Text as Data

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Naive Bayes and General Problem Setup

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Apply model to test data, classify those observations

Naive Bayes and General Problem Setup (Jurafsky Inspired Slide)

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Simple intuition about Naive Bayes:

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Simple intuition about Naive Bayes:

- Learn what documents in class j look like
- Find class k that document i is most similar to

Naive Bayes and Unigram Language Models

Assume the following data generating process (should look familiar)

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\begin{aligned}
\boldsymbol{\pi} &\sim \text{Dirichlet}(\boldsymbol{\alpha}) \\
\boldsymbol{\theta} &\sim \text{Dirichlet}(\boldsymbol{\lambda}) \\
\boldsymbol{\tau}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}) \\
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& \propto \overbrace{\widehat{\pi_k}}^{p(C_k)} \underbrace{\prod_{j=1}^{J} \left(\widehat{\theta}_{jk}\right)^{x_{ij}}}_{\text{Unigram model}}
\end{aligned}
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Some R Code

```
library(e1071)
dep<- c(labels, rep(NA, no.testSet))
dep<- as.factor(dep)
out<- naiveBayes(dep~., as.data.frame(tdm))
predicts<- predict(out, as.data.frame(tdm[-training.set,]))
```

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Basic intuition:

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Basic intuition:

- Examine joint distribution of characteristics (without making Naive Bayes like assumption)
- Focus on distributions (only) makes this analysis possible

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- 2^J possible vectors

ReadMe: Optimization for a Different Goal (Hopkins and King 2010)

Measure only presence/absence of each term [(Jx1) vector]

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\end{aligned}$$

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$$\underbrace{P(\boldsymbol{x})}_{2^J x 1} = \underbrace{P(\boldsymbol{x}|C)}_{2^J x K}\underbrace{P(C)}_{K x 1}$$

Matrix algebra problem to solve, for $P(C)$

Like Naive Bayes, requires two pieces to estimate

Complication $2^J >> $ no. documents

Kernel Smoothing Methods (without a formal model)

- $P(\boldsymbol{x})$ = estimate directly from test set
- $P(\boldsymbol{x}|C)$ = estimate from training set
  - Key assumption: $P(\boldsymbol{x}|C)$ in training set is equivalent to $P(\boldsymbol{x}|C)$ in test set
- If true, can perform biased sampling of documents, worry less about drift...

Algorithm Summarized

- Estimate $\hat{p}(\boldsymbol{x})$ from test set
- Estimate $\hat{p}(\boldsymbol{x}|C)$ from training set
- Use $\hat{p}(\boldsymbol{x})$ and $\hat{p}(\boldsymbol{x}|C)$ to solve for $p(C)$

Assessing Model Performance

Not classifying individual documents $\rightarrow$ different standards
Mean Square Error :

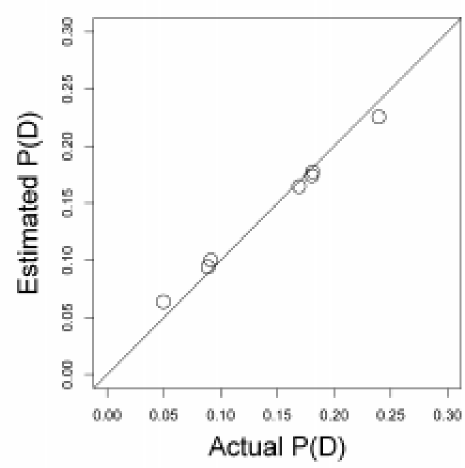
$$\mathrm{E}[(\hat{\theta} - \theta)^2] = \mathrm{var}(\hat{\theta}) + \mathrm{Bias}(\hat{\theta}, \theta)^2$$

Suppose we have true proportions $P(C)^{\text{true}}$. Then, we'll estimate Root Mean Square Error

$$\text{RMSE} = \sqrt{\frac{\sum_{j=1}^{J} (P(C_j)^{\text{true}} - P(C_j))}{J}}$$

$$\text{Mean Abs. Prediction Error} = \left|\frac{\sum_{j=1}^{J} (P(C_j)^{\text{true}} - P(C_j))}{J}\right|$$

Visualize: plot true and estimated proportions



Estimated P(D)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Actual P(D)

TABLE 1 Performance of Our Nonparametric Approach and Four Support Vector Machine Analyses

	Percent of Blog Posts Correctly Classified			
	In-Sample Fit	In-Sample Cross-Validation	Out-of-Sample Prediction	Mean Absolute Proportion Error
Nonparametric	—	—	—	1.2
Linear	67.6	55.2	49.3	7.7
Radial	67.6	54.2	49.1	7.7
Polynomial	99.7	48.9	47.8	5.3
Sigmoid	15.6	15.6	18.2	23.2

Notes: Each row is the optimal choice over numerous individual runs given a specific kernel. Leaving aside the sigmoid kernel, individual classification performance in the first three columns does not correlate with mean absolute error in the document category proportions in the last column.

Using the House Press Release Data

Method	RMSE	APSE
ReadMe	0.036	0.056
NaiveBayes	0.096	0.14
SVM	0.052	0.084

Code to Run in R

Control file:

filename	truth	trainingset
20July2009LEWIS53.txt	4	1
26July2006LEWIS249.txt	2	0

```
tdm<- undergrad(control=control, fullfreq=F)
process<- preprocess(tdm)
output<- undergrad(process)
output$est.CSMF ## proportion in each category
output$true.CSMF ## if labeled for validation set (but not
used in training set)
```