MINISTRY OF EDUCATION
MANDALAY TECHNOLOGICAL UNIVERSITY
Department of Mechatronic Engineering
2017-2018 Academic Year

Fourth Year **Second Semester Examination**

Date: 26.9.2018(TUE) **Time: 9:00 to 12:00 noon**

II. Solution & Marking of McE.42077 (Control Engineering II)

Ques No.	Solution	
1.(a.)	Solution: The closed-loop transfer function is $$\phi(s) = \frac{G(s)}{1+G(s)} = \frac{5K_A}{s^2 + 34.5s + 5K_A}$$ $$K_A = 200, \therefore \phi(s) = \frac{1000}{s^2 + 34.5s + 1000}$$ $$\therefore \omega_n^2 = 1000, \quad 2\zeta\omega_n = 34.5$$ $$\therefore \omega_n = 31.6(rad/s), \zeta = \frac{34.5}{2\omega_n} = 0.545$$ $$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.12(\sec)$$ $$t_s \approx \frac{3}{\zeta\omega_n} = 0.174(\sec)$$ $$\sigma\% = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\% = 13\%$$	3 Marks
	If $K_A = 200$, then $\omega_n = 34.5(rad/s)$; $\zeta = 0.545$ $\therefore t_p = 0.12(s)$, $\boxed{t_s=0.23s}$, $\sigma\% = 13\%$	
	If $K_A = 1500$, then $\omega_n = 86.2(rad/s)$; $\zeta = 0.2$ $\therefore t_p = 0.037(s)$, $\boxed{t_s=0.23s}$, $\sigma\% = 52.7\%$	3 Marks
	Thus, the greater the K_A, the less the ξ, the greater the ω_n, the less the t_p, the greater the $\sigma\%$, while the settling time t_s has no change.	
	$K_A = 13.5$ When $K_A = 13.5, \omega_n = 8.22(rad/s), \zeta = 2.1$	3 Marks
	$\zeta > 1$, Overdamped	
	When system is over-damped, there is no peak time, overshoot and oscillation.	1 Mark

When K_A increases, t_p decreases, t_r decreases, the speed of response increases, meanwhile, the overshoot increases.

1.(b.) For a zero steady-state error:

(a.) when the input is a step, one integration is needed, — 3 Mark

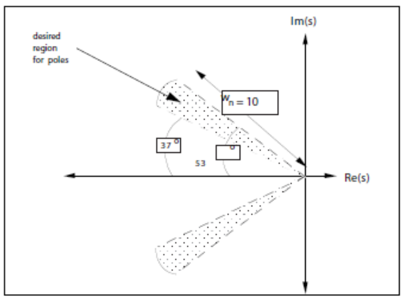
or a type 1 system.

(b.) A type 2 system is required for $e_{ss} = 0$ for a ramp input. — 2 Mark

1.(c.) i. $0.6 \le \zeta \le 0.8$, $\omega_n \le 10$,

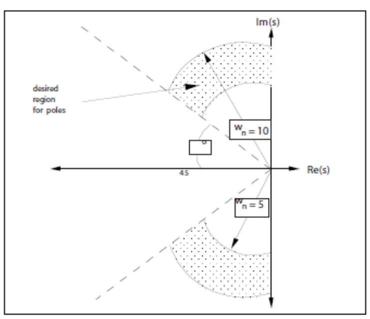
$\zeta = 0.6$, $\theta = 53.13^\circ$ $\zeta = 0.8$, $\theta = 36.87^\circ$

2 Marks



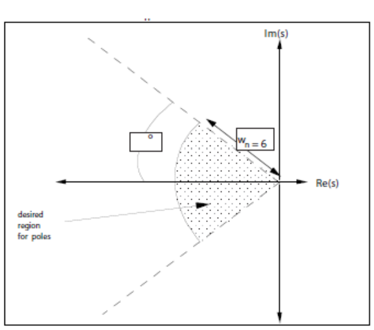
ii. $\zeta \le 0.707$, $5 \le \omega_n \le 10$,

$\zeta = 0.707$, $\theta = 45^\circ$ $\zeta = 0.8$, $\theta = 36.87^\circ$ — 1.5 marks



iii. $\zeta \ge 0.6$, $\omega_n \le 0.6$, — 1.5 marks

$\zeta = 0.6$, $\theta = 53.13^\circ$



Ques No	Solution	Marks
2.	P.O $\leq 10\ \%$ $100\ e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 10$ $e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 0.1$ $\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \leq \ln\ 0.1$ $\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \leq -2.302583$ $\frac{\pi\zeta}{\sqrt{1-\zeta^2}} \geq 2.302583$ Squaring both sides, $\frac{\pi^2\zeta^2}{1-\zeta^2} \geq 5.301898$ $1.8615\zeta^2 \geq 1- \zeta^2$ $2.8615\zeta^2 \geq 1$ $\zeta \geq 0.59$	3 Marks
	$T(s) = \frac{12K}{s^2+12s+12K}$ Compare with, $T(s) = \frac{\omega n^2}{s^2+2\zeta\omega_n\ s+\omega_n\ ^2}$ $2\zeta\omega_n = 12$ $\zeta\omega_n = 6$ $\omega_n = 10.169$ $\omega_n\ ^2 = 12K$ $K = \frac{\omega_n\ ^2}{2} = 8.6$	2 Marks
	(b.) K= 10, S_K^T =? $S_K^T = \frac{\partial T}{\partial K} \times \frac{K}{T}$ $S_K^T = \frac{\partial}{\partial K}\left[\frac{12K}{s^2+12s+12K}\right] \times \frac{K}{\left[\frac{12K}{s^2+12s+12K}\right]}$	

Ques No	Solution	Marks
2.	$=\frac{12s^2+144s+144K-144K}{s^2+12s+12K}\times\frac{1}{12}$	
	$S_K^T = \frac{s^2+12s}{s^2+12s+12K}$	
	$S_K^T = \frac{s(s+12)}{s(s+12)+12K}$	
	$= \frac{s(s+12)}{s(s+12)+120}$	5 Marks
	(c.) s=0, S_K^T =?	
	$S_K^T = \frac{s(s+12)}{s(s+12)+120} = \frac{0}{120} = 0$	2 Marks
	(d.) $\frac{60\ beats}{minute} = \frac{60\ beats}{60\ \ secs} = \frac{1\ beat}{1\ sec} = \mathrm{j}2\pi$	1Mark
	$S_K^T\ (\mathrm{j}2\pi) = ?$	
	$s = \mathrm{j}2\pi \times \frac{1\ beat}{1\ sec} = \mathrm{j}2\pi$	1 Mark
	$S_K^T = \frac{s(s+12)}{s(s+12)+120}$	
	$S_K^T = \frac{\mathrm{j}2\pi\ (\mathrm{j}2\pi\ + 12)}{\mathrm{j}2\pi\ (\mathrm{j}2\pi\ + 12)+120}$	
	$S_K^T = \frac{\mathrm{j}2\pi\ (\mathrm{j}2\pi\ + 12)}{\mathrm{j}2\pi\ (\mathrm{j}2\pi\ + 12)+120}$	
	$= \frac{-39.478+\mathrm{j}\ 75.398}{-39.478+\mathrm{j}\ 75.398+120}$	
	$S_K^T = \frac{-39.478+\mathrm{j}\ 75.398}{80.522+\mathrm{j}\ 75.398}$	
	$= \frac{85.11 < 117.6}{110 < 43.12} = 0.771 < 74.48$	
	$S_K^T = 0.771$	6 Marks

3.

$$G(s) = \frac{K(s+2)}{(s+10)} \times \frac{1}{s(s-1)}, \quad H(s) = 1$$

$$= \frac{K(s+2)}{(s+10)(s-1)s}$$

2 Marks

(a.) Characteristics Equation,

$1 + H(s)G(s) = 0$

$$1 + \frac{K(s+2)}{(s+10)(s-1)s} = 0$$

$$\frac{(s+10)(s-1)s + K(s+2)}{(s+10)(s-1)s} = 0$$

$$(s+10)(s-1)s + K(s+2) = 0$$

$$s^3 + 9s^2 - 10s + Ks + 2K = 0$$

3 Marks

Using Routh-hurwitz,

S^3	1	K-10
S^2	9	2K
S^1	$\frac{9K-90-2K}{9}$	0
S^0	2K	0

4 Marks

For the system to be stable,

$$\frac{7K-90}{9} > 0$$

3.

$$7K - 90 > 0$$

$$K > \frac{90}{7}$$

$$2K > 0$$

$$K > 0$$

4 Marks

(b.) When $K = \frac{90}{7}$

The system is marginally stable,

S^2 row ….. $q(s) = 9s^2 + 2K = 0$

$$9s^2 + 2\left(\frac{90}{7}\right) = 0$$

$$9s^2 = -\frac{180}{7}$$

5 Marks

	$s^2 = -\frac{180}{7} \times \frac{1}{9} = \frac{-20}{7}$ $s = \pm j\sqrt{\frac{20}{7}}$ The root of the characteristic equation is, $S_{1,2} = \pm j\sqrt{\frac{20}{7}}$	2 Marks
4.	$KG(s) = \frac{k(s+2)}{s(s+1)}$ (a) breakaway and entry point = ? (b) gain (k) and root ? real part or complex root s = ? (c) Sketch R. L. Solution: The characteristics equation is $q(s) = 1 + GH(s) = 0$ $1 + \frac{k(s+2)}{s(s+1)} = 0$ For breakaway point $k = -\left[\frac{s^2+s}{s+2}\right]$ $\frac{dk}{ds} = -\left[\frac{(s+2)(2s+1)-(s^2+s)}{(s+2)^2}\right] = 0$ $2s^2 + s + 4s + 2 - s^2 - s = 0$ $s^2 + 4s + 2 = 0$ $s = -0.585, -3.414$	20 Marks

Ques N0	Solution	Marks
4.	(i) Breakaway point , s = - 0.59 (ii) Entry point , s = -3.414 (b) K =? , s = ? at real part = -2 1+ KG(s) = 0 $1+\frac{K(s+2)}{s(s+1)} = 0$ Compare with $as^2+ bs +c= 0$ $s = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, a= 1 , b= K+1, c =2K $s = \frac{-(K+1)\pm\sqrt{(K+1)^2-4(1)(2K)}}{2\times1}$ $s = \frac{-(K+1)}{2} \pm \frac{\sqrt{(K+1)^2-4(1)(2K)}}{2}$ $\frac{-(K+1)}{2} = -2$ K = 3 When K =3 , $s^2+ (K+1)s+2K = 0$ $s^2+ 4s +6 = 0$ $s = -2 \pm j1.44$ The roots are $s= -2+ j1.44$ and $s= -2 - j1.44$. (b.) Sketch the root, 1+ KG(s) = 0 $1+\frac{K(s+2)}{s(s+1)} = 0$	

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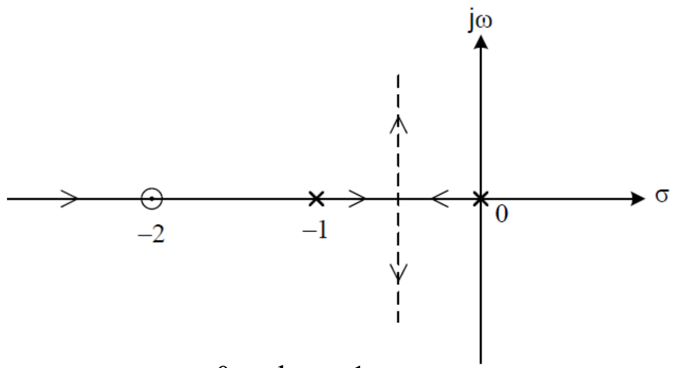
<tr><th>Ques N0</th><th>Solution</th><th>Marks</th></tr>
<tr><td>4.</td><td>

$1+\frac{K(s+2)}{s(s+1)} = 0$

$s(s+1)+K(s+2) = 0$

$n_p = 2,\ s = 0\ ,\ s = -1$

$n_z = 1,\ s = -2$



Segments are $s = 0$ and $s = -1$

$s = -2$ and $s = -\infty$

no of Separate root loci, $n_p = 2$

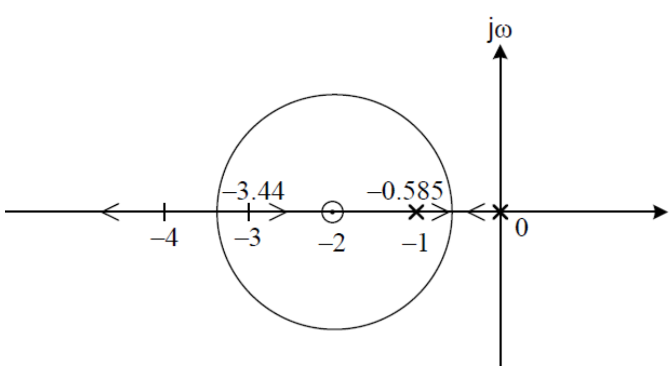
The root loci are symmetric with respect to the real axis

Center of Asymptotes,

$\emptyset_A = \frac{2q+1}{n_p + n_z} \times 180$, $q = n - m - 1 = 2 - 1 - 1 = 0$

When $q = 0$, $\emptyset_A = \frac{2(0)+1}{2-1} \times 180 = 180^\circ$

Breakaway point, $s = -0.59, -3.414$



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<tr><th>Ques
N0</th><th>Solution</th><th>Marks</th></tr>
<tr><td>5.(a.)</td><td>

The characteristic equation is,

$$1 + GH(s) = 1 + \frac{K(s+1)}{s(s+2)(s+4)^2}.$$

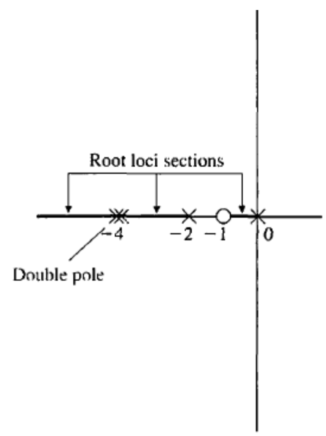
$$1 + GH(s) = 1 + \frac{K(s+1)}{s(s+2)(s+4)^2}. \quad = 0$$

When K= 0, the pole at s=0, s= -2, s = -4, s= -4

$\therefore \quad n_p = 4$

When K= ∞, the zeros at s= -1,

$\therefore \quad n_z = 1$



No of separate loci, SL= n_p = 4, n=4, M=1

The root loci are symmetric with respect to horizontal real axis.

The segment of Root Locus has at s=0 and s= -1, s=-2 and s= -4 ,

s=-4 and ∞

Angle of Asymptotes,

$$\phi_A = \frac{2q+1}{n_p + n_z} \times 180 \quad , q = n\text{-}m\text{-}1 = 4\text{-}1\text{-}1 = 2$$

When q = 0, q = 1, q = 1,	$\phi_A = +60^\circ \quad (k = 0)$, $\phi_A = 180^\circ \quad (k = 1)$, and $\phi_A = 300^\circ \quad (k = 2)$,

Centre of Asymptotes,

$$\sigma_A = \frac{\sum(-p) - \sum(-z)}{n_p - n_z}$$

</td><td>10
Marks</td></tr>
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$$\sigma_A = \frac{(-2) + 2(-4) - (-1)}{4 - 1} = \frac{-9}{3} = -3.$$

Characteristics Equation is,

$s(s+2)(s+4)^2 + K(s+1) = 0$

$(s^2+2s)(s+4)^2 + K(s+1) = 0$

$(s^2+2s)(s^2+8s+16) + K(s+1) = 0$

$s^4+10s^3+32s^2+(K+32)s+K=0$

By applying the Routh-Hurwitz criterion,

s^4	1	32	K
s^3	10	K+32	0
s^2	$\frac{288-K}{10}$	K	0
s^1	$\frac{9216+156K-K^2}{288-K}$	0	0
s^0	K	0	0

The system to be stable,

$$\frac{9216 + 156K - K^2}{288 - K} = 0$$

$$9216 + 156K - K^2 = 0$$

K = 201.7 or K = -45.7 (impossible)

$$\frac{288-K}{10} s^2 + K = 0$$

When K =201.7 ,

$$\frac{288-201.7}{10} s^2 + 201.7 = 0$$

$s = \pm j\,4.83$

$$G(s) = \frac{K}{(s+2)(s+4)}, \quad 1 + G(s) = 1 + \frac{K}{(s+2)(s+4)} = 0.$$

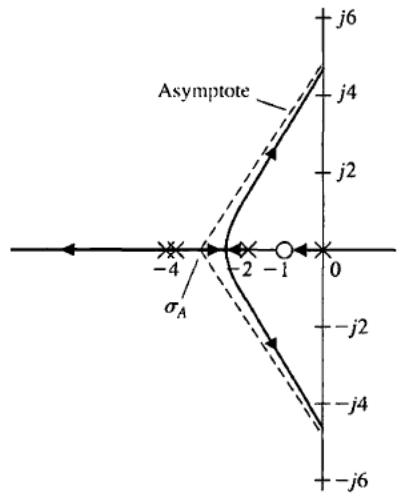
Breakaway point,

$$1 + GH(s) = 1 + \frac{K(s+1)}{s(s+2)(s+4)^2}.$$

$$K = \frac{-[\,s^4+10s^3+32s^2\ +32s]}{s+1}$$

$$\frac{dK}{ds} = \frac{d-}{ds}\ \ \frac{-[\,s^4+10s^3+32s^2\ +32s]}{s+1}$$

Breakaway point = - 2.6



5.(b.)

K=?, H(s)=1

10 Marks

$$G(s) = \frac{K(s+20)}{s(s+10)^2}$$

The characteristic equation,

1+G(s)H(s)=0

$$1 + \frac{K(s+20)}{s(s+10)^2} = 0$$

$s(s+10)^2$+K(s+20) = 0

$s(s+10)^2$+Ks+20K = 0

$s(s^2+100+20s)$+ Ks+ 20K = 0

$s^3 + 20s^2 + s(100+K) + 20K = 0$

Using the Routh-Hurwith Criterion,

S^3	1	(100+K)
S^2	20	20K
S^1	$\frac{20K+2000-20K}{20} = 100$	0
S^0	20K	

For the system to bestable,

20K > 0

K > 0