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Inverse Theory

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Practical 2: Least Squares Estimation

In this practical you will set up an inverse problem using data from a seismic refraction survey, and use Matlab to solve the inverse problem using least squares.

Background

A survey seismic refraction experiment has been carried out as shown in Figure 1. Your goal is to determine the seismic velocity of the three different layers and the thickness of the upper two.

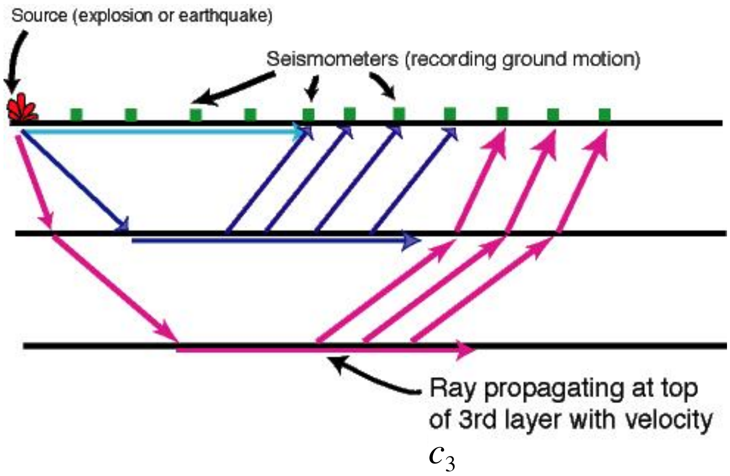

Figure 1. Seismic refraction survey set up.

At the geophones (seismometers) closest to the source, the first arrivals are the direct waves. In the middle distance, the first arrivals are the refracted waves that travel along the top of the middle layer. At the farthest geophones, the first arrivals are refracted and travel along the top of the deepest layer. A plot of first arrivals looks like Figure 2.

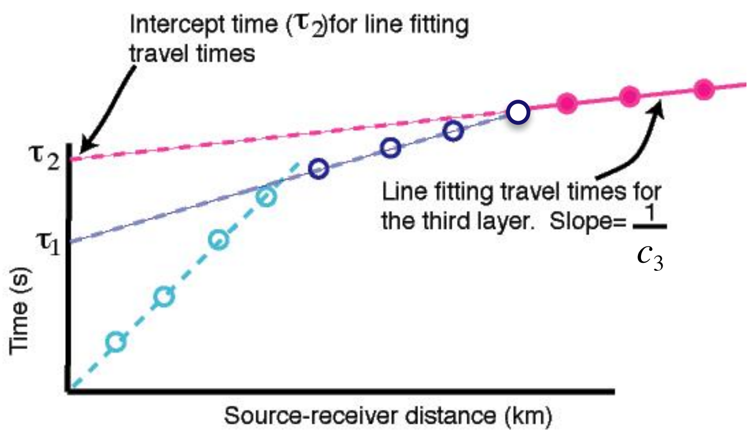

Figure 2. Time of first arrivals.

The slopes represent the "slowness" (reciprocal of velocity) of each layer. The intercepts with the y-axis are related to the velocities and the depths by the following formulae:

$$\tau_1 = 2z_1 \frac{\sqrt{c_2^2 - c_1^2}}{c_2 c_1} \qquad \tau_2 = 2z_1 \frac{\sqrt{c_3^2 - c_1^2}}{c_3 c_1} + 2z_2 \frac{\sqrt{c_3^2 - c_2^2}}{c_3 c_2}$$

where z_1 and z_2 are the thicknesses of the upper and middle layers, respectively, c_1 and c_2 are the corresponding velocities, and c_3 is the velocity of the bottom layer. The locations and measured first arrival times are given in Table 1.

Distance (m)	First arrival time (s)
80	0.0474
160	0.0979
240	0.1481
320	0.1949
400	0.2288
480	0.2563
560	0.2819
640	0.3108
720	0.3325
800	0.3529
880	0.3694

Table 1. Distance from source and time of first arrival, for each receiver.

N.B. Save your Matlab code as a Live Script, or Script, so you can rerun it

1) Plot time versus distance for the observations (use >>`plot` with the `'o'` option). Add a title and label the axes.
2) Write down the equation that relates the velocity of the upper layer, c_1, to the measurements at the first 4 geophones (in the format d_i=...). What are the unknown parameters (m_i)? Is this linear (i.e., is d_i the weighted sum of the unknown parameter(s))?
3) Now rewrite the equation using slowness of the upper layer, s_1 (where $s_1=1/c_1$), instead of velocity. Is this linear?
4) In Matlab, set up a vector of observations (**d1**) and a forward operator matrix (**G1**) based on the equation in (3). The columns of **G1** should correspond to the multipliers of m_i.
5) Use least squares to estimate s_1 (the function >>`inv` returns the inverse of a matrix). Calculate c_1 from s_1 and check your estimate on Minerva.
6) Add a line to your plot (in a different colour), based on your estimated value of s_1. Does it look visually like you have found the least squares solution?
7) Write down the equation that relates the slowness of the middle layer (s_2) and the intercept τ_1 to the measurements at the next 4 geophones (in the format d_i=...). What are the unknown parameters (m_i)?
8) In Matlab, set up a vector of observations (**d2**) and a forward operator matrix (**G2**) based on the equation in (7). Use least squares to estimate c_2 and τ_1 and add a line based on your estimate to your plot. Check your value for c_2 on Minerva.

9) What is the estimated thickness of the upper layer (z_1)?

10) Now estimate the velocity of the bottom layer (c_3) and the thickness of the middle layer (z_2) using least squares, and add an appropriate third line to your plot. Check your value for c_3 on Minerva.