SOEE3250/SOEE5675 2018/19

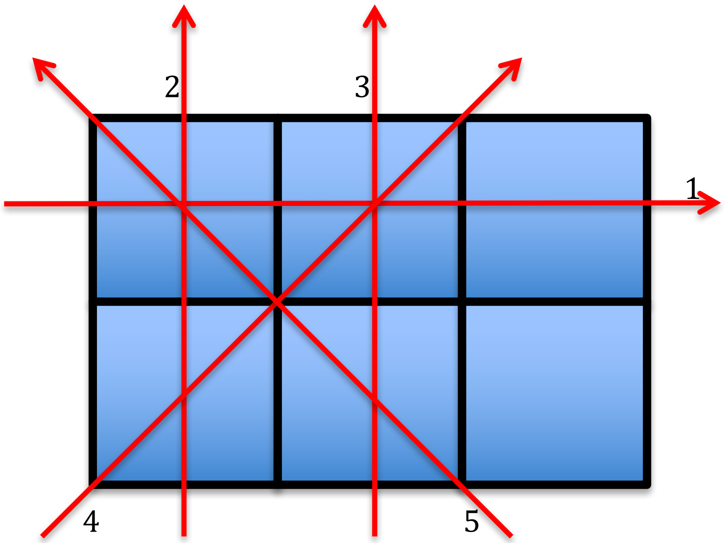
Inverse Theory

Andy Hooper

Practical 5: Mixed-determined problem and singular value decomposition

In this practical you will learn how to set up a mixed-determined tomography problem and solve it using singular value decomposition.

Inverse problem description



The figure above shows a 2-D seismic tomography experiment carried out on in a laboratory on a piece of rock measuring 30 mm by 20 mm. The red lines represent the paths of five seismic rays through the rock, the measured travel times for each of which is given below:

Ray Number	1	2	3	4	5
Time (μs)	5.43	3.18	3.65	5.50	4.35

In the model set-up, the rock is divided into six 10x10 mm blocks, as shown, and the goal is to determine the slowness of each the blocks (slowness is assumed constant in each block).

N.B. Save your matlab code into a script so you can rerun it

1. Write down the equation relating travel time for ray 1 to the slownesses in the relevant blocks. Is it linear? Now, assuming a reference velocity of 6km/s write down the equation in terms of Δs values, where Δs is the difference between the actual slowness and the reference slowness.
2. Write down the equations for rays 2 to 5, in terms of Δs values.
3. Write down **G**, the forward operator that maps the model parameters to the measurements and show it to a demonstrator. Set up **G** in Matlab.

4. Use `>>svd` to carry out the singular value decomposition of **G**. Check that $\mathbf{U}\times\mathbf{S}\times\mathbf{V}^T$ is equal to **G**.
5. Check that the columns in **U** are orthogonal by picking one column, and multiplying its transpose by another column. Do this with different pairs. Do the same for **V**. Look at the matrix of singular values (**S**). How many are non-zero? [N.B. count any number <1e-7 as zero]
6. Define $\mathbf{S}_p$ to contain the non-zero singular values of **S** only. It should be a square matrix with dimension equal to the number of non-zero singular values. Define $\mathbf{U}_p$ and $\mathbf{V}_p$ to contain the non-null columns of **U** and **V,** corresponding to the non-zero singular values in $\mathbf{S}_p$. Note that the number of columns in **U** and **V** should be equal to the number of non-zero singular values. Check that $\mathbf{U}_p\times\mathbf{S}_p\times\mathbf{V}_p^T$ is still equal to **G.**
7. Calculate **m_hat** as $\mathbf{G}^{-g}$ x **d**, where $\mathbf{G}^{-g}$ is the generalised Moore-Penrose inverse of **G**.
8. Visualise the estimated Δs values as square blocks (as in Fig. 1) using `>>imagesc` (`>>reshape` is useful to convert your **m_hat** vector into a 2 x 3 matrix). To make the blocks square use `>>axis equal tight`. Add a colorbar.
9. Visualise the two vectors in **V** that are in the model null space in the same way as **m_hat**. One of these should be obvious from the experiment set-up, the other probably not, but check that the travel time through it is zero for every ray.
10. Calculate predicted measurements, **d_hat,** from **G** x **m_hat**. Add 10x the first model null vector and -5x the second null vector to **m_hat** to create a new model vector, **m_new**. Visualise this vector in the same way as before. Now calculate the predicted measurements using this new model vector. Are your results identical to **d_hat** using your original **m_hat**? This demonstrates that the data are completely insensitive to any combination of the model null vectors.
11. Calculate the norm of **m_hat** (**m_hat'*m_hat**). Is it less than the norm of **m_new**? Satisfy yourself that adding any combinations of the null vectors always increases the norm. In other words, an infinite number of model vectors fit the data equally well, but the model vector returned by applying $\mathbf{G}^{-g}$ has the minimum norm.
12. Calculate the vector of data residuals (**d-d_hat**). Compare this residual vector to the data null vector in **U**. Is it a multiple? (Note that zero values may actually be set to a very small number rather than zero, due to numerical inaccuracy). Add 20x the data null vector to the data vector and recalculate **m_hat**. Is it the same as the original value? This shows that the combination of data values in the null vector cannot be fit by the model. It must represent either measurement error or errors in the assumptions made in defining the mathematical model (i.e., the forward operator).
13. Visualise the estimated velocity of each block in the same way as Fig. 1 (you will have to add the reference slowness to the estimated Δs values first).