6.5 Axis System Transformations (ref 6.6.2)

Transformation matrix for converting forces, velocities or accelerations from inertial (X, Y, Z) to body (*x*, *y*, *z*) axes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Expanding gives:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\theta\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

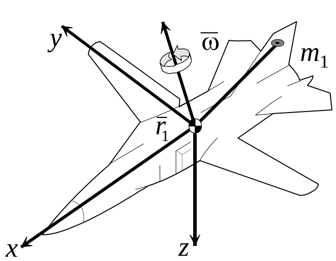
The inverse of the above transform matrix converts from the body axis to the inertial axis coordinate system

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Acceleration Transformations

- Convert body-axis angular rates & linear accelerations into total accelerations along body axes.
- Convert element (m_1) location & rates into specific angular momentum

$$a_x = \dot{u} + qw - rv$$
$$a_y = \dot{v} + ru - pw$$
$$a_z = \dot{w} + pv - qu$$



$$\frac{H}{m} = r_1 \times [\overline{\omega} \times \bar{r}_1] \Rightarrow$$
$$\left[\frac{H}{m}\right]^{a}_{l} = p\left(y^2 + z^2\right) - q(xy) - r(xz)$$
$$\Rightarrow \left[\frac{H}{m}\right]^{n}_{y} = q\left(x^2 + z^2\right) - r(yz) - p(xy)$$
$$\Rightarrow \left[\frac{H}{m}\right]^{n}_{k} = r\left(x^2 + y^2\right) - p(xz) - q(yz)$$