Triple Omni-Wheeled Mobile Robot which wheels' axles are along x-axis

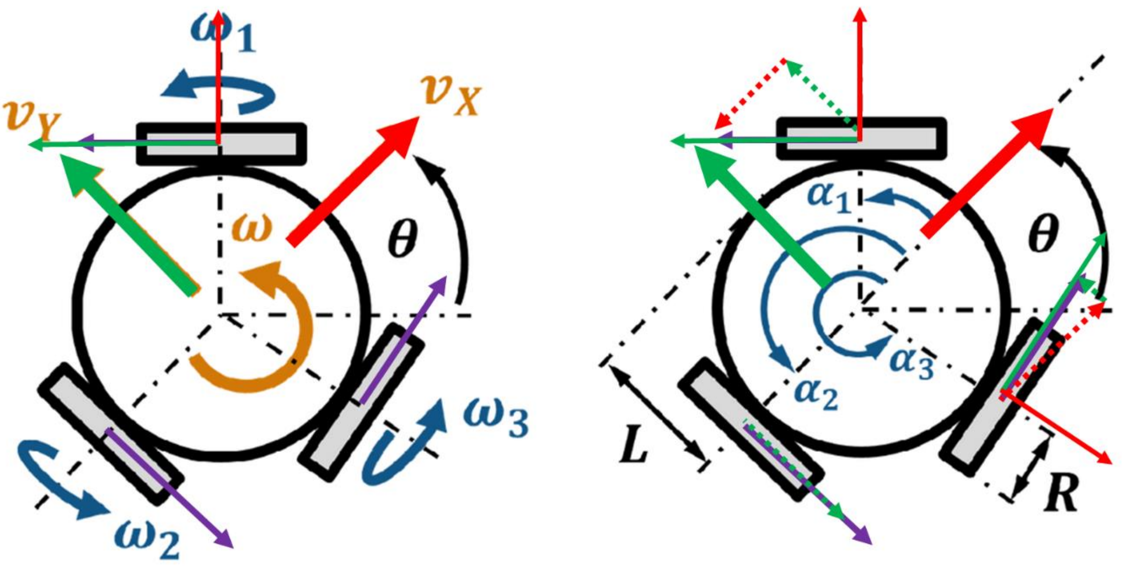


Fig. 1. Triple Omni-Wheeled Mobile Robot

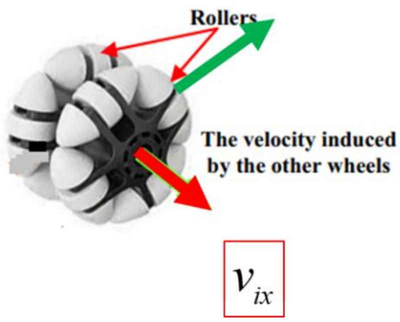
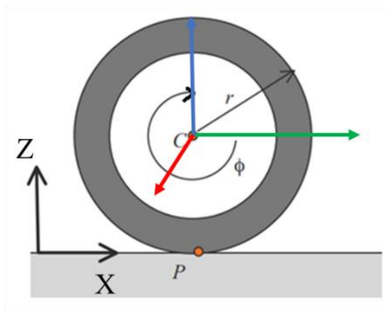


Fig. 2. The Omni-wheel

Since Omni-wheels are used, there are no constraints.

Consider the rolling wheel with a point of contact P with the ground,



(x, y, z) >>> (i, j, k)

Fig. 3. Wheel Frame

Since it is rolling, the velocity of the point P is zero ($v_P = 0$).

The velocity of the point C of the wheel can be obtained from the vector equation:

$$v_C = v_p + \omega \times \overrightarrow{PC} = 0 + \dot{\phi}\hat{i} \times r\hat{k} = -r\dot{\phi}\hat{j}$$

Therefore,

$$v_{C_1} = v_p + \omega \times \overrightarrow{PC} = 0 + \dot{\phi}_1 \hat{i} \times r\hat{k} = -r\dot{\phi}_1 \hat{j}$$

$$v_{C_2} = v_p + \omega \times \overrightarrow{PC} = 0 + \dot{\phi}_2 \hat{i} \times r\hat{k} = -r\dot{\phi}_2 \hat{j}$$

$$v_{C_3} = v_p + \omega \times \overrightarrow{PC} = 0 + \dot{\phi}_3 \hat{i} \times r\hat{k} = -r\dot{\phi}_3 \hat{j}$$

The rotation matrices of the wheel axis are related to the center of the robot as the following matrix:

$$Q(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Since there are three wheels,

$$Q_1 = \begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 \\ \sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos\alpha_3 & -\sin\alpha_3 \\ \sin\alpha_3 & \cos\alpha_3 \end{bmatrix}$$

Mapping wheels' centers in the robot's coordinate system,

$$v_{wheel1} = Q_1 v_{C_1} = \begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 \\ \sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \begin{bmatrix} 0 \\ -r\dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_1 \sin\alpha_1 \\ -r\dot{\phi}_1 \cos\alpha_1 \end{bmatrix}$$

$$v_{wheel2} = Q_2 v_{C_2} = \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{bmatrix} \begin{bmatrix} 0 \\ -r\dot{\phi}_2 \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_2 \sin\alpha_2 \\ -r\dot{\phi}_2 \cos\alpha_2 \end{bmatrix}$$

$$v_{wheel3} = Q_3 v_{C_3} = \begin{bmatrix} \cos\alpha_3 & -\sin\alpha_3 \\ \sin\alpha_3 & \cos\alpha_3 \end{bmatrix} \begin{bmatrix} 0 \\ -r\dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_3 \sin\alpha_3 \\ -r\dot{\phi}_3 \cos\alpha_3 \end{bmatrix}$$

The linear velocity of the mobile robot is calculated as follows:

$$v = \frac{v_{wheel1} + v_{wheel2} + v_{wheel3}}{3}$$
$$= \frac{r}{3} \begin{bmatrix} \dot{\phi}_1 \sin\alpha_1 & \dot{\phi}_2 \sin\alpha_2 & \dot{\phi}_3 \sin\alpha_3 \\ -\dot{\phi}_1 \cos\alpha_1 & -\dot{\phi}_2 \cos\alpha_2 & -\dot{\phi}_3 \cos\alpha_3 \end{bmatrix}$$

In matrix form,

$$v = \frac{r}{3} \begin{bmatrix} \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 \\ -\cos\alpha_1 & -\cos\alpha_2 & -\cos\alpha_3 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} = \frac{r}{3} \begin{bmatrix} \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 \\ -\cos\alpha_1 & -\cos\alpha_2 & -\cos\alpha_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Calculating angular velocity of the mobile robot is as follows:

Velocity equation is $v = r\omega$.

Since the angular motion of the robot center due to v_1 is along wheel axis (x-axis),

$$v_1 = v_O + \dot{\theta}_1 \hat{k} \times L\hat{j}$$

$$r\omega_1 = 0 - L\dot{\theta}_1 \hat{i} \qquad (\because v_O = 0)$$

$$\dot{\theta}_1 = -\frac{r\omega_1}{L}$$

Also,

$$\dot{\theta}_2 = -\frac{r\omega_2}{L}$$

$$\dot{\theta}_3 = -\frac{r\omega_3}{L}$$

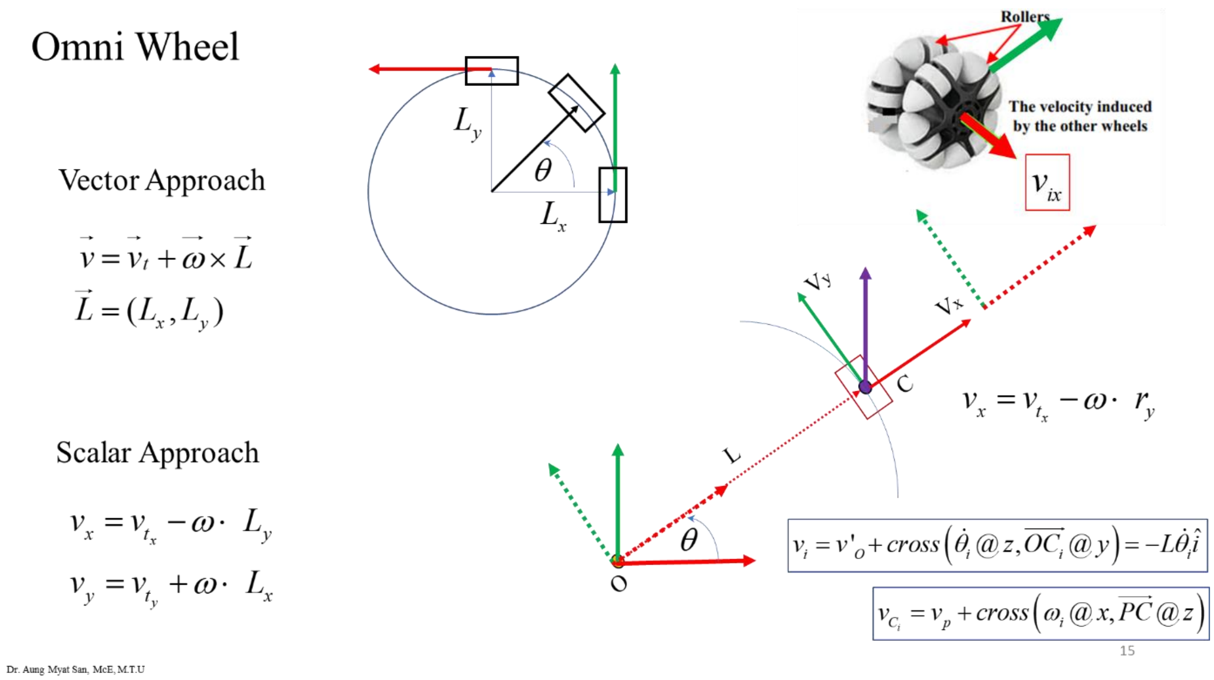


Fig. 4. Lateral movement of the Omni-wheeled mobile robot

Therefore, the angular velocity of the robot is

$$\dot{\theta} = \frac{\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3}{3} = -\frac{r\omega_1}{3L} - \frac{r\omega_2}{3L} - \frac{r\omega_3}{3L}$$

In matrix form,

$$\dot{\theta} = \begin{bmatrix} -\frac{r}{3L} & -\frac{r}{3L} & -\frac{r}{3L} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

The forward kinematics of the triple Omni-wheeled mobile robot is

$$\begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r\sin\alpha_1}{3} & \frac{r\sin\alpha_2}{3} & \frac{r\sin\alpha_3}{3} \\ -\frac{r\cos\alpha_1}{3} & -\frac{r\cos\alpha_2}{3} & -\frac{r\cos\alpha_3}{3} \\ -\frac{r}{3L} & -\frac{r}{3L} & -\frac{r}{3L} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Jacobian of forward kinematics is

$$J = \begin{bmatrix} \frac{r\sin\alpha_1}{3} & \frac{r\sin\alpha_2}{3} & \frac{r\sin\alpha_3}{3} \\ -\frac{r\cos\alpha_1}{3} & -\frac{r\cos\alpha_2}{3} & -\frac{r\cos\alpha_3}{3} \\ -\frac{r}{3L} & -\frac{r}{3L} & -\frac{r}{3L} \end{bmatrix} = \frac{r}{3}\begin{bmatrix} \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 \\ -\cos\alpha_1 & -\cos\alpha_2 & -\cos\alpha_3 \\ -\frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{bmatrix}$$

Inverse Jacobian is the inverse of the Jacobian of forward kinematics.

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = J^{-1}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = J^{-1}\begin{bmatrix} v \\ \dot{\theta} \end{bmatrix}$$

If $\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 & 120 & -120 \end{bmatrix}$,

$$J = \frac{r}{3}\begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{bmatrix}$$

Then, the inverse Jacobian is the inverse of the Jacobian of forward kinematics.

$$J^{-1} = \begin{bmatrix} 0 & -\frac{2}{r} & -\frac{L}{r} \\ \frac{\sqrt{3}}{r} & \frac{1}{r} & -\frac{L}{r} \\ \frac{\sqrt{3}}{r} & \frac{1}{r} & -\frac{L}{r} \end{bmatrix}$$

Checking Kinematics

Assume $r = 1$ and $L = 1$. Then, the forward Jacobian is

$$J = \frac{r}{3}\begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \\ -1 & -1 & -1 \end{bmatrix},$$

and the inverse Jacobian is

$$J^{-1} = \begin{bmatrix} 0 & -\frac{2}{r} & -\frac{L}{r} \\ \frac{\sqrt{3}}{r} & \frac{1}{r} & -\frac{L}{r} \\ \frac{\sqrt{3}}{r} & \frac{1}{r} & -\frac{L}{r} \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ \sqrt{3} & 1 & -1 \\ \sqrt{3} & 1 & -1 \end{bmatrix}$$

If $\omega_1 = 1$, $\omega_2 = 0$ and $\omega_2 = 1$,

$$\begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} = J \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = J \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{6} \\ -\frac{1}{6} \\ -\frac{2}{3} \end{bmatrix}$$

If $v_x = -\frac{\sqrt{3}}{6}$, $v_y = -\frac{1}{6}$ and $\dot{\theta} = \omega = -\frac{2}{3}$,

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = J^1 = \begin{bmatrix} 0 & -2 & -1 \\ \sqrt{3} & 1 & -1 \\ \sqrt{3} & 1 & -1 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{3}}{6} \\ -\frac{1}{6} \\ -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

The linear velocity of the robot is

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \frac{1}{3}$$

The orientation of the robot is

$$\theta = a\tan 2\left(v_y, v_x\right) = -\frac{5\pi}{6} = -150^\circ$$