Section 3 Universe/Earth/Atmospheric Properties

Section 3 Recurring Nomenclature

T = absolute temperature (Rankin or Kelvin)
T_R = absolute temperature, Rankin scale
T_o = standard day seal level absolute temperature
P = ambient pressure
P_o= standard day seal level ambient pressure
ρ = ambient density
ρ_o= standard day seal level ambient density

$\delta = P/P_o$ = atmospheric pressure/std day sea level pressure
$\theta = T/T_o$ = atmospheric absolute temp / std day sea level absolute temp
$\sigma = \rho/\rho_o$ = atmospheric density/std day sea level density

g = acceleration due to gravity
g_o = standard earth acceleration due to gravity
a_o = speed of sound at std day sea level temperature

Section 3.1 Universal Constants (reference 3.1)

Avogadros number, N_o	6.022169×10^{23} molecules/mole	
Boltzmann constant, k	1.380×10^{-23}	J/^{o}K
electron charge, e	$1,602 \times 10^{-19}$	coulomb
electron mass, m_e	9.109×10^{-31}	kg
gas constant, R	8.31434J/^{o}K	mole
gravitational constant, G	6.673×10^{-11}	Nm^2/kg^2
neutron mass, m_n	1.674×10^{-27}	kg
Planck constant, h	6.625×10^{-34}	J sec
proton mass, m_p	1.672×10^{-27}	kg
speed of light in a vacuum, c	2.998×10^{8}	m/sec
unified atomic mass constant, m_u	1.660×10^{-27}	kg
volume of ideal gas (std temp & press)	2.241 x 10	m^3/mol

Newtonian Gravity
The gravitaional field (g) near any mass can be calculated as

$$g = \frac{GM}{(R_A)^2}$$

where G is the universal gravitational constant and R_A is the absolute distance from the center of mass M

Section 3.2 Earth Properties (references 3.2, 3.3)

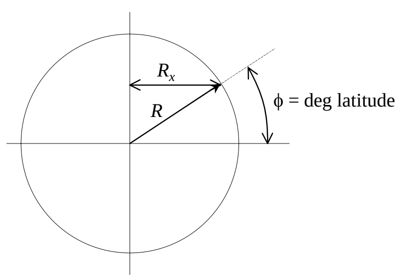
Std Earth gravitational acceleration, g_o = 9.8066 m/s^2 = 32.174 ft/s^2
mass = 5.98333 × 10^{24} *kg* = 13.22 x 10^{24} *lb*
rotation rate, ω = 7.292115 × 10^{-5} *rad/sec*
average density = 5.522 g/cm^3 = 344.7 lb/ft^3
radius average, R_{avg} = 6,367,444 *m* = 3956.538 *st. miles* = 20,890,522 *ft*
radius at the equator (R_e) is 6,378,137 *m* (±2)
radius at the poles R_p = 6,356,752 [*m*]
radius as a function of latitude, ϕ (assumes perfect ellipsoid):

$$R = \left[\left(\frac{\cos\phi}{R_e} \right)^2 + \left(\frac{\sin\phi}{R_p} \right)^2 \right]^{-\frac{1}{2}}$$

Centrifugal Relief from Gravity
The earth's "normal" gravity field includes both the Newtonian Law and a correction for the centrifugal force caused by the earth's rotation. The centrifugal relief correction is

$$\Delta CR = -\frac{V^2}{R_x} = -\frac{(R_x\omega)^2}{R_x} = R_x\omega^2$$

where ω is the earth’s rotation rate and R_x is the perpendicular distance from the earth's axis to the surface and can be calculated as $R_x = R \cos\phi$ (see figure below).



For any centrifugal relief calculations associated with aircraft performance, it is sufficiently exact (g ±0.00004 m/s^2) to use the average earth radius. An aircraft flying eastward contributes to centrifugal relief while a west-bound aircraft diminishes it.

The International Association of Geodesy publishes the following equation (accurate to 0.005%) to calculate *local sea level gravity including the effects of centrifugal relief for any point fixed to the earth's surface*

$$g_{lsl} = 9.780327\left(1 + 0.00530224\sin^2\phi - 0.000058\sin^2 2\phi\right)\left[\tfrac{m}{s^2}\right]$$

The above equation is tabulated below for quick reference.

Latitude (deg)	**Normal g_{local}**	
	(m/s^2)	**(ft/s^2)**
0	9.780327	32.088
15	9.783659	32.098
30	9.792866	32.188
45	9.805689	32.171
60	9.818795	32.214
75	9.828569	32.249
90	9.832185	32.258

The standard acceleration (g_o) corresponds to a latitude of 46.0625°.
g_{lsl} at the equator and the poles varies ±0.27% from g_o.

Altitude Effect on Gravitational Acceleration
R_A is the sum of the earth's local radius and the geometric distance (h_G) above the surface: $R_A = R + h_G$

Gravitational acceleration at any geometric altitude:

h_G (1000 ft)	
0	1
10	0.99904
20	0.99809
40	0.99618
60	0.99428
80	0.99238
100	0.99049

$$g_{alt} = g_{sls}\left(\frac{R}{R + h_G}\right)^2$$

$\frac{g_{alt}}{g_{lsl}}$

Actual Gravitational Pull on an Aircraft

Adding a centrifugal relief correction due to the aircraft's velocity, a complete calculation for its gravitational acceleration is

$$g_{A/C} = \left[g_{lsl} + \omega^2 R\cos\phi\right]\left[\frac{R}{R+h_G}\right]^2 - \left(\omega + \frac{V_G \sin\sigma}{R+h_G}\right)^2 (R+h_G)\cos\phi$$

where V_G = ground speed and σ = ground track angle (0° = true North, 90° = East, etc.).

Gravity Influence on Aircraft Cruise Performance

Even at the same altitude, changes in gravity due to latitude or centrifugal relief directly alter the required lift, drag, and fuel flow. For example, with sufficiently precise instrumentation, data collected heading West could show about 0.5% more drag and fuel flow than data collected heading East (centrifugal relief effect). After determining test and standard (or mission) values for g, flight test values for C_L, C_D, drag, and fuel flow can be corrected to standard as follows:

$$C_{L_{std}} = C_{L_t}\frac{N_{z_{\omega std}}}{N_{z_{\omega t}}}\left[\frac{g_{std}}{g_{A/C}}\right]$$

$$C_{D_{std}} = \frac{\left(C_{L_{std}}\right)^2}{\pi A \mathrm{Re}}$$

$$\Delta D = D_{std} - D_t = qS\left[C_{D_{i_{std}}} - C_{D_{i_t}}\right]$$

$$\dot{W}_{f_{std}} = \dot{W}_{f_t} + \Delta D \cdot TFSC$$

where N_z = normal load factor,
C_L = lift coefficient, C_D = drag coefficient,
AR = aspect ratio, e = Oswald efficiency factor,
ΔD change in drag force,
$TSFC$ = thrust specific fuel consumption, and

$\dot{W}_{f_{std}}$ = standard day fuel flow

Section 3.3 General Properties of Air (reference 3.1)

Gas constant, R = 53.35 ft lb/^{o}R lbm = 287.074 J/kg ^{o}K
= 1716 lb(ft)/slgs(o*R*) = 3089.7 lb(ft)/slgs(o*K*)

Speed of sound = $a_o(\theta)^{½}$
= 49.02 $(T_R)^{1/2}$ *ft/sec*
= 33.42 $(T_R)^{1/2}$ *miles/hr*
= 29.04 $(T_R)^{1/2}$ *knots*
= 20.05 $(T_R)^{1/2}$ *m/sec*

Density, ρ = .0023689 *slug/ft*^{3} = *1.225 kg/m*^{3}

Specific weight, g_ρ = .07647 *sec*^{2}*/ft*^{4}

Specific heat capacity at 59^o*F* (=T_o)
at constant pressure, c_p = .240 *BTU/lb* ^{o}*R* = 1004.76 *J/kg* ^{o}*K*
at constant volume, c_v = .1715 *BTU/lb* ^{o}*R* = 717.986 *J/kg* ^{o}*K*
specific heat ratio, $\gamma = \{c_p / c_v\}$ =1.4

Normal Composition of clean, dry atmospheric air near sea level

Nitrogen, N_2	78.084 % by volume
Oxygen, O_2	20.948 %
Argon, *A*	0.934 %
Carbon Dioxide, CO_2	0.031 %
Neon, *Ne*	0.002 %
total	99.9988 %

plus traces of helium, kryton, xenon, hydrogen, methane, nitrous oxide, ozone, sulfur dioxide, nitrogen dioxide, ammonia, carbon monoxide, and iodine.

Viscosities of Air

Coefficient of Viscosity, $\mu_c = \dfrac{7.3025x10^{-7}(T_R)^{3/2}}{T_R + 198.72}$ lb/ft sec

Kinematic viscosity, $v = \dfrac{\mu_c}{g\rho}$ ft^2/sec

Absolute Viscosity, lb $\mu = \rho v = \left[.317(T_R)^{3/2} \left(\dfrac{734.7}{T_R + 216} \right) \right] x10^{-10}$ sec/ft^2

Atmospheric Viscosity (U.S. Standard Atmosphere)

Pressure Altitude *ft*	Kinematic Viscosity υ *(ft^2/sec)*	Absolute Viscosity μ *(lb sec/ft^2)*
0	1.572 x 10^{-4}	3.737 x 10^{-7}
5,000	1.776	3.638
10,000	2.013	3.538
15,000	2.293	3.435
20,000	2.625	3.330
25,000	3.019	3.224
30,000	3.493	3.115
35,000	4.065	3.004
40,000	5.074	2.981
45,000	6.453	2.982
50,000	8.206	2.983
55,000	10.44	2.985
60,000	13.27	2.986
70,000	21.69	3.005
80,000	35.75	3.043
90,000	58.53	3.080
100,000	95.19	3.118
150,000	1066	3.572
200,000	6880	3.435

ClimateCraft

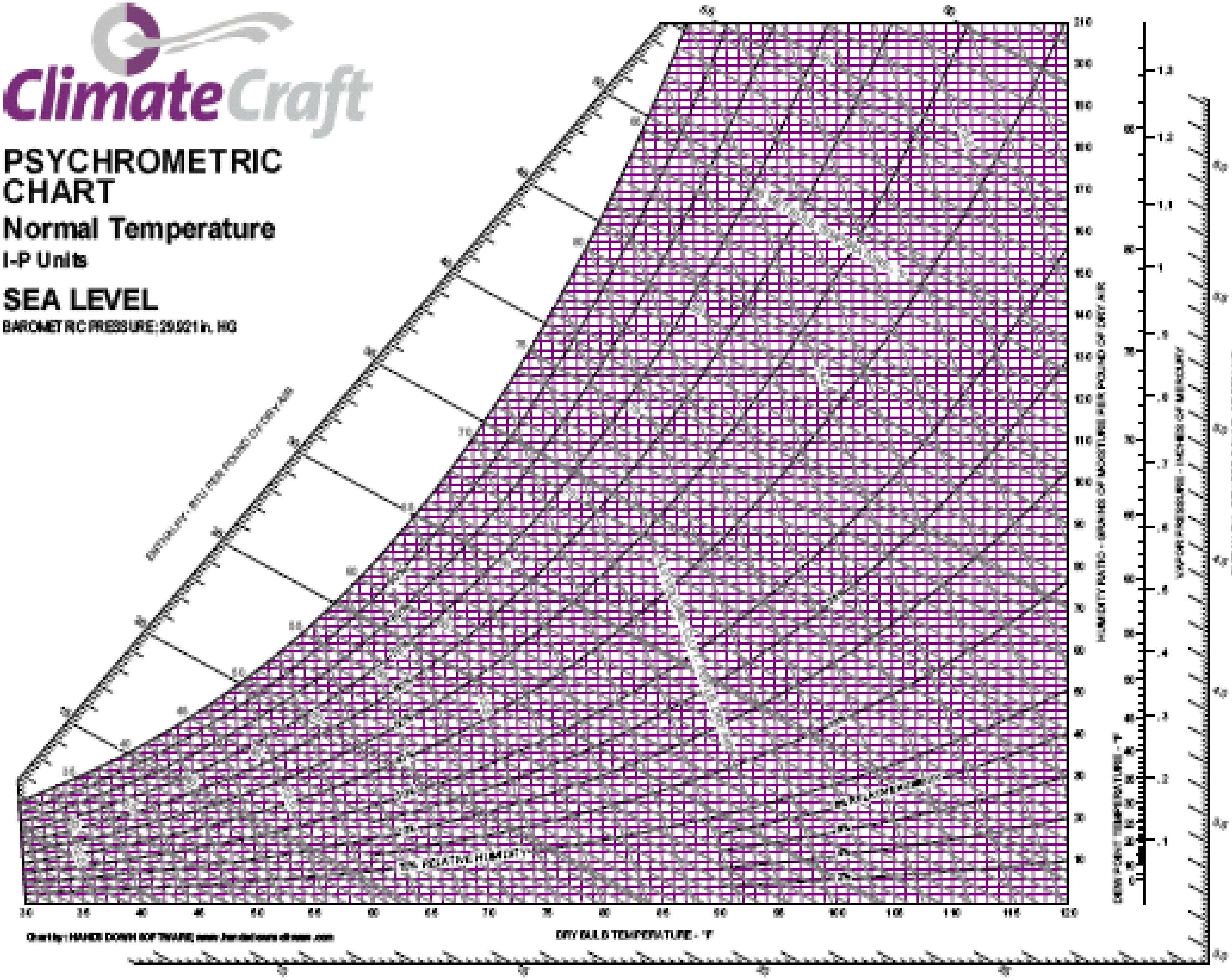
PSYCHROMETRIC CHART

Normal Temperature

I-P Units

SEA LEVEL

BAROMETRIC PRESSURE: 29.921 in. HG



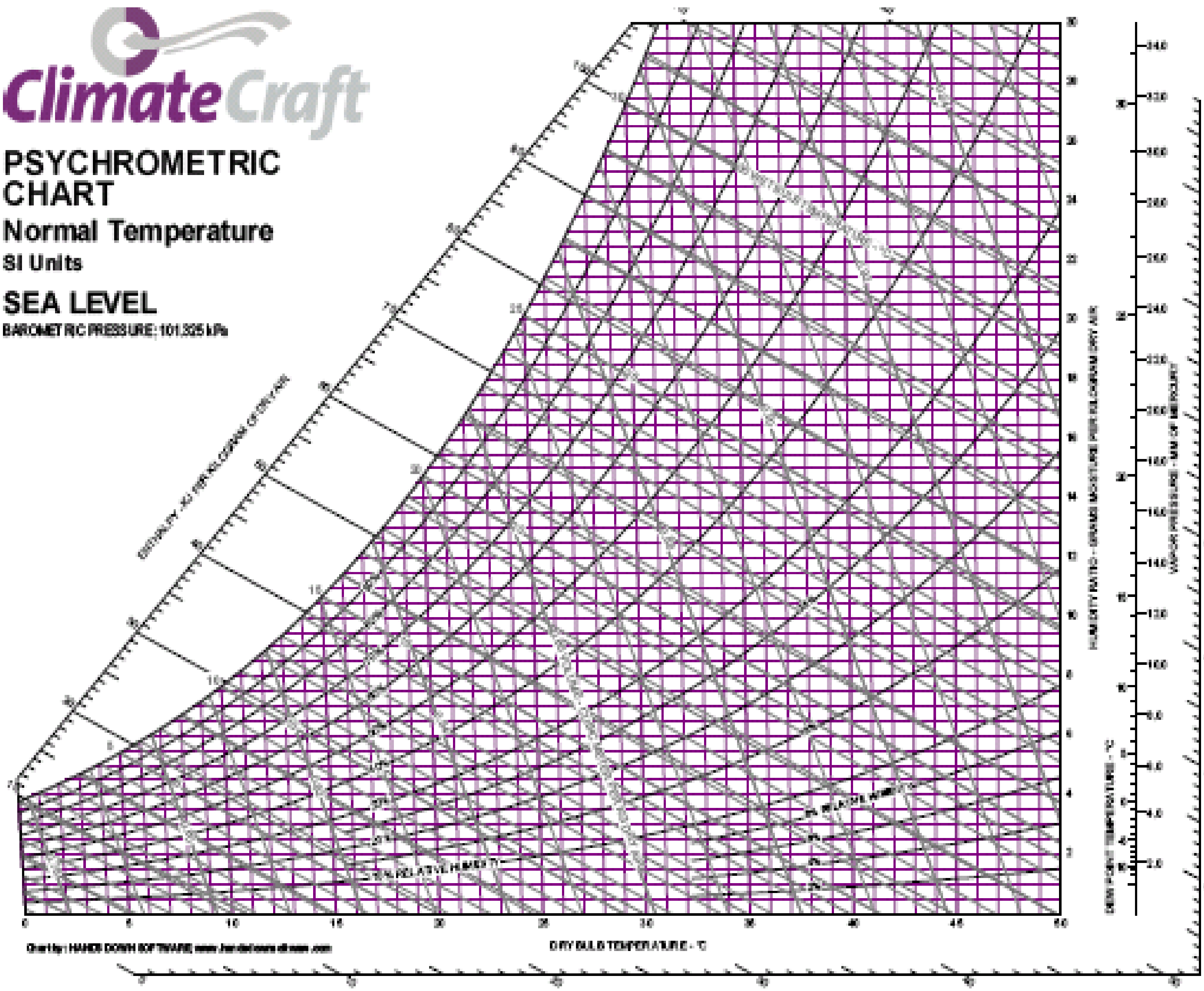
PSYCHROMETRIC CHART

Normal Temperature

SI Units

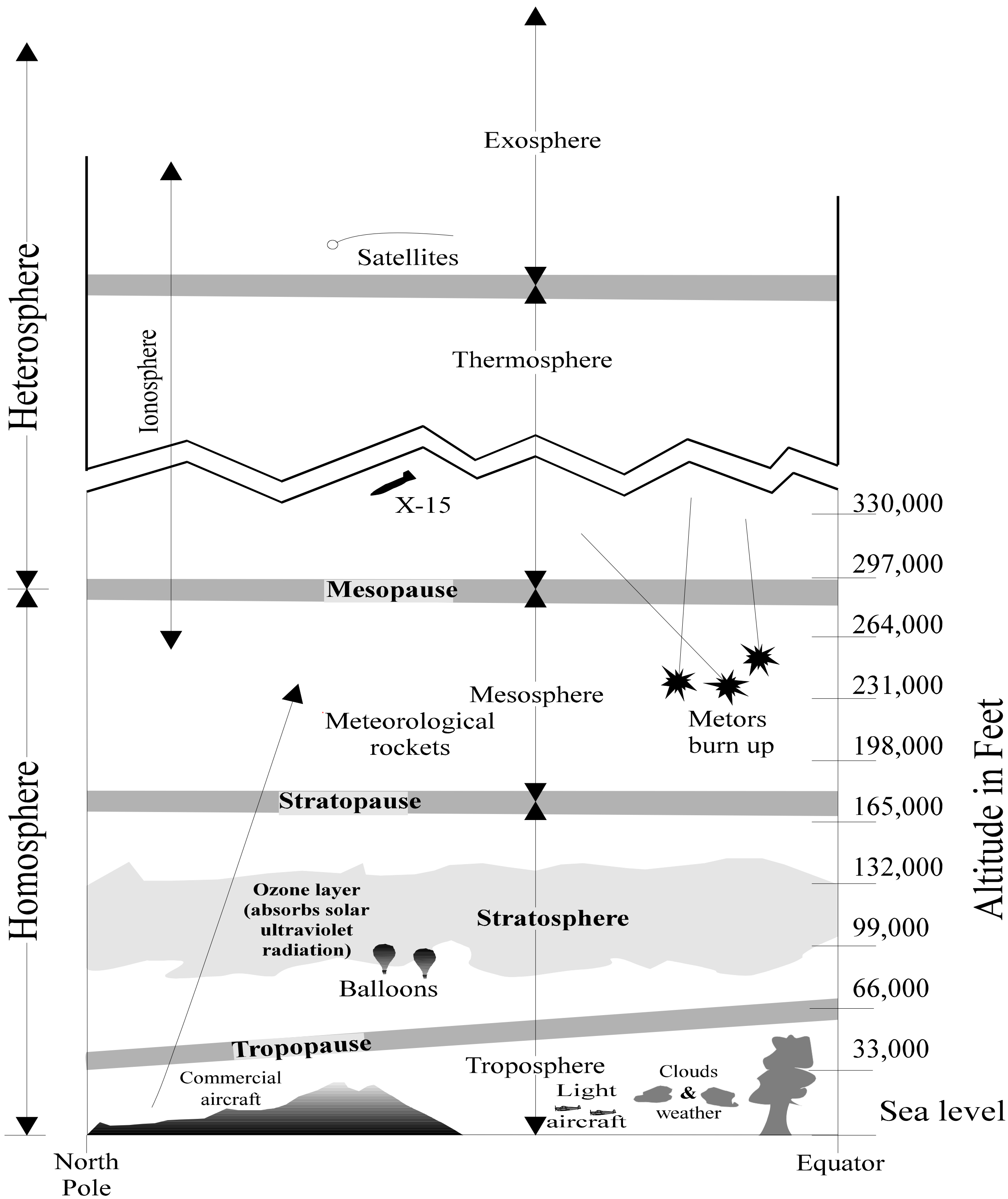
SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa



Section 3.4 Standard Atmosphere

Divisions of the Atmosphere



Constantly changing atmospheric conditions cannot be duplicated at will to provide the exact environment in which a flight takes place. A standard atmosphere provides a common basis to relate all flight test, wind tunnel results, aircraft design and general performance. Several models of "standard atmosphere" exist with minor differences based on mathematical constants used in the calculations.

Geometric altitude, h_G , is defined as the height of an aircraft above sea level (also called **tapeline** altitude)

Absolute altitude, h_a, is defined as the height of an aircraft above the center of the earth: (geometric altitude + radius of the earth).

Geopotential altitude, h, is required because g changes with height.If potential energy is calculated using sea level weight ($W_{SL} = mg_o$) instead of actual weight ($W = mg$), then the altitude must be lower.

$$W\,h_G = W_{SL}\;h$$

Pressure altitude is the altitude, on a standard day, at which the test day pressure would be found

Density altitude is the altitude, on a standard day, at which the test day density would be found

Temperature altitude is the altitude, on a standard day, at which the test day temperature would be found

Assumptions on which the standard atmosphere is built

1. The air is dry (only 0.4% per volume of water vapor)

2. The air is a perfect gas and obeys the equation of state,

$$P = \rho g R T$$

where $R = 53.35$ ft lb/^{o}R lbm

3. The gravitational field decreases with altitude

4. Hydrostatic equilibrium exists ($\Delta p = -\rho g_o \Delta h$)

Standard Day Sea Level Atmospheric Conditions

P_o = 2116.22 lb/ft^2 = 14.696 lb/in^2 = 29.921 in *Hg*
= 1013.25 HPa (mb) = 101325 Pa

T_o = 288.15 ^oK = 518.67 ^oR = 59 ^oF = 15 ^oC

ρ_o = 0.0023689 slgs/ft^3 = 0.07647 lbm/in^3 = 1.255 kg/m^3

a_o = 1116.45 ft/sec = 661 KTAS = 761.14 mph = 340.294 m/sec

g_o = 32.174 ft/sec^2 = 9.80665 m/sec^2

L = standard temperature lapse rate = 0.0019812 ^oK /ft

1976 U.S Standard Atmosphere Equations

Troposhere - below 36,089 ft (11,000 m) < *22636 Pa*

$$\theta = 1 - (L/ To)\, h = 1\text{-}(6.866 \times 10^{-6})\, h$$

$$\sigma = \theta^{n-1}$$

$$\delta = \theta^{n}$$

where n = 5.25585, h = geopotential altitude

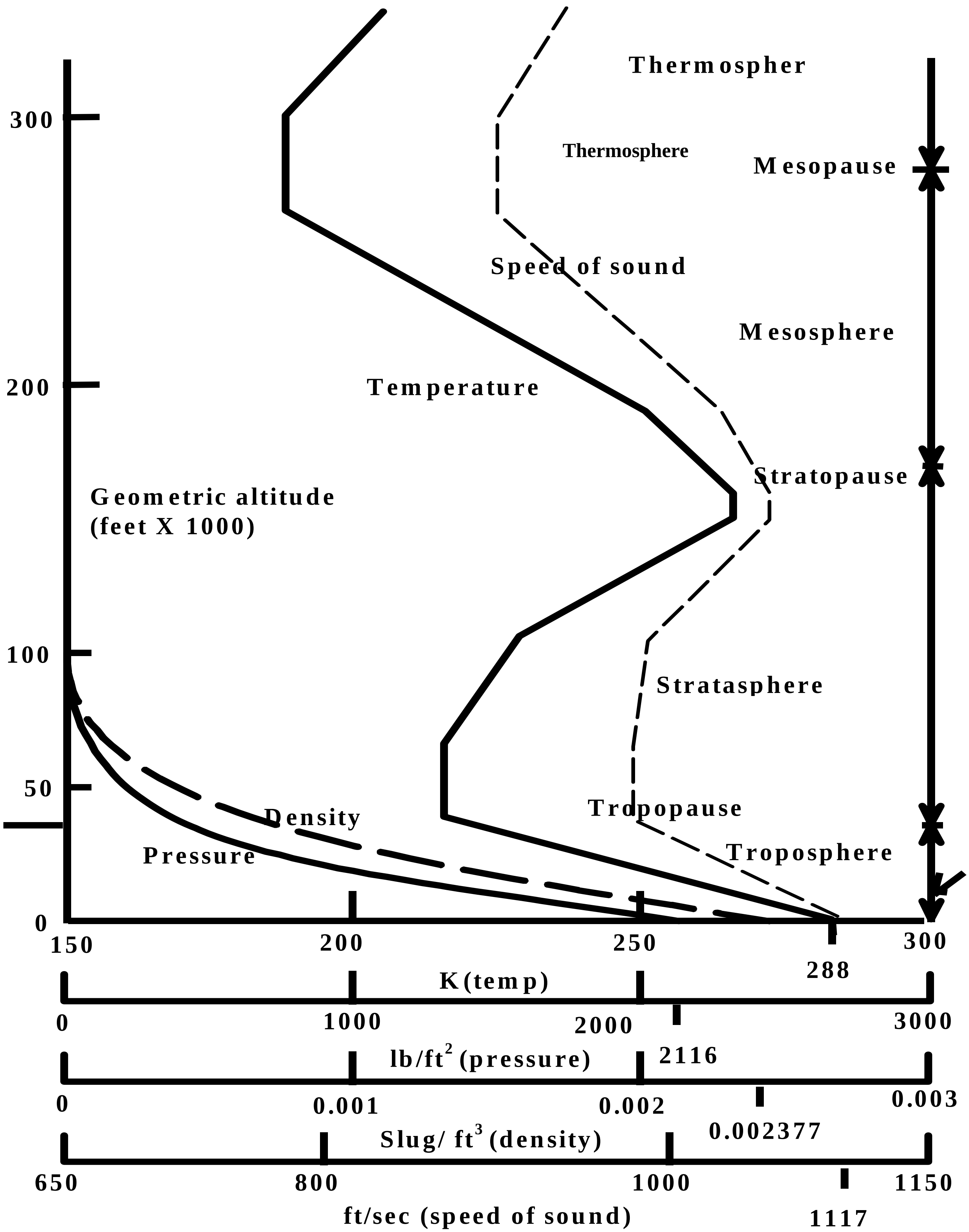
Stratosphere- between 36,089 ft and 65,616 ft (20,000 m) the standard day temperature is a constant 216.66 ^oK, therefore:

$$\theta = 0.7519$$

$$\sigma = .29707\; e^{\cdot -0.00004806\; [h-36,089]}$$

$$\delta = .223358\; e^{\cdot -0.00004806\; [h-36,089]}$$

1976 U.S. Standard Atmosphere Graph



1976 U.S. Standard Atmosphere Tables

H (ft)	δ (=Pa/Po)	θ (=Ta/To)	σ (=ρa/ρo)
-1,000	1.0367	1.0069	1.0296
0	1.0000	1.0000	1.0000
1,000	0.9644	0.9931	0.9711
2,000	0.9298	0.9862	0.9428
3,000	0.8962	0.9794	0.9151
4,000	0.8637	0.9725	0.8881
5,000	0.8320	0.9656	0.8617
6,000	0.8014	0.9587	0.8359
7,000	0.7716	0.9519	0.8106
8,000	0.7428	0.9450	0.7860
9,000	0.7148	0.9381	0.7620
10,000	0.6877	0.9312	0.7385
11,000	0.6614	0.9244	0.7156
12,000	0.6360	0.9175	0.6932
13,000	0.6113	0.9106	0.6713
14,000	0.5875	0.9037	0.6500
15,000	0.5643	0.8969	0.6292
16,000	0.5420	0.8900	0.6090
17,000	0.5203	0.8831	0.5892
18,000	0.4994	0.8762	0.5699
19,000	0.4791	0.8694	0.5511
20,000	0.4595	0.8625	0.5328
21,000	0.4406	0.8556	0.5150
22,000	0.4223	0.8487	0.4976
23,000	0.4046	0.8419	0.4807
24,000	0.3876	0.8350	0.4642
25,000	0.3711	0.8281	0.4481
27,000	0.3398	0.8144	0.4173
28,000	0.3250	0.8075	0.4025
29,000	0.3107	0.8006	0.3881
30,000	0.2970	0.7937	0.3741
31,000	0.2837	0.7869	0.3605
32,000	0.2709	0.7800	0.3473
33,000	0.2586	0.7731	0.3345
34,000	0.2467	0.7662	0.3220
35,000	0.2353	0.7594	0.3099
36,000	0.2243	0.7525	0.2981

1976 U.S. Standard Atmosphere Tables (cont.)

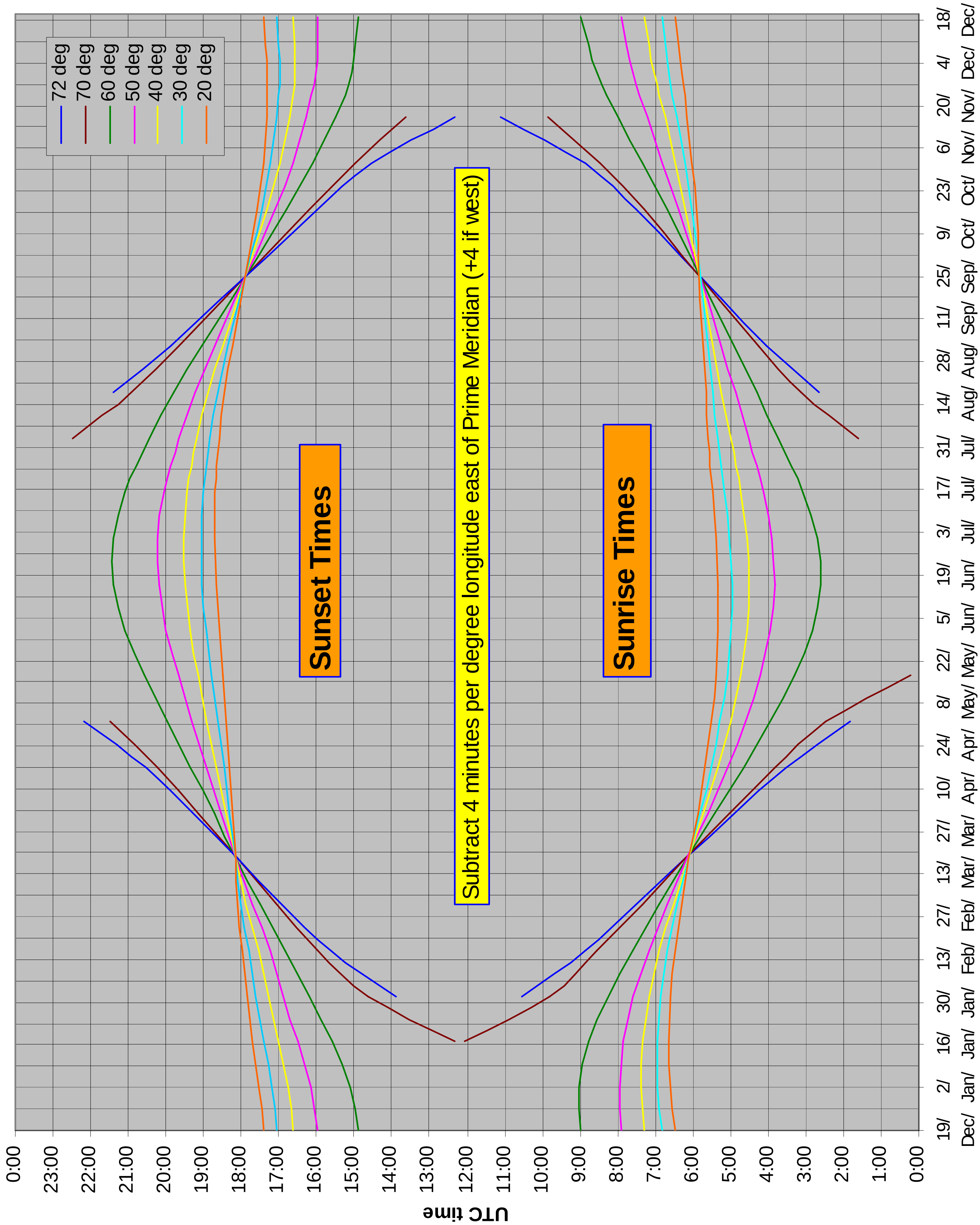
H (ft)	δ (=Pa/Po)	θ (=Ta/To)	σ (=ρa/ρo)
37,000	0.2138	0.7519	0.2843
38,000	0.2038	0.7519	0.2710
39,000	0.1942	0.7519	0.2583
40,000	0.1851	0.7519	0.2462
41,000	0.1764	0.7519	0.2346
42,000	0.1681	0.7519	0.2236
43,000	0.1602	0.7519	0.2131
44,000	0.1527	0.7519	0.2031
45,000	0.1455	0.7519	0.1936
46,000	0.1387	0.7519	0.1845
47,000	0.1322	0.7519	0.1758
48,000	0.1260	0.7519	0.1676
49,000	0.1201	0.7519	0.1597
50,000	0.1145	0.7519	0.1522
51,000	0.1091	0.7519	0.1451
52,000	0.1040	0.7519	0.1383
53,000	0.0991	0.7519	0.1318
54,000	0.0944	0.7519	0.1256
55,000	0.0900	0.7519	0.1197
56,000	0.0858	0.7519	0.1141
57,000	0.0818	0.7519	0.1087
58,000	0.0779	0.7519	0.1036
59,000	0.0743	0.7519	0.0988
60,000	0.0708	0.7519	0.0941
61,000	0.0675	0.7519	0.0897
62,000	0.0643	0.7519	0.0855
63,000	0.0613	0.7519	0.0815
64,000	0.0584	0.7519	0.0777
65,000	0.0557	0.7519	0.0740
70,000	0.0443	0.7563	0.0586
75,000	0.0350	0.7615	0.0459
80,000	0.0276	0.7667	0.0360
85,000	0.0219	0.7712	0.0284
90,000	0.0174	0.7773	0.0224
95,000	0.0138	0.7825	0.0177
100,000	0.0110	0.7877	0.0140
150,000	0.0013	0.9236	0.0015
200,000	0.0002	0.8811	0.0002

Section 3.5 Sea States

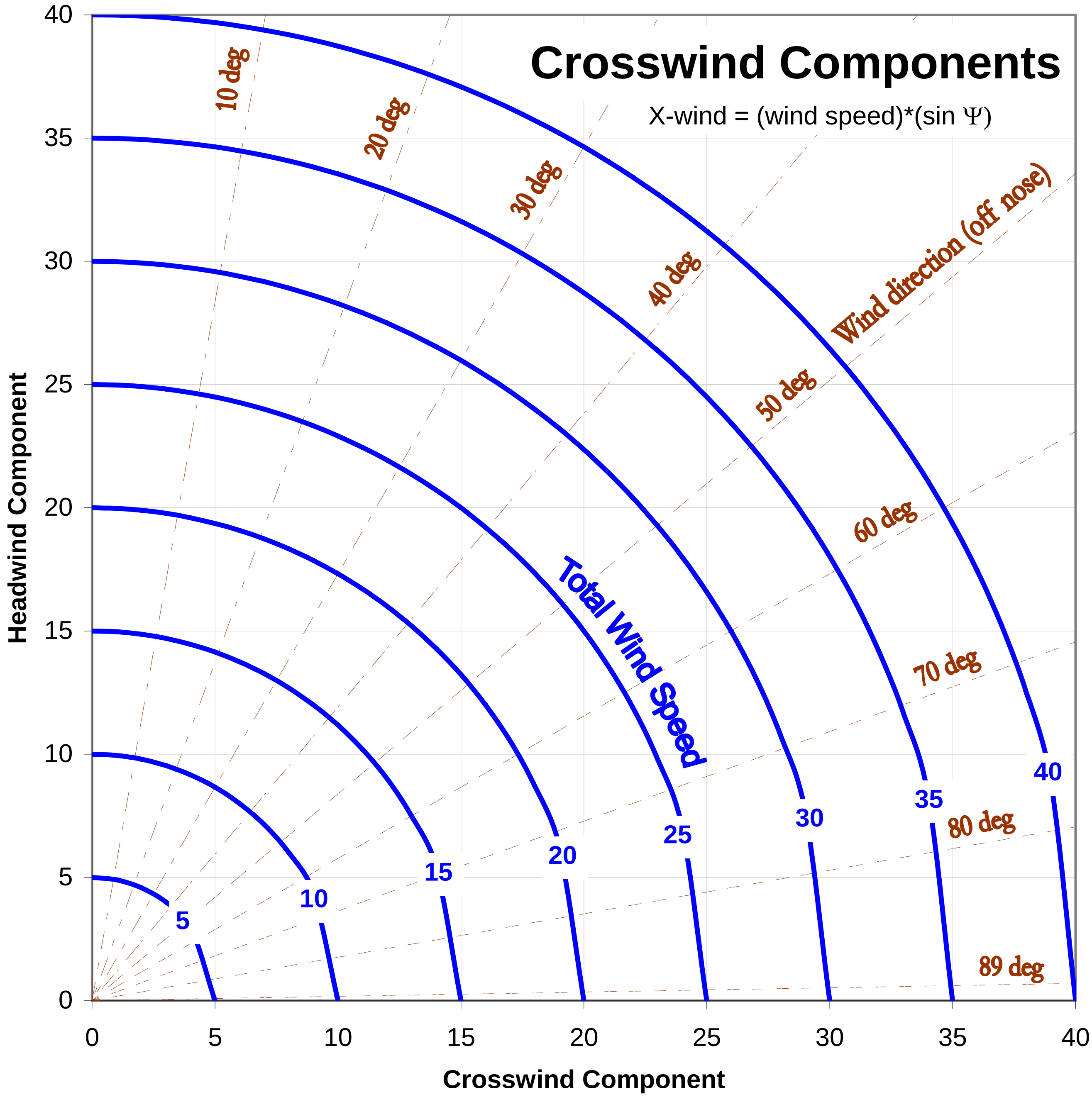
(ref 3.3) Sea Stale
International Swell Scale

Code	Sea	Wave Height, Crest to Trough (ft)
0	Calm	0
1	Smooth	Less than 1
2	Slight	1-3
3	Moderate	3-5
4	Rough	5-8
5	Very rough	8-12
6	High	12-20
7	Very high	20-40
8	Mountainous	40+
9	Confused, Used as additional description 1-8	
Code	**Swell**	**In Open Sea**
0	None	low
1	Short or average	
2	Long	
3	Short	Moderate height
4	Average	
5	Long	
6	Short	heavy
7	Average	
8	Long	
9	Confused, Used as additional description 1-8	

Section 3.6 Sunrise Sunset Times



Section 3.7 Crosswind Components



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