Predictive Information in a Sensory Population

Project Report for the Complexity Science Course at the University of Potsdam in WiSe 2021/22 held by Prof. Karoline Wiesner

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February 2022

It is important for living beings to predict changes in their surroundings. These predictions are possible because of information that living organisms receive from their sensory organs. In this context, the information, that can be used for the prediction, is called predictive information. If, for example, a ball is flying towards a soccer goal, the goalkeeper can predict the target point using the predictive information he receives from the trajectory of the ball.

In this project, the paper [Pal+15] was studied, which aims to clarify whether the retinal ganglion cells of a salamander start to calculate the prediction and how much predictive information the retinal cells carry about the future states of their own cell activity and consequently also about the surrounding of the salamander. The goals of this project are to find out if the results can be reproduced with a computer model and to discuss which results of the paper can be attributed to the retinal cells and which results may have a different origin.

Quantifying predictive information In general the predictive information can be measured by the mutual information between the past and future values of a time series [BNT01] (e.g. a time series of positions of an object). The mutual information is therefore the amount of information obtained from one random variable (e.g. the (future) position X of an object) by observing another random variable (e.g. the (past) neural signal W of a retina cell) [CT06]. This mutual information can be calculated by the equation 1.

$$I(W_t; X_{t'}) = \sum_{w_t, x_{t'}} P_W(w_t) P(x_{t'}|w_t) \log_2 \left(\frac{P(x_{t'}|w_t)}{P_X(x_{t'})} \right) \tag{1}$$

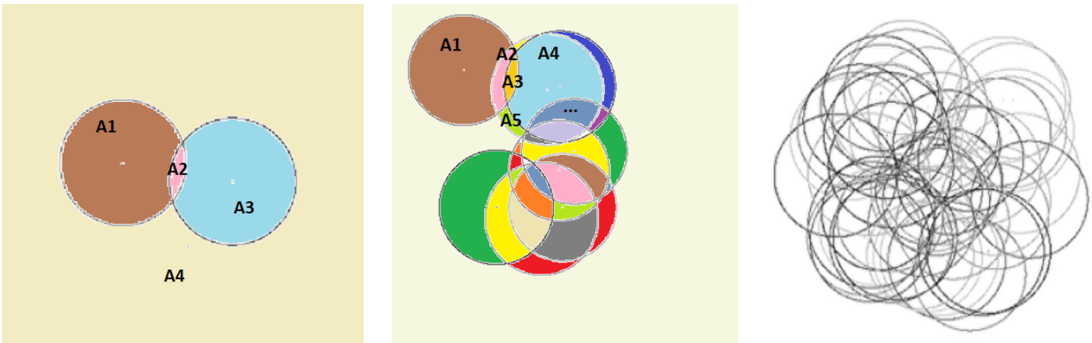
Methods and simulation setup In the original paper [Pal+15], videos with a moving bar were shown to the retina of a salamander and the electrical signals of the retina cells were measured. The position at time t of the 11x1 pixel bar was calculated by the equation 2. The iteration formula of the velocity of the bar (formula 3) can be derived by using Newton's law, a restoring force, a friction force and a random force. Here ξ_t is a Gaussian random variable with zero mean and unit variance, ω the frequency, Γ the damping, m

the mass[1], which is assigned to the bar, the timestep Δt and D, a dynamic parameter.

$$x_{t+1} = x_t + \dot{x}_t \Delta t \tag{2}$$

$$\dot{x}_{t+1} = \dot{x}_t + \Delta \dot{x}_t = \left(1 - \frac{\Gamma}{m}\right) \dot{x} - \frac{\omega^2}{m} x \Delta t + \frac{\sqrt{D \Delta t}}{m} \xi_t \tag{3}$$

The neural signals w_t at time t are called words and consist of ones and zeros, which indicate whether a retinal cell has fired or has not reacted. For our simulation, we define for every simulated retina cell a position with a radius, which define a circular receptive field. If the moving bar is inside this receptive field, we get a one from this cell. The simulated cells are randomly arranged around zero and thus form together a sensory population which can determine the position of the bar. The inaccuracy of the position decreases with the increase of the number of cells, as the distinguishable areas A_n of the array increase in quantity and decrease in size. (1).



(a) color-coded distinguishable areas A_n for 2 simulated retinal cells

(b) color-coded distinguishable areas A_n for 8 simulated retinal cells

(c) circular receptive fields of 50 simulated retinal cells

Figure 1: Field of view with receptive fields for a different number of simulated cells. For a higher number of cells, the sensor array becomes more precise because of the higher number of distinguishable areas A_n

This model is based on the shape and arrangement of the receptive fields of the biphasic OFF cells which were studied in [Mar+12] and shown in figure 2.

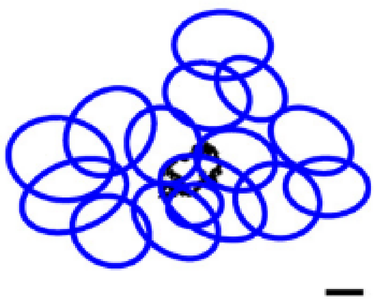
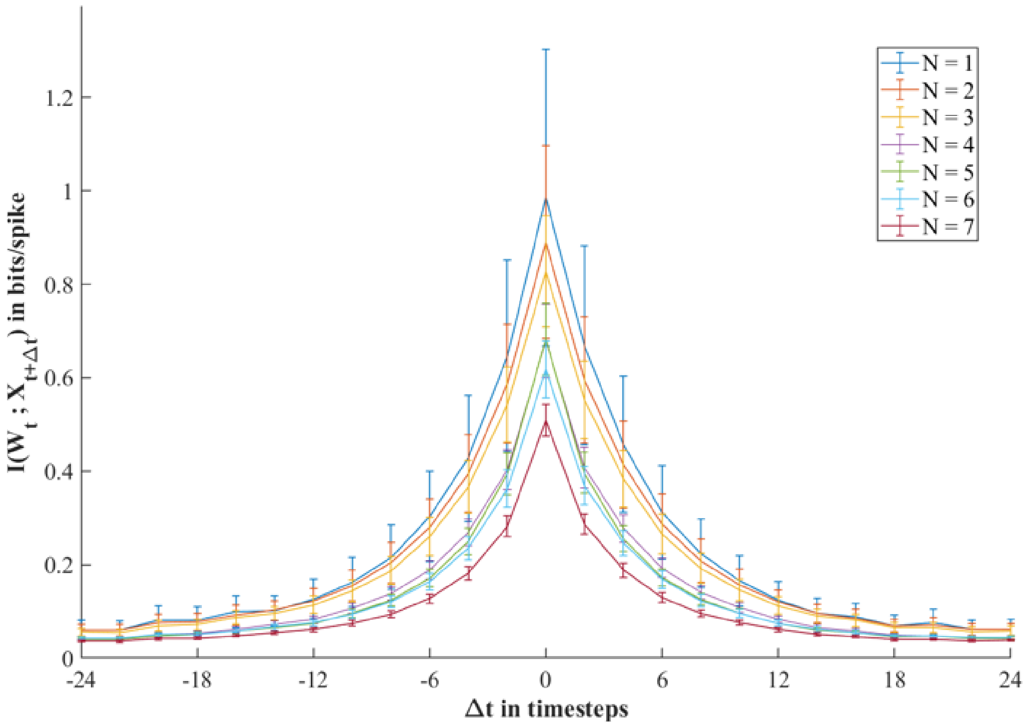


Figure 2: „Ellipses fitted to the spatial receptive field profile for the biphasic OFF cells. Scale bar, 100μm“[Mar+12]

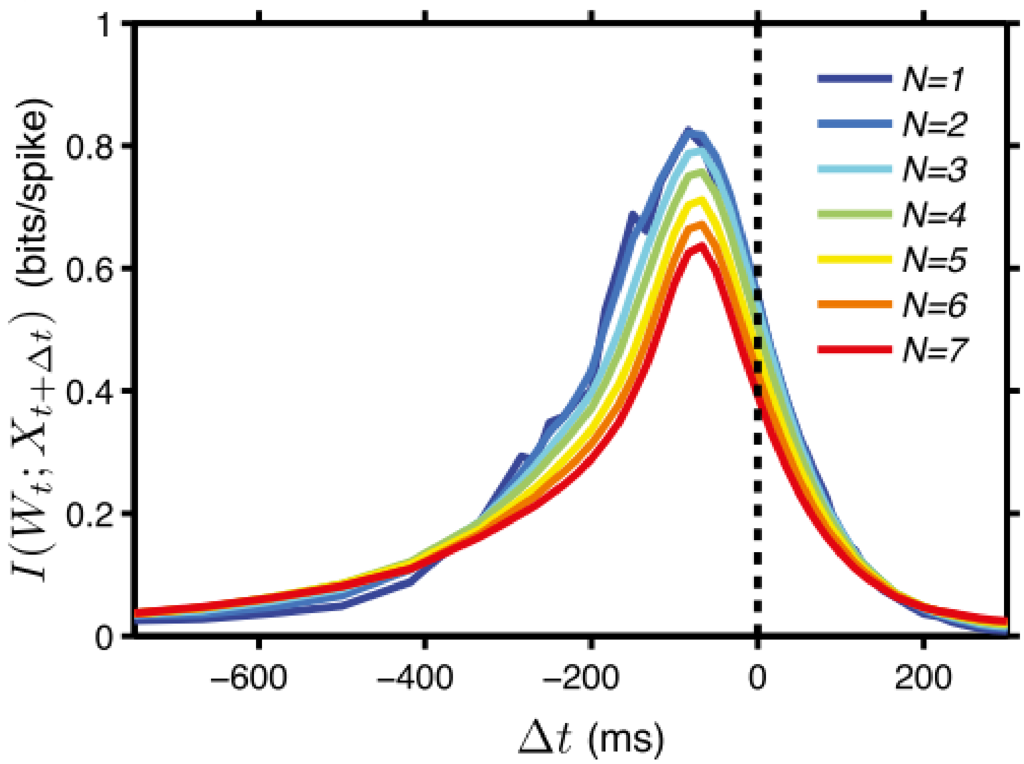
[1] The mass was added for the simulation to investigate the influence of inertia and was not included at [Pal+15].

Measured mutual information If we look at figure 3(a) with the mutual information between signal and position calculated using the formula 1, we can see a symmetrical shape with its maximum at 0 and also that mutual information extends to more distant times.

If we compare these simulated results to the experimental results in figure 3(b), we see at both figures that the mutual information per spike decreases for groups with a larger number of cells. This can be explained by the increasing redundancy of the signals [Pal+15] and may be related to the increasing frequency of cells covering the same area as other cells with almost the same receptive field.



(a) 50 retinal cells were simulated. For different groups of these 50 cells with N members, the mutual information was calculated individually and then the mean value was determined. The number of groups was set to 25 because of the large calculation time. This small number leads to large errors.



(b) Results from [Pal+15]. The latency in the response of retina cells is around 80ms[Pal+15], which explains the shift.

Figure 3: Calculated mutual information using formula 1 for a movement of a bar described by formula 2 and 3 with the original parameters from [Pal+15] and a mass $m = 1$. 100ms of the experimental data from 3(b) corresponds to 6 timesteps in 3(a).

Intertia as the source of predictability The simulated result shows great similarity to the experimentally determined results (figure 3). Through comparison, it is clear that the presence of predictive information in the future alone does not indicate that the retinal cells actively start to calculate the prediction for the future.
The predictive information of the future times is due to the fact that the considered system (the moving bar) is a subject, which is affected by inertia and thus this movement is predictable. Cells which only serve as a sensor, that record the predictable motion (like the simulated cells), also carry predictive information of the bar position at future times.

In the figure 4 we can see that a higher influence of inertia, so a higher mass of the bar, leads to a higher predictability (meaning more mutual information at future times).

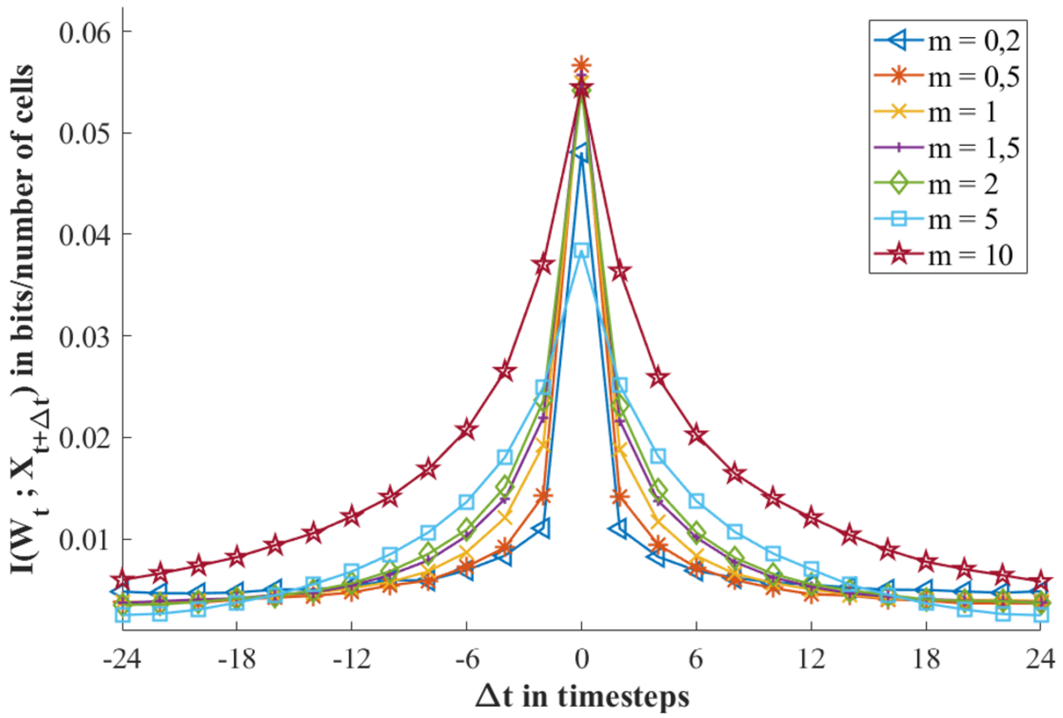


Figure 4: Calculated mutual information for a movement of a bar described by formula 2 and 3 with the original parameters from [Pal+15] for different masses

Different sources of predictive information Some difference can be recognized between the experimental results from figure 3(b) and figure 3(a): While the data of the simulated retinal cells is symmetric, the mutual information of the experimental data decreases more slowly towards the past than towards the future and for long times the mutual information for cell groups with more cells is larger than the mutual information for smaller cell groups. One reason could be, that the retinal cells do not only act as simple sensors, but also use other information sources to obtain a larger predictive information. In [CI20] it was pointed out, that by a synergy of different information sources (current neural activity, past neural activity, past environmental state) more predictive information can be obtained (figure 5). In conclusion, retinal cells that have a certain memory of the past could receive more predictive information than retinal cells that only look at the current position of the bar, as is the case with the simulated cells.

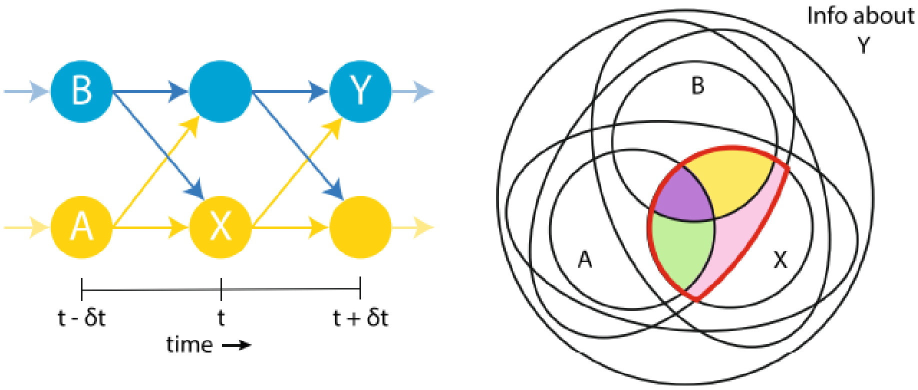


Figure 5: Graphic from [CI20] - Agent-environment interaction unrolled over time and „Partial information diagram for calculating the sources of predictive information in an agent-environment system. The total information that X has about Y is a combination of information that is available uniquely from A alone (green), uniquely from B alone (yellow), synergistically from their combination [A, B] (pink), and redundantly from both of them (purple).“
With **X** - Current neural activity, **A** - Past neural activity, **B** - Past environmental state, **Y** - future stimulus.

Inspiration for future work Up to now, it is very difficult to determine which result can be assigned to the function of the specific retinal cells or is valid for each sensory population that observes a bar which is influenced by inertia. Therefore, the focus of the next work should be to study the system of an inertially influenced object observed by a sensor array in more detail and to analyze the consequences for the predictive information from the change of different parameters.

First, the complicated two-dimensional system should be simplified to one dimension. In one dimension, mainly the influence of the different sizes and arrangements of the receptive fields (in one dimension: intervals) and the size of the observed object can be investigated.

Next, in two dimensions the investigation of the object shape and the different arrangements and shape of the receptive fields should be continued. In addition, it should be clarified which if it would make a difference, if we allow movements in all two dimensional directions.

In [Pal+15], the influence of different videos on the amount of predictive information was also investigated. It was found that the predictive information is greater in natural videos (videos of fishes) and extends further into the future, compared to videos with only one moving bar. It should be investigated whether this effect is related to the fact that there is more information in a natural video and therefore more predictive information can be extracted from this natural videos. Such a natural video may be compared to a bar video with several moving bars (representing natural objects like fishes). In this case, an influence of the arrangement of the sensors is also possible because several bars could interfere with each other if the distinguishable areas of the sensors are too large.

So far, only a simple model for a retinal cell has been used where the simulated cells fire when an object is inside their receptive fields. Here it should be investigated, what influence more complicated models have on the mutual information. These includes, for example, that a minimum time interval between two activations of the cell could be simulated and that there is an activation probability that depends on the distance of the object from the center of the receptive field. In addition, a cell memory could be simulated by make the probability of activation dependent on past activations.

Conclusion and reference to further sources In summary, using our own computer model of the retinal cells, it was possible to determine that the main features of the mutual information curve originate from the nature of the bar, i.e., from inertia. The question which causes the slight asymmetry and the higher mutual information when considering larger cell groups and longer times remains unanswered, but could be related to the different sources for mutual information. This should be investigated with further computer models by modifying individual parameters of the simulated cells.

For future work, more scientific work should be considered. This includes, for example, the influence of the bottleneck theory [TPB99] or comparable work, such as [Liu+21] and works on synergistic coding.

Used Code All data was generated by: https://github.com/SoenBeier/predictive_informations

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