# Censored regression, selection model, weak IV, and quantile regression

Tutorial 1

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### **Tutorials**

- 7 TA sessions
  - 6 TA sessions are about lecture material.
  - the last session is primarily about exam and remaining questions about the course material (TBA)
- Send me **any** questions you want to discuss before each TA session
  - use Canvas or send me an email (s.avdeev@tinbergen.nl)
  - alternately, leave your questions anonymously here: https://onlinequestions.org
     (enter the event code 18631)

## Assignments

- Due date: 11:59pm on Sundays (the first assignment is an exception: 11:59am on Tuesday)
- Assignments are graded within a week from the deadline
- Solutions will not be shared so if you want to discuss a specific exercise, let me know before the TA session (you submit your solutions on Sunday, thus, we can discuss any questions on the following TA session on Tuesday)

## Course objective

- The key objective of the course is **applying** microeconometric techniques rather than **deriving** econometric and statistical properties of estimators
- In other words, there's way less of this

$$ext{plim} \hat{eta}_{OLS} = eta + ext{plim} (rac{1}{N} X' X)^{-1} ext{plim} rac{1}{N} X' arepsilon = eta + Q^{-1} ext{plim} rac{1}{N} X' arepsilon$$

And way more of this

• If you would like to go deeper into the former, take Advanced Econometrics I and II next year

## Goal for today's tutorial

- 1. Use a tobit model to estimate censored data
- 2. Discuss a sample selection model and implement the selection mechanism
- 3. Work with strong and weak instrumental variables
- 4. Test instrumental variables
- 5. Work with a quantile regression and discuss inference tools

## Censored regression

- Censoring occurs when the value of a variable is limited due to some constraint
  - $\circ$  for example, we tend not to see some values of self-declared earnings with discrete categories (if you earn **at least** 3500 euro per month, write 3500)
- In this case OLS estimates are biased
  - a standard method to account for censoring is to combine a probit model with OLS,
     i.e. tobit model

## Censored regression: simulation

- The clearest way to understand how a certain estimator works is to generate data yourself so you know the true **data generating process** DGP
- Let's estimate returns to education: does education increase wages?
- Let's assume the following model

$$Y_i = \alpha + X_i' \beta_1 + U_i$$

where  $Y_i$  are monthly wages,  $X_i$  are years of education

- But suppose that we do not observe wages above a specific threshold (due to the features of a questionnaire, privacy concerns, coding, etc.)
  - how can we estimate the model in this case?
- First, we need to generate data containing years of education and wages

## Censored regression: simulation

education	wage_star	wage
5.062510	1837.496	1837.496
5.036464	1878.439	1878.439
5.260247	1889.825	1889.825
5.266903	1913.437	1913.437
14.988792	4081.234	3500.000
14.931072	4167.111	3500.000
14.849343	4174.288	3500.000
14.900142	4201.639	3500.000

## Censored regression: OLS

Now let's pretend that we do not know the DGP and simply apply OLS

ols\_model ← lm(wage ~ education, df)

	Model 1
(Intercept)	1255.542***
	(14.018)
education	167.686***
	(1.340)
Num.Obs.	1000
+ p < 0.1, * p < 0.05, **	* p < 0.01, *** p < 0.001

- ullet Using these OLS estimates, we would wrongly conclude that "an additional year of education is associated with 167.686 increase in monthly wages"
  - $\circ$  if we think that we **causally** identified the effect we'd say "an additional year of education **causes** 167.686 increase in monthly wages"

## Censored regression: tobit model

- But these are biased estimates since we know the true effect is 200 (remember DGP)
- Let's try to recover unbiased effects of education on wages by using a tobit model
- The solution provided by a tobit model is to
  - use a probit model to account for the censoring
  - estimate OLS on the non-censored data
- Tobit model estimator is easy to implement with censReg package

## Censored regression: tobit model

ullet Remember that we have right censored data: wages above 3500 are coded as 3500

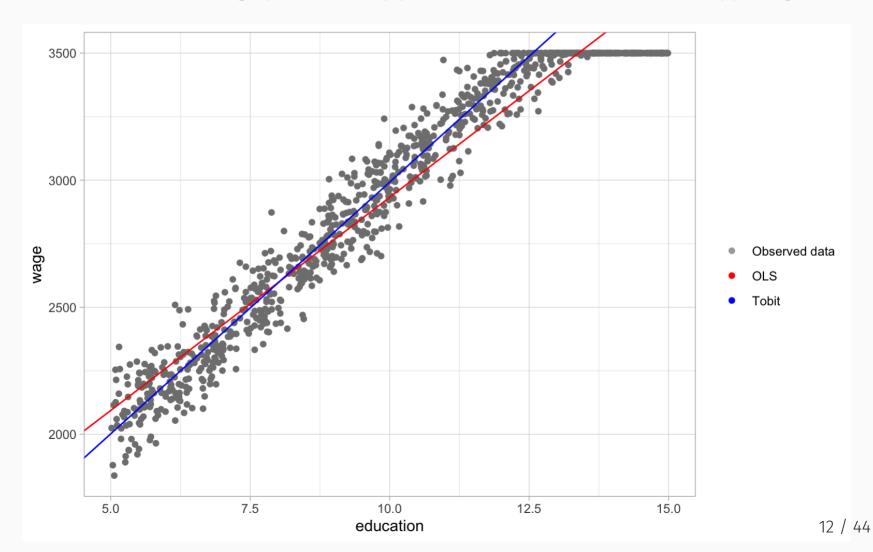
```
library(censReg)
tobit_model ← censReg(wage ~ education, data = df, right = 3500)
```

	Model 1	
(Intercept)	1011.629***	
	(13.998)	
education	198.053***	
	(1.495)	
logSigma	4.586***	
	(0.026)	
Num.Obs.	1000	
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.00		

• We recovered the **unbiased** estimates of returns to education

## Censored regression: graphically

• We will use a lot of graphs since they provide more intuition of what is happening



## Censored regression: some remarks

- You can specify both left and right censoring using censReg() function
- Important assumption of a tobit model is that the unobserved term is normally distributed (which is the case in our simulated data set)
- What if the data is missing not because the outcome variable is **above (or below)** some threshold but because individuals in the data have made a **choice** such that we can't observe their outcome variable?
- In this case censoring cannot be applied because the availability of data is **influenced** by the choice of agents
  - it is called **selection on unobservables**
  - it is a typical sample selection problem

## Sample selection model

- Let us consider the case of studying female's wages
  - usually, wages are observed for a fraction of women in the sample, whereas the remaining part of women are observed as unemployed or inactive
  - if we run an OLS regression using the observed wages, this would deliver consistent estimations only if working females are a **random sample** of the population
- However, theory of labor supply suggests that this may not be the case, since (typically) female labor supply is sensitive to household decisions
  - that is, female workers **self-select** into employment, and the self-selection is not random
  - this difference may lead us to underestimate the gender wage gap

## Sample selection model

ullet Suppose a female worker decides to work or not based on a latent variable  $I_i^*$  (say, utility derived from working), which depends on a set of observed  $Z_i$  and unobserved  $V_i$  characteristics

$$I_i^* = Z_i' \gamma + V_i$$

ullet The indicator function (decision to work or not), based on  $I_i^*$ , takes two values

$$I_i = egin{cases} 1 ext{ (working)} & ext{if } I_i^* > 0 \ 0 ext{ (not working)} & ext{if } I_i^* \leq 0 \end{cases}$$

• Suppose there is a latent outcome  $Y_i^*$ , i.e. wages of female workers, which depend on a set of observed  $X_i$  and unobserved  $U_i$  characteristics

$$Y_i^* = X_i' eta + U_i$$

ullet However, we observe wages only for females who decide to work:  $Y_i$  are observed wages

$$Y_i = egin{cases} Y_i^* & ext{if } I_i = 1 \ ext{missing} & ext{if } I_i = 0 \end{cases}$$

## Sample selection model: assumptions

- As always we need to have some assumptions, for example, in an OLS regression we usually assume  $U_i \sim \mathcal{N}(0,\sigma^2)$
- To estimate the sample selection model, we make an assumption that disturbances terms are bivariate normal

$$\left[egin{array}{c} U_i \ V_i \end{array}
ight] \sim \mathcal{N}\left(0, \left[egin{array}{ccc} \sigma^2 & 
ho\sigma \ 
ho\sigma & 1 \end{array}
ight]
ight)$$

 Note that the variance of the normal distribution is not identified in the probit model so it is set to 1

## Sample selection model: simulation

Let's simulate a data set with the selection mechanism

```
## # A tibble: 6 * 6
## z x uv[,1] [,2] i_star y_star y
## <dbl> <3.39
## 2 0.398 0.813 0.190 -0.283 2.20 3.28 3.28
## 3 0.116 0.278 -0.960 0.310 2.46 5.47 5.47
## 4 0.0697 0.968 -2.19 -1.84 1.46 1.25 1.25
## 5 0.244 0.247 -0.613 0.670 2.17 5.93 5.93
## 6 0.792 0.905 -2.65 -0.582 -2.61 2.70 0</pre>
```

## Sample selection model: simulation

ullet The true effect of Z on I (decision to work) is -5 and the effect X on Y (wages) is -3

```
selection_equation \leftarrow glm(I(y > 0) ~ z, df, family = binomial(link = "probit")) wage_equation \leftarrow lm(y ~ x, df)
```

	Model 1	Model 2
(Intercept)	3.013***	4.575***
	(0.178)	(0.137)
Z	-3.770***	
	(0.251)	
X		-2.111***
		(0.239)
Num.Obs.	1000	1000
+ p < 0.1, * p < 0.	05, ** p < 0.01	, *** p < 0.001

ullet Clearly, the estimates are biased since  $\mathrm{cov}(U_i,V_i)
eq 0$ 

## Sample selection model: Heckman

- ullet To solve the sample selection problem, one needs to use the **Heckman selection model** (left as an exercise in the  $oldsymbol{1}^{st}$  assignment)
  - the Heckman estimator is very similar to the Tobit estimator
  - the difference is that this estimator allows for a set of characteristics that
     determine whether or not the outcome variable is censored

#### IV

- The basic idea of the IV estimator is to
  - use the instrument to predict treatment
  - use that predicted treatment to predict the outcome
- We need a separate equation for each of these steps
  - $\circ$  **First stage**: predict treatment  $X_1$  with the instrument Z and a control variable  $X_2$

$$X_1=\gamma_0+\gamma_1Z+\gamma_2X_2+V$$

 $\circ$  **Second stage**: use these predictions to predict Y

$$Y = \beta_0 + \beta_1 \hat{X}_1 + \beta_2 X_2 + U$$

• Notice that we need to use all exogenous variables in both stages

#### IV: conditions

- For the IV to work, we need two things to hold
  - Validity: the instrument must actually be exogenous (or at least exogenous after adding controls)

$$cov(Z, U) = 0$$

• **Relevance**: the instrument must be a strong predictor of the treatment. It can't be trivial or unimportant

$$\operatorname{cov}(X_1,Z) 
eq 0$$

## IV: small-sample bias

- IV is actually a **biased** estimator
  - the mean of its sampling distribution is not the population parameter
  - it would be the population parameter at infinite sample size, but we don't have that
  - in small samples, the **bias of IV** is

$$rac{\mathrm{cov}(Z,U)}{\mathrm{cov}(X_1,Z)} = rac{\mathrm{corr}(Z,U)}{\mathrm{corr}(X_1,Z)} rac{\sigma_U}{\sigma_{X_1}}$$

- ullet If Z is valid, then in infinite samples  $\mathrm{cov}(Z,U)=0$  and this goes away
- But in a non-infinite sample, it will be nonzero by chance, inducing some bias
- The bias is smaller
  - $\circ$  the stronger the relationship between  $X_1$  and Z
  - the smaller the sum of squared errors
  - $\circ$  the bigger the variation in  $X_2$
  - the bigger the sample
- What happens when  $\mathrm{corr}(X_1,Z)$  is small?

#### Weak IV

- ullet If Z has only a trivial effect on X, then it's not **relevant** (even if it's truly **exogenous**)
  - our **small-sample bias** will be big (remember the formula on the previous slide)
- Thus, weak IV means that we probably shouldn't be using IV in small samples
  - $\circ$  this also means that it's really important that  $\mathrm{corr}(X_1,Z)$  is not small
- There are rules of thumb for how strong IV must be to be counted as "not weak"
  - t-statistic above 3
  - $\circ$  F-statistic from a joint test of the instruments that is 10 or above
- These rules of thumb aren't great
  - selecting a model on the basis of significance naturally biases your results
  - $\circ$  what you really want is to know the **population effect** of Z on  $X_1$  you want the F-statistic from that to be bigger than 10. Of course, we don't actually know that

#### Weak IV: estimation

- There are a bunch of ways to do the IV analysis
  - the classic one is ivreg() in the AER package
- Other functions are more fully-featured, including robust SEs, clustering, and fixed effects

```
    feols() in fixest
    felm() in lfe
    tsls() in sem
    multile functions in ivpack
```

• We'll be using feols() from fixest

• Let's create a data set with an instrument

z1	u1	х1	у1
2.2872472	1.2465863	6.8585432	26.8085610
-1.1967717	-0.7655089	-5.4047210	-20.0417075
-0.6942925	0.2161769	-0.3578382	0.0073698
-0.4122930	-0.3643673	-1.9102661	-7.5526346

```
# The true effect is 3
library(fixest)
ols_model ← lm(y1 ~ x1, df)
iv_model ← feols(y1 ~ 1 | x1 ~ z1, df, se = 'hetero')
```

	Model 1	Model 2
x1	4.171***	
	(0.009)	
fit_x1		3.479***
		(0.348)
Num.Obs.	1000	1000
+ p < 0.1, * p < 0	.05, ** p < 0.0°	1, *** p < 0.001

- ullet  $Z_1$  is a pretty effective instrument even if the correlation between  $Z_1$  and  $X_1$  is small
  - $\circ$  check validity:  $\operatorname{corr}(Z_1,U_1)=$  0.027, pretty close to zero
  - $\circ$  check relevance:  $\mathrm{corr}(X_1,Z_1)=$  0.068, so it's a weak instrument

- Remember that usually we can't test the **validity** assumption when we have one instrument, but we know the DGP in this case
- ullet Now let's see what happens when there is a small correlation between Z and U
- ullet Imagine there is some additional explanatory variable V which is unobserved but partially explains the instrument

```
set.seed(7)  
df \leftarrow tibble(v = rnorm(1000), \\ z2 = -v + rnorm(1000), \\ u2 = 0.1*v + rnorm(1000), \\ x2 = 0.2*z2 + 4*u2 + rnorm(1000), # all coefficients stay the same here \\ y2 = 3*x2 + 5*u2) # and here
```

v	z2	u2	х2	y2
2.2872472	-1.0406609	1.6434732	7.567896	30.92105
-1.1967717	0.4312628	-2.2230083	-9.868396	-40.72023
-0.6942925	0.9104694	-1.1531165	-5.119197	-21.12317
-0.4122930	0.0479257	-0.4115676	-3.494073	-12.54006

```
# The true effect is 3 0ls_{model2} \leftarrow lm(y2 \sim x2, df) iv_{model2} \leftarrow feols(y2 \sim 1 \mid x2 \sim z2, data = df, se = 'hetero')
```

	Model 1	Model 2
x2	4.163***	
	(0.009)	
fit_x2		0.944
		(4.365)
Num.Obs.	1000	1000
+ p < 0.1, * p < 0	0.05, ** p < 0.0°	1, *** p < 0.001

- In this case we get a better estimate by using the OLS estimator than by using IV
- Why? Because of the weak instrument problem and the bias
  - $\circ$  check validity:  $\mathrm{corr}(Z_2,U_2)=$  -0.036
  - $\circ$  check relevance:  $\operatorname{corr}(X_2,Z_2)=$  0.021

ullet These results are primarily a function of the weakness of Z at explaining X. Let's see what happens if Z has more explanatory power

v	z2	u2	хЗ	y2
2.2872472	-1.0406609	1.6434732	4.654045	22.17950
-1.1967717	0.4312628	-2.2230083	-8.660860	-37.09762
-0.6942925	0.9104694	-1.1531165	-2.569883	-13.47523
-0.4122930	0.0479257	-0.4115676	-3.359881	-12.13748

```
# The true effect is 3 ols_model3 \leftarrow lm(y2 \sim x3, df) iv_model3 \leftarrow feols(y2 \sim 1 \mid x3 \sim z2, data = df, se = 'hetero')
```

	Model 1	Model 2
х3	3.578***	
	(0.020)	
fit_x3		2.956***
		(0.036)
Num.Obs.	1000	1000
+ p < 0.1, * p < 0	.05, ** p < 0.0	1, *** p < 0.001

- ullet Even though the correlation between Z and U is the same as previously, the strength of the instrument in explaining X wins out and gives us a better estimator than OLS
  - $\circ$  check validity:  $\operatorname{corr}(Z_2,U_2)=$  -0.036
  - $\circ$  check relevance:  $\operatorname{corr}(X_3,Z_2)=$  0.697

#### Weak IV: F-test

• Let's look at the F-test from the output of feols()

```
## TSLS estimation, Dep. Var.: y2, Endo.: x3, Instr.: z2
## Second stage: Dep. Var.: y2
## Observations: 1,000
## Standard-errors: Heteroskedasticity-robust
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.006571 0.162236 -0.040501 0.9677
## fit_x3 2.955645 0.036370 81.266072 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## RMSE: 5.12118 Adj. R2: 0.939727
## F-test (1st stage), x3: stat = 941.5, p < 2.2e-16, on 1 and 998 DoF.
## Wu-Hausman: stat = 8,467.5, p < 2.2e-16, on 1 and 997 DoF.</pre>
```

- 941.48 is way above 10
- $\bullet$  Lee, D., et al. (2021) discuss the potentially severe large-sample distortions from using conventional value of the F-test equal to 10 and they suggest to use as a rule of thumb the minimum value of the F-test equal to 104.7, which is needed to ensure a test with a significance level 0.05

#### Weak IV: overidentification test

- Overidentification just means we have more identifying conditions (validity assumptions) than we actually need
  - we only need one instrument, but we have two (or more)
  - so we can compare what we get using each instrument individually
- If we assume that **at least one of them is valid**, and they both produce similar results, then that's evidence that **both** are valid

#### Weak IV: overidentification test

• We can do this using fitstat() in fixest

## Sargan: stat = 248.7, p < 2.2e-16, on 1 DoF.

• The null hypothesis of the **Sargan test** is that the covariance between the instruments and the error term is zero

$$corr(Z, U) = 0$$

- Thus, rejecting the null indicates that at least one of the instruments is not valid
- So we reject the null, indicating that one of the instruments is endogenous (although without seeing the true DGP we couldn't guess if it were  $Z_1$  or  $Z_2$ )

## Weak IV: overidentification test

• The true effect is 2

```
iv1 \leftarrow feols(y \sim 1 \mid x \sim z1, df, se = 'hetero')

iv2 \leftarrow feols(y \sim 1 \mid x \sim z2, df, se = 'hetero')
```

	Model 1	Model 2
(Intercept)	0.029	0.012
	(0.043)	(0.049)
fit_x	2.058***	2.912***
	(0.044)	(0.046)
Num.Obs.	1000	1000
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001		

## Quantile regression

• Consider a very simple OLS

$$Y_i = \alpha + D_i' \beta_1 + U_i$$

where  $Y_i$  is an outcome variable,  $D_i$  is a treatment variable

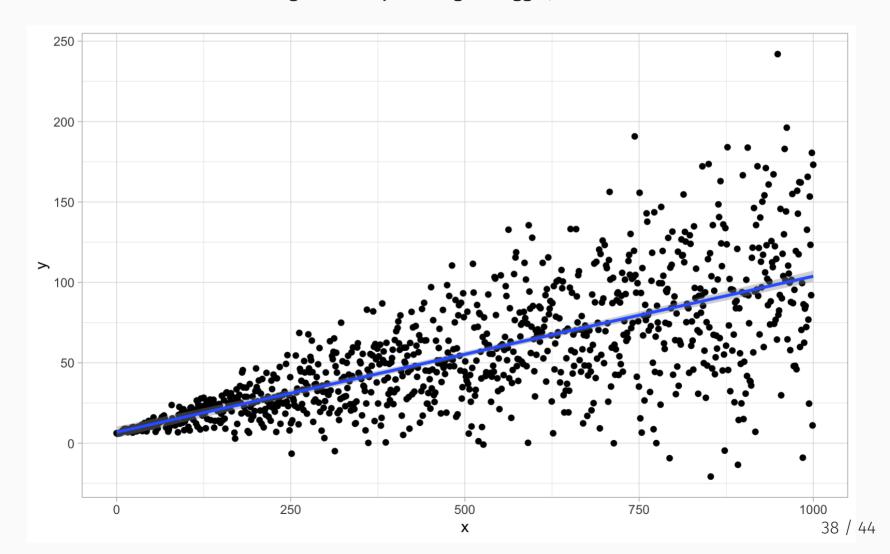
- ullet What is the interpretation of the effect of  $D_i$  on  $Y_i$ ?
  - it is the expected change in the outcome for a person moving from untreated to treated
  - in other words, it characterizes the **mean** of our outcome variable

## Quantile regression

- What if we care about other things but the mean?
  - what are the effects of a subsidized insurance policy on medical expenditures for people with low-, medium-, and high- expenditures?
  - what are the effects of a training program on employment opportunities for people with different years of education?
- Quantile regression can handle such questions
- Quantile regression also solves problems with
  - skewed variables no more worrying about logs or outliers in the outcome variable
  - censoring
- But it has its own issues
  - it is noisier
  - it is challenging to interpret in an intuitive way
- If you have underlying theory that has implications for distribution of the effects, the quantile regression is the right tool for empirical analysis

• Let's simulate a data set with normal random errors with a non-constant variance

ullet We can see the increasing variability: as X gets bigger, Y becomes more variable



- ullet The estimated mean of an OLS regression 0.097 is still unbiased
  - $\circ$  but it doesn't tell us much about the relationship between X and Y
  - $\circ$  especially as X gets larger
- ullet To perform quantile regression, use the <code>quantreg</code> package and specify tau a quantile

```
library(quantreg)
qr ← rq(y ~ x, df, tau = 0.9)

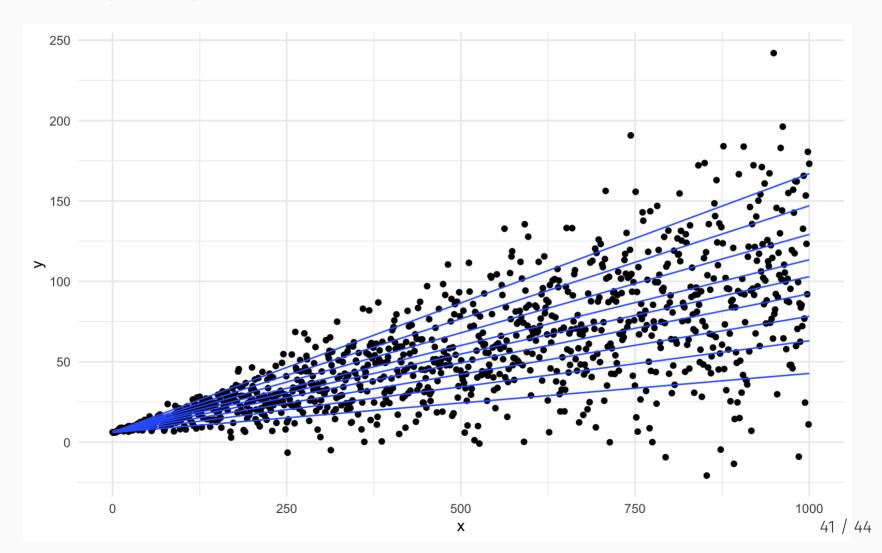
##
## Call: rq(formula = y ~ x, tau = 0.9, data = df)
##
## tau: [1] 0.9
##
## Coefficients:
## coefficients lower bd upper bd
## (Intercept) 6.28754 5.97387 7.43619
## x 0.16066 0.15581 0.16472
```

ullet The X coefficient estimate of 0.161 says that "one unit increase in X is associated with 0.161 increase in the  $90^{
m th}$  quantile of Y"

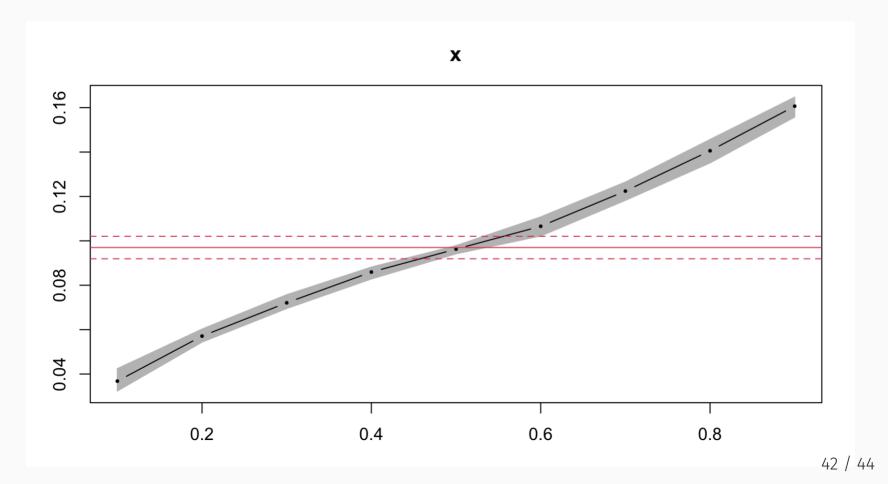
• Let's look at different quantiles at once

• The intercept estimates don't change much but the slopes steadily increase

• Let's plot our quantile estimates



- Each black dot is the slope coefficient for the quantile indicated on the x axis
- The red lines are the OLS estimate and its confidence interval
- Lower and upper quantiles are well beyond the OLS estimate



## Quantile regression: inference

- There are several alternative methods of conducting inference about quantile regression coefficients
  - rank-inversion confidence intervals: summary.rq(qr)
  - o more conventional standard errors: summary.rq(qr, se = "nid")
  - bootstraped stanard errors: summary.rq(qr, se = "boot")
- To read more about calculating confidence intervals, use ?summary.rq

#### References

#### Books

- Huntington-Klein, N. The Effect: An Introduction to Research Design and Causality, Chapter 19: Instrumental Variables
- Cunningham, S. Causal Inference: The Mixtape, Chapter 7: Instrumental Variables

#### Slides

- Huntington-Klein, N. Econometrics Course, Week 8: Instrumental Variables
- Goldsmith-Pinkham P. Applied Empirical Methods Course, Week 7: Linear Regression III:
   Quantile Estimation

#### **Articles**

• Lee, D. S., McCrary, J., Moreira, M. J., & Porter, J. R. (2021). Valid t-ratio Inference for IV (No. w29124). National Bureau of Economic Research