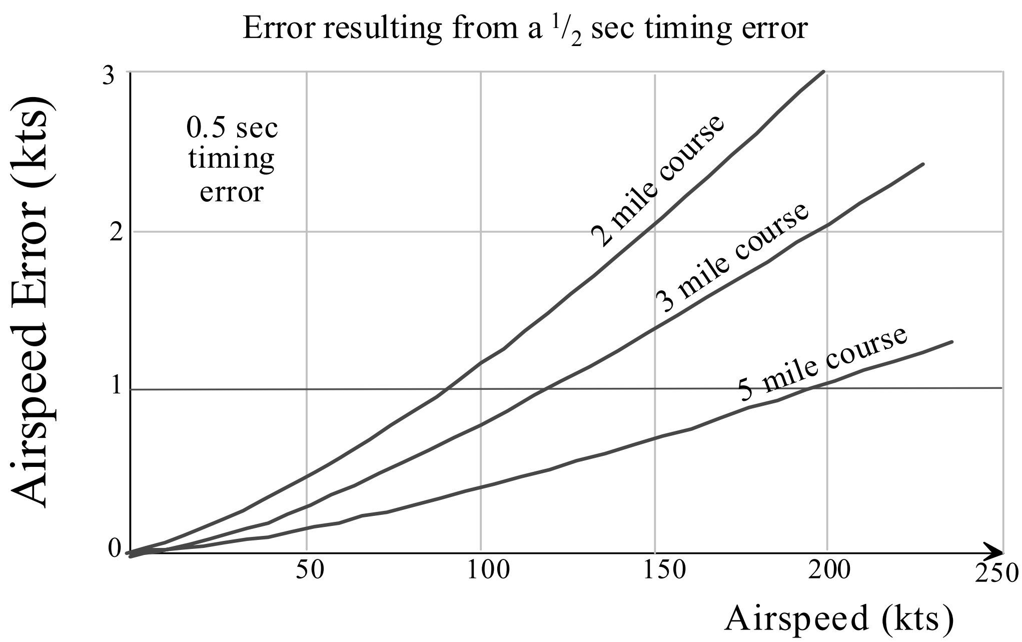
$$\Delta V_{pc} = V_c - V_{ic}$$

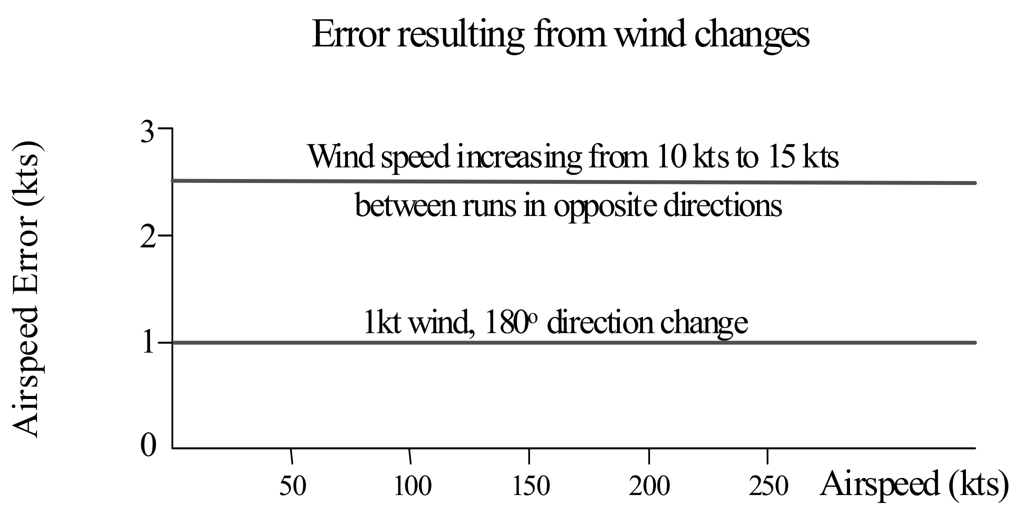
To determine altimeter error assume $\Delta P_T = 0$

$$1/2\, \rho_0 [V_c^2 - V_{ic}^2] = +\Delta P_S$$

$$\Delta P_S = +\rho g \Delta H$$

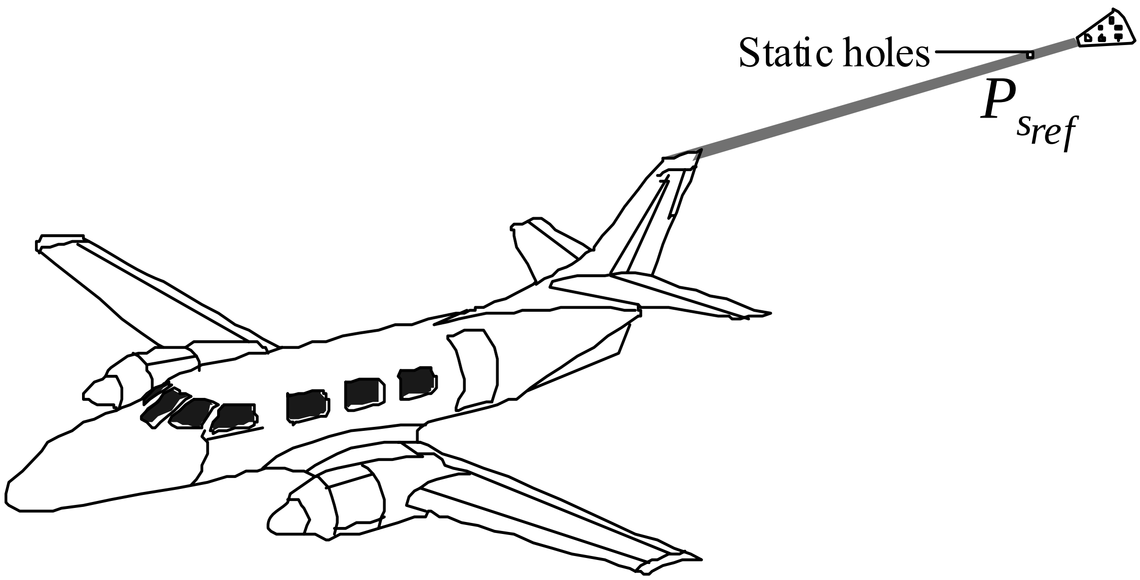
If $\Delta P_T \neq 0$, then $\Delta H_{pc} = +\dfrac{\Delta P_S - \Delta P_T}{\rho g}$





This airspeed-based Test method determines airspeed corrections directly. Accurately converting this airspeed error to a static source error requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined using one of the altitude-based methods that directly yield altitude corrections (tower fly-by, trailing cone or bomb). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated values for static pressure error and altimeter. correction

Section 4.7.3 Trailing Cone/Bomb Method



1. Measure P_S (ref) away from pressure field of aircraft
2. Cone is used to stabilize static line
3. No speed limitations
4. Inexpensive—can be trailed on landing
5. Consider lag effects during rapid altitude changes

$$\Delta P_S = \Delta P_{S_{A/C}} - \Delta P_{S_{REF}}$$

$$\Delta H_{pc} = +\frac{\Delta P_S}{\rho g} = \text{altimeter correction}$$

$$\Delta P_T - \Delta P_S = \Delta P_D = q_{ic} - q_c \text{ assuming } \Delta P_T = 0, M < .2$$

If pitot errors do exist, then they must be included in calculations for ΔV_{pc} (see flowchart)

Using a trailing cone during stall testing may give airspeed errors due to lag errors during the deceleration.

This altitude-based Test method determines altimeter corrections and therefore static error directly. Accurately converting this static source error to an airspeed correction also requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined either through direct comparison with a calibrated noseboom pitot pressure or from one of the air-speed-based methods that directly yield airspeed corrections (pace, ground course, GPS). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated value for airspeed correction.

4.7.4 GPS Methods

- The attraction
  - ~ no aircraft modification required
    - ›› no trailing cone or aircraft plumbing mod
    - ›› no flight test boom
  - ~ no limitation on speed or altitude
    - ›› can be done down to near stall,
    - ›› any altitude
  - ~ easy data reduction
    - ›› no correlation with pace aircraft, ground radar, or other references required

Various methods available, all assume steady winds and ambient temperature. You must determine wind speed and direction to calculate V_T and T_0 and to ensure steady winds existed during test series.

GPS accuracies are variable. Know tolerances before accepting GPS as a truth model.

If exact (± 10º) winds are calculated inflight, you can fly one pass directly into/away from the wind
$V_T = V_G + V_{Headwind}$

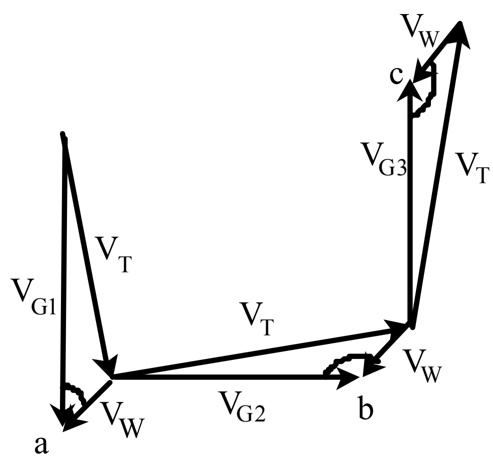
This airspeed-based Test method determines airspeed corrections directly. Accurately converting this airspeed error to a static source error requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined using one of the altitude-based methods that directly yield altitude corrections (tower fly-by, trailing cone or bomb). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated values for static pressure error and altimeter. correction

Graphs in Section 4.11 separately show the effect of measurement errors in ΔP_T, air temperature, or true air-speed on ΔV_C calculations. The last chart (Page 04-29) shows how each knot of accumulated ΔV_C uncertainty affects the ΔH_C uncertainty at various altitudes and temperatures.

Flying four legs instead of three allows four separate calculations of wind speed & direction to confirm stable winds at that test airspeed. If several real-time calculations of winds confirm constant direction and velocity, then testing may be shortened by flying only one pass directly into or away from the wind. If this is done, an end-of-test wind calibration must be performed to confirm steady winds throughout the test series. To minimize temperature and wind variations, testing should be accomplished within a relatively small area.

Horseshoe Track GPS Method

- Horseshoe track method
  - ~ fly three legs with each perpendicular ground tracks, noting GPS ground speed on each
  - ~ determine true airspeed by solving three equations in three unknowns
- Practical problem
  - ~ need to fly close to the ground, tracking perpendicular ground references



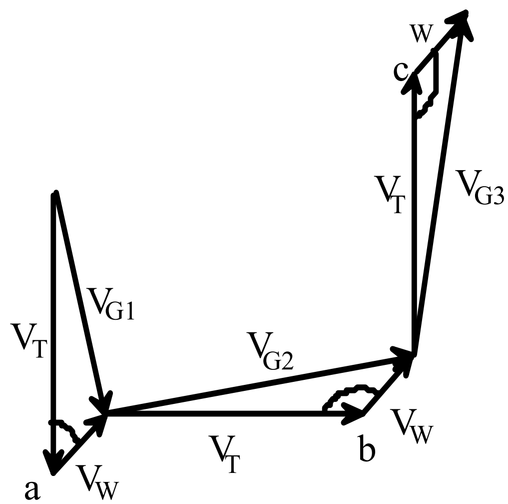
$$\text{True airspeed: } V_T = \frac{1}{2}\sqrt{\left(V_1^2 + V_2^2 + V_3^2 + V_1^2 \times \left(\frac{V_3^2}{V_2^2}\right)\right)}$$

$$\text{Wind velocity} V_W = \sqrt{\left(\frac{V_1 - V_3}{2}\right)^2 + \left(\frac{V_2 - V_1 \times V_3 / V_2}{2}\right)^2}$$

$$\text{Wind direction} \psi_W = \tan^{-1}\frac{(V_2 - V_1 \times V_3 / V_2)}{(V_1 - V_3)}$$

Horseshoe Heading GPS Method

- Horseshoe Heading Method
  - ~ Fly three legs with perpendicular headings, noting GPS ground speed on each
  - ~ Determine true airspeed by solving three equations in three unknowns



Angle a = ψ
Angle b = 90 + ψ
Angle c = 180 - ψ

$$\text{Wind direction } \psi = \tan^{-1}\left[\frac{-V_{G1}^2 + 2V_{G2}^2 - V_{G3}^2}{V_{G3}^2 - V_{G1}^2}\right]$$

$$\text{Wind velocity } V_W = \tfrac{1}{2}\left[V_{G3}^2 + V_{G1}^2 \pm \sqrt{\left(V_{G3}^2 + V_{G1}^2\right)^2 +/- \left(\frac{-V_{G1}^2 + 2V_{G2}^2 - V_{G3}^2}{\sin\psi}\right)^2}\,\right]^{\frac{1}{2}}$$

$$\text{True airspeed } V_T = \sqrt{\frac{V_{G3}^2 + V_{G1}^2}{2} - V_W^2}$$

Cloverleaf Method

(Microsoft Excel spreadsheet adapted from Doug Gray, NSW Australia)

Fly three legs with approximately 90-120 degree difference between headings.
~ Can be accomplished in a broad turn as with the horseshoe method, or
~ Directly over a single point (cloverleaf maneuver).

Accurate results require
~ Identical values for indicated airspeed (and TAS) for all legs.
~ Constant winds throughout data collection (single W/S vector in figure).
~ Approx. 10 seconds stable ground speed, Vg, (G/S in figure) during each leg.

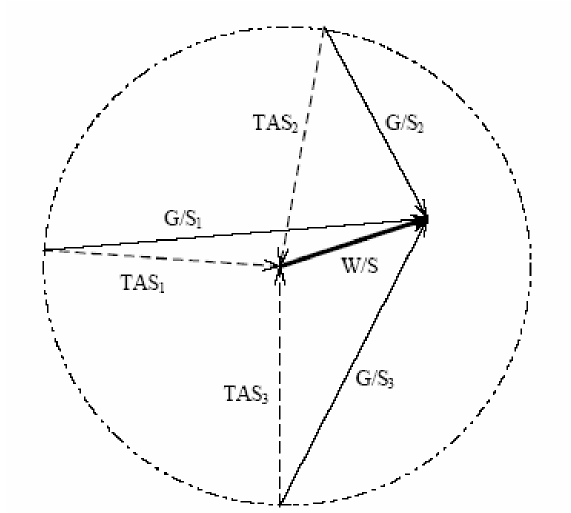
Aircraft heading results for each leg entail an airborne compass swing.

Inputs for each 3-leg data set

Vg_1	Vg_2	Vg_3
Trk_1	Trk_2	Trk_3

Intermediate calculations

$X_1 = Vg_1$*SIN(PI()*(360-Trk_1)/180)
$Y_1 = Vg_1$*COS(PI()*(360-Trk_1)/180)
$X_2 = Vg_2$*SIN(PI()*(360-Trk_2)/180)
$Y_2 = Vg_2$*COS(PI()*(360-Trk_2)/180)
$X_3 = Vg_3$*SIN(PI()*(360-Trk_3)/180)
$Y_3 = Vg_3$*COS(PI()*(360-Trk_3)/180)
$M_1 = -(X_2 - X_1)/(Y_2 - Y_1)$
$M_2 = -(X_3 - X_1)/(Y_3 - Y_1)$
$B_1 = (Y_1 + Y_2)/2 - M_1 * (X_1 + X_2)/2$
$B_2 = (Y_1 + Y_3)/2 - M_2 * (X_1 + X_3)/2$
$V_{Wx} = (B_1 - B_2)/(M_2 - M_1)$
$V_{Wy} = M_1 * V_{Wx} + B_1$

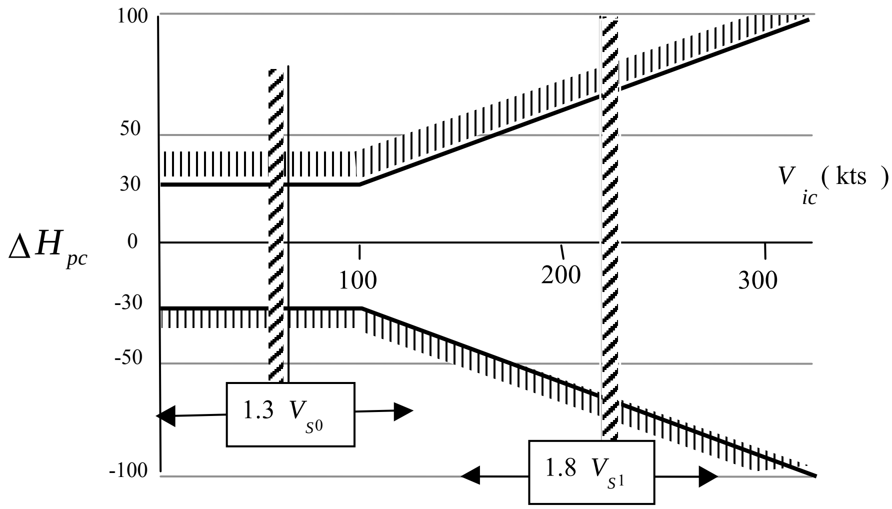


Vector Triangles for three G/S tracks.

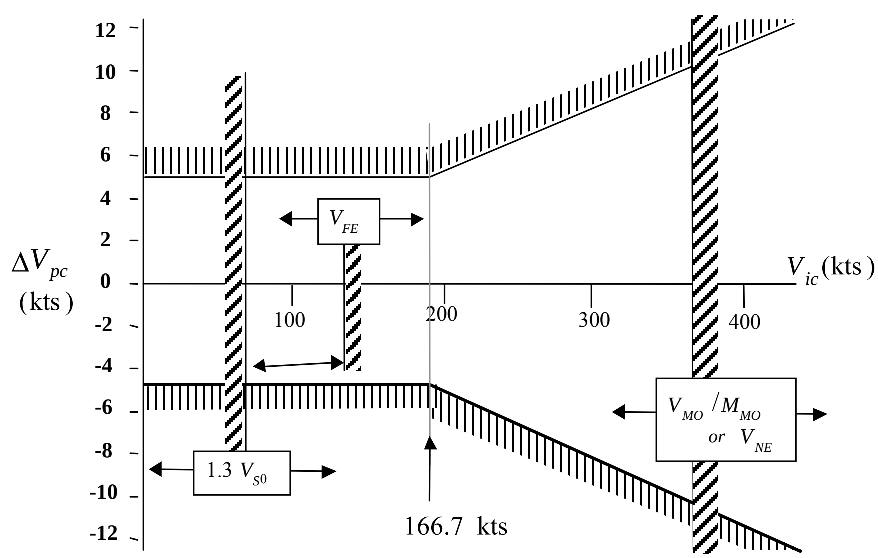
Results

Aircraft true airspeed = $V_T = [(X_1 - V_{Wx})^2 + (Y_1 - V_{Wy})^2]^{0.5}$
Total wind speed = $V_w = [(V_{Wx}{}^2 + V_{Wy}{}^2)]^{0.5}$
Wind direction = $\psi_w = Psi_w$ = MOD(540-(180/PI()*ATAN2(V_{Wy}, V_{Wx})),360)
1st leg a/c heading = $\psi_1 = Psi_1$ = MOD(540-(180/PI()*ATAN2($V_{Wy} - Y_1$, $V_{Wx} - X_1$)),360)
2nd leg a/c heading = $\psi_2 = Psi_2$ = MOD(540-(180/PI()*ATAN2($V_{Wy} - Y_2$, $V_{Wx} - X_2$)),360)
3rd leg a/c heading = $\psi_3 = Psi_3$ = MOD(540-(180/PI()*ATAN2($V_{Wy} - Y_3$, $V_{Wx} - X_3$)),360)

Section 4.8 Position Error FAR 23/25.1323 and .1325/JAR Certification Requirements



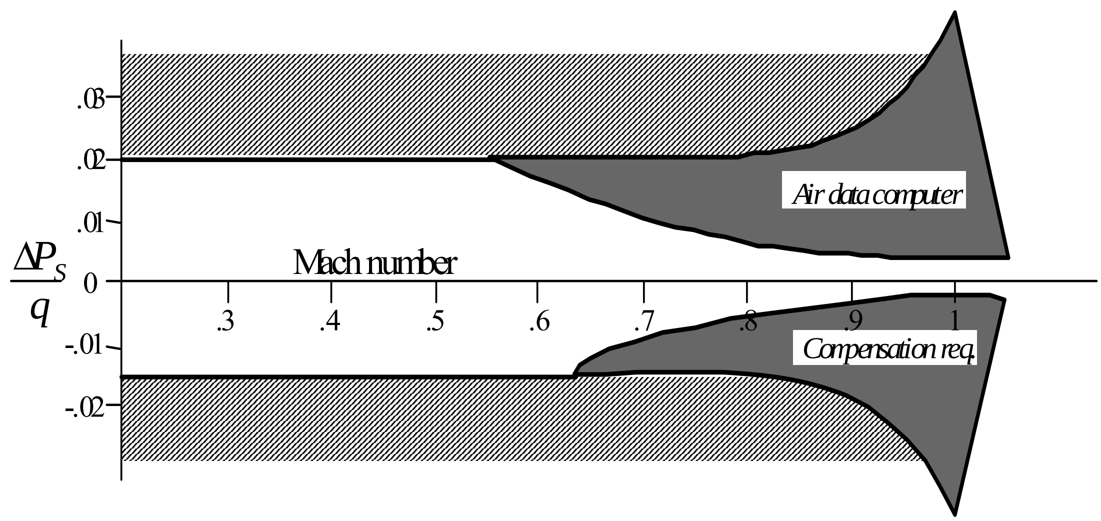
Maximum error at sea level must be less than ± 30 ft/100 kts between 1.3 V_{SO} and 1.8 V_{SI}
V_{SO} = Full flap, gear down, power off, stall speed
V_{SI} = *Stall speed in a specific configuration*



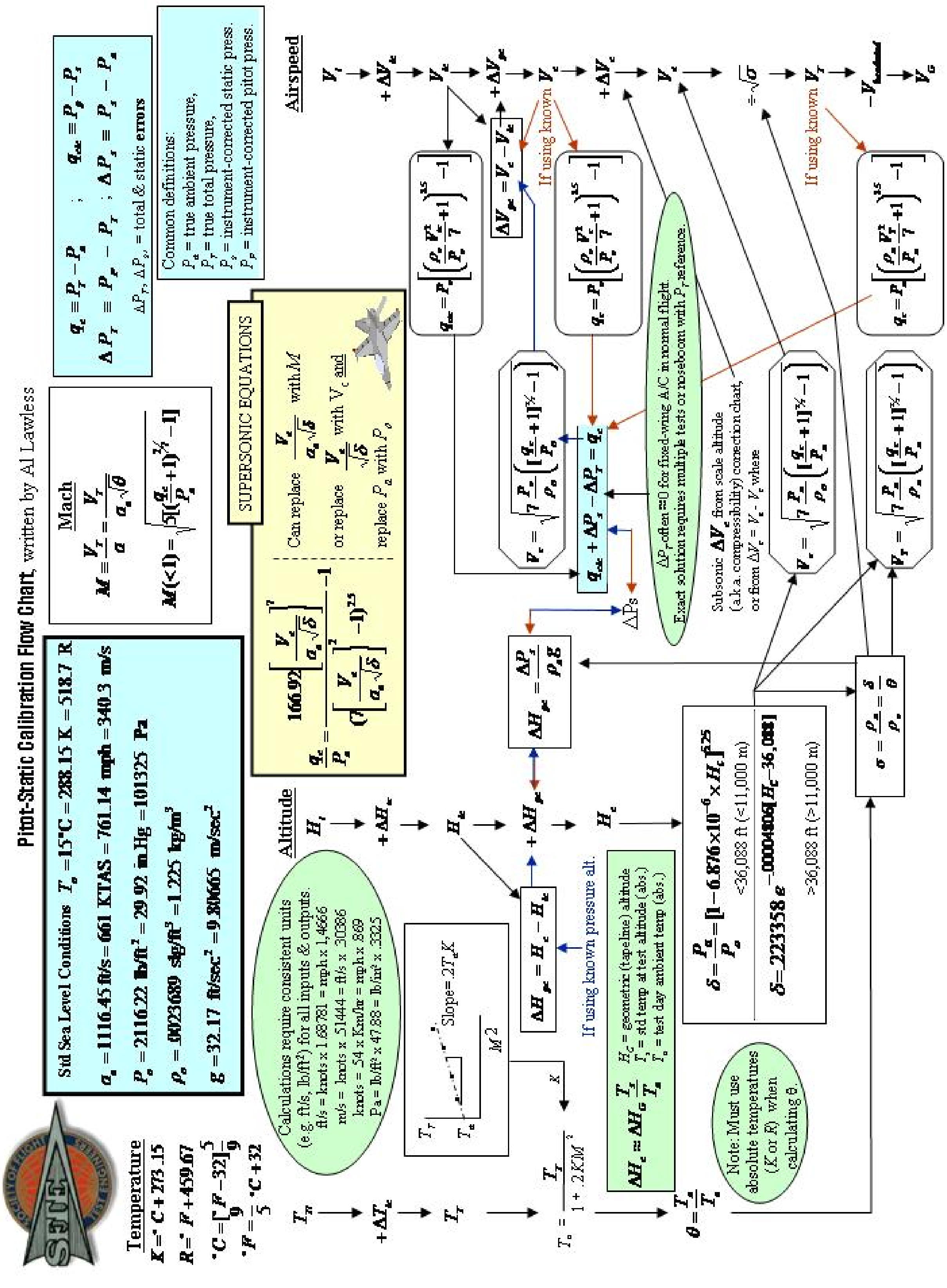
Errors must be equal to or less than $\pm$3%
of V_c or $\pm$5kts whichever is greater

Mil Spec P-26292 C (USAF)

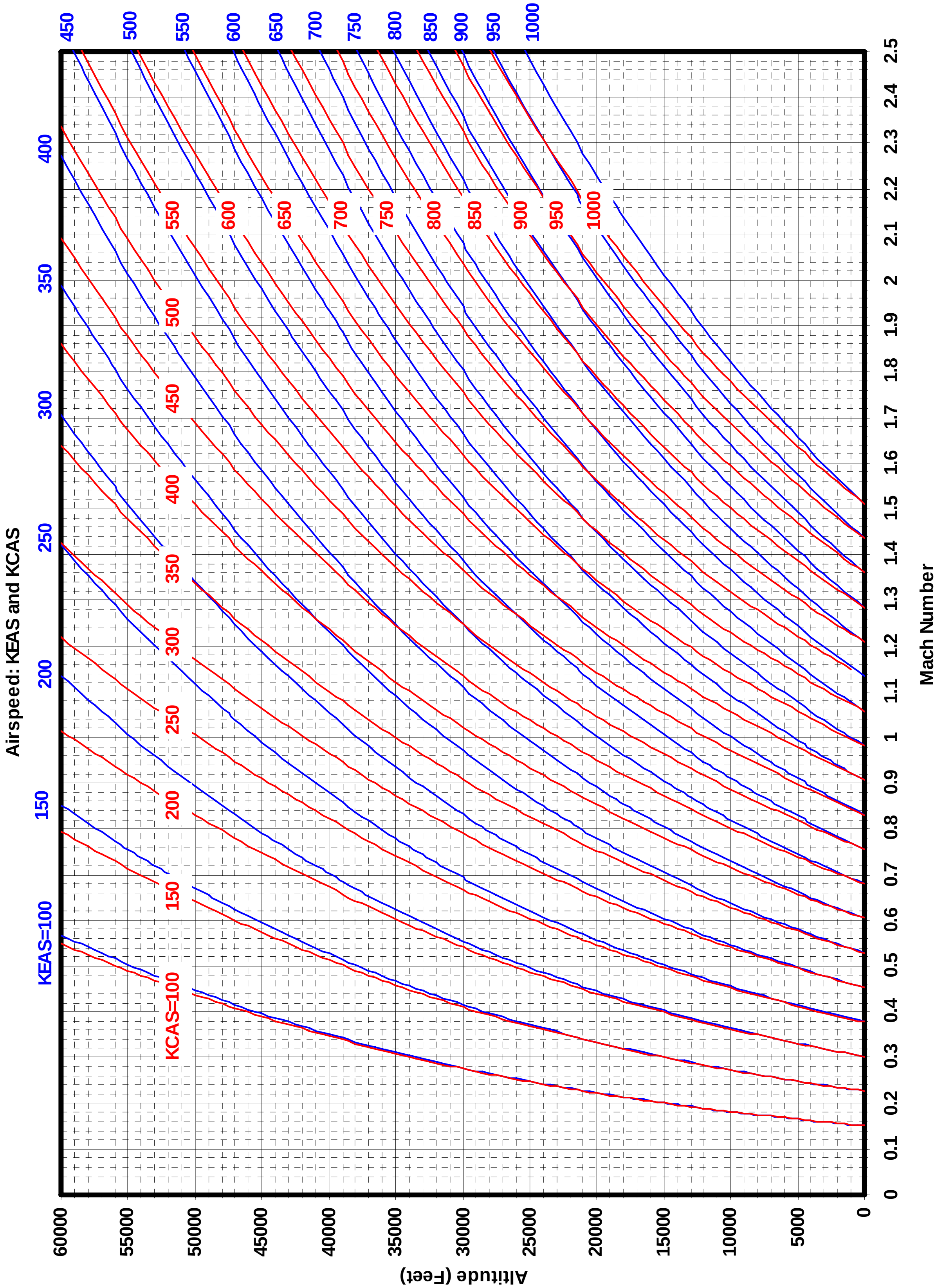
Landing configuration: $\Delta H_{pc} \pm 30$ ft.

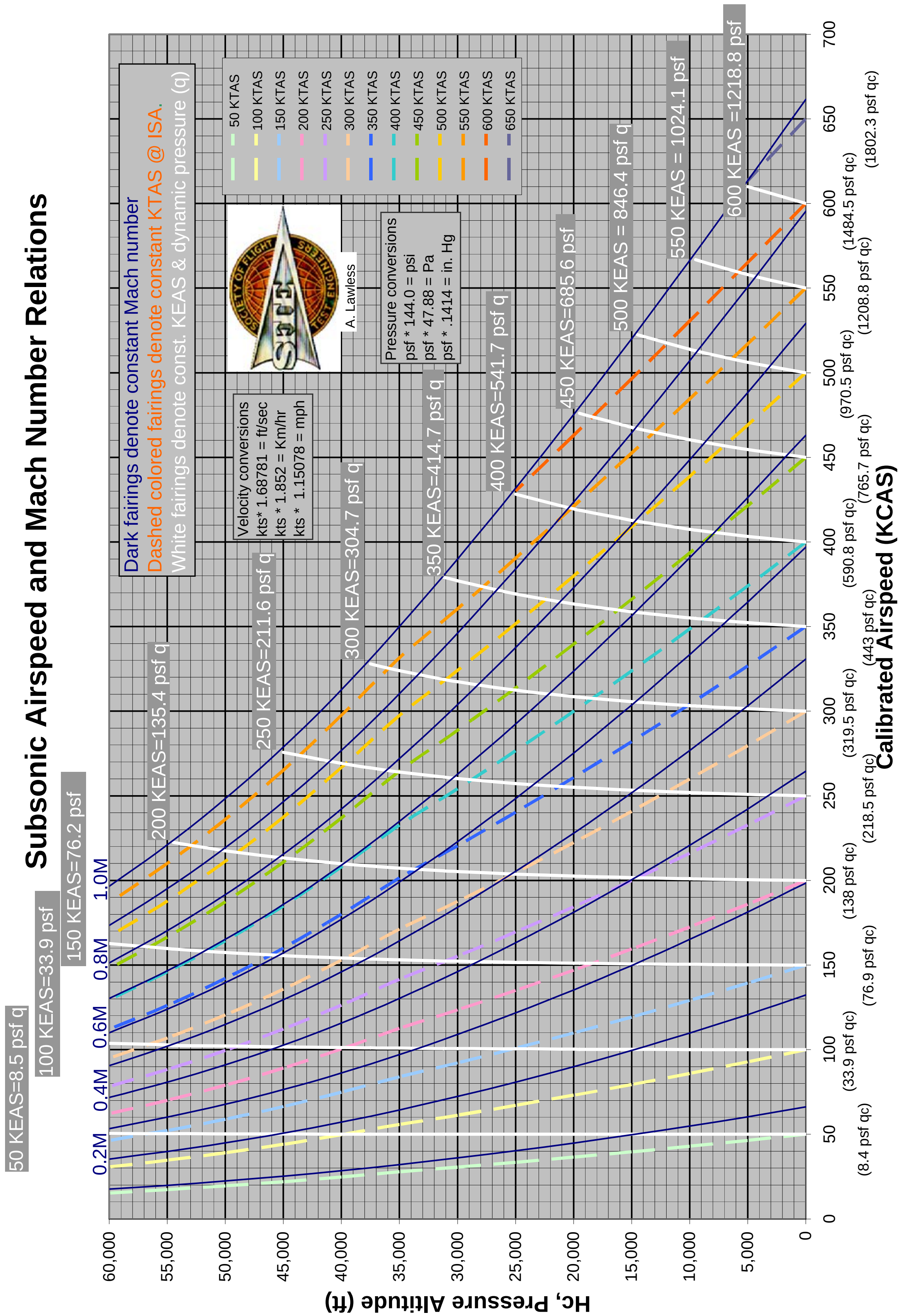


4.9 PEC Correction Process Flow Chart

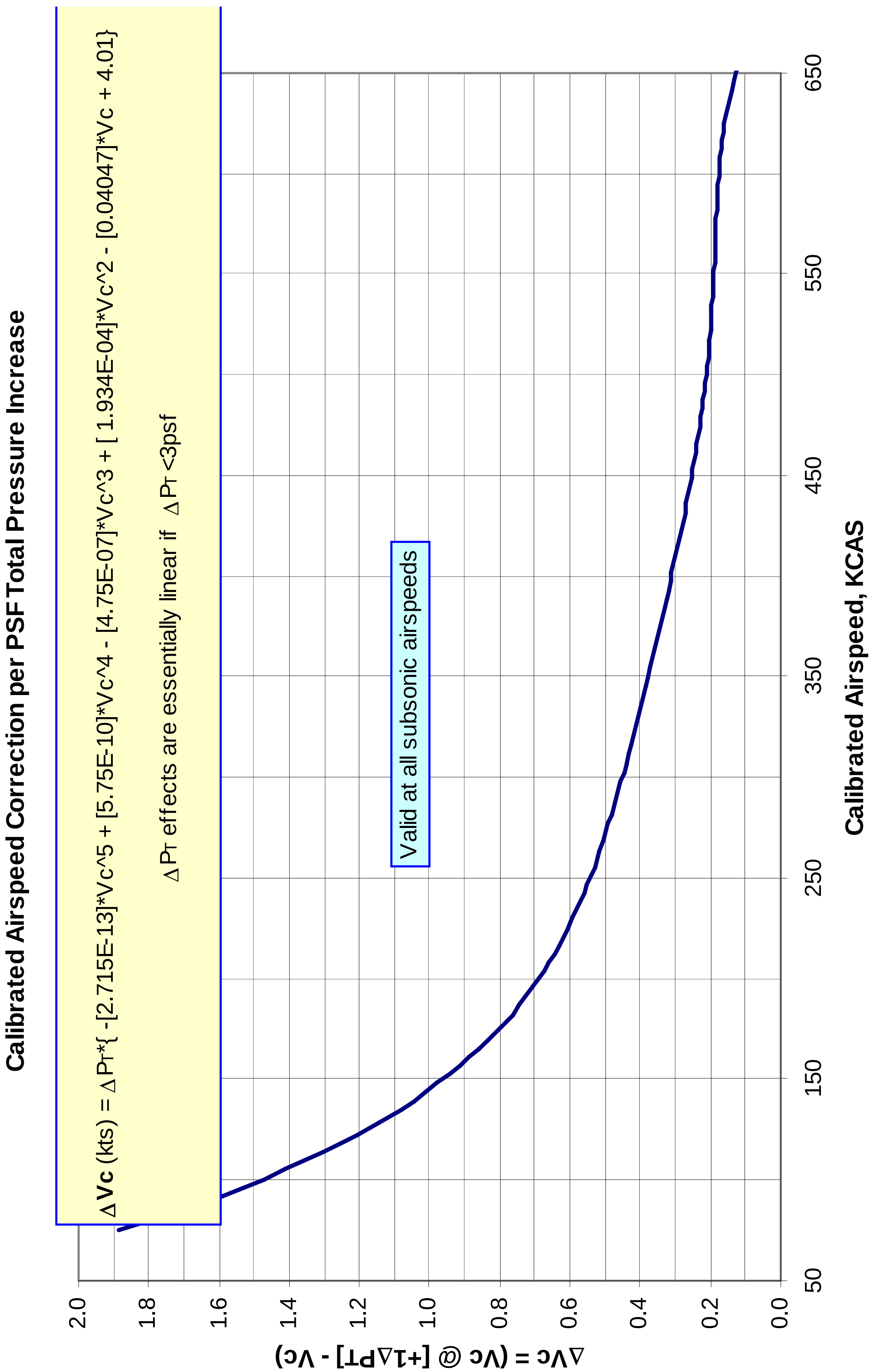


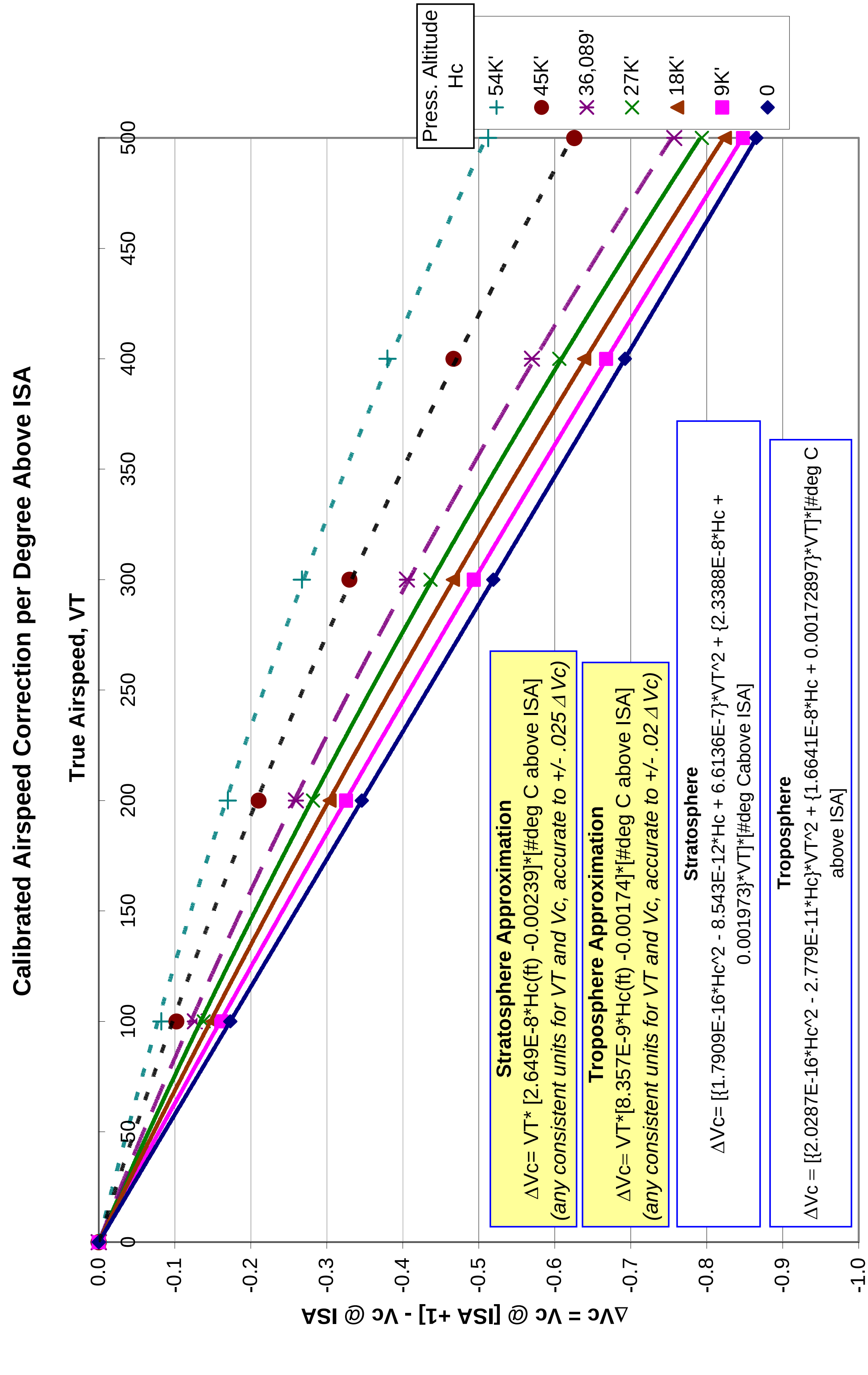
4.10 Airspeed/Altitude/Mach Graphic Relation

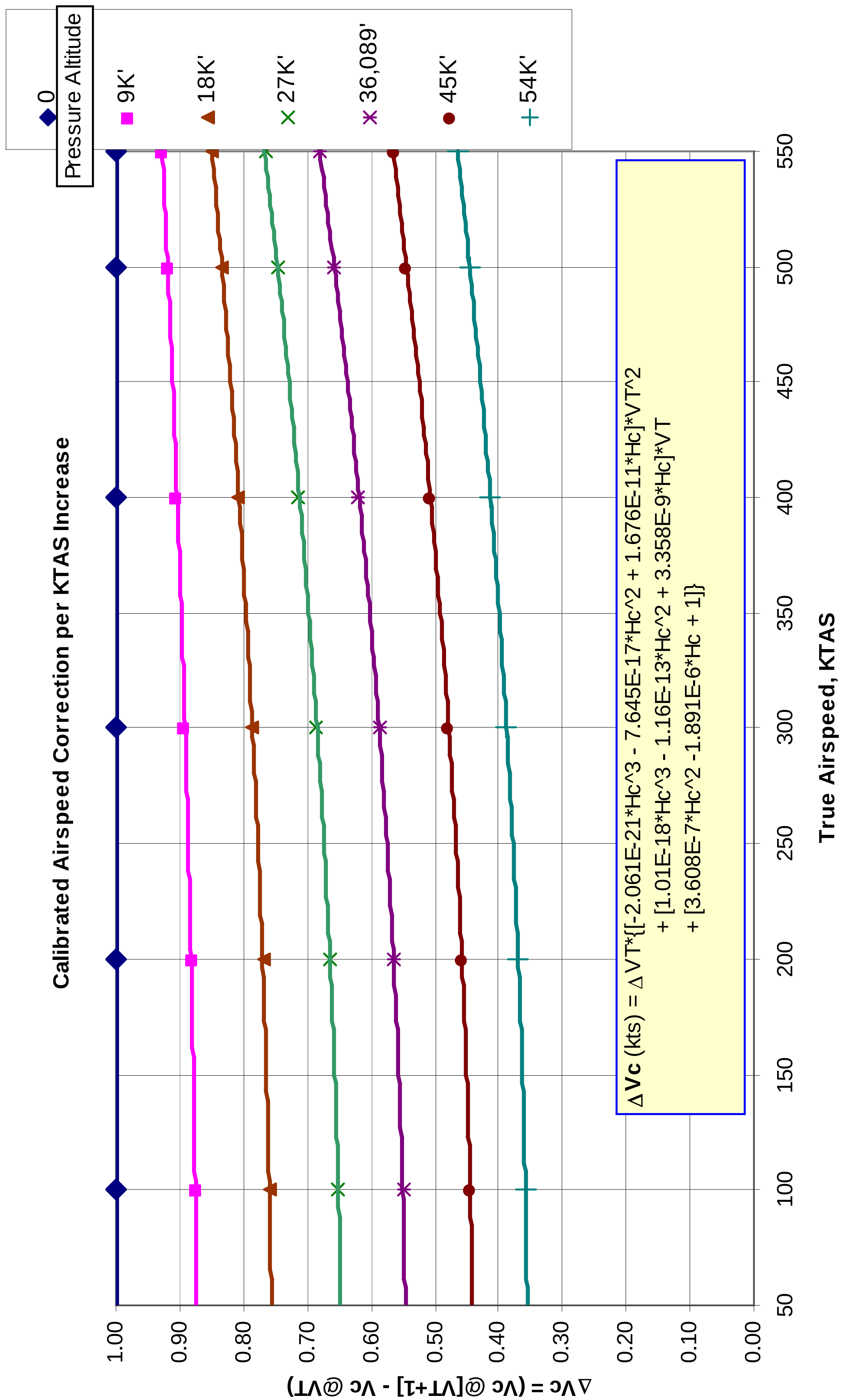



Subsonic Airspeed and Mach Number Relations
Dark fairings denote constant Mach number
Dashed colored fairings denote constant KTAS @ ISA.
White fairings denote const. KEAS & dynamic pressure (q)
50 KTAS
100 KTAS
150 KTAS
200 KTAS
250 KTAS
300 KTAS
350 KTAS
400 KTAS
450 KTAS
500 KTAS
550 KTAS
600 KTAS
650 KTAS
A. Lawless
Velocity conversions
kts* 1.68781 = ft/sec
kts * 1.852 = Km/hr
kts * 1.15078 = mph
Pressure conversions
psf * 144.0 = psi
psf * 47.88 = Pa
psf * .1414 = in. Hg
50 KEAS=8.5 psf q
100 KEAS=33.9 psf
150 KEAS=76.2 psf
200 KEAS=135.4 psf q
250 KEAS=211.6 psf q
300 KEAS=304.7 psf q
350 KEAS=414.7 psf q
400 KEAS=541.7 psf q
450 KEAS=685.6 psf
500 KEAS = 846.4 psf q
550 KEAS = 1024.1 psf
600 KEAS =1218.8 psf
0.2M
0.4M
0.6M
0.8M
1.0M
Hc, Pressure Altitude (ft)
Calibrated Airspeed (KCAS)
(8.4 psf qc)
(33.9 psf qc)
(76.9 psf qc)
(138 psf qc)
(218.5 psf qc)
(319.5 psf qc)
(443 psf qc)
(590.8 psf qc)
(765.7 psf qc)
(970.5 psf qc)
(1208.8 psf qc)
(1484.5 psf qc)
(1802.3 psf qc)

4.11 Effect of Errors on Calibrated Airspeed and Altitude




Calibrated Airspeed Correction per Degree Above ISA
True Airspeed, VT
ΔVc = Vc @ [ISA +1] - Vc @ ISA
0 50 100 150 200 250 300 350 400 450 500
0.0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1.0
Press. Altitude Hc
54K'
45K'
36,089'
27K'
18K'
9K'
0
Stratosphere Approximation
ΔVc= VT* [2.649E-8*Hc(ft) -0.00239]*[#deg C above ISA]
(any consistent units for VT and Vc, accurate to +/- .025 ΔVc)
Troposphere Approximation
ΔVc= VT*[8.357E-9*Hc(ft) -0.00174]*[#deg C above ISA]
(any consistent units for VT and Vc, accurate to +/- .02 ΔVc)
Stratosphere
ΔVc= [{1.7909E-16*Hc^2 - 8.543E-12*Hc + 6.6136E-7}*VT^2 + {2.3388E-8*Hc + 0.001973}*VT]*[#deg Cabove ISA]
Troposphere
ΔVc = [{2.0287E-16*Hc^2 - 2.779E-11*Hc}*VT^2 + {1.6641E-8*Hc + 0.00172897}*VT]*[#deg C above ISA]


Calibrated Airspeed Correction per KTAS Increase
Pressure Altitude
0
9K'
18K'
27K'
36,089'
45K'
54K'
Δ Vc (kts) = Δ VT*{[-2.061E-21*Hc^3 - 7.645E-17*Hc^2 + 1.676E-11*Hc]*VT^2
+ [1.01E-18*Hc^3 - 1.16E-13*Hc^2 + 3.358E-9*Hc]*VT
+ [3.608E-7*Hc^2 -1.891E-6*Hc + 1]}
True Airspeed, KTAS
ΔVc = (Vc @[VT+1] - Vc @VT)
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
50
100
150
200
250
300
350
400
450
500
550

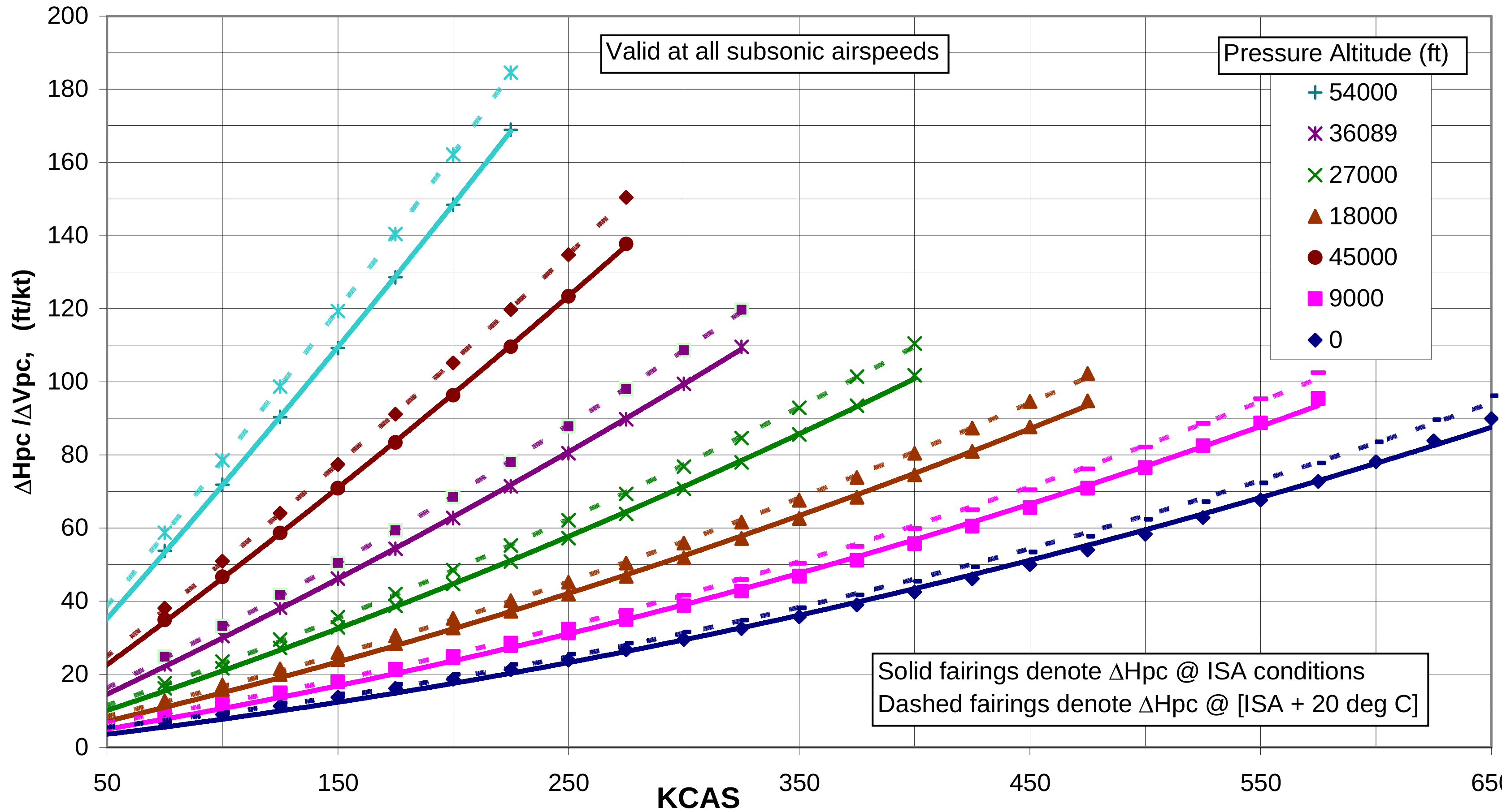
Altimeter Correction per Knot Airpseed Correction

ISA ΔHpc = ΔVpc{[(-1.184E-18]*Hc^3 + (4.004E-14)*Hc^2 + (-1.533E-09)*Hc - 0.000105]*Vc^2
+ [(-5.034E-15)*Hc^3 + (1.502E-10)*Hc^2 + (-5.019E-06)*Hc - 0.06421]*Vc }

Non-ISA:

Troposphere ΔHpc =[ISA ΔHpc]*[1+{3.1059E-08*Hc + 0.00344}*(deg C above ISA)]

Stratosphere ΔHpc =[ISA ΔHpc]*[1+0.004616*(deg C above ISA)]



Notes

Section 5 Aerodynamics

5.0 Recurring Terminology

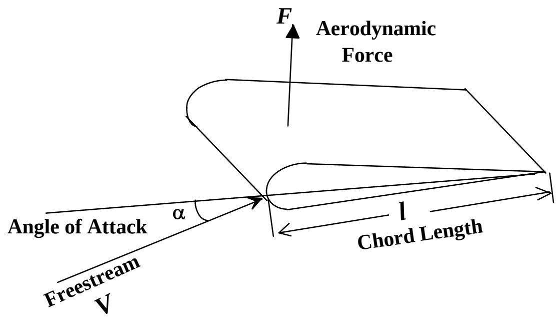
a slope of lift curve, $dC_L/d\alpha$
$a.c.$ aerodynamic center, location along the chord where pitching moments about this center do not change with angle of attack (25% MAC for airfoils in subsonic flow, 50% MAC for airfoils in supersonic flow)
AOA angle of attack
AR aspect ratio = [wing span]^2/ [reference wing area] = b^2/S
B wing span
b_t horizontal tail span
C coefficient, a non-dimensional representation of an aerodynamic property
c wing chord length Camber maximum curvature of an airfoil, measured at maximum distance between chord line and amber line, expressed in % of MAC.Camber line theoretical line extending from an air foil's leading edge to the trailing edge, located halfway between the upper and lower surfaces.
C_D drag coefficient
C_{Di} induced drag coefficient
C_{Do}, C_{Dpe} parasitic drag coefficient
c_f friction coefficient
Chord straight-line distance from an airfoil's leading edge to its trailing edge.
C_L lift coefficient
Cp pressure coefficient = $\Delta p/q$
e Oswald efficiency factor
l distance traveled by flow, or characteristic length of surface
M Mach number
MAC mean aerodynamic chord, chord length of location on wing where total aerodynamic forces can be concentrated.
MGC mean geometric chord, the average chord length, derived only from a plan form view of a wing (similar to MAC if wing has no twist and constant cross section & thickness-to-chord ratio).
P pressure
$P_{req'd}$ power required
q dynamic pressure = $\frac{1}{2} \rho_a V_T^2 = \frac{1}{2} \rho_o V_T^2$
R gas constant
R_n, R_e Reynolds number
S reference wing area, includes extension of wing to fuselage centerline.
S_t horizontal tail surface area
S_W wetted area of surface
T temperature
V true velocity
V_e equivalent velocity
α angle of attack
α_i induced angle of attack
δ depth of boundary layer, or surface wedge angle
μ viscosity, or wave angle
ν flow turning angle
θ shock wave angle
ρ density

- Perfect Fluid
  - ~ incompressible, inelastic, and non-viscous
  - ~ used in flow outside of boundary layers at M < .7
- Incompressible, inelastic, viscous
  - ~ used for boundary layer studies at M < .7
- Compressible, non-viscous, elastic fluid
  - ~ used outside boundary layers up to M = 5

5.1 Dimensional Analysis Interpretations (ref 5.2)

Aerodynamic force = F

- $F = f\,(\rho, \mu, T, V$, shape, orientation, size, roughness, gravity)
- For aircraft ignore R, K & hypersonic effects



- Initially assume similar body orientations, shapes & roughness.

- Dimensional Analysis reveals four non-dimensional (π) parameters:

Force Coefficient $\pi_1 = \dfrac{F}{\rho V^2 l^2}$

Reynolds Number $\pi_2 = \dfrac{\rho V l}{\mu}$

Mach Number $\pi_3 = \dfrac{V}{a}$

Froude Number $\pi_4 = \dfrac{V}{\sqrt{l\,g}}$

A closer look at the force coefficient:

$$C_F = \frac{F}{\rho V^2 l^2} \Rightarrow \frac{F}{\frac{1}{2}\rho V^2 S}$$

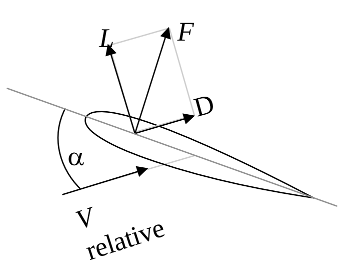
where $^1/_2\, \rho_a V_T{}^2 = {}^1/_2\, \rho_o V_e{}^2$ =dynamic pressure, q
dimensions of reference wing area, S are the same as l^2

A feel for *q*

- Kinetic energy of a moving object = ½ mV_T^2
- Block of moving air kinetic energy = ½ ρ (volume)V_T^2
- Dividing through by volume yields KE per volume of moving air = ½ $\rho\ V_T^2$
- "Dynamic pressure" or "*q*" = potential for converting each cubic foot of the airflow's kinetic energy into frontal stagnation pressure
- Feel *q* by extending your hand out the window of a moving car
- Dividing the above by 2 equates the flow's density & velocity to kinetic energy

A feel for coefficients

- $C_F = (F/S)/q$ = the ratio between the total force pressure and the flow's dynamic pressure
- Lift is the component of the total force perpendicular to the freestream flow
- Drag is the component along the flow
- Break total into lift and drag coefficients:
  $C_L = (L/S)/q \qquad C_D = (D/S)/q$
- Increasing dynamic pressure
  generates a larger total force, lift and drag

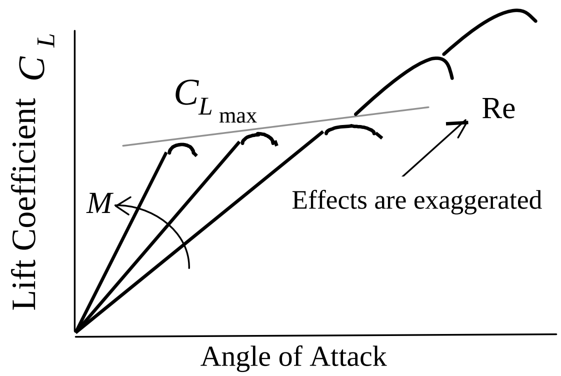


- Froude number is not significant in aerodynamic phenomena
- Recall that forces are aslo a function of angle of attack, shape & surface roughness, therefore

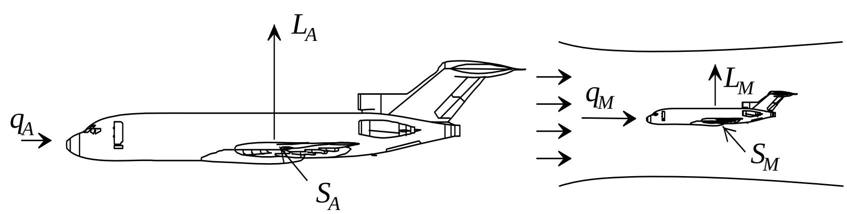
Froude number is not significant in aerodynamic phenomena

Recall that forces are also a function of angle of attack, shape & surface roughness

$C_L, C_D = f\,[M, \mathrm{R}e, \alpha]$ for a given shape, roughness



To compare test day and standard day aircraft or to match wind tunnel C_F data to actual aircraft; the shape, roughness, M, R_n and α must be equal for both aircraft



$$\frac{L_A}{q_A S_A} = C_L = \frac{L_M}{q_M S_M}$$

5.2 General Aerodynamic Relations (refs 5.1, 5.2, 5.10)

Lift & Drag forces can be described using two approaches:

1) Change in momentum of airstream, *F = d{mv]/dt*
2) "Bernoulli" approach which requires the continuity and conservation of energy equations

Continuity Equation

Fluid Mass in = Fluid Mass out

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For subsonic (incompressible) flow $\rho_1 = \rho_2$

$$V_1 A_1 = V_2 A_2$$

Conservation of Energy (Bernoulli) **Equation:**

Potential + Kinetic + Pressure = constant
(changes in Potential energy are negligible)
Energy per unit volume is pressure then
Dynamic Pressure + Static Pressure = Total Pressure

$$\tfrac{1}{2}\rho V^2 + p_s = \text{constant}$$

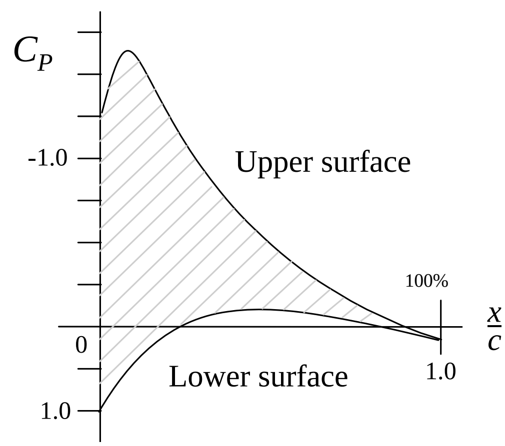
$$\tfrac{1}{2}\rho V^2 + p_s = p_t$$

- This classic approach only applies in the "potential flow" region and not in the boundary layer where energy losses occur

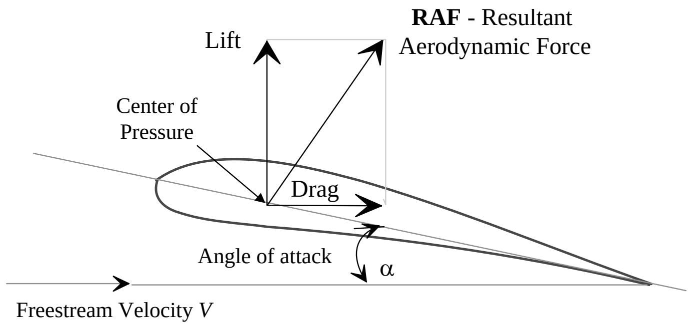
- Pressures around a surface can be calculated or measured from tests and converted into pressure coefficients,

$$c_p = (p_{\text{local}} - p_{\text{ambient}})/\text{dynamic pressure} = \Delta p/q$$

- c_p values can be mapped out for all surfaces



- Summation of all pressures perpendicular to surface yield the pitching moments and the "**Resultant Aerodynamic Force**" which is broken into lift and drag components

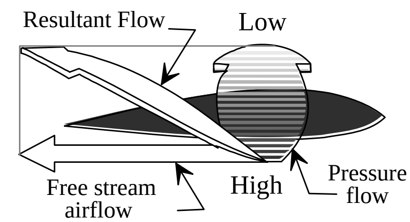


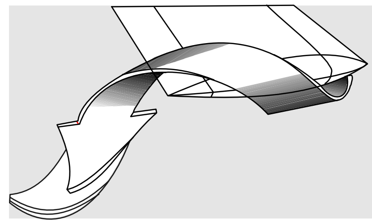
- Lift & drag forces are referred to the aerodynamic center (*ac*) where the pitching moment is constant for reasonable angles of attack.
- Pitching moments increase with airfoil camber, are zero if symmetric.
- Aerodynamic center is located at 25% MAC for fully subsonic flow and at 50% MAC for fully supersonic flow.

5.3 Wing Design Effects on Lift Curve Slope (refs 5.1, 5.2, 5.10)

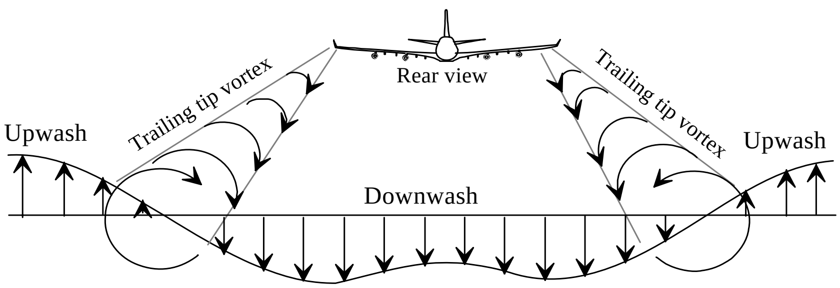
Aspect Ratio Effect

- Pressure differential at wingtip causes tip vortex

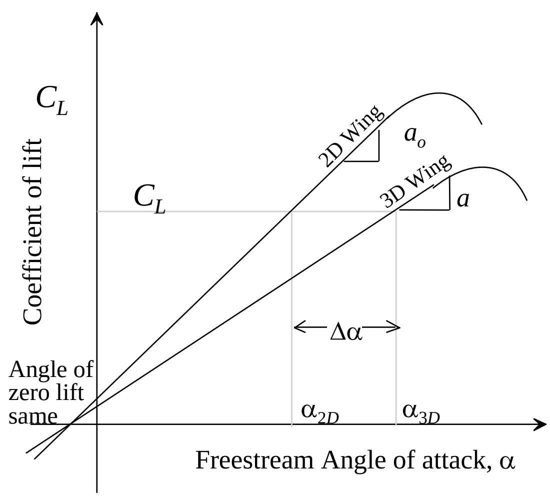




- Vortex creates flow field that reduces AOA across wingspan



- Local AOA reductions decrease average lift curve slope



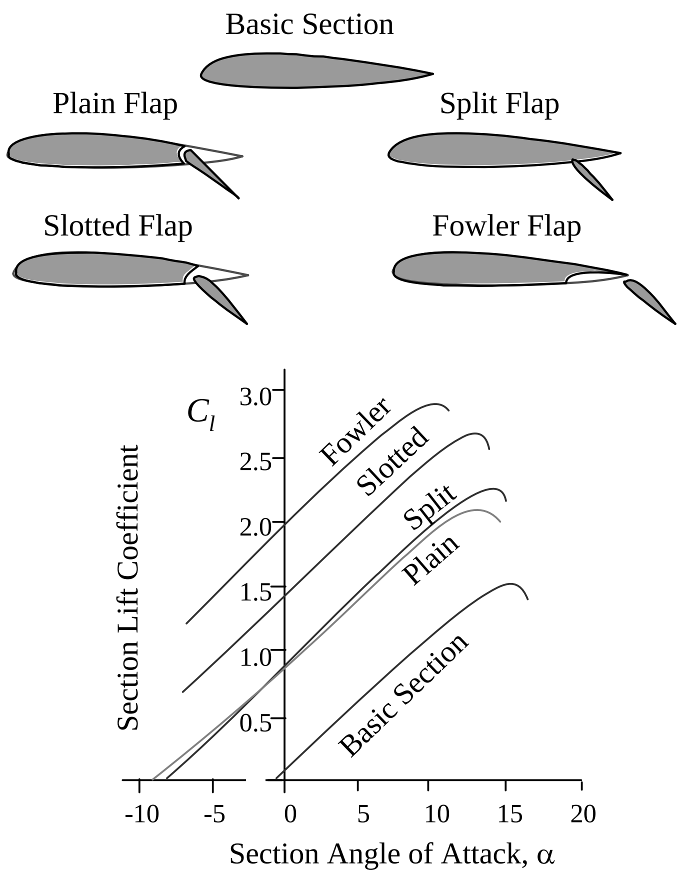
2D wing = wind tunnel airfoil extending to walls (infinite aspect ratio).

a_o = Lift curve slope for an infinite wing

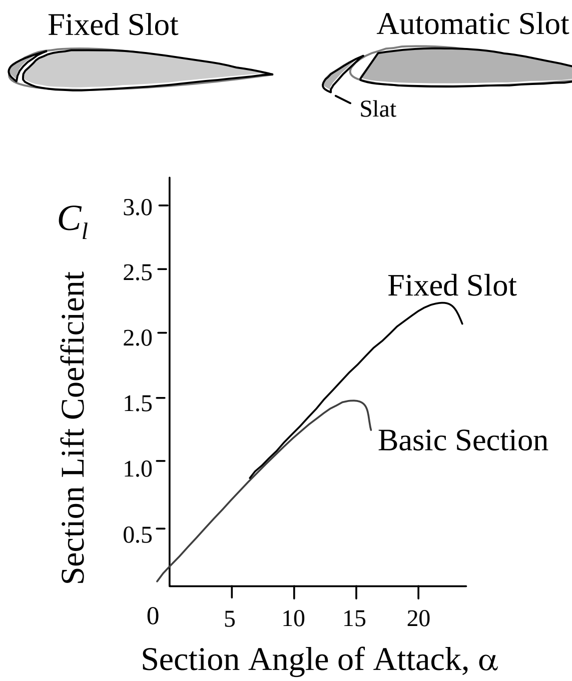
a = Lift curve slope for a finite wing

- Above relationship estimated as $$a = \frac{dC_L}{d\alpha} = \frac{a_o}{1 + \frac{57.3 a_o}{\pi AR}}$$

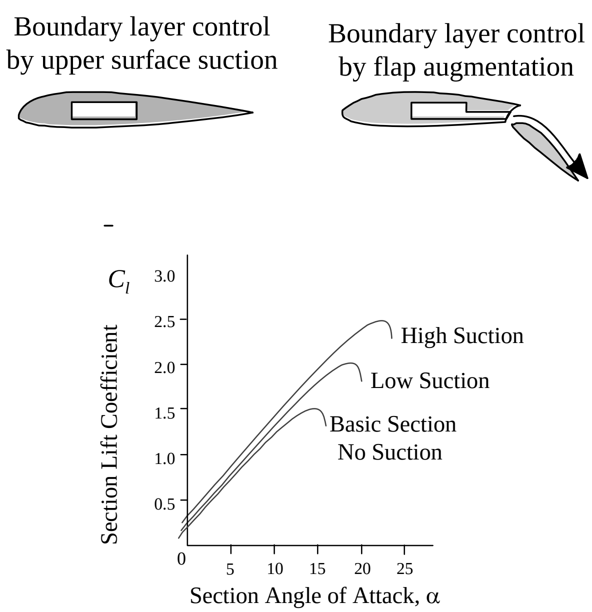
Trailing Edge Flap Effects



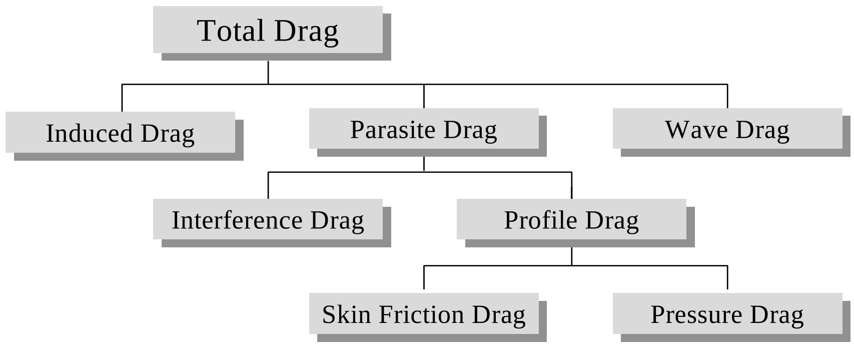
Leading Edge Flap Effects



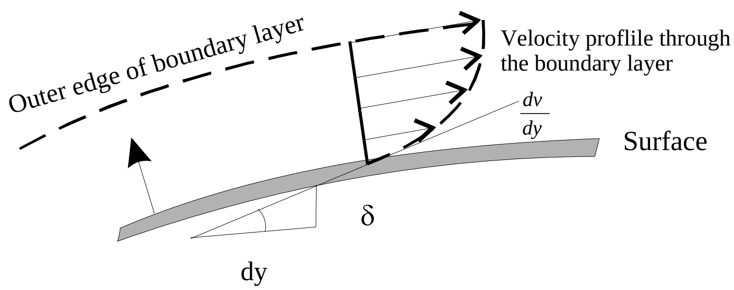
Boundary Layer Control Effects



5.4 Elements of Drag (refs 5.1, 5.2, 5.10)



- Skin friction shear stress is a function of velocity profile at surface



$$\text{Shear stress} \quad \Im_w = \mu \left(\frac{dv}{dy} \right)_{y=0}$$

- **Viscosity** (μ) increases with temperature (ref 5.9)

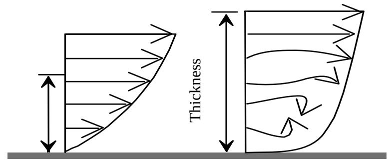
Sutherland law: $$\mu = \mu_o \frac{\left(\frac{T}{T_o}\right)^{1.5} (T_o + S)}{(T+S)}$$ Power law: $$\mu = \mu_o \left(\frac{T}{T_o}\right)^n$$

Where T_o = 273.15 *K* = 518.67 *R*.
For air: *S* =110.4 *K* = 199 *R; n=.67*
For air at 273 K: μ_o= 1.717x10^{-5} [kg/m s] = 3.59x10^{-7} [slug/ft s]
Inserting air values (T_K=Kelvin and T_R=Rankin) into Sutherland law gives

$$\mu = 1.458x10^{-6} \frac{T_K^{1.5}}{T_K + 110.4} \left[\frac{kg}{s \cdot m}\right] = 2.2x10^{-8} \frac{T_R^{1.5}}{T_R + 199} \left[\frac{s\lg}{s \cdot ft}\right]$$

Reynolds Number Effects (ref 5.10)

- Laminar boundary layers have more gradual change in velocity near surface than turbulent boundary layers.
- High Reynolds numbers help propagate turbulent flow.



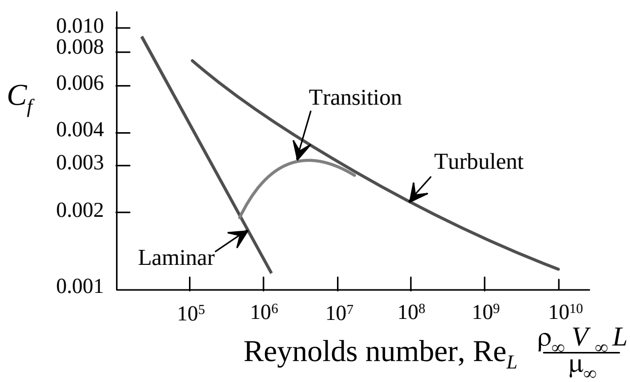
Laminar Turbulent

Shearing stress $\Im_w = \mu \left(\frac{dv}{dy} \right)_{y=0}$

Skin friction coefficient $C_f = \frac{\Im_w}{\frac{1}{2}\rho_\infty V_\infty^{\;2}} = \frac{\Im_w}{q_\infty}$

Laminar boundary layer Total $C_f = \frac{1.328}{(\mathrm{Re}_L)^{1/2}}$

Turbulent boundary layer Total $C_f = \frac{.455}{(\log \mathrm{Re}_L)^{2.58}} \approx \frac{0.074}{(\mathrm{Re}_L)^{1/5}}$



Re$_L$ based on total length of flat plate

- Depth of boundary layer (δ) depends on local Reynolds number (Re_x) and whether the flow is turbulent or laminar.

$$\mathrm{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} \equiv \frac{\text{Inertia Forces}}{\text{Viscous Forces}}$$

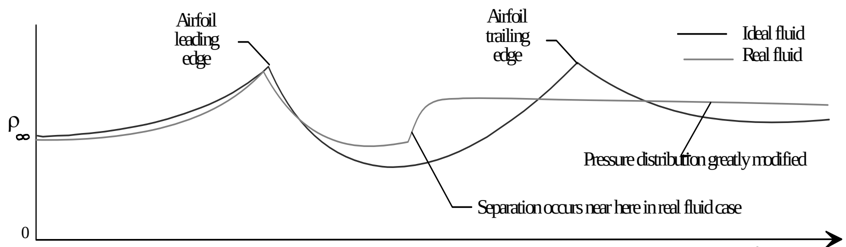
x= distance traveled to point in question

$$\delta_{lam} = \frac{5.2\,x}{\sqrt{\mathrm{Re}_x}}$$

$$\delta_{turb} = \frac{.37\,x}{\mathrm{Re}_x^{\;.2}}$$

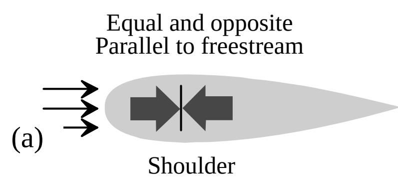
5.4.2 Pressure Drag

- Ideal frictionless flow has no losses and leads to zero pressure drag
- Real fluids have friction and energy losses along surface
- Energy losses negate total pressure recovery, lead to decreasing total pressure along surface

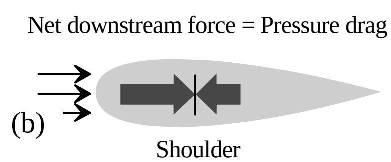


(a) Airfoil upper surface static presure distributions

- Imbalance of pressures on surfaces causes pressure drag

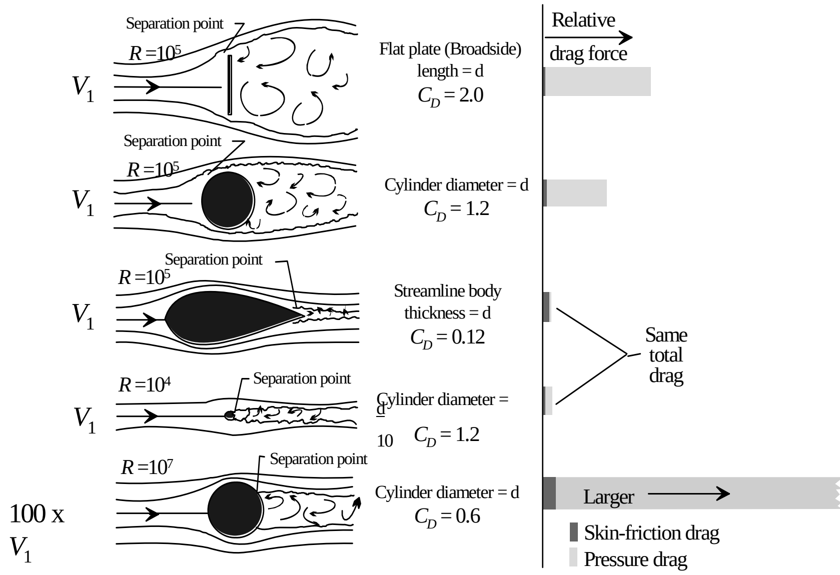


(b) Ideal fluid air foil (no pressure drag)
Sum of horizontal pressures = 0



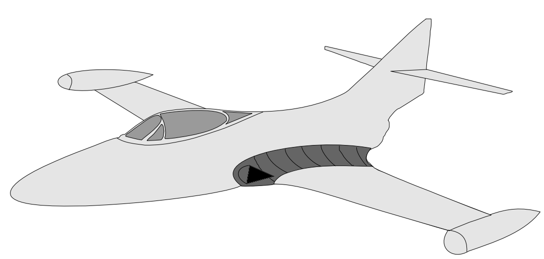
(c) Real fluid airfoil (net pressure drag) more drag pressure than thrust pressure

- Profile streamlining reduces pressure drag



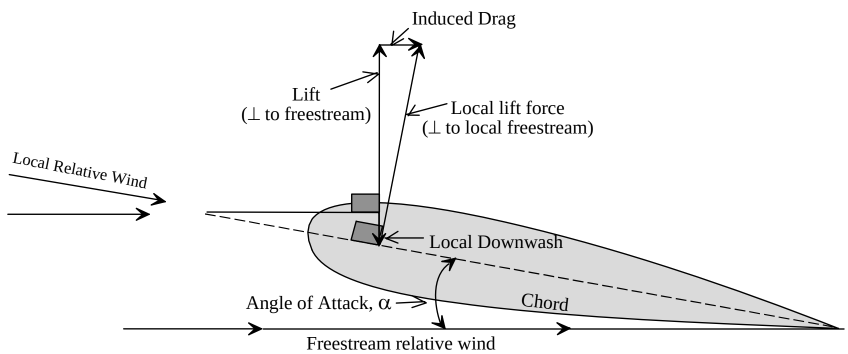
5.4.3 Interference Drag

- Occurs with multiple surfaces approximately parallel to flow
- Caused by flow’s interference with itself or by excessive adverse pressure gradient due to rapidly decreasing vehicle cross section
- Most severe with surfaces at acute angles to each other
- Effects often reduced by fillets around contracting surfaces



5.4.4 Induced Drag

- Wingtip vortex reduces local AOA at each station along wing
- Local lift vector is perpendicular to local AOA
- Local lift vector is therefore tilted back relative to freestream lift
- Induced drag defined as rearward component of local lift vector



$$\text{Induced Drag }(D_i) = \text{Lift}(L)\sin\alpha_i = L\alpha_i$$

$$\text{Induced Drag }(D_i) = L(\alpha_i)$$

$$\text{For elliptical lift distributions} \quad \alpha_i = \frac{C_L}{\pi AR}$$

$$\therefore D_i = L\left(\frac{C_L}{\pi AR}\right) \quad \text{but} \quad L = qSC_L$$

$$C_{D_i} = \frac{D_i}{qS} = \frac{C_L{}^2}{\pi AR} = \text{induced drag coefficient}$$

Oswald efficiency factor, *e*, accounts for losses in excess of those predicted above (due to uneven downwash and changing interference drag effects).

$$\therefore C_{D_i} = \frac{C_L{}^2}{\pi ARe}$$

5.5 Aerodynamic Compressibility Relations (reference 5.8)

Prandtl/Glauert Approximation
Approximates Mach effects on aerodynamics below critical Mach

$$C_{P_{compressible}} = \frac{1}{\sqrt{1-M^2}} C_{P_{incompressible}}$$

Total vs Ambient Property Relations for Adiabatic Flow

$$\frac{T_T}{T} = 1 + \frac{\gamma - 1}{2} M^2$$ Isentropic flow not required

$$\frac{P_T}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$ Isentropic (shockless) flow required

$$\frac{\rho_T}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma - 1}}$$ Isentropic flow required

Normal Shock Relations

Assumes isentropic flow on each side of the shock
Assumes flow across shock is adiabatic
Property changes occur in a constant area (throat)

$$\frac{P_2}{P_1} = \frac{1 - \gamma + 2\gamma M_1^2}{1 + \gamma}$$

$$\frac{\rho_2}{\rho_1} = \left[\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}\right]^{-1}$$

$$\frac{T_2}{T_1} = \left[\frac{1 - \gamma + 2\gamma M_1^2}{1 + \gamma}\right]\left[\frac{2 + (\gamma - 1)M_1^2}{(1 + \gamma)M_1^2}\right]$$

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

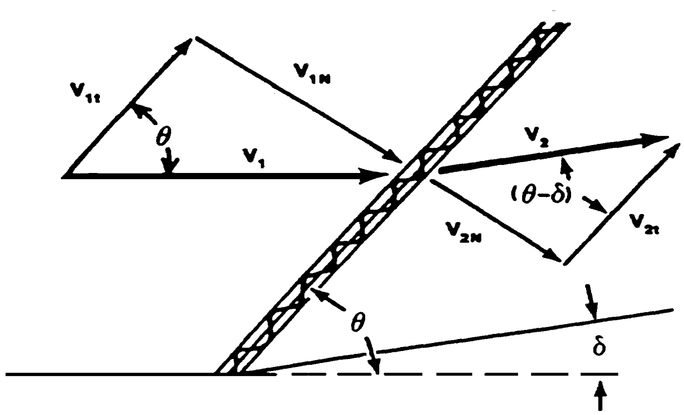
Normal shock summary

$$P_{T_1} > P_{T_2} \quad \rho_{T_1} > \rho_{T_2} \quad T_{T_1} = T_{T_2} \quad M_1 > M_2$$

$$P_1 < P_2 \quad \rho_1 < \rho_2 \quad T_1 < T_2 \quad s_1 < s_2$$

5.5.1 Oblique Shocks

Oblique Shock Description



δ = surface turning angle

θ = shock wave angle

Subscript 1 denotes upstream conditions

Subscript 2 denotes downstream conditions

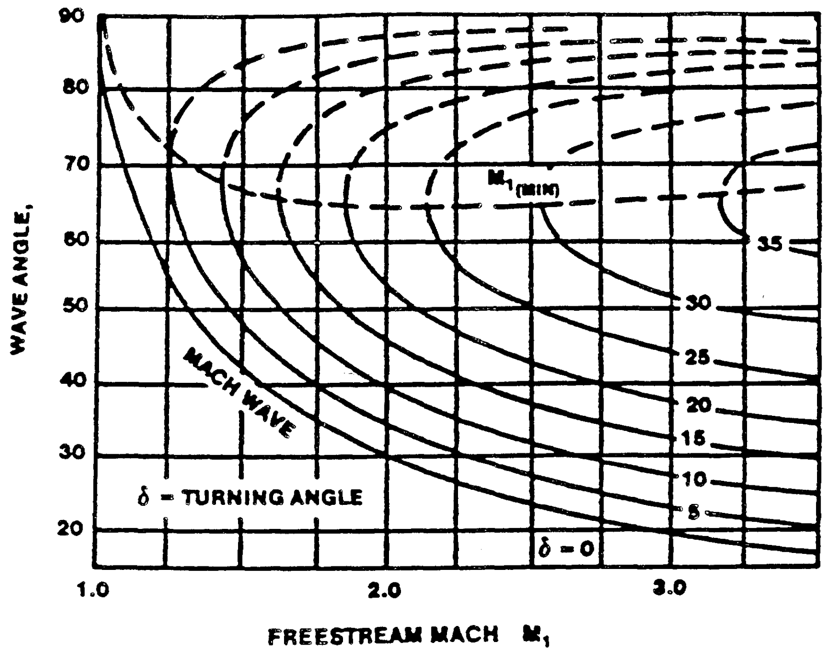
Oblique Shock Relations

- Calculate P_2/P_1, T_2/T_1, and ρ_2/ρ_1 across oblique shocks by using normal shock equations and substituting $M_1 \sin\theta$ in place of M_1
- Calculate total pressure loss across oblique shock as
- Calculate relation between Mach number and angles as

$$\frac{P_{T_2}}{P_{T_2}} = \left\{ \left[\frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1) M_1^2 \sin^2\theta} \right]^{\gamma} \left[\frac{2\gamma}{\gamma + 1} M_1^2 \sin^2\theta - \frac{\gamma - 1}{\gamma + 1} \right] \right\}^{\frac{1}{1-\gamma}}$$

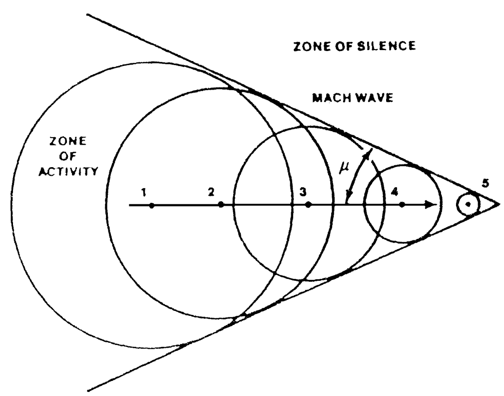
$$M_2^2 \sin^2(\delta - \theta) = \frac{M_1^2 \sin^2\theta + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 \sin^2\theta - 1}$$

Oblique Shock Turning Angle as a Function of Wave Angle



- Two θ solutions exist for every M_1 & δ combination
  - These represent the strong and weak shock solutions
  - Weak shocks normally occur in nature
- There is a minimum Mach number for each turning angle
- The wave angle of a weak shock decreases with increased Mach
- For a given Mach number, θ approaches μ as δ decreases

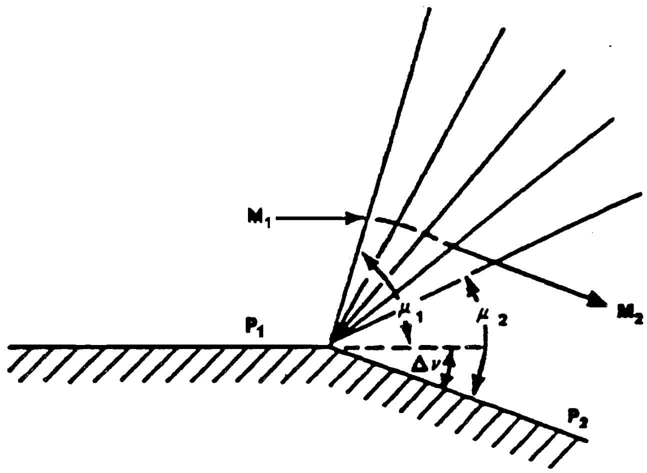
Mach Cone Angle



Minimum Wave Angle

$\mu = \sin^{-1}(1/M)$

5.5.2 Supersonic Isentropic Expansion Relations



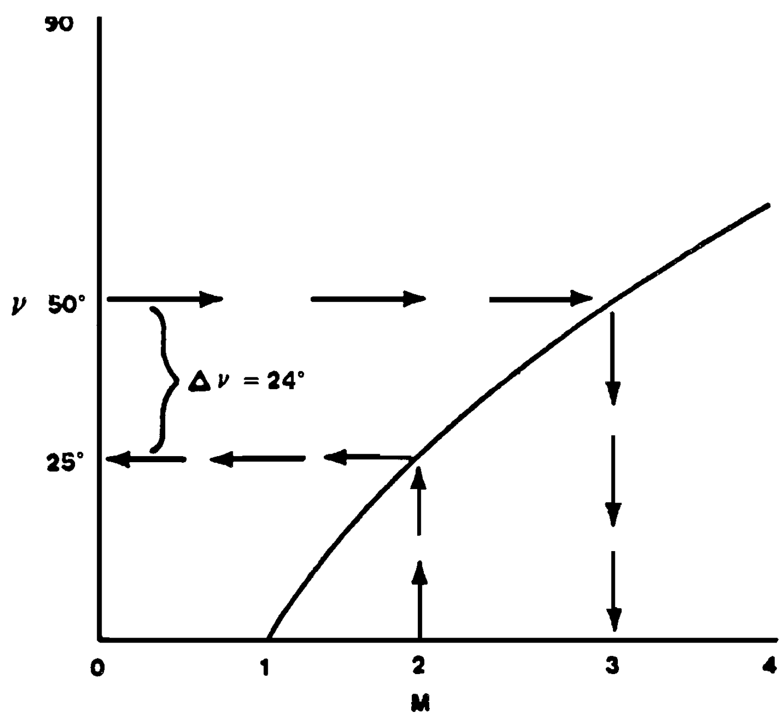
- The wave angle μ determines where the lower pressure can be felt and thus where the flow can be accelerated
- As the flow accelerates, a new wave angle forms and the subsequent lower pressure further accelerates the flow
- Results in a series of Mach waves forming a “fan” until the flow turns and accelerates so that it is parallel to the new boundary

Prandtl-Meyer Function

Shows flow's required turning angle (ν) to accelerate from one Mach number to another

$$\nu_M = \sqrt{\frac{\gamma+1}{\gamma-1}}\left[\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)}\right] - \tan^{-1}\sqrt{M^2-1}$$

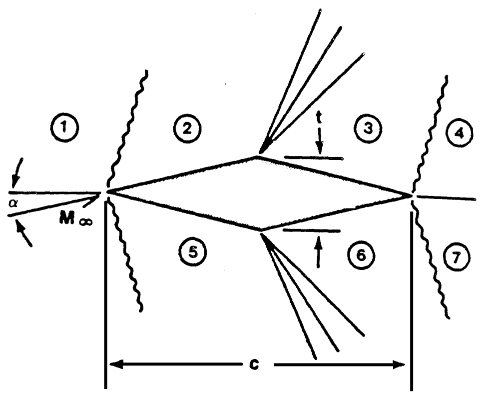
- If upstream Mach (M_1) =1, then $\nu_1 = 0$, and equation directly relates downstream Mach (M_2) to surface turning angle ($\Delta\nu$)
- If M_1>1, determine M_2 as follows:
  Calculate upstream ν_1 from above equation
  Calculate $\nu_2 = \nu_1 + \Delta\nu$
  Reverse above equation to obtain corresponding M_2
- Above equation is tabulated in NACA TR 1135 and is plotted below



Example: Flow initially at M_1 = 2.0 accelerates through an expansion corner of 24 *deg*. Exit Mach number is 3.0

5.5.3 Two-Dimensional Supersonic Airfoil Approximations

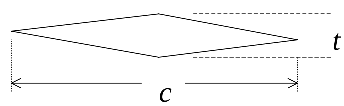
- Determine surface static pressures by calculating changes through obliques shocks and expansion fans



- Ackert approximations for thin wings are based on

$$C_p = \frac{\Delta P}{q} \cong \pm \frac{2\delta}{\sqrt{M^2 - 1}}$$

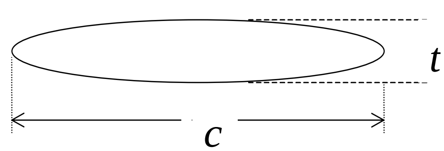
- Double wedge airfoil approximations



$$C_L \cong \frac{4\alpha}{\sqrt{M^2 - 1}}$$

$$C_D \cong \frac{4\alpha^2}{\sqrt{M^2 - 1}} + \frac{4}{\sqrt{M^2 - 1}}\left(\frac{t}{c}\right)^2$$

- Biconvex wing approximations



$$C_L \cong \frac{4\alpha}{\sqrt{M^2 - 1}}$$

$$C_D \cong \frac{4\alpha^2}{\sqrt{M^2 - 1}} + \frac{5.33}{\sqrt{M^2 - 1}}\left(\frac{t}{c}\right)^2$$

5.6 Drag Polars (ref 5.2)

5.6.1 Drag Polar Construction and Terminology

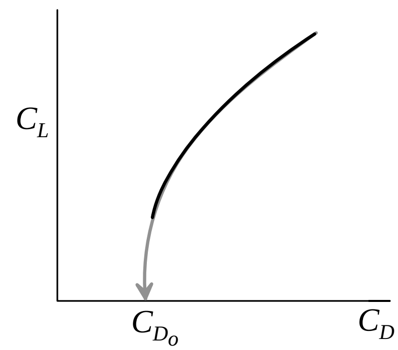
C_L = lift coefficient
C_D = drag coefficient
C_{Di} = induced drag coefficient
C_{Do} = parasitic drag coefficient
AR = aspect ratio
e = Oswald efficiency factor
l = length flow has traveled
S_{wet} = wetted area of surface
S = reference wing area

Simple Drag Polar Equation Limitations

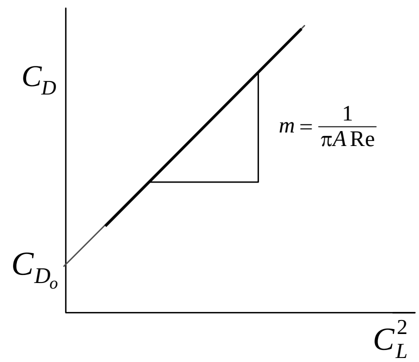
- No separated flow losses
- Symmetric Camber
- Applies at one Mach, Altitude, *cg*

$$C_D = C_{D_o} + \frac{C_L^2}{\pi A \mathrm{R} e} = C_{D_o} + C_{D_i}$$

"Polar" form of simple drag polar

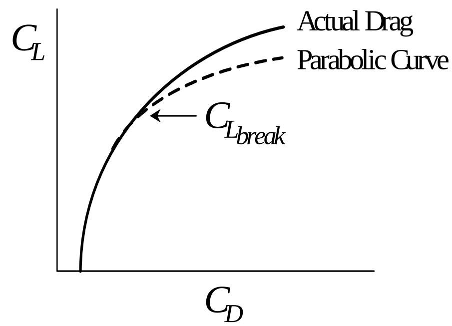


Linearized form of simple drag polar



5.6.2 Complicating Factors

Airflow Separation Effects



Drag Polar Equation Accounting for Flow Separation:

$$C_D = C_{D_{\min}} + \frac{\left(C_L - C_{L_{\min}}\right)^2}{\pi A \,\mathrm{Re}} + k_2\left(C_L - C_{L_{break}}\right)$$

- Delete last term if $C_L < C_{lbreak}$
- Determine k_2 from flight test

Reynolds Number Effects (refs 5.4, 5.11)

- Calculate length Re_L and friction coefficient (c_f) for each surface as

$$\mathrm{Re}_L = \frac{\rho V l}{\mu} = 7.101 \times 10^6 M \left[\frac{\delta}{\theta^2}\right]\left[\frac{T_K + 110}{398}\right] l$$

(T_K = Kelvin,
l= total length, ft)

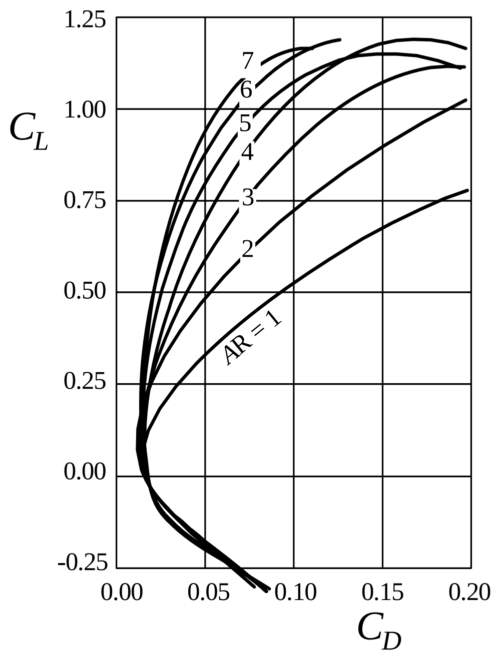
$$c_f = \left\{\frac{1.328}{\sqrt{\mathrm{Re}_L}}\right\}\left[1 + 0.1305 M^2\right]^{-0.12} \text{ laminar, or} = \left[\frac{.074}{\left(\mathrm{Re}_L\right)^2} - \frac{1700}{\mathrm{Re}_L}\right] \text{transition}$$

or $$c_f = 0.455\left\{\log \mathrm{Re}_L\right\}^{-258}\left\{1 + 0.144 M^2\right\}^{-0.65} \text{ turbulent}$$

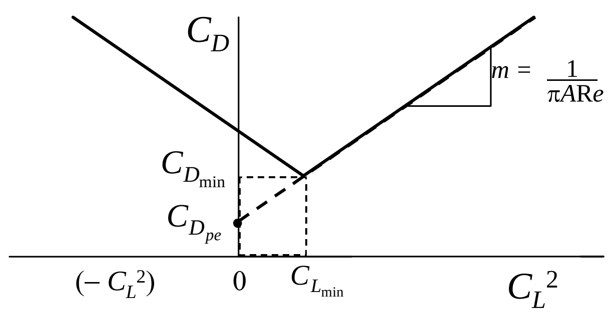
- In general, c_f decreases as R_n increases (unless transitioning from laminar to turbulent flow)
- Friction drag = $c_f\, q\, S_{wet}$ for each component (S_{wet} = wetted area)
- Correct from test day to standard day aircraft drag coefficient by summing differences of each component's drag change

$$\Delta C_D = \frac{\Sigma\left(c_{f_s} - c_{f_t}\right) S_{wet}}{S}$$

Wing Camber or Incidence Angle Effects



Note slight increase in drag as lift decreases towards zero



Linearized drag polar for aircraft with wing camber and/or incidence

Revised drag polar equation accounting for wing camber or incidence

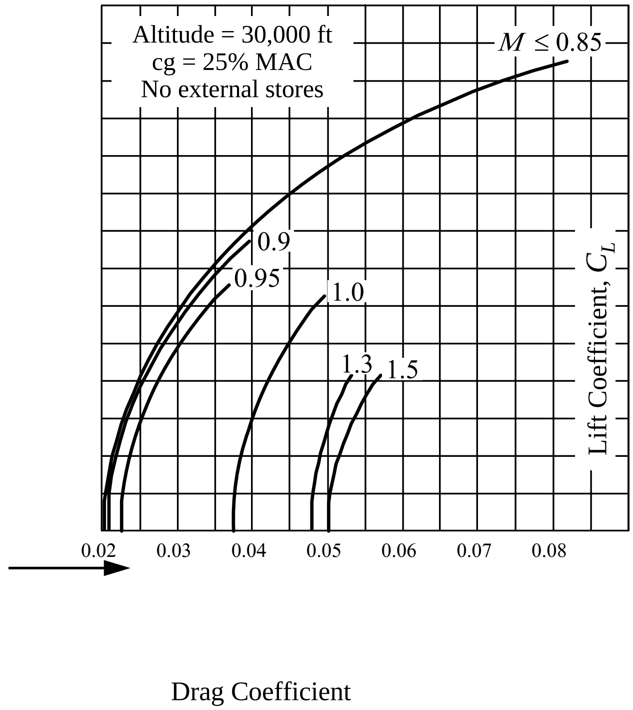
$$C_D = C_{D_{min}} + \frac{\left(C_L - C_{L_{min}}\right)^2}{\pi A \mathrm{Re}}$$

- Generally not necessary since most flight occurs above C_{Lmin}

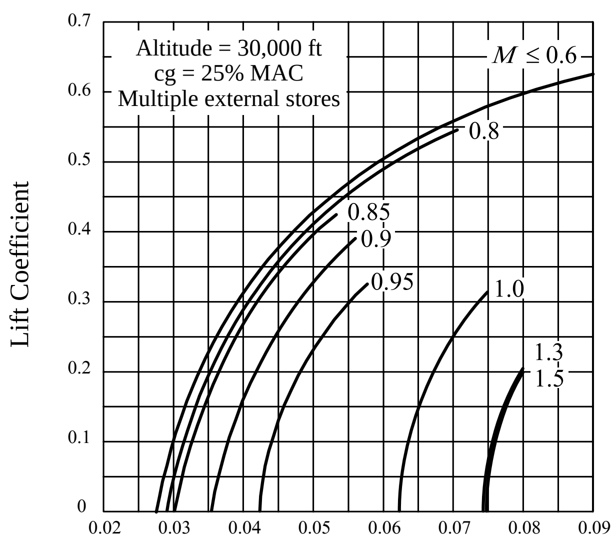
Mach Number Effects

- Aircraft with low parasitic drag coefficients and high fineness ratios pay a relatively small "wave drag" penalty.

Modern fighter-type aircraft



- With external stores, same aircraft pays larger Mach penalty

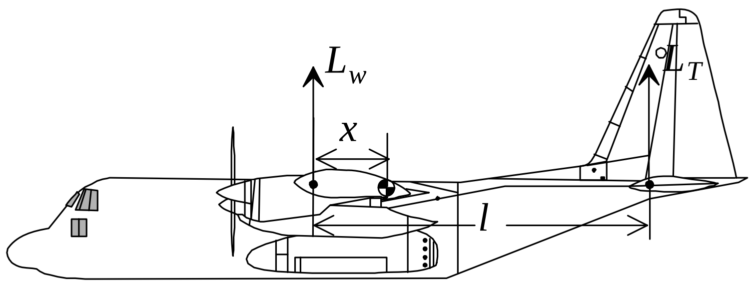


Propeller Slipstream Effects

- a.k.a "scrubbing" drag
- Propwash increases flow speed over surface within slipstream
- More drag is created by higher q and vorticity.
- Function of prop speed and power absorbed (C_p)or thrust (C_T)
- Problem should be addressed in airframe or propeller models

Trim Drag Effects (reference 5.4)

e = wing Oswald efficiency factor
e_t = tail Oswald efficiency factor
b = span, b_t = tail span
x = wing *ac*-to-*cg* distance
l= wing *ac*-to tail *ac* dist.

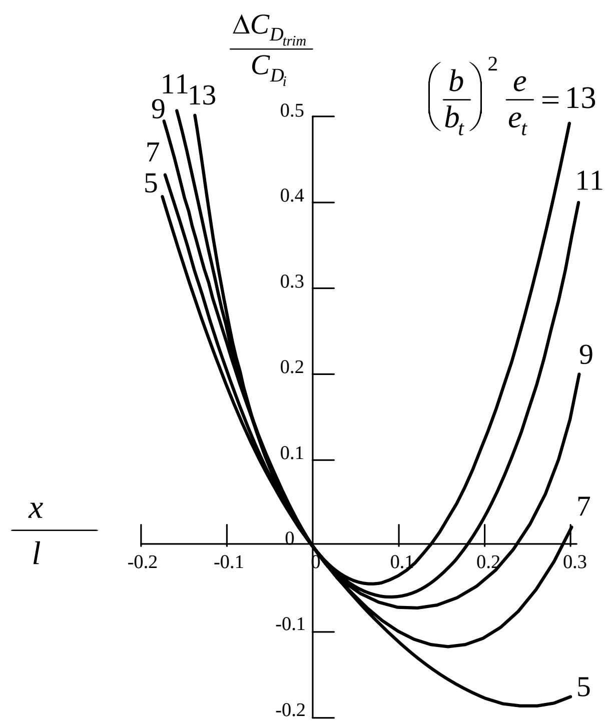


$$\Delta C_{D_{trim}} = \frac{W^2}{\pi q^2 S b^2 e}\left\{\frac{2}{lW}\left[x_0 - x_1\right] + \frac{1}{l^2}\left[1 + \frac{S}{S_t}\frac{e}{e_t}\left(\frac{b}{b_t}\right)^2\right]\left[x_0^2 - x_1^2\right]\right\}$$

Trim drag change relative to total induced drag:

$$\frac{\Delta C_{D\,trim}}{\Delta C_{D_i}} = \frac{x}{l}\left[\frac{x}{l}\left(\frac{b}{b_t}\right)^2\frac{e}{e_t} - 2\right]$$

Plot of above equation



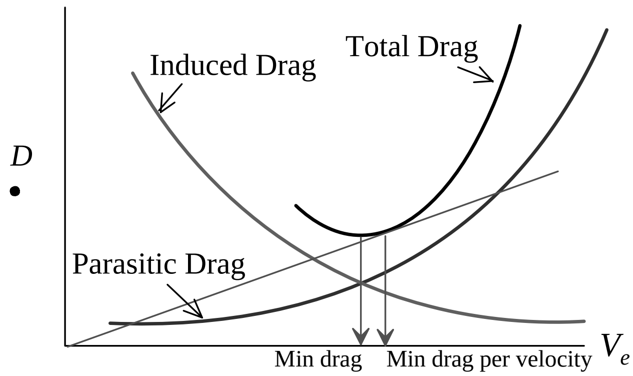
5.6.3 Drag Polar Analysis

$$D = \bar{q} S C_D = \bar{q} S \left[C_{D_o} + \frac{C_L^2}{\pi A \operatorname{Re}} \right] = \tfrac{1}{2} \rho_o V_e^2 S \left[C_{D_o} + \frac{W^2}{\pi A \operatorname{Re} \left(\tfrac{1}{2} \rho_o V_e^2 S \right)^2} \right]$$

- For a given configuration (C_{Do}, S, AR, e)

$$D = k_1 V_e^2 + k_2 \frac{W^2}{V_e^2}$$ first term = parasitic drag, second term = induced drag

- For any given weight, $D = f$(equivalent airspeed) only



- Minimum total drag occurs when $D_{induced} = D_{parasitic}$
  - same as speed where $C_{Di} = C_{Do}$
  - occurs at max C_L / C_D ratio (same as max L/D ratio)
- Minimum drag/velocity occurs at min slope of Drag vs V curve
  - same as speed where $3C_{Di} = C_{Do}$
  - occurs at max $C_L^{1/2} / C_D$ ratio

Power required = drag x true airspeed

$$P_{req} = D V_T = D \frac{V_e}{\sqrt{\sigma}} = k_1 \frac{V_e^3}{\sqrt{\sigma}} + k_2 \frac{W^2}{\sqrt{\sigma} V_e}$$

Minimum total $P_{req'd}$ occurs when $P_{induced} = P_{parasitic}$
- same as speed where $C_{Di} = 3C_{Do}$
- occurs at max $C_L^{3/2} / C_D$ ratio

Minimum power/velocity occurs at min slope of $P_{req'd}$ vs V curve
- same as speed where $C_{Di} = C_{Do}$
- occurs at max C_L / C_D ratio

Optimum Aerodynamic Flight Conditions

Gliders/ Engine-Out Flight

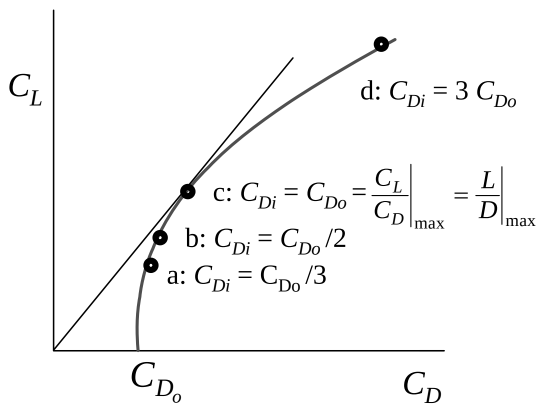
- Max range (minimum glide slope) occurs at max C_L/C_D
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic
- Min sink rate (minimum power req'd) occurs at max $C_L^{3/2}/C_D$ ratio same as condition where $3C_{Do} = C_{Di}$ *if* drag polar is parabolic

Reciprocating Engine Aircraft (assuming constant BSFC & prop η)

- Max range (minimum power/velocity) occurs at max C_L/C_D ratio
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic
- Max endurance (minimum power req'd) occurs at max $C_L^{3/2}/C_D$
  same as condition where $3C_{Do} = C_{Di}$ *if* drag polar is parabolic

Turbine Jet Engine Aircraft (assuming constant TSFC)

- Max range at constant altitude (minimum drag/velocity)
  occurs at max $C_L^{1/2}/C_D$ ratio
  same as condition where $C_{Do} = 3C_{Di}$ *if* drag polar is parabolic
- Best cruise/climb range (maximum [*M* x *L/D*] ratio)
  occurs at max $C_L/C_D^{3/2}$ ratio
  same as condition where $C_{Do} = 2C_{Di}$ *if* drag polar is parabolic
- Best endurance (minimum drag)
  occurs at max C_L/C_D ratio
  same as condition where $C_{Do} = C_{Di}$ *if* drag polar is parabolic



To calculate optimum speed V_2 for configuration$_2$ & weight$_2$ based on optimum speed V_1 at configuration$_1$ & weight$_1$

$$V_2 = \left(\frac{C_{D_{o_1}}}{C_{D_{o_2}}}\right)^{\frac{1}{4}} \left(\frac{W_2}{W_1}\right)^{\frac{1}{2}} V_1$$

5.7 References

5.1 Roberts, Sean “Aerodynamics for Flight Testers” *Chapter 3*, *Subsonic Aerodynamics*, National Test Pilot School, Mojave, CA, 1999

5.2 Lawless, Alan R., et al, “Aerodynamics for Flight Testers” *Chptr 4, Drag Polars*, National Test Pilot School, Mojave ,CA, 1999

5.3 Hurt Hugh H., “Aerodynamics for Naval Aviators“, University of Southern California, Los Angeles, CA, 1959.

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5.5 Stinton, Darryl, “The Design of the Aeroplane“, BSP Professional Books, Oxford, 1983

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5.9 White, Frank M. “Fluid Mechanics” pg 29, McGraw-Hill, 1979, ISBN 0-07-069667-5.

5.10 Anderson, John D. Jr, “Introduction to Flight” pg 142, Mcraw-Hill, 1989, ISBN 0-07-001641-0.

5.11 Twaites, Bryan, Editor, “Incompressible Aerodynamics: An Account of the steady flow of incompressible Fluid Past Aerofoils, Wings, and Other Bodies,” Dover Publications, 1960.

NOTES

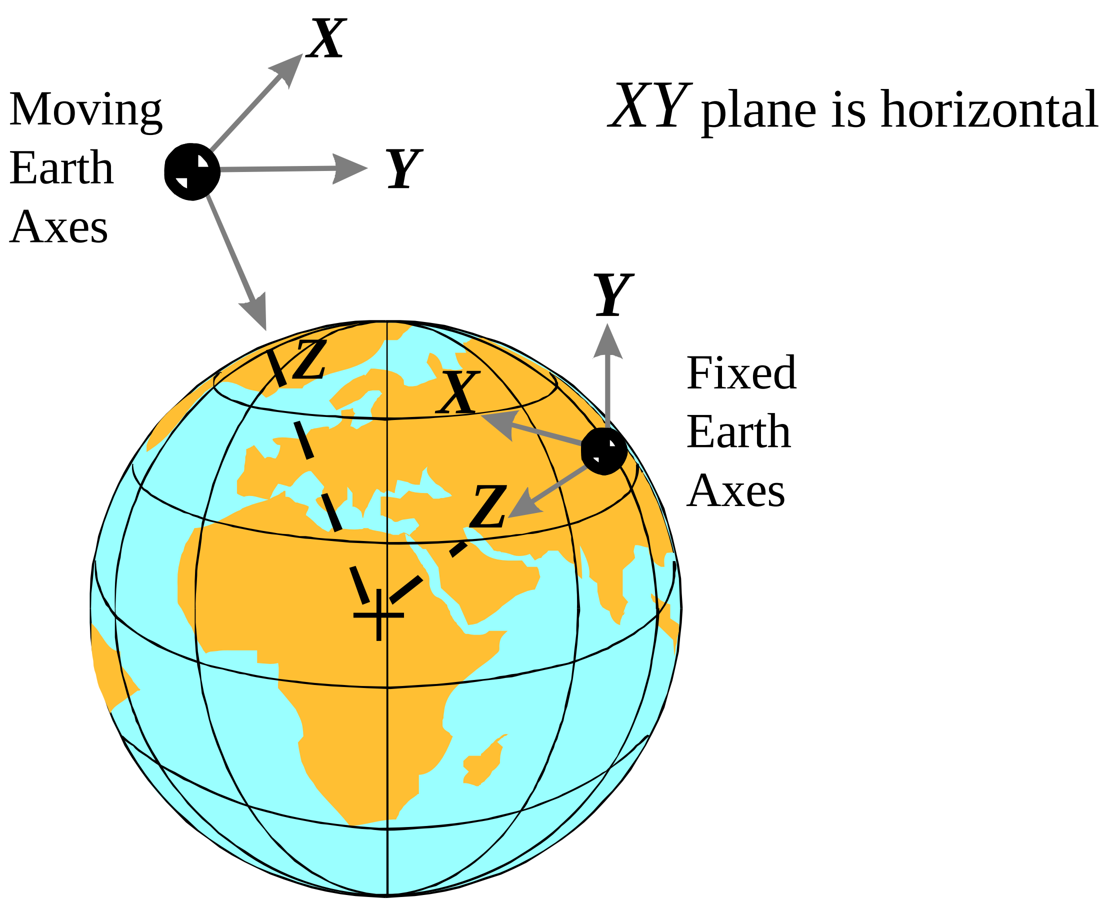
NOTES

Section 6 Axis Systems and Transformations

6.1 Earth Axis Systems (ref 6.6.1)

Both fixed-Earth and moving-Earth axis systems keep constant *orientation* with respect to the Earth. The Z-axis point towards the center of the Earth.

- The origin of a fixed-Earth system does not move relative to the Earth. (such as a ground radar site)
- The origin of a moving Earth system does not move relative to its host (such as an aircraft inertial reference unit) .



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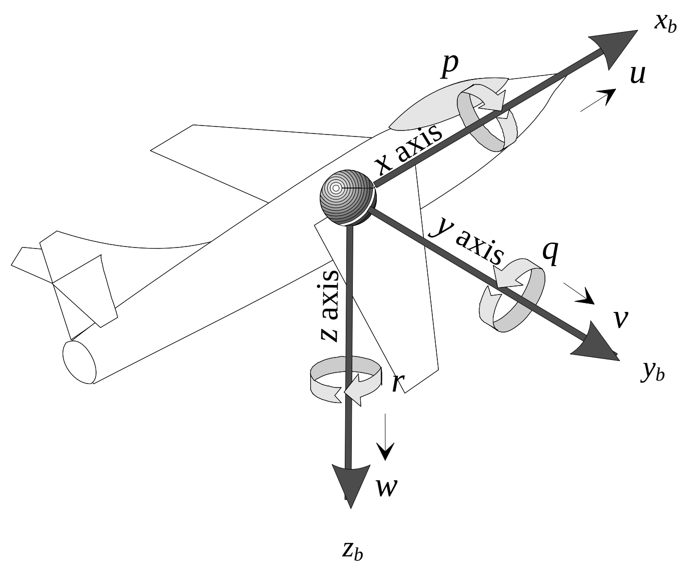
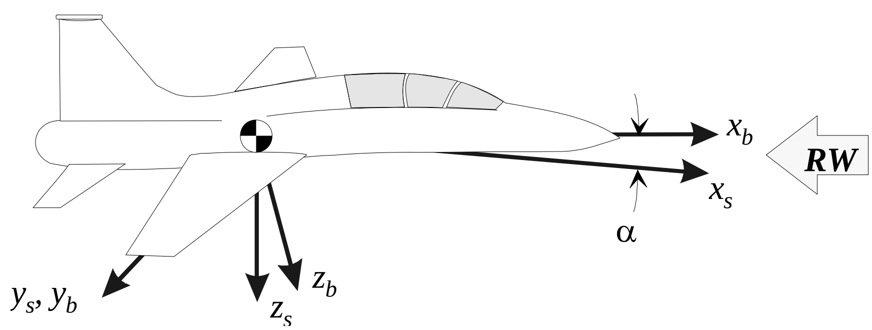
6.2 Aircraft Axis Systems (ref 6.6.2)

- The **body axis** system originates at the aircraft's reference center of gravity. The $+x_b$ direction is towards the front, the $+y_b$ direction is towards the right wing tip, and the $+z_b$ direction is towards the bottom of the aircraft.



- The **stability axis system** is similar to the body axis system except that it is rotated about the y-axis through the angle of attack (α)

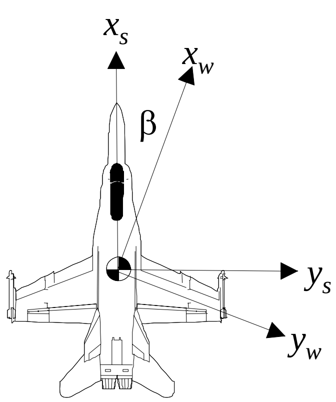


Forces, velocities or accelerations along the stability axes are related to the body axes as follows

$$x_b = x_s \cos\alpha - z_s \sin\alpha$$
$$z_b = z_s \cos\alpha + x_s \sin\alpha$$
$$y_b = y_s$$

- The **wind axis** system is similar to the stability axis system except it is rotated about the z_s axis through the angle of sideslip (β).

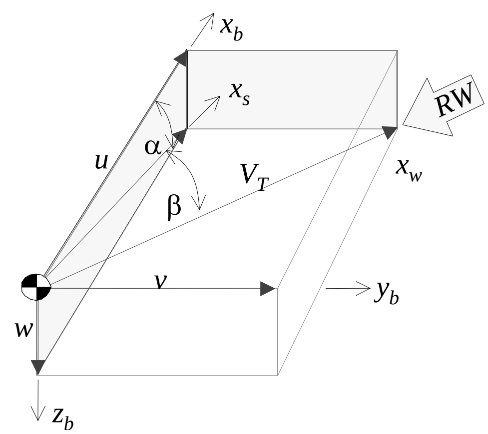
The term "wind" refers to the fact that the freestream relative wind approaches the aircraft directly along the x_w axis. This dictates that the true airspeed also lies along the x_w axis.



Forces, velocities or accelerations along the wind axes are related to the stability axes as follows

$x_s = x_w \cos\beta - y_w \sin\beta$
$y_s = y_w \cos\beta + x_w \sin\beta$
$z_s = z_w$

The geometric relations between body, stability and wind axis velocities are illustrated here.



$$\sin\alpha = \frac{\omega}{V_T\cos\beta},\ \sin\beta = \frac{V}{V_T}$$

if β is small, then $\sin\alpha = \dfrac{\omega}{V_T},\ \beta = \dfrac{V}{V_T}$

if α is small, then $\alpha = \dfrac{\omega}{V_T}$

Most aircraft sideslip vanes do not measure β directly. They measure the flanking angle, which is the projection of the relative wind into the aircraft's *x-y* plane. The difference between these two angles increases with angle of attack. Ignoring upwash, boom bending, and body axis rate corrections, calculate true sideslip as a function of vane α and β as follows:

$$\beta_{true} = \tan^{-1}\ [\ \tan(\beta_{vane})\ \cos\alpha\ ]$$

- **Wind-Body Axis Transformations** (ref 6.6.1)

Combining the two previous transformations, forces, velocities or accelerations along the wind axes are related to the body axes as follows

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix}$$

After expansion,

$x_b = \cos\alpha\,(x_w\cos\beta - y_w\sin\beta) - z_w\sin\alpha$
$y_b = x_w\sin\beta + y_w\cos\beta$
$z_b = \sin\alpha\,(x_w\cos\beta - y_w\cos\beta) + z_w\cos\alpha$

The inverse transform, converting from the body to the wind axis system is

$$\begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}$$

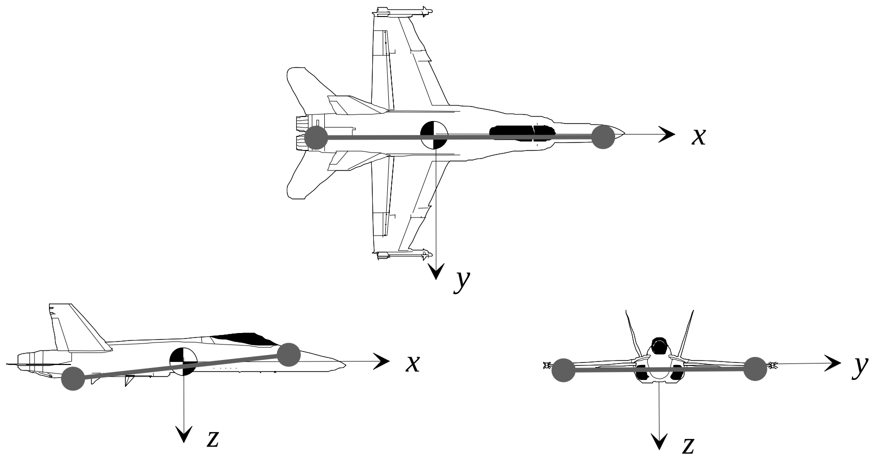
After expansion,

$x_w = \cos\beta\,(x_b\cos\alpha + z_b\sin\alpha) + y_b\sin\beta$
$y_w = -\sin\beta\,(x_b\cos\alpha + z_b\sin\alpha) + y_b\cos\beta$
$z_w = -x_b\sin\alpha + z_b\cos\alpha$

Note that these equations apply to the sign convention with z+ down. If sign convention (and instrumentation calibration) use z+ upward, then the above equations become:

$x_w = \cos\beta(x_b\cos\alpha - z_b\sin\alpha) + y_b\sin\beta$
$y_w = -\sin\beta(x_b\cos\alpha + z_b\sin\alpha) + y_b\cos\beta$
$z_w = x_b\sin\alpha + z_b\cos\alpha$

- The **Principle axes** are those about which the products of inertia are zero. They can be equated to the axis of “dumbells” which represent concentrated mass elements. Neglecting aerodynamic and gyroscopic effects, an aircraft rotating about one of its principle axes will <u>not</u> tend to cross-couple into motion about any other axis.



Wind to Body Axes Matrix Transformation

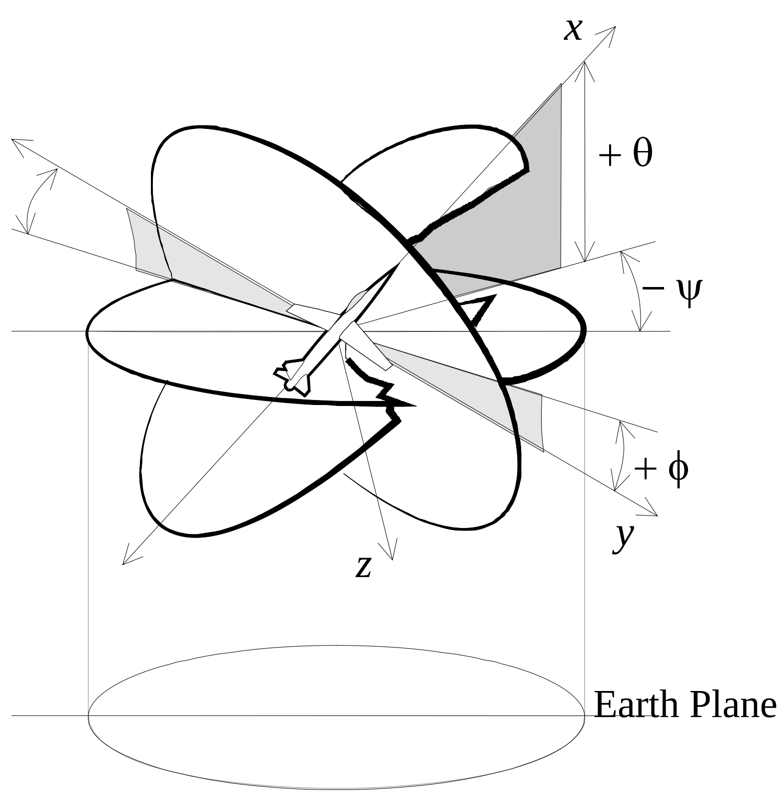
6.3 Euler Angles (ref 6.6.1)

Euler angles are expressed as yaw, pitch, and roll. The sequence: first yaw, then pitch, then roll; must be maintained to arrive at the proper orientation angles. The Euler angles are defined as follows:

$\psi \equiv$ Yaw Angle: The angle between the projection of the vehicle x_b- axis onto the horizontal reference plane and some initial reference position of the Earth *x*-axis. Yaw angle equals the vehicle heading only if the initial reference is North.

$\theta \equiv$ Pitch angle (in vertical plane) between x_b and horizon.

$\phi \equiv$ Bank angle, the angle (measured in the *y*-*z* plane of the body-axis system) between the *y*-axis and the horizontal reference plane. Also known at the roll angle, it is a measure of the rotation (about the *x*-axis) to return the aircraft to a wings level condition.



6.4 Flight Path Angles (ref 6.6.3)

Just as the Euler angles define the attitude of the aircraft with respect to the Earth, three flightpath angles describe the vehicle's *cg* trajectory relative to the Earth (not the air mass).

σ = Flight path heading angle; also known as ground track heading, is the horizontal angle between some reference direction (usually North) and the projection of the velocity vector on the horizontal plane. Positive rotation is from North to East.

γ = Flightpath elevation angle; the vertical angle between the flightpath and the horizontal plane. Positive rotation is up. During a descent, this parameter is commonly known as glide path angle.

μ = Flightpath bank angle; the angle between the plane formed by the velocity vector and the lift vector and the vertical plane containing the velocity vector. Positive rotation is clockwise about the velocity vector, looking forward.

The first two parameters above are easily measured using ground-based radar, or onboard GPS or inertial reference systems. If only α, β, and the Euler angles are available, then **assuming zero winds**, the flightpath angles can be calculated as

$$\gamma = \sin^{-1}[(\sin\theta\cos\alpha - \cos\theta\cos\phi\sin\alpha)\cos\beta - \cos\theta\sin\phi\sin\beta$$

$$\sigma = \sin^{-1}[\frac{\cos\phi\sin\beta - \sin\phi\sin\alpha\cos\beta}{\cos\gamma}] + \psi$$

$$\mu = \sin^{-1}[\frac{\cos\theta\sin\phi\cos\beta + (\sin\theta\cos\alpha - \cos\theta\cos\phi\sin\alpha)\sin\beta}{\cos\gamma}]$$

Technically, the above equations describe the **velocity vector** (angles relative to the air mass). If the air mass is moving relative to the Earth, as is usually the case, the above equations <u>do not</u> describe the flight path.

Editor's note: not knowing the difference between flightpath and velocity vector angles can cause considerable confusion when analyzing data from different sources.

6.5 Axis System Transformations (ref 6.6.2)

Transformation matrix for converting forces, velocities or accelerations from inertial (X, Y, Z) to body (*x, y, z*) axes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Expanding gives:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\theta\sin\psi+\sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

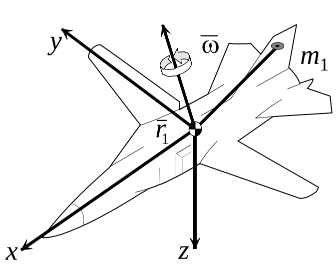
The inverse of the above transform matrix converts from the body axis to the inertial axis coordinate system

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Acceleration Transformations

- Convert body-axis angular rates & linear accelerations into total accelerations along body axes.
- Convert element (m_1) location & rates into specific angular momentum

$$a_x = \dot{u} + qw - rv$$
$$a_y = \dot{v} + ru - pw$$
$$a_z = \dot{w} + pv - qu$$



$$\frac{H}{m} = r_1 \times [\bar{\omega} \times \bar{r}_1] \Rightarrow$$
$$\left[\frac{H}{m}\right]^a_l = p\left(y^2 + z^2\right) - q\left(xy\right) - r\left(xz\right)$$
$$\Rightarrow \left[\frac{H}{m}\right]^n_y = q\left(x^2 + z^2\right) - r\left(yz\right) - p\left(xy\right)$$
$$\Rightarrow \left[\frac{H}{m}\right]^n_k = r\left(x^2 + y^2\right) - p\left(xz\right) - q\left(yz\right)$$

Transformations between body axis rates and Euler angle rates

$$p = \dot{\phi} - \dot{\psi} \sin \theta$$
$$q = \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi$$
$$r = \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi$$
$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$
$$\dot{\theta} = q \cos \phi - r \sin \phi$$
$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}$$

Transformations from Euler & aerodynamic angles to the aircraft stability and wind axis angular rates.
Subscripts *b*, *s*, and *w* denote the body, stability and relative wind axis systems.

$$(p_s, q_s, r_s, p_w, q_w, r_w) = f(p_b, q_b, r_b, \alpha, \beta)$$
$$p_s = p_b \cos\alpha + r_b \sin\alpha \qquad p_w = p_s \cos\beta + q_s \sin\beta$$
$$q_s = q_b \qquad q_w = q_s \cos\beta - p_s \sin\beta$$
$$r_s = r_b \cos\alpha - p_b \sin\alpha \qquad r_w = r_s$$

Transformations from Euler angles to the three aircraft axis angular accelerations (ref 6.6.3)

$$(\dot{p}_b, \dot{p}_s, \dot{p}_w, \dot{q}_b, \dot{q}_s, \dot{q}_w, \dot{r}_b, \dot{r}_s, \dot{r}_w) = f(\theta, \dot{\theta}, \ddot{\theta}, \phi, \dot{\phi}, \ddot{\phi}, \psi, \dot{\psi}, \ddot{\psi}, \alpha, \dot{\alpha}, \beta, \dot{\beta})$$
$$\dot{p}_b = \ddot{\phi} - \ddot{\psi} \sin\theta - \dot{\psi}\dot{\theta}\cos\theta$$
$$\dot{q}_b = \ddot{\theta}\cos\phi - \dot{\theta}\dot{\phi}\sin\phi + \ddot{\varphi}\cos\theta\sin\phi - \dot{\psi}\dot{\theta}\sin\theta\sin\phi + \dot{\psi}\dot{\phi}\cos\theta\cos\phi$$
$$\dot{r}_b = \ddot{\psi}\cos\theta\cos\phi - \dot{\psi}\dot{\theta}\sin\theta\cos\phi - \dot{\psi}\dot{\phi}\cos\theta\sin\phi - \ddot{\theta}\sin\phi - \dot{\theta}\dot{\phi}\cos\phi$$
$$\dot{p}_s = \dot{p}_b \cos\alpha + \dot{\alpha} p_b \sin\alpha + \dot{r}_b \sin\alpha + \dot{\alpha} r_b \cos\alpha$$
$$\dot{q}_s = \dot{q}_b$$
$$\dot{r}_s = \dot{r}_b \cos\alpha - \dot{\alpha} r_b \sin\alpha - \dot{p}_b \sin\alpha - \dot{\alpha} p_b \cos\alpha$$
$$\dot{p}_w = \dot{p}_s \cos\beta - p_s\dot{\beta}\sin\beta + \dot{q}_s \sin\beta + q_s\dot{\beta}\cos\beta$$
$$\dot{q}_w = \dot{q}_s \cos\beta - q_s\dot{\beta}\sin\beta - \dot{p}_s \sin\beta - p_s\dot{\beta}\cos\beta$$
$$\dot{r}_w = \dot{r}_s$$

6.6 References

6.6.1 Lawless, Alan R., *Math and Physics for Flight Testers* "Chapter 7, Axis Systems and Transformations", National Test Pilot School, Mojave CA, 1998.

6.6.2 Anon., *Aircraft Flying Qualities, Chapter 4, Equations of Motion*, USAF TestPilot School notes, AFFTC Edwards AFB CA, March 1991.

6.6.3 Kalviste, Juri, *Flight Dynamics Reference Handbook*, Northrop Corp. Aircraft Division, April 1988.

NOTES

Section 7 Mass Properties

7.1 Abbreviations and Terminology Abbreviations

a	perpendicular distance from the spring line of action to the oscillation axis (*ft*).
cg	center of gravity
f	measured frequency of oscillation (cycles/sec)
g	Earth's acceleration due to gravity (g = 32.172 ft/sec^2)
h	vertical component of the perpendicular distance from the *cg* to the oscillation axis.
I_{cg}	moment of inertia any axis passing through the *cg*
I_{xb}	moment of inertia about aircraft body x-axis
I_{yb}	moment of inertia about aircraft body y-axis
I_{zb}	moment of inertia about aircraft body z-axis
I_{xz}	product of inertia in aircraft body x-z plane
I_o	moment of inertia about oscillation axis
K_o	component of spring stiffness perpendicular to vehicle motion.
k	spring constant (*lb/ft*)
K_{tot}	total radius of gyration (includes effect of offset pivot)
L	rolling moment
l_{eff}	effective pendulum length
MAC	mean aerodynamic chord
METO	maximum power (except for takeoff)
m	mass
N	yawing moment
T	period of oscillation
p	roll rate
r	yaw rate
ΔI_{te}	correction for test equipment mass (slug-*ft*2)
ΔI_{am}	equivalent moment of inertia contribution of the air (slug-*ft*2)
δ	tilt of spring assembly (measured positive if tilted nose-down relative to local horizontal).
ε	inclination of principle axis (positive if tilted down).
ϕ	angle between aircraft y-axis and line connecting aircraft *cg* with spring attach points.
ω_d	damped frequency of oscillation (rad/sec) $=2\pi f$
ω_n	natural frequency of oscillation (rad/sec)

Terminology

Allowable *cg* range	Documented on Type Certificate Data Sheet. May be different for takeoff vs landing. Forward limit usually determined by control power limitations, aft limit usually deter mined by stability requirements.
datum	The manufacturer defined reference plane used for distance calculations.
empty weight	Basic aircraft weight with only equipment on board. (without crew, passengers, or fuel). This weight may or may not inlcude oil weight, depending on civil certification date.
empty weight *cg* range	The allowable *cg* locations for an empty aircraft. This is defined by the manufacturer to help assure that a normally loaded aircraft will have an acceptable total *cg* location.
lateral	Along the aircraft y-axis.
longitudinal	Along the aircraft x-axis
maximum weight	Maximum allowable weight. Usually implies takeoff weight, but may apply to landing or in-flight weight. conditions after aerial refueling.
minimum fuel	A calculated value that represents the minimum amount of fuel any airplane should have while retaining appropriate flight reserves. Calculated as $\text{min fuel } [lbs] = 0.5 METO\ [hp]$
moment arm	Distance between datum plane and *cg* of object.
moment	Product of moment arm and force (weight)
tare	The bias in weight scales due to test equipment weight or due to scale calibration errors.
useful load	Maximum takeoff weight minus empty weight.
weighing point	Location where aircraft is supported during the *cg* measurement.

7.2 Longitudinal & Lateral Center of Gravity Measurement

Test set-up procedures for empty weight *cg* determination

1. Clean aircraft including any mud or water seepage.
2. Ensure all aircraft equipment in place.
3. Drain to only residual oil (for older aircraft that include only residual oil as part of empty weight calculations) or fill to full oil (newer aircraft include full oil for calculations).
4. Drain to residual fuel only.
5. Rig all equipment in a closed building to eliminate wind effects.

6 Calibrate scales.

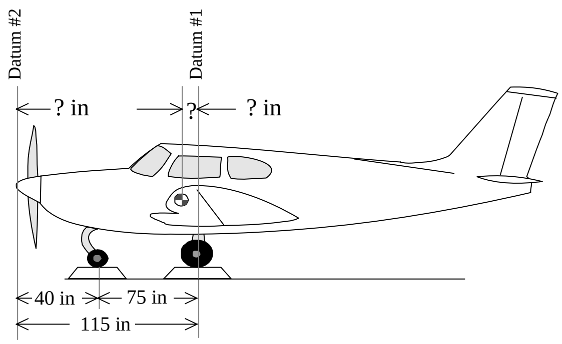
7. Record tare (bias in scale readings, may be due to wheel chocks, jack stands, or other test equipment)
8. Place aircraft on scales and level according to manufacturer's specifications.
9. Measure longitudinal and lateral distances between scale centers and datum. NOTE: distances behind datum are positive, distances ahead of datum are negative.
10. Record each scale weight

Calculating longitudinal center of gravity

11. Subtract the tare from each scale reading to get the correct weight.
12. Multiply each scale's corrected weight by its distance to the datum. This product is the moment for each scale.
13. Sum the moments in step 12.
14. Sum the corrected weights.
15. Divide the total moment by the total weight [step 13/step 14] to obtain the distance from the datum to the *cg*. Some aircraft use this distance for all *cg* references (typically presented in inches).
16. Other aircraft refer *cg* location to a percent of the mean aerodynamic chord (*MAC*). In this case, subtract the distance between the datum and the leading edge of the *MAC* from the distance in step 15.
17. Divide the distance in step 16 by the length of the MAC. This number is the fraction of the cg location along the MAC.

18 To present the above fraction in terms of % *MAC*, multiply by 100.

Example *cg* calculations



320 *lbs*

816 *lbs* right
810 *lbs* left

Example Using Datum at Main Landing Gear					
Item	Weight	×	Arm	=	Moment
Right wheel	816 lb	×	0	=	0
Left wheel	810 lb	×	0	=	0
Nose wheel	320 lb	×	-75	=	-24,000 inlb
Total	**1,946 lb**	×		=	**-24,000 inlb**

$$\frac{24{,}000}{1{,}946} = 12.33\,\text{in (fwd of MWCL)}$$

Example Using Datum at Prop Spinner					
Item	Weight	×	Arm	=	Moment
Right wheel	816 lb	×	115	=	93,840
Left wheel	810 lb	×	115	=	93,150
Nose wheel	320 lb	×	40	=	12,800
Total	**1,946 lb**	×		=	**199,790**

$$\frac{199{,}790}{1{,}946} = 102.67\,in$$

Example Lateral *cg* Calculation					
Item	Weight	×	Lat. Arm	=	Moment
Right wheel	816 lb	×	+70	=	57,120
Left wheel	810 lb	×	-70	=	-56,700
Nose wheel	320 lb	×	0	=	0
Total weight	**1,946 lb**	×		=	**420**

$$\text{then} \;=\; \frac{420}{1{,}946} = .216\;in$$

Correcting empty weight *cg* for changes in fuel, passengers, equipment or stores.

1. Note aircraft empty weight & empty weight *cg*. Multiply these values to obtain the empty weight moment.
2. Note the weight and moment arm for each item added to or subtracted from the aircraft (items subtracted are listed as negative weights).
3. Multiply each item's weight and arm to determine its moment.
4. Sum each item's moment in step 3 with the aircraft empty weight moment.
5. Sum each item's weight with the aircraft empty weight.
6. Calculate the new *cg* as [step 4/step 5].

Example *cg* corrections:
Given aircraft with empty weight = 1,075 *lbs* and *cg* @ 84 inches. Add pilot (170 *lbs*, @85.5"), fuel (75 *lbs* @94"), and oil (15 *lbs* @ 31.7").

Item	Wt	×	Arm	=	Moment
Airplane (empty)	1,075	×	84	=	90,300.0
Pilot	170	×	85.5	=	14,535.0
Fuel	75	×	94	=	7,050.0
Oil	15	×	31.7	=	475.5
Total	**1,335**	×		=	**112,360.5**

then $= \frac{112,360.5}{1,335} = 84.16\ in$

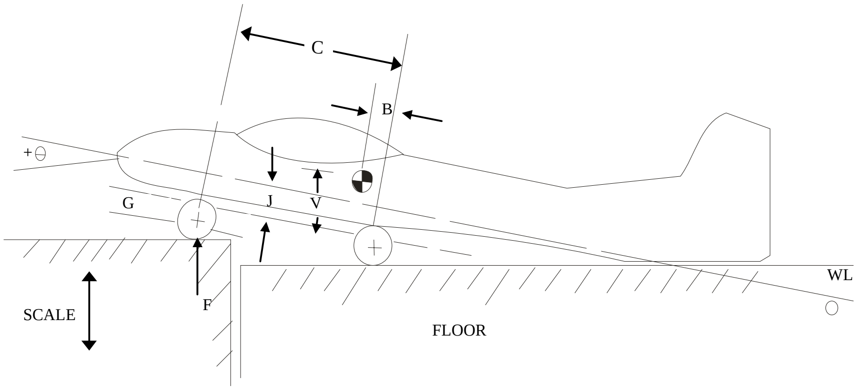
Given aircraft with empty weight = 1,220 *lbs* and *cg* @ 25 inches. Add radio (15 *lbs*, @ 65"), and replace 11 *lb* generator with 14 *lb* generator at same 21.5" location (in front of datum-located on firewall).

Item	Wt	×	Arm	=	Moment
Airplane (empty)	1,220	×	25.0	=	30,500.0
Radio	15	×	65.0	=	975.0
Generator (removed)	-11	×	-21.5	=	236.5
Generator (installed)	+14	×	-21.5	=	-294.0
New empty weight	1,238	×		=	31,417.5

$\frac{31,417.5}{1,238} = 25.38\ in$ the new cg (empty)

7.3 Vertical Center of Gravity Measurement

1. Drain or block landing gear struts to keep distances *G*, *J*, and *V* constant.
2. Level the fuselage and measure the weight on the nose wheel (F_o).
3. Tilt the aircraft at various (θ) measure nose wheel weight (F).

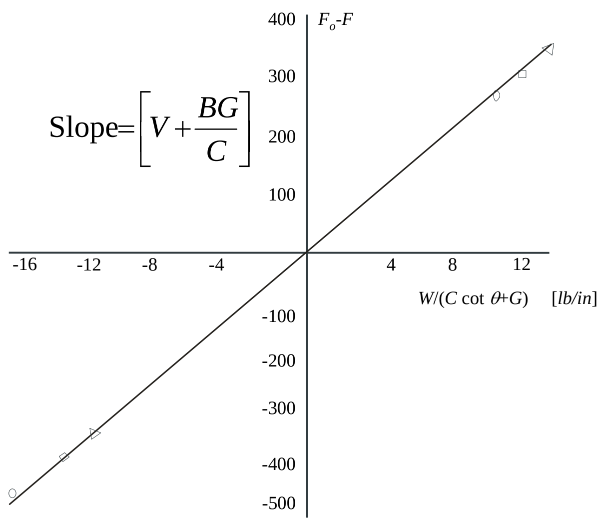


The change in nose wheel force can be written as

$$\left(F_O - F\right) = \left[V + \frac{BG}{C}\right]\left(\frac{W}{C\cot\theta + G}\right)$$

4. Plot (F_o-F) vs the term in parenthesis.
5. Slope of line equals term in brackets.
6. Solve for *V* after measuring *B*, *C*, *G*, and the slope.

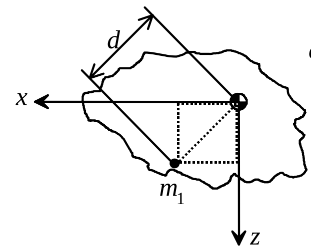
- This method applies to “gear down” *cg* .
- For “gear up” add the manufacturer's prediction of the *cg* shift to this result.



7.4 Moment & Product of Inertia Measurement

The moment of inertia about any axis of a body is the summation of the product of every element's mass and distance squared. Moments of inertia represent the resistance to rotational momentum changes.

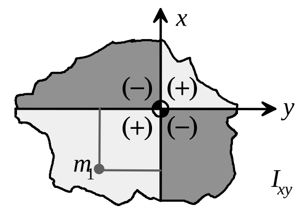
$$I_{xb} \equiv \int \left(y^2 + z^2\right) dm$$
$$I_{yb} \equiv \int \left(x^2 + z^2\right) dm$$
$$I_{zb} \equiv \int \left(x^2 + y^2\right) dm$$



$d^2 = x^2 + z^2$

Products of inertia are also calculated about any body axes. They represent the symmetry of mass distribution (comparing opposing quadrants).

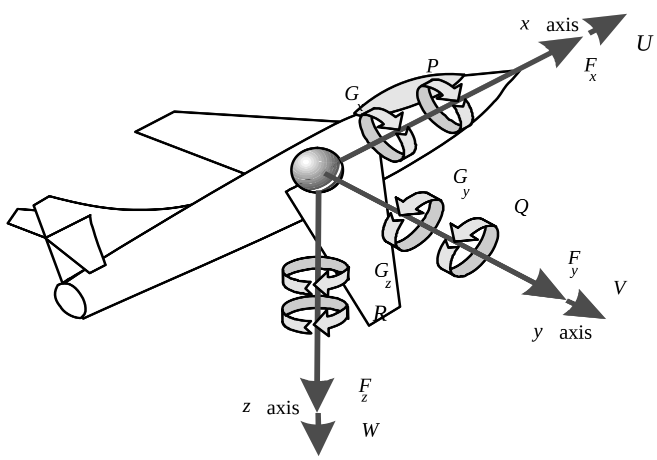
$$I_{xy} = I_{yx} \equiv \int xydm$$
$$I_{yz} = I_{zy} \equiv \int yzdm$$
$$I_{xz} = I_{zx} \equiv \int xzdm$$



$I_{xy} = (+)$

- Aircraft moments and products of inertia are calculated using body axes as the reference system.

- Careful Documentation can yield inertial predictions within about 1-5% of actual.



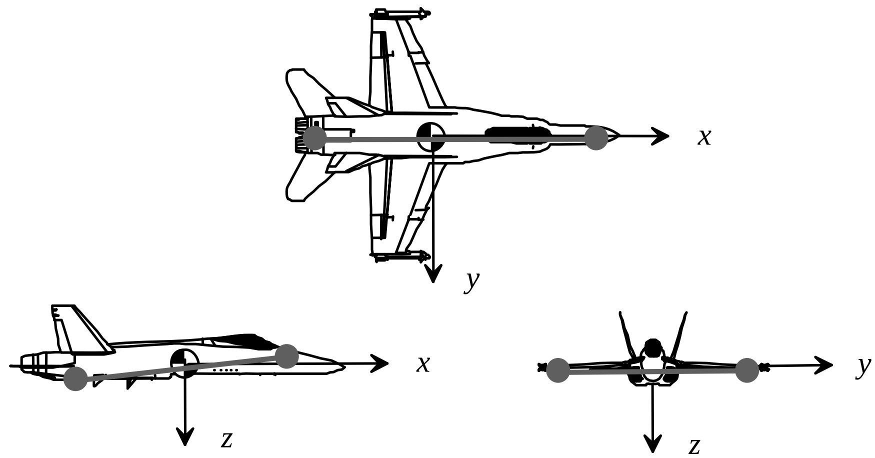
7.4.1 Radius of Gyration

- Dimensions of all *I* terms are [slugs-ft^2].
- Aircraft moments & products of inertia are generally assumed to be constant.
- For moments of inertia, mathematically replace *I* with the product of total mass times some constant with the dimensions of square feet.
- This constant is called the "radius of gyration" (*k*)
- If all the mass were concentrated at this radius, it would have the same moment of inertia as the actual body.

$$I_x \equiv \int \left(y^2 + z^2\right) dm = m k_{yz}^2$$

Aircraft moment of inertia summary

I_{zb} is always the largest value.
$I_{yb} > I_{xb}$ for fuselage-loaded aircraft
$I_{xb} > I_{yb}$ for wing-loaded aircraft.



- Vehicle mass distribution can be represented with concentrated "dumbbell" masses.
- The lines connecting the opposing dumbbells are the **principle axes**.
- When the principle axis lies along some line different from the body axis, the products of inertia are non-zero.
- If the orientation of the principle axes and the moments of inertia about theses axes are known, then the moments of inertia about any other axis system can be calculated.

7.4.2 Parallel Axis Theorem Applications

Shows how to **determine modified moment of inertia** after some
component alterations. Example: Correct original aircraft roll inertia, I_{xb}, to a modified value that accounts for the addition of wingtip fuel tanks.

$$I_{xbmod} = I_{xb\,orig} + \Delta I_{x\,comp}$$

- $\Delta I_{x\,comp}$ is composed of two components

$$\Delta I_{x\,comp} = I_{x\,comp} + m_{comp} r^2$$

- $I_{x\,comp}$ is the moment of inertia of just the new component about an axis which is parallel to the aircraft axis in question (this axis should run through the component's *cg*).
- $I_{x\,comp}$ can be determined analytically by summing the inertias of every mass element throughout the component (documented for simple shapes in various engineering handbooks).
- $I_{x\,comp}$ can be determined experimentally by "swinging," (sect'n 7.4.8).
- $I_{x\,comp}$ is usually much smaller than $m_{comp} r^2$ and can often be omitted.
- m_{comp} is the component's mass
- r is the distance from the axis in question to the component's *cg*.

Similarly, the parallel axis theorem shows how to **determine modified products of inertia** after some component alterations

$$I_{xz\,\mathrm{mod}} = I_{xz\,orig} + [I_{xz\,comp} + m_{comp} xz]$$

x and z are the distances from the component *cg* to the reference axes.

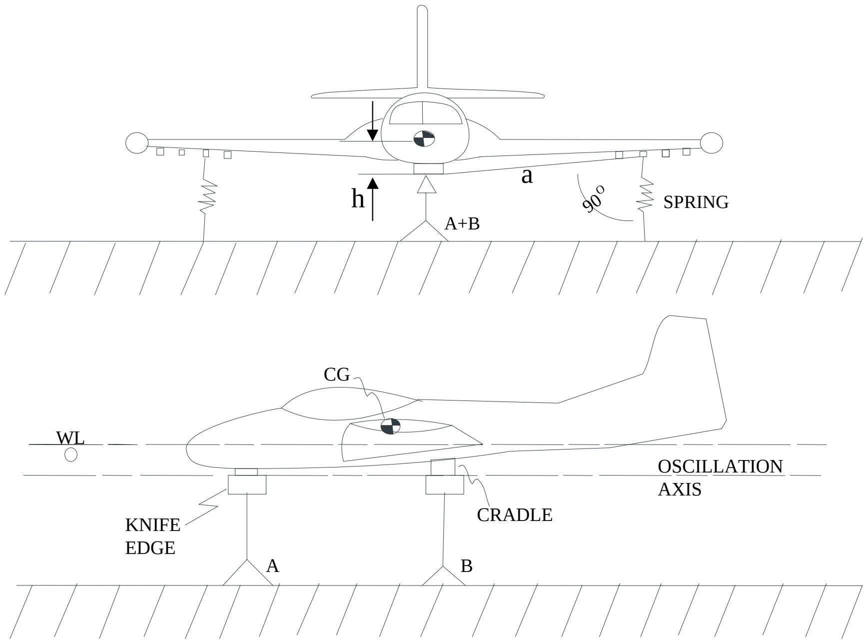
Moments of inertia are calculated about a set of reference axes which all intersect at the reference *cg*. In general, however, the actual *cg* does not lie exactly at this reference *cg*. The parallel axis theorem shows how to **correct the moment of inertia to the actual *cg* axis**. As an example, the rolling moment of inertia about the actual *cg* (I_{xcg}) is calculated from the reference I_{xb} as follows.

$$I_{x\,cg} = I_{xb} - m[y^2 + z^2]$$

where m = total mass
y = lateral distance between *cg* and aircraft x-axis.
z = vertical distance between *cg* and aircraft x-axis.

7.4.3 **Measuring Roll Inertia, I_{xb}** (spring oscillation method)

1. Balance aircraft on prism-like "knife edges" that support wooden cradles that conform to aircraft shape.
2. Attach springs (tension springs illustrated here) so they are parallel to line *a* as shown.



3. Allow aircraft to oscillate freely in roll after a small disturbance.
4. Use **automatic recording** to determine period of oscillations (*T*).
5. Calculate damped frequency as

$$\omega_d = \frac{2\pi}{T}$$

6. Record peak magnitude of each oscillation.
7. Calculate ζ using transient peak ratio method (see "Motion Analy sis," Section 8 of this handbook).
8. Calculate natural frequency as

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

Measuring Roll Inertia (continued)

9. Measure the spring's stiffness (K_o)
10. Calculate inertia about oscillation axis (I_o) as $I_o = \frac{K_o}{\omega_n^2}$
11. Use parallel axis theorem to correct I_o to a parallel axis passing through the *cg*.
12. Springs, cradles and attachments hardware will change the moment of inertia. Sum their total into a combined "test equipment inertia" (ΔI_{te}) and subtract this from the above result.

Combining steps 10-12 gives the complete moment of inertia equation

$$I_{cg} = \frac{K_o a^2 - mgh}{\omega_n^2} - md^2 - \Delta I_{te}$$

where *h* is the vertical distance between the *cg* and axis of oscillation and *d* is the total distance between the *cg* and the axis of oscillation ($d = h$ in the illustrated roll inertia test setup).

If automatic recording is not available...

Accomplish steps 1-3 as described for automatic recording case.

4. Use a stopwatch to time several oscillations and calculate ω_d as before
5. Measure the spring's rotational stiffness (K_o)
6. Approximate inertia using the damped frequency using

$$I_o \approx \frac{K_o}{\omega_d^2}$$

7. Use parallel axis theorem to correct I_o to a parallel axis passing through the *cg*.
8. Correct for effects of test hardware moment of inertia (ΔI_{te}).
9. To correct for errors in the I_o approximation, apply an "additional mass correction" (ΔI_{am}) which equates the flate plate area damping effect to additional inertia. This correction is detailed in ref. 7.5.3.

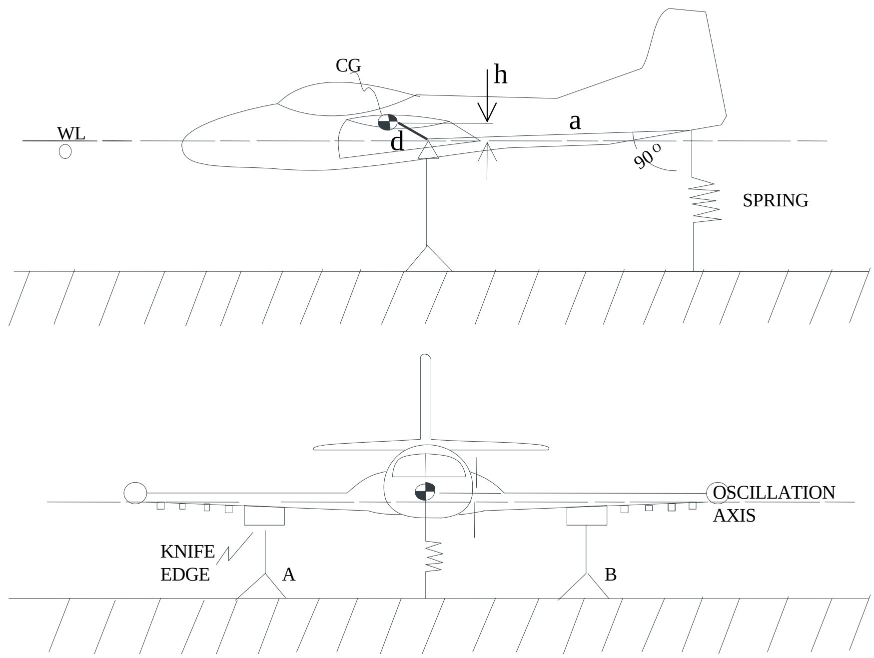
Combining steps 6-9 gives $$I_{cg} = \frac{K_o a^2 - mgh}{\omega_d^2} - md^2 - \left(\Delta I_{te} + \Delta I_{am}\right)$$

Above methods can also be used to determine I_{yb} and I_{zb}

7.4.4 Measuring Pitch Inertia, I_{yb} (spring oscillation method)

1. Balance aircraft on knife edges as shown.

2. Attach spring perpendicular to line *a*. Only one spring is required since the aircraft *cg* is off-center. Spring must be stiff enough to hold the aircraft in equilibrium as well as provide a restoring moment during oscillations.



Repeat steps 3-12 and apply the moment of inertia equation to determine I_{yb}

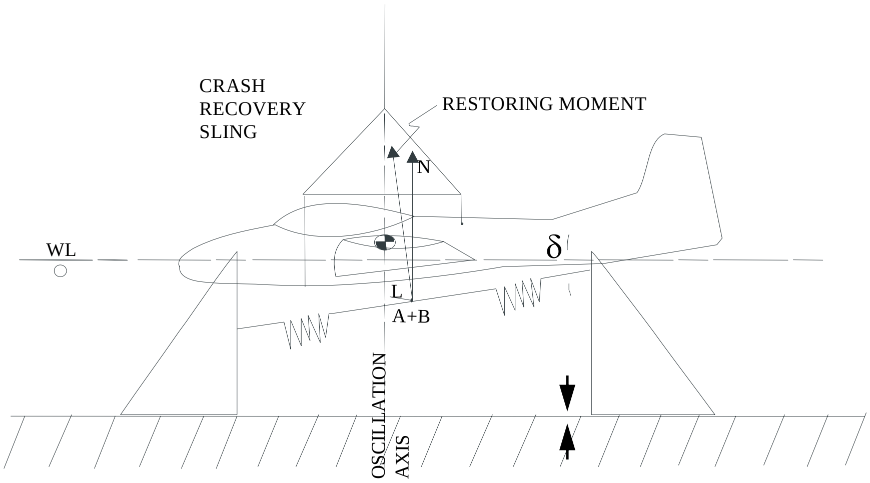
$$I_{cg} = \frac{K_o a^2 - mgh}{\omega_n^2} - md^2 - \Delta I_{te}$$

or, for an uninstrumented aircraft,

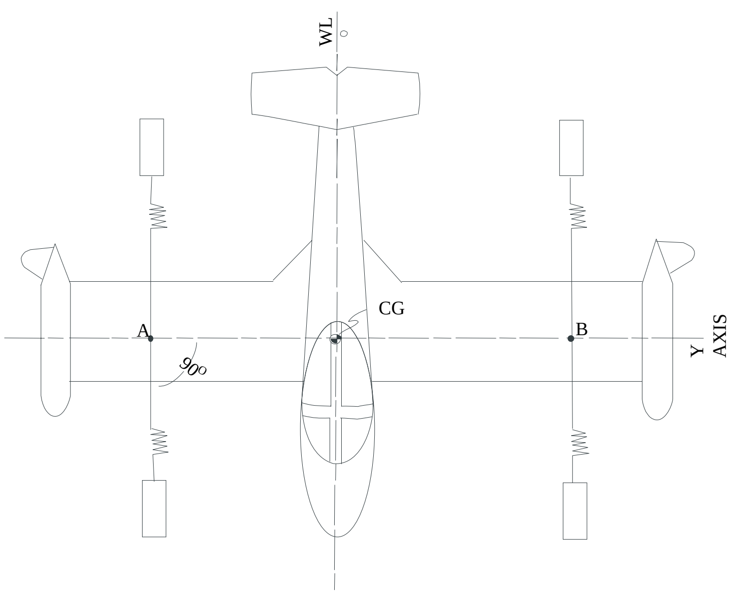
$$I_{cg} = \frac{K_o a^2 - mgh}{\omega_d^2} - md^2 - \left(\Delta I_{te} + \Delta I_{am}\right)$$

7.4.5 Measuring Yaw Inertia, I_{zb} (spring oscillation method)

1. Suspend aircraft as shown: fuselage reference line parallel with floor, z-body axis coincides with the oscillation axis ($h = d = 0$).
   For I_{zb} test, set spring assembly tilt angle parallel to floor ($\delta = 0$).



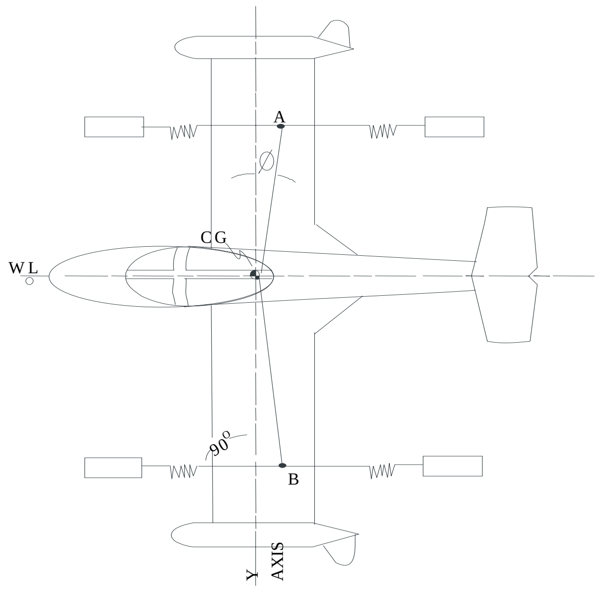
2. Ideal case is where the wing attach points are in line with the aircraft cg as shown. Springs are perpendicular to the AB line and are parallel.



Measuring Yaw Inertia (continued)

If not possible to rig wing attach points in line with the *cg*, then keep the springs parallel to each other and measure the angle ϕ.

- Calculate $K_o = k \cos \phi$ where k is the actual spring constant



Repeat steps 3-12 and apply the moment of inertia equation to determine $I_{zb.}$ Note that distances h and d are zero for a properly suspended aircraft.

$$I_{cg} = \frac{K_o a^2}{\omega_n^2} - \Delta I_{te}$$

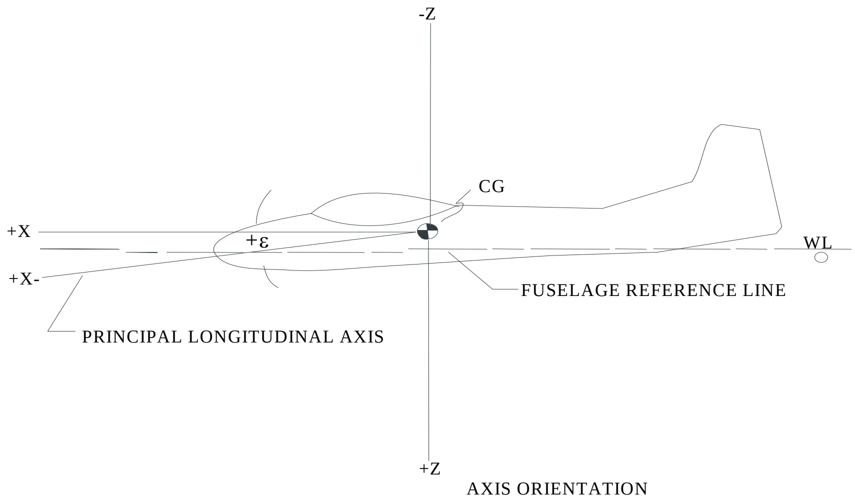
or, for an uninstrumented aircraft,

$$I_{cg} = \frac{K_o a^2}{\omega_d^2} - \left(\Delta I_{te} + \Delta I_{am}\right)$$

7.4.6 Measuring Axis of Inclination and I_{xz}

- Inclination of principle axis (ε) is positive if it lies in the +*xz* plane as shown.

- I_{xz} positive if ε is positive.



- From equations of motion

$$L = I_x \dot{p} - I_{xz}(\dot{r} + pq) + (I_z - I_y)qr \quad \text{and} \quad N = I_z \dot{r} - I_{xz}(\dot{p} - qr) + (I_y - I_x)pq$$

- When forcing small motions about only the yaw axis, *pq* and *qr* are negligible, giving

$$L = I_x \dot{p} - I_{xz} \dot{r} \quad \text{and} \quad N = I_z \dot{r} - I_{xz} \dot{p}$$

- If ε (and therefore I_{xz}) is positive, then yawing to the right will generate a left rolling moment.

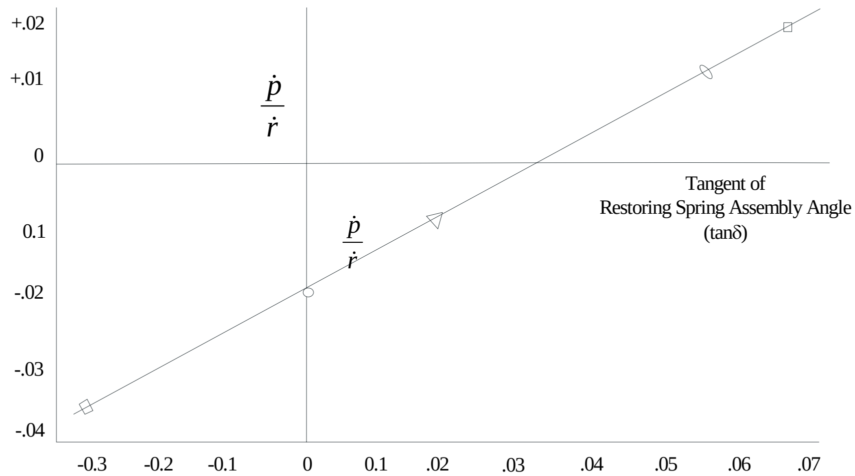
- If ε and I_{xz} are negative, a right yaw will generate a *right* moment.

Measuring Axis of Inclination and I_{xz} (continued)

1. Determine I_{xz} and ε by repeating yaw experiment for different spring angles (δ). Use instrumentation to note for each δ.

2 Graphically determine the angle δ_o at which the restoring vector produced by the springs completely counteracts the roll motion.

Plot [tan δ] vs $\frac{\dot{p}}{\dot{r}}$ as shown.



3. Determine the point where $\frac{\dot{p}}{\dot{r}}$ equals zero. This occurs at tanδ_o.

4. Calculate the product of inertia using $I_{xz} = I_z \tan\delta_o$

5. Calculate the inclination of the principle axis using

$$\tan 2\varepsilon = \frac{2I_{xz}}{I_{zb} - I_{xb}}$$

Note: Since the I_{xz} test objective is to interpolate to a condition where $\frac{\dot{p}}{\dot{r}}$ equals zero, only the ratio is necessary

and the absolute magnitudes of the accelerations are not reuired. In other words, the roll acceleration sensitivity can be increased to allow for easier measurement of the ratio.

7.4.7 Guidelines for Spring Oscillation Method

- Use only small magnitude oscillations
- Keep size and radius of knife edges as small as practical.
- Springs are typically linear except around zero load. Best results occur if springs are pre-loaded.
- Choose spring stiffness so oscillation frequency will be within instrumentation limits.
- If using a hand recorded stopwatch, best spring stiffness gives about one cycle per second (*f* = 1).

∗ Estimate the desired spring rate using

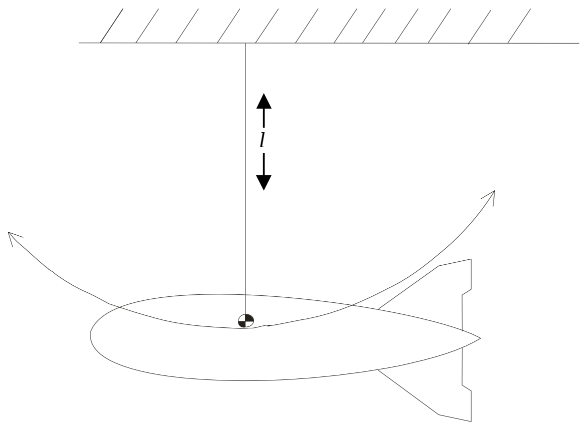
$$k = \frac{\left(I_{cg} + md^2\right)(2\pi f)^2 + hmg}{a^2}$$

- The purpose of instrumentation is to provide a means for measuring frequency and magnitude. Any of several parameters will be sufficient, including angular displacement, rate or acceleration; or linear acceleration.

7.4.8 Swing Method

1. Suspend component as shown.

2. Measure pendulum length (l), component mass (m), and the period of oscillation (T).



- The observed period is a function of the effective pendulum length.

$$\omega = \sqrt{\frac{g}{l_{eff}}} \quad \text{and} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l_{eff}}{g}} \qquad \text{or} \qquad l_{eff} = \left(\frac{T}{2\pi}\right)^2 g$$

- This effective length is the total radius of gyration $l_{eff} = K_{tot}$

- The total moment of inertia about the pivot point is the product of mass and radius of gyration squared. The parallel axis theorem states that this also is the sum of the component's moment of inertia about its *cg* plus its mass times the distance between the pivot and the component's *cg*.

$$I_{eff} = mK_{tot}^2 = I_{cg} + ml^2$$

3. Solving for the component's moment of inertia about its own *cg* gives

$$I_{cg} = mK_{tot}^2 - ml^2 = m(K_{tot}^2 - l^2) = m(\left[\left(\frac{T}{2\pi}\right)^2 g\right]^2 - l^2)$$

7.5 References

7.5.1 Lawless, Alan R. et al, “Special Topics and Aircraft Subsystems Flight Testing,” *Chapter 1, Mass Properties*, National Test Pilot School, Mojave, CA, 1999.

7.5.2 Bradfield, Edward N., "Experimental Determination of the Moments of Inertia, Product of Inertia, and Inclination of the Principle Axis of Conventional Aircraft by the Spring Oscillation Method" FTC-TIM-1001, AFFTC, Edwards AFB, CA, 1971.

7.5.3 Malvestuto, S. F., et al, "Formulas for Additional Mass Corrections to the Moments of Inertia of Air planes" TN 1187, Langley Memorial Aeronautical Laboratory, Langley Field Virginia, 1947.

7.5.4 Lawless, Alan R., "Fixed Wing Flying Qualities Flight Testing" *Chapter 7, Equations of Motion*, Na tional Test Pilot School Mojave CA, 1998.

Additional Reading

7.5.5 Tanner H.L., "Measurement of the Moments of Inertia of an Airplane by a Simplified Method" NACA2201, Ames Aeronautical Laboratory, Moffet Field, CA, 1950.

7.5.6 Woodward, C.R., et al "Handbook of Instructions for Experimentally Determining the Moments of Iner tia of Aircraft by the Spring Oscillation Method" TB-822-F-2, ASTIA AD97104, Cornell Aeronautical Laboratory, Buffalo, New York, 1955.

Section 8 Motion/Vibration Analysis

8.1 Recurring Abbreviations

$C_{1/x}$ number of cycles to achieve $1/x$ amplitude
D damping
D_1, D_2 peak-to-peak displacement (subsequent)
FV final value
$F(t)$ *forcing function*
f frequency, cycles/sec = $\omega/(2\pi)$
$HCAR$ half cycle amplitude ratio (i.e., x_2/x_1, x_3/x_2, etc.)
Im imaginary axis
M mass
MP peak overshoot
Re real axis
rms root mean square
s_1, s_2 equation roots of second order
T period = $1/f = 2\pi/\omega_d$ (seconds)
T_d delay time (i.e., time to 50% of FV)
T_r rise time (i.e., time from 10% to 90% of FV)
T_p time to peak amplitude
TPR transient peak ratio
T_s settling time (time to settle within x% of FV)
$T_{1/2}$ time to achieve $1/x$ amplitude
x displacement
x_1, x_2 peak displacements (subsequent)
v velocity
v_o peak velocity

ε $=\zeta\omega_n / \omega_d$
ϕ phase lag (radians)
ζ damping coefficient (non-dimensional)
σ damping rate $=\zeta\omega_n = 1/\tau$
τ time constant $=1/\zeta\omega_n$
ω frequency, radians/sec
ω_d damped natural frequency (rad/sec)
ω_n natural frequency (rad/sec)

8.2 First Order Motion

Found in classical aircraft roll and spiral modes. Named first-order because the motions are described by mathematics using the first derivative of a parameter.

8.2.1 Elements of First Order Motion

- Mechanical analogy contains elements of mass, damping and sometimes a forcing function.
- Example: Determine the vertical velocity of a diver as she hits the water at 10 ft/s (assume constant body position & neutral buoyancy)

Summing vertical forces

$$\sum F_{vert} : M\frac{dV}{dt} + DV = 0$$

$$\frac{dV}{V} + \frac{D}{M}dt = 0$$

$$\int \frac{dV}{V} = -\int \frac{D}{M}dt$$

Since *D* & *M* are constant

$$\text{n}V = \frac{-D}{M}t + C = \frac{-D}{M}t + \ln c$$

$$V_C = ce^{\frac{-D}{M}t} \quad \text{(complementary solution)}$$

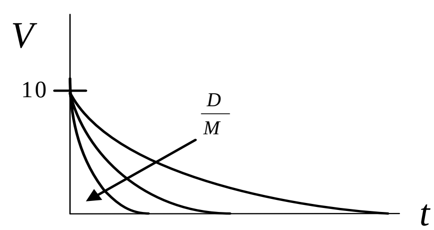
Apply initial condition

$$V(t = 0) = 10 \ ft/s \Rightarrow$$

$$10 = C$$

$$V_T = 10e^{\frac{-D}{M}t}$$

Plot response over time



- Exponential rate of decay described by *D/M* ratio

- Example 2: Diver with 20 lb submerged weight releases from zero velocity at top of pool (quiescent condition).

Solve using Laplace analysis methods:

$$M\dot{V} + DV = 20 \text{ (step input)}$$

$$M\left[sV(s) - V(0)\right] + DV(s) = \frac{20}{s}$$

$$sV(s) - V(0) + \frac{D}{M}V(s) = \frac{20}{M}\frac{1}{s}$$

$$V(s)\left(s + \frac{D}{M}\right) = \frac{1}{s}\frac{20}{M} + V(0) \qquad V(s) = \frac{\frac{1}{s}\frac{20}{M} + V(0)}{s + \frac{D}{M}} = \left\{\frac{\frac{20}{M}}{s\left(s + \frac{D}{M}\right)}\right\} + \frac{V(0)}{s + \frac{D}{M}}$$

use partial fraction $= \left\{\frac{\frac{20}{M}}{s\left(s+\frac{D}{M}\right)}\right\} = \frac{A}{s} + \frac{B}{s + \frac{D}{M}}$ $\qquad \left\{\frac{\frac{20}{M}}{s\ \left(s+\frac{D}{M}\right)}\right\} = \frac{A\left(s+\frac{D}{M}\right)}{s\left(s+\frac{D}{M}\right)} + \frac{Bs}{s\left(s+\frac{D}{M}\right)} = \frac{A\left(s+\frac{D}{M}\right) + Bs}{s\left(s+\frac{D}{M}\right)}$

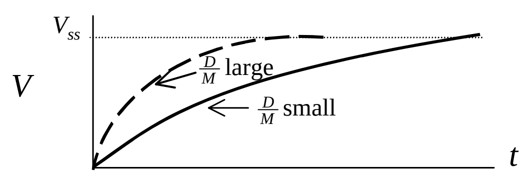
let $s = \frac{-D}{M}$; $\quad \frac{20}{M} = B\frac{-D}{M} \Rightarrow B = \frac{-20}{M}$, let $\quad s = 0 : \frac{20}{M} = A\frac{D}{M} \Rightarrow A = \frac{20}{D}$

$$V(s) = \left\{\frac{\frac{20}{D}}{S} + \frac{-\frac{20}{D}}{s + \frac{D}{M}}\right\} + \frac{V_{(0)}}{s + \frac{D}{M}},$$

$$v(t) = \left\{\frac{20}{D} - \frac{20}{D}e^{-\left(D/M\right)t}\right\} + V_{(0)}e^{-\left(D/M\right)t} \text{ since } V(0) = 0 \text{ then } v(t) = \frac{20}{D}\left[1 - e^{-\left(D/M\right)t}\right]$$

Note that $\frac{20}{D}$ is the steady state value i.e. $\qquad v(t) = V_{ss}\left[1 - e^{\frac{-D}{M}t}\right]$

This "force/damping" ratio is merely a scaling factor for the steady state.



- Several methods can be used to describe the quickness of convergence toward steady state (i.e., time to 99.999 % of V_{ss}, time to 1/2 V_{ss}).
- By convention, we use a % that directly reflects the exponent.
- Establish a time constant τ based on *D/M*.

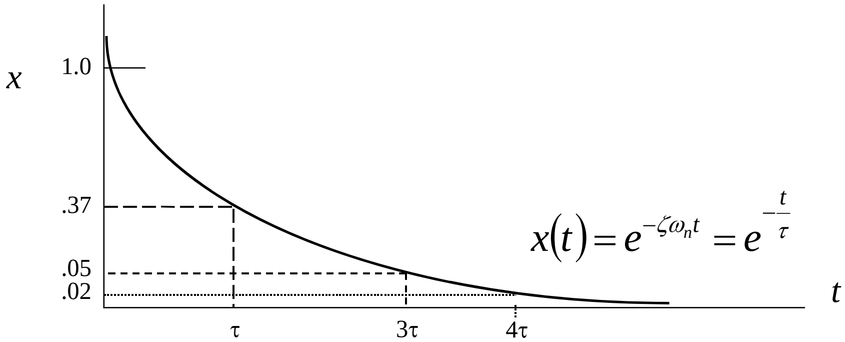
$$\frac{D}{M} \equiv \frac{1}{\tau} \quad \text{so} \quad e^{-\frac{D}{M}t} = e^{-\frac{t}{\tau}}$$

when $\quad t = \tau, \quad$ then $\quad e^{-\frac{t}{\tau}} = e^{-\frac{\tau}{\tau}} = e^{-1} = 0.36788$

so, after $\tau\left[\left(\frac{D}{M}\right)^{-1}\right]$ seconds have elapsed, $V = V_{ss}\left[1 - 0.36788\right] = 63.212\% V_{ss}$

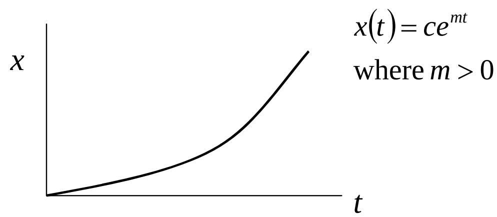
τ = time constant (time for parameter to reach 63% of its steady state value)

8.2.2 First Order Motion Descriptive Parameters



when $t = 0.6931\tau$: $x = e^{-.6931}$ = 0.5 (time to half amplitude)
when $t = \tau$: $x = e^{-1}$ = 0.37
when $t = 3\tau$ $x = e^{-3}$ = 0.05
when $t = 4\tau$: $x = e^{-4}$ = 0.02

- If exponent > 0, then motion is divergent.



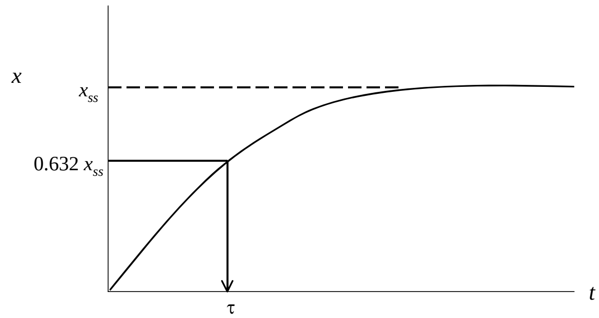
- τ again describes the exponential rate of divergence.
- By convention, the "time to double amplitude" (t_2) is usually applied as the evaluation metric.
- $x_{(t2)} = 2x_{(0)}$ where $x_{(0)} = ce^{m0}$

$$\text{Therefore} \quad x_2 = 2c$$
$$2c = ce^{mt_2}$$
$$2 = e^{mt_2}$$
$$\ln 2 = mt_2$$
$$\therefore \text{Time to double amplitude} \quad \frac{0.6931}{m} = t_2 = 0.693\tau$$

8.2.3 Determining τ from Step Input Time History

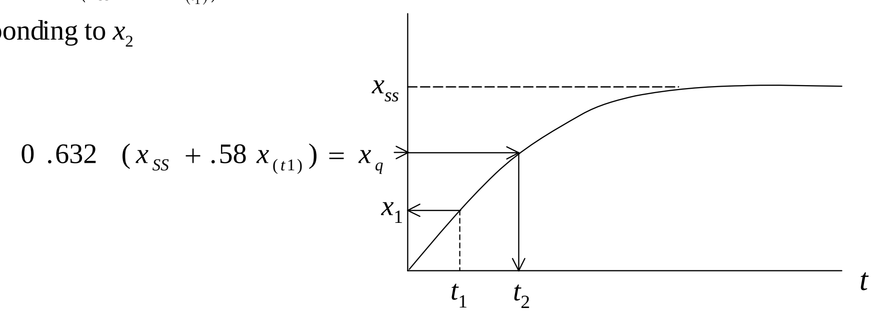
Method #1

τ = time to reach 0.632 x_{ss}



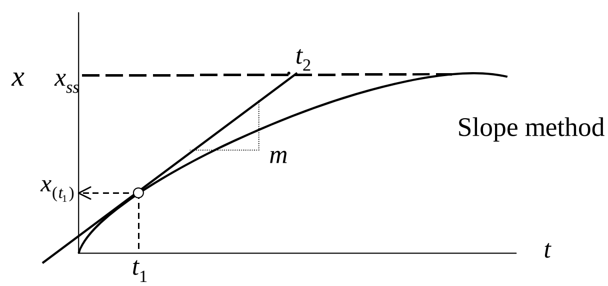
Method #2

1. Define x_{ss}
2. Measure x_1 at some time t_1
3. Calculate $x_2 = 0.632\left(x_{ss} + .58x_{(t_1)}\right)$
4. Find t_2 corresponding to x_2
5. $t_2 - t_1 = \tau$



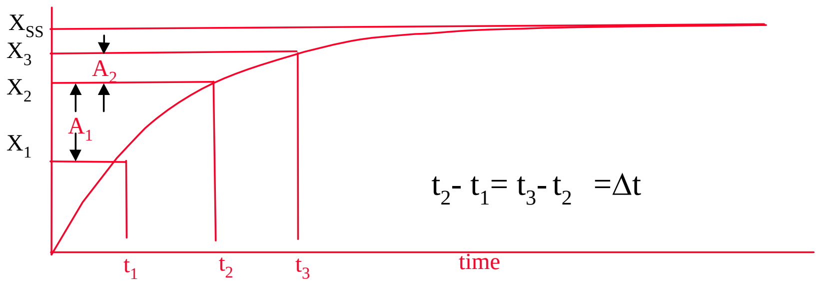
Method #3

1. Pick any time t_1.
2. Draw tangent at t_1.
3. Note t_2 where tangent intercepts x_{ss}.
4. $\tau = t_2 - t_1$



Method #4 When X_{SS} is unknown use

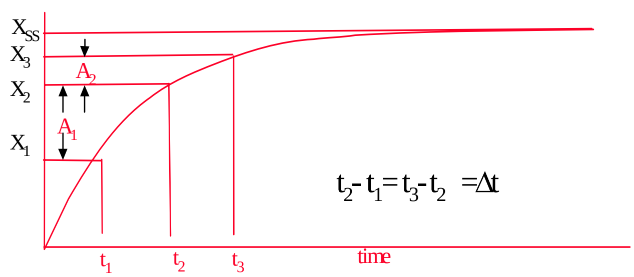
$$\tau = \frac{\Delta t}{\ln\left(\frac{A_1}{A_2}\right)}$$



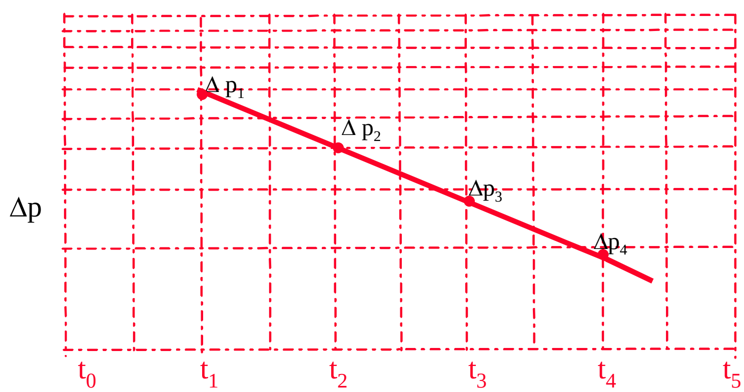
Method #5 When *Xss* is known, use $\tau = \frac{-\Delta t}{\ln\left[\frac{X_{SS} - X_1}{X_{SS} - X_2}\right]}$

Linearity check:
Note parameter change between even time increments.

Plot parameter changes vs elapsed time on semi-log scale



Slope of line equals τ

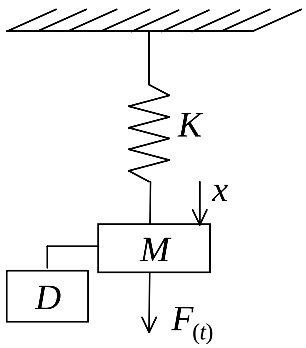


8.3 Second Order Motion

Found in classical aircraft phugoid, Dutch roll and short period modes as well as noise filter and vibration testing. Named second-order because the motions are described by mathematics using the second derivative of a parameter.

8.3.1 Elements of Second Order Motion

Mechanical systems have elements of spring, mass, and usually damping. Forcing functions can be included (see illustration).



K	= spring stiffness (F/x)
x	= displacement from equilibrium
M	= mass
$F_{(t)}$	= forcing function
D	= damping

$$\sum F_{vert} : M\ddot{x} + D\dot{x} + Kx = F(t)$$

Natural character is observed when system is allowed to move with no external input [$F(t) = 0$]

$$\sum F_{vert} : M\ddot{x} + D\dot{x} + Kx = 0$$

Apply operator technique: let $x = e^{st}, \dot{x} = se^{st}, \ddot{x} = s^2 e^{st}$

$$\Rightarrow e^{st}\left(Ms^2 + Ds + K\right) = 0$$

Divide out e^{st}, since it never equals zero, the characteristic equation remains:

$$Ms^2 + Ds + K = 0 \quad \text{or} \quad s^2 + \frac{D}{M}s + \frac{K}{M} = 0$$

The values of s that satisfy this equation are called the roots

$$\left[x = c_1 e^{s_1 t} + c_2 e^{s_2 t}\right]$$

Solve for the roots using the quadratic equation

8.3.2 Second Order Motion Descriptive Parameters

Solution (x) calculated as

$$x = c_1 e^{s_1 t} + c_2 e^{s_2 t} \quad \text{where} \quad s_1, s_2 = \frac{-D}{2M} \pm j\sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)^2}$$

Apply Euler's identity for complex conjugate roots

$$x = Ae^{\frac{-D}{2M}t} \sin\left(\sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)^2}\, t + \phi\right)$$

- ϕ defines the **phase shift**.
- A defines the **initial amplitude**.
- The real part of the root [$D/2M$] defines the **envelope** of the motion.
- The imaginary part of the root identifies the **damped frequency** $\sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)^2}$ of the oscillations, ω_d (rad/sec).

$$\omega_d = \sqrt{\tfrac{K}{M} - \left(\tfrac{D}{2M}\right)^2}$$

- If damping is reduced to $D = 0$ then only $[K/M]^{1/2}$ remains. This is the undamped or **"natural" frequency** (ω_η).

$$\omega_n \equiv \sqrt{\tfrac{K}{M}}$$

- If $\left(\frac{D}{2M}\right)^2 = \frac{K}{M}$ then D is conisidered to be critical [just enough to prevent oscillations]

$$D_{crit} = \sqrt{\frac{K}{M}}\, 2M = 2\sqrt{KM}$$

- For oscillatory motion, actual system damping is typically expressed as a fraction critical damping. Define **damping ratio** as

$$\zeta \equiv \frac{D}{D_{crit}} = \frac{D}{2\sqrt{KM}}$$

Combining $\omega_d \equiv \sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)}^2$

with $\omega_n \equiv \sqrt{\frac{K}{M}}$ and $\zeta = \frac{D}{2\sqrt{KM}}$

gives $\omega_d \equiv \omega_n \sqrt{1-\zeta^2}$ and $\frac{D}{2M} = \zeta\omega_n$

The values can be substituted to give

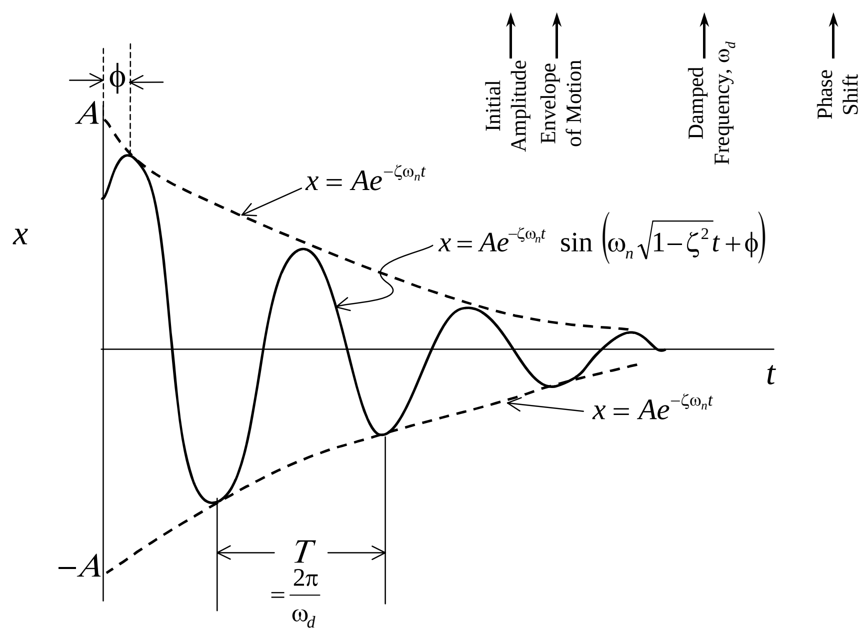
$$x = Ae^{-\frac{D}{2M}t} \ \sin\left(\sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)^2} + \phi\right)$$

$$= Ae^{-\zeta\omega_n t} \ \sin\left(\omega_d + \phi\right)$$

$$= Ae^{-\zeta\omega_n t} \ \sin\left(\omega_n \sqrt{1-\zeta^2} + \phi\right)$$

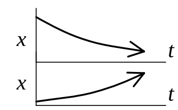
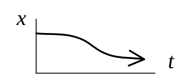
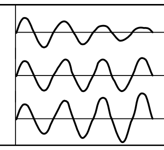
where $\phi = \tan^{-1}\frac{\omega_d}{\zeta\omega_n} = \tan^{-1}\frac{\omega_n\sqrt{1-\zeta}}{\omega_n} = \tan^{-1}\frac{\sqrt{1-\zeta}}{\zeta}$

$$x = Ae^{-\zeta\omega_n t} \ \sin\left(\omega_n \sqrt{1-\zeta^2}\,t + \phi\right)$$



$$\text{Roots} \quad s_{1,2} = \frac{-D}{2M} \pm j\sqrt{\frac{K}{M} - \left(\frac{D}{2M}\right)^2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Possible Solutions:

if $\left(\frac{D}{2M}\right)^2$	then roots are	and system response is	
$> \frac{k}{m}$	Real and unequal	Non-oscillatory ~ convergent if $D > 0$ ~ divergent if $D < 0$	
$= \frac{k}{m}$	Real and equal	Non-oscillatory ~ convergent if $D > 0$ ~ divergent if $D < 0$	
$< \frac{k}{m}$	Complex (purely imaginary if $D = 0$)	Non-oscillatory ~ convergent if $D > 0$ ~ neutral if $D = 0$ ~ divergent if $D < 0$	

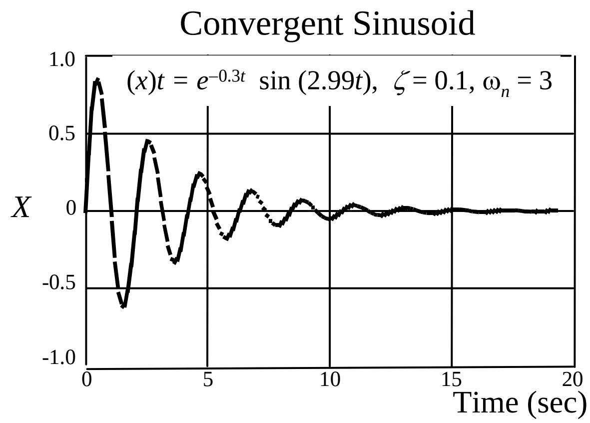
- The various combination of K, M, and D and their effects on system response can be related to damping ratio ζ as follows:

$\zeta > 1$	Real & unequal roots exponential, convergent	$x(t) = c_1 e^{-\omega_n\left(\zeta+\sqrt{\zeta^2-1}\right)t} + c_2 e^{-\omega_n\left(\zeta-\sqrt{\zeta^2-1}\right)t}$
$\zeta = \pm 1$	Real & equal roots exponential, conv or div	$x(t) = c_1 e^{-\zeta\omega_n t} + c_2 t e^{-\zeta\omega_n t}$
$0 < \zeta < 1$	Complex pair roots sinusoidal, convergent	$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$
$\zeta = 0$	Imaginary pair roots sinusoidal, neutral	$x(t) = A\sin(\omega_n t + \phi)$ $= A\cos\omega_n t$
$-1 < \zeta < 0$	Complex pair roots sinusoidal, divergent	$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$
$\zeta < -1$	Real & unequal roots exponential, divergent	$x(t) = c_1 e^{-\omega_n\left(\zeta+\sqrt{\zeta^2-1}\right)t} + c_2 e^{-\omega_n\left(\zeta-\sqrt{\zeta^2-1}\right)t}$

Damping ratio effect on second order system

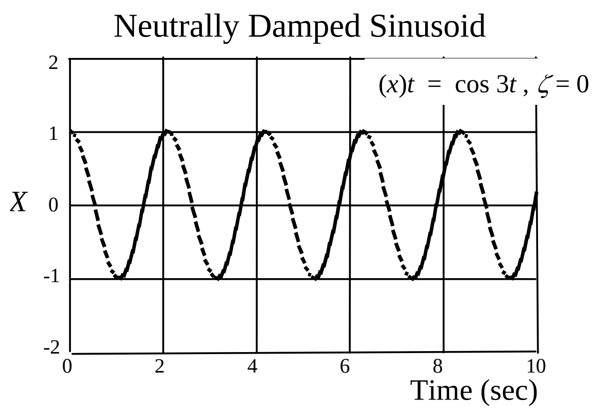
Response of various second order systems to an impulse input.

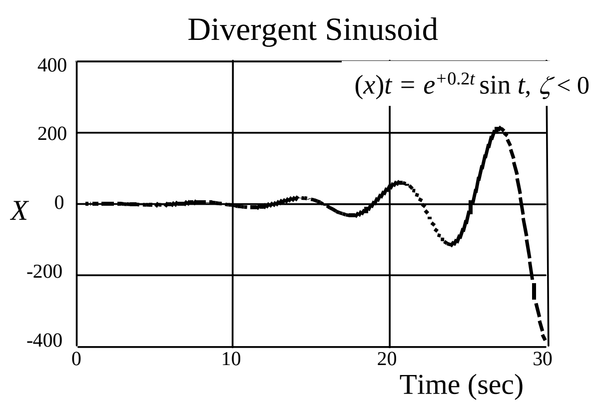
Second-order systems are oscillatory if
-1 > ζ > 1.



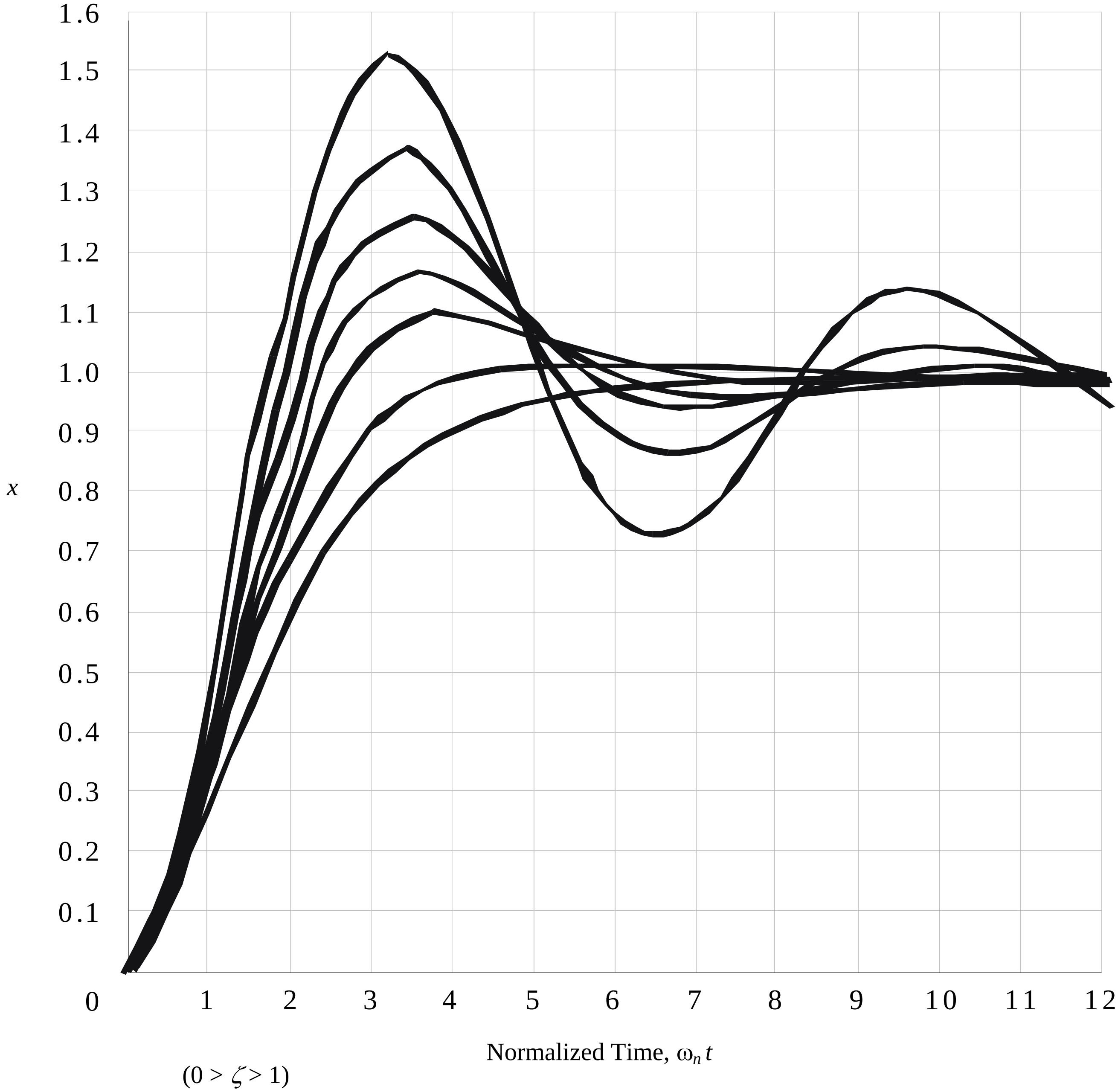
Motion typically described by
ω_n and ζ

T, ω_d, ω_n and ζ are linked such that knowledge of any two will yield the other two.





2nd order system response to **unit step input** for **underdamped systems**



Decay rates (for 0 < ζ <1) and Useful Insights

- Time to decay $$\zeta\omega_n = \frac{\ln \frac{x_1}{x_2}}{t_2 - t_1} \Rightarrow \Delta t = \frac{\ln \frac{x_1}{x_2}}{\zeta\omega_n}$$

  Time to $\frac{1}{2}$ amplitude $$\equiv \Delta t_{\frac{1}{2}} = \frac{\ln \frac{2}{1}}{\zeta\omega_n} = \frac{0.6931}{\zeta\omega_n}$$

  Time to $\frac{1}{10}$ amplitude $$\equiv \Delta t_{\frac{1}{10}} = \frac{\ln \frac{10}{1}}{\zeta\omega_n} = \frac{2.3026}{\zeta\omega_n}$$

  Time to $\frac{1}{x}$ amplitude $$\equiv \Delta t_{\frac{1}{x}} = \frac{\ln x}{\zeta\omega_n}$$

- # cycles to $\frac{1}{2}$ amplitude # cycles $$= \frac{\Delta t}{T} = \frac{\Delta t\,\omega_d}{2\pi}$$

  # cycles to $\frac{1}{2}$ amplitude $$\equiv C_{\frac{1}{2}} = \frac{0.6931}{\zeta\omega_n}\frac{\omega_d}{2\pi} = \frac{0.6931}{2\pi}\frac{\omega_d}{\zeta\omega_n} = \frac{0.1103}{\varepsilon}$$

  # cycles to $\frac{1}{10}$ amplitude $$\equiv C_{\frac{1}{10}} = \frac{2.3026}{\zeta\omega_n}\frac{\omega_d}{2\pi} = \frac{2.3026}{2\pi}\frac{\omega_d}{\zeta\omega_n} = \frac{0.3665}{\varepsilon}$$

  # cycles to $\frac{1}{x}$ amplitude $$\equiv C_{\frac{1}{x}} = \frac{\ln x}{2\pi}\frac{\omega_d}{\zeta\omega_n}$$

- Same analysis applies for exponentia l growth

  For same Δt $$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} \quad \text{etc}$$

  Therefore $$\frac{x_1}{x_3} = \frac{x_1}{x_2}\frac{x_2}{x_3} = \left(\frac{x_1}{x_2}\right)^2 \quad \Rightarrow \quad HCAR = \sqrt{\text{full cycle amp ratio}}$$

HCAR $$\equiv \frac{x_1}{x_2} = e^{\zeta\omega_n(t_2 - t_1)} = e^{\zeta\omega_n(\Delta t)} = e^{\zeta\omega_n\left(\frac{T}{2}\right)} = e^{\zeta\omega_n\left(\frac{2\pi}{2\omega_d}\right)}$$

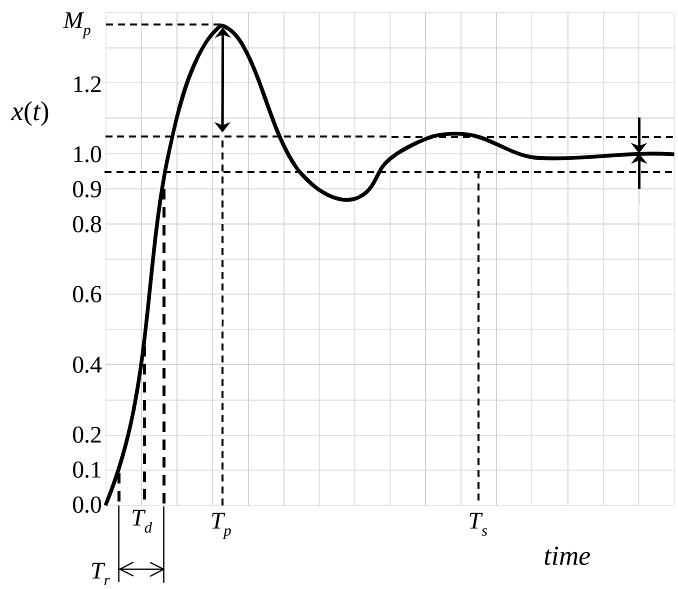
$$= e^{\zeta\omega_n\left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}\right)} = e^{\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$ Note: $HCAR = f(\text{only })$

$$\Rightarrow \ln\frac{x_1}{x_2} = \frac{\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = \sqrt{\frac{\left(\ln\frac{x_1}{x_2}\right)^2}{\pi^2 + \left(\ln\frac{x_1}{x_2}\right)^2}}$$

This can be applied in graphical form (transient peak ratio method)

8.3.3 Determining Descriptive Parameters

Time domain metrics



Peak Value, *MP*: largest value

Final Value, *FV*: steady state value

Delay Time, T_d: 50% of final value

Rise Time, T_r: 10% - 90% of FV

Peak Time, T_p: time to MP

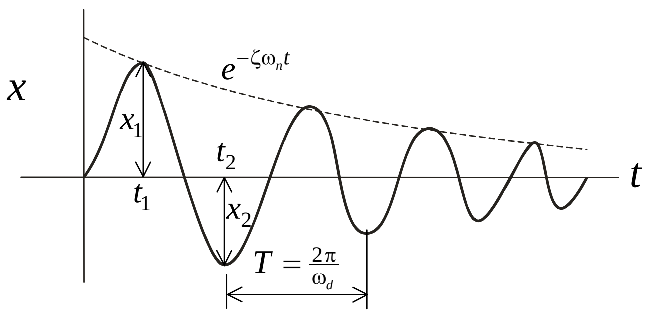
Settling Time, T_s: time to reach some defined % of final value

% Overshoot, *PO*: $\frac{MP-1}{1} \times 100\%$ *target value = unity*

Method #1 Basic Analysis

Note $x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + 0)$

$[\zeta\omega_n]$ and $[\omega_d]$ describe a time history



$$f_d = \frac{1}{T} \quad \omega_d = \frac{2\pi}{T} \quad \boxed{\zeta\omega_n = \frac{\ln\frac{x_1}{x_2}}{(t_2 - t_1)}}$$

Easily measured values: can use any points on envelope

Define $\varepsilon \equiv \dfrac{\zeta\omega_n}{\omega_d} = \dfrac{\frac{\ln\frac{x_1}{x_2}}{(t_2-t_1)}}{\frac{2\pi}{T}} = \dfrac{T\ln\frac{x_1}{x_2}}{2\pi(t_2-t_1)}$

where $(t_2 - t_1) = \dfrac{T}{2} \Rightarrow \dfrac{\zeta\omega_n}{\omega_d} = \dfrac{\ln\frac{x_1}{x_2}}{\pi} = \varepsilon = \ln[x_1/x_2]/\pi$

recall $\omega_d = \omega_n\sqrt{1-\zeta^2}$

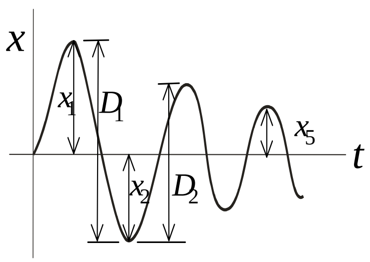
$$\varepsilon = \frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}} = \frac{\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \varepsilon^2 = \frac{\zeta^2}{1-\zeta^2} \Rightarrow \varepsilon^2(1-\zeta^2) = \zeta^2 \Rightarrow \varepsilon^2 = \zeta^2[1+\varepsilon^2]$$

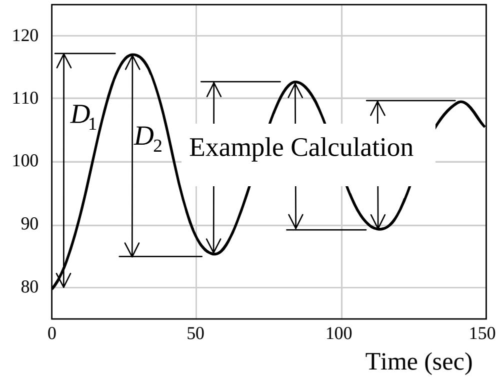
$$\zeta = \sqrt{\frac{\varepsilon^2}{1+\varepsilon^2}} \qquad \omega_n = \frac{\zeta\omega_n}{\zeta} = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

$\left|\dfrac{x_1}{x_2}\right| \equiv$ Half - Cycle Amplitude Ratio (*HCAR*) $= e^{\zeta\omega_n(t_2-t_1)}$

Method #2 Transient Peak Ratio Analysis

1) Measure either D or x distances as shown.





2) Note ratio of adjacent peak values (transient peak ratios).
3) Average several TPRs.
4) Use equation to find ζ:

$$\zeta = \sqrt{\frac{\left(\ln \frac{x_1}{x_2}\right)^2}{\pi^2 + \left(\ln \frac{x_1}{x_2}\right)^2}}$$

$$\text{First TPR}: \quad \frac{D_2}{D_1} = \frac{117-86}{117-80} = \frac{31}{37} = 0.8378$$

$$\text{Second TPR}: \quad \frac{D_3}{D_2} = \frac{112-86}{117-86} = \frac{26}{31} = 0.8387$$

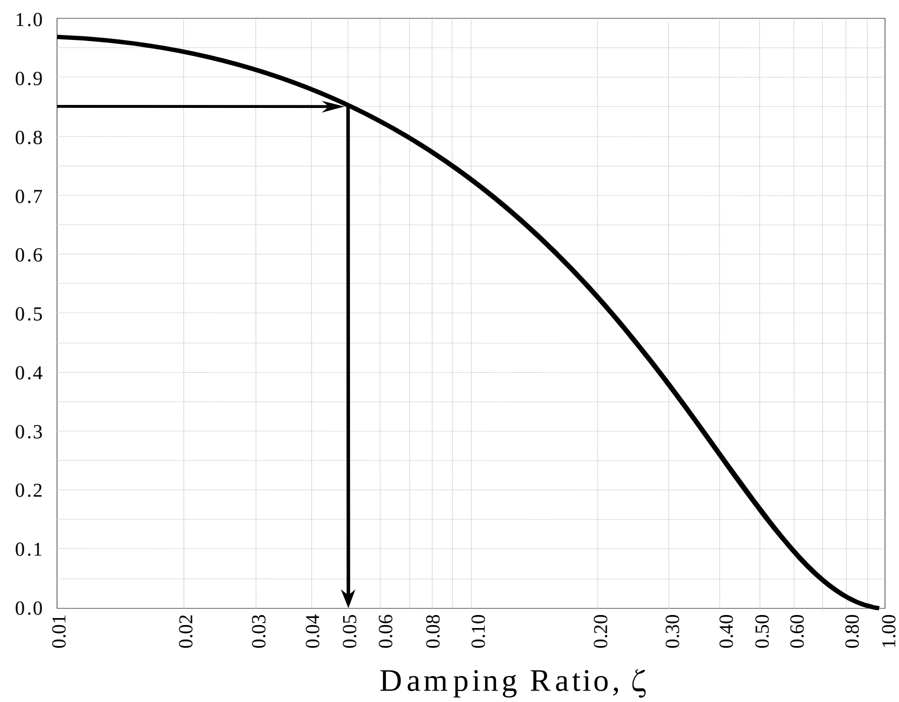
$$\text{Third TPR}: \quad \frac{D_4}{D_3} = \frac{112-90}{112-86} = \frac{22}{26} = 0.8462$$

Average TPR = 0.8409

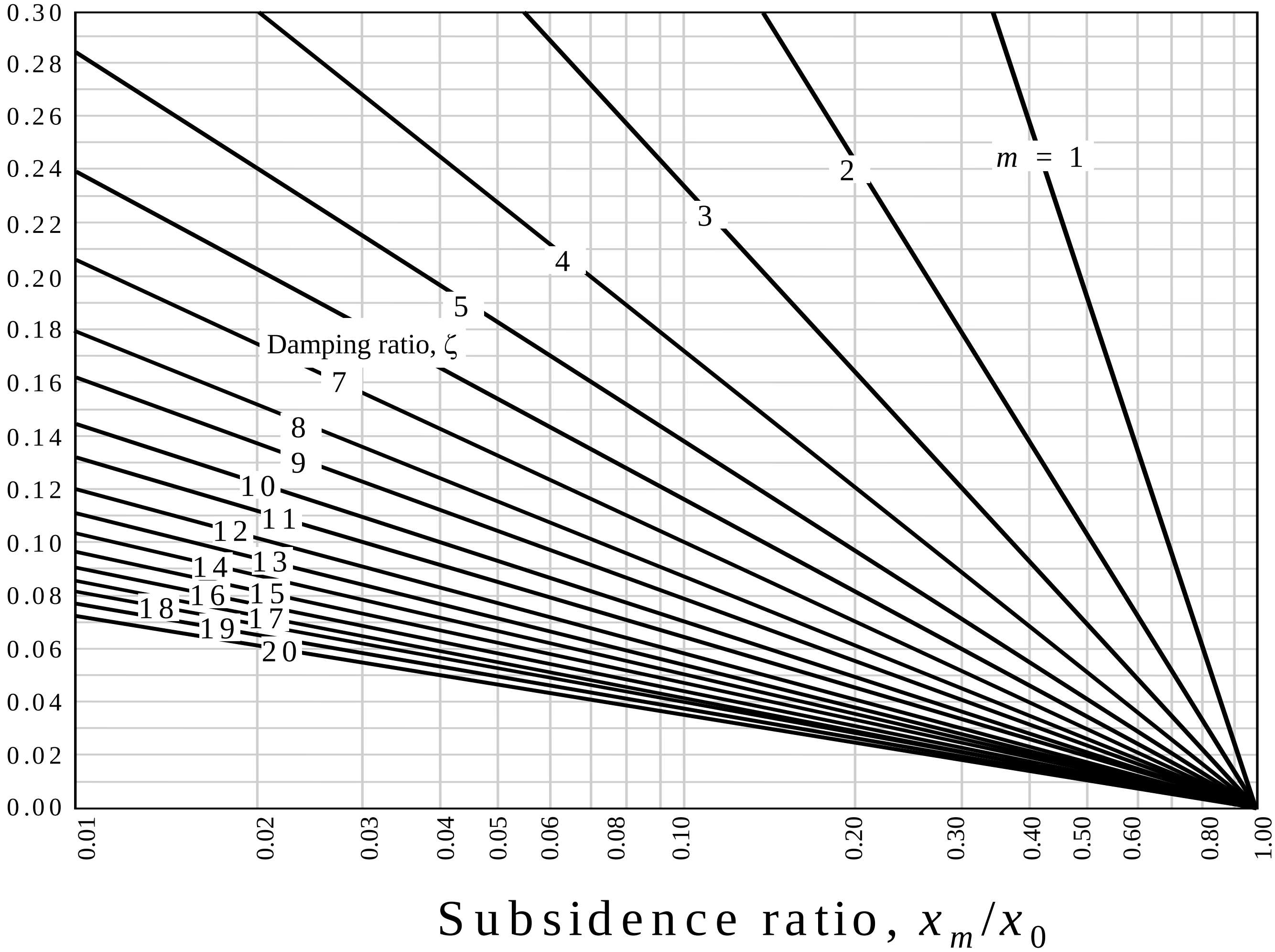
4a) Can use
D_1/D_2 or x_1/x_2
ratios in above equation.

4b) In lieu of equation, use adjacent look-up curve to find ζ.

4c) Time ratio method works better with heavy damping.



Method #3 Multiple *TPR* Analysis



To determine damping ratio

~ Use the m = 1 line when comparing the next ratio.

$$\frac{\Delta x_1}{\Delta x_0} = \frac{2.5}{4.5} = 0.56 \qquad \zeta = 0.16$$

$$\frac{\Delta x_2}{\Delta x_1} = \frac{1.5}{2.5} = 0.60 \qquad \zeta = 0.14$$

~ Use the m = 2 line for comparing every other peak ratio.

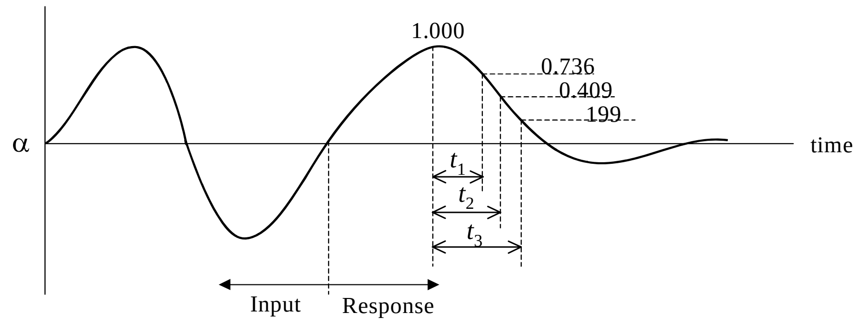
$$\frac{\Delta x_2}{\Delta x_0} = \frac{1.5}{4.5} = 0.33 \qquad \zeta = 0.16$$

$$\frac{\Delta x_3}{\Delta x_1} = \frac{1.0}{2.5} = 0.40 \qquad \zeta = 0.14$$

Method #4 Time Ratio Analysis

- If the damping ratio is between 0.5 and 1.0 (two or less overshoots), then the time ratio method can be used to determine frequency and damping ratio. Select a peak where the response if free.

- Note times for amplitude to reduce to 73.6%, 40.9%, and 19.9% of the peak value.



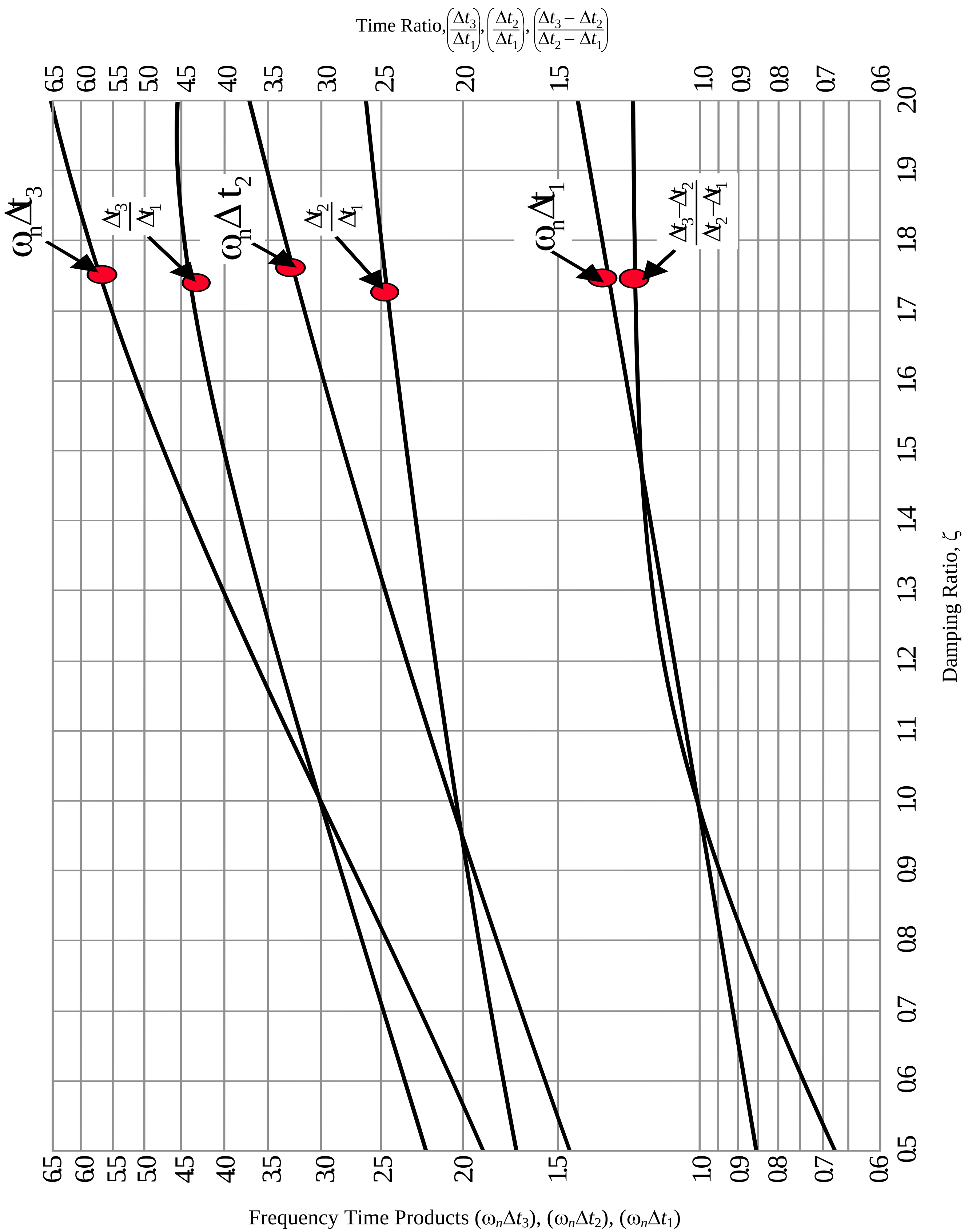
- Form the time ratios t_2/t_1, t_3/t_1, and $[t_3-t_2]/[t_2-t_1]$

- Enter the next figure at the time ratio side to find ζ for each time ratio.

- This technique is valid if the system transfer function has no zeros.

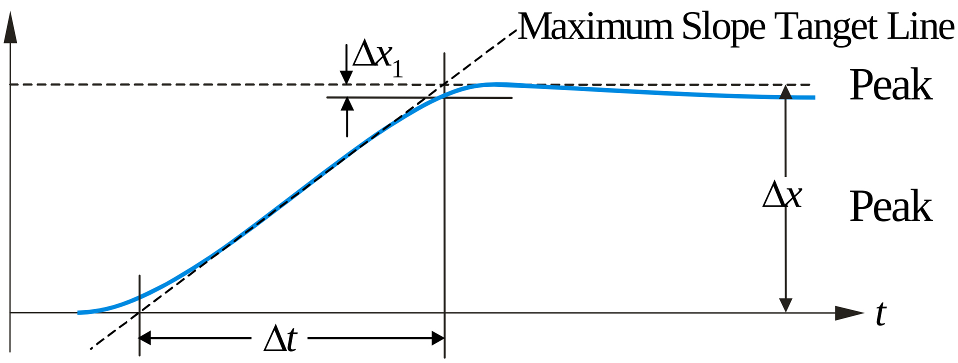
- If recorded measurements are not available and if the number of overshoots is between 2 and 6, then

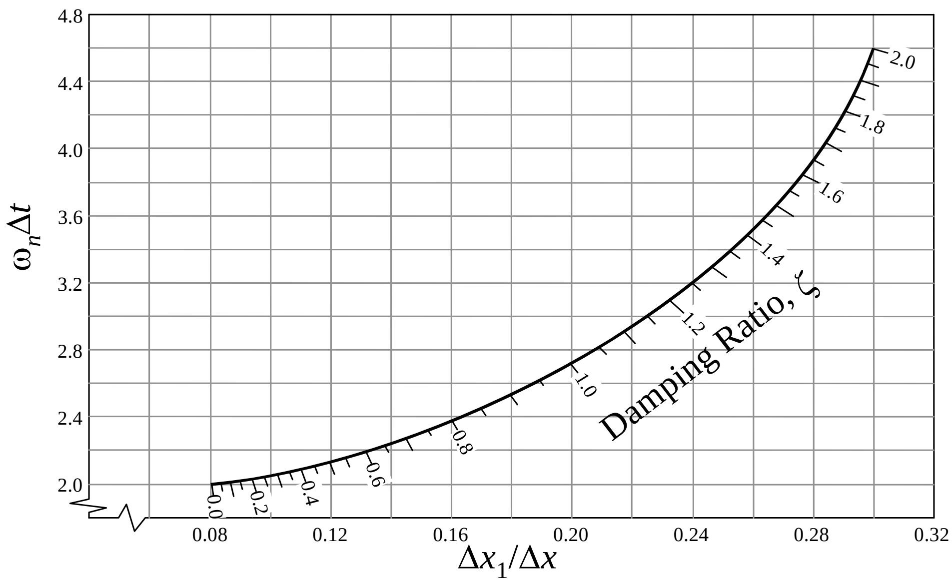
$$\zeta \approx \frac{7 - \#overshoots}{10}$$



Time Ratio, $\left(\frac{\Delta t_3}{\Delta t_1}\right), \left(\frac{\Delta t_2}{\Delta t_1}\right), \left(\frac{\Delta t_3 - \Delta t_2}{\Delta t_2 - \Delta t_1}\right)$
$\omega_n \Delta t_3$
$\frac{\Delta t_3}{\Delta t_1}$
$\omega_n \Delta t_2$
$\frac{\Delta t_2}{\Delta t_1}$
$\omega_n \Delta t_1$
$\frac{\Delta t_3 - \Delta t_2}{\Delta t_2 - \Delta t_1}$
6.5 6.0 5.5 5.0 4.5 4.0 3.5 3.0 2.5 2.0 1.5 1.0 0.9 0.8 0.7 0.6
0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
Damping Ratio, ζ
Frequency Time Products $(\omega_n \Delta t_3)$, $(\omega_n \Delta t_2)$, $(\omega_n \Delta t_1)$

Method #5 Maximum Slope Analysis



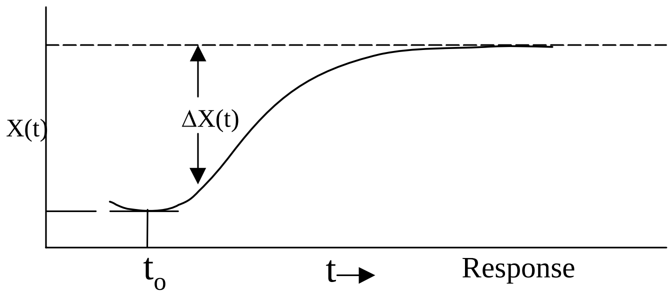


- Calcu- lation of ζ somewhat sensitive to Δx_1 measurement
- $\omega_n = \dfrac{\omega_n \Delta t}{\Delta t}$ not too sensitive to Δx_1
- Best if $0.5 \le \zeta \le 1.4$
- Initial overshoot approximation: let (step inputs only) $K \equiv \dfrac{X_{pK}}{X_{ss}} - 1$

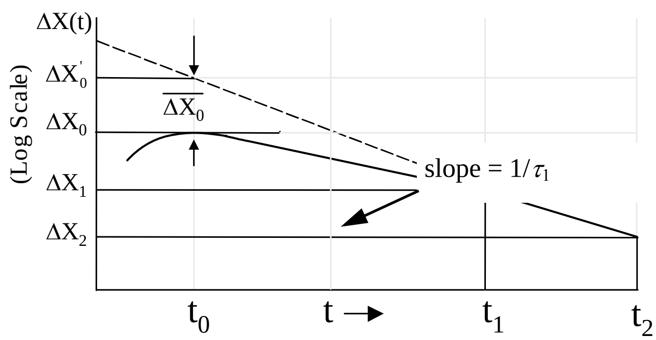
$$\zeta = \left[\frac{\left(\frac{-1}{\pi} \ln K \right)^2}{1 + \left(\frac{-1}{\pi} \ln K \right)^2} \right]^{\frac{1}{2}}$$

Method #6 Separated Real Root Analysis (when $\zeta>1$)

1) Determine several steady state $\Delta X(t)$ values from time history



2) Plot ΔX vs t on semi-log scale



3) After the faster root has decayed, the semi-log plot will be a straight line whose slope determines the slower root ($1/\tau_1$)

$$\tau_1 = \frac{t_1 - t_2}{\ln\left(\frac{\Delta\chi_1}{\Delta\chi_2}\right)}$$

4) Determine by extrapolating the straight line portion of the response to establish the values

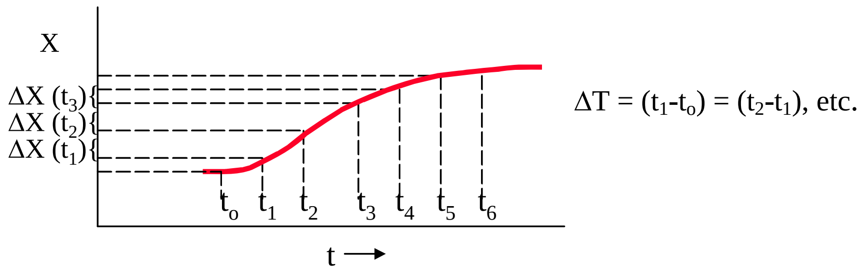
$$\left(\frac{1}{\tau_2}\right) \qquad \overline{\Delta\chi_0} \ \& \ \Delta\chi_0'$$

$$\tau_2 = \tau_1\left(\frac{\overline{\Delta\chi_0}}{\Delta\chi_0'}\right) \qquad \boxed{\omega_n = \sqrt{\frac{1}{\tau_1}\frac{1}{\tau_2}}} \text{ and } \boxed{\zeta = \frac{-\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)}{2\omega_n} = \frac{-\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)}{2\sqrt{\frac{1}{\tau_1}\frac{1}{\tau_2}}}}$$

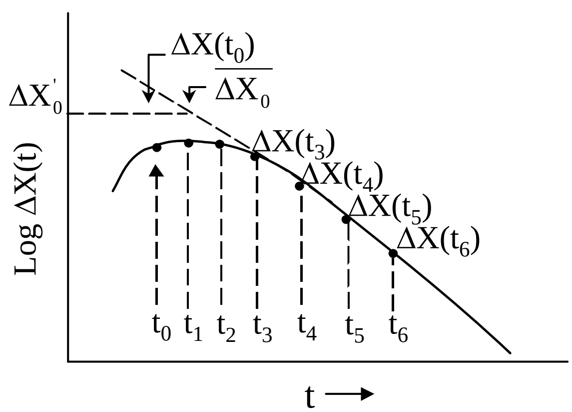
Method #7 Modified Separated Real Root Analysis

- Method #6 is sensitive to errors in determining steady state values
- Alternate method is to avoid need for steady state value
- Define $\Delta X(t) \equiv [\, x(t+\Delta T) - x(t)]$ where ΔT is a time increment

1) From time history, measure ΔX values according to definition



2) Plot $\Delta X\ (t)$ vs time on semi-log scale



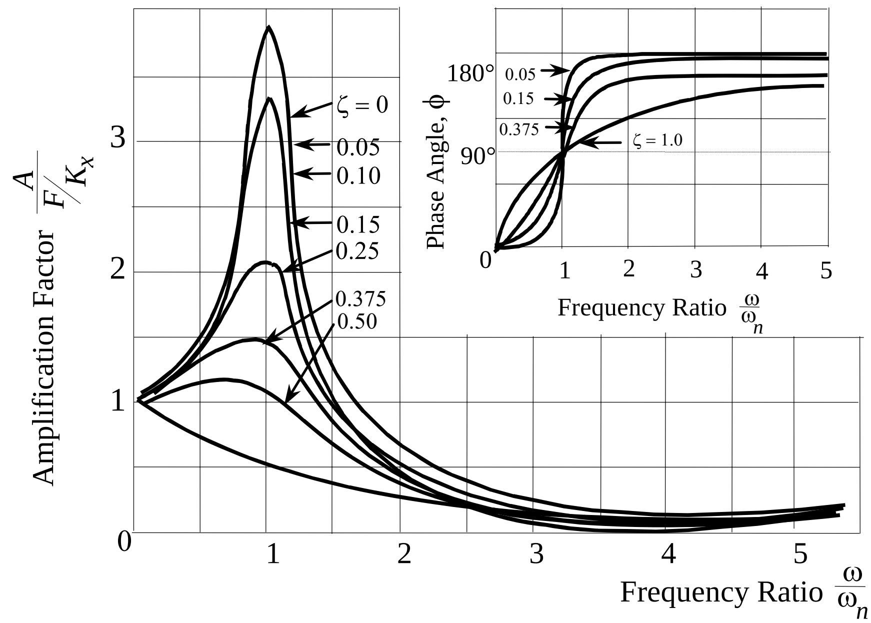
3) Use previous method to determine roots and characteristics

- Gross error will result if ζ is actually <1
- If ζ is near 1, check results using time ratio or slope method

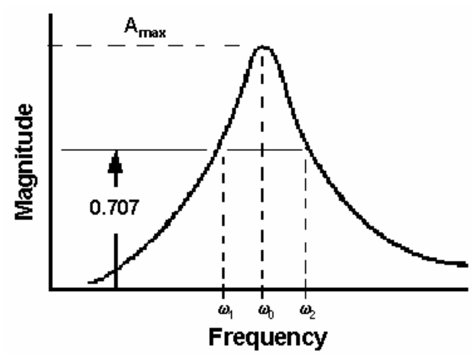
Method # 8 Frequency Sweep Analysis

Determine ω_n and ζ using sinusoidal inputs.

- This “forced response” method most useful when damping is heavy.
- For a second order system, output/input amplitude ratio and phase shift are a function of input frequency.



- Amplitude ratio peaks at “resonant” frequency, ω_r.
- Resonant peaks increase as ζ decreases below 0.707.
- Peak amplitude ratio “rolls off” as ζ increases above 0.707.
- Resonant frequency approaches natural fre quency as damping decreases: $\omega_r = \omega_n[1 - 2\zeta^2]^{.5}$
- Phase shift = 90° if excited at ω_n, regardless of damping ratio.

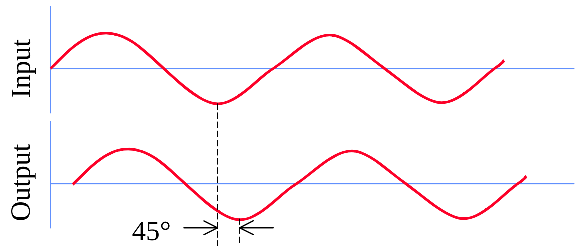


$$\zeta = 0.5(\omega_2 - \omega_1)/\omega_n$$

Frequency Sweep Analysis (continued)

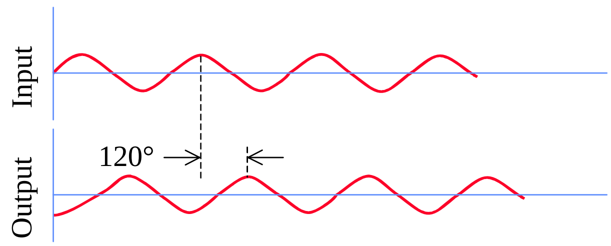
1. Using sinusoidal inputs excite system @ ω near ω_n

2. Measure phase lag (ϕ) of $\frac{output}{input}$

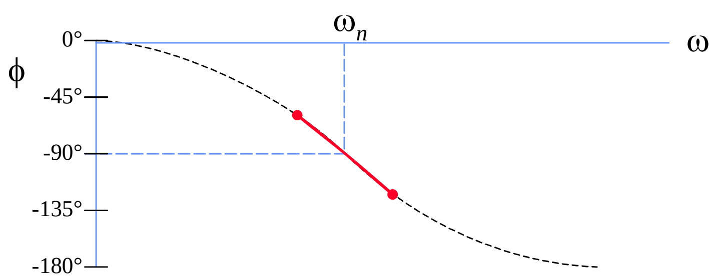


3. Excite system @ another ω near ω_n

4. Again Measure phase lag ϕ



5. Plot ϕ vs input frequency

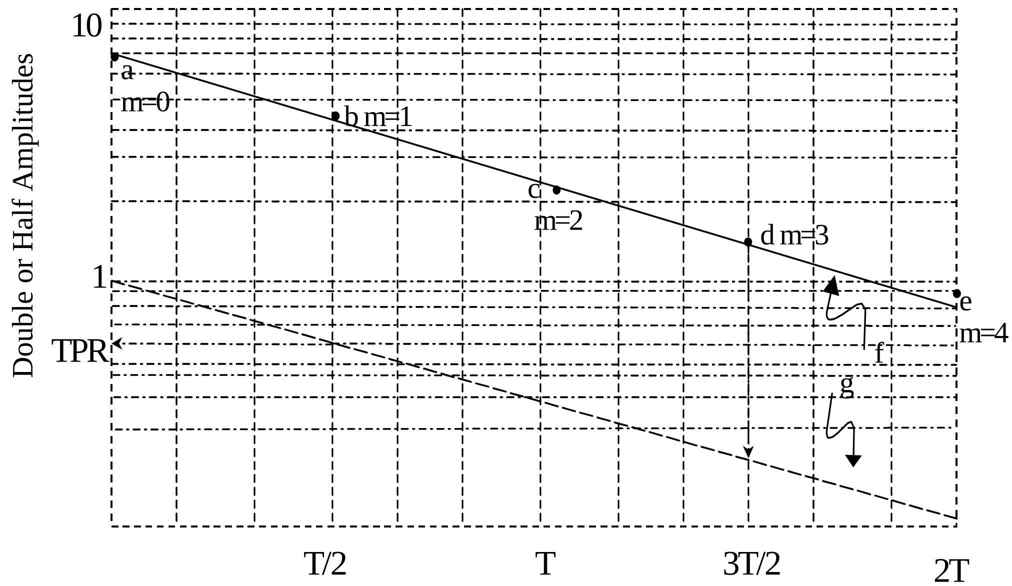


6. ω_n occurs at $\phi = 90°$

Linearity Check /Accuracy Improvement

1) On semi-log scale, plot ratio of initial amplitude to subsequent peak amplitudes at each half cycle (points *a*-*e*).

2) Fair straight line (*f*) through these points.



3) Draw line (*g*) parallel (*f*) intercepting the ordinate at *TPR*=1

4) Average *TPR* occurs at *T*/2 on line *g*

8.4 Complex Plane

Begin with sum of forces in spring-mass-damper example

$$M\ddot{x} + D\dot{x} + Kx = f(t)$$

let $x = e^{st}$, find transient solution

$$\left(Ms^2 + Ds + K\right)e^{st} = 0$$

Apply quadratic equation to solve for roots

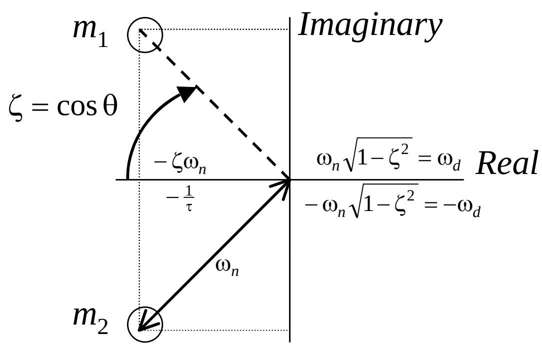
$$s_{1,2} = \tfrac{-D}{2M} \pm \sqrt{\left(\tfrac{D}{2M}\right)^2 - \tfrac{K}{M}}$$

Recall previous analogy

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\tau = \frac{1}{\zeta\omega_n}$$

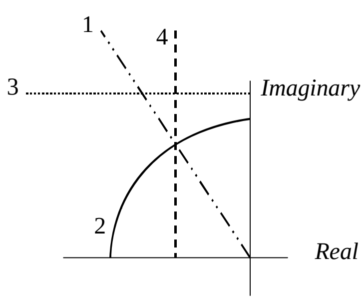
$$-\frac{1}{\tau} \pm j\omega_d$$



Location of Roots on Complex Plane

1. Line of constant damping ratio ζ – varying $C_{1/n}$ and ω_n
2. Line of constant ω_n – varying ζ
3. Line of constant ω_d and period (T)
4. Line of constant real part ($\zeta\omega_n$) and time to damp ($T_{1/n}$)

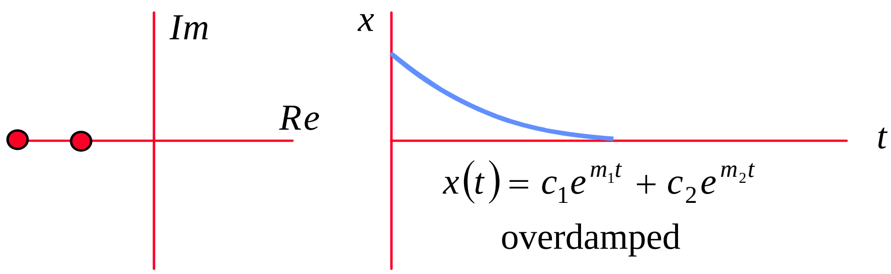
$\sigma = \zeta\omega_n = 1/\tau$ = damping rate

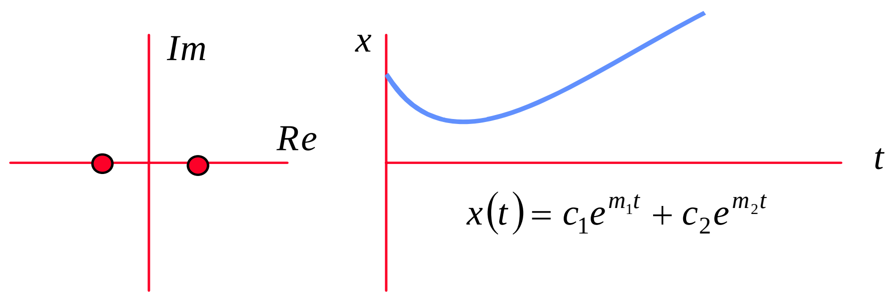


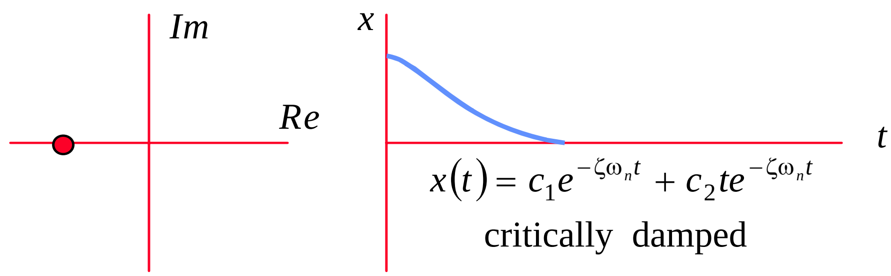
Sample second order root plots and corresponding time histories

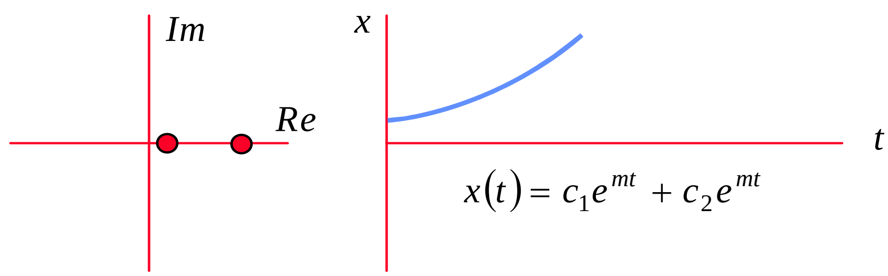
(time histories represent trends only)

Examples of “two real roots”



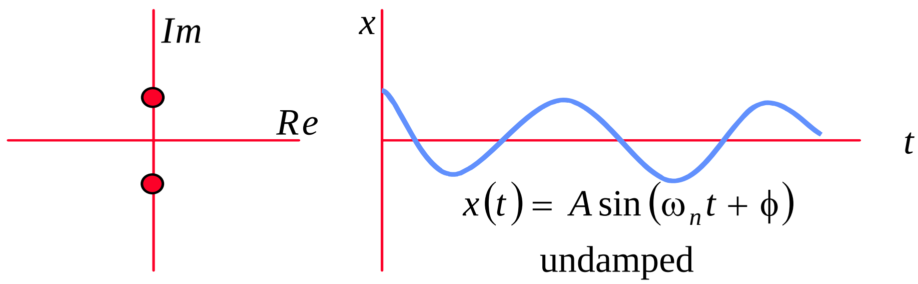


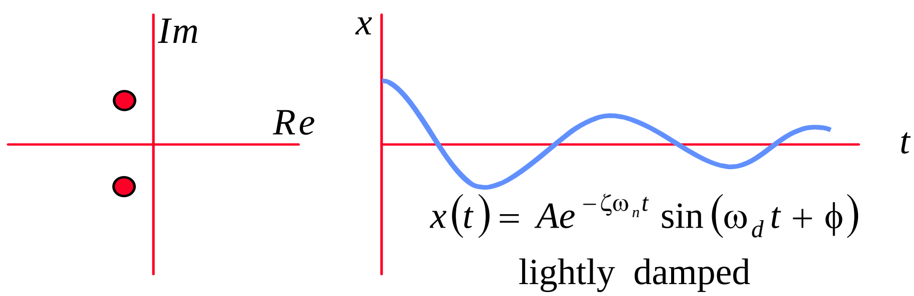


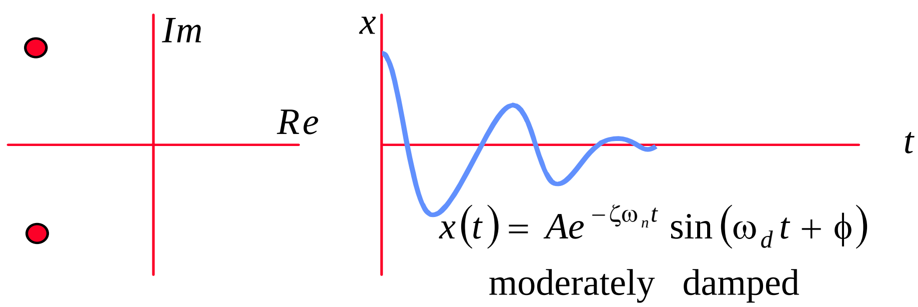


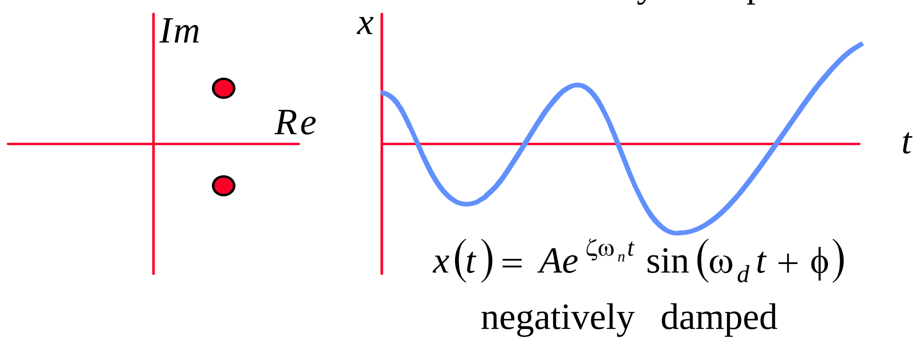
More sample second order root plots and corresponding time histories

Examples of “imaginary roots”









8.5 Parameter Conversions

For conversion of accelerometer measurements.

- For magnitude conversion substitute $2\pi f$ for $j\omega$.
- Assumes linear spectra.
- Conversion factor should be squared for power spectra.

Acceleration to velocity

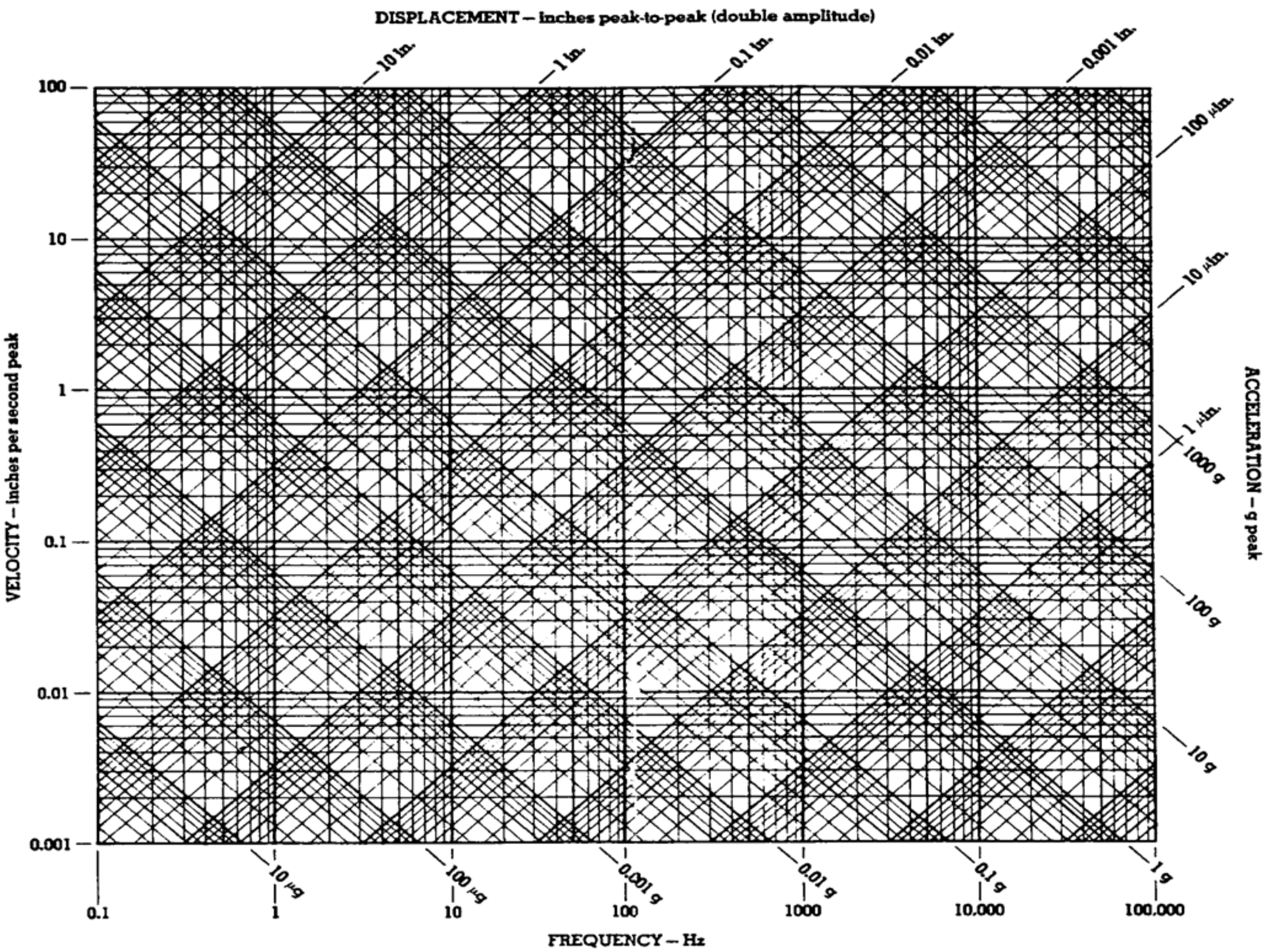
to convert from	to	multiply by
ft/s² rms	*ft/s rms*	$1/j\omega$
ft/s² rms	*in/s rms*	$12/j\omega$
ft/s² rms	*in/s peak*	$16.97/j\omega$
g rms	*in/s rms*	$386/j\omega$
g rms	*in/s peak*	$545.8/j\omega$
m/s² rms	*mm/s rms*	$1000/j\omega$
m/s² rms	*mm/s peak*	$1414/j\omega$
g rms	*mm/s rms*	$9806/j\omega$
g rms	*mm/s peak*	$13865.7/j\omega$

Acceleration to Displacement

to convert from	to	multiply by
ft/s² rms	*in rms*	$12/(j\omega)^2$
ft/s² rms	*in p-p*	$33.9/(j\omega)^2$
ft/s² rms	*mil p-p*	$33.9\ E\ 03/(j\omega)^2$
g rms	*in rms*	$386/(j\omega)^2$
g rms	*in p-p*	$1091.6\ E\ 03/(j\omega)^2$
g rms	*mil p-p*	$1091.6\ E\ 03/(j\omega)^2$
m/s² rms	*mm rms*	$1000/(j\omega)^2$
m/s² rms	*mm p-p*	$2828/(j\omega)^2$
m/s² rms	micron *p-p*	$2828\ E\ 03/(j\omega)^2$

E= engineering exponent (x10 __)
g = 32.174 ft/sec2
in= inches
mil = thousandths of an inch
mm = milimeters
p-p = peak-to-peak
rms = root mean square

8.6 Vibration Nomograph



8.7 References

8.7.1 Lawless, Alan R., *Math and Physics for Flight Testers*, "Chapter 9, Motion Analysis," National Test Pilot School, Mojave CA, 1999.

8.7.2 Ward, Don, *Introduction to Flight Testing*, Texas A&M, Elsevier, 1993.

8.7.3 Lang, George F., *Understanding Vibration Measurements*, Application Note 9, Rockland Scientific Corporation, Rockleigh, New Jersey, December 1978.

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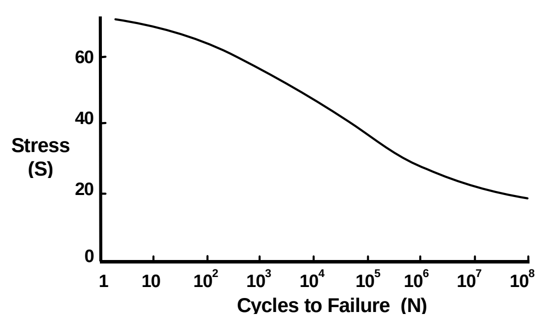
Section 9 Material Strength

Section 9 Abbreviations

A	cross-sectional area (ft^2)
DLL	design load limit
E	modulus of elasticity or Young's Modulus (lb/ft^2)
e	strain (non-dimensional)
EK	gage factor
GW	gross weight
KU	effective gust velocity (ft/sec)
L	lift force
L	length (ft)
N_{zb}	normal load factor, along aircraft z-axis
P	applied load (lb)
R	unstrained resistance
ΔR	change in resistance due to load
S	wing area (ft^2)
V	flight speed
V_s	stall speed
V_e	equivalent airspeed
W	aircraft weight
W/S	wing loading
ν	Poisson's ratio
σ	stress (lb/ft^2)
σ	air density ($slugs/ft^3$)

9.1 Loads Terminology

Annealing	A heat treatment that eliminates the effects of cold working.
Brittleness	Measure of a material's lack of ductility (by one definition breakage at five percent or less strain implies brittleness.
Creep rate	The rate at which a material continues to stretch when stress is applied at high temperature.
Cold Working	Deformation of a metal below its recrystallization temp., thereby strengthening and reshaping it.
Design Load Limit	Maximum loads expected in normal service.
Ductility	Ability of a material to deform without breaking.
Durability	Ability to resist cracking, corrosion, thermal degradation, delamination, wear, and the effects of foreign object damage over time.
Elastic Deformation	Deformation of the material that is recovered when the applied load is removed.
Elasticity	Ability of a material to return to its undeformed shape after all loads have been removed.
Endurance Limit	The stress below which a material will not fail in a fatigue test.
Factor of Safety	Ratio of the predicted failure stress to the maximum stress anticipated in normal operation (*DLL*). For aircraft, the Factor of Safety is typically 1.5 *DLL*.
Fatigue	The failure of a material when subjected to repeated loads less than the ultimate sustainable load. This effect is presented in an *S*-*N* diagram such as



Fatigue life The number of cycles at a particular stress before a material fails by fatigue.

Hardness Resistance to plastic deformation resulting from impact loads.

Impact Energy The energy required to fracture a specimen when the load is suddenly applied.

Limit Stress The maximum stress where the Modulus of Elasticity remains constant (proportional limit).

Margin of Safety Any load-bearing capability greater that the ultimate load, calculated as

$$\frac{\text{failure load as a factor of } DLL}{1.5\, DLL} - 1$$

Notch Sensitivity Measure the effect of a notch on impact energy

Plastic Deformation Permanent deformation of a material applied load. Plasticity Material deformation charac teristics beyond its elastic limit.

Resilience A measure of the amount of energy a material can absorb elastically in a unit volume of the material.

Rupture time The time required for a specimen to fail by creep at a particular temperature and stress.

Stiffness A qualitative of the elastic deformation produced.

Strain (e) The deformation of a material under an applied load.

Strength Ability to withstand external loads without failure.

Stress (σ) The ability of a material to react a force distributed over some area.

Thermal stress Stress resulting from expansion (strain) of a material subjected to heating.

Tempering A low-temp. heat treatment which reduces hardness.

Tensile strength The stress that corresponds to the maximum load in a tensile test.

Toughness Total energy absorbed before failure occurs (area under the stress-strain curve).

Transition Temperature The temperature below which a material behaves in a brittle manner in an impact test.

True Strain The actual strain produces when a load is applied.

Ultimate Stress The stress point at which additional load cannot be reacted.

Wing Loading Aircraft weight per wing area, *W/S*, a ready measure of air loads for steady level flight.

Yield Stress The stress applied to a material that just causes permanent plastic deformation.

9.2 Material Stress & Strain

Stress (σ) is the ability of a material to react a force distributed over some area. In the simple axial load case this can be presented as

$$\sigma = P/A$$

where P = the applied axial load
A = cross-sectional area over which the load is applied

Strain (e) is the deformation of a material under an applied load. In the basic form this can be presented as

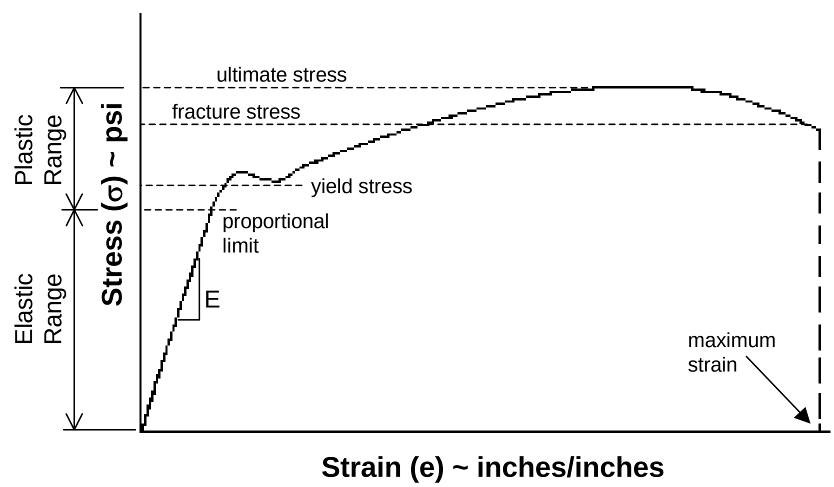
$$e = \Delta L/L$$

where ΔL is the change in dimension due to some load, and L is the original dimension

The *stress-strain relationship* is linear (proportional) for a large percentage of the applied load to the maximum, as expressed by the *Modulus of Elasticity* (*Young's Modulus*)

$$E = \sigma/e$$

A typical stress & strain relationship for a material is illustrated as



9.3 V-n Diagram

Flight Path Normal Load Factor (N_{zw}) can be expressed during level flight, as

$$N_{zw} = 1/cos\phi = L/W$$

where C_L = lift coefficient
F_n = net thrust
L = lift force = wing lift + thrust lift = $C_L qS + F_n sin\alpha_F$
q = dynamic pressure
S = wing area
W = gross weight
α_F = incidence angle between thrust line and relative wind
ϕ = angle of bank

Body Axis Normal Load Factor (N_{zb}) is calculated as

$$N_{zb} = [N_{zw} - N_{xb}\sin\alpha]/\cos\alpha$$

where N_{zb} = load factor along aircraft body x-axis
α = angle of attack

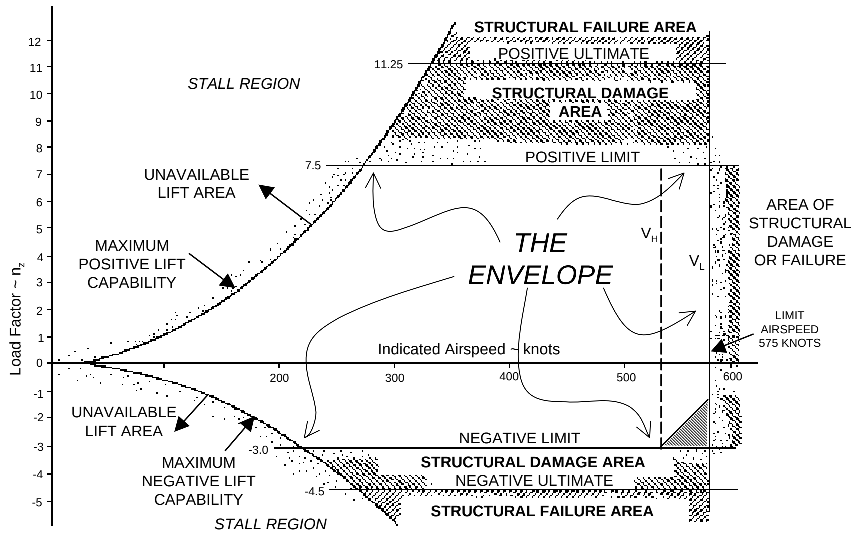
For the simplified case of negligible thrust lift, the maximum achievable N_{zb} at any flight speed can be calculated as

$$N_{zb} = (V/V_s)^2$$

where both speed must be the same units
(i.e., true, equivalent, calibrated)
V = flight airspeed
V_s = stall speed

A general normal load flight envelope (V-n diagram) would appear as

- The envelope typically varies with: asymmetric loading; aircraft configuration; for air loads other than along the normal axis; and other structural, system, and safety considerations.



- It is frequently desirable to correct measured (test) N_{zb} data to a standard weight or design gross weight (GW) using the relationship

$$N_{zb} = (\text{test } N_{zb})(W_t/W_s)$$

where W_t = test weight
W_s = standard weight

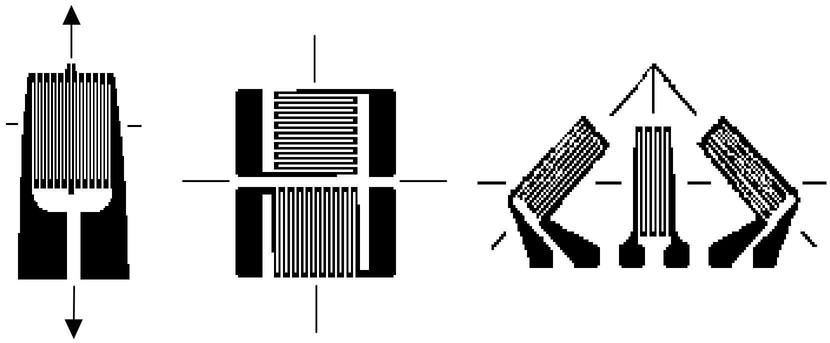
- The increase in load factor due to a vertical gust (Δn) is calculated as

$$\Delta n = 0.115 m V_e (KU)/(W/S)$$

where m = slope of $C_{L\alpha}$ curve
V_e = equivalent airspeed (knots)
KU = effective gust velocity (fps)
W/S = wing loading (psf)

9.4 Strain Gages

The three strain gage configurations most commonly used are



Strain (e) is measured using the electrical resistance measured via the strain gage in a material subject to load. For the uniaxial gauge

$$K = (\Delta R/R)/e$$

where K = gage factor (provided by manufacturer)
R = unstrained resistance
ΔR = change in resistance due to load
(+ΔR for tension)

- For the bi-axial gage oriented coincident with the principal axes (maximum strain), each leg of the gage is analyzed as a uniaxial gage using the above equation for the principle strains. The associated stresses are

$$\sigma_{max} = E(e_{max} + \upsilon e_{min})/(1 - \upsilon^2)$$
$$\sigma_{min} = E(e_{min} + \upsilon e_{max})/(1 - \upsilon^2)$$

where e_{max} and e_{min} are the measured principal strains in the appropriate legs of the bi-axial gage,
E is the Young's Modulus of the material, is Poisson's ratio for the material.
(ratio of compression and tension strains)

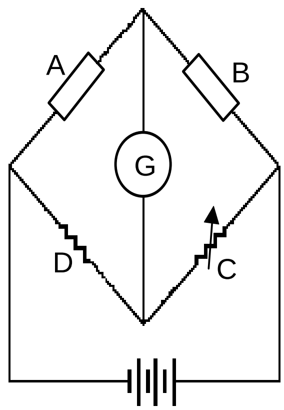
- For the Rosette gage, the principal strains and stresses are derived as

$$e_{max,\ min} = 0.5(e_a + e_c) \pm 0.5\sqrt{(e_a - e_c)^2 + (2e_b - e_a - e_c)^2}$$

$$\sigma_{max,\ min} = E/2\left[(e_a + e_c)/(1-u)\right] \pm \sqrt{(e_a - e_c)^2 + (2e_b - e_a - e_c)^2}/(1-u)$$

where *e*'s denote the strains in each of the three legs of the Rosette (+ is used for the maximum and - for the minimum).

To accurately measure the very small resistance changes in a strain gage, a *Wheatstone Bridge* is typically used



A = active or strain-measuring gage
B = temperature compensating (dummy) gage
C = D = internal resistance in instrument
G = galvanometer

9.5 References

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9.2 Norton, William J., *Structures Flight Test Handbook*, AFFTC-TIH-90-001, Air Force Flight Test Center, Edwards AFB, California, November 1990.

Additional Reading

Military Specification Airplane Strength and Rigidity - *General Specification*, MIL-A-8860.

Military Specification Airplane Strength and Rigidity, Sonic Fatigue, MIL-A-008893.

NOTES

NOTES

Section 10 Reciprocating Engines

10.1 Recurring Abbreviations and Terminology

(references 10.5.1-10.5.5)

Abbreviations

BHP brake horsepower (measured at engine crankshaft)
BHP_s brake horsepower at standard conditions
BHP_t brake horsepower at test conditions
BHP_{alt} brake horsepower at maximum altitude where test RPM and MP can be maintained.
BHP_{slmax} maximum brake horsepower at standard sea level conditions (for any given RPM)
BHP_c chart brake horsepower
$BSFC$ brake specific fuel consumption (fuel flow/horsepower/ hour)
C manifold pressure correction factor
HP horsepower (= 550ft-lb/sec)
M freestream Mach number
MP manifold pressure, also MAP
SHP shaft horsepower (measured at propeller shaft)
P power output [ft-lb/sec or HP]
P_a ambient pressure
P_{ts} standard day total (ram) pressure
P_{tt} test day total (ram) pressure
Q torque [ft-lbs]
q dynamic pressure
RPM revolutions per minute
T_{ct} test day carburetor temperature (absolute)
T_{cs} standard day carburetor temperature (absolute)
T_{as} standard day ambient temperature (absolute)
T_{at} test day ambient temperature (absolute)
V_T freestream true velocity
ΔBHP_{cat} change in brake horsepower due to carb. temp. change
ΔBHP_{mp} change in brake horsepower due to manifold pressure change
$\Delta MP_{\Delta t}$ change in manifold pressure due to temperature change
Δt change between test and standard temperature (T_{at}-T_{as})
η_r carburetor ram inlet efficiency
ρ_a ambient density
In Hg inches of mercury (manifold pressure)

Terminology

Bore Piston diameter

Critical Altitude

The altitude at which a supercharged (or turbocharged) engine can no longer:
a) maintain sea level manifold pressure, or
b) maximum allowable horsepower.

Detonation

An operating condition where combinations of excessive temperature, high manifold pressure, and l ow RPM cause explosive fuel burn, large internal pressure pulses, and subsequent engine damage.

Displacement

Total volume swept by all cylinders, measured in either cubic inches or liters.

Manifold Pressure

Pressure of fuel-air mixture passing through intake manifold, typically measured in absolute gauge pressure (inches of mercury or lb/in^2).

Mixture Ratio

Ratio of [fuel weight/air weight] passing through the intake manifold.

- This ratio must be between .05 and .125 to burn.
- Best power typically occurs at mixture ratio of 0.075 to 0.08.
- Best economy typically occurs at a ratio of .0625
- To provide sufficient cooling, the mixture ratio is usually greatly increased from best economy when operating at very high or very low power settings (a.k.a. auto rich).

Reduction gear

Gearing between the engine crankshaft and propeller shaft that reduces the rotation speed going to the propeller.

Stroke Linear distance traveled by piston.

Supercharger

A mechanically driven compressor that boosts the ambient air pressure to provide the engine with higher power output.

Turbocharger

Also known as a turbo supercharger, it is similar to a super charger except that the compressor is driven by engine exhaust pressure.

10.2 Reciprocating Engine Modeling (ref 10.5.3)

For any given engine design, power output is primarily a function of the product of engine speed (*RPM*) and intake manifold pressure (*MP*). Smaller but significant effects are due to the fuel/air density (ρ_{fa}) and exhaust back pressure (which is essentially ambient pressure, P_a). Less significant effects are due to the condition of the engine itself and include such factors as ignition quality & timing, piston ring leakage, fuel grade, and oil viscosity.

Engine models have various levels of sophistication which can account for the four most significant factors listed above. Theses models can be presented graphically or analytically. Figure 10.2a shows a typical **graphic power model** for determining reciprocating engine BHP.

1) The left-hand chart shows the fundamental relation between *BHP* and the product of *RPM* and *MP*. Enter with *MP* and *RPM* to obtain point "B" and the associated "base brake horse power" (BHP_B) at sea level standard day pressure and temperature.

2) Transfer this BHP_B value to point "B^1" on the ordinate of the right hand chart.
3) Enter the right hand chart with the same *MP* & *RPM* to obtain point "A" and the associated brake horsepower at altitude (BHP_A) .

4) Connect points B^1 and A with a straight line.

5) Enter the abscissa at the test pressure altitude, locate point "C", and read the corresponding "chart horsepower" (BHP_C). BHP_C. is the sea level power corrected to the reduced back pressure con ditions at altitude. It does not account for non-standard temperatures.

6) To correct for non-standard air temperature, subtract 1% from BHP_C for each 6^0C warmer than test altitude standard temperature. Conversely, add 1% to BHP_C for each 6^0C cooler than standard. For convenience, the lower right hand chart of Figure 10.2a illustrates standard temperature as a function of pressure altitude.

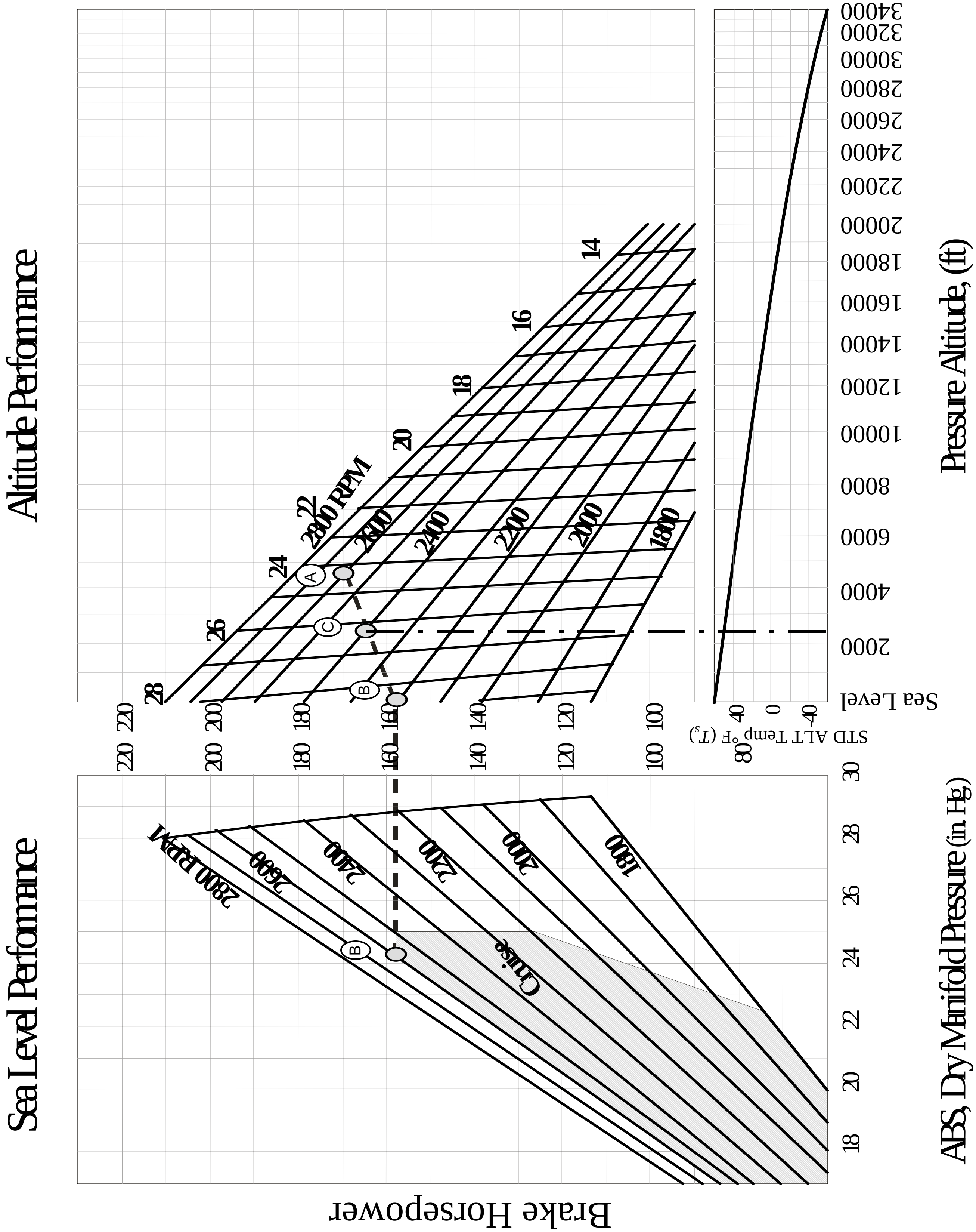

Figure 10.2a Engine Performance Chart for Continental IO360D

An **analytic power model** of a reciprocating engine should match the graphic model. Its principle application is in automating the power determination process rather than manually performing graphic lookups.

1) The left hand chart shows that BHP_B is a linear function of MP, but it is not necessarily a linear function of RPM. Extrapolating the *RPM* curves towards low manifold pressure illustrates their convergence to a common point. This left hand chart can be summarized with the equation

$$BHP_B = [a(RPM)^2 + b(RPM) + c][MP - e] + d$$

a through *e* are determined from the graph or from separate engine tests.

2) In a similar fashion, calculate $BHP_A = [a(RPM)^2 + b(RPM) + c][MP]$

3) Based on the direct relation between available power and density, calculate chart horsepower as

$$BHP_C = BHP_B[BHP_A - BHP_B][(1-\sigma_A)/(1-\sigma_D)]$$

where σ_D is the standard atmosphere density ratio at the operating pressure altitude (H_c). For convenience, this calculation is presented below for flight in the tropopause

$$\sigma_D = [1 - 6.876\, x10^{-6} H_c]^{4.2558} \quad (H_c \text{ in feet})$$

σ_A is the density ratio corresponding to point A and is calculated as

$$\sigma_A = .117 + \frac{BHP_A}{BHP_{sl\,\max}}$$

where $BHP_{sl\,max}$ is the full-throttle sea level power at the *RPM* in question. This value is located towards the right side of the BHP_B chart.

4) The final step in determining test day power (BHP_t) is to correct for non-standard ambient absolute temperature (T_a)

$$BHP_t = BHP_C \sqrt{\frac{T_{as}}{T_{at}}}$$

where T_{as} is the standard absolute ambient temperature at the test altitude, and, below the tropopause, is calculated as T_{as} = 288.15-.0019812H_c .

The **fuel flow model** is centered around the brake specific fuel consumption (*BSFC*) defined as

$$BSFC \equiv \frac{fuel\ \ flow}{power} \quad \left(\frac{lb/hr}{BHP}\right)$$

Figure 10.2b shows the basic effect of *RPM* & *BHP* on *BSFC* (ref 10.5.4).

- At any given *BHP*, operating at lower *RPM*s reduces mechanical friction and therefore *BSFC*.
- At any given *RPM*, operating at very low *BHP* increases the percentage of piston work overcoming friction and therefore increases *BSFC*.
- Operation at high *BHP* also increases *BSFC*, but this is due to the fuel enrichment required to prevent detonation at high loads.

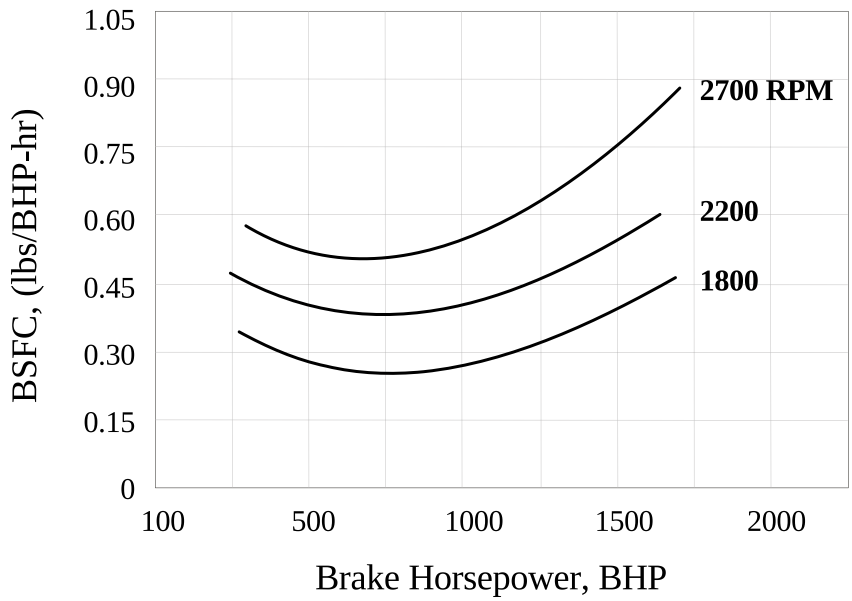

Figure 10.2b Effect of *RPM* and *BHP* on *BSFC*

The same effects can be modeled as shown in Figure 10.2c (ref 10.5.5).

- Not shown on these figures are the possible altitude and temperature effects. Flight testers rarely need to validate these models throughout the engine's working range. Instead, testers typically evaluate *BSFC* only at the combinations of *RPM* and *MP* recommended by the manufacturer to give the desired power output.

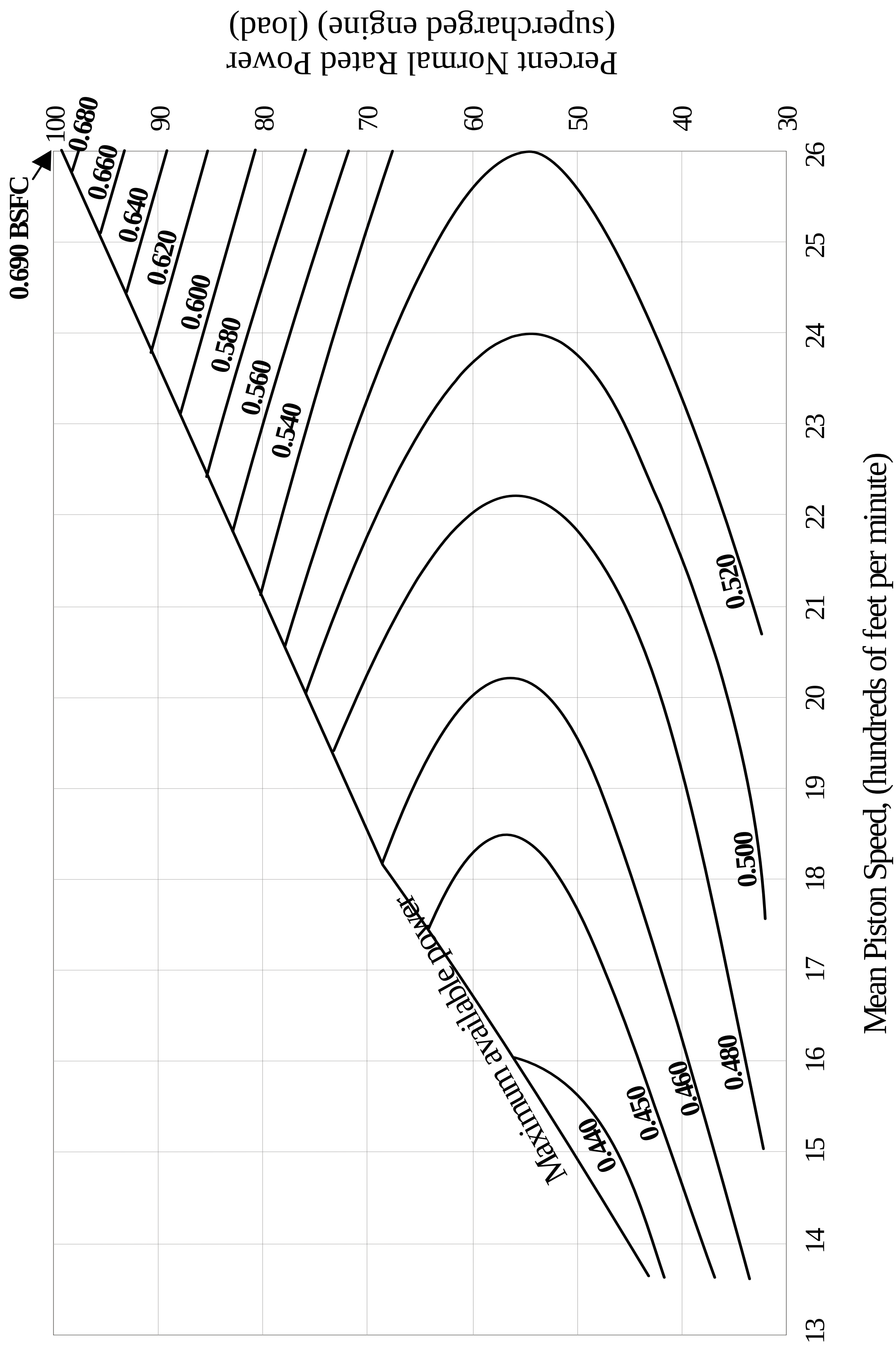

Figure 10.2c BSFC Curves for a Typical Supercharged Airplane Engine

10.3 Reciprocating Engine Power Standardization (ref 10.5.4)

Correcting from test day to standard day power available uses one of several methods, depending on the test conditions.

Some useful insights are summarized below.

- In all cases, test day *RPM* must equal standard day *RPM*. There are no corrections if this requirement is violated.
- The effect of density on power output at wide open throttle has been empirically shown to be (ref. 10.5.6)

$$BHP_{alt} = BHP_{sl\,\max}(1.1324\,\sigma - .1324)$$

- The above correction is not typically applied to test data since test and standard day pressure altitudes (H_c) are usually the same. Instead, most standardization requirements center around correcting to standard temperature.
- Engine power is actually related to the carburetor air temperature. The difference between test and standard day carburetor temperature equals the difference between test and standard day ambient temperature (T_{ct}-T_{cs} = T_{at}-T_{as}).
- With proper design, most of the freestream dynamic pressure (q) is converted into additional pressure at the carburetor and is known as "ram" effect. Above the critical altitude (where full throttle operation does not generate maximum manifold pressure), ram effect can be used to increase manifold pressure and therefore power output.

The different power standardization methods are described below.

Partial Throttle Standardization. If the test engine is set at some partial throttle setting to achieve a specific manifold pressure, then the same *MP* should be achievable on standard day with a slightly different throttle setting. Calculate standard day power (BHP_s) as

$$BHPs = BHP_t\left(\frac{T_{ct}}{T_{cs}}\right)^n$$

T_{cs} is the standard day carburetor absolute temperature = T_{as}-T_{at} +T_{ct}

n is the power exponent, usually = 0.5

This correction does not address changes in back pressure, so $H_{c\ test} = H_{c\ std}$ std

Full Throttle Standardization applies only if test and standard pressure altitude are equal. If the engine is operated full throttle on a test day, then the change in ambient temperature would generate a power change due to carburetor air temperature (ΔBHP_{cat}) and manifold pressure changes (ΔBHP_{mp}). Calculate standard day power (BHP_s) as

$$BHP_s = BHP_t + \Delta BHP_{cat} + \Delta BHP_{mp}$$

The first correction is another form of the previous constant *MP* correction

$$\Delta BHP_{cat} = BHP_t \left[\left(\frac{T_{ct}}{T_{cs}} \right)^n - 1 \right]$$

The second (manifold pressure) correction accounts for two effects:
1) For supercharged engines, correct for the change in pressure ratio of the supercharger due to inlet temperature changes.
2) For all engines operating below maximum *MP*, correct for the change in air inlet ram pressure ratio due to flight Mach number changes.

$$\Delta BHP_{mp} = BHP_t \left(\frac{MP_s}{MP_t} - 1 \right)$$

MP_t is the test manifold pressure.
MP_s is the manifold pressure corrected to standard temperature and flight Mach number:

$$MP_s = MP_{\Delta t} \frac{P_{ts}}{P_{tt}}$$

$MP_{\Delta t}$ is the correction of manifold pressure due to changes in ambient temperature and is approximated as

$$\Delta MP_{\Delta t} = MP_t C \Delta t$$

C is a constant depending upon the pressure ratio (P_2/P_1), carburetor air inlet temperature, and whether or not the fuel is vaporized during process.

- If only the air is compressed, or if the inlet temperature is measured *after* fuel vaporization, then determine C using Figure 10.3a .
- If the fuel is injected after the temperature is taken but *before* the charge is compressed, then determine C using Figure 10.3b.
- By use of Figures 10.3a and 10.3b, any combination of induction processes for air only or for a fuel air mixture may be evaluated.

Δt is the difference between test and standard day carburetor air temperature and was previously described as the change in ambient air temperature

$$\Delta t = T_{at} - T_{as}$$

P_{ts}/P_{tt} is the ratio between standard and test day total (ram) inlet pressures at the standard and test Mach numbers. The first step in determining this ratio is to recognize

$$\frac{P_{ts}}{P_{tt}} = \frac{P_{ts}/P_a}{P_{tt}/P_a}$$

P_a is the pressure altitude and must be the same for test and standard days. Calculate P_{tt}/P_a using test Mach number and the equation

$$\frac{P_t}{P_a} = \eta_r \left[\left(1 + .2M^2\right)^{3.5} - 1 \right] + 1$$

η_r is the carburetor inlet ram efficiency and is usually between 0.7 and 0.75. A more exact value may be calculated as

$$\eta_r = \frac{P_t(actual) - P_a}{P_t(theoretical) - P_a}$$

Calculate P_{ts}/P_a using the same equations and standard Mach number.

- This last calculation may be iterative because standard Mach number cannot be exactly determined from the drag polar until power output is known.
- This correction is not normally made unless the flight Mach number is above 0.6 and the power change causes a speed change of more than 3 knots.
- To get a feel for the dynamic pressure change (and therefore ram effect change) due to Mach number change, recall

$$q\left[\frac{lb}{ft^2}\right] = \frac{1}{2}\rho_a V_T^2 = 1481\delta M^2$$

The final standard day power curves are presented in a form similar to that shown in Figure 10.3c

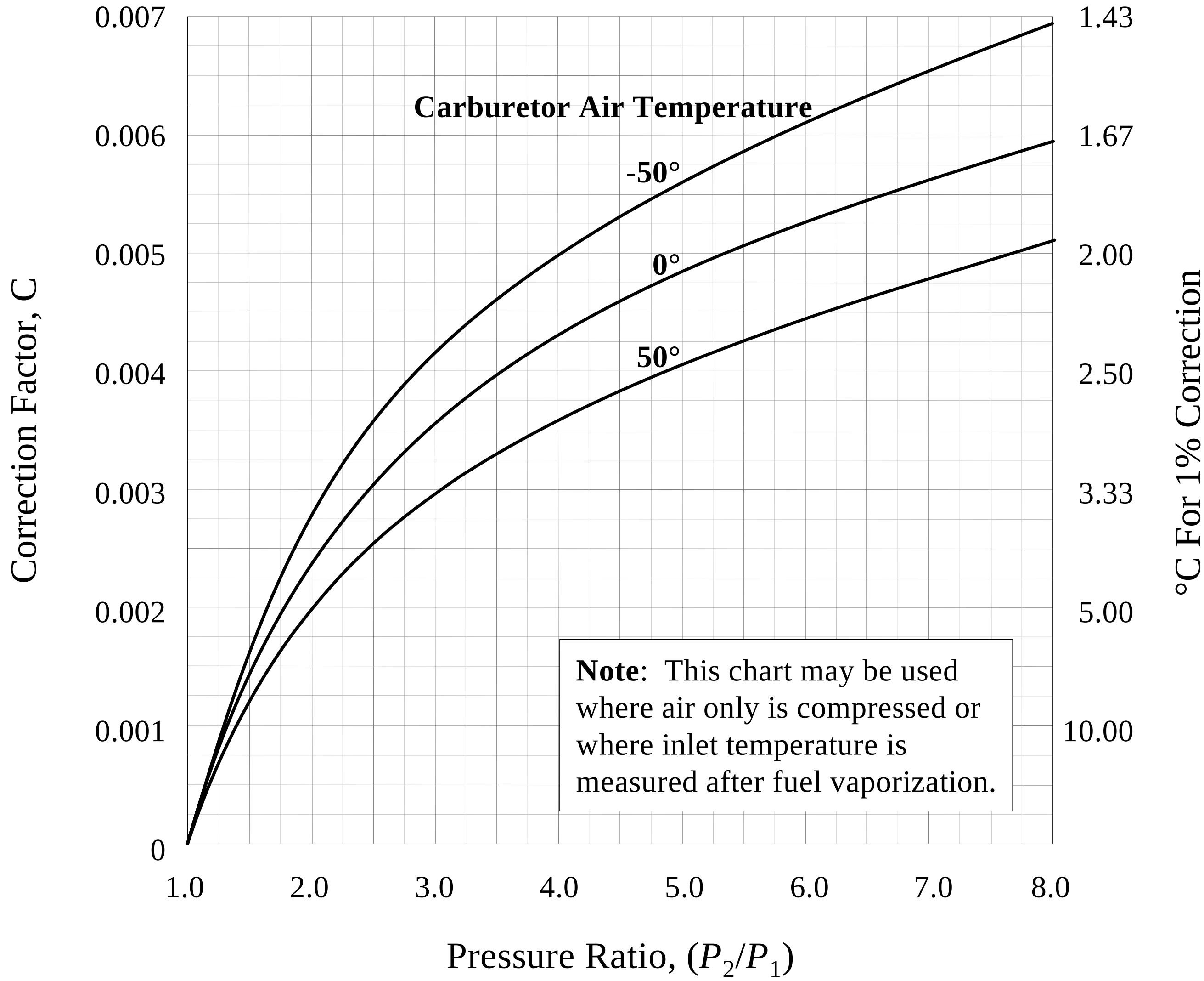

Figure 10.3a
Manifold Pressure Correction When Temperature is Measured

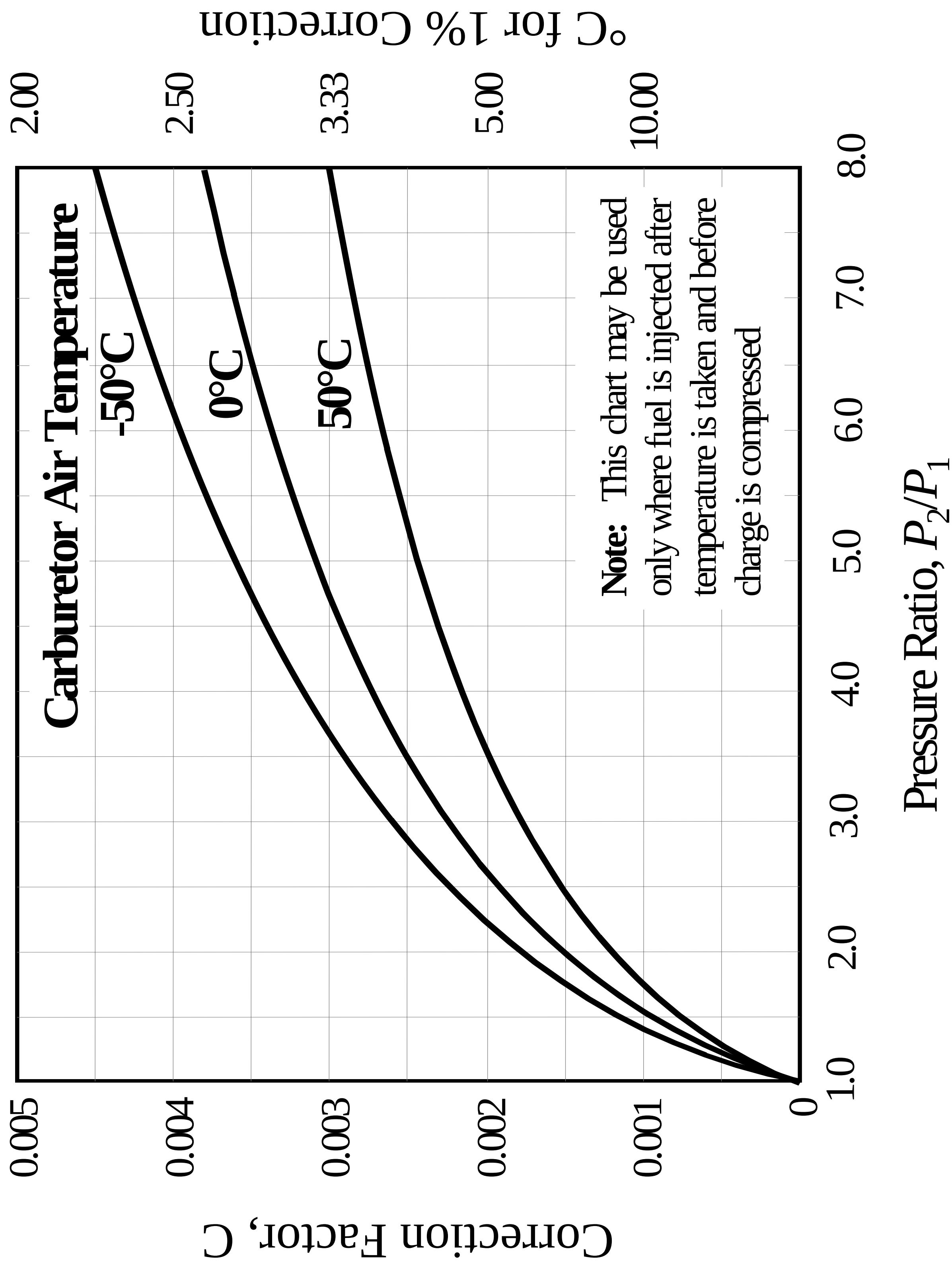

Figure 10.3b
Manifold Pressure Correction When Temperature is Measured Before Fuel Injection

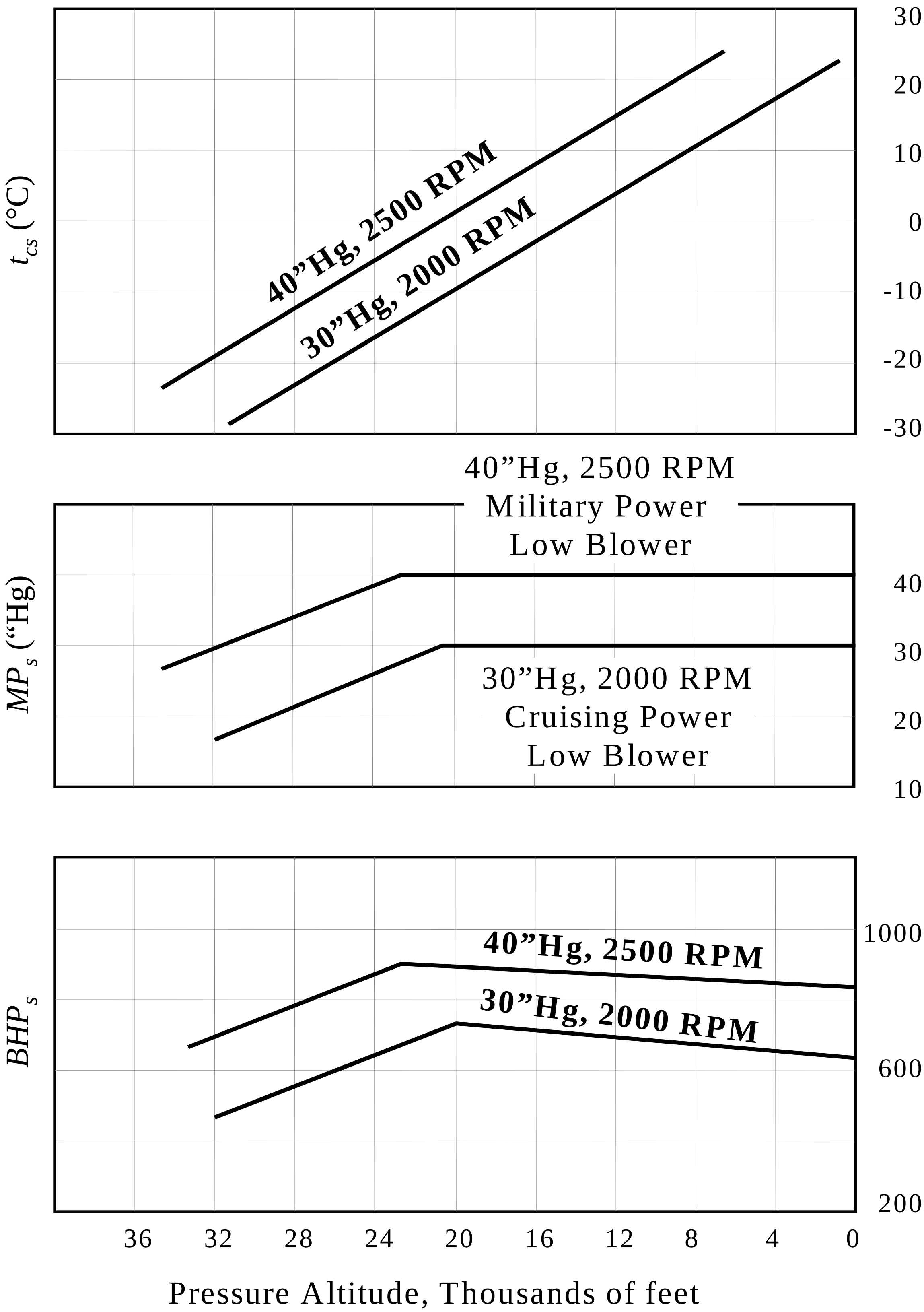

Figure 10.3c Example Standard Day Supercharged Engine Performance Data

10.4 FAA Approved Engine Temperature Corrections

The intent is to ensure that the critical engine parts, (i.e., cylinder head and cylinder barrel) do not exceed the engine manufacturer's specified limits during worst-case climb operating conditions on a 100 ^{o}F hot day.

Test procedures are detailed in AC 23-A. The basic idea is best illustrated with the single-engine airplane procedure:

1) Trim in level flight at the lowest practical altitude with at least 75% maximum continuous power. Allow
2) temperatures to stabilize.
2) Increase engine power to takeoff rating and climb at a speed not greater than V_y (best climb speed). Maintain takeoff power for one minute.
3) At the end of one minute, reduce engine power to maximum continuous and continue climb for at least five minutes after temperatures peak or the maximum operating altitude is reached. Flight manual lean ing procedures should be used.

Correct the peak test day cylinder barrel temperature (T_{bt}) to hot day conditions (T_{bh}) as follows

$$T_{bh} = T_{bt}+0.7[100-0.0036H_c-T_{at}]$$

Correct the peak test day cylinder head or other temperature (T_{ht}) to hot day conditions (T_{hh}) as follows

$$T_{hh} = T_{ht}+100-0.0036H_c-T_{at}$$

H_c is the pressure altitude in feet
T_{at} is the outside air temperature in degrees F

This method is known to be quite conservative More satisfactory temperatures may be achieved by actually testing during hot weather.

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10.5.2 Lawless, Alan R, "Fixed Wing Aircraft Performance Testing" Chapters 3 and 4, National Test Pilot School, Mojave CA, 1996.

10.5.3 Baughn, J. W., "A Method for computing Brake Horsepower from the Performance Charts of Reciprocating Aircraft Engines," AIAA- 94-2148-CP, from 7th Biennial AIAA Flight Test Conf., 1994.

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10.5.6 Gagg, R.F., and Farrar, E.V., "Altitude Performance of Aircraft Engines Equipped with Gear-Driven Superchargers," SAE Transcripts, Vol 29, pg 217-223, 1934.

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10.5.9 Hamlin, B., "Flight Testing Conventional and Jet-Propelled Airplanes," The Macmillan Co., New York, NY, 1946.

10.5.10 Operators Manual for Series IO-360 Aircraft Engines, form No. X-30032, FAA Approved March 1979.

Section 11 Propellers

11.1 Abbreviations and Terminology

Abbreviations

AF activity factor = $\frac{100,000}{16}\int_{.15}^{1.0}\left(\frac{b}{D}\right)x^3 dx$

B number of blades
b blade section width (feet)
BHP brake horsepower (measured at engine crankshaft)
C_{LD} blade section design lift coefficient

C_{Li} integrated design lift coefficient = $4\int_{.15}^{1.0}(C_{LD})x^3 dx$

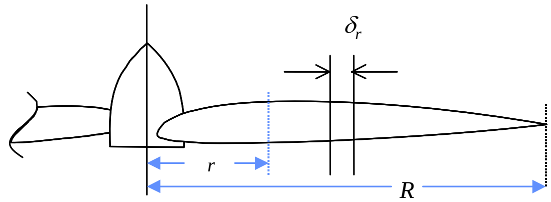
C_P power (absorbed) coefficient
C_T thrust coefficient
D propeller diameter (feet)
f_c ratio of speed of sound at standard day sea level to speed of sound at operating condition $f_c = \frac{1}{\sqrt{\theta}}$
HP horsepower (1 HP = 550 ft-lb/sec)
G.R. gear ratio, propeller speed/engine speed
J Propeller advance ratio $=V_T/nD$ (nondimensional)
M aircraft Mach number
N propeller speed, revolutions per minute (RPM)
n propeller speed, revolutions per second
N_e engine speed, RPM
P_a ambient pressure
P power output (ft-lb/sec)
Q torque (ft-lb)
q dynamic pressure
T thrust
T_a absolute ambient temperature
R blade radius at propeller tip (feet)
r radius at blade element (feet)
SHP shaft horsepower (measured at propeller shaft)
T propeller thrust (pounds)
V_T freestream velocity (ft/sec)
V_K freestream velocity (knots)
x fraction of propeller tip radius, r/R
V_{tan} tangential velocity
V_R resultant velocity
V_{tip} tip speed
α local angle of attack
β local blade twist angle, measured between chord and plane of rotation, same as θ (degrees).
ΔM Mach number adjustment for effect of blade camber
ϕ propeller disk angle of attack
η isolated propeller efficiency.
η_{comp} composite prop efficiency (includes tip and blockage corrections)
$\theta^{3/4}$ propeller blade twist angle at x =3/4 (degrees), same as $\beta^{3/4}$
σ ratio of operating density to sea level standard density = ρ_a/ρ_o.
ω propeller rotation speed (radians/second)

Terminology

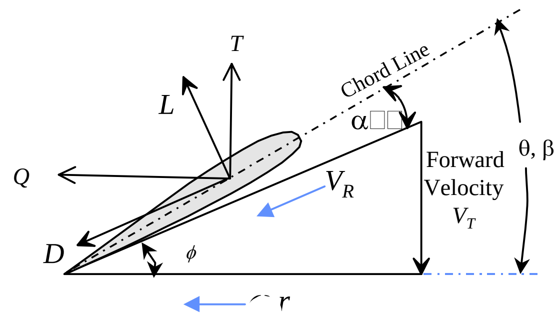
blade aspect ratio	measured as [*R* / max blade width].
effective pitch	actual advance per revolution.
experimental pitch	necessary advance to generate zero thrust.
geometric pitch (*p*)	advance per revolution if blade element moves according to β (i.e., with no slip).
reduction gear	gearing between the engine crankshaft and prop shaft that reduces the propeller rotation speed .
right-handed	moves clockwise (viewed from the slipstream).
solidity	fraction of prop disk covered by blade area = $2\pi R/Bb$.
total width ratio (*TWR*)	measured as [*WR* x *B*]
thickness ratio (*TR*)	blade thicknessmeasured locally or at .75*R* to represent entire prop.
width ratio (*WR*)	calculated as {*b* @ .75*R* } / *D*

11.2 Propeller Geometry

- δ_r is the width of any element along blade radius.



- $x = r/R$, the local fraction of prop tip radius



- Prop blade chord extends from leading edge to trailing edge.
- Blade twist angle θ, measured between rotation plane and local chord.
- Relative wind is the resultant velocity (V_R), comprised of aircraft forward speed and tangential speed at radial location along blade.

$$V_R = \sqrt{V_T^2 + (2\pi r n)^2}$$

$$\phi = \tan^{-1}\frac{V_T}{r\omega} = \tan^{-1}\frac{V_T}{r\,2\pi n} = \tan^{-1}\frac{V_T}{xD\,\pi n}: \qquad \phi_{tip} = \tan^{-1}\frac{1}{\pi}\frac{V_T}{nD}$$

- Angle ϕ is measured between plane of rotation and local V_R

$$\alpha^x = \theta^x - \phi = \theta^x - \tan^{-1}\frac{1}{\pi}\frac{V_T}{r2n} = \theta^x - \tan^{-1}\frac{1}{\pi}\frac{V_T}{xDn} = \theta^x - \tan^{-1}\frac{J}{\pi x}$$

- Advance ratio (J) is defined as $J = V_T/nD$.
- Local angle of attack at any fraction of radius(α^x) is measured between the local chord line and relative wind
- Lift and drag are perpendicular and parallel to V_R, respectively
- Thrust (T) and torque (Q) are perpendicular and parallel to the plane of rotation, respectively.

11.3 Propeller Coefficients

Integrating lift and drag along a blade gives the thrust (T) and torque (Q). Multiply by number of blades (B) to determine total T and Q.

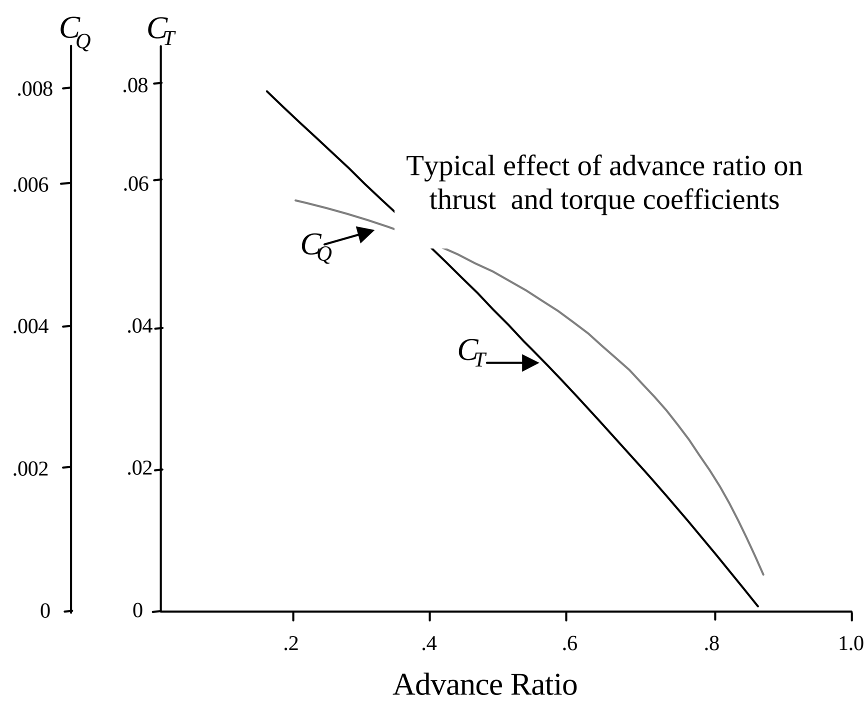
$$T = qB\int_{R_1}^{R_2} \frac{c}{\sin^2\phi}\left(C_L \cos\phi - C_D \sin\phi\right)dr$$

$$Q = qB\int_{R_1}^{R_2} \frac{cr}{\sin^2\phi}\left(C_L \sin\phi + C_D \cos\phi\right)dr$$

Thrust Coefficient, C_T $\equiv \dfrac{T}{\rho n^2 D^4}$

Torque Coefficient, C_Q $\equiv \dfrac{Q}{\rho n^2 D^5}$

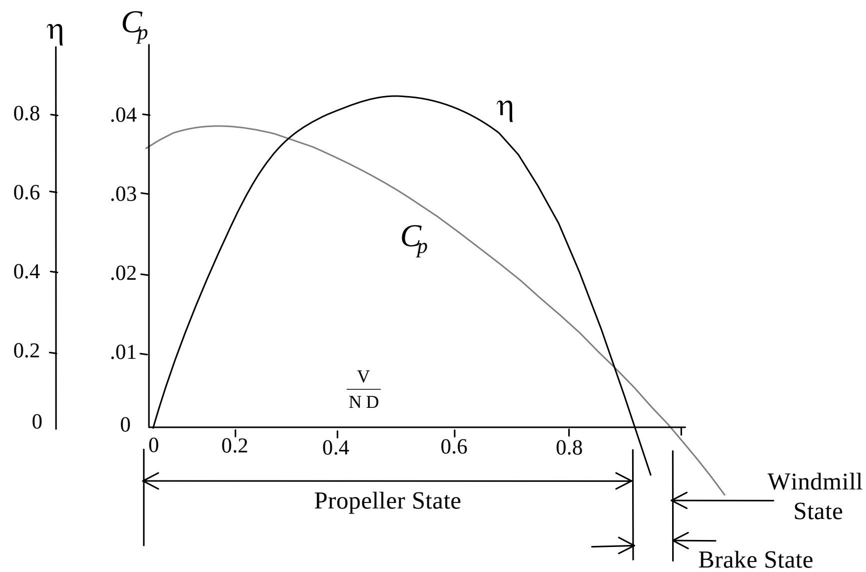
Power Coefficient, C_P $\equiv \dfrac{P}{\rho n^3 D^5} = \dfrac{Q\times\omega}{\rho n^3 D^5} = \dfrac{Q\times 2\pi n}{\rho n^3 D^5} = 2\pi\dfrac{Q}{\rho n^2 D^5} = 2\pi C_Q$



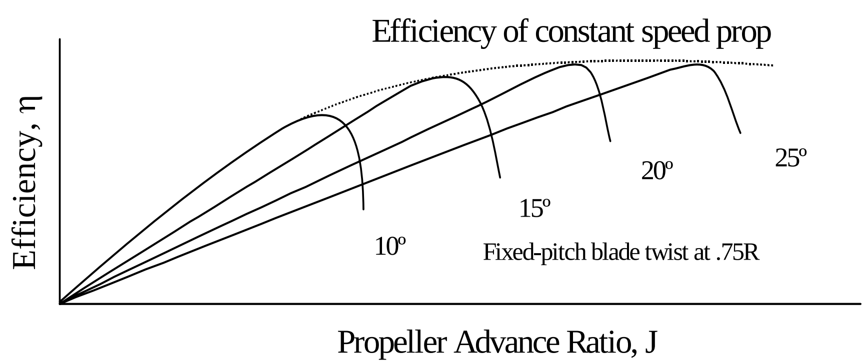
11.4 Propeller Efficiency and States

Propeller efficiency (η)

$$\eta \equiv \frac{P_{out}}{P_{in}} = \frac{Thrust \times V_T}{Q \times \omega} = \frac{C_T \rho n^2 D^4 \times V_T}{C_Q \rho n^2 D^5 \times 2\pi n} = \frac{1}{2\pi}\frac{C_T}{C_Q}\frac{V_T}{nD} = \frac{C_T}{C_P} J$$



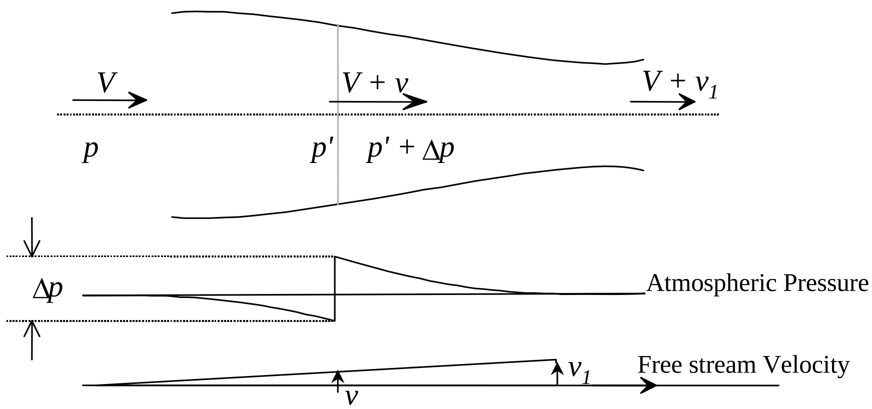
Propeller state: positive thrust & efficiency, power supplied by engine.
Brake state: negative thrust & efficiency, power supplied by engine.
Windmill state: negative thrust & η, power supplied by freestream.



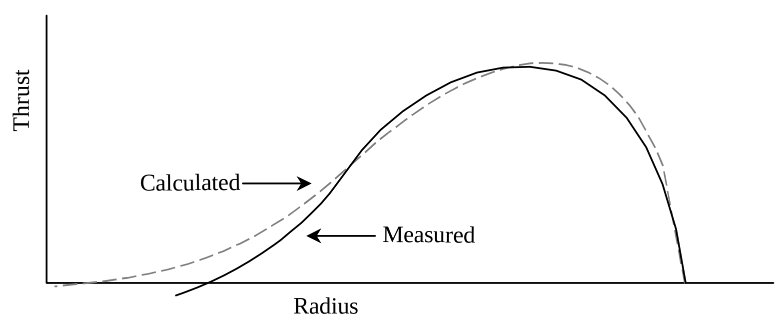
11.5 Propeller Theory

Simple momentum theory describes pressure jump (ΔP) across propeller disk .

- The downstream velocity increment (v_1) is twice the velocity increment at the disk (v) .
- Thrust (ΔP) = ΔP x disk area
- Froude's momentum theory: efficiency = $\eta \equiv \frac{TV_T}{T(V_T + v)} = \frac{V_T}{V_T + v}$



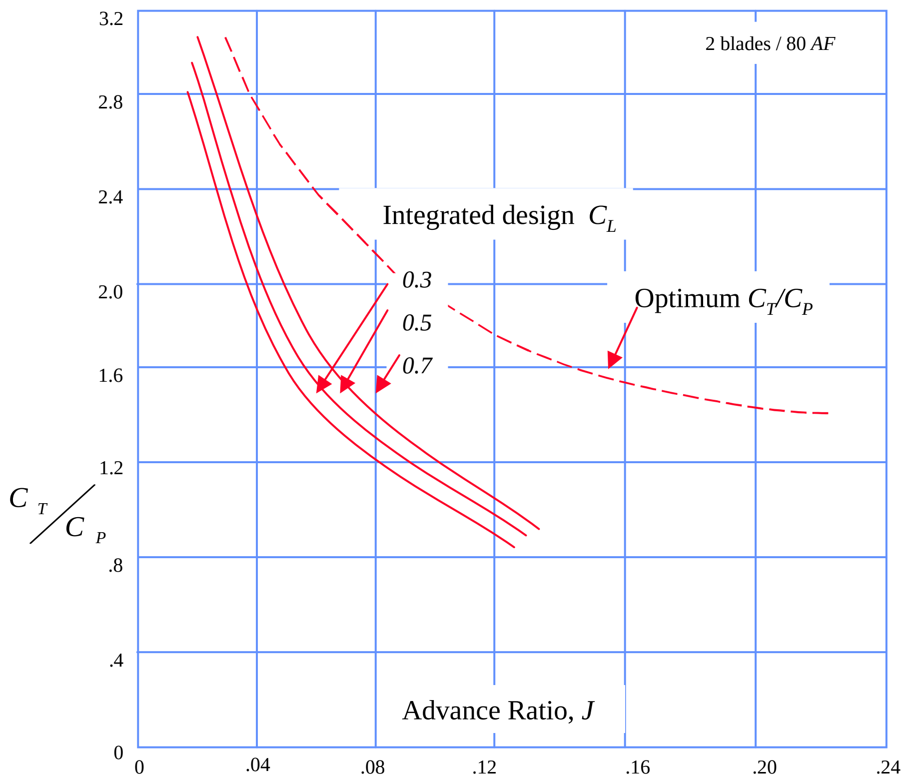
Blade element theory tends to be more complex and may include corrections for tip swirl losses, Mach effects, nacelle blockage, etc. Below is a comparison of typical calculated and measured thrust distribution.



11.6 Propeller Modeling

- For a specified propeller geometry; C_T , C_P, J, and blade angle (θ) are interrelated such that knowledge of any two defines the other two.
- Calculate propeller efficiency as $\eta = JC_T / C_P$.
- Models assume isolated conditions, i.e., without nacelle blockage.
- Models assume negligible Mach effects at propeller tips.
- Different models required for static and "in-flight" conditions.

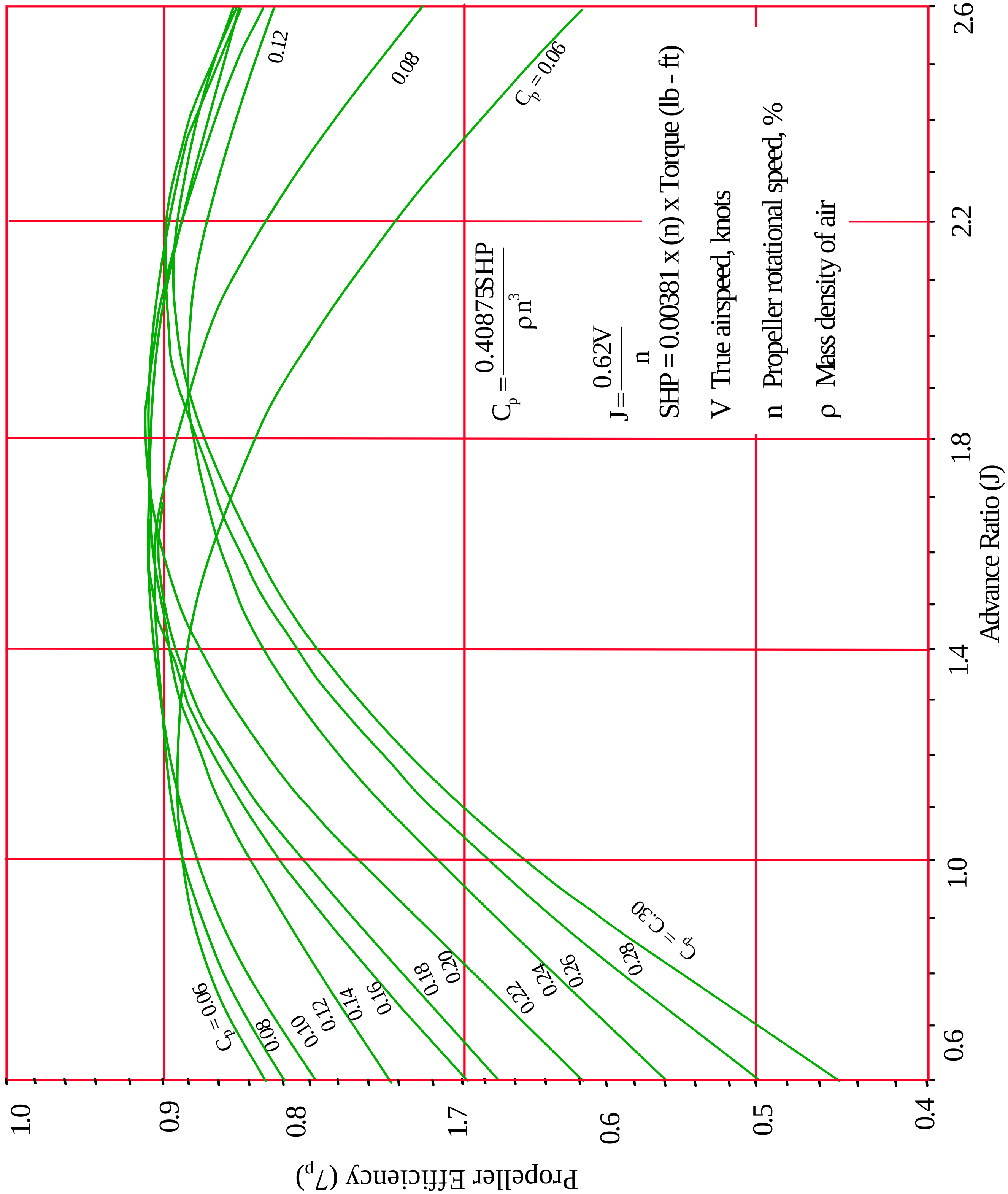
Determine static C_T and C_P using "**Static Thrust Chart**" (ref 11.2)



~ Separate charts exist for each combination of *AF* and # of blades (*B*).
~ Enter chart at appropriate J & C_{li}

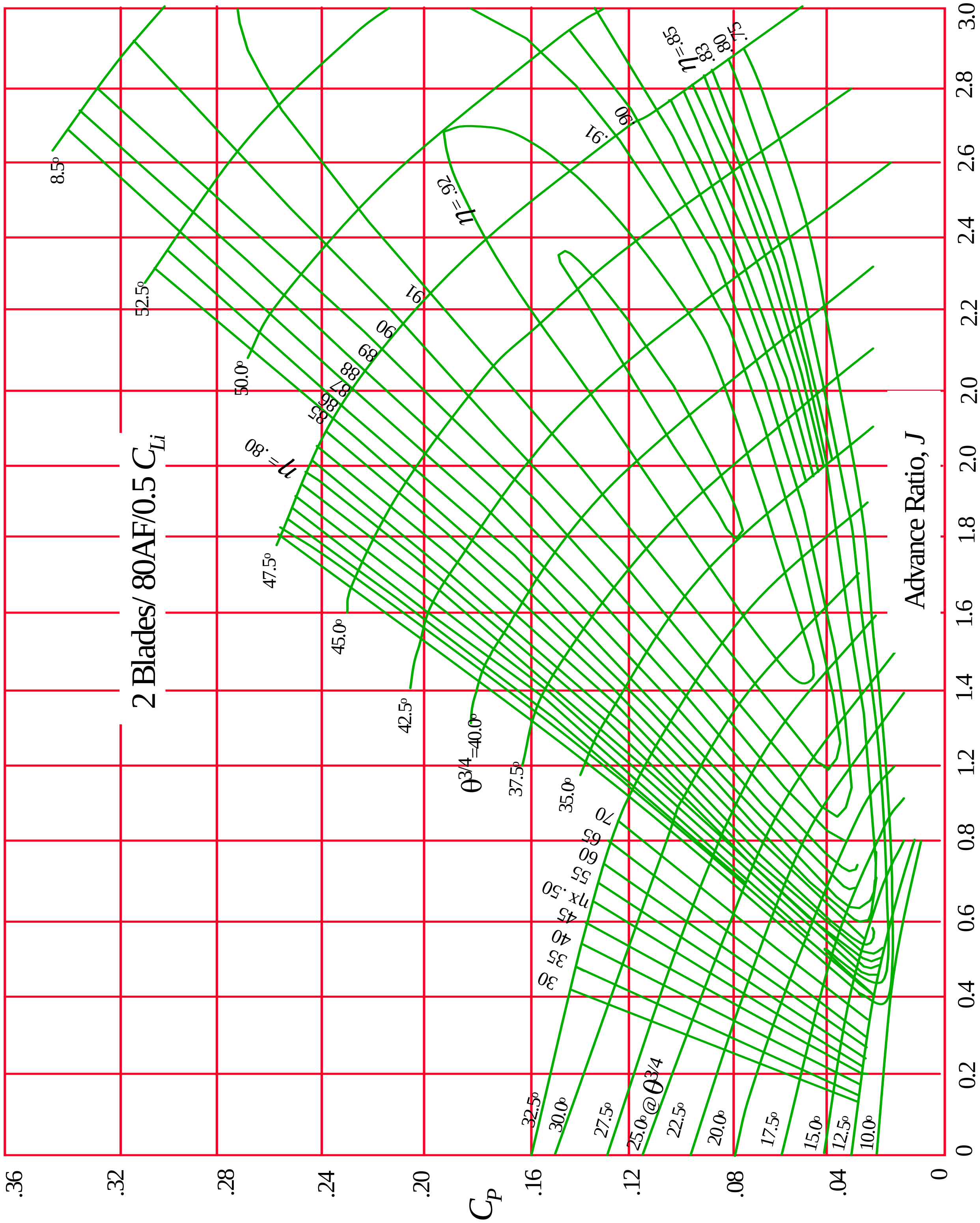
Static Thrust = $T_{static} = \frac{C_T}{C_P} \frac{SHP}{ND} 33{,}000$ where N = Propeller *RPM* Determine isolated propeller in-flight effi

ciency (η) from the appropriate **"Flight Charts."** They are typically presented in one of two forms.

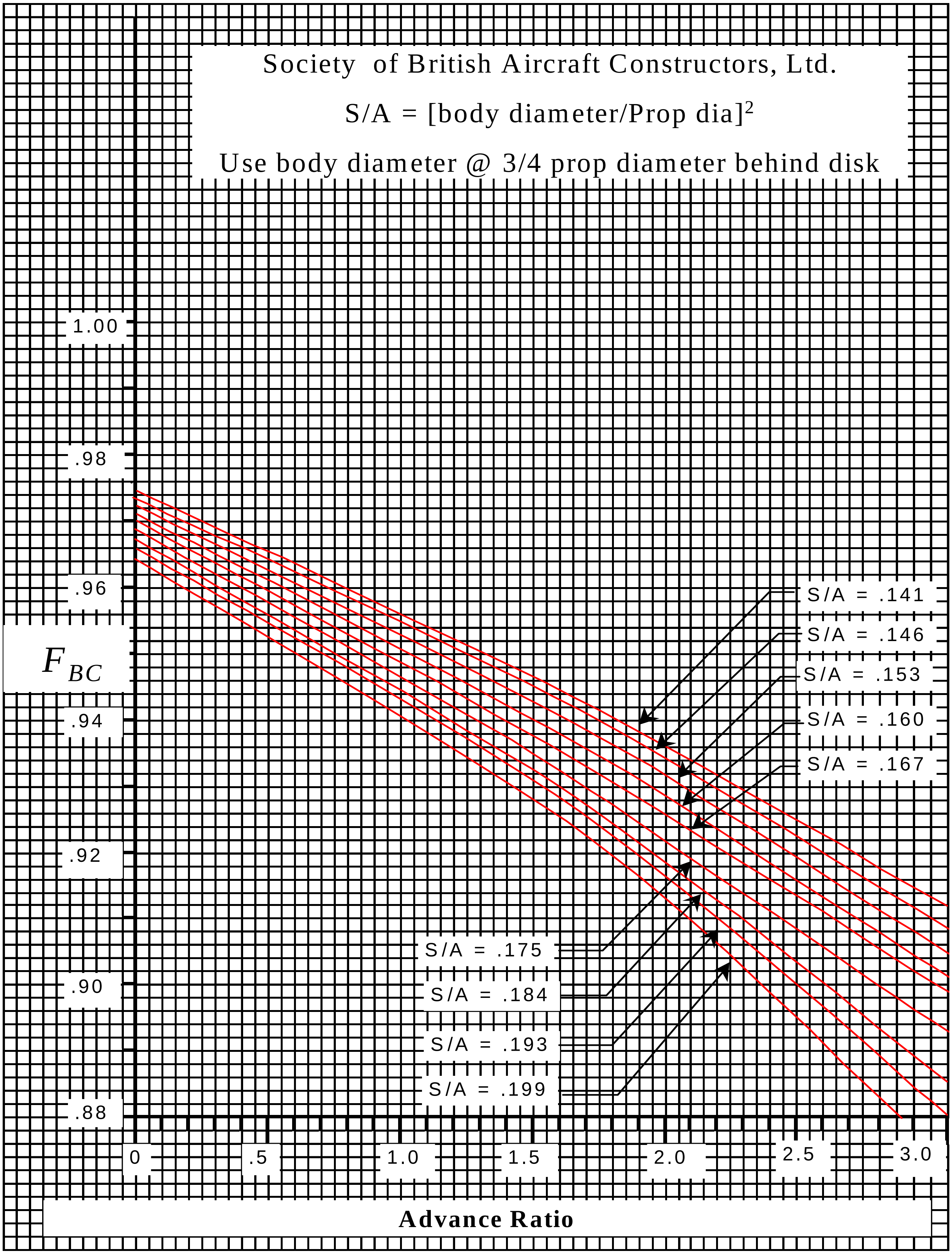


Above example for AiResearch TPE331-3U-303G engines and
Hartzell T10282HDB-4R 3-blade, constant speed, feathering propellers.

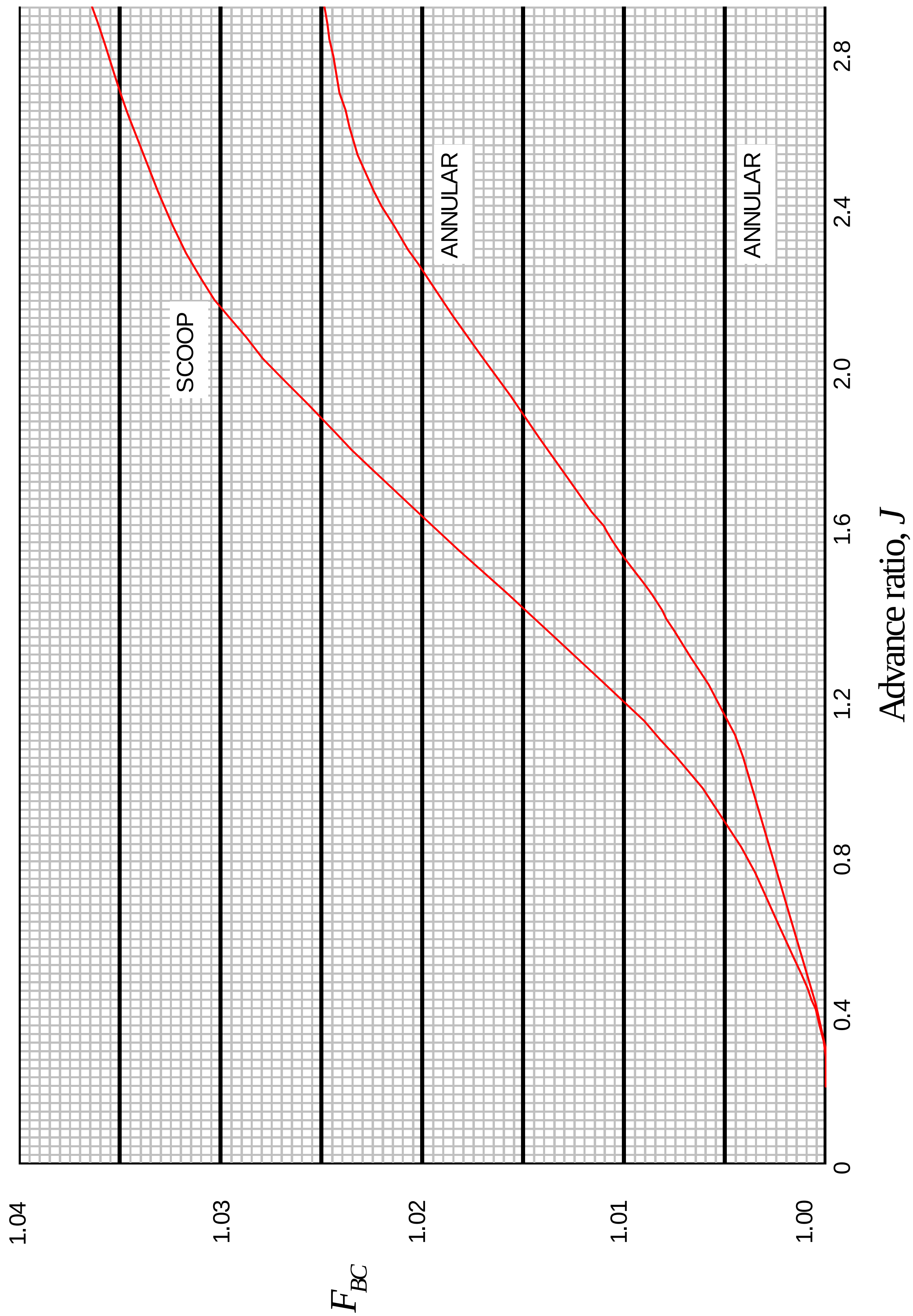
The other "in-flight η" format also requires calculation of C_P and J. Below is a typical flight chart published by Hamilton Standard (Ref 11.2). This applies to a propeller with 2 blades, AF= 80, and C_{Li} =0.5



A **body correction factor** (F_{BC}) should be applied to account for reduced efficiency due to body flow blockage immediately behind the propeller. Two examples follow.



Hamilton Standard also publishes a generalized nacelle blocking correction for typical scoop and annual inlet nacelles used on typical turboprops.

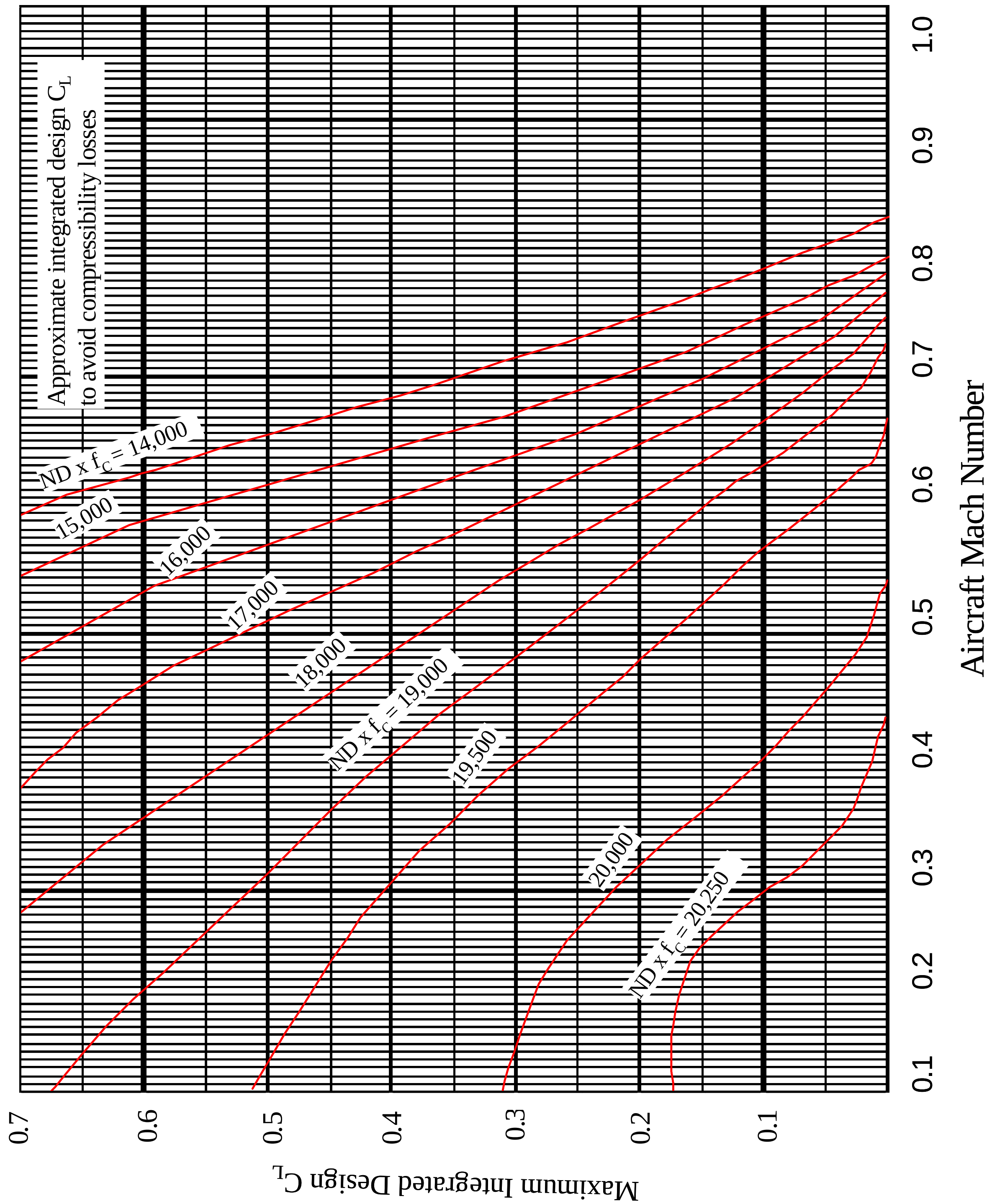


To determine if tip compressibility corrections are appropriate, find the maximum integrated design lift coefficient, C_{Limax} from the graph below.

~ Enter at flight Mach number, and move across at appropriate NDf_c.

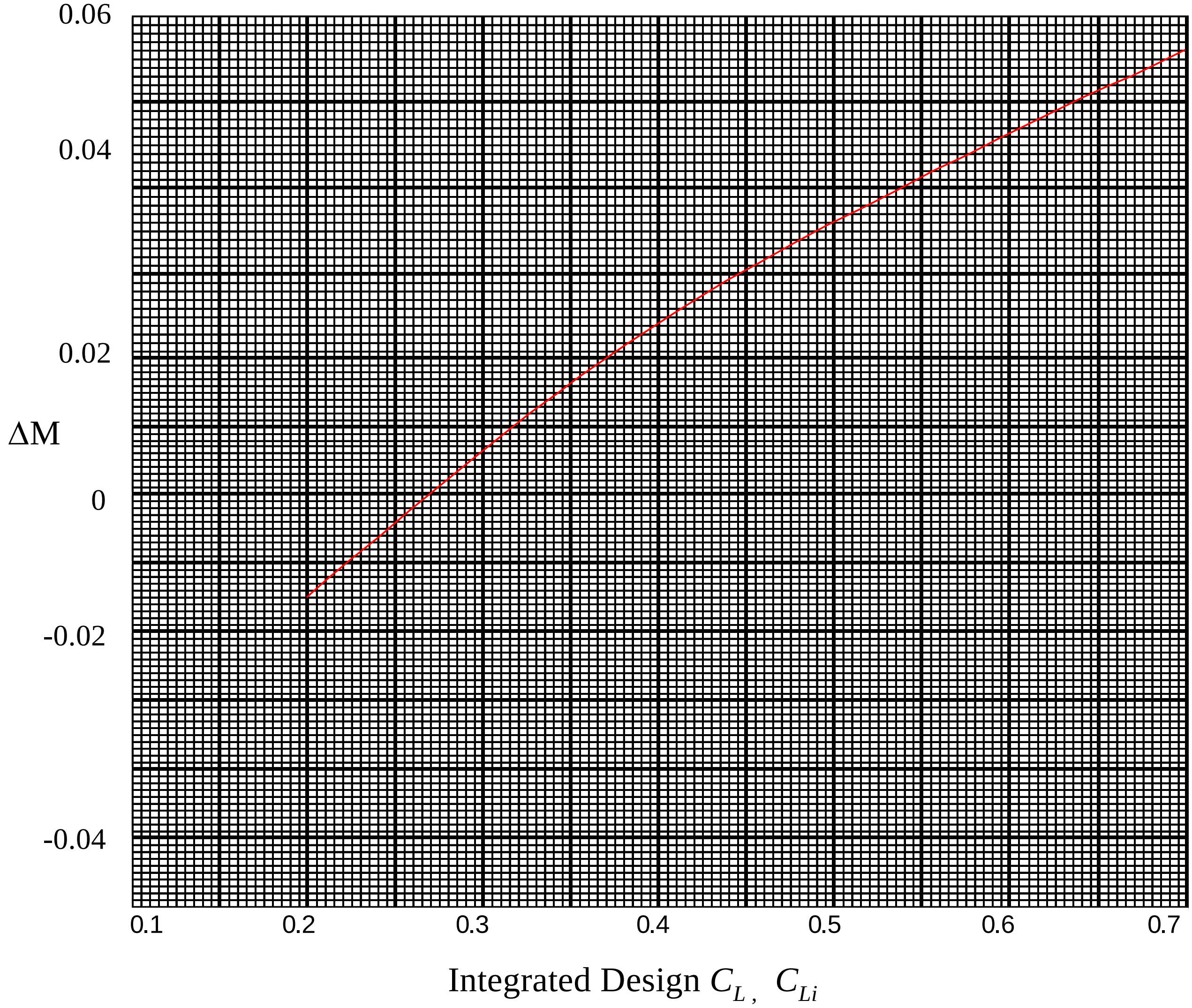
$$f_c = \frac{1}{\sqrt{\theta}}$$

~ If C_{Limax} is below calculated C_{li}, then corrections are required.



If tip compressibility corrections are necessary, then the first step is to

- Determine the **Mach number adjustment for the effect of blade camber** (ΔM) from the figure below.

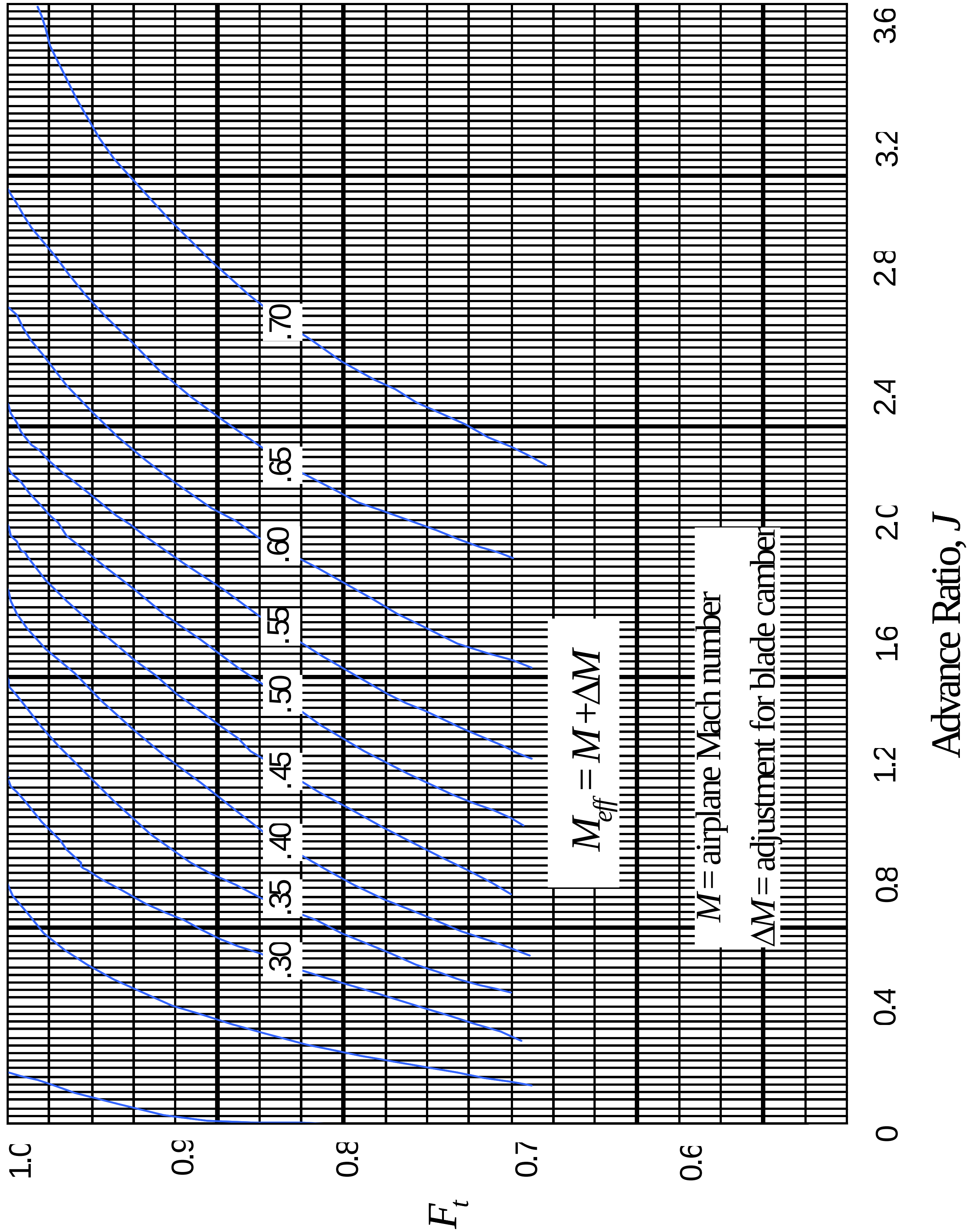


- Next, add ΔM from above to flight Mach number to get M_{eff}.
- Enter adjacent **generalized compressibility correction chart** to determine propeller efficiency tip factor (F_t)
- Calculate composite propeller efficiency as

$$\eta_{comp} = \eta \text{ x } F_t \text{ x } F_{BC}$$

- Calculate in-flight thrust as

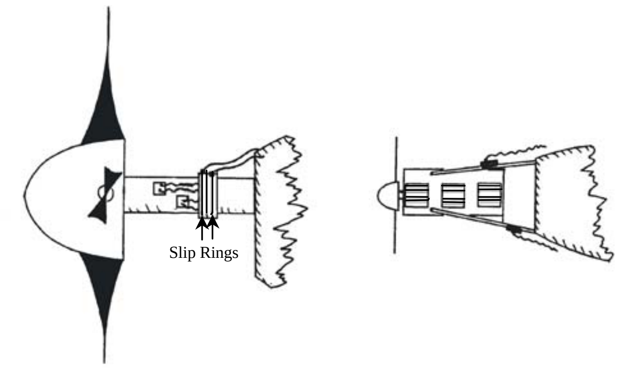
$$T = \frac{\eta_{comp} SHP}{V_T} = \frac{326\,\eta_{comp} SHP}{KTAS}$$



.30
.35
.40
.45
.50
.55
.60
.65
.70
$M_{eff} = M + \Delta M$
M = airplane Mach number
ΔM = adjustment for blade camber
1.0
0.9
0.8
0.7
0.6
F_t
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
Advance Ratio, J

11.7 Propeller Flight Test

The best method for determining η_{comp} is to instrument the prop shaft and/or engine mounts to measure thrust and torque.



Calculate efficiency as

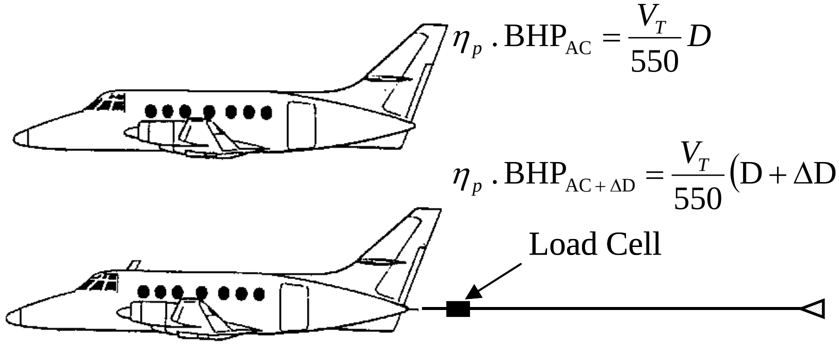
$$\eta_{comp} = \frac{T\,V_T}{Q\;\omega}$$

As an alternate, the **incremental drag method** requires an accurate engine power model, a load cell and a small drag device.

- Trim the aircraft at test *RPM*, V_T , & altitude. Note *SHP* required.
- Repeat above test with drag device and load cell attached. Note the power requirement change (ΔSHP) and load cell drag (ΔD).

$$\eta_p \,.\, \mathrm{BHP}_{\mathrm{AC}} = \frac{V_T}{550} D$$

$$\eta_p \,.\, \mathrm{BHP}_{\mathrm{AC}+\Delta\mathrm{D}} = \frac{V_T}{550}\left(\mathrm{D} + \Delta\mathrm{D}\right)$$



- Calculate aircraft drag and prop efficiency as

$$D = \frac{\Delta D(SHP)}{\Delta SHP} \qquad\qquad \eta = \frac{V_T}{550(SHP)D}$$

- This technique assumes the same η for both tests and is valid if J is constant and the C_P change is small. The drag device must therefore be small enough to not violate this assumption, yet large enough for the change in *SHP* to be measurable on engine instruments.

11.8 References

11.8.1 Roberts, Sean, "Light Aircraft Performance for Test Pilots and Flight Test Engineers," Flight Research Inc., Mojave CA, 1982.

11.8.2 anon., Hamilton Standard Propeller Efficiency Charts (a.k.a.Redbook), PDB 6101.

11.8.3 Von Mises, Richard, "Theory of Flight," McGraw-Hill, 1945.

NOTES

NOTES

Section 12 Fixed-Wing Performance Standardization

12.1 Recurring Abbreviations (references 10.5.1-10.5.5)

a acceleration
BHP brake horsepower
$BSFC$ brake specific fuel consumption (fuel flow per horsepower per hour)
C_D drag coefficient
$C_{Di\text{OGE}}$ induced drag coefficient out of ground effect
$C_{Di\text{IGE}}$ induced drag coefficient in ground effect
cg center of gravity
C_L lift coefficient
$C_{L\text{IGE}}$ lift coefficient in ground effect
$C_{L\text{OGE}}$ lift coefficient out of ground effect
D drag
F_n net thrust
F_g gross thrust
F_e ram thrust
F_{ex} excess thrust
F/δ corrected thrust
g reference acceleration due to gravity (32.174 ft/sec²)
GE_{CL} ground effect correction factor for lift coefficient
GE_{CD} ground effect correction factor for drag coefficient
H, h geopotential altitude
H_c pressure altitude
L lift
L_W lift of the wing
M Mach number
m mass
N_{xw} longitudinal load factor along flight path (wind axis)
N_{zw}, n_z load factor normal to flight path
P power output
P_a ambient pressure
P_o std ambient pressure sea level (2116.22 lb/ft² = 29.921 in Hg)
P_{iw} standard day sea-level power required
P_m mission-day power required
P_s specific excess power
P_s standard power required
P_t test-day power required
q dynamic pressure

R	range
R/C	rate of climb
R_n	Reynolds number
RF	range factor
S	reference wing area
S_a	horizontal air distance
S_g	ground roll
S_{LD}	total landing distance
SR	specific range
S_{TO}	total takeoff distance
T	ambient air temperature (absolute)
T	thrust
T_o	ambient temperature sea level standard (288.15 °K=15.0 °C)
V	inertial speed
V_c	calibrated airspeed
V_e	equivalent airspeed
V_{iw}	standard day sea-level true airspeed
V_T	true airspeed
V_{Tt}	test true airspeed
V_{Ts}	test true airspeed
W_t	test weight
W_s	weight standard
$\dot{W}_f$	fuel flow
$\dot{W}_{f\,iw}$	standard day sea-level fuel flow
α	angle of attack
β	sideslip angle
δ	ambient air pressure ratio
σ	ambient air density ratio
ι_T	thrust incidence angle
γ	flightpath angle
ϕ	bank angle
μ	rolling coefficient of friction
ω	turn rate (rad/sec)

12.2 Standardization Techniques (ref 12.5)

Performance data is usually corrected to “standard” conditions which are specified values of weight, altitude, *cg* and Mach number. The process also corrects data to some standard ambient air temperature, usually defined by the 1976 U.S. Standard Atmosphere. In some cases the data is corrected to “standard hot” day or “standard cold” day conditions which are a specified increment relative to the true standard conditions.

The standardization process usually relies on models of drag, thrust (or power), fuel flow, and propeller efficiency if appropriate. The overall principle is to collect test data as near as practical to standard conditions (+/-10%) and correct the results to standard using the models. Even with a 10% modeling error, correcting test data that is 10% from standard leads to only 1% total error in the standardized results.

The most common of the two standardization methods is the **difference method** which adds a correction to the test day parameter. This correction is the difference between the model predictions for standard and test conditions:

$$P_s = P_t + (P_s' - P_t') \qquad \text{(Eq'n 12.1)}$$

where
P_s = standardized parameter
P_t = test day parameter
P_s' = standard day parameter predicted by models
P_t' = test day parameter predicted by models

The parameter of interest can be one of the basic modeling values such as thrust, drag, power, or fuel flow. The parameter can also be the end result of the predictive process, and may include values like takeoff/landing distance, climb/sustained turning capability, or cruise range.

The **ratio method** is the other standardization process. It corrects to standard conditions by multiplying the test values by a correction factor. This factor is the ratio of the model predictions for standard and test conditions.

$$P_s = P_t \, (P_s' / P_t') \qquad \text{(Eq'n 12.2)}$$

The preferred approach is whichever gives the lowest total error. If the prediction models are in error by approximately a constant percentage, then the ratio method yields the least error. If the models are in error by approximately a constant magnitude, then the increment method yields the least error. Less exact empirical methods can also be used.

12.3 Takeoff Distance (refs 12.1, 12.2, 12.5)

The total takeoff distance is the sum of the ground roll distance (S_g, from brake release to main wheel lift-off) and the horizontal component of the air distance (S_a, from liftoff to main gear reaching either 35 or 50 feet altitude-depending on the requirements).

$$S_{TO} = S_g + S_a$$

Both S_g and S_a can be standardized using the increment or ratio method, or by empirical relations. The empirical methods are useful when detailed aircraft models are not available. The more exact process of predicting takeoff distance using models is described in section 12.3.2.

12.3.1 Empirical Standardization Method

First correct for the effects of the test day wind. Define headwind velocity as V_w, liftoff true airspeed as V_{LO}, and test day ground roll as S_{gw}. With a typical variation of thrust per headwind, estimate the test day zero-wind ground roll (S_{gzw}) using the following empirical equation:

$$S_{g_{zw}} = S_{g_w}\left(\frac{V_{LO}}{V_g}\right)^{1.85} \qquad \text{(Eq'n 12.3)}$$

If the average thrust is not appreciably affected by velocity, then the exponent should be 2.0 in lieu of 1.85. The zero-wind air distance (S_{at}) correction is

$$S_{a_t} = S_{a_w} + V_w t \qquad \text{(Eq'n 12.4)}$$

where t is the time from liftoff to 35 (or 50) feet altitude.

The second correction is for the effect of runway slope (θ, positive *uphill*) and therefore applies only to the ground roll. Correct the above zero-wind distance (S_{gzw}) to the test day zero-slope distance (S_{gt}) as follows:

$$S_{g_t} = S_{g_{zw}}\left[1 - \frac{W \sin\theta}{[F_{ex}]_{avg}}\right] \qquad \text{(Eq'n 12.5)}$$

If the average excess thrust is not known, then approximate F_{ex} as that at 70% of the liftoff airspeed or from the zero-wind ground roll distance:

$$[F_{ex}]_{avg} \approx \frac{mV_{LO}^2}{2S_{g_{zw}}} \qquad \text{(Eq'n 12.6)}$$

After correcting the test day distance to zero wind and slope, use the
following empirical equations to correct for non-standard weight, density, and temperature. Any desired values can be treated as the "standard" conditions.

Aircraft Propulsion Type	Standard Distance
Fixed pitch propellers *neglect temp correction for constant rpm evaluation	$S_{gs} = S_{gt}\left(\frac{W_s}{W_t}\right)^{2.4}\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.4}\left(\frac{T_{as}}{T_{as}}\right)^{0.5}$ $S_{as} = S_{at}\left(\frac{W_s}{W_t}\right)^{2.2}\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.2}\left(\frac{T_{as}}{T_{at}}\right)^{0.6}$
Turbo-propeller aircraft *for heavy aircraft, replace 2.3 & -1.2 with 2.6 & -1.5 respectively	$S_{gs} = S_{gt}\left(\frac{W_s}{W_t}\right)^{2.6}\left(\frac{\sigma_s}{\sigma_t}\right)^{-1.7}\left(\frac{N_s}{N_t}\right)^{-0.7}\left(\frac{P_s}{P_t}\right)^{-0.9}$ $S_{as} = S_{at}\left(\frac{W_s}{W_t}\right)^{2.3}\left(\frac{\sigma_s}{\sigma_t}\right)^{-1.2}\left(\frac{N_s}{N_t}\right)^{-0.8}\left(\frac{P_s}{P_t}\right)^{-1.1}$
Large jet aircraft *for lights jets, replace 2.3 & 0.7 exponents with 2.6 & 1 respectively	$S_{gs} = S_{gt}\left(\frac{W_s}{W_t}\right)^{2.3}\left(\frac{\sigma_t}{\sigma_s}\right)\left(\frac{F_{nt}}{F_{ns}}\right)^{1.3}$ $S_{as} = S_{at}\left(\frac{W_s}{W_t}\right)^{2.3}\left(\frac{\sigma_t}{\sigma_s}\right)^{0.7}\left(\frac{F_{nt}}{F_{ns}}\right)^{1.6}$

where
P_t = Test day pressure
N_t = Test day propeller RPM
$F_{nt} = T_{ot}$ = average test net thrust (approx .94 *x* static thrust).

These empirical corrections are valid only for small (<10%) changes.
(above equations from ref 12.3)

12.3.2 Takeoff Distance Prediction (refs 12.1, 12.5)

With reasonably precise models available, the takeoff distance can be predicted through calculation. Test distances can then be standardized using either the increment or ratio method (Eqn's 12.1 and 12.2). The following **direct approximation of takeoff ground roll** requires a value for the average net thrust (T_{avg}) across the takeoff roll speed range:

$$S_g = \frac{W}{g\rho_a S\left(\mu C_{L\,IGE} - C_{D\,IGE}\right)} \ln\left[V_{TO}^2 + \frac{T_{avg} - \mu W}{\frac{\rho_a S}{2}\left(\mu C_{L\,IGE} - C_{D_{IGE}}\right)}\right] \quad \text{(Eq'n 12.7)}$$

where

μ is the rolling friction coefficient (typically between 0.015 and 0.025 for hard dry runways), and

C_{LIGE} is the lift coefficient in ground effect while at ground roll attitude.

Estimate C_{LIGE} by determining the out-of-ground-effect lift coefficient (C_{LOGE}) at the ground roll angle of attack and correcting it as follows: n

$$C_{LIGE} = C_{LOGE}\ \mathrm{GE_{CL}} \qquad \text{(Eq' 12.8)}$$

where the ground effect factor, $\mathrm{GE_{CL}}$ = [0.8609 –0.6282 $\log_{10}(h/b)$]

and h is the wing height above the surface and b is the wingspan.

The above correction is *not used* above the height that predicts

$$\mathrm{GE_{CL}} < 1$$

C_{DIGE} is the induced drag coefficient while in ground effect. Estimate this by determining the out-of-ground-effect drag coefficient ($C_{Di\mathrm{OGE}}$) at the appropriate angle of attack and correcting it as follows:

$$C_{Di\mathrm{IGE}} = C_{Di\mathrm{OGE}}\ \mathrm{GE_{CD}} \qquad \text{(Eq'n12.9)}$$

where the ground effect factor, $\mathrm{GE_{CD}}$ = [0.2412 $\ln(h/b)$+1.0829]

The above correction is *not used* above the height that predicts

$$\mathrm{GE_{CD}} > 1$$

A **direct approximation of takeoff air distance** requires the desired speeds at liftoff and at 50 feet (typically $1.1V_s$ and $1.2V_s$, respectively). It also requires an estimate of the average excess thrust as the aircraft climbs out of ground effect.

$$S_a = \frac{W}{(T-D)_{avg}}\left[\frac{\left(V_{50}{}^2\right)-\left(V_{LO}{}^2\right)}{2g}+50\right] \qquad \text{(Eq'n 12.10)}$$

A **direct approximation of the total takeoff distance** (S_{TO}) can be calculated as the sum of the ground and air distances or can be estimated by multiplying the ground roll distance by a "planform factor" (F_{pl}).

$$S_{TO} = S_g F_{pl} \qquad \text{(Eq'n 12.11)}$$

F_{pl} combines the effects of wing type, thrust-to-weight ratio, and pilot technique. The following values characterize the typical aircraft.

straight wing: $F_{pl} = 1.15$
swept wing: $F_{pl} = 1.36$
delta wing: $F_{pl} = 1.58$

A more **exact prediction of takeoff performance** (ref 12.5) requires accurate thrust and drag models and an integration of the aircraft's velocity over the takeoff time. This is equivalent to a double integration of the aircraft's acceleration or its specific excess thrust.

$$S_{TO} = \int V_T dt = \iint a dt = \iint \frac{F_{ex}}{m} dt = \frac{1}{m}\iint F_{ex} dt \qquad \text{(Eq'n 12.12)}$$

This double integration can be performed numerically or graphically. Alternately, use planar kinematics and sum the distances required to accelerate between incremental true airspeeds from brake release (V_0) to the true airspeed when the aircraft reaches the takeoff altitude ($V_{50'}$).

$$S_{TO} = \frac{m}{2}\sum_{V_0}^{V_{50'}} \frac{V_2{}^2 - V_1{}^2}{F_{ex}} \qquad \text{(Eq'n 12.13)}$$

Both methods above are typically split into pre-rotation ground roll, rotation/post-rotation ground roll, and airborne segments. Both methods require calculation of the excess thrust, addressed below.

Solving for the excess net thrust *during the ground roll* for either takeoff or landing cases requires a simultaneous solution of the three equations of motion along the aircraft's longitudinal & vertical axes and about the pitch axis. These equations (in the above order) are as follows:

$$F_{ex} + \mu_{nw}R_{nw} + \mu_m R_m = [F_g \cos\iota_T + F_e - D_{wb} - D_t - \sin\theta_{rw}] \qquad \text{(Eq'n 12.14)}$$

$$R_{nw} + R_m = [W\cos\theta_{rw} - L_w - L_t] \qquad \text{(Eq'n 12.15)}$$

$$(X_1 + X_2)R_{nw} = [W\cos\theta_{rw}X_2 + W\sin\theta_{rw}Z_1 + \{F_g\cos(\theta+\iota_T) - F_e\}Z_1 + L_t(X_3 + X_4 - X_2) - L_w(X_2 - X_3) - D_t(Z_1 + Z_2)] \qquad \text{(Eq'n 12.16)}$$

where

F_{ex} = excess net thrust
μ_{nw} = nose wheel coefficient of friction
(about 0.02 for takeoff, 0.5 for maximum dry runway braking)
R_{nw} = reaction force (weight) on nose wheel (positive)
μ_m = main wheel coefficient of friction (positive)
R_m = reaction force (weight) on nose wheel (positive)
F_g = gross engine thrust (positive, aligned with engine axis)
ι_T = thrust incidence angle (positive denotes thrust that generates lift)
F_e = ram thrust (or drag) due to momentum change of the air outside the engine, measured along drag axis-aligned with relative wind (typically negative at low speed, positive at high speed)
D_{wb} = aerodynamic drag of wing and body (excludes horizontal tail drag)
D_t = aerodynamic drag of horizontal tail (positive aft)
W = aircraft weight (positive)
θ_{rw} = runway slope (positive denotes uphill)
L_w = main wing lift (positive denotes up)
L_t = main wing lift (positive denotes up)
X_1 = distance from nose gear to aircraft cg (positive)
X_2 = distance from the main gear to aircraft cg (positive)
Z_1 = distance from the ground plane to the aircraft body axis (positive)
θ = aircraft pitch attitude (positive denotes nose up)
X_3 = horiz. dist. from wing's aerodynamic center to aircraft *cg* (positive)
X_4 = horizontal distance from the wing's aerodynamic center to the horizontal tail's aerodynamic center (positive)
Z_2 = vertical distance from the horizontal tail's aerodynamic center to the aircraft body axis (positive)

The previous equations were arranged so that the right hand side of each can be abbreviated as A_1, A_2, and A_3 respectively. This step allows for a compact matrix form of the equations using a 3x3 matrix

$$\begin{bmatrix} 1 & \mu_{nw} & \mu_m \\ 0 & 1 & 1 \\ 0 & X_1 + X_2 & 0 \end{bmatrix} \begin{bmatrix} F_{ex} \\ R_{nw} \\ R_m \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

Solve for F_{ex} by pre-multiplying both sides by the inverse of the first matrix

$$\begin{bmatrix} F_{ex} \\ R_{nw} \\ R_m \end{bmatrix} = \begin{bmatrix} 1 & \mu_{nw} & \mu_m \\ 0 & 1 & 1 \\ 0 & X_1 + X_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

Although wheel reaction forces are not required for takeoff distance prediction, they are useful for accurate calculation of rotation capability and for braking effectiveness during landing ground rolls. For takeoff calculations, several simplifying assumptions can be made such as:

$$\mu_{nw} = \mu_m = 0.02$$
$$F_g >> F_e$$
$$D_t = 0$$

This above approach can be repeated for the segment between rotation and liftoff. This is slightly more complicated because the changing angle of attack alters drag and acceleration.

Precise **predictions of the takeoff air distance** can be made by applying Equation 12.10 in small increments using accurate models that describe thrust as a function of airspeed and the lift & drag changes due to climbing out of ground effect.

Along with the incremental S_g and S_a calculations, the time to accelerate between the corresponding incremental velocities can be calculated as

$$\Delta t = m \frac{V_2 - V_1}{F_{ex}}$$

12.4 Landing Distance (refs 12.1, 12.2)

The total landing distance (S_{LD}) is the sum of the ground roll distance (S_g, from touchdown to full stop) and the horizontal component of the air distance (S_a, from the screen height to touchdown). The screen height can be either 35 or 50 feet above the surface, depending on the requirements

$$S_{LD} = S_g + S_a$$

Both S_g and S_a can be standardized according to the increment or ratio methods described by equations 12.1 and 12.2, or by empirical relations. The empirical methods are useful when detailed aircraft models are not available. The more exact process of predicting landing distance using models is described in section 12.4.2.

12.4.1 Empirical Standardization Method

First correct for the effects of the test day wind. Define headwind velocity as V_w, touchdown true airspeed as V_{TD}, and test day ground roll as S_{gw}. With a typical variation of thrust per headwind, estimate the test day zero-wind ground roll (S_{gzw}) using the approach applied to takeoff ground roll:

$$S_{g_{zw}} = S_{g_w}\left(\frac{V_{TD} + V_w}{V_{TD}}\right)^{1.85} \qquad \text{(Eq'n 12.17)}$$

If the average thrust is not appreciably affected by velocity, then the exponent should be 2.0 in lieu of 1.85.

Apply Equation 12.4 to correct to the zero-wind air distance (where t is the time to descend from the screen height to touchdown).

To **correct to a zero-slope runway**, apply Equation 12.5 to the zero-wind ground roll distance (note that F_{ex} is negative). If the average excess thrust is not known, then approximate $[F_{ex}]_{avg}$ as that at 70% of the touchdown airspeed. Alternately, approximate $[F_{ex}]_{avg}$ from the zero-wind ground roll distance using

$$[F_{ex}]_{avg} \approx \frac{mV_{TD}^2}{2S_{g_{zw}}} \qquad \text{(Eq'n 12.18)}$$

After correcting the test day distance to zero wind and slope, use the following empirical equations to correct the ground roll to standard weight and air density.

$$S_{g_s} = S_{g_t}\left[\frac{W_s}{W_t}\right]^2 \frac{\sigma_t}{\sigma_s} \qquad \text{(Eq'n 12.19)}$$

Any desired values can be treated as the "standard" conditions.

Correct the air distance to standard weight and air density using the zero-wind air distance as follows (for a 50-foot screen height)

$$S_{a_s} = S_{a_t}\left[\frac{W_s}{W_t}\right]^{2+\frac{h_v}{h_v+50}}\left[\frac{\sigma_t}{\sigma_s}\right]^{\frac{h_v}{h_v+50}} \qquad \text{(Eq'n 12.20)}$$

where h_v is the specific kinetic energy change during the air phase. For the case of a 50-foot screen height, this term is calculated as

$$h_v = \frac{V_{50'}^2 - V_{TD}^2}{2g} \qquad \text{(Eq'n 12.21)}$$

12.4.2 Landing Distance Prediction

With reasonably precise models available, the landing distance can be predicted through calculation. Test distances can then be standardized using either the increment or ratio method (Equations 12.1-12.2).

A **direct approximation of landing ground roll** can be obtained by applying the same Equation 12.7 used for the takeoff case. This method requires a value for the average net thrust (T_{avg}) across the landing roll speed range and reasonable values for the wheel braking friction coefficient ($0.35<\mu<0.5$ for typical dry runway max braking). The same equations for estimating ground effect also apply.

A **direct approximation of takeoff air distance** including the flare requires the desired lift and associated drag coefficients, the thrust, and the applied normal load factor during the landing flare (n = 1.15).

$$S_a = \frac{50}{\left(\frac{C_D}{C_L} - \frac{T}{W}\right)} + \frac{\frac{W}{S}\left(\frac{C_D}{C_L}\frac{T}{W}\right)}{T\rho_o g(n-1)C_L} \qquad \text{(Eq'n 12.22)}$$

A more **exact prediction of landing performance** requires accurate thrust and drag models and an integration of the aircraft's velocity across the landing time. This is equivalent to a double integration of the aircraft's acceleration as shown in Equation 12.12. This double integration can be performed numerically or graphically. Similarly, Equation 12.13 can be adapted for landing as follows:

$$S_{LD} = \frac{m}{2}\sum_{V_0}^{V_{50'}} \frac{V_2^{\,2} - V_1^{\,2}}{F_{ex}} \qquad \text{(Eq'n 12.23)}$$

As with the takeoff case, this equation is usually broken into the air phase and the ground roll phase. Calculation of excess thrust during the ground roll needs to consider the changing weight on the wheels and associated braking force. This requires a simultaneous solution of the three equations of motion along the aircraft's longitudinal & vertical axes and about the pitch axis, previously shown as Eq'ns 12.14-12.16.

Precise calculation of excess thrust during the air phase must consider the change in normal and longitudinal load factor during the flare and the changes in lift and drag coefficients due to entering ground effect (previously described).

If the desired flare technique is some schedule of flight path angle (γ) versus altitude, then the normal load factor can be calculated from γ and the rate of γ using

$$N_z = \cos\gamma + \frac{V_T\,\dot{\gamma}}{g}$$

The longitudinal load factor can be calculated as $N_x = \frac{\dot{H}}{V_T} + \frac{\dot{V}_T}{g}$

where $\sin\,\gamma = \frac{\dot{H}}{V_T}$

An alternate method of calculating distance is the **fixed time increment approach**. The following air distance example is based on a *constant angle of attack* landing technique (ref 12.5).

Fixed inputs	Initial inputs
angle of attack, α	initial ground speed, V_{go}
wing area, S	initial air distance, $S_{ao} = 0$
air density, ρ	initial altitude, h_o
weight, W	initial sink rate, $h_o$dot
wingspan, b	initial lift coefficient, C_{LOGE}
head wind, V_w	wing aspect ratio, AR
net thrust, F_n ($F_n = F_g \cos\alpha - F_e$)	wing Oswald efficiency factor, e

time increment, Δt (.05 sec works well)
height of wing above ground when on gear, h_{wing}

Initial calculations
initial true airspeed, $V_{To} = V_{go} + V_w$
initial glide slope, $\gamma_o = \sin^{-1}[h_o\text{dot}]/V_{To}$
initial load factor, $N_z = \cos\gamma_o$ (assumes $d\gamma_o/dt = 0$)
initial trim speed, $V_T = [2N_zW/\rho C_L S]^{1/2}$

Incremental calculations
(values with prime symbols represent the result of the previous iteration).

1) $C_L = C_{LOGE}\,\text{GE}_{\text{CL}} = C_{LOGE}[0.8609 - 0.6282\log_{10}(h/b)]$
2) $L = 0.5 N_z W \rho C_L S V_T^2$
3) $N_z = L/W$
4) $\gamma\text{dot} = g(N_z - \cos\gamma)/V_T$
5) $\gamma = [\gamma dot]'\Delta t + \gamma'$
6) $h\text{dot} = V_T \sin\gamma$
7) $h = (h\text{dot} + [h\text{dot}]')\Delta t/2 + h'$
8) $C_{Di\text{IGE}} = C_{Di\text{OGE}}\,\text{GE}_{\text{CD}} = [C_L^2/\pi AR e][0.2412\ln(h/b) + 1.0829]$
9) $C_D = C_{Do} + C_{Di\text{IGE}}$
10) drag, $D = C_D \rho S V_T^2/2$
11) $F_{ex} = F_n - D$
12) $N_x = F_{ex}/W$
13) $V\text{dot} = g(N_x - \sin\gamma)$
14) $V_g = V_g' + V\text{dot}(\Delta t)$
15) $S_a = (V_g + V_g')\Delta t/2 + S_a'$

12.5 Climb/Descent/Level Acceleration (ref 12.4)

Standard performance can be determined either by *predicting* results using (flight test validated) models or by *correcting* individual flight test climb/acceleration results to standard conditions

Performance predictions require accurate net thrust and aerodynamic models. Net thrust is the sum of the gross thrust and ram drag, while the aero model includes the drag polar and lift curve.

Corrections to individual climb/accelerations tests require models that only show the *change* in thrust & drag between test and standard conditions. The following sections address both the prediction and correction approaches.

12.5.1 Climb/Descent/Acceleration Prediction

According to basic energy theory, an aircraft's specific excess power (P_s) is related to the change in kinetic and potential energy as fol- lows

$$P_s = \frac{P_{ex}}{W} = \frac{(F_n - D)V}{W} = \frac{dH}{dt} + \frac{H}{W}\frac{dW}{dt} + \frac{V}{g}\frac{dV}{dt} + \frac{V^2}{2Wg}\frac{dW}{dt} \qquad \text{(Eq'n 12.24)}$$

where V is technically inertial speed. True airspeed and an assumption of zero wind is usually used instead of inertial speed. Since aircraft typically have negligible weight change during a maneuver, the above reduces to

$$P_s = \frac{(F_n - D)V}{W} = \frac{dH}{dt} + \frac{V}{g}\frac{dV}{dt} \qquad \text{(Eq'n 12.25)}$$

This shows not only how climb rate *or* acceleration performance can be predicted, but also shows how the climb and acceleration capabilities can be *exchanged* at any given specific excess power. Dividing this equation through by V shows the relation between specific excess thrust and **climb angle, γ**

$$\frac{P_S}{V} = \frac{F_n - D}{W} = \frac{\dot{H}}{V} + \frac{1}{g}\frac{dV}{dt} = \sin\gamma \qquad \text{(Eq'n 12.26)}$$

When predicting climb performance capability using this approach, iterations may be required because the resulting climb angle affects the normal load factor ($N_z = \cos\gamma$) and the therefore the induced drag.

12.5.2 Correcting to Standard Climb Rate

The below sequence corrects results at the test q and (usually) pressure altitude (i.e., $V_{e\ test} = V_{e\ std}$*, and* $H_{test} = H_{std}$*).*

1) If the test day vertical velocity is measured by timing pressure altitude changes, then first correct the altimeter readings for instrument error and then convert the indicated pressure altitude rate to geometric (tapeline) climb rate as follows

$$\left.\frac{dH}{dt}\right|_t = \left(\frac{T_t}{T_s}\right)\left.\frac{dH}{dt}\right|_{indicated}$$

2) Equation 12.25 yields the climb rate correction that accounts for the change in power (or thrust) between test and standard days (at the test weight and velocity)

$$\Delta\dot{H}\Big|_P = \Delta P_s = \frac{\Delta P}{W} = \frac{\Delta F_n V}{W}$$

where ΔP or ΔF_n comes from engine models. For reciprocating engines without models that can predict this power change, estimate the correction using only a standard day power chart and the following equation

$$\Delta\dot{H}\Big|_P = \frac{550\eta BHP_s}{W_t}\left[1 - \sqrt{\frac{T_s}{T_t}}\right]$$

3) A changing horizontal headwind with altitude will alter climb results. If this change (dV_w/dH) is known, then add the following correction to the tapeline climb rate

$$\Delta\dot{H}\Big|_{hw} = \frac{V}{g}\left(\frac{dV}{dH}\right)\left(\frac{dH}{dt}\right)_t$$

Usually the exact wind shear profile is unknown. In this case, fly perpendicular to the known crosswind direction and repeat each climb speed at the reciprocal heading. After completing the remaining corrections listed below, average the reciprocal results to obtain a standard climb rate.

4) If the climb is flown at constant indicated airspeed or Mach, then true airspeed will change with air density. Correct for any change in true airspeed with the following "acceleration factor" correction

$$\Delta\dot{H}\Big|_{AF} = \frac{V}{g}\frac{dV}{dt} = \frac{V}{g}\left(\frac{V_{final} - V_{initial}}{\text{time to climb}}\right)$$

5) Combine the previous corrections then multiply this by the "inertial correction" factor that accounts for the inertial effects of changing the weight from test to standard conditions

$$\dot{H}\Big|_I = \frac{W_t}{W_s}$$

6) To the above result, add a correction for the change in induced drag due to weight change.

$$\Delta\dot{H}\Big|_{Ind} = \frac{2}{\pi A \mathrm{Re} \rho_{alt} V_T S}\left[\frac{W_t^2 - W_s^2}{W_s}\right]$$

Summary of climb rate corrections

$$\dot{H}\Big|_{std} = \left\{\left(\frac{T_t}{T_s}\right)\frac{dH}{dt}\Big|_{indicated} + \Delta\dot{H}\Big|_P + \Delta\dot{H}\Big|_{hw} + \Delta\dot{H}\Big|_{AF}\right\}\dot{H}\Big|_I + \Delta\dot{H}\Big|_{Ind} \text{ (Eq'n 12.27)}$$

Equation 12.27 can also be used to correct descents, level accelerations, and level decelerations to a standard climb rate. The primary difference is that for level accelerations, the accelerations factor is the dominant term while the indicated climb rate is near zero.

12.5.3 Weight/Altitude/Temperature (WAT) Limits

To ensure safety, aviation authorities specify minimum climb gradients ($\gamma_{req'd}$) for many aircraft operations. The most straight forward way to comply with the specified gradients is to document the maximum allowable weight at various pressure altitude/temperature combinations.

Assuming the test day C_L for best γ equals that for any other day, calculate the maximum allowable weight by applying the following correction to the best test γ results.

$$W_{\max} = \frac{\left[\sin\gamma_t + \frac{C_{D_O}}{C_L} + \frac{C_L}{\pi A \mathrm{Re}}\right]W_t + \Delta F_n}{\sin\gamma_{req'd} + \frac{C_{D_O}}{C_L} + \frac{C_L}{\pi A \mathrm{Re}}}$$

where ΔF_n (= F_{nstd} - F_{ntest}) comes from the engine model. To ensure accuracy, the test configuration (i.e., one engine inoperative) must equal the standard configuration. Level acceleration results are not an acceptable substitute for actual climb data.

12.6 Level Turn Performance (ref 12.1)

Standard level turn performance can be determined either by *predicting* results using (flight test validated) models or by *correcting* individual turn results to standard conditions. It is possible to predict turn performance using climb or level acceleration data, but this approach is not always accurate and should be validated with actual turn results.

Performance predictions require accurate net thrust and aero models (drag polar and lift curve). Corrections to test day turn results require models that only show the *change* in thrust & drag between test and standard conditions.

The following sections address both the prediction and correction approaches. For either approach, load factor (n_{zw}) is usually determined first, then the corresponding turn rate (ω-radians/sec) and radius (*R*-ft) are calculated using the equations below.

$$R = \frac{V_T^2}{g\sqrt{n_{zw}^2 - 1}} \qquad \omega = \frac{g\sqrt{n_{zw}^2 - 1}}{V_T} \qquad n_{zw} = \sqrt{\left(\frac{\omega V}{g}\right)^2 + 1} \quad \text{(Eq'n}$$

12.6.1 Sustained Level Turn Performance Prediction

1) At the desired speed, altitude, temperature, and throttle setting use the engine model to determine the gross thrust (F_g). Sophisticated models may show this to be a function of the inlet angle of attack as well.

2) At the same conditions, use the engine and airframe models to determine the ram drag (F_e).

3) Calculate net thrust as $F_n = F_g \cos \alpha_F + F_e$ where $\alpha_F = (\alpha + \iota_T)$ and is ι_T the incidence angle of the thrust line (TED positive).

4) The total lift is the sum of the wing lift and the thrust lift:

$$L = L_W + F_g \sin\alpha_F.$$

Since $L = n_{zw}W$, then $L_W = n_{zw}W - F_g \sin\alpha_F$

5) For any sustained turn, the net thrust equals the drag

$$F_n = D = qSC_D = qS\left[C_{Do} + \frac{\left(\frac{L_W}{qS}\right)^2}{\pi A \operatorname{Re}}\right] = qS\left[C_{Do} + \frac{\left(n_{zw}W - F_g \sin\alpha_F\right)^2}{(qS)^2 \pi A \operatorname{Re}}\right]$$

Solving for load factor gives

$$n_{zw} = \frac{1}{W}\left(\left\{\left[\frac{F_g \cos(\alpha + \iota_T) + F_e}{qS} - C_{Do}\right](qS)^2 \pi A \operatorname{Re}\right\}^{1/2} + F_g \sin(\alpha + \iota_T)\right) \quad \text{(Eq'n 12.29)}$$

For any combination of weight, altitude, and airspeed, calculation of the standard sustained load factor requires knowledge of the gross thrust, ram drag, drag polar (C_{D0}, e), and angle of attack.

6) To determine the standard angle of attack, start with the lift curve slope model

$$C_L = C_{L_{\alpha=0}} + \frac{dC_L}{d\alpha}\alpha = C_{L_{\alpha=0}} + C_{L_\alpha}\alpha$$

Rearrange to solve for α

$$\alpha = \frac{C_L - C_{L_{\alpha=0}}}{C_{L_\alpha}} = \frac{\left(\frac{n_{zw}W - F_g \sin(\alpha + \iota_T)}{qS}\right)^2 - C_{L_{\alpha=0}}}{C_{L_\alpha}} \quad \text{(Eq'n 12.30)}$$

Because α cannot be solved for explicitly, calculate it using successive iterations of Equations 12.29 and 12.30.

In cases where $\frac{F_g}{W}\sin\alpha_F < \frac{n_{zw}}{10}$

the angle of attack can be roughly estimated without significant error to the final result.

12.6.2 Sustained Level Turn Performance Correction

The best method for obtaining standardized sustained level turn data is to correct actual level turn results to standard conditions. It is also possible to correct level acceleration or climb data to give standard level turn results. This approach may not work as well since any drag polar or engine model errors will be magnified. Additionally, inlet distortion that accompanies actual turn thrust is different during (low angle of attack) climbs and accelerations.

The equation below corrects any combination of test day climb, turn, and acceleration to a load factor for a sustained turn at the same dynamic pressure but at standard conditions.

$$n_{zw\,std} = \frac{F_{g_{std}}}{W_s}\sin\alpha_{F_s} + \left\{\left(n_{zw_t}\frac{W_t}{W_s} - \frac{F_{g_t}}{W_s}\sin\alpha_{F_t}\right)^2 + \frac{\pi A R e\, qS}{W_s^2}\left[\frac{W_t \dot{V}_{T_t}}{g} + \frac{W_t \dot{H}_t}{V_{T_t}} + F_{g_{std}}\cos\alpha_{F_{std}} - F_{g_t}\cos\alpha_{F_t}\right]\right\}^{\frac{1}{2}} \quad \text{(Eq'n 12.31)}$$

If $\frac{F_g}{W}\sin\alpha_F < \frac{n_{zw}}{10}$ then the above equation can be closely approximated as

$$n_{zw_{std}} = \frac{1}{W_S}\sqrt{\left(n_{zw_t}W_t\right)^2 + \pi A R e q S\left[\frac{\dot{V}_{T_t}W_t}{g} + \frac{\dot{H}_t W_t}{V_T} + \Delta F_{ex}\right]} \quad \text{(Eq'n 12.32)}$$

where $\Delta F_{ex} = F_{gstd}(\cos\alpha_{\mathrm{Fstd}}) - F_{gt}(\cos\alpha_{\mathrm{Ft}})$

The primary difference between using turn, accel, or climb test data is the dominant term in the above corrections. In all cases, the test and standard day thrust values come from engine models.

12.6.3 Level Limit Turn Performance Correction

A limit turn is one in which the aircraft performs a level turn beginning from maximum speed and maximum load factor and continues to decelerate at the N_{zb} limit until reaching the maximum C_L. At this point, the aircraft continues its level turning deceleration at the lift limit. This maneuver is also known as a "slow-down" turn.

Test day limit turn data is corrected to a standard specific excess power (P_s) for each given combination of altitude, Mach number and load factor (or AOA) limit. The following correction accounts for changes in trim drag, weight, and atmospheric affects on thrust.

$$P_{s_s} = P_{s_t} + \Delta P_s$$

where $$P_{s_t} = \frac{(F_{ex_t})V_{T_t}}{W_t} = \frac{(m_t a_{xw_t})V_{T_t}}{W_t} = \frac{\left(\frac{W_t a_{xw_t}}{g}\right)V_{T_t}}{W_t} = N_{xw_t} V_{T_t} = M a_o \sqrt{\theta_t} N_{xw_t}$$

and

$$\Delta P_s = M a_o \left\{ \left(F_{g_s} \cos\alpha_{F_s} + F_e\right)\frac{\sqrt{\theta_s}}{W_s} - \left(F_{g_t} \cos\alpha_{F_t} + F_e\right)\frac{\sqrt{\theta_t}}{W_t} + S C_{D_o}\left[\frac{q_t\sqrt{\theta_t}}{W_t} - \frac{q_s\sqrt{\theta_s}}{W_s}\right]\right.$$
$$\left. + \frac{S q_t \sqrt{\theta_t}}{W_t}\left(m\left[\frac{N_{zw_t} W_t - F g_{n_t} \sin\alpha_{F_t}}{q_t S}\right]^2 + \Delta C_{D_{trim_t}}\right) - \frac{S q_s \sqrt{\theta_s}}{W_s}\left(m\left[\frac{N_{zw_s} W_s - F_{g_s} \sin\alpha_{F_s}}{q_s S}\right]^2 + \Delta C_{D_{trim_s}}\right)\right\}$$

where $$\alpha_{F_t} = \iota_{T_t} - \frac{C_{L_{o\alpha}}}{a} + \frac{N_{zw_t} W_t - F_{g_t} \sin\alpha_{F_t}}{a q_t S} \qquad \text{(Eq'n 12.33)}$$

and $$\alpha_{F_s} = \iota_{T_s} - \frac{C_{L_{o\alpha}}}{a} + \frac{N_{zw_s} W_s - F_{g_s} \sin\alpha_{F_s}}{a q_s S}$$

As with the sustained level turn case, one cannot solve explicitly for α_F, so either assume an approximate value or iterate until a solution converges.

In For the simplified case where $\delta_t = \delta_s$, $cg_t = cg_{std}$, and $\sin\alpha_F = 0$, then the above equation reduces to

$$\Delta P_s = M a_o \left\{ \frac{\sqrt{\theta_s}}{W_s}\left[F_{n_s} - qSC_{D_o} - \frac{(N_{zw_s} W_s)^2}{qS\pi ARe}\right] - \frac{\sqrt{\theta_t}}{W_t}\left[F_{n_t} - qSC_{D_o} - \frac{(N_{zw_t} W_t)^2}{qS\pi ARe}\right]\right\} \qquad \text{(Eq'n 12.34)}$$

12.7 Reciprocating Engine Cruise Performance (ref 12.1)

Cruise performance standardization consists of correcting test day range and endurance results to standard conditions. Standard conditions are typically the standard aircraft weight & *cg* location, the nearest 5,000 ft increment of pressure altitude, and standard ambient temperature at that altitude. Although not included in this section, additional corrections can be made to adjust fuel flow to a standard
heating value and to adjust the thrust and fuel flow for the slight gravity effects due to changes in latitude and centrifugal relief (see section 3.2).

Although any weight can be called “standard,” several are quite common. General aviation aircraft typically have the test data corrected to the maximum takeoff weight. Transport aircraft often use a mid-mission weight (maximum payload and one-half fuel) as standard, and fighter/attack aircraft typically use full ordnance and half internal fuel as standard for any given configuration.

Once standard cruise results are documented, mission planning can be conducted by reversing the standardization equations to the desired “mission” conditions. If desired, test day results can be directly corrected to mission conditions by simply treating the mission conditions as standard. These options are shown below.

It is common practice to correct test data for only minor changes in altitude & temperature conditions. Because large changes in Mach and Reynolds numbers alter drag polars and engine efficiency, it is not common practice to correct results across altitude differences of more than 5,000 feet. This leads to a series of results separated by altitude.

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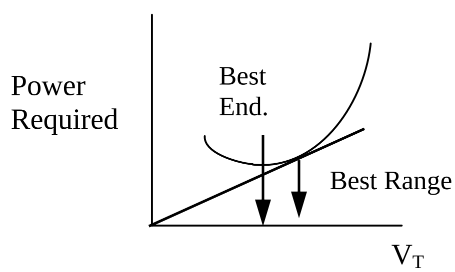
12.7.1 Power Standardization

If fuel flow is directly proportional to power output only, the power and optimal velocity for cruise performance can be determined from a power required curve as shown below.



To correct the power required curve to any standard altitude/weight condition, the usual approach is to treat the lift coefficient as the anchor ($C_{L\ test} = C_{L\ std}$). This leads to the following power and velocity standardization equations

$$P_s = P_t\left(\frac{W_s}{W_t}\right)^{\frac{3}{2}}\left(\frac{\sigma_t}{\sigma_s}\right)^{\frac{1}{2}} \qquad V_{T\,s} = V_{T\,t}\left(\frac{W_s}{W_t}\frac{\sigma_t}{\sigma_s}\right)^{\frac{1}{2}} \quad \text{(Eq'ns 12.35)}$$

Because the drag polar of low performance propeller aircraft generally collapses (generalizes) well to a single curve, it is often acceptable to correct <u>all</u> power required data to a single standard altitude/weight condition. When this condition is chosen to be standard day sea level at maximum weight, the above correction simplifies to what is known as the "P_{iw}~V_{iw}" values.

$$P_s = P_{iw} = P_t\left(\frac{W_s}{W_t}\right)^{\frac{3}{2}}(\sigma_t)^{\frac{1}{2}} \qquad V_{T\,s} = V_{iw} = V_{e\,t}\left(\frac{W_s}{W_t}\right)^{\frac{1}{2}} \quad \text{(Eq'ns 12.36)}$$

Although all points along the test day power curve can be standardized, the most useful points are those for best range and endurance. When corrected to standard conditions, the performance of the test aircraft can be fairly compared to that of another aircraft which has also been corrected to the same flight conditions.

Additionally, once the standard power and velocity are known and documented, the required power and airspeed for any "mission" conditions can be predicted by reversing Equations 12.36 as follows

$$P_m = P_{iw}\left(\frac{W_m}{W_s}\right)^{\frac{3}{2}}\left(\frac{1}{\sigma_m}\right)^{\frac{1}{2}} \qquad V_{T\,m} = V_{iw}\left(\frac{W_s}{W_t}\right)^{\frac{1}{2}}\left(\frac{1}{\sigma_m}\right)^{\frac{1}{2}} \quad \text{(Eq'ns 12.37)}$$

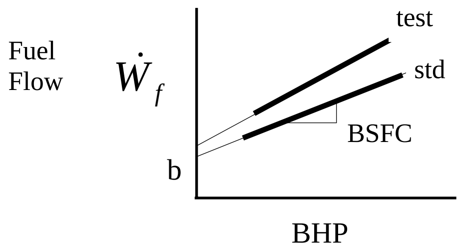
The power & optimal speed for best mission range (and mission endurance) are determined by applying the above equations to correct the points noted on the above figure. To correct directly from test conditions to mission conditions, apply Equation 12.35 and substitute mission weight and density in lieu of standard values.

12.7.2 Fuel Flow Standardization

Because reciprocating engine fuel flow is essentially proportional to power output, Equations 12.35 and 12.36 can be modified to correct the test fuel flow to standard values. For the following standardization equations to be accurate, the propeller efficiency and brake specific fuel consumption (*BSFC*) must be the same for test and standard days.

$$\dot{W}_{f\,s} = \dot{W}_{f_t}\left(\frac{W_s}{W_t}\right)^{\frac{3}{2}}\left(\frac{\sigma_t}{\sigma_s}\right)^{\frac{1}{2}} \quad \text{or} \quad \dot{W}_{f\,iw} = \dot{W}_{f_t}\left(\frac{W_s}{W_t}\right)^{\frac{3}{2}}(\sigma_t)^{\frac{1}{2}} \qquad \text{(Eq'ns 12.38)}$$

Because *BSFC* is affected by engine RPM (due to friction losses), fuel flow results at one engine RPM are never corrected to another RPM. Separate tests must be performed for each engine speed of interest. *BSFC* may also be affected by ambient air pressure and temperature. If the relation between fuel flow and power can be represented with a model as shown, then the fuel flow is a linear function of *BSFC*.



$$\dot{W}_f = b + BSFC \bullet BHP$$

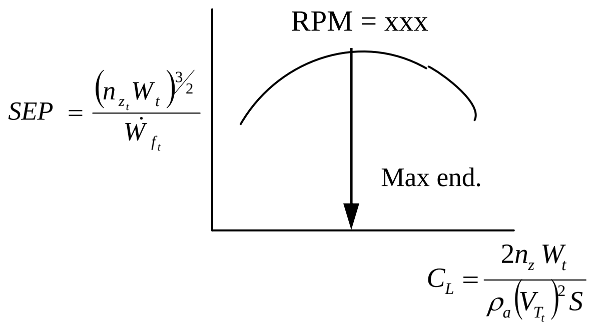
If the values for *b* and *BSFC* are known for both the test and standard conditions, then test fuel flow can be more exactly corrected to standard weight and density conditions as follows

$$\dot{W}_{f\,s} = b_s + \left(\dot{W}_{f\,t} - b_t\right)\frac{\eta_t}{\eta_s}\frac{BSFC_s}{BSFC_t}\left(\frac{W_s}{W_t}\right)^{\frac{3}{2}}\left(\frac{\sigma_t}{\sigma_s}\right)^{\frac{1}{2}} \qquad \text{(Eq'n 12.39)}$$

Note that this correction requires only a knowledge of the *ratio* of test and standard *BSFC* values. If both values have the same percent error, then the effect is self-canceling. The above equation also corrects for changes in fuel flow due to changing propeller efficiency.

12.7.3 Endurance Optimization and Prediction

To determine the optimum endurance flight profile and time aloft for any condition (at the same RPM as the test condition), plot the test day specific endurance parameter (*SEP*) vs the test day lift coefficient (C_L)



The maximum endurance occurs at the peak of the *SEP* curve. The associated lift coefficient is the optimum endurance condition for the aircraft (at that same RPM). The results of this test change with engine speed. If the aircraft operates at this optimum C_L or any other constant C_L , then the total endurance time (t) while *at constant altitude* can be calculated from this test day data using

$$t = SEP\left(\frac{\sigma_s}{\sigma_t}\right)^{\frac{1}{2}}\left[\frac{2}{\sqrt{W_F}} - \frac{2}{\sqrt{W_I}}\right] \qquad \text{(Eq'n 12.40)}$$

where the *SEP* comes from the above test day curve at whatever C_L is chosen. W_I is the total aircraft weight at the start of the endurance segment and W_F is the final weight of the endurance segment. This equation accounts for the effect of how a change in air density alters the power required and the subsequent fuel flow, but does not account for changes in propeller efficiency, *BSFC*, or the fuel flow intercept, *b*.For endurance at a constant C_L and V_T, use the following equation and the *SEP* and test weight associated with the lift coefficient at the start of the endurance segment.

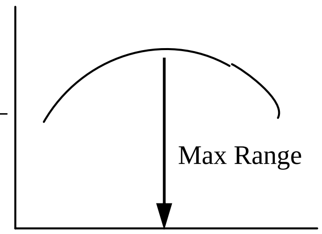
$$t = \frac{SEP}{\sqrt{W_t}} \ln \frac{W_I}{W_F} \qquad \text{(Eq'n 12.41)}$$

12.7.4 Range Optimization and Prediction (ref 12.1)

To determine the optimum range flight profile and distance for any condition (at the same RPM as the test condition), plot the test day range factor (*RF*) vs the test day lift coefficient (C_L)

RPM = xxx

$$RF = \frac{V_{T_t} W_t}{\dot{W}_{f_t}}$$



$$C_L = \frac{2 n_z W_t}{\rho_a \left(V_{T_t}\right)^2 S}$$

The maximum range occurs at the peak of the *RF* curve. The associated lift coefficient is the optimum range condition for the aircraft (at that same RPM). The results of this test change with engine speed. If the aircraft operates at this optimum C_L or any other constant C_L , then the range at constant *altitude* can be calculated from this test day data using

$$R = RF \ \ln \frac{W_I}{W_F} \qquad \text{(Eq'n 12.42)}$$

where the *RF* comes from the above test day curve at whatever C_L is chosen. W_I is the total aircraft weight at the start of the range segment and W_F is the final weight of the range segment. Although not explicitly shown in this equation, the correction does account for changes in air density, but does not account for changes in propeller efficiency, *BSFC*, or the fuel flow intercept, *b*.

For cruise at constant *airspeed and altitude*, use the following equation and the *RF* associated with the lift coefficient at the start of the cruise segment.

$$R = RF \bullet 2\tan^{-1}\left[\frac{1 - \frac{W_F}{W_I}}{1 + \frac{W_F}{W_I}}\right] \qquad \text{(Eq'n 12.43)}$$

12.8 Jet Aircraft Cruise Performance (ref 12.1)

Refer to section 12.7 for a general discussion of cruise performance standardization.

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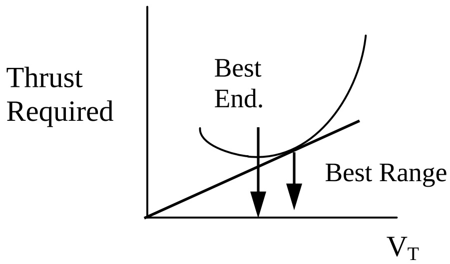
12.8.1 Thrust Standardization

If fuel flow is directly proportional to net thrust output only, the thrust and optimal velocity for cruise performance can be determined from a thrust required curve as shown below.



Because jet aircraft typically cruise at speeds where changes in Mach number affect the drag polar, it is customary to treat both the lift coefficient and Mach numbers as anchors ($C_{L\ test} = C_{L\ std}$, $M_{test} = M_{std}$). In terms of Mach number, cruise ($n_z = 1$) lift coefficient is calculated as

$$C_L = \frac{W/\delta}{1481 M^2 S} \qquad \text{(Eq'n 12.44)}$$

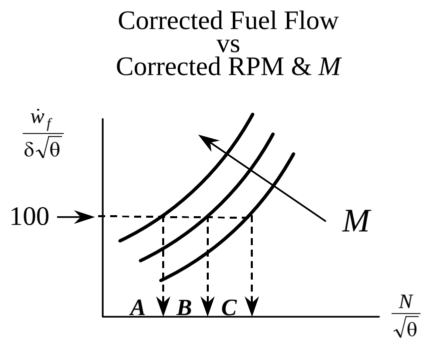
From this relation, the only way to match test & standard values for both C_L and M is to match test & standard values for W/δ. In this case, the test day net thrust required curve can be corrected to standard conditions as follows

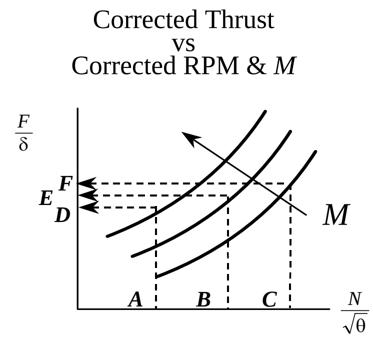
$$F_{n_s} = F_{n_t} \frac{W_s}{W_t} \qquad V_{T_s} = V_{T_t} \sqrt{\frac{\theta_s}{\theta_t}} \qquad \text{(Eq'ns 12.45)}$$

Although all points along the test day thrust curve can be standardized, the most useful points are those for best range and endurance. When corrected to standard conditions, the performance of the test aircraft can be fairly compared to that of another aircraft which has also been corrected to the same flight conditions. Additionally, once the standard thrust and velocity are known and documented, the required power and airspeed for any "mission" conditions can be predicted by reversing Equations 12.45.

12.8.2 Fuel Flow Standardization

Both the thrust and fuel flow of a simple (fixed-geometry turbojet) turbine engine are functions of engine speed (N), Mach number (M), ambient pressure (δ), and ambient temperature (θ). Dimensional analysis and experimental results show these parameters to be related approximately as illustrated in the figures below.





Thrust specific fuel consumption ($TSFC$) is defined as the fuel flow per thrust. At any given level of corrected fuel flow, the above figures can be cross-plotted onto a single figure that relates corrected thrust (F/δ) to corrected fuel flow at various Mach numbers.

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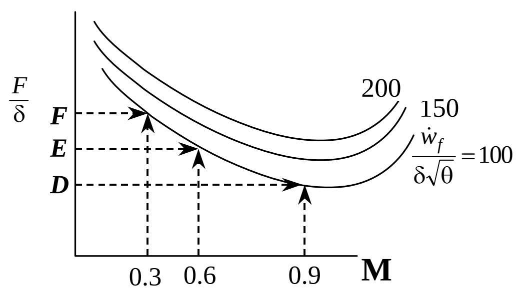


The slopes of the above figure exaggerate the typical case where $TSFC$ changes with Mach number. If, at any given Mach number, steady increments of corrected fuel flow are evenly spaced vertically, then

$$\frac{TSFC}{\sqrt{\theta}} \approx \text{ constant at that Mach number.}$$

Standard fuel flow can be determined from these relations. If flight test data is to be corrected from test to standard conditions at the same C_L and M , then the C_D will also be the same for both test & standard conditions. Because thrust equals drag* during cruise, the following relations show that corrected thrust (F/δ) must be the same for test and standard conditions

$$C_D = \frac{D/\delta}{1481 M^2 S} = \frac{F/\delta}{1481 M^2 S}$$

- technically $F_n \cos(\alpha+\iota_T)+F_e = D$, where F_n = net thrust, F_e = ram thrust, and ι_T is the thrust incidence angle

If Mach number and F/δ are equal for both test & standard conditions, then the previous cross plot shows that corrected fuel flow must also be the same for both conditions.

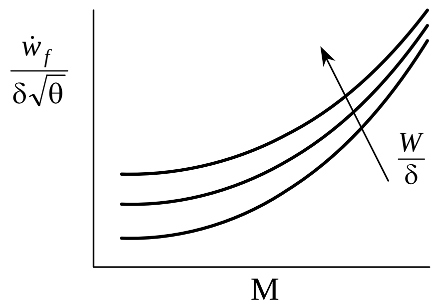
$$\frac{\dot{W}_{f_t}}{\delta_t \sqrt{\theta_t}} = \frac{\dot{W}_{f_s}}{\delta_s \sqrt{\theta_s}}$$

This relation allows standard fuel flow to be calculated as

$$\dot{W}_{f_s} = \dot{W}_{f_t} \frac{\delta_s \sqrt{\theta_s}}{\delta_t \sqrt{\theta_t}} \quad \text{if } C_{L\ test} = C_{L\ std},\ M_{test} = M_{std} \qquad \text{(Eq'n 12.46)}$$

12.8.3 Endurance Optimization and Prediction

Test day results are corrected to standard results at the same Mach, C_L (and therefore the same W/δ, according to Eq‘n 12.44) as the test condition. For each W/δ ratio tested, plot the test day corrected fuel flow vs the test Mach number.



At any given W/δ, the maximum endurance occurs at the Mach corresponding to the bottom of the curve. This optimal Mach and W/δ define the optimum lift coefficient for endurance (Eq‘n 12.44).

The corrected fuel flow for any desired Mach & W/δ combination can be interpolated from the above figure. Calculate the actual fuel flow using Equation 12.46.

Even with simple turbojets, experience has shown that the above curves do not generalize well if the desired standard altitudes

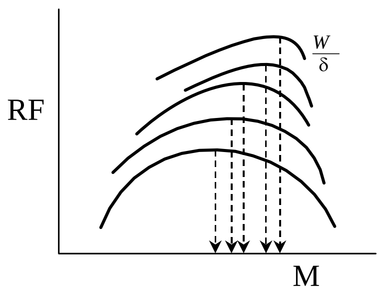
If the aircraft maintains flight at any combination of constant Mach & W/δ, then the corrected fuel flow will be constant. For flight at a constant C_L, endurance time can be calculated using

$$t = \frac{1}{\sqrt{\theta}} \frac{\sqrt{\theta}}{c} \frac{C_L}{C_D} \ln \frac{W_I}{W_F} \qquad \text{(Eq'n 12.47)}$$

where c is the thrust specific fuel consumption at sea level std conditions. W_I is the total aircraft weight at the start of the endurance segment and W_F is the final weight of the endurance segment. Although not explicitly shown in this equation, the correction process does account for changes in aircraft weight, and ambient pressure & temperature.

12.8.4 Range Optimization and Prediction

As with endurance analysis, test day range results are standardized at a common Mach & W/δ. Using the same corrected fuel flow vs Mach test data illustrated in section 12.8.3, create a cross plot of range factor (*RF*) vs Mach number for each W/δ tested.



where range factor can be calculated as

$$RF = SR_t \cdot W_t = \frac{V_{T_t}}{\dot{W}_{f_t}} W_t = \frac{M}{\dfrac{\dot{W}_{f_t}}{\delta_t \sqrt{\theta_t}}} \frac{W_t}{\delta_t} a_o \qquad \text{(Eq'ns 12.48)}$$

The optimum C_L for range at any given W/δ occurs at the Mach corresponding to the top of the curve. The best overall W/δ is the highest. These curves do not usually generalize well if the desired standard altitudes are more than about 5,000 ft away from the test altitudes.

If the aircraft cruises at any combination of constant Mach & W/δ, then the range factor will be constant, and range is calculated as

$$R = RF \ln \frac{W_I}{W_F} \qquad \text{(Eq'n 12.49)}$$

where *RF* comes from the above test day figure at whatever Mach & W/δ is chosen. It is often reasonable to interpolate the above test data to define a *RF* for the desired standard conditions. W_I and W_F are the total aircraft weights at the start and end of the range segment.

For cruise at constant *altitude*, fly at a constant C_L by allowing the airspeed to decrease with weight. Calculate range from test day results using

$$R = 2\sqrt{W_t} \frac{V_{T_t}}{\dot{W}_{f_t}} \left(\sqrt{W_i} - \sqrt{W_f}\right) \qquad \text{(Eq'n 12.50)}$$

For this equation to be valid, use the V_{Tt} and fuel flow corresponding to the same C_L and altitude of the desired standard conditions. Both of the above correction equations account for changes in aircraft weight and ambient temperature.

12.9 References

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12.3 Lush, Kenneth J “Standardization of Take-Off Performance Measuresments for Airplanes,” AFFTC Technical Note R-12, USAF Air Research and Development Command, Edwards AFB, CA, circa 1955.

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Section 13 Acoustics

13.1 Abbreviations and Terminology

Abbreviations

ANSI Acoustic National Science Institute
dB decibels
f frequency, cycles/sec
Hz Hertz
nm 10^{-9} meters
P sound power
p pressure
pW 10^{-12} Watts
x RMS value of quantity
x_o reference value of quantity
μPa 10^{-6} Pascals

Terminology

decade band with the upper frequency x10 that of thelower.
decibels measure of a magnitude, $dB = 10 log_{10}$(mag).
far field beyond the near field (region where sound level drops -6 *dB* as distance from the source doubles).
Hertz frequency in cycles/second.
narrow band band whose width is less than one-third octave but less than 1% of the center frequency near field range within a distance equal to the wavelength of the lowest frequency emitted or twice the greatest dimension of the subject.
octave a band with the upper freq exactly twice the lower freq. (common octaves include .0375-.075, .075-.15, 15-.3, .6-1.2, 1.2-2.4, 2.4-4.8, 4.8-9.6 k*Hz*).
pink noise has equal energy in each octave from 20 to 20,000 Hz, or with an energy content inversely proportional to frequency.
random noise does not have a uniform frequency spectrum and has an amplitude, as a function of time, consis tent with a Gaussian distribution curve.
third-octave highest frequency =1.26 x lower frequency (ratio= $2^{1/3}$)
white noise has a constant spectrum level over the entire band of audible frequencies (need not be random).

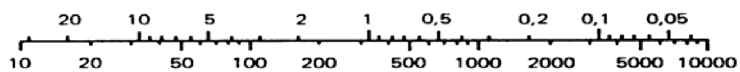
13.2 Acoustic Velocities, Spectrum, and Reference Levels

Acoustic Velocity (speed of sound)

Medium	Approximate Velocity
Air (20° C)	343 m/s
Fresh water	1,480 m/s
Aluminum	5,150 m/s
Concrete	3,600 m/s
Glass	5,300 m/s
Steel	6,000 m/s

Wavelength (λ) $= \dfrac{\text{acoustic velocity}}{\text{frequency}}$

wavelength (m) in air



frequency (Hz)

Human hearing range is approximately 20 to 20,000 Hz

- Ultrasound lies above 20,000 Hz
- Infrasound lies below 20 Hz

Acoustic Reference Levels

Quantity	Formula	
Velocity (L_v)	$20\log(v/v_0)$	$v_o = 10\ nm/s^2$
Intensity (L_I)	$10\log(I/I_0)$	$I_o = 1\ pW/m^2$
Sound Power Level (L_W)	$10\log(P/P_0)$	$Po = 1\ pW$
Sound Pressure Level "*SPL*" (L_p)	$20\log(p/p_0)$	$20\mu Pa$ (air)
Pressure Spectrum Level (*PSL*)*	SPL – $10 log\Delta f$	(dB)
Pressure Band Level (*PBL*)	$PSL + 10 log\Delta f$	(dB)
Overall SPL (*OASPL*)	$10 log_{10}\ \Sigma 10^{SPL/10}$	20 μPa (air)

* the *SPL* contained within a band 1 *Hz* wide

13.3 Acoustic Pressure and Intensity

Sound Pressure from Sound Power

Transmission Environment	L_p
Free Field	$L_W + \log Q - 20 \log r - 10.8\ dB$
Reflecting Plane	$L_W + \log Q - 20 \log r - 7.8\ dB$
Reverberant Room	$L_W + \log Q - 20 \log R - 6.2\ dB$

where r = distance from source
Q = directivity index of source
R = room constant

Acoustic Intensity

$$I - \frac{\text{Imaginary}[G_{yx}(f)]}{4\pi\rho_0 \Delta r f} = \frac{\text{Im}[G_{yx}(f)]}{16.25\ \Delta r f} \text{(for air)}$$

where ρ_0 = fluid density = 1.293 kg/m^3 for air
Δr = microphone spacing (meters)
f = frequency

Intensity Spectrum Level (ISL)

Intensity level of a sound contained within a band 1*Hz* wide

$$ISL = 10 \log \frac{I}{I_o \Delta f} = IL - 10 \log \Delta f \quad (dB)$$

where f = center frequency of band
I = sound intensity (watts/m^2)
$I_o = 10^{-12}$ watt/m^2 reference intensity
Δf = bandwidth (*Hz*)

13.4 Acoustic Weighting Curves (ANSI S1.4 1983)

Weighting for SPL

Nominal Freq (*Hz*)	Exact Freq (*Hz*)	A (*dB*)	B (*dB*)	C (*dB*)
10	10.00	-70.4	-38.2	-14.3
12.5	12.59	-63.6	-33.3	-11.3
16	15.85	-56.4	-28.3	-8.4
20	19.95	-50.4	-24.2	-6.2
25	25.12	-44.8	-20.5	-4.4
31.5	31.62	-39.5	-17.1	-3.0
40	39.81	-34.5	-14.1	-2.0
50	50.12	-30.3	-11.6	-1.3
63	63.10	-26.2	-9.4	-0.8
80	79.43	-22.4	-7.3	-0.5
100	100.0	-19.1	-5.6	-0.3
125	126.9	-16.2	-4.2	-0.2
160	158.5	-13.2	-2.9	-0.1
200	199.5	-10.8	-2.0	.0
250	251.2	-8.7	-1.4	.0
315	316.2	-6.6	-0.9	.0
400	398.1	-4.8	-0.5	.0
500	501.2	-3.2	-0.3	.0
630	631.0	-1.9	-0.1	.0
800	794.3	-0.8	.0	.0
1,000	1,000	.0	.0	.0
1,250	1,259	0.6	.0	.0
1,600	1,585	1.0	.0	-0.1
2,000	1,995	1.2	-0.1	-0.2
2,500	2,512	1.3	-0.2	-0.3
3,150	3,162	1.2	-0.4	-0.5
4,000	3,981	1.0	-0.7	-0.8
5,000	5,012	0.6	-1.2	-1.3
6,300	6,310	-0.1	-1.9	-2.0
8,000	7,943	-1.1	-2.9	-3.0
10,000	10,000	-2.5	-4.3	-4.4
12,500	12,589	-4.3	-6.1	-6.2
16,000	15,849	-6.7	-8.5	-8.6
20,000	19,953	-9.3	-11.2	-11.3

13.5 1/3 Octave Center Frequencies

(ANSI S1.6 1984)

Band No.	Nominal Center (*Hz*)	Exact Center (*Hz*)	Octave Center (*Hz*)
1	1.25	1.26	
2	1.60	1.58	
3	2.00	2.00	2.0
4	2.50	2.51	
5	3.15	3.16	
6	4.00	3.98	4.0
7	5.00	5.01	
8	6.30	6.31	
9	8.00	7.94	8.0
10	10.00	10.00	
11	12.5	12.59	
12	16.0	15.58	16.0
13	20.0	19.95	
14	25.0	25.12	
15	31.5	31.62	31.5
16	40.0	39.81	
17	50.0	50.12	
18	63.0	63.10	63.0
19	80.0	79.43	
20	100.0	100.00	
21	125.0	125.89	125.0
22	160.0	158.49	
23	200.0	199.53	
24	250.0	251.19	250.0
25	315.0	316.23	
26	400.0	398.11	
27	500.0	501.19	500.0
28	630.0	630.96	
29	800.0	794.33	
30	1,000	1,000.0	1,000
31	1,250	1,258.9	
32	1,600	1,584.9	
33	2,000	1,995.3	2,000
34	2,500	2,511.9	
35	3,150	3,162.3	
36	4,000	3,981.1	4,000
37	5,000	5,011.9	
38	6,300	6,309.6	
39	8,000	7,943.3	8,000
40	10,000	10,000.0	
41	12,500	12,589.3	
42	16,000	15,848.9	16,000
43	20,000	19,952.6	

13.6 References

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Additional Reading

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NOTES

Section 14 Electromagnetic Compatibility

14.1 Electromagnetic Compatibility (EMC)

This section gives the Flight Test Engineer a basic introduction to terms and concepts used by EMC engineers and insight into good testing philosophy and appropriate practices thus improving the interaction between the EMC and Flight Test engineers.

14.2 Abbreviations

A	Area	m^2
c	Speed of Light	3.0E8 m/s
E	Electric Field Intensity	Volts/meter, V/m
f	Frequency	Hertz, Hz
H	Magnetic Field Intensity	Ampere/meter, A/m
I	Current	Ampere, A
L	Inductance	Henries
Q	Charge	Coulomb, C
V	Electric Potential	Volt, V
X_C	Capacitive Impedance	Ohms
X_L	Inductive Impedance	Ohms
λ	Wavelength	meter

14.3 Terms

AC	Alternating Current
DC	Direct Current
EMC	Electromagnetic Compatibility
EMI	Electromagnetic Interference
Far Field	Distance beyond 10λ
HIRF	High Intensity Radiated Fields
RF	Radio Frequency

Decibel Logarithmic (base 10) expression for amplitude ratios.

$$dB(power) = 10 \, Log_{10} \, (P_1/P_2)$$

$$dB(voltage) = 20 \, Log_{10} \, (V_1/V_2)$$

$$dB(current) = 20 \, Log_{10} \, (I_1/I_2)$$

Commonly used decibels for EMC:

dBm	decibels relative to 1 milliwatt
dBW	decibels relative to 1 watt
dBμV	decibels relative to 1 microvolt
dBi	antenna gain relative to an isotropic antenna

dB	Power Ratio	V or I Ratio
0	1	1
3	2.0	1.4
6	4	2
10	10	3.2
20	100	10
30	1000	32

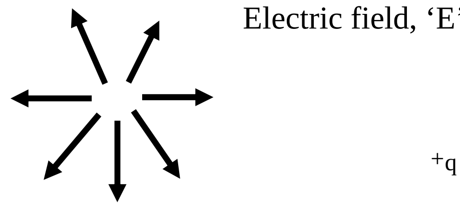
Common Decibel Values

The sensitivity of a radio receiver can be on the order of 1 μV/m, while RF field strengths for HIRF can be 1000V/m, a factor of a billion or 180dBμV/m.

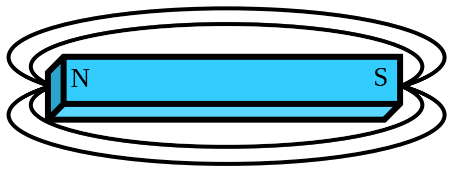
14.4 Fundamentals

14.4.1 Electric and Magnetic Fields

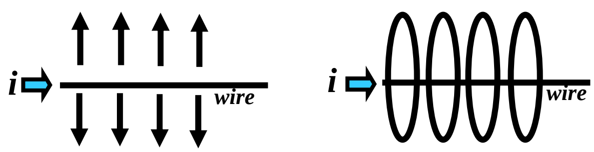
A static charge *Q*, creates a static Electric field, ‘E’.



A common magnet produces a static magnetic field, ‘H’.

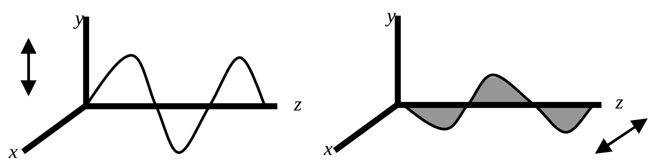


Transferring charge *Q*, i.e. DC current on a wire creates both a constant magnitude Electric and Magnetic field.



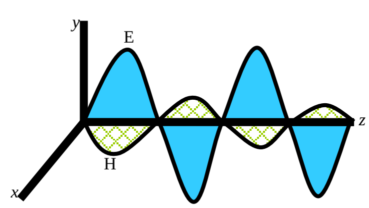
Electric Field ‘E’ Magnetic Field ‘H’

An amplitude varying charge, i.e. changing current (AC) will generate a time varying Electric and Magnetic Field in different planes.



Electric Field ‘E’ Magnetic Field ‘H’

Plane Waves that are self sustaining Electric and Magnetic Fields and combine in the far field, are commonly called an Electromagnetic Wave.



14.4.2 Antennas

Antennas can transmit and/or Receive RF equally well. Electrical length determines effectiveness.

$\lambda / 2$ is an efficient antenna element length

where: $\lambda = \dfrac{c}{f}$

Frequency, *f*	Wavelength, λ
3 MHz	100m
30 MHz	10 m
150 MHz	2 m
300 MHz	1 m
3 GHz	0.10 m

Common Wavelengths

Slots that have favorable electrical lengths are effective antenna elements also, i.e. hatches, doors, avionics metal enclosure seams and ventilation holes.

Loop Area is the area encapsulated between the signal line and its return path that can be an effective antenna. The larger the loop (capture) area, the better the antenna effectiveness is.

14.4.3 Spectra are the frequency content of the electronic signals and are an important consideration in understanding EMI issues. Periodic signals contain energy at various frequencies and as such, a frequency domain approach is needed. How much energy at what frequency depends largely on the type of periodic signal, (i.e. square wave or sine wave), initial frequency and rise/fall times of the signal. The faster the rise/fall times are, the more spectral content will be developed in the signal, most of which will be unintentional and unwanted. This is mathematically demonstrated by the use of a trigonometric Fourier series.

14.4.4 Non-Ideal behavior of components can exists in discrete components such as resistors, capacitors, inductors and even wire when operated at off nominal conditions, for example temperature. Another condition is frequency. For example, a short grounding wire from a DC perspective is a dead short, neglecting the extremely small inductance. But at some frequency, this inductance gets large enough to be a factor, for example on a bonding strap for lightning protection. A 26 gauge wire, 1 inch above a ground plane will have 0.028 µH per inch of inductance (L).

Where: $\mathbf{X_L=2\pi fL}$

For f = 150 MHz; X_L=26.4 Ohms per inch of wire which can be significant. To reduce this, replace the ground wire with a wide strap.

14.5 Electromagnetic Interference (EMI)

Electromagnetic Compatibility (EMC) is defined as systems that:

a) don't interfere with other systems;
b) are tolerant of interference from other systems;
c) don't interfere with itself.

Broadband Interference is interfering signals over a large range of frequencies. These can be associated with spark producing equipment like motors that can create signals with lots of spectral content.

Narrowband Interference is interfering signals that have a limited range of frequencies, usually a single frequency along with it's harmonics. These can be associated with digital devices that have periodic characteristics like clocks.

14.5.1 Interference Model

The classic interference model is:

Source → Path → Victim

To reduce the interference you can:

–Reduce the emissions from the **Source**
–Disrupt the **Path**
–Harden the **Victim**

Sources of interference can be clocks, switching power supplies, CPUs, data buses, network systems, relays, local oscillators, and transmitter harmonics.

Coupling Paths can be signal and power lines, radiating wires, apertures or slots on LRUs, windows, door and hatch openings or antennas themselves.

Front Door coupling is meant to be interference coming in the normal path to the system, i.e. through the antenna ports to the radio, and can cause interference at extremely low power levels (-100dBm).

Back Door coupling is interference coming into the system with the wires leading to the system and is of relatively higher power.

Capacitive coupling primarily involves electric waves in the near field and is due to voltages on wires.

Inductive coupling primarily involves magnetic waves in the near field and is due to current on either wires or chassis.

The aircraft fuselage is sometimes incorrectly thought of as a Faraday Cage encapsulating the RF energy inside or preventing it from entering because of its aluminum structure, but actually it is not. All of the windows, doors and hatches allow RF energy to travel through quite easily.

Victims of interference can be radio receivers, VHF, HF, VOR, ILS, ADF, Display systems, Audio and Passenger Address system, smoke and fire detection circuits, fuel quantity systems. Typically, low energy systems can be susceptible.

The reduction or elimination of EMI can be done in three areas; the systems end; by modifying the emissions and/or susceptibility requirements; or at the aircraft end by modifying the aircrafts wiring or structure.

14.5.2 Conducted Emissions

Current/signal on wires that are not the intended or primary signal is considered conducted emissions. This ‘extra’ current will be passed along to other systems and/or can radiate on those wires acting like antennas.

Differential Mode current is made up of the intended signal or information and/or noise that goes out on the signal wires and comes back on the return lines.

Common Mode current is usually just noise that goes out on two or more signal/return lines and returns via some other path. This is usually the most troublesome in terms of emissions and should be eliminated whenever possible.

14.5.3 Radiated Emissions

RF energy emanating from the unit/LRU itself through holes, slots and apertures or from the interconnecting wires is considered radiated emissions.

14.5.4 Aviation Frequency Spectrum

The table below lists the frequency spectrum of interest to the aviation community. The range is from 100kHz to 10GHz, a factor of 10^8 , (90dB). The primary interest is with equipment that is sensitive to RF energy, i.e. radio receivers, which are primarily intended to detect small signals (-105dBm). Emission requirements are set at a low level that will still allow proper operation of the radio receivers. For EMI purposes, emissions from equipment should stay clear of these frequencies.

Band	Frequency
ADF	190–1750 kHz
HF	2–30 MHz
Marker Beacon	75 MHz
VHF Nav	108-118 MHz
VHF Comm	118–138 MHz
Glideslope	328-335 MHz
DME, ATC, TCAS	960-1220 MHz
GPS	1227, 1558, 1575 MHz
Glonass	1609 MHz
Radio Altitude	4.2-4.4 GHz
MLS	5.0-5.25 GHz
WXR	5.4, 8.8, 9.0-9.3 GHz

Aviation Frequencies of Interest

14.6 Testing

Regulations and Industry Guidance

The following references are regulations and industry guidelines that address procedures and acceptable limits for interference testing.

-RTCA DO160D, Chapter 21
-FARs Part 25.1353 and 25.1431
-MIL STD 461
-CISPR
-FCC Part 15
-Aircraft manufacturers own standards

14.6.1 Lab Testing

Lab testing of the unit using established standards and practices is the first and best means of testing. Not only is this where you will find the trouble spots (i.e. frequencies) but also is a place where some troubleshooting could alleviate potential problem areas. Contracts with LRU vendors should be written to require the equipment pass these tests, identified above, before delivery. A list of frequencies that exceed an established limit is the result.

14.6.2 Aircraft Ground Testing

After lab testing, the unit should be installed in the airplane and be tested with the installed shops wiring. Testing will consist of measuring conducted emissions with current probes on wire bundles associated with the new equipment.

Radiated emissions are tested by using the aircrafts antennas hooked to test equipment to determine how much RF energy is getting into these sensitive systems. Again, a list of frequencies that exceed an established limit is the result.

14.6.3 Aircraft Flight Testing

Only after both lab and ground testing is accomplished can a meaningful flight test occur. The results of the ground test should produce a list of frequencies of some exceedance or observed interference. It is usually only these frequencies that need to be cleared in flight. The appropriate systems should be tuned to those frequencies and with the equipment to be tested in its' operating mode, determine if there is objectionable interference, (usually a pilots subjective opinion). Pilots can evaluate systems only if adequate lab/ground testing has been done beforehand. EMI issues that are found in Flight Test are very difficult and expensive to fix at this stage, and can typically only reduce or mask the problem.

14.6.4 Avionics changes and EMI testing

Changes in the hardware/wiring of a piece of avionics that could affect EMI testing are:
-Processor speeds
-Power Supply changes
-Frequency sensitive components, capacitors and inductors
-Circuit card layout and repackaging changes

Software changes typically don't affect EMI unless software controls/switches hardware related functions, i.e. speeds, options, peripherals etc.

14.7 Lightning

Lightning is a very large electrical transient that can impart thousands of Amperes of current through an aircraft structure. The structure needs to present a low impedance path for the lightning current so that no damage causing arcing and/or over-heating occurs. Additionally nearby wiring needs to be shielded to protect against the induced current produced by the ever changing magnetic fields.

14.7.1 Aircraft Lightning Zones

The aircraft is divided into different areas that relate to the probability of a lightning attachment. The nose, tail, wingtips and engine nacelles (extremities) are more likely areas.

14.7.2 Direct Effects

Direct effects of a lightning attachment can be in the form of heating, arcing and acoustic issues. Designing the structure to handle the current flow and providing a low impedance path for the lightning current will greatly minimize these effects.

14.7.3 Indirect Effects

Indirect effects considers the current that is induced by the transient and coupled onto aircraft wiring that is parallel to the main lightning current flow. The protection is two fold. Systems are designed and tested to handle these types of transients as well as the wiring is addressed to minimize the induced transient to these systems. Shielding and good grounding with short pigtails at both ends is a good method to reduce the induced current.

14.7.4 Instrumentation Precaution

Any flight test instrumentation wiring that lies outside the protective fuselage needs to be evaluated for both direct and indirect effects of a nearby lightning attachment. The sensor itself must be protected from the direct attachment and the wiring must be protected from induced current onto that wiring. This current may damage the data system equipment and/or, other aircraft systems that are also instrumented. Good shielding and grounding techniques will minimize these effects. For more information see the 10-6 Reference at the end of this handbook section.

14.8 High Intensity Radiated Fields (HIRF)

Aircraft can be exposed to large RF energy produced by high powered radio transmitters or military/airport surveillance radars. These RF fields can penetrate the aircraft fuselage through windows and doors/slots which could couple with aircraft wiring and/or systems and potentially interfere. This threat is addressed by both the aircraft and systems approach.

The systems themselves are designed and tested to be immune to a particular level of RF. These levels are determined by the criticality of the systems and are specified in regulatory material. Testing is usually done in a laboratory environment.

From the aircraft side, the internal wiring for critical systems is protected with appropriate shielding and grounding. Aircraft ground testing is done at special facilities that can radiate the vehicle with large RF fields with instrumentation inside to measure the penetration and to verify correct system operation.

14.9 Precipitation Static (Pstatic)

This occurs due to a buildup of static charges that discharge by noisy arcing from/to various parts of the aircraft. The static buildup is caused by tribo-electric charging from the aircraft impacting snow/rain/ash particles in the air while flying. This charge should gracefully exit the aircraft through static wicks installed on the wingtips and empennage tips. If it doesn't the problem shows up as broad banded noise (white noise) heard on receivers such as ADF, HF and to some extent VHF as the aircraft flies through the precipitation.

Typical causes are access panels (composite and metal), cowling and fairings that are not properly grounded. Ground straps do a good job of not isolating parts. (Note: these straps should not be used for lightning protection as they usually are not sized to handle the current).

14.10 Reference Material

10.1) Paul, C. R., "Introduction to Electromagnetic Compatibility", John Wiley & Sons Publishing, 1992

10.2) Ott, H. W, "Noise Reduction Techniques in Electronic Systems", John Wiley & Sons Publishing, 1988

10.3) Hrehov, D. W. and Walen, D. B., "What Flight Test Crews Need to Know About EMI/EMC", *34th Annual SFTE Symposium Workshop,* 2003

10.4) Federal Aviation Regulations, Part 25

10.5) RTCA DO160D, "Environmental Conditions and Test Procedures for Airborne Equipment", 1997

10.6) Hrehov, D. W., "What Instrumentation Engineers Need to Know About Lightning", *31st Annual SFTE Symposium,* 2000

10.7) Fisher, F. A., Perala, F. A., and Plumer, J A., "Lightning Protection for Aircraft", Lightning Technologies Inc., 1990

Section 15 Handling Qualities

15.1 Cooper-Harper Rating Related Figures

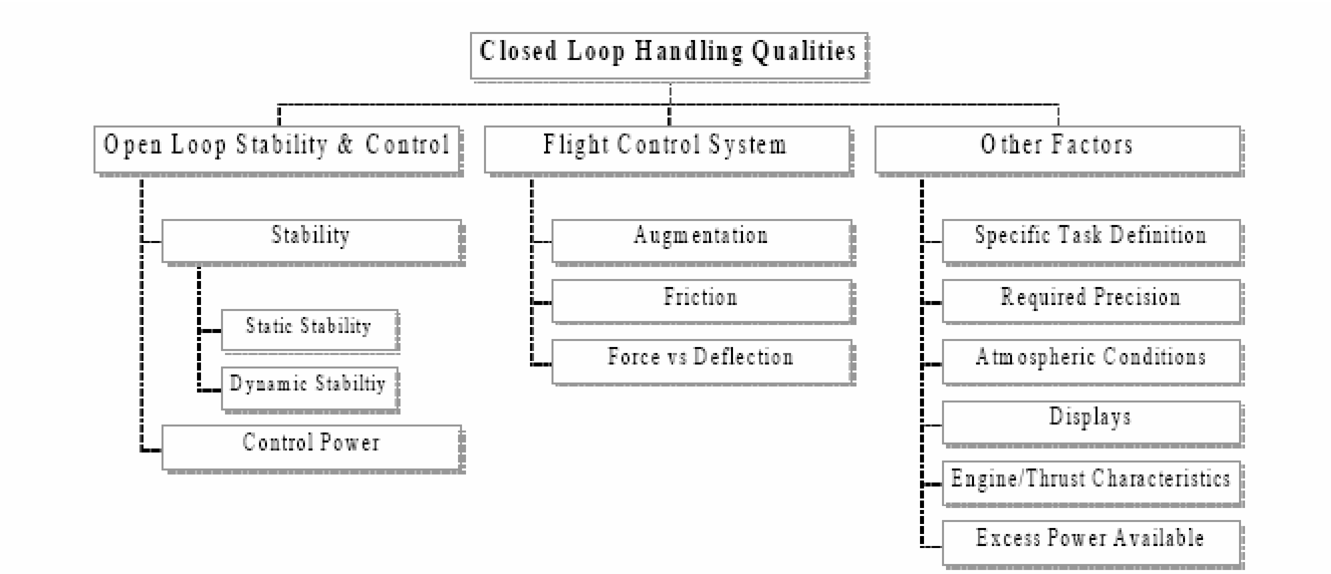


Figure 15.1-1 Elements of Closed-Loop Handling Qualities

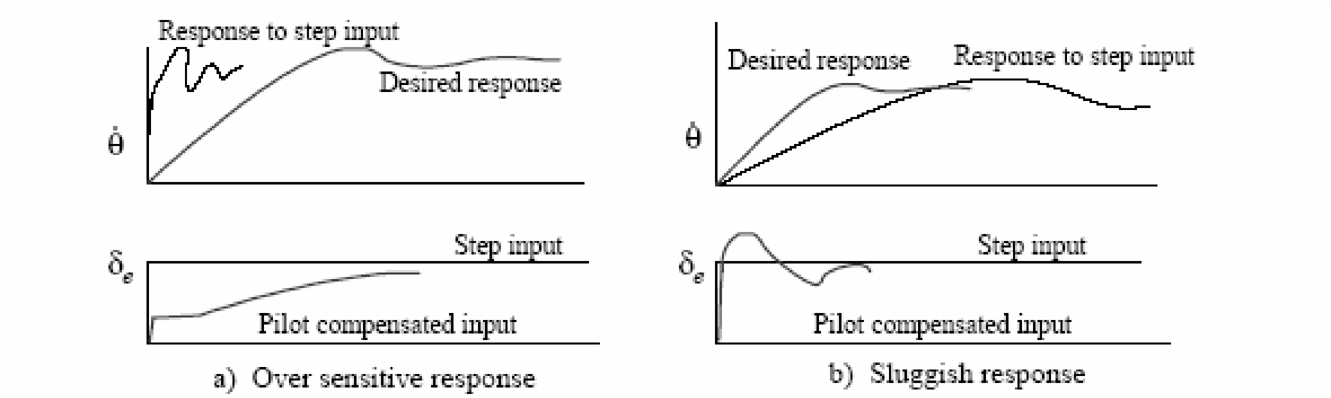


Figure 15.1-2 Undesirable step input responses and pilot compensation to achieve desired response a) Lag compensation, b) lead-lag compensation.

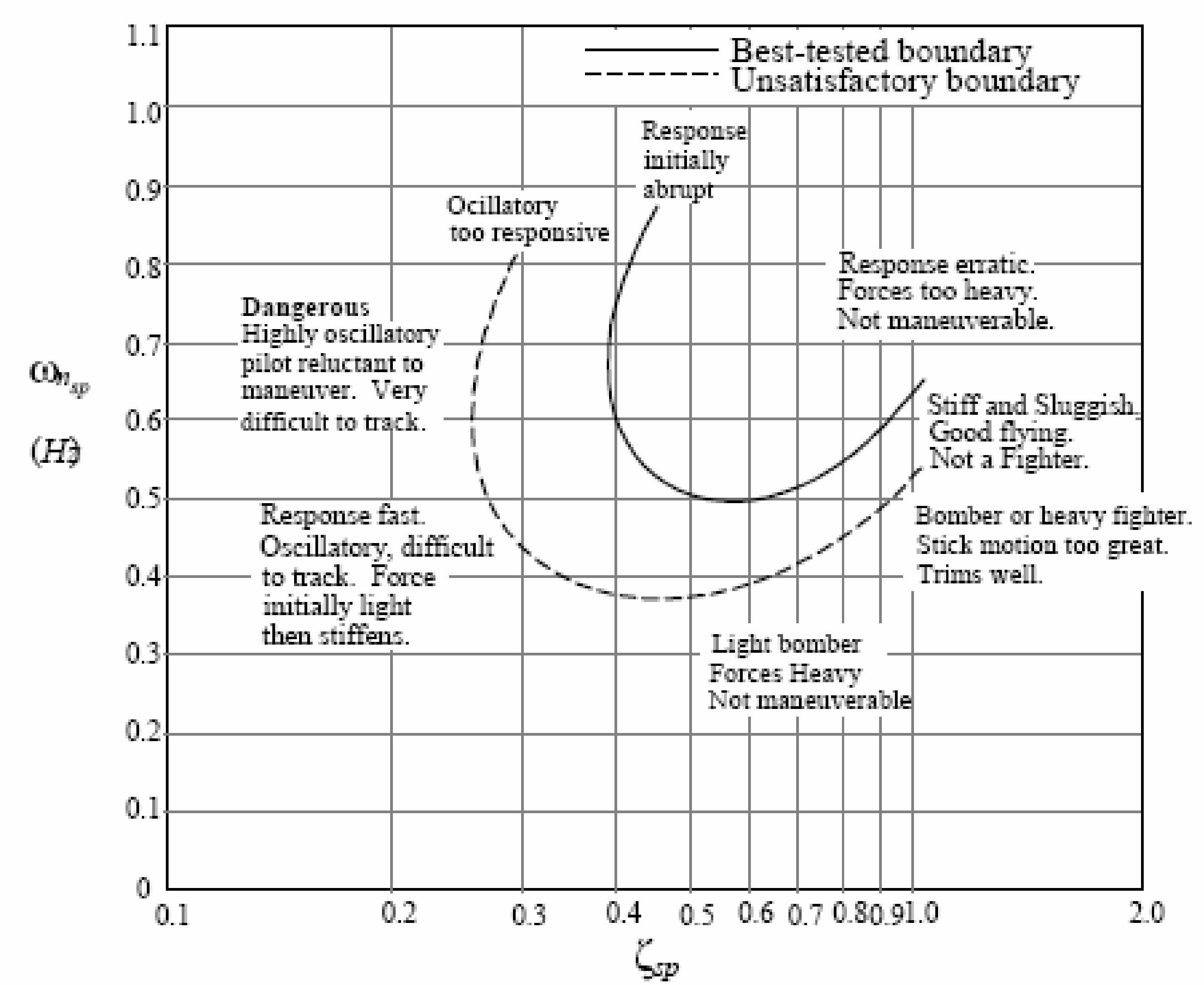


Figure 15.1-3 Optimum Short Period Frequency and Damping Based on Pilot Opinion

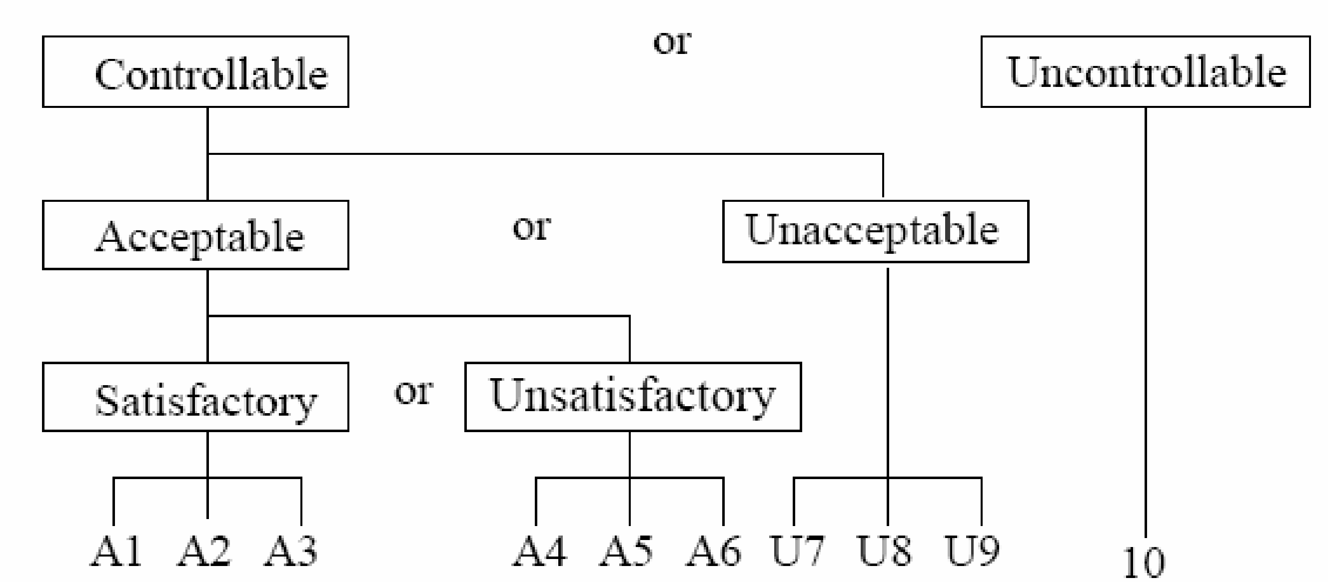


Figure 15.1-4 General Pilot Ratings for Handling Qualities

Closed Loop Handling Qualities Test Requirements

1) Explicit **mission definition:** what the pilot must accomplish. Identify the circumstances and operating conditions.

2) Define mission **tasks**. Tasks should be repeatable, require sufficient control input frequency to stress the system, and they should be of adequate duration to differentiate transient from steady state responses.

3) Establish **desirable** and **acceptable** criteria for task performance. Criteria established should be quantifiable, recordable, and realistic. Desirable criteria specify a satisfactory level of performance. Acceptable criteria specify the level of performance that is marginally adequate.

4) Test should include realistic typical **distractions and disturbances**.

5) Record task **performance** relative to the criteria established (comments, video, audio, pipper movement, etc.)

6). Measuring & record pilot **workload** and compensation.

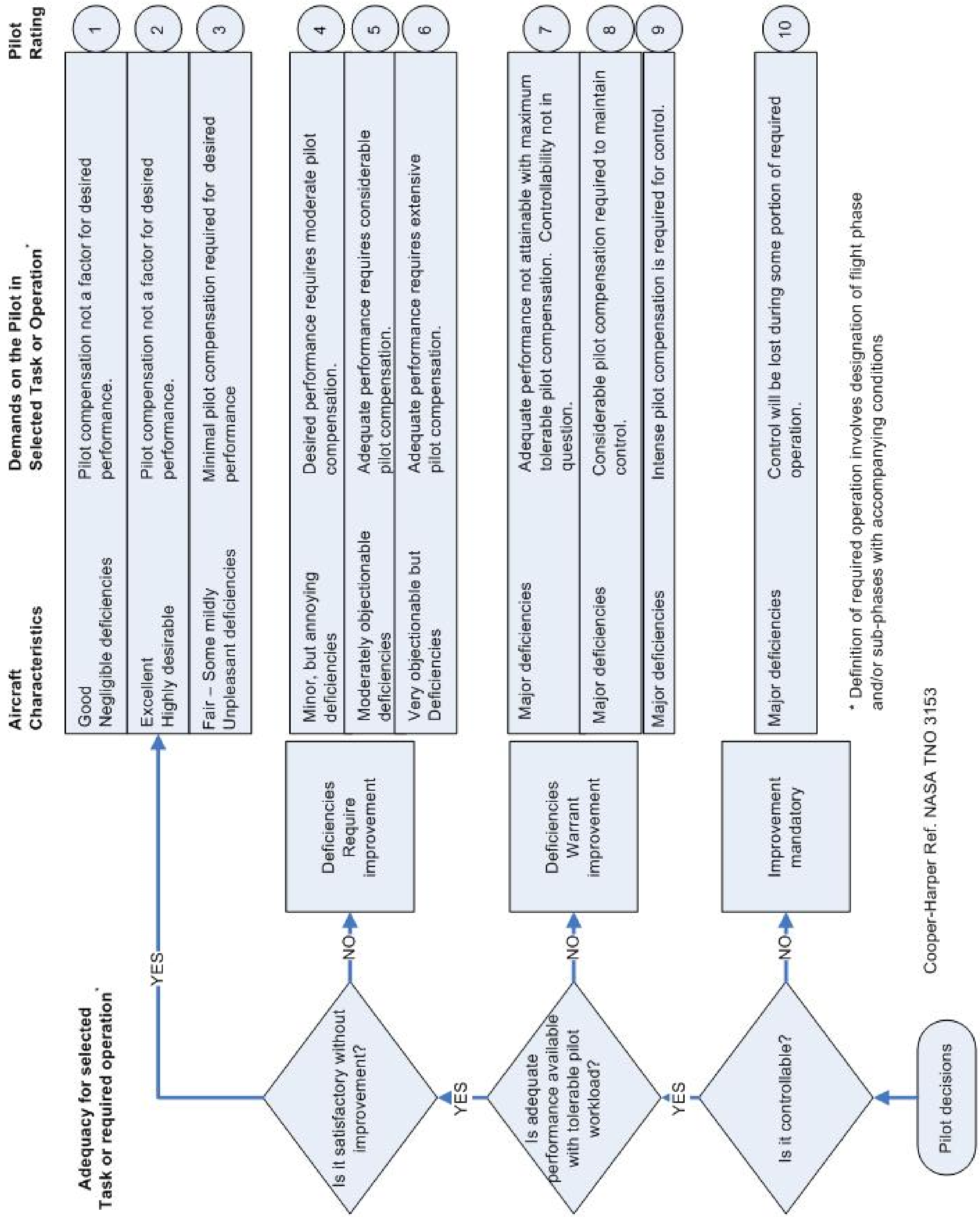


Figure 15.1-5 Cooper Harper Workload and Handling Qualities Rating Scale

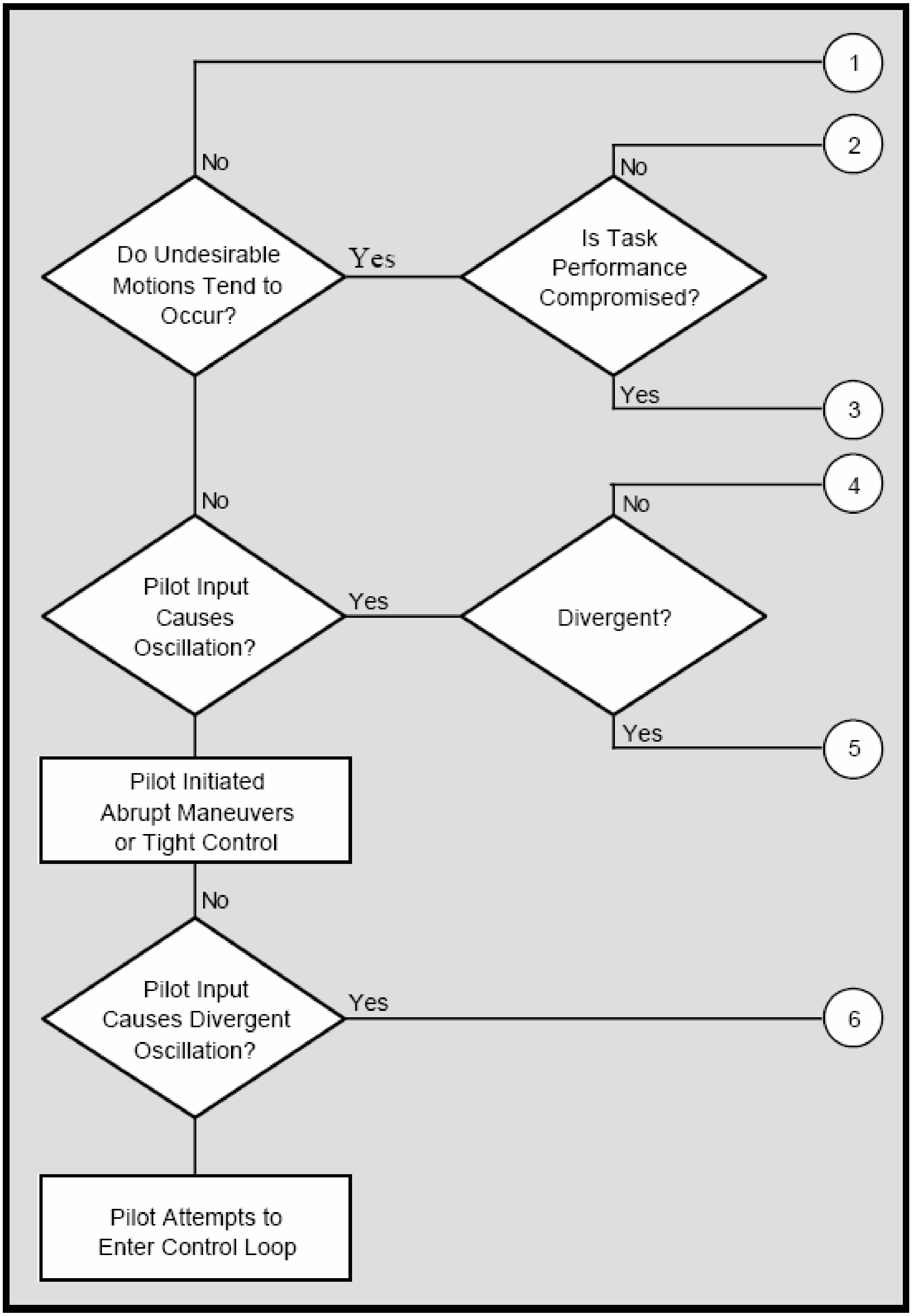


Figure 15.1-6 Pilot Induced Oscillations Rating Scale

NOTES

Section 16 Rotary Wing

16.1 Principal Aeroderivatives

16.2 Forward Flight Static And Dynamic Stability

16.1 PRINCIPAL AERODERIVATIVES

Derivative	Common Name	Principal Contributors	Typical Sign
		CONTROL POWER	
M_{B_1}	Pitch control power	• MR Thrust vector • Mast bending moment • Control gearing • Rotor type • Effective hinge offset	-
L_{A_1}	Roll control power	• MR Thrust vector • Mast bending moment • Control gearing • Rotor type • Effective hinge offset	-
$N_{\theta_{TR}}$	Yaw control power	• TR thrust • TR moment arm • Control gearing	-
Z_{θ_C}	Heave control power	• MR thrust • Control gearing	-
		STATIC STABILITY	
M_u	Speed stability	• MR flap back • Mast bending moment • Horizontal tailplane	+
M_w	Static/Incidence/Angle of Attack stability	• MR flap back • Mast bending moment • Horizontal tailplane • Fuselage	
L_v	Lateral static stability (dihedral effect)	• MR 'flap back' • TR vertical moment arm • Fuselage	-
N_v	Directional static stability (weathercock effect)	• TR thrust • Vertical tailplane • Fuselage	+
		DAMPING	
X_u	Drag damping	• Rotor drag • Fuselage drag	-
Y_v	Side force	• Rotor drag • Fuselage drag	-
Z_w	Heave damping	• MR characteristics	-
L_p	Roll damping	• Main rotor • Effective hinge offset	-
M_q	Pitch damping	• Main rotor • Effective hinge offset • Horizontal tailplane	-
N_r	Yaw damping	• Tail rotor • Vertical tailplane • Fuselage	-
		CROSS COUPLING	
$L_{\theta_{TR}}$	Tail rotor roll	• Tail rotor vertical position	+

Derivative	Common Name	Principal Contributors	Typical Sign
M_{θ_C}	Pitch change with power	• Forward speed • Main rotor	+
N_{θ_C}	Torque reaction	• Torque	
$Y_{\theta_{TR}}$	Tail rotor drift	• Tail rotor	

References:

Padfield, G.D., (2007), *Helicopter Flight Dynamics*, 2nd Edition, Blackwell Publishing, UK.

Cooke, A., Fitzpatrick, E., (2002), *Helicopter Test and Evaluation*, Wiley Blackwell, UK.

Leishman, J.G., (2006), *Principles of Helicopter Aerodynamics*, 2nd Edition, Cambridge University Press, UK.

16.2 FORWARD FLIGHT STATIC AND DYNAMIC STABILITY

Stability Characteristic	Principal Influences	Typical Test	Role Relation
Longitudinal Static Stability	• M_w • M_u • M_{θ_C} • $M_{\theta_{TR}}$	• Trimmed flight control positions • Trimmed flight control positions - collective	• Control margins • Control inputs progressive, predictable, and in correct sense
		• Apparent static stability	• Speed selection
		• Collective fixed static stability	• Speed maintenance
Manoeuvre Stability	• M_w • M_q • M_{θ_C}	• Apparent manoeuvre stability • Collective fixed manoeuvre stability • Pull-ups/push-overs	• Aggressive turning and manoeuvring flight
Longitudinal Dynamic Stability	• M_w • M_u • M_q	• Excitation of dynamic long term mode • Natural turbulence, release to trim, pulse input	• IMC flight • Transit • Nuisance mode
Lateral-Directional Static Stability	• L_v • N_v	• Trimmed flight control positions	• Control margins • Control inputs progressive, predictable, and in correct sense
	• Y_v	• Steady heading sideslip (SHSS)	• Sideforce cues • Maintaining balanced flight
Lateral Static Stability (Dihedral)	• L_v	• SHSS • Turns on one control – pedal	• Transit • Lateral and out-of-wind transitions • Instrument approaches
Directional Static Stability	• N_v	• SHSS • Turns on one control - cyclic	• Transit • Instrument approaches
Lateral-Directional Dynamic Stability – Lateral-Directional Oscillations (Dutch Roll Mode	• L_v • N_v	• Excitation of LDO via doublet, pulse, or SHSS release to trim	• IMC flight • Transit • Nuisance mode
Lateral-Directional Dynamic Stability – Spiral Stability	• L_v • N_r • N_v • L_r	• Turns on one control – cyclic • Time to half/double bank angle	• IMC flight • Turns • Lateral gust response

References:

Padfield, G.D., (2007), *Helicopter Flight Dynamics*, 2nd Edition, Blackwell Publishing, UK.

Cooke, A., Fitzpatrick, E., (2002), *Helicopter Test and Evaluation*, Wiley Blackwell, UK.

Leishman, J.G., (2006), *Principles of Helicopter Aerodynamics*, 2nd Edition, Cambridge University Press, UK.