

Section 4 Pitot Statics

- 4.1 Subsonic Airspeed and Mach Equations
- 4.2 Subsonic Scale Altitude (Compressibility) Correction Chart
- 4.3 Subsonic Relations Between Compressible and Incompressible Dynamic Pressure
- 4.4 Supersonic Airspeed and Mach Equations
- 4.5 Total Temperature Equation
- 4.6 Altimeter Equation
- 4.7 PEC Test Methods
 - 4.7.1 Tower Fly-by
 - 4.7.2 Ground Course Method
 - 4.7.3 Trailing Bomb, Cone Method
 - 4.7.4 GPS Methods
- 4.8 Position Error Correction Certification Requirements
- 4.9 PEC Correction Process Flow Chart
- 4.10 Airspeed/Altitude/Mach Graphic Relations
- 4.11 Effect of Errors on Calibrated Airspeed and Altitude

Editor's Note

In an effort to reduce confusion and conflict regarding pitot and static pressure nomenclature, SFTE has elected to change two definitions and symbols since the first edition of this handbook was released. Henceforth, ΔP_s shall indicate static pressure ERROR ($\Delta P_s = P_s - P_a$) and ΔP_T shall indicate total (i.e. pitot) pressure ERROR ($\Delta P_T = P_p - P_T$). This nomenclature eliminates the ΔP_p symbol and confusion as to whether it indicates position error or pitot error.

Section 4 Common Nomenclature

a = speed of sound

a_o = speed of sound at sea level on a std day

M = Mach number

P_a = ambient pressure

P_o = ambient pressure at sea level on a std day

($=2116.2 \text{ lb/ft}^2 = 29.92 \text{ in Hg}$)

P_p = pitot pressure corrected for instrument error only

P_s = static pressure (indicated at static port)

P_T = total pressure

q = incompressible dynamic pressure

q_c = compressible differential pressure ($P_T - P_a$)

q_{cic} = instrument corrected differential pressure ($= P_p - P_s$)

T_a = ambient temperature (absolute scale)

T_o = ambient temperature at sea level on a std day

($=288.15 \text{ }^\circ\text{K} = 15 \text{ }^\circ\text{C} = 518.7 \text{ }^\circ\text{R} = 59 \text{ }^\circ\text{F}$)

T_T = total temperature (absolute scale)

V_c = calibrated airspeed

V_e = equivalent airspeed

V_g = ground speed

V_i = indicated airspeed

V_{ic} = instrument corrected airspeed

V_T = true airspeed

ΔH_{ic} = altimeter instrument correction

ΔH_{pc} = altimeter position error correction

ΔP_D = dynamic pressure error ($= P_T - \Delta P_s$)

ΔP_T = total (pitot) pressure error ($= P_p - P_T$)

ΔP_s = static pressure error ($= P_s - P_a$)

ΔV_{ic} = airspeed instrument correction

ΔV_{pc} = airspeed position error correction

δ = pressure ratio between ambient and sea level std ($= P_a / P_o$)

θ = temperature ratio between ambient and sea level std ($= T_a / T_o$)

ρ_o = ambient density at sea level on a std day ($= .002377 \text{ slg/ft}^3$)

σ = density ratio between ambient and sea level std ($= \rho_a / \rho_o$)

γ = ratio of specific heats ($= 1.4$ for air)

Section 4.1 Subsonic Airspeed and Mach Equations

True Airspeed

$$V_T = \left[\frac{2\gamma}{\gamma-1} \frac{P_a}{\rho_a} \left(\left[\frac{P_T - P_a}{P_a} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{1}{2}}$$

Equivalent Airspeed

(= V_T equation with assumption of std day sea level density)

$$V_e = \sqrt{7 \frac{P_a}{\rho_o} \left(\left[\frac{P_T - P_a}{P_a} + 1 \right]^{\frac{2}{\gamma}} - 1 \right)} = V_T \sqrt{\frac{\rho_a}{\rho_o}} = V_T \sqrt{\sigma}$$

Calibrated Airspeed

(= V_e equation with assumption of std day sea level pressure)

$$V_c = \left[\frac{2\gamma}{\gamma-1} \frac{P_o}{\rho_o} \left(\left[\frac{P_T - P_a}{P_o} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{1}{2}}$$

$$\sqrt{7 \frac{P_o}{\rho_o} \left(\left[\frac{P_c}{P_o} + 1 \right]^{\frac{2}{\gamma}} - 1 \right)}$$

Applying British units (lb/ft^2) and converting from ft/sec to knots yields

$$V_c = 1479 \sqrt{\left[\frac{P_T - P_a}{2116} + 1 \right]^{\frac{2}{\gamma}} - 1} \quad (\text{kts})$$

Mach Number

$$M = \frac{V_T}{a} = \sqrt{\frac{2}{\gamma-1} \left(\left[\frac{P_T - P_a}{P_a} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right)} = \sqrt{5 \left(\left[\frac{q_c}{P_a} + 1 \right]^{\frac{2}{\gamma}} - 1 \right)}$$

Section 4.2 Scale Altitude (Compressibility) Correction

The name comes from the fact that although the equivalent airspeed equation does correct for compressibility, the sea level pressure assumption used for calibrated airspeed makes the compressibility correction valid only for that (sea level) pressure. Above sea level, the calibrated airspeed must be re-scaled for pressure effects on compressibility. The mathematical method for determining V_e from V_c is to first solve the calibrated airspeed equation for q_c

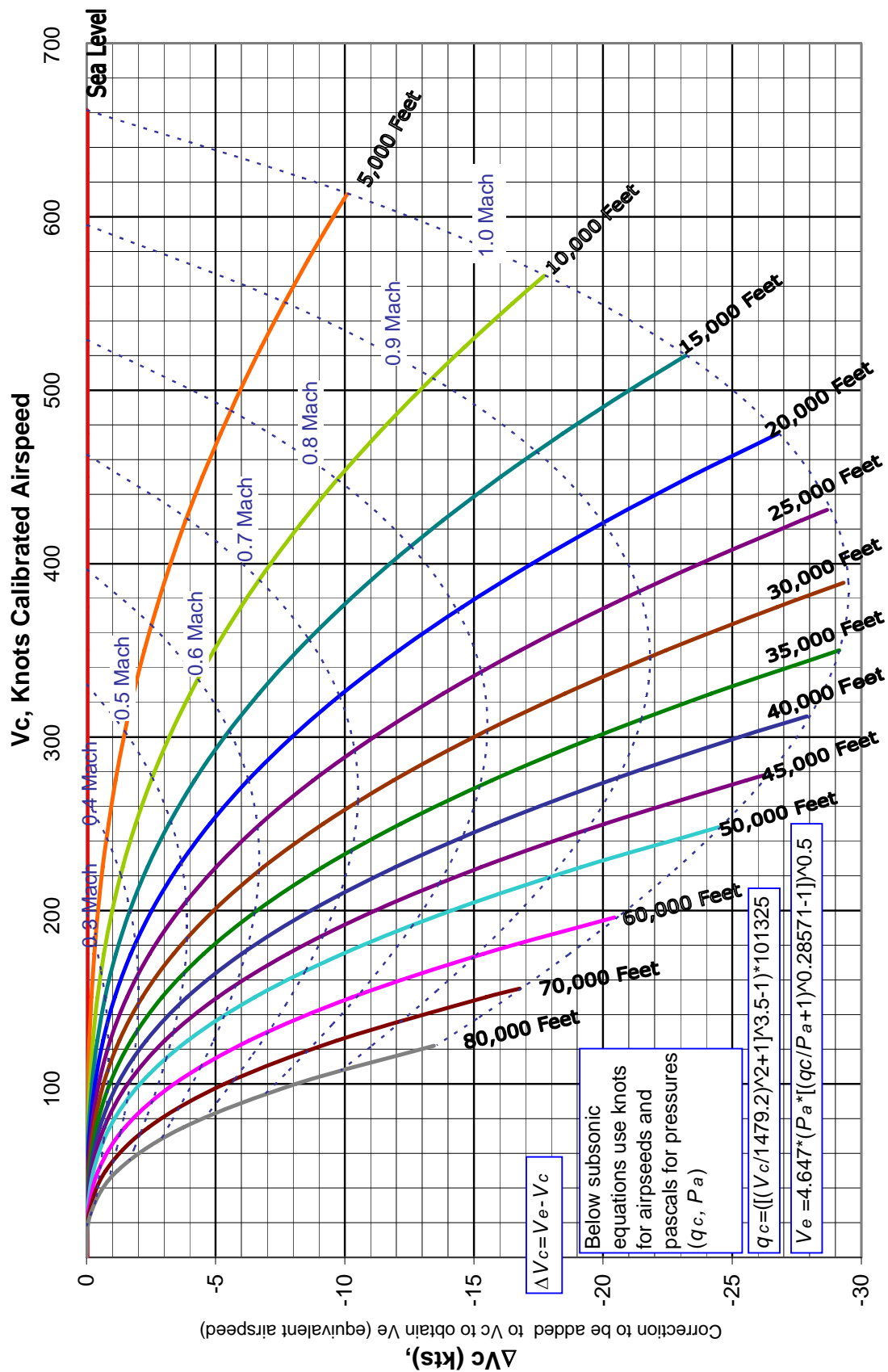
$$q_c = P_o \left[\left(\frac{\rho_o}{P_o} \frac{V_c^2}{7} + 1 \right)^{3.5} - 1 \right]$$

Next, substitute this value and the ambient pressure (P_a) into the equivalent airspeed equation. ($q_c = P_T - P_a$)

$$V_e = \sqrt{7 \frac{P_a}{\rho_o} \left(\left[\frac{q_c}{P_a} + 1 \right]^{\frac{2}{7}} - 1 \right)}$$

The adjacent chart does this graphically for all subsonic airspeeds.

$$V_e = V_c + \Delta V_c$$



Subsonic Scale Altitude (Compressibility) Correction

Section 4.3 Subsonic Relations Between Compressible and Incompressible Dynamic Pressure

For constant density (incompressible) flow Bernoulli's equation reduces to

$$V_T = \sqrt{\frac{2}{\rho_a}(P_T - P_a)} = \sqrt{\frac{2q}{\rho_a}}$$

Where incompressible dynamic pressure q is defined as $P_T - P_a$.

As airflow speed increases, its density at the stagnation point increases thereby increasing the sensed pressure.

The ratio between compressible & incompressible dynamic pressure can be written as a function of Mach number

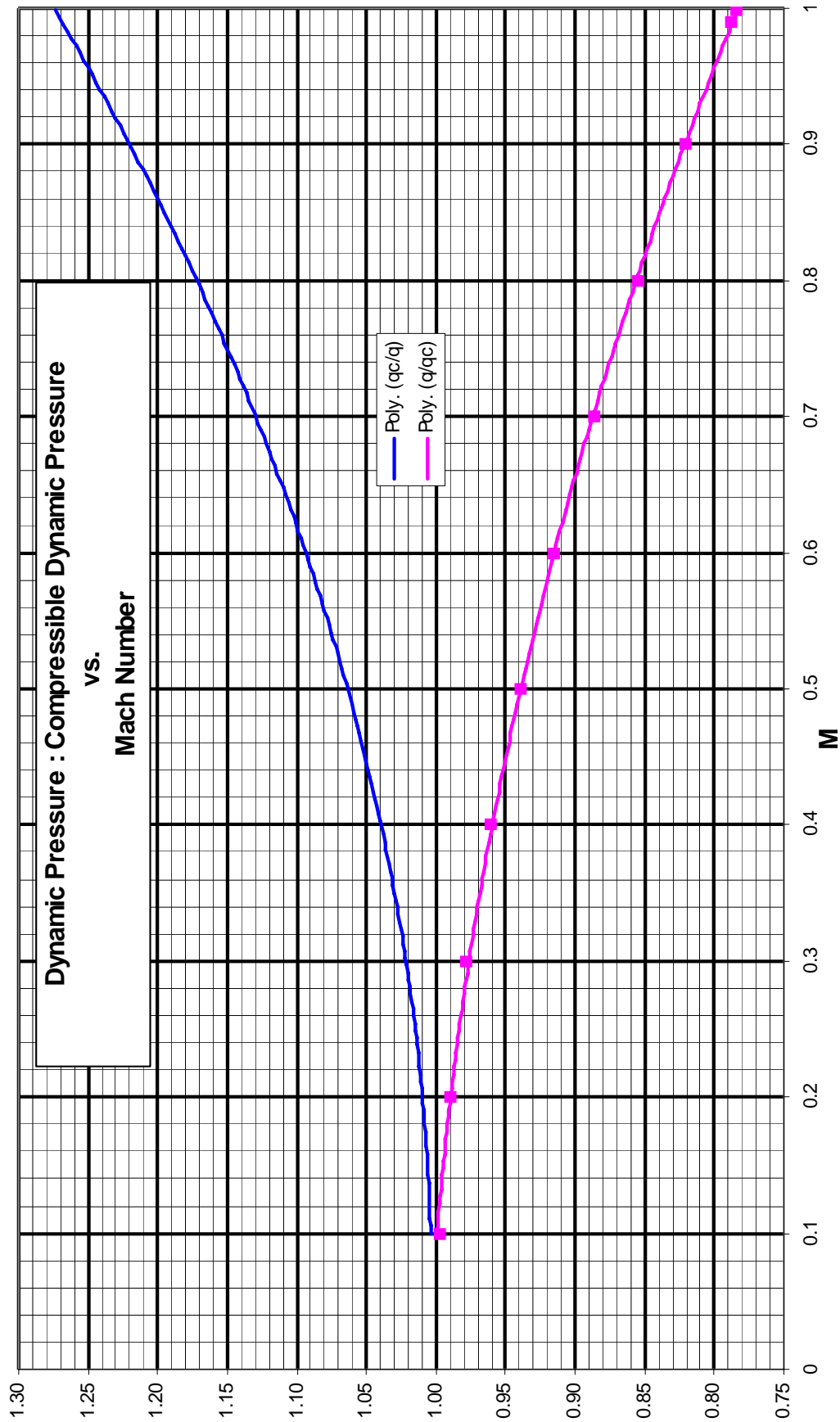
$$q_c = q \left[1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^6}{1600} + \dots \right]$$

True dynamic pressure q (as used in modeling) is defined in dimensional analysis as:

$$q = \frac{1}{2} \rho_a V_T^2 = \frac{1}{2} \rho_o V_e^2$$

This value for q should not be confused with compressible (a.k.a. impact or differential) pressure, $q_c (= P_T - P_a)$

$$q = \frac{1}{2} \rho_o \gamma \frac{P_a}{P_o} \left(\left[\frac{q_c}{P_a} + 1 \right]^{\frac{2}{\gamma}} - 1 \right)$$



Section 4.4 Supersonic Airspeed and Mach Equations

P_T' denotes pitot pressure behind shock wave

True Airspeed

$$\frac{P_T' - P_a}{P_a} = \frac{q_c}{P_a} = \left[\frac{\gamma + 1}{2} \left(\frac{V}{a} \right)^2 \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{1}{\frac{2\gamma}{\gamma + 1} \left(\frac{V}{a} \right)^2 - \frac{\gamma - 1}{\gamma + 1}} \right]^{\frac{1}{\gamma - 1}} - 1$$

Equivalent Airspeed (valid if $q_c/P_a > 0.892929158$)

$$\frac{q_c}{P_a} = \frac{166.92 \left[\frac{V_e}{a_o \sqrt{\delta}} \right]^7}{\left(7 \left[\frac{V_e}{a_o \sqrt{\delta}} \right]^2 - 1 \right)^{2.5}} - 1$$

Calibrated Airspeed (valid if $V_c > a_o$)

$$\frac{q_c}{P_o} = \frac{166.92 \left[\frac{V_c}{a_o} \right]^7}{\left(7 \left[\frac{V_c}{a_o} \right]^2 - 1 \right)^{2.5}} - 1$$

Mach Number

$$\frac{q_c}{P_a} = \frac{166.92 [M]^7}{(7[M]^2 - 1)^{2.5}} - 1$$

Section 4.5 Total Temperature Equation

Since stagnation exists at the probe, it absorbs the energy of the air



Apply Bernoulli:
$$\frac{V^2}{2} + \frac{\gamma}{\gamma-1} \cdot \frac{P_s}{\rho_s} = \frac{\gamma}{\gamma-1} \cdot \frac{P_p}{\rho_p}$$

also $P/\rho = RT$ and $a^2 = \gamma RT$

$$\therefore \frac{T_T}{T_a} = 1 + \left(\frac{\gamma-1}{2} \right) M^2$$

Use K (probe recovery factor) to account for heat losses:

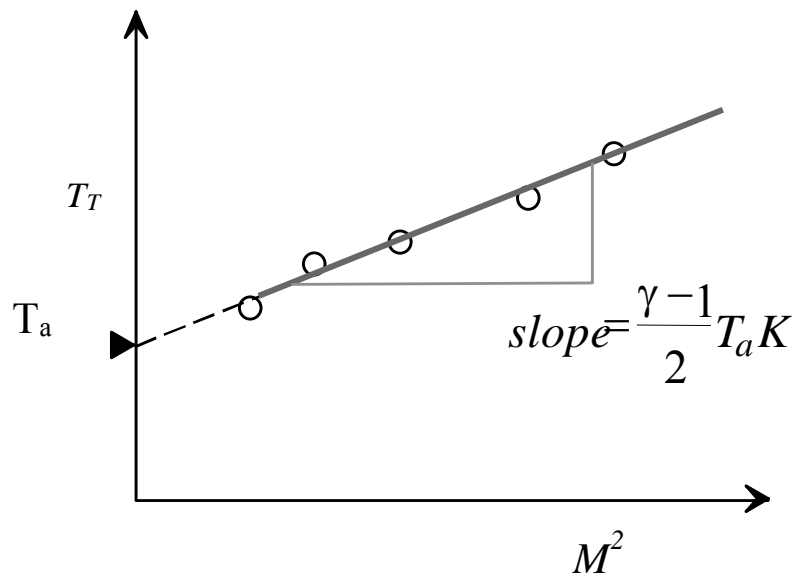
$$\frac{T_T}{T_a} = \left[1 + \frac{K(\gamma-1)}{2} M^2 \right]$$

During position error flight testing, measure T_{i*}

From V_c and H_{pc} determine M

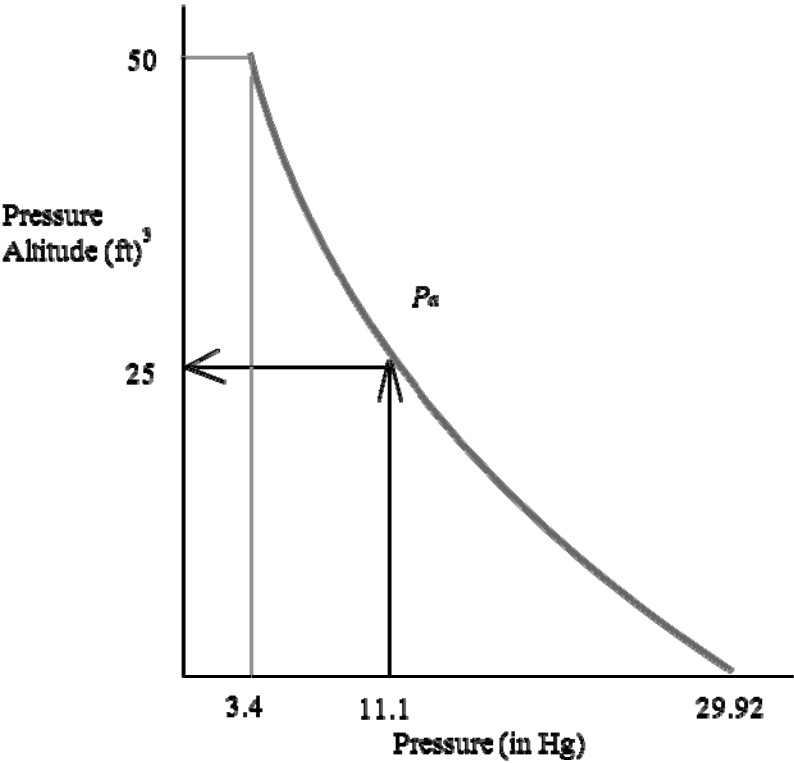
$$T_i + \Delta T_{ic} = T_T = T_a + T_a K M^2$$

plot $T_i \sim M^2$



Section 4.6 Altimeter Equation

$$P_a = P_o (1 - 6.87535 \times 10^{-6} H)^{5.256} \quad \text{below 36,089 ft}$$
$$P_a = P_o (.22335) e^{-.00004806[H - 36,089]} \quad \text{above 36,089 ft}$$

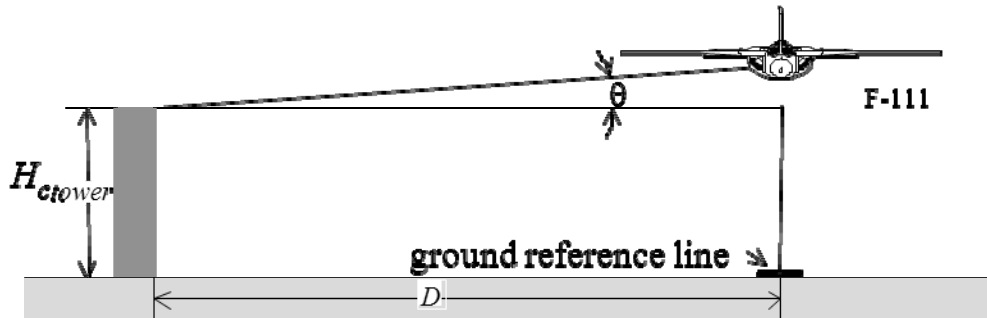


H_i	Indicated pressure altitude (29.92" Hg)
ΔH_{ic}	Instrument error correction
H_{ic}	Altimeter corrected of instrument error
ΔH_{pc}	Position error correction
H_c	Calibrated pressure altitude

Section 4.7 Position Error Test Methods

4.7.1 Fly-by

As depicted below, the flyby method originally used some sort of viewing platform with surveyed distances and a grid or other device for determining the aircraft's relative angle above the platform's altimeter. This information combined to give the aircraft's actual pressure altitude. Modern methods replace the tower system with a radar altimeter or GPS unit to determine tapeline height above the flyby line (H_g). This geometric height is converted to a pressure altitude change using a temperature correction. When added to the aircraft's pressure altitude on the runway, this change provides the actual pressure altitude during the flyby (Actual H_c = runway pressure altitude + $H_g(T_s/T_t)$).



Assumptions

1. No errors in total head.
2. Constant height runs
3. Surveyed course

$$\text{Actual } H_c = H_{ctower} + \left(D \tan \theta \cdot \frac{T_s}{T_t} \right)$$

$$\Delta H_{pc} = \text{Actual } H_c - (H_i + \Delta H_{ic})$$

$$\Delta P_s = -\rho g \Delta H_{pc}$$

$$\Delta P_s = q_c - q_{ic}$$

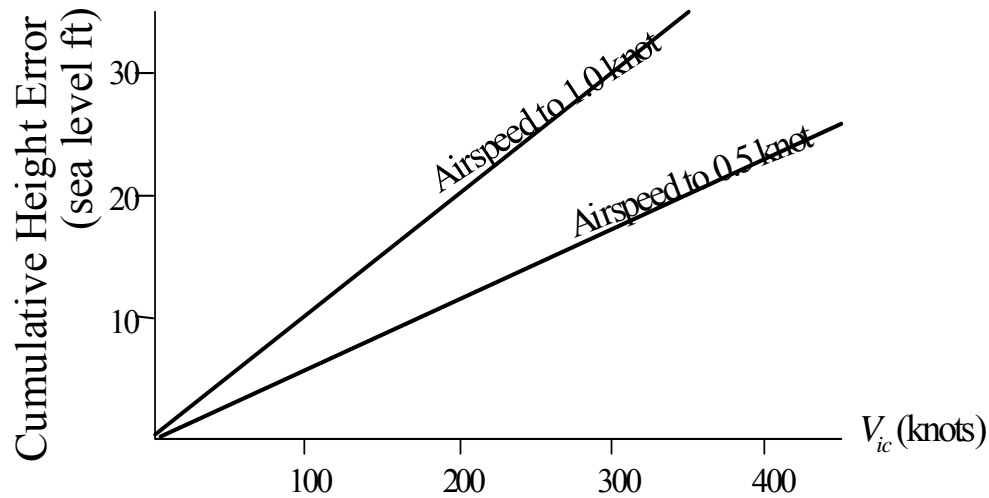
$$\Delta P_s = \frac{1}{2} \rho_0 V_c^2 - \frac{1}{2} \rho_0 V_{ic}^2 \quad (\text{low Mach only})$$

Solve for V_c

$$\Delta V_{pc} = V_c - V_{ic}$$

See flowchart for high mach or $\Delta P_T \neq 0$ cases.

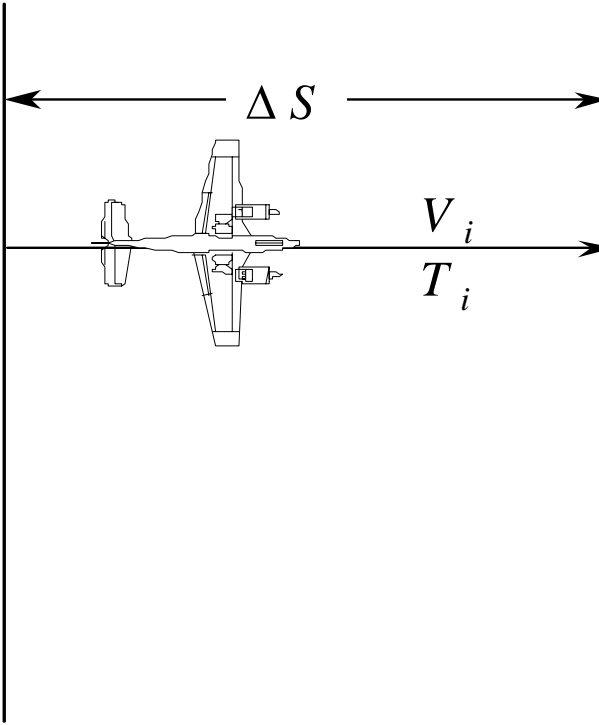
Error Analysis



Note: A check on basic instrument calibration is easily accomplished using a “ground block” where a parked test aircraft compares altimeters with tower. Any error can be treated as a bias.

This altitude-based Test method determines altimeter corrections and therefore static error directly. Accurately converting this static source error to an airspeed correction also requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined either through direct comparison with a calibrated noseboom pitot pressure or from one of the airspeed-based methods that directly yield airspeed corrections (pace, ground course, GPS). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated value for airspeed correction.

Section 4.7.2 Ground Course Method

$$\begin{aligned}
 & V_i \\
 + & \Delta V_{ic} \\
 = & V_{ic} \\
 + & \Delta V_{pc} \\
 = & V_c \\
 \div & \sqrt{\sigma} \\
 = & V_T \\
 = & \frac{\Delta S}{\Delta T}
 \end{aligned}$$


Fly known course at constant V_i

Elapsed time = ΔT , $\therefore V_T = \Delta S / \Delta T$

Use H_i and T_i to compute $V_e = V_T(\sqrt{\sigma}) = V_c$ for low altitude.

Correct V_i for instrument error corrections (ΔV_{ic}) using

$$V_{ic} = V_i + \Delta V_{ic}$$

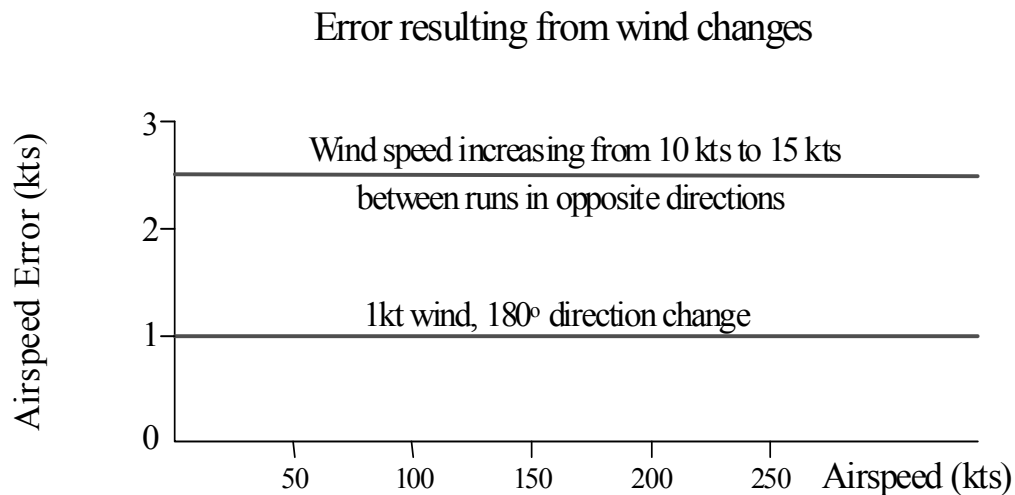
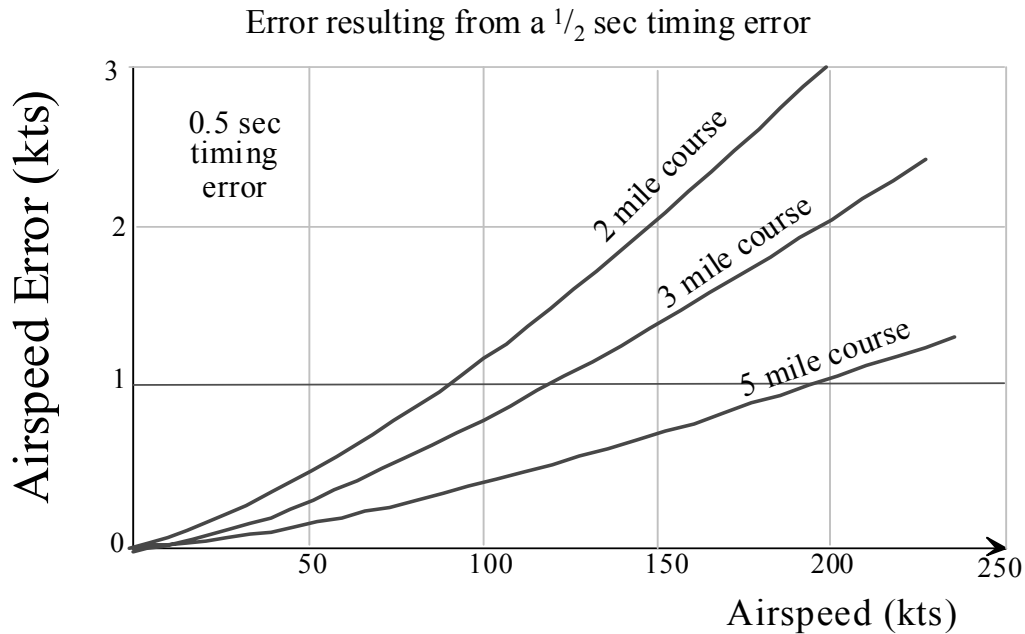
$$\Delta V_{pc} = V_c - V_{ic}$$

To determine altimeter error assume $\Delta P_T = 0$

$$\frac{1}{2} \rho_0 [V_c^2 - V_{ic}^2] = +\Delta P_S$$

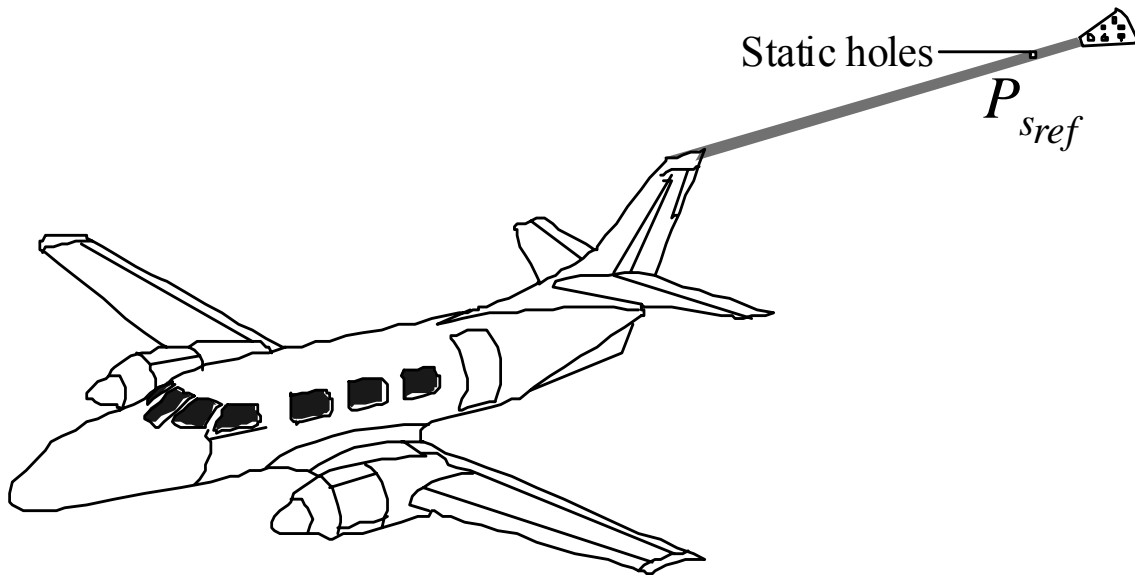
$$\Delta P_S = +\rho g \Delta H$$

$$\text{If } \Delta P_T \neq 0, \text{ then } \Delta H_{pc} = + \frac{\Delta P_S - \Delta P_T}{\rho g}$$



This airspeed-based Test method determines airspeed corrections directly. Accurately converting this airspeed error to a static source error requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined using one of the altitude-based methods that directly yield altitude corrections (tower fly-by, trailing cone or bomb). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated values for static pressure error and altimeter correction.

Section 4.7.3 Trailing Cone/Bomb Method



1. Measure P_s (ref) away from pressure field of aircraft
2. Cone is used to stabilize static line
3. No speed limitations
4. Inexpensive—can be trailed on landing
5. Consider lag effects during rapid altitude changes

$$\Delta P_s = \Delta P_{sA/C} - \Delta P_{sREF}$$

$$\Delta H_{pc} = + \frac{\Delta P_s}{\rho g} = \text{altimeter correction}$$

$$\Delta P_T - \Delta P_s = \Delta P_D = q_{ic} - q_c \text{ assuming } \Delta P_T = 0, M < .2$$

If pitot errors do exist, then they must be included in calculations for ΔV_{pc} (see flowchart)

Using a trailing cone during stall testing may give airspeed errors due to lag errors during the deceleration.

This altitude-based Test method determines altimeter corrections and therefore static error directly. Accurately converting this static source error to an airspeed correction also requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined either through direct comparison with a calibrated noseboom pitot pressure or from one of the airspeed-based methods that directly yield airspeed corrections (pace, ground course, GPS). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated value for airspeed correction.

4.7.4 GPS Methods

- The attraction
 - ~ no aircraft modification required
 - >> no trailing cone or aircraft plumbing mod
 - >> no flight test boom
 - ~ no limitation on speed or altitude
 - >> can be done down to near stall,
 - >> any altitude
 - ~ easy data reduction
 - >> no correlation with pace aircraft, ground radar, or other references required

Various methods available, all assume steady winds and ambient temperature. You must determine wind speed and direction to calculate V_T and T_0 and to ensure steady winds existed during test series.

GPS accuracies are variable. Know tolerances before accepting GPS as a truth model.

If exact ($\pm 10^\circ$) winds are calculated inflight, you can fly one pass directly into/away from the wind

$$V_T = V_G + V_{Headwind}$$

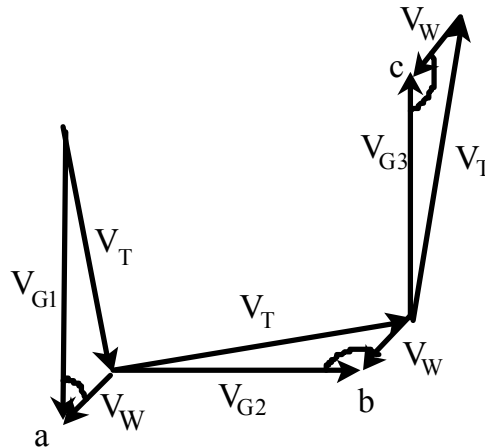
This airspeed-based Test method determines airspeed corrections directly. Accurately converting this airspeed error to a static source error requires knowledge of the test pitot tube's total pressure error (ΔP_T). This can be determined using one of the altitude-based methods that directly yield altitude corrections (tower fly-by, trailing cone or bomb). In lieu of these options, the pitot error may assumed to be zero, but this will reduce confidence in the calculated values for static pressure error and altimeter. correction

Graphs in Section 4.11 separately show the effect of measurement errors in ΔP_T , air temperature, or true airspeed on ΔV_C calculations. The last chart (Page 04-29) shows how each knot of accumulated ΔV_C uncertainty affects the ΔH_C uncertainty at various altitudes and temperatures.

Flying four legs instead of three allows four separate calculations of wind speed & direction to confirm stable winds at that test airspeed. If several real-time calculations of winds confirm constant direction and velocity, then testing may be shortened by flying only one pass directly into or away from the wind. If this is done, an end-of-test wind calibration must be performed to confirm steady winds throughout the test series. To minimize temperature and wind variations, testing should be accomplished within a relatively small area.

Horseshoe Track GPS Method

- Horseshoe track method
 - ~ fly three legs with each perpendicular ground tracks, noting GPS ground speed on each
 - ~ determine true airspeed by solving three equations in three unknowns
- Practical problem
 - ~ need to fly close to the ground, tracking perpendicular ground references



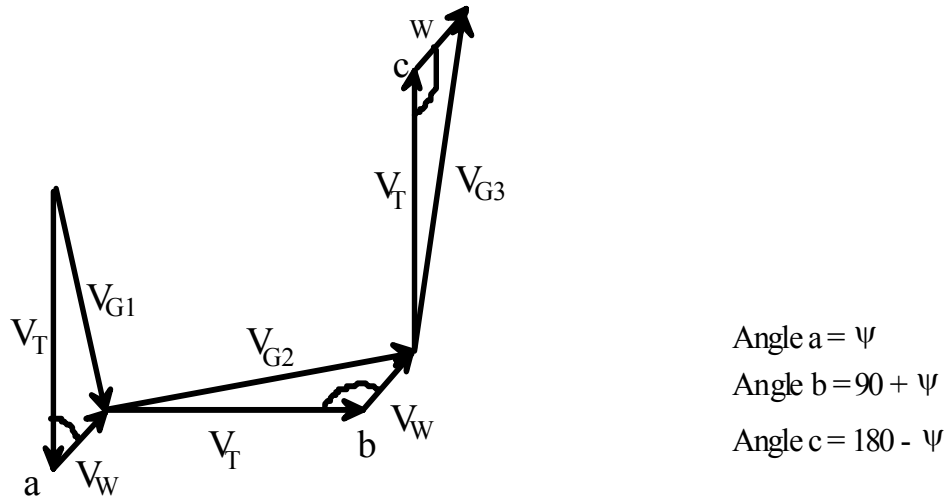
$$\text{True airspeed: } V_T = \frac{1}{2} \sqrt{\left(V_1^2 + V_2^2 + V_3^2 + V_1^2 \times \left(\frac{V_3^2}{V_2^2} \right) \right)}$$

$$\text{Wind velocity } V_W = \sqrt{\left(\frac{V_1 - V_3}{2} \right)^2 + \left(\frac{V_2 - V_1 \times V_3 / V_2}{2} \right)^2}$$

$$\text{Wind direction } \psi_W = \tan^{-1} \frac{(V_2 - V_1 \times V_3 / V_2)}{(V_1 - V_3)}$$

Horseshoe Heading GPS Method

- Horseshoe Heading Method
 - ~ Fly three legs with perpendicular headings, noting GPS ground speed on each
 - ~ Determine true airspeed by solving three equations in three unknowns



$$\text{Wind direction } \psi = \tan^{-1} \left[\frac{-V_{G1}^2 + 2V_{G2}^2 - V_{G3}^2}{V_{G3}^2 - V_{G1}^2} \right]$$

$$\text{Wind velocity } V_w = \frac{1}{2} \left[V_{G3}^2 + V_{G1}^2 \pm \sqrt{(V_{G3}^2 + V_{G1}^2)^2 + \left(\frac{-V_{G1}^2 + 2V_{G2}^2 - V_{G3}^2}{\sin \psi} \right)^2} \right]^{1/2}$$

$$\text{True airspeed } V_T = \sqrt{\frac{V_{G3}^2 + V_{G1}^2}{2} - V_w^2}$$

The “Windbox” method consists of flying four legs instead of three. The extra leg provides a fourfold increase in wind calculations to improve result confidence. The “Orbis” method extends this advantage by collecting data at every heading throughout a level turn.

Cloverleaf Method

(Microsoft Excel spreadsheet adapted from Doug Gray, NSW Australia)

Fly three legs with approximately 90-120 degree difference between headings.

- ~ Can be accomplished in a broad turn as with the horseshoe method, or
- ~ Directly over a single point (cloverleaf maneuver).

Accurate results require

- ~ Identical values for indicated airspeed (and TAS) for all legs.
- ~ Constant winds throughout data collection (single W/S vector in figure).
- ~ Approx. 10 seconds stable ground speed, V_g , (G/S in figure) during each leg.

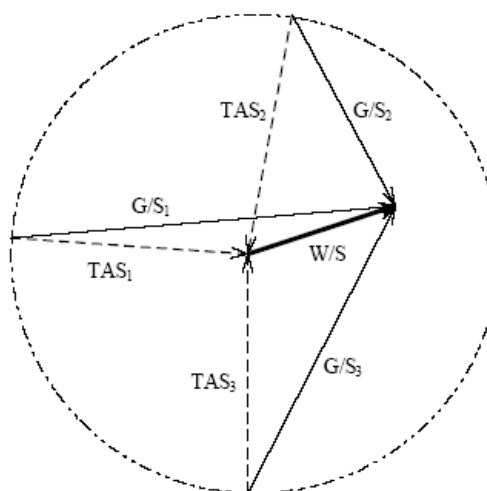
Aircraft heading results for each leg entail an airborne compass swing.

Inputs for each 3-leg data set

Vg_1	Vg_2	Vg_3
Trk_1	Trk_2	Trk_3

Intermediate calculations

$$\begin{aligned}
 X_1 &= Vg_1 * \sin(\pi * (360 - Trk_1) / 180) \\
 Y_1 &= Vg_1 * \cos(\pi * (360 - Trk_1) / 180) \\
 X_2 &= Vg_2 * \sin(\pi * (360 - Trk_2) / 180) \\
 Y_2 &= Vg_2 * \cos(\pi * (360 - Trk_2) / 180) \\
 X_3 &= Vg_3 * \sin(\pi * (360 - Trk_3) / 180) \\
 Y_3 &= Vg_3 * \cos(\pi * (360 - Trk_3) / 180) \\
 M_1 &= -(X_2 - X_1) / (Y_2 - Y_1) \\
 M_2 &= -(X_3 - X_1) / (Y_3 - Y_1) \\
 B_1 &= (Y_1 + Y_2) / 2 - M_1 * (X_1 + X_2) / 2 \\
 B_2 &= (Y_1 + Y_3) / 2 - M_2 * (X_1 + X_3) / 2 \\
 V_{wx} &= (B_1 - B_2) / (M_2 - M_1) \\
 V_{wy} &= M_1 * V_{wx} + B_1
 \end{aligned}$$



Vector Triangles for three G/S tracks.

Results

$$\text{Aircraft true airspeed} = V_T = [(X_1 - V_{wx})^2 + (Y_1 - V_{wy})^2]^{0.5}$$

$$\text{Total wind speed} = V_w = [(V_{wx}^2 + V_{wy}^2)]^{0.5}$$

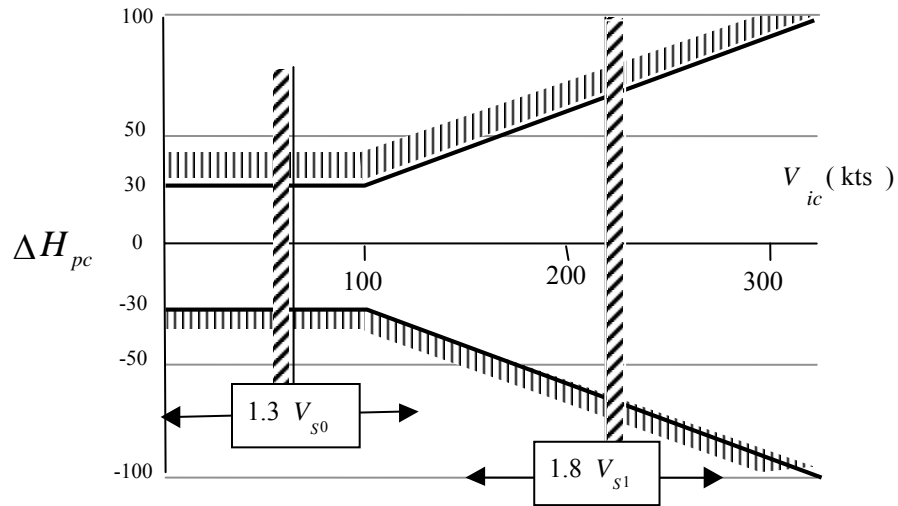
$$\text{Wind direction} = \psi_w = \text{Psi}_w = \text{MOD}(540 - (180/\pi) * \text{ATAN2}(V_{wy}, V_{wx}), 360)$$

$$1^{\text{st}} \text{ leg a/c heading} = \psi_1 = \text{Psi}_1 = \text{MOD}(540 - (180/\pi) * \text{ATAN2}(V_{wy} - Y_1, V_{wx} - X_1), 360)$$

$$2^{\text{nd}} \text{ leg a/c heading} = \psi_2 = \text{Psi}_2 = \text{MOD}(540 - (180/\pi) * \text{ATAN2}(V_{wy} - Y_2, V_{wx} - X_2), 360)$$

$$3^{\text{rd}} \text{ leg a/c heading} = \psi_3 = \text{Psi}_3 = \text{MOD}(540 - (180/\pi) * \text{ATAN2}(V_{wy} - Y_3, V_{wx} - X_3), 360)$$

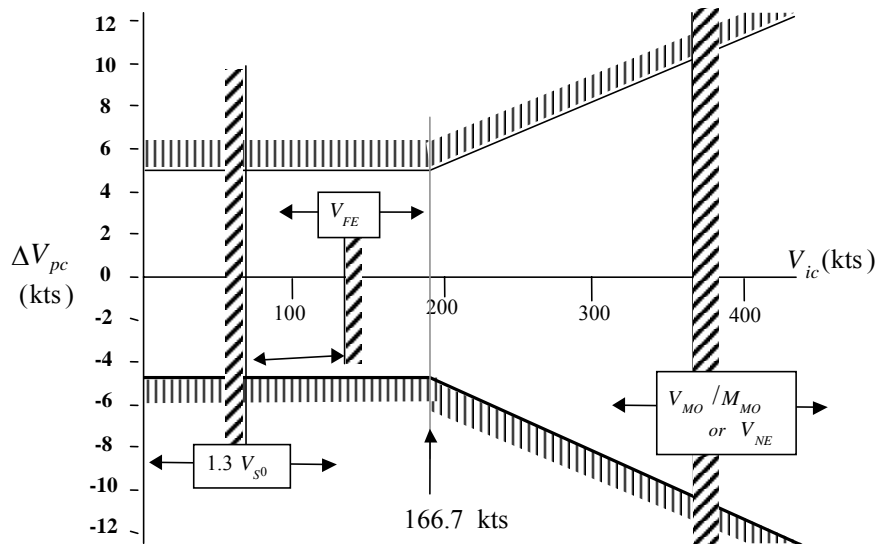
Section 4.8 Position Error FAR 23/25.1323 and .1325/JAR Certification Requirements



Maximum error at sea level must be less than ± 30 ft/100 kts between $1.3 V_{S0}$ and $1.8 V_{S1}$

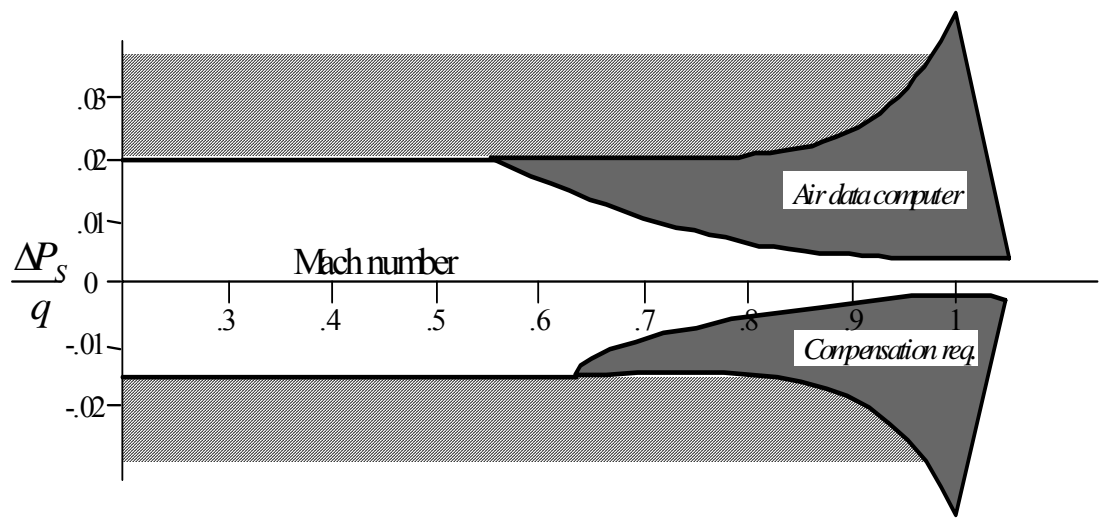
V_{S0} = Full flap, gear down, power off, stall speed

V_{S1} = Stall speed in a specific configuration

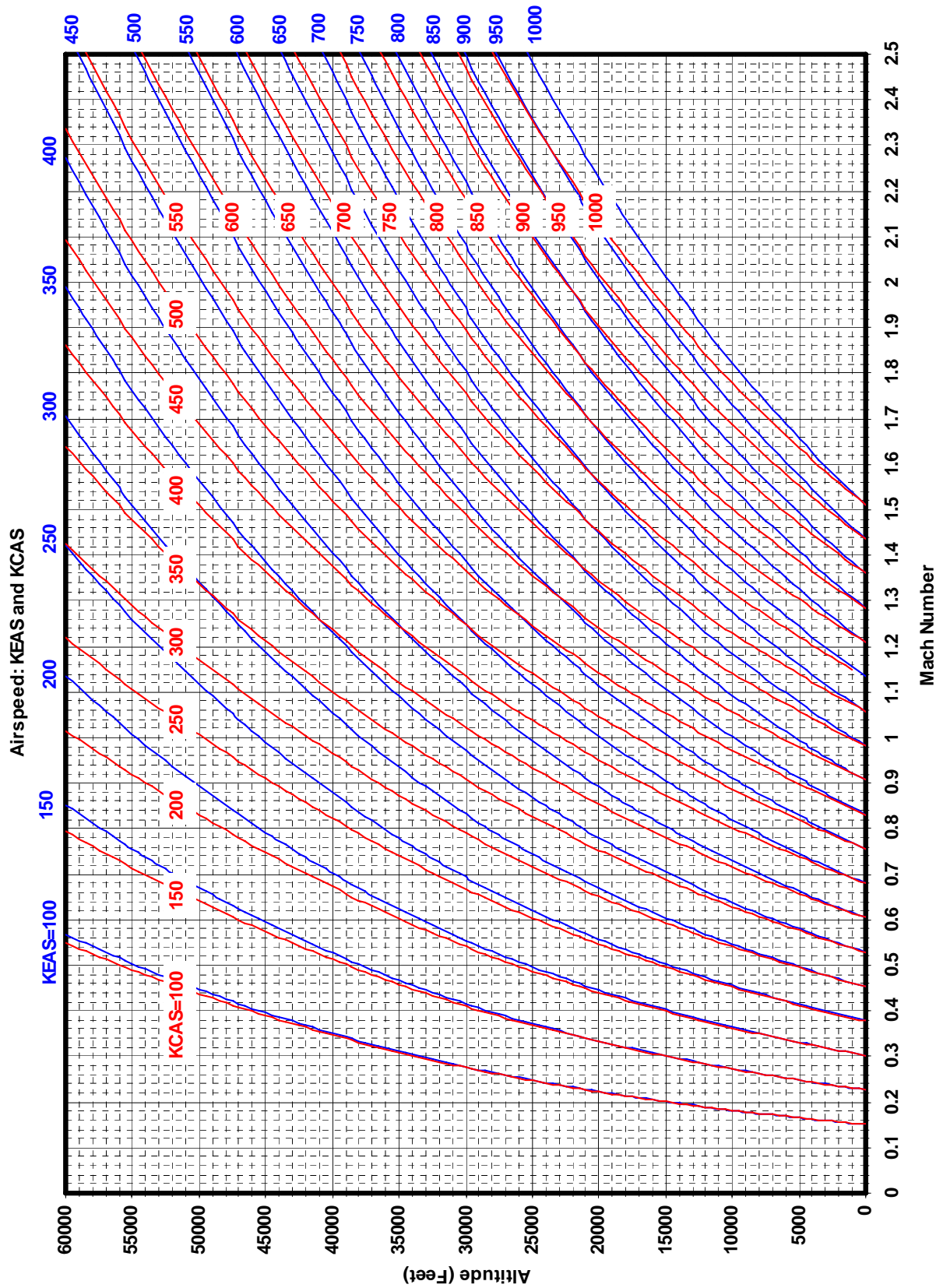


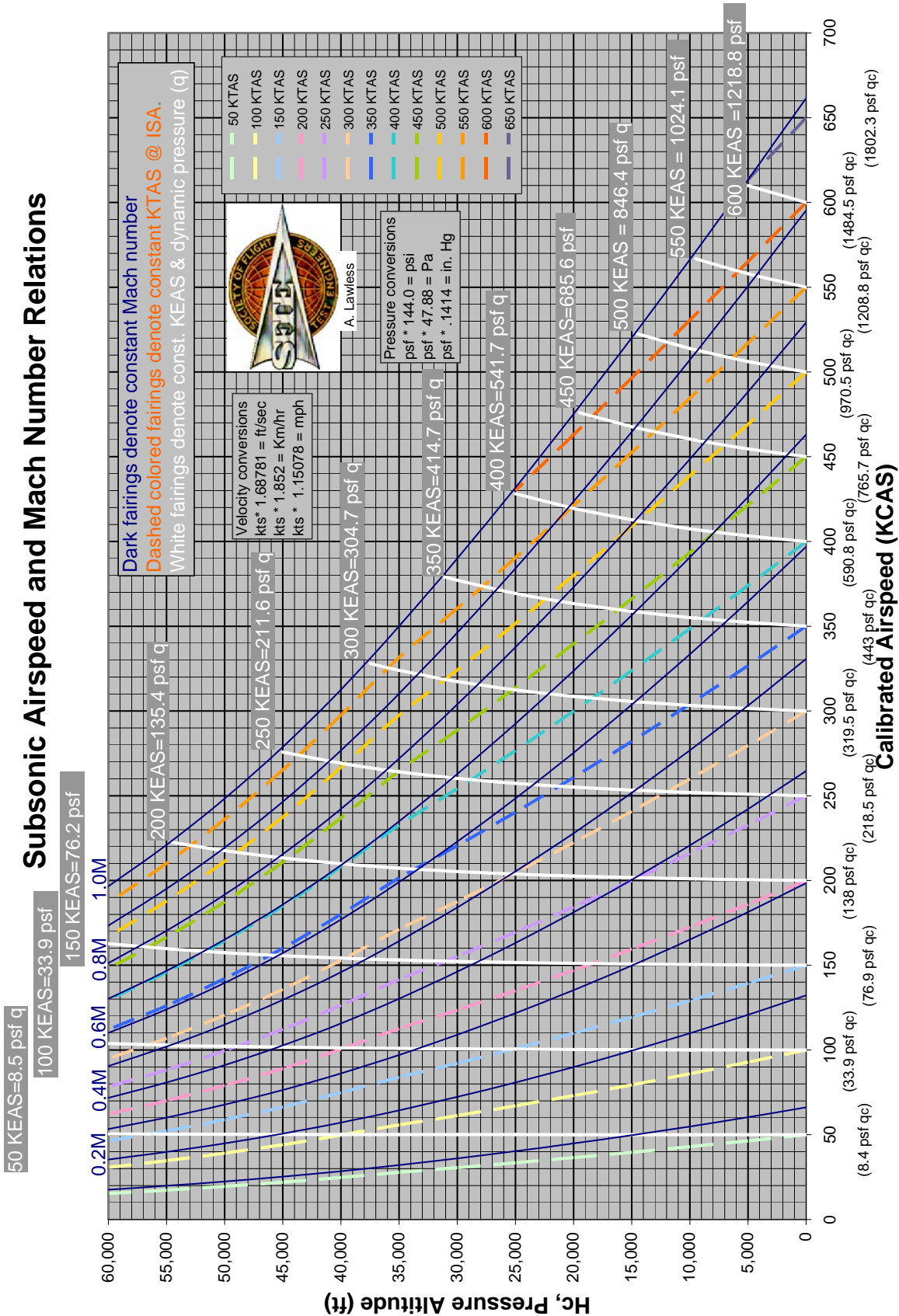
Errors must be equal to or less than $\pm 3\%$ of V_c or ± 5 kts whichever is greater

Mil Spec P-26292 C (USAF)
Landing configuration: $\Delta H_{pc} \pm 30$ ft.

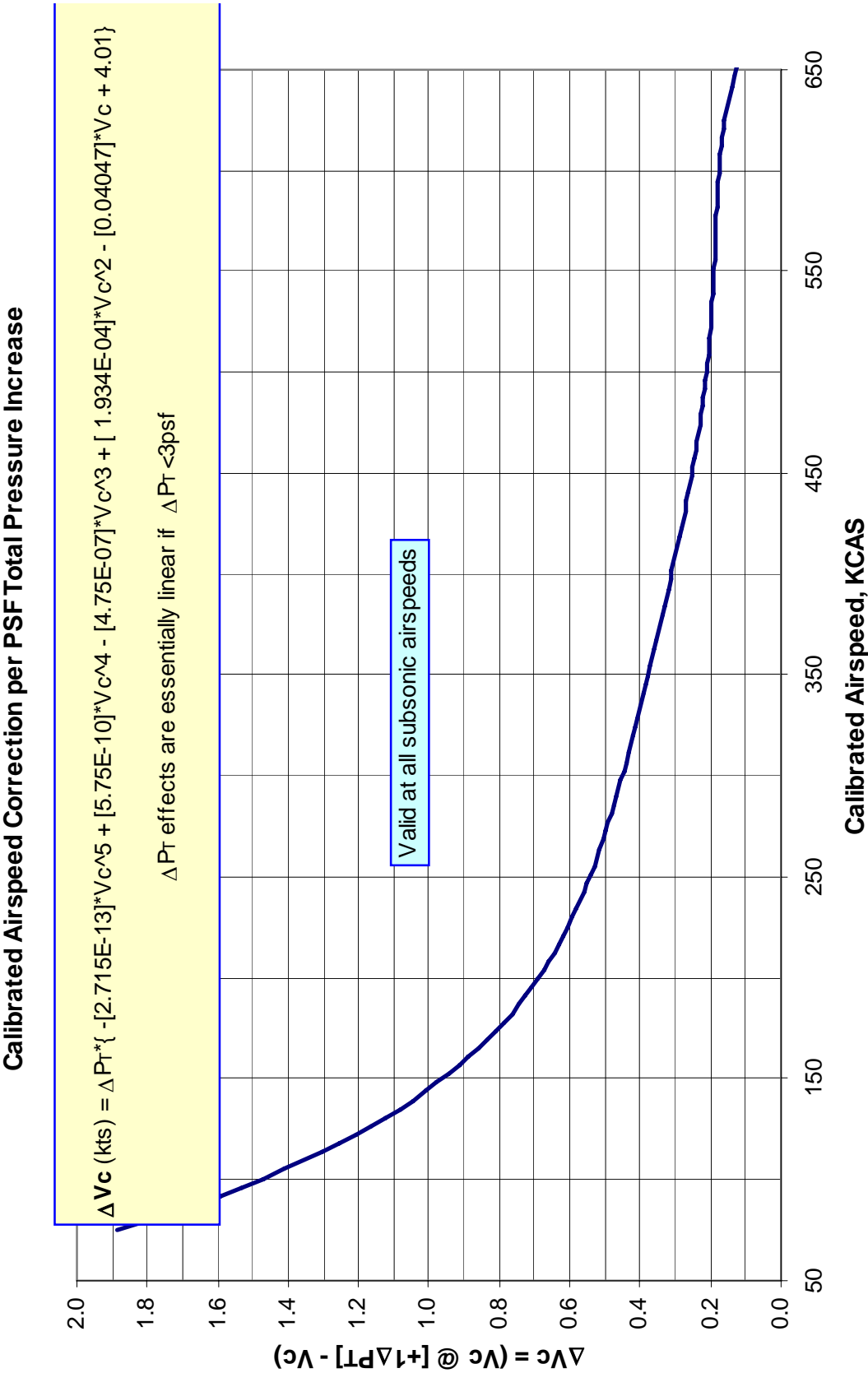


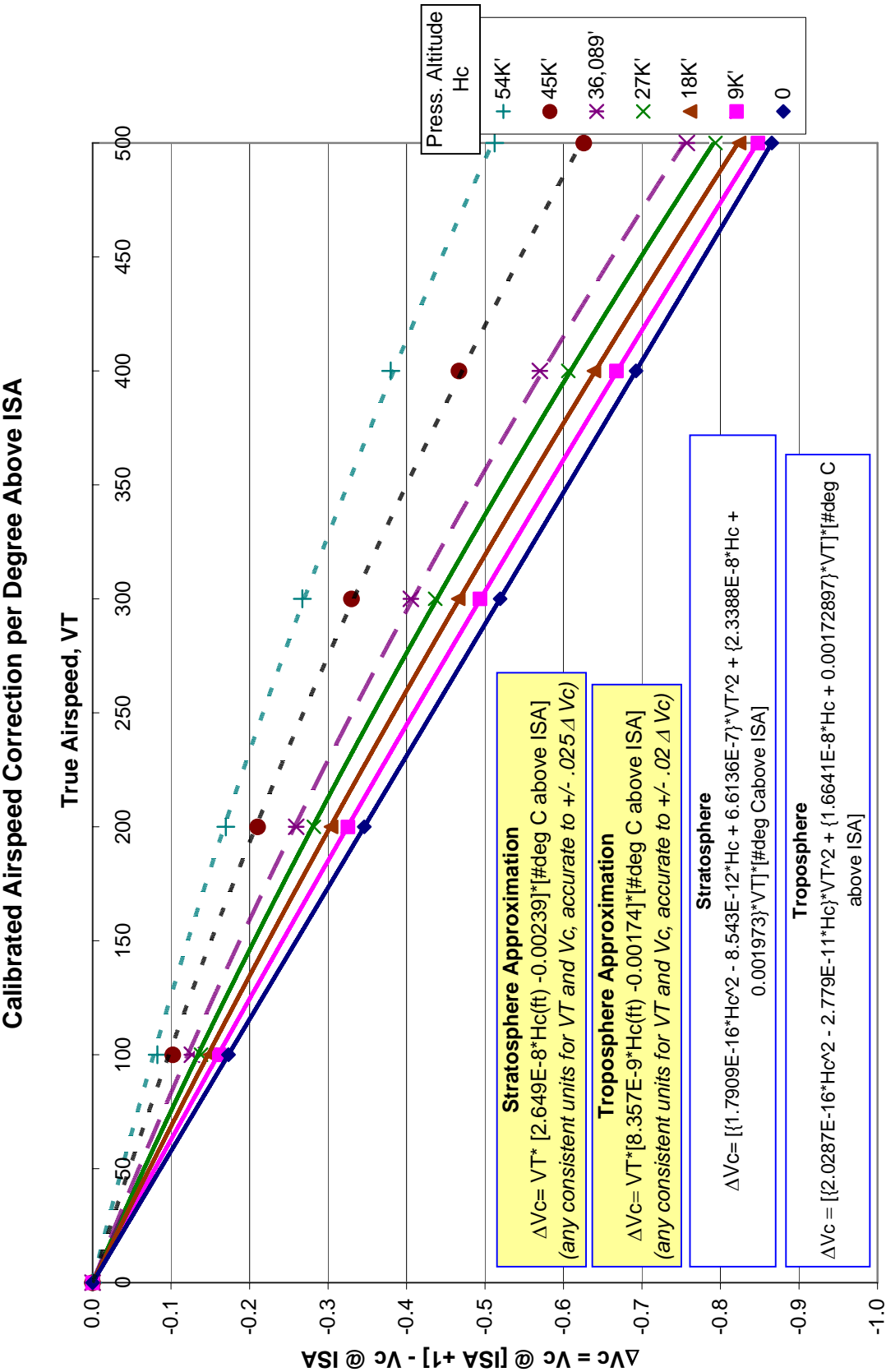
4.10 Airspeed/Altitude/Mach Graphic Relation

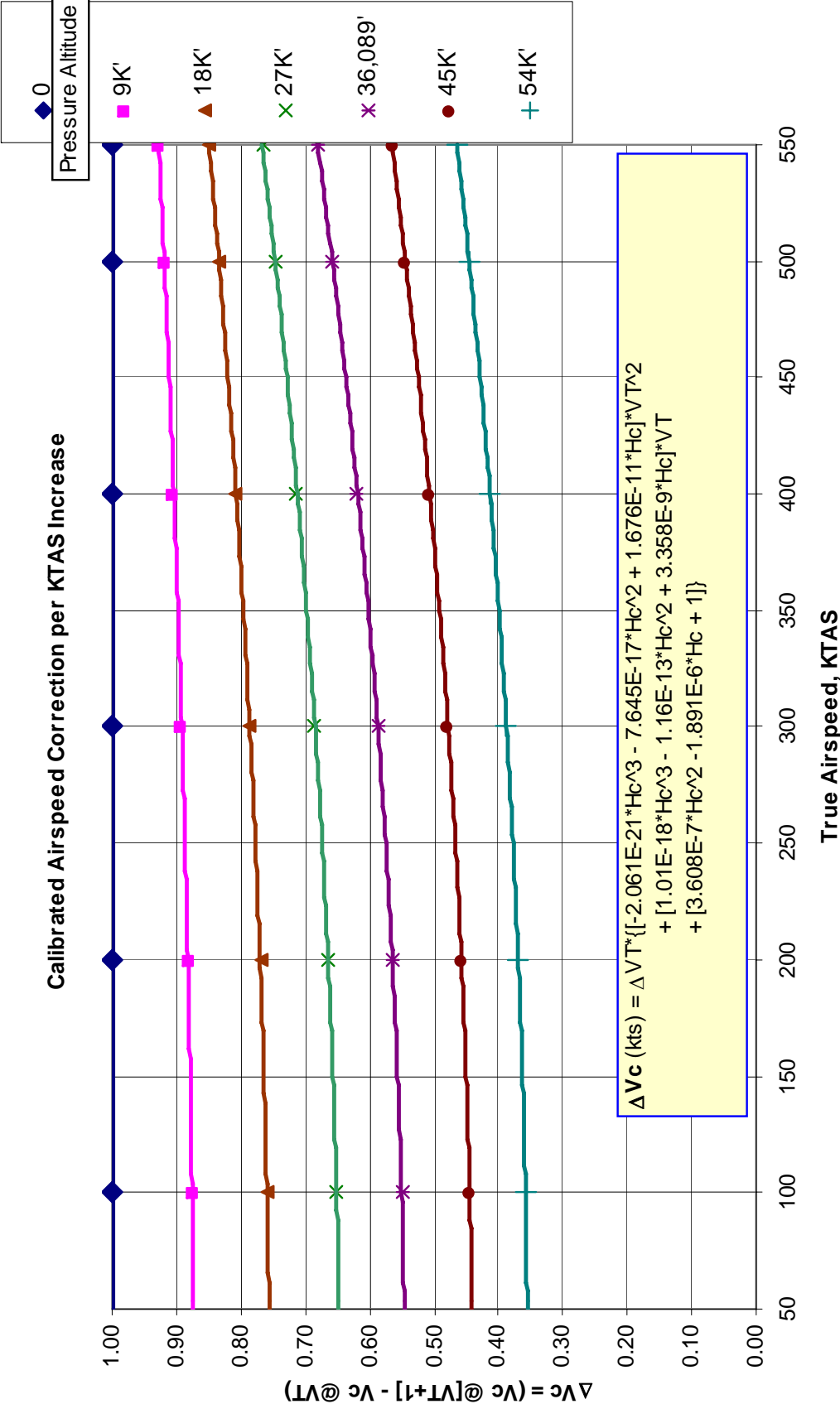




4.11 Effect of Errors on Calibrated Airspeed and Altitude







Altimeter Correction per Knot Airspeed Correction

$$\text{ISA } \Delta H_{pc} = \Delta V_{pc} \{ [(-1.184E-18) * H_c^3 + (4.004E-14) * H_c^2 + (-1.533E-09) * H_c - 0.000105] * V_c^2 + [(-5.034E-15) * H_c^3 + (1.502E-10) * H_c^2 + (-5.019E-06) * H_c - 0.06421] * V_c \}$$

Non-ISA:

$$\text{Troposphere } \Delta H_{pc} = [\text{ISA } \Delta H_{pc}] * [1 + \{3.1059E-08 * H_c + 0.00344\} * (\text{deg C above ISA})]$$

$$\text{Stratosphere } \Delta H_{pc} = [\text{ISA } \Delta H_{pc}] * [1 + 0.004616 * (\text{deg C above ISA})]$$

