2.7 Standard Series

(reference 2.4)

Taylor's

$$f(x)=f(a)+f'(a)\frac{x-a}{1}+f''(a)\frac{(x-a)^2}{2!}+f'''(a)\frac{(x-a)^3}{3!}+\ldots f^{(n-1)}(a)\frac{(x-a)^{(n-1)}}{(n-1)!}+R_n$$

Maclaurin's (Taylor series with $a = 0$):

$$f(x)=f(0)+f'(0)\frac{x}{1}+f''(0)\frac{(x)^2}{2!}+f'''(0)\frac{(x)^3}{3!}+\ldots f^{(n-1)}(0)\frac{(x)^{(n-1)}}{(n-1)!}+R_n$$

Binomial:

$$(a+x)^n=a^n+na^{n-1}x+\frac{n(n-1)}{2!}a^{n-2}x^2+\frac{n(n-1)(n-2)}{3!}a^{n-3}x^3+\ldots. \qquad [x^2<a^2]$$

Exponential:

$$a^x=1+x\ln a+\frac{(x\ln a)^2}{2!}+\frac{(x\ln a)^3}{3!}+\ldots.$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots.$$

$$\frac{1}{2}(e^x+e^{-x})=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots.$$

$$\frac{1}{2}(e^x-e^{-x})=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots.$$

$$e^{-x^2}=1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\frac{x^8}{4!}\ldots.$$

Logarithmic:

$$\ln x=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\ldots. \qquad [0<x<2]$$

$$\ln x=\frac{(x-1)}{x}-\frac{1}{2}\left(\frac{x-1}{x}\right)^2+\frac{1}{3}\left(\frac{x-1}{x}\right)^3-\ldots. \qquad \left[x>\frac{1}{2}\right]$$

$$\ln x=2\left[\frac{x-1}{x+1}-\frac{1}{3}\left(\frac{x-1}{x+1}\right)^3+\frac{1}{5}\left(\frac{x-1}{x+1}\right)^5+\ldots.\right] \qquad [0<x]$$