A laboratory Fourier Synthesiser Using Hybrid (analog/digital) techniques
(Square Wave)

A Report Submitted to the Faculty of the
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATION ENGINEERING

<u>A laboratory Fourier Synthesiser</u>
<u>Using Hybrid (analog/digital) techniques</u>
(<u>Square Wave</u>)

Submitted by

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Introduction:

The purpose of the work described in this report is to develop a piece of electronic hardware that could be used in a communications laboratory to demonstrate convincingly the principles of Fourier Analysis . A primary goal is to create a laboratory instrument of sufficient precision and flexibility to generate a wide variety of waveshapes.

Formulas :

$$f(t) = a_o + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

where the coefficients, a_n and b_n, can be calculated as follows:

$$a_o = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(t)\, dt$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(t) \cos n\omega_o t\, dt$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(t) \sin n\omega_o t\, dt$$

where T is the period of repetition and

$$\omega_o = \frac{2\pi}{T}, \text{ and } n = 1, 2, 3, \ldots$$

Theory : It can be shown that the Fourier series converges to f(t), except at points of discontinuity, if f(t) satisfies the Dirichlet conditions . Any physically realisable signal satisfies the strong Dirichlet condition by remaining finite in amplitude and containing only a finite number of minima and maxima per period.

The real time Fourier Synthesizer described in this report allows the operator to generate any waveform that can be described by the following equation -

$$f(t) = a_o + \sum_{n=1}^{12} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

where: -10 volt < a < +10 volt and -10 volt < b < +10 volt
A choice had to be made between designing an instrument to

synthesise a Fourier series in the sine-cosine form previously shown or its alternate form represented by the following -

$$f(t) = C_o + C_n \sum_{n=1}^{\infty} \cos(n\omega_o t + \theta_n)$$

$$\text{where } C_n = \sqrt{a_n^2 + b_n^2}$$

$$C_o = a_o$$

$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

The amplitude coefficient, C , is very easy to synthesise with analog computing circuitry but the phase, θ , is difficult to synthesise, with suitable accuracy The major source of phase error would be the active RC bandpass filter circuits.

Fourier Series--Square Wave

Since the Square wave function f(x) is odd , so $a_0 = a_n = 0$, and

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

reduces to

$$\begin{aligned} b_n &= \frac{2}{L}\int_0^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx \\ &= \frac{4}{n\pi}\sin^2\left(\tfrac{1}{2}n\pi\right) \\ &= \frac{2}{n\pi}[1-(-1)^n] \\ &= \frac{4}{n\pi}\begin{cases} 0 & n \text{ even} \\ 1 & n \text{ odd.} \end{cases} \end{aligned}$$

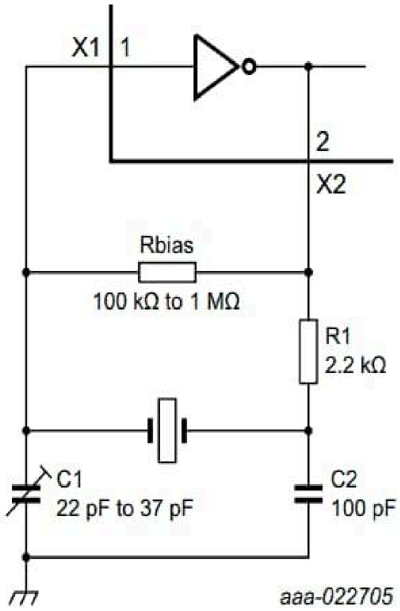
The Fourier series is therefore

$$x(t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4}{n\pi}\sin(n\omega_0 t)$$

A simplified block diagram for the basic system is shown in Below Fig . The 5 harmonically related frequencies are derived from a single stable reference oscillator with digital counter circuits

A sine wave with amplitude proportional to the potentiometer , is generated by rejecting all harmonic Fourier components of the

square wave with a bandpass filter tuned to the fundamental frequency of the square wave The phase, θ, of the sine wave is determined by the relative amplitudes of the cosine coefficient, a^n, and the sine coefficient, b^n , The outputs of the 5 bandpass
filters are summed to be displayed on an oscilloscope.

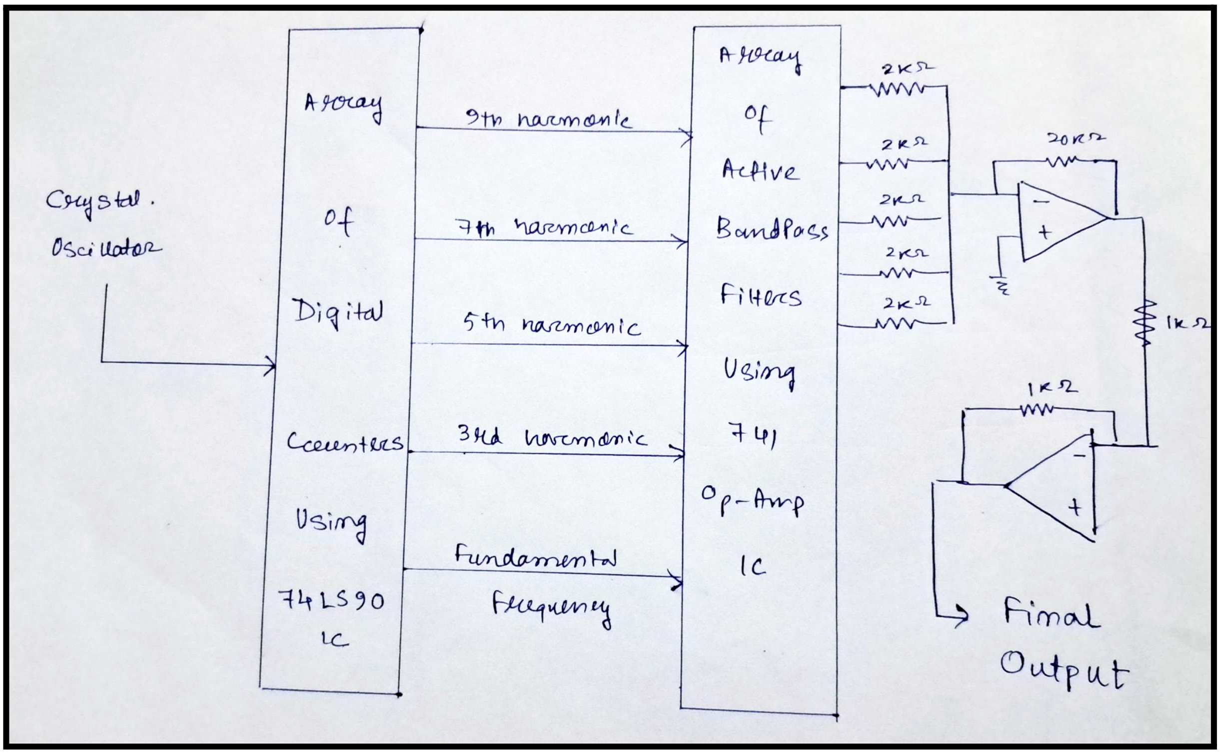


Apparatus List :

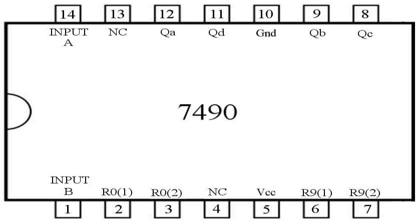
- Resistors (330 , 10M , 22.6 K , 100K , 620 , 1K ohm)
- Crystal Oscillator (11.2896 MHz)
- Capacitors (33pF , 0.33 uF , 0.1uF , 0.056 uF , 0.01 uF , 0.047 uF , 0.033 uF)
- Veroboard
- IC 741 (Opamp IC) , IC 7490 (Decade Counter IC) , IC 7408 (And Gate IC)
- Single strand wire

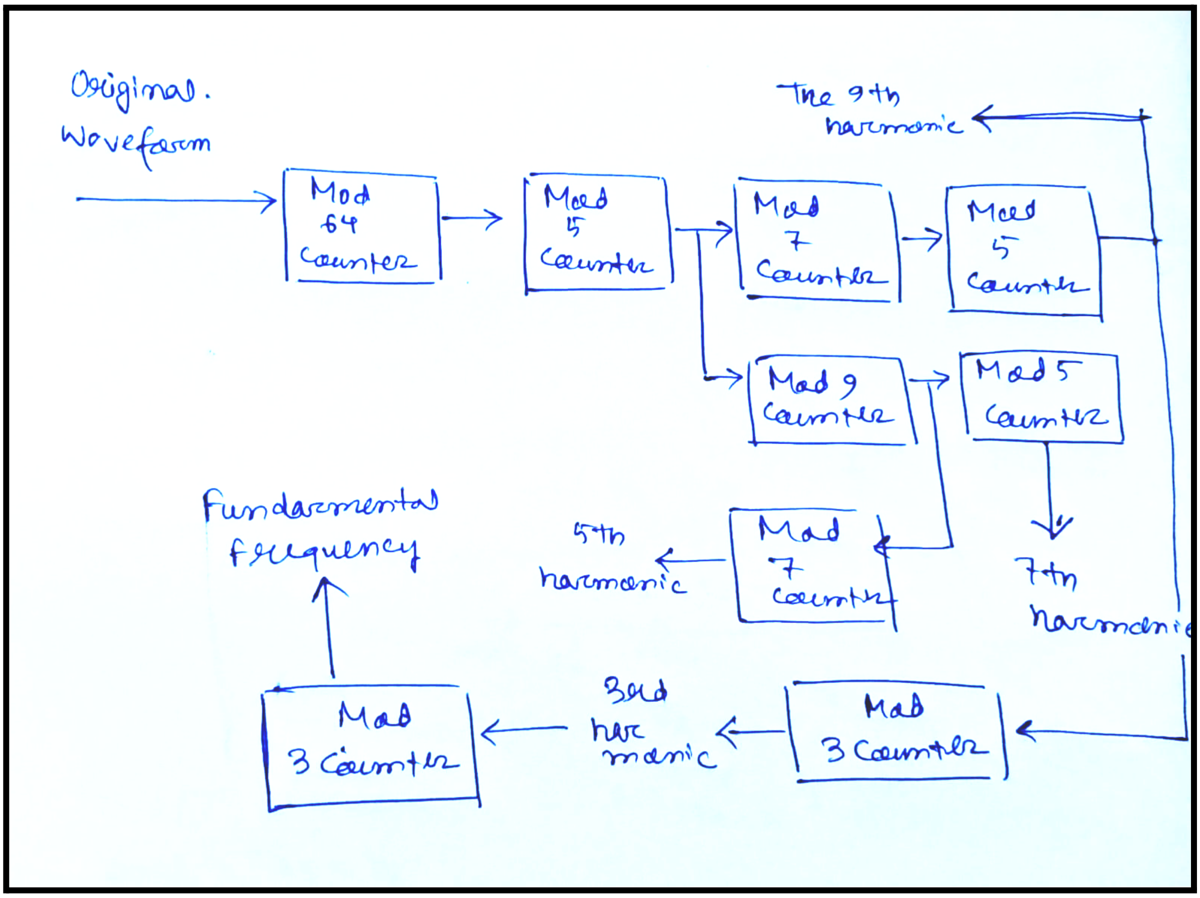
Oscillator selection :-

The crystal oscillator frequency value is chosen to accommodate all odd harmonics and to keep it possible to separate them from each other using counters while keeping the fundamental frequency at 112 Hz and keeping focus on market availability of oscillators of different values.



Digital Counter Design : We utilised the versatile IC 7490 to construct a series of counters with moduli of 5, 9, 3, and 7. These counters effectively divide the original waveform into multiple frequency-specific segments. By passing these segmented waveforms through active bandpass filters, we selectively isolate the odd sine harmonics of the fundamental frequency while excluding all other unwanted harmonics. This meticulous design ensures that only the desired frequency components are retained, resulting in a refined output signal free from unwanted interference





Bandpass Filter

The input to the active RC bandpass filter is a composite square wave, that is, composed of the sum of many square waves of same amplitude but ofdifferent frequency. Since the bandpass filter represented by the transfer function

$$H(s) = \frac{E_2}{E_1} = \frac{\alpha H_o \omega_o s}{s^2 + \alpha \omega_o S + \omega_o^2}$$

where

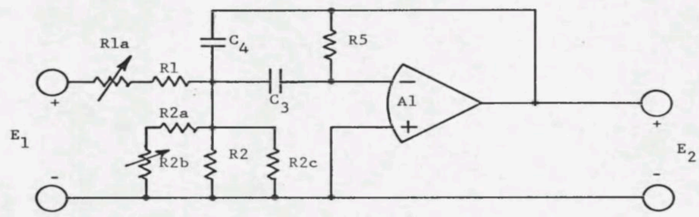
$$s = j\omega \,, \quad Q = \frac{1}{\alpha}, \quad \omega_o = 2 f_o, \quad \left.\frac{E_2}{E_1}\right|_{S=j\omega_o} = H_o$$

The multiple feedback bandpass active filter that realises the desired transfer function was chosen for the small number of components required The low impedance of its operational amplifier output allows external circuits to be driven if required . The transfer function is

$$\frac{E_2}{E_1} = \frac{-\,s\,\frac{1}{CR1}}{s^2 + s\,\frac{2}{CR5} + \frac{1}{C^2 R5}\left(\frac{1}{R1} + \frac{1}{R2}\right)}$$

where $C = C_3 = C_4$, $s = j\omega$

To optimise the performance of the bandpass filters, meticulous attention was paid to the adjustable parameters, particularly R2 and R1. Given that R2 predominantly influences Q and phase without impacting gain, it was outfitted with a trimmer for fine-tuning the centre frequency or phase adjustment. This adjustment proved crucial, as the centre frequency is inversely proportional to capacitance (denoted as G) facilitated the creation of nearly identical bandpass filters for each harmonic, differing only in capacitance values. Conversely, R1 significantly influences transfer gain, thus requiring careful calibration for ensuring uniform amplitude matching between harmonics. By implementing trimmers for both R1 and R2 adjustments, we achieved precise control over the bandpass filter characteristics, enhancing the fidelity and consistency of waveform synthesis.

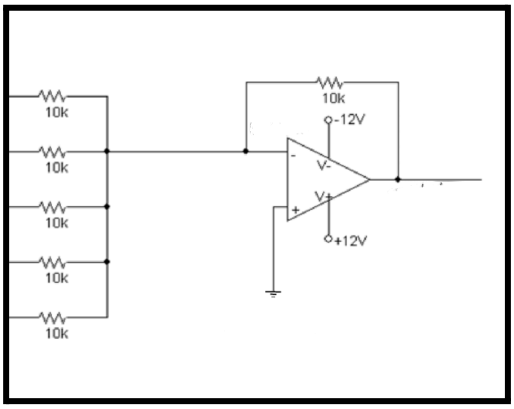


A1	B-B 3050
R1	22.6K
R1a	20 K Pot
R2	332 Ω
R2a	10 K
R2b	2 K Pot
R2c	hand select
R5	100 K

Harmonic Number	$C = C_3 = C_4$ (μf)
1	.33
3	.10
5	.056//.01
7	.047
9	.033

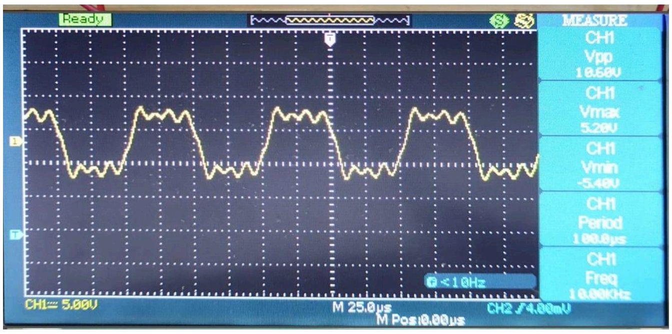
Summing Network :

In our setup, we incorporated two operational amplifier ICs. The first op-amp serves to summate the outputs originating from the active bandpass filters. Meanwhile, the second op-amp functions as an active inverting buffer, effectively restoring the original polarity of the summed signal obtained from the adder circuit. Through this process, we achieve the generation of the desired square waveform by amalgamating all the odd sine harmonics derived from an oscillator. This comprehensive approach ensures the accurate reconstruction of the intended waveform, culminating in a precise output representation.



OUTPUT:

Square Wave



Coefficients for Square Wave :

$b_1 = 10.00$

$b_3 = 3.33$

$b_5 = 2.00$

$b_7 = 1.47$

$b_9 = 1.11$

(All units in Volt)

<u>Conclusion</u> :

In conclusion, our experiment employing IC 7490 counters, active bandpass filters, and operational amplifier circuits proved instrumental in achieving our objective of isolating and reconstructing specific frequency components from an original waveform. By strategically dividing the waveform into frequency-specific segments using the counters and filtering out unwanted harmonics with active bandpass filters, we successfully extracted the odd sine harmonics of the fundamental frequency. The subsequent combination of these harmonics through op-amp circuits facilitated the generation of the desired square waveform. Through meticulous design and implementation, we demonstrated the effectiveness of this approach in signal processing and waveform manipulation, underscoring its potential applicability in various engineering and scientific domains.

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