Discussion

INFO/STSCI/ILRST 3900: Causal Inference

20 Sep 2023

Open or blocked?

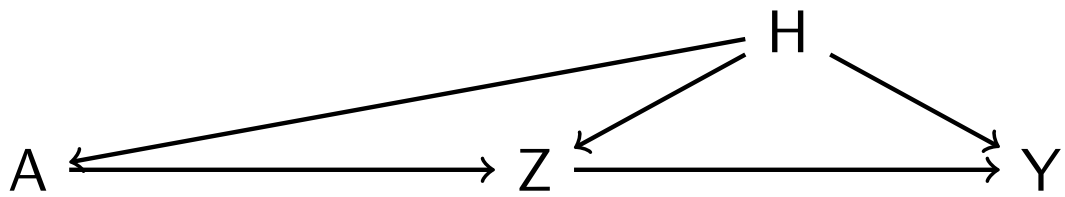
How to check if a path is open or blocked:

1. Traverse the path node by node
2. If any node is blocked, the entire path is blocked
3. If all nodes are open, then entire path is open

How to check if a node is open or blocked:

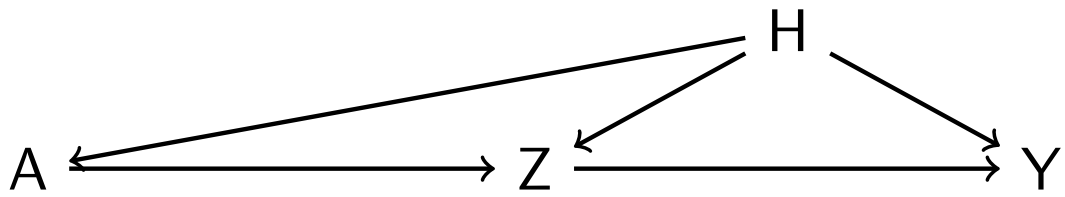
- If non-collider:
  - Open if it is not in the conditioning set
  - Blocked if it is in the conditioning set
- If collider:
  - Open if it or any of its descendants are in the conditioning set
  - Otherwise it is blocked

Practice



- What are the paths from A to Y?
- Determine if each of the paths is causal or non-causal
- Determine whether each node on each path is a collider or non-collider

Practice



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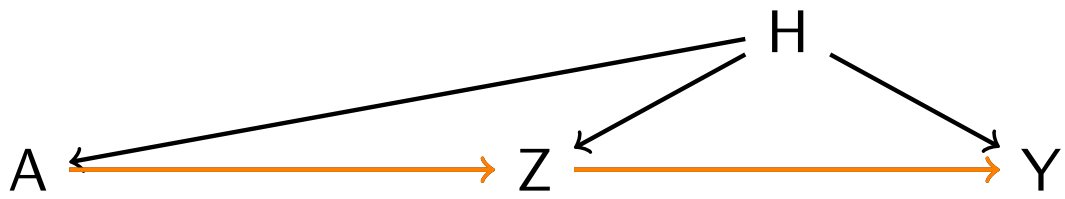
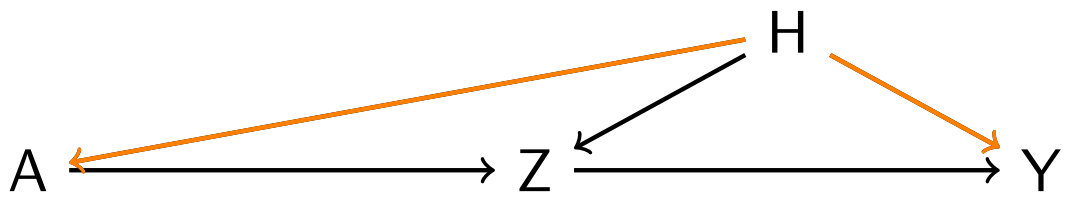
Practice



- $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ causal path

Practice



- $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ causal path
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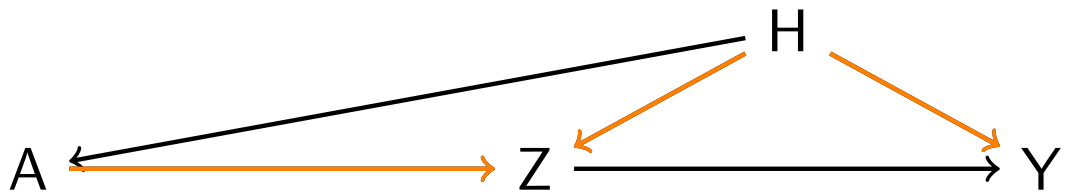

- $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ causal path
- $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ non-causal
- $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ non-causal

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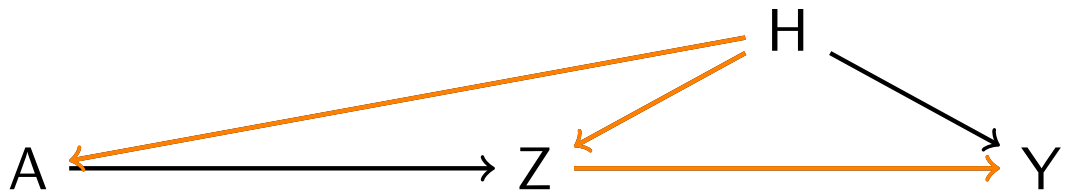

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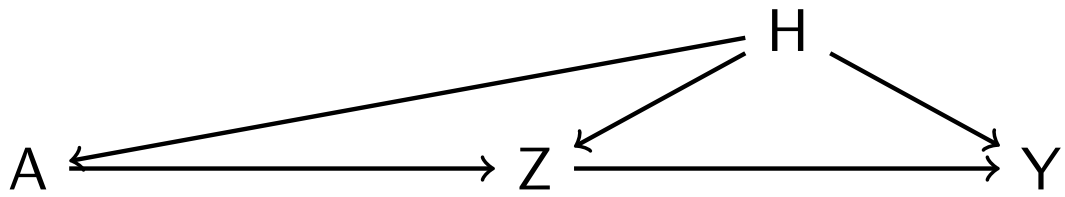

If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

- $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$
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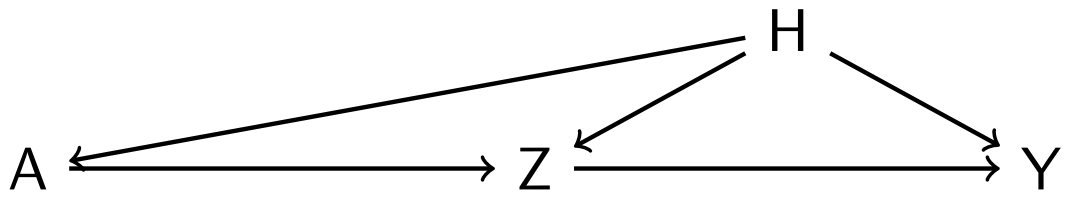

If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

- $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Open
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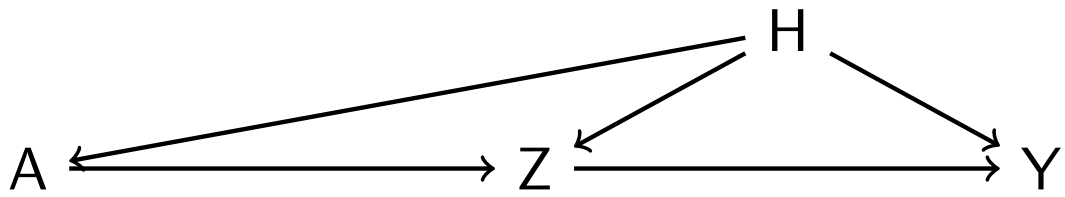

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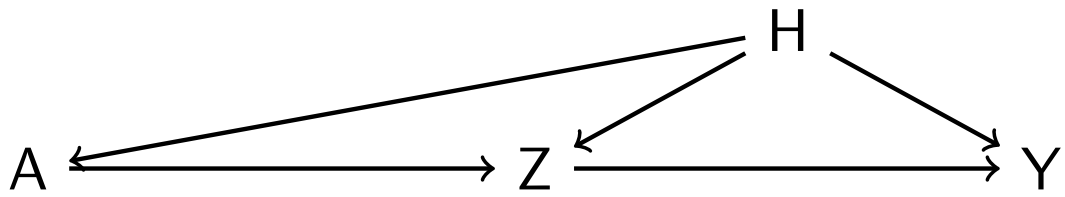

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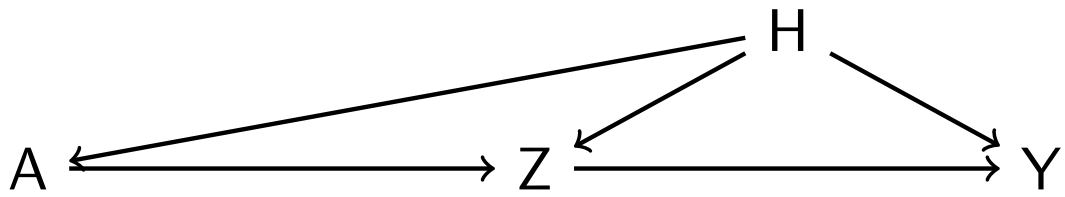

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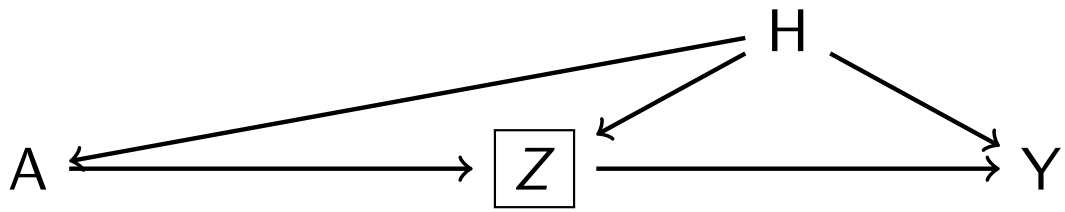
Practice



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

- $A \rightarrow \underbrace{\boxed{Z}}_{\text{NC}} \rightarrow Y$
- $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- $A \rightarrow \underbrace{\boxed{Z}}_{\text{CoI}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{\boxed{Z}}_{\text{NC}} \rightarrow Y$

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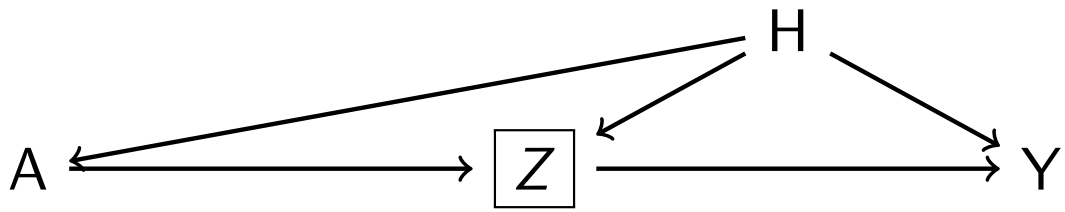

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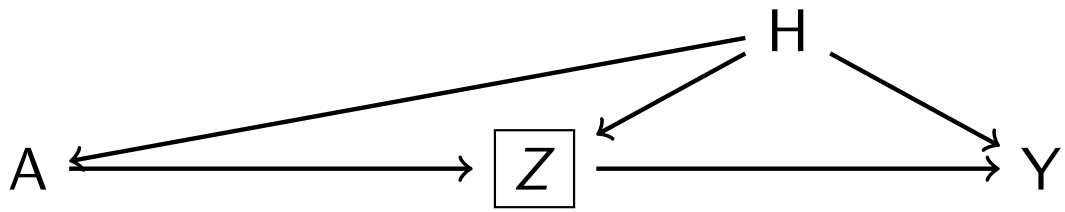

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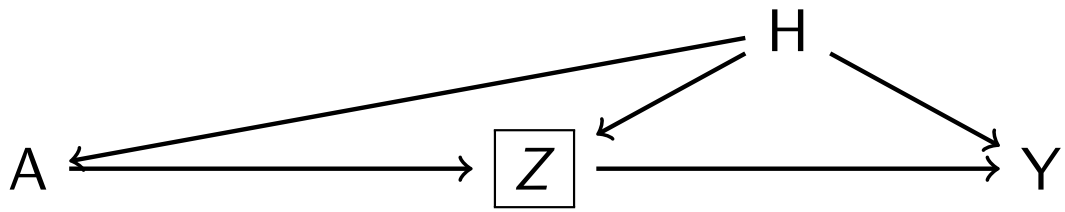

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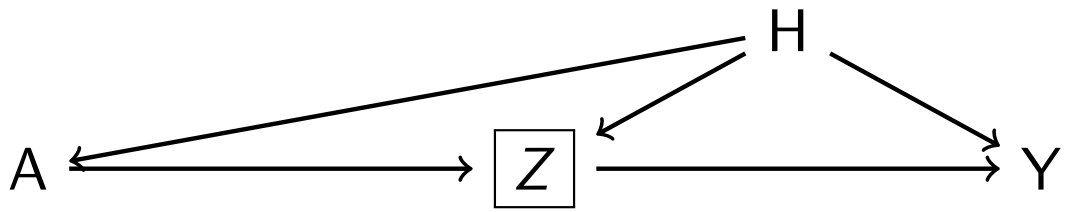

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Causal Discovery

- So far, we have assumed the DAG is known from expert knowledge
- DAG tells us about conditional independence we would observe in data

$$\text{DAG} \Rightarrow \text{Conditional independence in data}$$

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$$\text{DAG} \Rightarrow \text{Conditional independence in data}$$

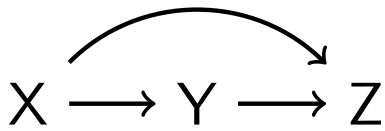
- Conditional independence is a observational quantity (i.e., not causal)
- Can be tested in observed data
- Can we go in the opposite direction?

$$\text{Conditional independence in data} \overset{?}{\Rightarrow} \text{DAG}$$

Causal Discovery

Can we tell which nodes are/aren't connected by an edge?

Causal Discovery



- $X \perp\!\!\!\perp Y$?
- $X \perp\!\!\!\perp Z$?
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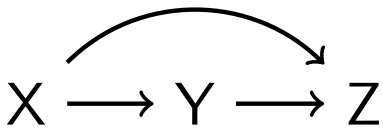

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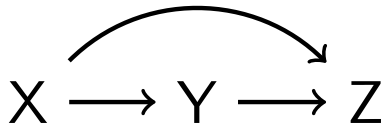
Causal Discovery



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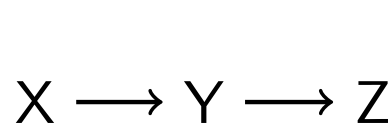

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- $X \perp\!\!\!\perp Y \mid Z$? No
- $Y \perp\!\!\!\perp Z \mid X$? No
- $X \perp\!\!\!\perp Z \mid Y$? Yes

If there is an edge between two nodes, they cannot be made conditionally independent!

Rule 1

- Start with (undirected) edges between every pair of nodes
- If you can find a set L such that $X \perp\!\!\!\perp Y \mid L$, take away the edge between X and Y

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Allows us to find where the edges are, but not necessarily direction

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Allows us to find where the edges are, but not necessarily direction

A **skeleton** is the DAG where we have made all edges undirected

$$\text{DAG} : X \to Y \to Z \qquad \text{Skeleton} : X - Y - Z$$

Causal Discovery

Can we also tell which direction an edge points?

Causal Discovery

$X \longrightarrow Y \longrightarrow Z$

- $X \perp\!\!\!\perp Y$? No
- $X \perp\!\!\!\perp Z$? No
- $Z \perp\!\!\!\perp Y$? No
- $X \perp\!\!\!\perp Y \mid Z$? No
- $Y \perp\!\!\!\perp Z \mid X$? No
- $X \perp\!\!\!\perp Z \mid Y$? Yes

$X \longrightarrow Y \longleftarrow Z$

- $X \perp\!\!\!\perp Y$?
- $X \perp\!\!\!\perp Z$?
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Causal Discovery

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Colliders can sometimes tell us the direction of an edge

Rule 2

- Suppose we have $X - Y - Z$ and no edge between X and Z
- Suppose $X \not\perp\!\!\!\perp Y \mid L$ for some set L that does not contain Y

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- Suppose $X \not\perp\!\!\!\perp Y \mid L$ for some set L that does not contain Y
- Then, $X \rightarrow Y \leftarrow Z$

Rule 2

- Suppose we have $X - Y - Z$ and no edge between X and Z
- Suppose $X \not\perp\!\!\!\perp Y \mid L$ for some set L that does not contain Y
- Then, $X \rightarrow Y \leftarrow Z$
- **Unshielded collider**: $X \rightarrow Y \leftarrow Z$ and X and Z do not have an edge

Causal Discovery

How far can we go?
Can we fully determine the graph from data?

Causal Discovery

$X \longrightarrow Y \longrightarrow Z$

- $X \perp\!\!\!\perp Y$? No
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- $Z \perp\!\!\!\perp Y$? No
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$X \longleftarrow Y \longleftarrow Z$

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- $X \perp\!\!\!\perp Z \mid Y$? Yes

Some graphs have the exact same set of conditional independence statements and cannot be distinguished from data alone!

Rule 3

Graphs have the same conditional independence statements if

- Same skeleton: edges in the same location, but possibly different direction (from Rule 1)
- Same unshielded colliders: $X \rightarrow Y \leftarrow Z$ and X and Z do not share an edge (from Rule 2)