**Section 5 Aerodynamics**

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**5.0 Recurring Terminology**

*a* slope of lift curve, *dCL/da*

*a.c*. aerodynamic center, location along the chord where pitching moments about this center do not change with angle of attack (25% MAC for airfoils in subsonic flow, 50% MAC for airfoils in supersonic flow)

*AOA* angle of attack

AR aspect ratio = [wing span]2/ [reference wing area] = *b2/S*

B wing span

bt horizontal tail span

*C* coefficient, a non-dimensional representation of an aerodynamic property

*c* wing chord length Camber maximum curvature of an airfoil, measured at maximum distance between chord line and amber line, expressed in % of MAC.Camber line theoretical line extending from an air foil’s leading edge to the trailing edge, located halfway between the upper and lower surfaces.

*CD* drag coefficient

*CDi* induced drag coefficient

*CDo,CDpe* parasitic drag coefficient

*cf* friction coefficient

Chord straight-line distance from an airfoil’s leading edge to its trailing edge.

*CL* lift coefficient

*Cp* pressure coefficient = *Dp/q*

*e* Oswald efficiency factor

*l* distance traveled by flow, or characteristic length of surface

*M* Mach number

MAC mean aerodynamic chord, chord length of location on wing where total aerodynamic forces can be concentrated.

MGC mean geometric chord, the average chord length, derived only from a plan form view of a wing (similar to MAC if wing has no twist and constant cross section & thickness-to-chord ratio).

*P* pressure

*Preq’d* power required

*q* dynamic pressure = *½ ra VT2 = ½ ro VT2*

*R* gas constant

*Rn,Re* Reynolds number

*S* reference wing area, includes extension of wing to fuselage centerline.

*St* horizontal tail surface area

*SW* wetted area of surface

*T*  temperature

*V* true velocity

*Ve* equivalent velocity

*a* angle of attack

*ai* induced angle of attack

d depth of boundary layer, or surface wedge angle

*m* viscosity, or wave angle

n flow turning angle

*q* shock wave angle

*r* density

• Perfect Fluid

~  incompressible, inelastic, and non-viscous

~  used in flow outside of boundary layers at M < .7

• Incompressible, inelastic, viscous

~  used for boundary layer studies at M < .7

• Compressible, non-viscous, elastic fluid

~ used outside boundary layers up to M = 5

**5.1 Dimensional Analysis Interpretations** (ref 5.2)

Aerodynamic force = *F*

· *F = f* *(r, m, T*, *V*, shape, orientation, size, roughness, gravity)

· For aircraft ignore *R*, *K* & hypersonic effects

· Initially assume similar body orientations, shapes & roughness.

· Dimensional Analysis reveals four non-dimensional (p) parameters:

  Force Coefficient

 Reynolds Number

  Mach Number

  Froude Number

A closer look at the force coefficient:



 where 1/2 ρ*aVT*2 = 1/2 ρ*oVe*2 =dynamic pressure, *q* dimensions of reference wing area, *S* are the same as *l*2

**A feel for *q***

· Kinetic energy of a moving object = ½ mVT2

· Block of moving air kinetic energy = ½ ρ (volume) V T2

· Dividing through by volume yields KE per volume of moving air = ½ ρ V T2

· "Dynamic pressure” or “*q*” = potential for converting each cubic foot of the airflow's kinetic energy into frontal stagnation pressure

· Feel *q* by extending your hand out the window of a moving car

**A feel for coefficients**

· *C F* = (*F /S)/q* = the ratio between the total force pressure and the flow 's dynamic pressure

· Lift is the component of the total force perpendicular to the free stream flow

· Drag is the component along the flow

· Break total into lift and drag coefficients:

*C L = (L/S)/q CD = (D/S)/q*

· Increasing dynamic pressure generates a larger total force, lift and drag



· Froude number is not significant in aerodynamic phenomena

· Recall that forces are aslo a function of angle of attack, shape & surface roughness, therefore



Effects are exaggerated



To compare test day and standard day aircraft or to match wind tunnel *CF* data to actual aircraft; the shape, roughness, *M, Rn* and *a* must be equal for both aircraft



**5.2 General Aerodynamic Relations** (refs 5.1, 5.2, 5.10)

Lift & Drag forces can be described using two approaches:

1) Change in momentum of airstream, *F = d{mv]/dt*

2) “Bernoulli” approach which requires the continuity and conservation of energy equations

**Continuity Equation**

Fluid M ass in = Fluid Mass out

ρ 1*V* 1*A* 1 = ρ 2*V* 2*A* 2

For subsonic (incompressible) flow ρ 1 = ρ 2

*V* 1*A* 1 = *V* 2*A* 2

**Conservation of Energy** (Bernoulli) **Equation:**

Potential + Kinetic + Pressure = constant

(changes in Potential energy are negligible)

Energy per unit volume is pressure then

Dynamic Pressure + Static Pressure = Total Pressure



· This classic approach only applies in the “potential flow” region and not in the boundary layer where energy losses occur

· Pressures around a surface can be calculated or measured from tests and converted into pressure coefficients,

*cp* = (*p*local-*p*ambient)/dynamic pressure = *Dp/q*

· *cp* values can be mapped out for all surfaces



 · Summation of all pressures perpendicular to surface yield the pitching moments and the “**Resultant Aerodynamic Force**” which is broken into lift and drag components



 · Lift & drag forces are referred to the aerodynamic center (*ac*) where the pitching moment is constant for reasonable angles of attack.

· Pitching moments increase with airfoil camber, are zero if symmetric.

· Aerodynamic center is located at 25% MAC for fully subsonic flow and at 50% MAC for fully supersonic flow.

**5.3 Wing Design Effects on Lift Curve Slope** (refs 5.1, 5.2, 5.10)

**Aspect Ratio Effect**

· Pressure differential at wingtip causes tip vortex





· Vortex creates flow field that reduces AOA across wingspan



 · Local AOA reductions decrease average lift curve slope



2D wing = wind tunnel

airfoil extending to walls (infinite aspect ratio).

*ao* = Lift curve slope for

an infinite wing

*a*  = Lift curve slope for

a finite wing

 · Above relationship estimated as



**Trailing Edge Flap Effects**





**Leading Edge Flap Effects**





**Boundary Layer Control Effects**

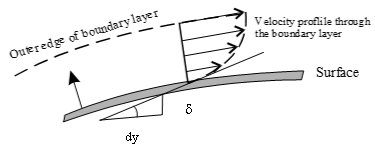




**5.4 Elements of Drag** (refs 5.1, 5.2, 5.10)



· Skin friction shear stress is a function of velocity profile at surface

****



d

dy



·  **Viscosity** (m) increases with temperature (ref 5.9)

Sutherland law:



Power law:



Where *To* = 273.15 *K* = 518.67 *R*.

For air: *S* =110.4 *K* = 199 *R; n=.67*

For air at 273 K: mo= 1.717x10-5 [kg/m s] = 3.59x10-7 [slug/ft s]

Inserting air values (*TK*=Kelvin and T*R*=Rankin) into Sutherland law gives



**Reynolds Number Effects** (ref 5.10)

· Laminar boundary layers have more gradual change in

velocity near surface than

turbulent boundary layers.

· High Reynolds numbers help

 propagate turbulent flow.





*ReL* based on total length of flat plate

· Depth of boundary layer (d) depends on local Reynolds number (Rex) and whether the flow is turbulent or laminar.



*x*= distance traveled to point in question

**5.4.2 Pressure Drag**

· Ideal frictionless flow has no losses and leads to zero pressure drag

· Real fluids have friction and energy losses along surface

· Energy losses negate total pressure recovery, lead to decreasing total pressure along surface



· Imbalance of pressures on surfaces causes pressure drag



· Profile streamlining reduces pressure drag



**5.4.3 Interference Drag**

· Occurs with multiple surfaces approximately parallel to flow

· Caused by flow’s interference with itself or by excessive adverse pressure gradient due to rapidly decreasing vehicle cross section

· Most severe with surfaces at acute angles to each other

· Effects often reduced by fillets around contracting surfaces



**5.4.4 Induced Drag**

· Wingtip vortex reduces local AOA at each station along wing

· Local lift vector is perpendicular to local AOA

· Local lift vector is therefore tilted back relative to freestream lift

· Induced drag defined as rearward component of local lift vector





Oswald efficiency factor, *e*, accounts for losses in excess of those predicted above (due to uneven downwash and changing interference drag effects).



**5.5 Aerodynamic Compressibility Relations** (reference 5.8)

**Prandtl/Glauert Approximation**

Approximates Mach effects on aerodynamics below critical Mach



**Total vs Ambient Property Relations for Adiabatic Flow**



Isentropic flow not required

Isentropic (shockless) flow required

Isentropic flow required

**Normal Shock Relations**

Assumes isentropic flow on each side of the shock

Assumes flow across shock is adiabatic

Property changes occur in a constant area (throat)

·

·



Normal shock summary



**5.5.1 Oblique Shocks**

**Oblique Shock Description**



d = surface turning angle

q = shock wave angle

Subscript 1 denotes upstream conditions

Subscript 2 denotes downstream conditions

**Oblique Shock Relations**

· Calculate *P2/P1, T2/T1*, and *r2/r1* across oblique shocks by using normal shock equations and substituting *M1 sinq* in place of *M1*

· Calculate total pressure loss across oblique shock as

· Calculate relation between Mach number and angles as



**Oblique Shock Turning Angle as a Function of Wave Angle**



· Two q solutions exist for every M1 & *d* combination

These represent the strong and weak shock solutions

Weak shocks normally occur in nature

· There is a minimum Mach number for each turning angle

· The wave angle of a weak shock decreases with increased Mach

· For a given Mach number, *q* approaches *m* as *d* decreases

**Mach Cone Angle**



Minimum Wave Angle

m = sin-1 (1/M)

**5.5.2 Supersonic Isentropic Expansion Relations**



· The wave angle *m* determines where the lower pressure can be felt and thus where the flow can be accelerated

· As the flow accelerates, a new wave angle forms and the subsequent lower pressure further accelerates the flow

· Results in a series of Mach waves forming a “fan” until the flow turns and accelerates so that it is parallel to the new boundary

**Prandtl-Meyer Function**

Shows flow’s required turning angle (n) to accelerate from one Mach number to another



· If upstream Mach (M1) =1, then n1 = 0, and equation directly relates downstream Mach (M2) to surface turning angle (Dn)

· If M1>1, determine M2 as follows:

Calculate upstream n1 from above equation

Calculate n2 = n1 + Dn

Reverse above equation to obtain corresponding M2

· Above equation is tabulated in NACA TR 1135 and is plotted below



Example: Flow initially at *M1* = 2.0 accelerates through an expansion corner of 24 *deg*. Exit Mach number is 3.0

**5.5.3 Two-Dimensional Supersonic Airfoil Approximations**

· Determine surface static pressures by calculating changes through obliques shocks and expansion fans



· Ackert approximations for thin wings are based on



· Double wedge airfoil approximations



· Biconvex wing approximations



**5.6 Drag Polars** (ref 5.2)

**5.6.1 Drag Polar Construction and Terminology**

*CL* = lift coefficient

*CD* = drag coefficient

*CDi* = induced drag coefficient

*CDo* = parasitic drag coefficient

AR = aspect ratio

*e* = Oswald efficiency factor

*l*  = length flow has traveled

*Swet* = wetted area of surface

*S* = reference wing area

**Simple Drag Polar Equation Limitations**

· No separated flow losses

· Symmetric Camber

· Applies at one Mach, Altitude, *cg*



“Polar” form of simple drag polar

 Linearized form of

simple drag polar

**5.6.2 Complicating Factors**

**Airflow Separation Effects**



Drag Polar Equation Accounting for Flow Separation:

· Delete last term if *CL<Clbreak*

· Determine *k*2 from flight test

**Reynolds Number Effects** (refs 5.4, 5.11)

· Calculate length *ReL* and friction coefficient (*cf* ) for each surface as

*TK* = Kelvin*,*

*l=* total length*,* ft)



· In general*, cf*  decreases as *Rn* increases (unless transitioning from laminar to turbulent flow)

· Friction drag *= cf qSwet* for each component (*Swet* = wetted area)

· Correct from test day to standard day aircraft drag coefficient by summing differences of each component’s drag change



**Wing Camber or Incidence Angle Effects**



Note slight increase in drag as lift decreases towards zero

Linearized drag polafor aircraft with

wing camber and/or

incidence



Revised drag polar equation accounting for wing camber or incidence



· Generally not necessary since most flight occurs above *CLmin*

**Mach Number Effects**

· Aircraft with

low parasitic drag

coefficients and

high fineness ratios

pay a relatively

small “wave drag”

 penalty.

· With external stores, same aircraft pays larger Mach penalty



Drag Coefficient

Lift Coefficient

**Propeller Slipstream Effects**

·  a.k.a “scrubbing” drag

·  Propwash increases flow speed over surface within slipstream

·  More drag is created by higher *q* and vorticity.

·  Function of prop speed and power absorbed (*Cp*)or thrust (*CT*)

· Problem should be addressed in airframe or propeller models

**Trim Drag Effects** (reference 5.4)

e = wing Oswald efficiency factor

et = tail Oswald efficiency factor

b = span, bt = tail span

*x* = wing *ac-*to-*cg* distance

*l*= wing *ac*-to tail *ac* dist.

S = Area





Trim drag change relative to

total induced drag:



Plot of above equation



**5.6.3 Drag Polar Analysis**

· For a given configuration (*CDo, S, AR, e*)



first term *=* parasitic drag,

second term = induced drag

· For any given weight, *D* = *f*(equivalent airspeed) only



· Minimum total drag occurs when *Dinduced = Dparasitic*

same as speed where *CDi = CDo*

occurs at max *CL /CD* ratio (same as max *L/D* ratio)

· Minimum drag/velocity occurs at min slope of Drag vs V curve

same as speed where 3*CDi =CDo*

occurs at max *CL 1/2 /CD* ratio

Power required = drag x true airspeed



Minimum total *Preq’d* occurs when *Pinduced = Pparasitic*

· same as speed where *CDi = 3CDo*

· occurs at max *CL 3/2 /CD* ratio

Minimum power/velocity occurs at min slope of *Preq’d* vs *V* curve

·  same as speed where *CDi =CDo*

·  occurs at max *CL /CD* ratio

**Optimum Aerodynamic Flight Conditions**

*Gliders/ Engine-Out Flight*

· Max range (minimum glide slope) occurs at max *CL/CD*

same as condition where *CDo* = *CDi* *if* drag polar is parabolic

· Min sink rate (minimum power req‘d) occurs at max *CL3/2 /CD* ratio same as condition where 3*CDo* = *CDi* *if* drag polar is parabolic

*Reciprocating Engine Aircraft (assuming constant BSFC & prop h)*

· Max range (minimum power/velocity) occurs at max *CL/CD* ratio

same as condition where *CDo* = *CDi* *if* drag polar is parabolic

· Max endurance (minimum power req‘d) occurs at max *CL3/2 /CD*

same as condition where 3*CDo* = *CDi* *if* drag polar is parabolic

*Turbine Jet Engine Aircraft (assuming constant TSFC)*

· Max range at constant altitude (minimum drag/velocity)

occurs at max *CL 1/2 /CD* ratio

same as condition where *CDo* = 3*CDi* *if* drag polar is parabolic

· Best cruise/climb range (maximum [*M* x *L/D*] ratio)

occurs at max *CL/CD 3/2* ratio

same as condition where *CDo* = 2*CDi* *if* drag polar is parabolic

· Best endurance (minimum drag)

occurs at max *CL/CD* ratio

same as condition where *CDo* = *CDi* *if* drag polar is parabolic



To calculate optimum speed *V*2 for configuration2 & weight2 based on optimum speed *V*1 at configuration1 & weight1



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