Metric Space Q Practice

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1 Questions from Shane Sirs Slides

Proof. Q1
To prove: $A \subset f^{-1}(f(A))$ where f is $X \to Y$ and $A \subset X$
Proof: if $a \in A$
$f(A) = \{b : b \in B | a \in A\}$ Where f(A)=B
$f^{-1}(S) = \{x : x \in X | y \in S\}$ Where $S \subseteq Y$
$f^{-1}(f(A)) = \{a : a \in A | b \in f(A)\}$
Therefore a in $f^{-1}(f(A))$ when $a \in A$. Therefore $A \subset f^{-1}(f(A))$

Proof. For inverse function:
T.P $f : A \to B$ be a function. Then f is bijective if and only if the inverse relation f^{-1} is a function from B to A.
Proof: Since f is a function $\forall a \in A, \exists! b \in B : f(a) = b$
But the relation $f^{-1} := \{(b, a) : (a, b) \in R\}$. Therefore for f^{-1} to be a function
$\forall b \in B, \exists! a \in A : f{-}1(b) = a$
$(b, a) \in f^{-1} \implies (a, b \in f)$. Since b is any element of B, f is onto
Assume $f(a_1) = f(a_2) = b$ then implies that $(b, a_1), (b, a_2) \in f^{-1}$
Since $\forall b \in B, \exists! a \in A$. It follows that since $f^{-1}(b) = a_1$ and $f^{-1} = a_2$, $a_2 = a_1$. Therefore f is one-one.

Proof. Q2.
T.P : $f : X \to Y$ is continuous iff for every sequence $a_n \in X$ converging to l the sequnce $f(a_n) \in Y$ converges to f(l)
Proof:
For continutiy of a function $f : X \to Y$:
Given an $\epsilon > 0, \exists \delta > 0 : d_X(x, y) < \delta \implies d_Y(f(x), f(y)) < \epsilon$
$x_n \to l := \forall \epsilon \exists N : n > N, d(x_n, l) < \epsilon$
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