

I used a random seed to ensure results were predictable. I tested several seeds, but had trouble finding one with a validation error under .6 on the ensemble model. Most seeds resulted in an ensemble training error of 0, but occasional increased to .1.

## Output

Model 1

```
[[0, 0, 1, '+'],  
 [0, 0, 1, '+'],  
 [1, 1, 0, '-'],  
 [1, 0, 1, '+'],  
 [1, 1, 0, '-'],  
 [1, 0, 0, '+'],  
 [1, 1, 0, '-'],  
 [1, 1, 0, '-'],  
 [1, 0, 1, '+'],  
 [1, 1, 0, '-']]
```

Tree 1

root: 1

left child: +

right child: -

Model 2

```
[[1, 0, 0, '+'],  
 [0, 1, 1, '-'],  
 [0, 0, 0, '+'],  
 [0, 1, 0, '+'],  
 [1, 0, 0, '+'],  
 [0, 0, 0, '+'],  
 [0, 1, 0, '+'],  
 [1, 0, 1, '+'],  
 [0, 1, 0, '+'],  
 [0, 0, 1, '+']]
```

Tree 2

root: 2

left child: +

right child: 1

left child: +

right child: -

Model 3

```
[[0, 0, 1, '+'],
```

[1, 0, 1, '+'],  
[1, 0, 0, '+'],  
[0, 1, 1, '-'],  
[1, 1, 0, '-'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 1, 0, '-'],  
[1, 1, 0, '-'],  
[0, 1, 0, '+']]

Tree 3

root: 1

left child: +

right child: 0

left child: 2

left child: +

right child: -

right child: -

Model 4

[[0, 1, 0, '+'],  
[1, 0, 0, '+'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 0, 1, '+'],  
[0, 1, 1, '-'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 0, 0, '+'],  
[0, 1, 0, '+']]

Tree 4

root: 1

left child: +

right child: 0

left child: 2

left child: +

right child: -

right child: -

Model 5

[[1, 1, 0, '-'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 1, 0, '-'],  
[1, 0, 1, '+'],  
[1, 0, 0, '+'],  
[0, 1, 1, '-'],

[0, 0, 0, '+'],  
[0, 1, 0, '+'],  
[1, 1, 0, '-']]

Tree 5

root: 1

left child: +

right child: 0

left child: 2

left child: +

right child: -

right child: -

Model 6

[[0, 0, 0, '+'],  
[0, 1, 0, '+'],  
[1, 1, 0, '-'],  
[0, 1, 1, '-'],  
[1, 0, 1, '+'],  
[0, 0, 1, '+'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 0, 0, '+'],  
[1, 1, 0, '-']]

Tree 6

root: 1

left child: +

right child: 0

left child: 2

left child: +

right child: -

right child: -

Model 7

[[1, 0, 1, '+'],  
[0, 1, 0, '+'],  
[0, 1, 1, '-'],  
[1, 1, 0, '-'],  
[1, 1, 0, '-'],  
[0, 0, 0, '+'],  
[1, 1, 0, '-'],  
[1, 0, 0, '+'],  
[1, 1, 0, '-'],  
[0, 0, 1, '+']]

Tree 7

root: 1

left child: +

right child: 0  
left child: 2  
left child: +  
right child: -  
right child: -

#### Model 8

[[1, 1, 0, '-'],  
[0, 0, 1, '+'],  
[1, 0, 0, '+'],  
[1, 0, 1, '+'],  
[1, 1, 0, '-'],  
[0, 1, 1, '-'],  
[0, 0, 0, '+'],  
[0, 1, 0, '+'],  
[1, 1, 0, '-'],  
[1, 0, 1, '+']]

#### Tree 8

root: 1  
left child: +  
right child: 0  
left child: 2  
left child: +  
right child: -  
right child: -

#### Model 9

[[1, 0, 1, '+'],  
[0, 1, 1, '-'],  
[1, 0, 0, '+'],  
[0, 0, 1, '+'],  
[1, 0, 1, '+'],  
[1, 0, 0, '+'],  
[1, 0, 0, '+'],  
[1, 1, 0, '-'],  
[1, 1, 0, '-'],  
[1, 1, 0, '-']]

#### Tree 9

root: 1  
left child: +  
right child: -

#### Model 10

[[1, 1, 0, '-'],  
[1, 0, 0, '+'],  
[0, 0, 0, '+'],

[0, 0, 1, '+'],  
[1, 0, 0, '+'],  
[0, 0, 1, '+'],  
[1, 1, 0, '-'],  
[0, 1, 1, '-'],  
[1, 0, 1, '+'],  
[0, 0, 1, '+']]

Tree 10

root: 1

left child: +

right child: -

Ensemble training error: 0.0

Ensemble validation error: 0.6

## Source Code

```
#####  
# Author: John Soderstrom  
# Due: 5/1/2020  
#  
# Creates an ensemble model (composed of a given number of models)  
# to classify data.  
#  
# Data is expected to be in the format [0, 1, ... , "+"] with n columns  
# where columns from 1 to n-1 are 0 or 1, and column n is "+" or "-".  
#  
# Once all models are generated, using testEnsemble(data) on the Bagging object  
# will query each data point on all models and get the majority classification.  
# When complete, it will return the error rate (0 - 1) of the given data  
# on the ensemble model.  
#  
# Use print() on the bagging object to print out all trees used to  
# form the ensemble and the data used to form it.  
#  
# Printed trees used indentation to show the depth, and label  
# each node as the root, or a left/right child of its parent.  
# The labels are printed with them, where a number indicated the  
# column/attribute used to split the node, and a classification (+/-)  
# means the line has ended and that is the result of the decision tree.  
#####  
  
import math, random, pprint  
  
###  
# Contains a list of decision trees used to form an ensemble model.  
# Requires the original training data, number of rounds or models to form,  
# and a random seed to make sure each run is predictable.  
#  
# Stores the data used to create each tree for easy access with them.  
###  
class Bagging:  
    ###  
    # Set random seed, store original training data, and prepare  
    # to create a given number of trees from rounds.  
    ###  
    def __init__(self, trainingData, rounds, randomSeed):  
        random.seed(randomSeed)  
        self.trainingData = trainingData  
        self.rounds = rounds  
        self.trees = []
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self.randomData = []
self._generateTrees()

###
# Called on initialization, creates a given number of trees.
# After each is finished, it will get the training error
# and if error is over 50% (.5), it will create a replacement.
# Successful trees are stored.
###
def _generateTrees(self):
    for i in range(self.rounds):
        error = 1
        # Repeat until given a tree with success rate over 50%.
        while error >= .5:
            randomData = self.baggingData()
            temp = Tree(self.randomData[i])
            error = temp.getError(self.trainingData)
            self.trees.append(temp)

###
# After trees are completed, test given data on all trees/ensemble model.
# Tallies a majority vote from each tree for each data point, then checks
# the vote against the actual classification.
###
def testEnsemble(self, data):
    error = 0
    # Test each point 1 by 1
    for i in data:
        numPlus = 0
        numMinus = 0
        # Tally classification results from each tree
        for j in self.trees:
            label = j.query(i)
            if label is "+":
                numPlus += 1
            else:
                numMinus += 1
        # Get the majority vote and test classification
        label = ""
        if numMinus > numPlus:
            label = "-"
        else:
            label = "+"
        if i[-1] is not label:
            error += 1
        print("Actual Classification: '{}' Ensemble Guess: '{}".format(i[-1], label))

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    # Return error rate of the model on given data
    return error / len(data)

####
# Randomly select points from original training data to use in
# each tree's creation. Uniform random distribution from bagging notes.
####
def baggingData(self):
    newData = []
    num = len(self.trainingData)
    for count, i in enumerate(self.trainingData):
        index = random.randint(0, num - 1)
        newData.append(self.trainingData[index])
    self.randomData.append(newData)

####
# Print model information including the randomized data used to create
# it and the model itself.
####
def print(self):
    for i in range(self.rounds):
        print("Model {}".format(i + 1))
        pprint.pprint(self.randomData[i])
        print("Tree {}".format(i + 1))
        self.trees[i].print()
        print()

####
# Given a set of training data, Tree will create a decision tree.
# Expects all lists in data to have the same length. The final column
# should be a +/- classifier. All other columns should contain 0/1.
####
class Tree:
    ####
    # Sets the root node for the tree and passes data to build.
    ####
    def __init__(self, data):
        attCols = [i for i in range(len(data[0]) - 1)]
        self.root = Node(-1, 0, None, None)
        rootLabel = self.majority(data)
        self.buildTree(self.root, data, attCols, rootLabel)

    ####
    # Build a full decision tree starting from the root node.
    # Continues each line until a stopping condition is met.
    # Tests for stopping conditions are checked through endLine.

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# Narrows down list of columns to check and data as the tree splits.
###
def buildTree(self, node, data, cols, parLabel):
    # Check if this node is an end
    if self.endLine(node, data, cols, parLabel):
        return;

    # Check gain for each input column not already used
    bestCol = -1
    bestSplit = -1
    for i in cols:
        newSplit = self.branchGain(data, i)
        if newSplit > bestSplit:
            bestCol = i
            bestSplit = newSplit
    # Remove the column number for later splits
    cols.remove(bestCol)

    # Separate data in this node by value on split
    dataLeft = []
    dataRight = []
    # References to elements in data are preferred here, no changes made
    for i in data:
        # Always move values of 0 in best column to left child
        if i[bestCol] is 0:
            dataLeft.append(i)
        # Always move values of 1 in best column to right child
        else:
            dataRight.append(i)

    # Assign proper label to the current node, then create children
    node.label = bestCol
    node.gain = bestSplit
    node.left = Node(-1, 0, None, None)
    node.right = Node(-1, 0, None, None)

    # Get majority label in case a child has no data
    parLabel = self.majority(data)

    # Data references are split, column removed from further checking
    self.buildTree(node.left, dataLeft, cols, parLabel)
    self.buildTree(node.right, dataRight, cols, parLabel)

###
# Test for ways to split the current node
# If no data, use the parent's majority label

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# If there is no way to split, use the current node's majority label
####
def endLine(self, node, data, cols, parLabel):
    # If there is no data, use the parent's label and return
    if len(data) is 0:
        node.label = parLabel
        return True

    # If there are attributes and columns to split on,
    # do not end the line.
    if self.testAttribute(data, cols):
        return False

    # If no columns remain to split, get the majority vote.
    # In the event of a tie, it will be "+"
    newLabel = self.majority(data)

    # Assign label to final node in the line
    node.label = newLabel
    return True

####
# Generates the gain of a node split by a given attribute
# When comparing gain values, the highest result is the best choice.
# data expects a list with at least 2 columns
# col expects an integer between 0 and length of list in data minus 1
####
def branchGain(self, data, col):
    # List 0 is left node, list 1 is right node
    # First entry in either node is "-", second is "+"
    branch = [[0, 0],
               [0, 0]]
    for i in data:
        # Set class to add for to "-" by default
        incClass = 0
        if i[-1] is "+":
            incClass = 1
        # Add to the chosen node, then by class
        branch[i[col]][incClass] += 1

    # Generate entropy for both children nodes
    childError = []
    for i in branch:
        totNode = i[0] + i[1]
        # Prevent an error when a child has no data
        if totNode is 0:

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        childError.append(0)
    else:
        first = i[0] / totNode
        second = i[1] / totNode
        err = 0
        # Prevent errors when a child is pure
        if first > 0:
            err -= first * math.log(first)
        if second > 0:
            err -= second * math.log(second)
        childError.append(err)

# Get values to weight entropy
leftSum = sum(branch[0])
rightSum = sum(branch[1])
totalSum = len(data)

# Subtracted weighted entropy from 1 to get gain for the current node on that split
return 1 - (leftSum * childError[0] + rightSum * childError[1]) / totalSum

####
# Returns the classification with the majority of remaining data
# In event of a tie, "+" is used.
# Data is expected to have already been checked for length > 0
####
def majority(self, data):
    if len(data) is 0:
        print("Error: no data to get majority of")
        return -1
    numPlus = 0
    numMinus = 0
    for i in data:
        if i[-1] is "+":
            numPlus += 1
        else:
            numMinus += 1
    if numMinus > numPlus:
        return "-"
    else:
        return "+"

####
# Test if there are attributes and classifications to
# split data on for another step in the decision tree.
####
def testAttribute(self, data, cols):

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# Test presence of both + and -
# Fails test if all classifications are one option
# There is nothing to split in that case
numPlus = 0
numMinus = 0
for i in data:
    if i[-1] is "+":
        numPlus += 1
    else:
        numMinus += 1
if numPlus is 0 or numMinus is 0:
    return False

# Test presence of an attribute to split on in all remaining columns
# If no columns have values to split on, tell endLine to end it
for i in cols:
    pres0 = False
    pres1 = False
    for j in data:
        if j[i] is 0:
            pres0 = True
        else:
            pres1 = True
    if pres0 and pres1:
        return True

# Even if there are classifications to split, if no column has
# multiple attributes, there is no way to split it.
return False

###
# On a complete decision tree, test data on it. Returns the error
# rate from 0 to 1.
###
def getError(self, data):
    error = 0
    # Query each data point one by one and test for misclassification
    # Increment error count if it was misclassified.
    for i in data:
        label = self.query(i)
        if label is not i[-1]:
            error += 1
    # Convert error to a decimal
    return error / len(data)

###

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# Given a single data point, query the decision tree to get
# the result, either "+" or "-"
####
def query(self, data):
    currentNode = self.root
    # Continue through decision tree until at an end node
    while currentNode.label is not "+" and currentNode.label is not "-":
        # Go to left child on 0, right child on 1
        if data[currentNode.label] is 0:
            currentNode = currentNode.left
        else:
            currentNode = currentNode.right
    # Return the final result of the query
    return currentNode.label

####
# Preorder print starting from the root node. Node's print
# will travel through the remaining nodes in the tree.
####
def print(self):
    self.root.print()

####
# Node class stores access to left and right child
# as well as the label the node splits on (column number),
# or +/- if the line ends.
#
# Print functionality expects starting at the root (as the
# tree class automatically does), and prints each node in
# preorder, showing the label and indenting at each depth.
####
class Node:
    ####
    # Label should be a column number or classification
    # shown in last column of the data.
    # left and right give access to children.
    ####
    def __init__(self, label, gain, left, right):
        self.label = label
        self.gain = gain
        self.left = left
        self.right = right

    ####
    # Preorder print from given starting node (called root,
    # even if it's just for a subtree) and its children.

```

```

###
def print(self, loc = "root", depth = 0):
    line = "{} {}: {}".format(" " * depth, loc, self.label)
    print(line)
    if(self.left):
        self.left.print("left child", depth + 1)
    if(self.right):
        self.right.print("right child", depth + 1)

# Columns: (index = Instance) A, B, C, Class
#  A,B,C = 0 or 1, Class = +, -
training = [[0, 0, 0, "+"],
            [0, 0, 1, "+"],
            [0, 1, 0, "+"],
            [0, 1, 1, "-"],
            [1, 0, 0, "+"],
            [1, 0, 0, "+"],
            [1, 1, 0, "-"],
            [1, 0, 1, "+"],
            [1, 1, 0, "-"],
            [1, 1, 0, "-"]]
validation = [[0, 0, 0, "+"],
              [0, 1, 1, "+"],
              [1, 1, 0, "+"],
              [1, 0, 1, "-"],
              [1, 0, 0, "+"]]

results = Bagging(training, 10, 67)
results.print()
print("Ensemble training error: {}".format(results.testEnsemble(training)))
print("Ensemble validation error: {}".format(results.testEnsemble(validation)))

```