

109065711 朱高如

#### Homework 4 (計算方法設計 · Design and Analysis of Algorithms)

註: 所有的作業皆以紙本的方式, 在截止日以前繳交給助教(台達館 737 或 738 室), 請注意不接受遲交。 All homework assignments should be submitted to the TAs (Room 737 or 738 at Delta Building) as hard copy (handwriting or paper printout) by the due date. Please note that late assignment submissions will not be accepted.

**Due date: May 21, 2021**

1. (30%) Refer to Section 5-6 in the text book for the definition of the personal assignment problem. Consider the following example:  $P = \{P_1, P_2, P_3, P_4\}$  with  $P_1 < P_2 < P_3 < P_4$ ,  $J = \{J_1, J_2, J_3, J_4\}$ , the partial ordering of  $J$  is  $J_1 \leq J_3$ ,  $J_1 \leq J_4$  and  $J_2 \leq J_3$ , and the cost matrix  $C_{ij}$  is shown as follows.

Job Person	1	2	3	4
	1	2	3	4
1	18	13	10	15
2	3	21	7	9
3	32	30	26	28
4	29	19	17	12

Please use the branch and bound strategy to solve this instance of the personal assignment problem. Note that in your answer, you need to also draw your searching tree for the given instance.

2. (70%) Use a programming language you familiar with to implement the brute force method and the branch and bound algorithm, both of which were already introduced in the class, for solving the traveling salesperson problem (50%). Compare their running time when the input size  $n$  is from 5 to 15 in steps of 1 (20%). Note that for each  $n$ , you should generate three problem instances and average the running time of solving these three instances. To verify



the accuracy of your program, please take an  $n \times n$  cost matrix (corresponding to a complete graph of  $n$  cities) as the input of your program and produce a shortest Hamiltonian cycle and its total cost as the output.

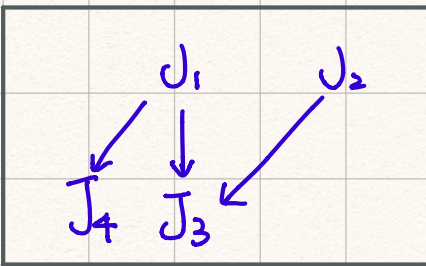


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1.



A Partial Ordering of Jobs

$J_1 \rightarrow J_2 \rightarrow J_3 \rightarrow J_4$

$J_1 \rightarrow J_2 \rightarrow J_4 \rightarrow J_3$

$J_1 \rightarrow J_4 \rightarrow J_2 \rightarrow J_3$

$J_2 \rightarrow J_1 \rightarrow J_3 \rightarrow J_4$

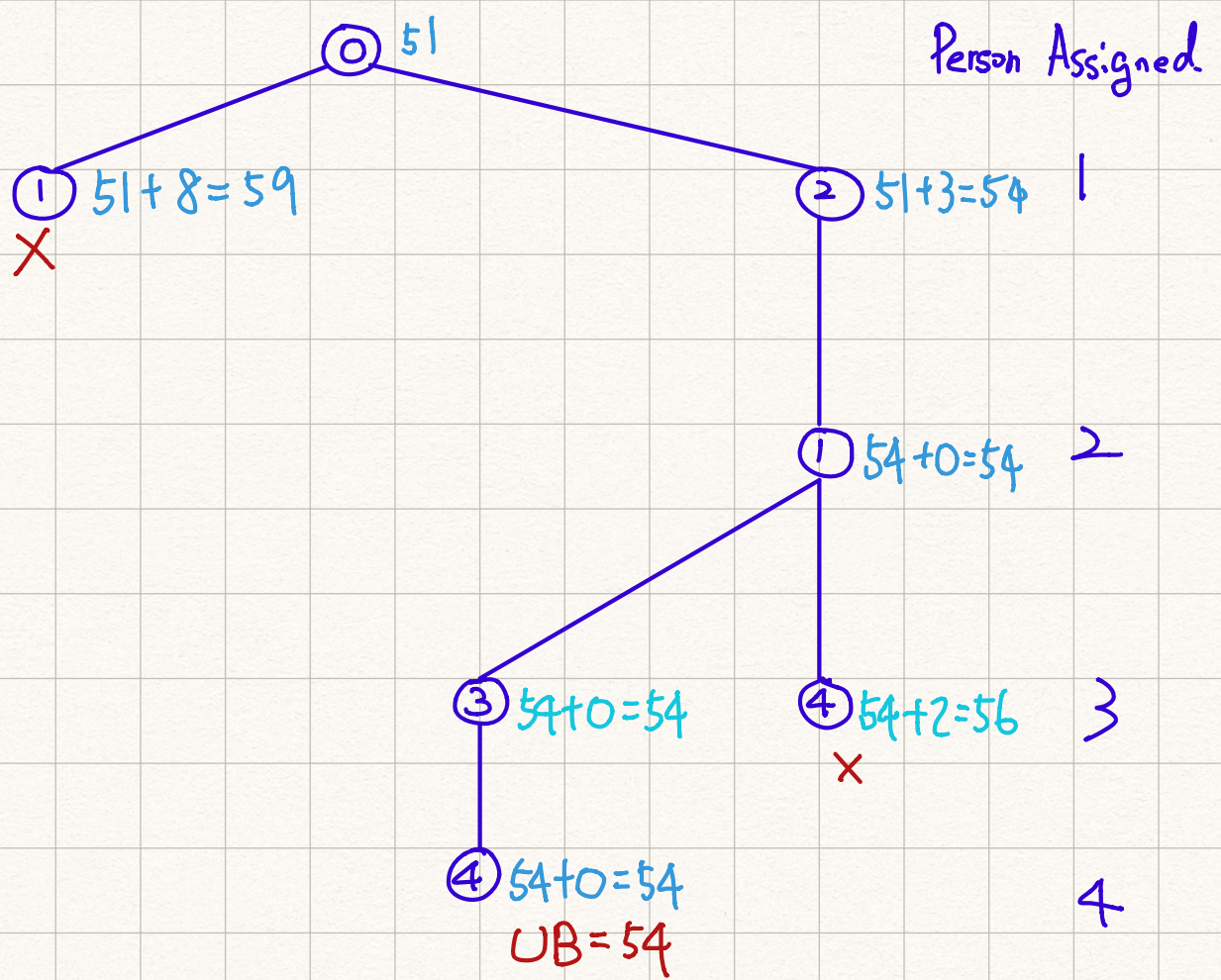
$J_2 \rightarrow J_1 \rightarrow J_4 \rightarrow J_3$

The Topologically Sorted Sequences

Jobs \ Person	1	2	3	4	Total: 51
1	8	3	0	5	-10
2	0	18	4	6	-3
3	6	4	0	2	-26
4	17	7	5	0	-12

TABLE 1. A Reduced Cost Matrix





Ans:  $P_1 \rightarrow J_2$   $P_2 \rightarrow J_1$   $P_3 \rightarrow J_3$   $P_4 \rightarrow J_4$