Department of Mechatronic Engineering

2018-2019Academic Year

Fourth Year **First Semester Examination**

McE-41077 Control Engineering I

Date: 5.4.2019(FRI) **Time: 1:00 to 4:00 pm**

Ques No.	Solution	Marks

1.(a) 10 Marks

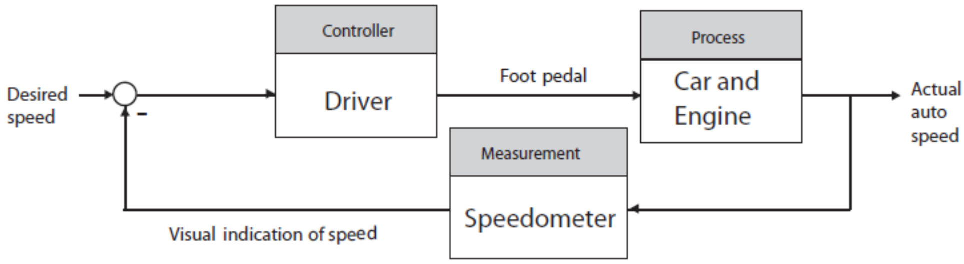


Figure: Block Diagram of the Speed Control System of a Motorcycle

(b.) On the mechanical side, the torque $T_{(s)}$, developed by the motor, is balanced by the load and disturbance torques.

$$T_{m\,(s)} = T_{l(s)} + T_{d(s)}$$

$$T_{l(s)} = Js^2\theta_{(s)} + b\,s\,\theta_{(s)} \quad \text{[J= inertia, b = friction]}$$

The magnetic flux $\phi(t)$ is proportional to the field current $i_f(t)$

$$\phi(t) = k_1\, i_f(t)$$

The torque produced by a motor is proportional to the product of the magnetic flux $\phi(t)$ and the armature current $i_a(s)$

$$T_{m\,(t)} = (K_1 K_f I_{f(s)})\, I_{a(s)} = K_m\, I_{a(s)}$$

$i_{f(s)} = I_{f(s)}$ is a constant field current and K_m = motor constant

The armature current is related to the input voltage applied to the armature.

$$V_{a(s)} = (R_a + L_{a(s)})I_{a(s)} + V_{b(s)}$$

$V_{b(s)}$ = back electromotive_force voltage proportional to the motor speed.

$$V_{b(s)} = K_b\, \omega_{(s)} \quad , \quad \omega_{(s)} = s\,\theta(s)$$

$$I_{a(s)} = \frac{V_{a(s)} - K_b\,\omega(s)}{(R_a + La_s)}$$

$T_{(t)}$ is also related to the rotational speed $\omega(t)$ by the differential equation

$$T_{m\,(s)} = T_{l(s)} + T_{d(s)}$$

$$K_m I_{a(s)} = Js^2\theta_{(s)} + b\,s\,\theta_{(s)} \qquad [Td(s) = 0]$$

$$K_m I_{a(s)} = \theta_{(s)}\,[Js^2 + bs]$$

$$\frac{K_m[V_{a(s)} - K_b\,\omega(s)]}{(R_a + La_s)} = \theta_{(s)}\,[Js^2 + bs]$$

$$\theta_{(s)} = \frac{K_m[V_{a(s)} - K_b\,\omega(s)]}{(R_a + La_s)[Js2 + bs]}$$

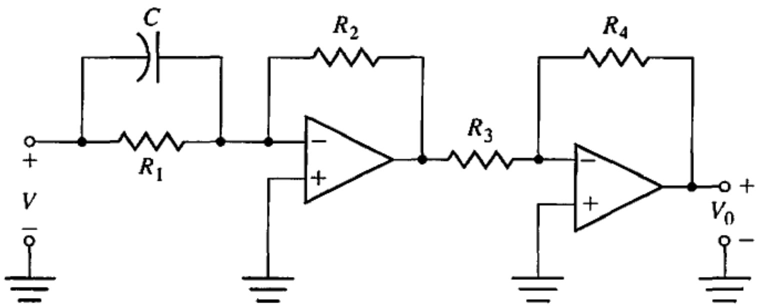
$$\theta_{(s)}(R_a + La_s)[Js2 + bs] = K_m V_{a(s)} - K_m K_b\,\omega_{s)}$$

$$\{\theta_{(s)}(R_a + La_s)[Js2 + bs]\} + K_m K_b\,s\theta\,(s) = K_m V_{a(s)}$$

$$\theta_{(s)}\,[\{(R_a + La_s)(Js2 + bs)\} + K_m K_b\,s] = K_m V_{a(s)}$$

$$\frac{\theta\,(s)}{V_{a(s)})} = \frac{K_m}{[\{(R_a + La_s)(Js2 + bs)\} + K_m K_b\,s]} \;\#\#$$

2.(a)



(a.)Let R_1 =167 kΩ, **(10.Marks)**

R_2 = 250 kΩ,

R_3 = 1 kΩ,

R_4 = 200 kΩ,

C = 1 μ F

2.(a)	$V_1 = V_3 = 0$ $C \,//\, R_1$ $\frac{1}{R_5} = \frac{1}{C} + \frac{1}{R_1}$ $= \frac{1}{1/CS} + \frac{1}{R_1}$ $= CS + \frac{1}{R_1}$ $= \frac{R_1CS+1}{R_1}$ $R_5 = \frac{R_1}{R_1CS+1}$ Apply KCL at node V_1, $i_{R_5} + i_{R_2} = 0$ $\frac{V_1 - V}{\frac{R_1}{R_1CS+1}} + \frac{V_1 - V_2}{R_2} = 0$ $\frac{-V}{\frac{R_1}{R_1CS+1}} - \frac{V_2}{R_2} = 0$ $\frac{-V}{\frac{R_1}{R_1CS+1}} = \frac{V_2}{R_2}$ $\frac{V_2(s)}{V(s)} = -\frac{R_2(R_1CS+1)}{R_1}$ $V_2(s) = -\frac{R_2(R_1CS+1)}{R_1} \text{ x } V(s)$ Apply KCL at node V_3 , $i_{R_3} + i_{R_4} = 0$ $\frac{V_3 - V_2}{R_3} + \frac{V_3 - V_0}{R_4} = 0$ $-\frac{V_2}{R_3} - \frac{V_0}{R_4} = 0$ $-\frac{V_2}{R_3} = \frac{V_0}{R_4}$ $\frac{V_0}{R_4} = \frac{\frac{R_2(R_1CS+1)}{R_1} xV(s)}{R_3}$ $\frac{V_0(s)}{V(s)} = \frac{R_2R_4(R_1CS+1)}{R_1R_3}$	

When $R_1 = 167\ k\Omega$, $R_2 = 250\ k\Omega$, $R_3 = 1\ k\Omega$, $R_4 = 200\ k\Omega$, $C = 1\ \mu F$

$$\frac{V_{o(s)}}{V(s)} = \frac{250\times10^{3}\times200\times10^{3}\left(167\times10^{3}\times1\times10^{6}\times s+1\right)}{167\times10^{3}\times1\times10^{3}}$$

$$= \frac{250\times200\times\left(0.167s+1\right)}{167}$$

$$= \frac{50\times10^{3}\times\left(0.167s+1\right)}{167} = 50s + 299.4 \quad \#\#$$

2.(b)

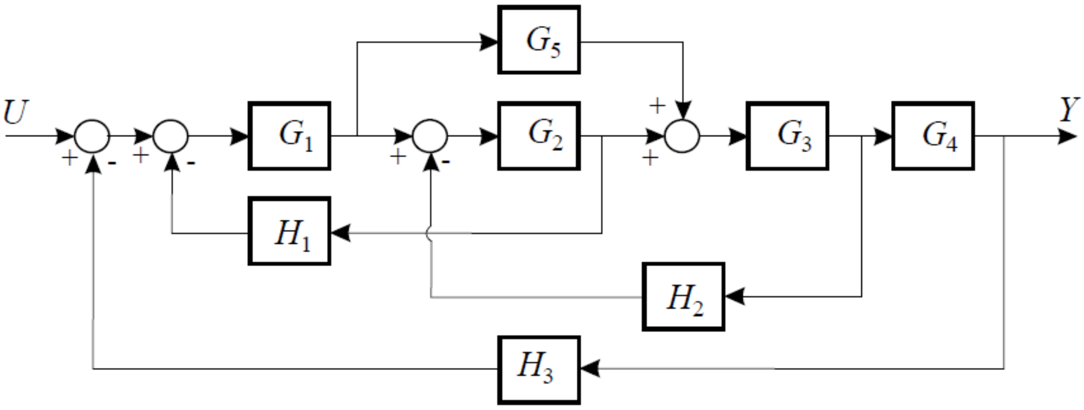
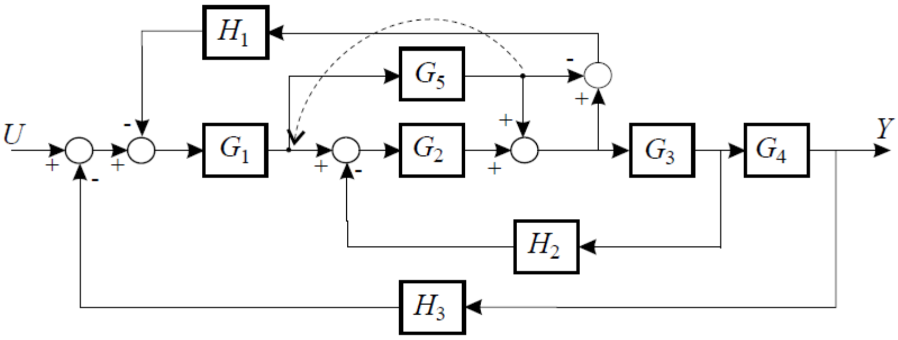
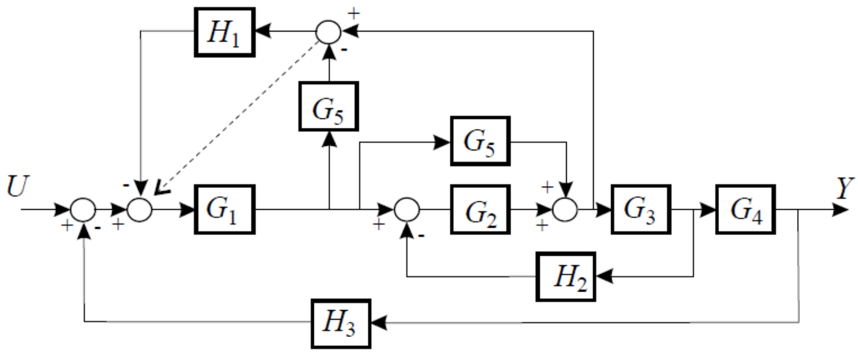


Figure 2.(b) : Multiple-loop feedback control system



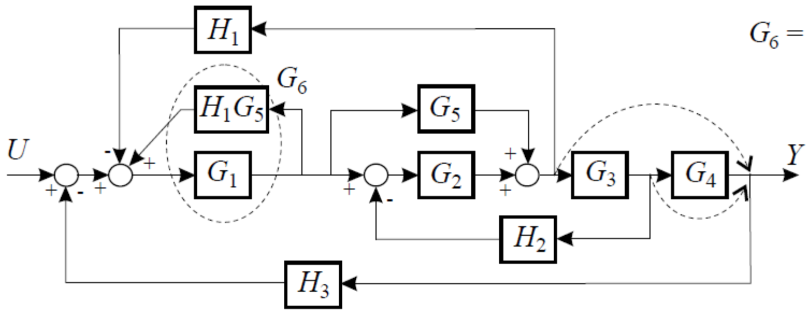
(b)



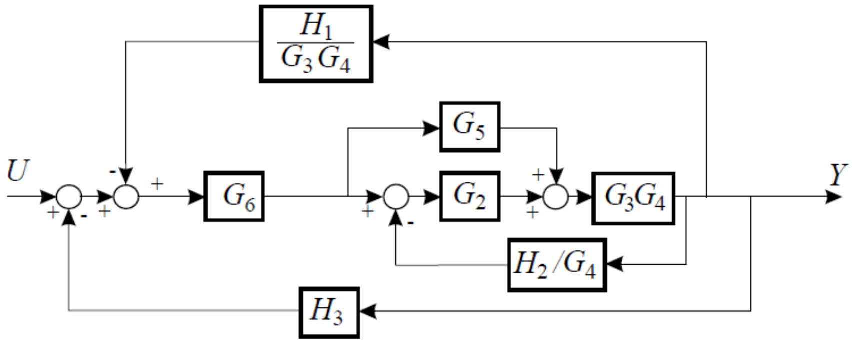
(c)

2.(b)

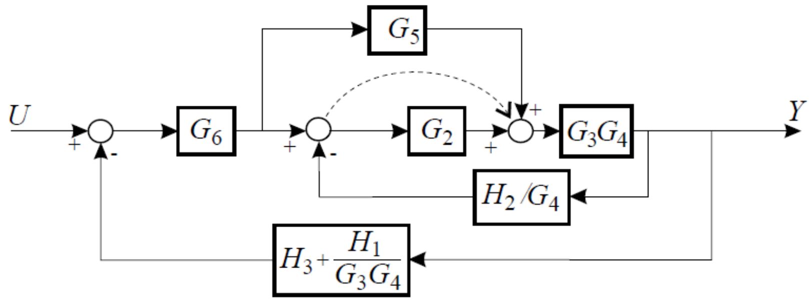
$$G_6 = \frac{G_1}{1 - G_1 H_1 G_5}$$



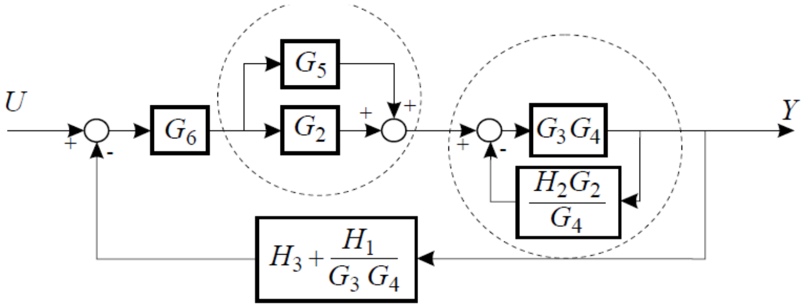
(d)



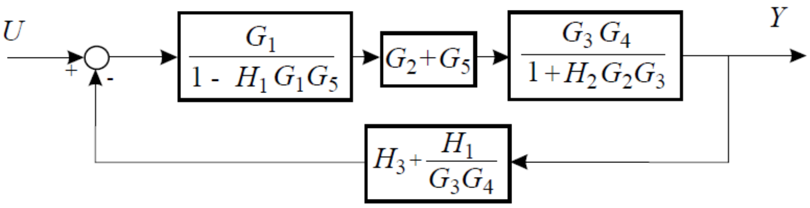
(e)



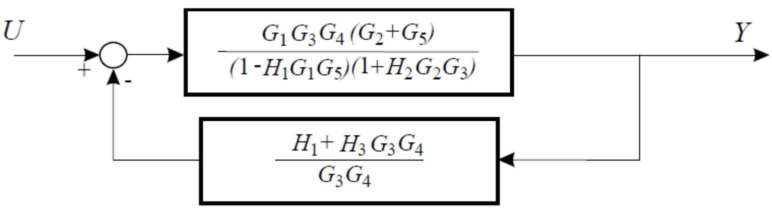
(f)



(g)



(h)



(i)

3.(a)	$\overset{\circ}{X} = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$ and $y = \begin{bmatrix} 10 & 0 \end{bmatrix} x$ $G(s) = Y(s) / U(s) = ?$ $A = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 10 & 0 \end{bmatrix}$ **$G(s) = C\,\phi(s)\,B$** **$\phi_{(s)} = (SI - A)^{-1}$** **$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} s & -2 \\ 3 & s+4 \end{bmatrix}$** **$\phi_{(s)} = (SI - A)^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s+4 & 2 \\ -3 & s \end{bmatrix}$** $\Delta(s) = (s+4)(s) - (2)(-3) = s^2 + 4s + 6$ **$G(s) = C\,\phi(s)\,B = \begin{bmatrix} 10 & 0 \end{bmatrix} x \frac{1}{\Delta(s)} \begin{bmatrix} s+4 & 2 \\ -3 & s \end{bmatrix} x \begin{bmatrix} 0 \\ 2 \end{bmatrix}$** $= \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 2s \end{bmatrix} \frac{1}{\Delta(s)} = \frac{40}{s^2 + 4s + 6}$ $G(s) = Y(s) / U(s) = \frac{40}{s^2 + 4s + 6}$ ## --	
3.(b)	$\overset{\circ}{X} = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$ and $y = \begin{bmatrix} 10 & 0 \end{bmatrix} x$ $G(s) = Y(s) / U(s) = ?$ $A = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 10 & 0 \end{bmatrix}$ **$G(s) = C\,\phi(s)\,B$** **$\phi_{(s)} = (SI - A)^{-1}$** **$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} s & -2 \\ 3 & s+4 \end{bmatrix}$** **$\phi_{(s)} = (SI - A)^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s+4 & 2 \\ -3 & s \end{bmatrix}$** $\Delta(s) = (s+4)(s) - (2)(-3) = s^2 + 4s + 6$	

$$\mathbf{G(s)= C\,\phi(s)\,B} = \begin{bmatrix}10 & 0\end{bmatrix} \times \frac{1}{\Delta(s)} \begin{bmatrix} s+4 & 2 \\ -3 & s \end{bmatrix} \times \begin{bmatrix}0\\2\end{bmatrix}$$

$$= \begin{bmatrix}10 & 0\end{bmatrix} \begin{bmatrix}4\\2s\end{bmatrix} \frac{1}{\Delta(s)} = \frac{40}{s^2+4s+6}$$

$$G(s) = Y(s)/U(s) = \frac{40}{s^2+4s+6} \quad \#\#$$

4.

$$A = \begin{bmatrix} 0 & 6 \\ -3 & -5 \end{bmatrix}$$

$$\phi_{(s)} = (SI - A)^{-1}$$

$$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} s & -6 \\ 3 & s+5 \end{bmatrix}$$

$$\phi_{(s)} = (SI - A)^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s+5 & 6 \\ -3 & s \end{bmatrix}$$

$\Delta(s) = (s+5)(s) - (6)(-3) = s^2 + 5s + 18 = (s+2.5 + 3.43j)(s+2.5 - 3.43j)$

The roots of the characteristic equation; $s = -2.5 - 3.43j,\ -2.5 + 3.43j$

$$\phi_{(s)} = (SI - A)^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s+5 & 6 \\ -3 & s \end{bmatrix} = \frac{1}{s^2+5s+18} \begin{bmatrix} s+5 & 6 \\ -3 & s \end{bmatrix}$$

$$\phi 11_{(s)} = \frac{s+5}{s^2+5s+18} = \frac{s+5}{(s+2.5+3.43j)(s+2.5-3.43j)}$$

$$= \frac{A}{(s+2.5+3.43j)} + \frac{B}{(s+2.5-3.43j)}$$

$$\mathbf{A} = \left.\frac{(s+5)(s+2.5+3.43j)}{(s+2.5+3.43j)(s+2.5-3.43j)}\right|_{s=(-2.5-3.43j)}$$

$$\mathbf{A} = \frac{-2.5-3.43j+5}{(-2.5-3.43j+2.5-3.43j)} = \frac{2.5-3.43j}{-6.86j} = 0.5 + 0.36j$$

$$\mathbf{B} = 0.5 - 0.36j$$

$$\phi_{11(s)} = \frac{0.5+0.36j}{(s+2.5+3.43j)} + \frac{0.5-0.36j}{(s+2.5-3.43j)}$$

$$= (0.5 + 0.36j)e^{-2.5-3.43j\,t} + (0.5 - 0.36j)e^{-2.5+3.43j\,t}$$

$$\phi_{11(t)} = (-0.72 + 1j)e^{-2.5t} \sin 3.43t$$

$$\phi_{12(s)} = \frac{6}{s^2+5s+18} = \frac{A}{(s+2.5+3.43j)} + \frac{B}{(s+2.5-3.43j)}$$

$$A = \left.\frac{6(s+2.5+3.43j)}{(s+2.5+3.43j)(s+2.5-3.43j)}\right|_{s=(-2.5-3.43j)}$$

$$= \frac{6}{(-2.5-3.43j+2.5-3.43j)} = \frac{6}{-6.86j} = 0.875j$$

$$B = -0.875j$$

$$\phi_{12(s)} = \frac{0.875j}{(s+2.5+3.43j)} + \frac{-0.875j}{(s+2.5-3.43j)}$$

$$= (0.875j)\, e^{-2.5-3.43j}{}^{t} + (0.875\,j) e^{-2.5+3.43j}{}^{t}$$

$$= (1.75)\ e^{-2.5t} \sin 3.43t$$

$$\Phi_{21(s)} = \frac{-3}{s^2+5s+18} = \frac{A}{(s+2.5+3.43j)} + \frac{B}{(s+2.5-3.43j)}$$

$$A = \frac{(-3)(s+2.5+3.43j)}{(s+2.5+3.43j)(s+2.5-3.43j)}\Bigg|_{s=(-2.5-3.43j)}$$

$$= \frac{-3}{(-2.5-3.43j+2.5-3.43j)} = \frac{-3}{-6.86j} = -0.437j$$

$$B = 0.437j$$

$$\Phi_{21(s)} = \frac{-0.437j}{(s+2.5+3.43j)} + \frac{0.437j}{(s+2.5-3.43j)}$$

$$= (-0.437j)\, e^{-2.5-3.43j}{}^{t} + (0.437j) e^{-2.5+3.43j}{}^{t}$$

$$= (-0.87)\ e^{-2.5t} \sin 3.43t$$

$$\Phi_{22(s)} = \frac{s}{s^2+5s+18} = \frac{A}{(s+2.5+3.43j)} + \frac{B}{(s+2.5-3.43j)}$$

$$A = \frac{s(s+2.5+3.43j)}{(s+2.5+3.43j)(s+2.5-3.43j)}\Bigg|_{s=(-2.5-3.43j)}$$

$$= \frac{-2.5-3.43j}{(-2.5-3.43j+2.5-3.43j)} = \frac{-2.5-3.43j}{-6.86j} = 0.5 \text{ “ } 0.36j$$

$$B = 0.5 + 0.36$$

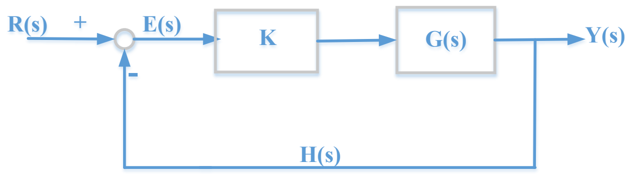
$$\Phi_{22(s)} = \frac{0.5 - 0.36j}{(s+2.5+3.43j)} + \frac{0.5+0.36j}{(s+2.5-3.43j)}$$

$$= (0.5 - 0.36j)\, e^{-2.5-3.43j}{}^{t} + (0.5 + 0.36j) e^{-2.5+3.43j}{}^{t}$$

$$\Phi_{22(t)} = (0.72 + j)\ e^{-2.5t} \sin 3.43t$$

$$\Phi_{(t)} = \begin{vmatrix} (-0.72 + 1j)e^{-2.5t} \sin 3.43t & (1.75)\ e^{-2.5t} \sin 3.43t \\ (-0.87)\ e^{-2.5t} \sin 3.43t & (0.72 + j)\ e^{-2.5t} \sin 3.43t \end{vmatrix} \#\#$$

$G(s) = \frac{10}{s(\tau s+1)}$, $\tau = 0.001$ second



(a) e_{ss} =?

$e_{ss} = \lim_{s\to 0} sE(s)$

$E_{(s)} = R(s)\,[\,1 - T_{(s)}\,]$

$$T_{(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{KG(s)}{1+KG(s)}$$

$$= \frac{K[\frac{10}{s(\tau s+1)}]}{1+[\frac{K10}{s(\tau s+1)}}$$

$$= \frac{K10}{s(\tau s+1)} \text{ X } \frac{s(\tau s+1)}{s(\tau s+1)+10K}$$

$$= \frac{10K}{s(\tau s+1)+10K}$$

$E_{(s)} = R_{(s)}\,[\,1 - T(s)\,]$

$$= \frac{1}{s}\,[\,1 - \frac{10K}{s(\tau s+1)+10K}\,] = \frac{1}{s}\,[\,\frac{s(\tau s+1)+10K - 10K}{s(\tau s+1)+10K}\,]$$

$$= \frac{1}{s}\,[\,\frac{s(\tau s+1)}{s(\tau s+1)+10K}\,]$$

$e_{ss} = \lim_{s\to 0} sE(s)$

$$= \lim_{s\to 0} s \times \frac{1}{s}\,[\,\frac{s(\tau s+1)}{s(\tau s+1)+10K}\,] \;=\; 0 \text{ ###}$$

(b) K = ?, e_{ss} = 0.1 mm = 0.01cm

$R_{(s)} = \frac{1}{s^2} \times 20$ cm/sec

$E_{(s)} = R_{(s)}\,[\,1 - T_{(s)}\,]$

$$=\frac{20}{s^2}\left[1-\frac{10K}{s(\tau s+1)+10K}\right]$$

$$=\frac{20}{s^2}\left[\frac{s(\tau s+1)}{s(\tau s+1)+10K}\right]$$

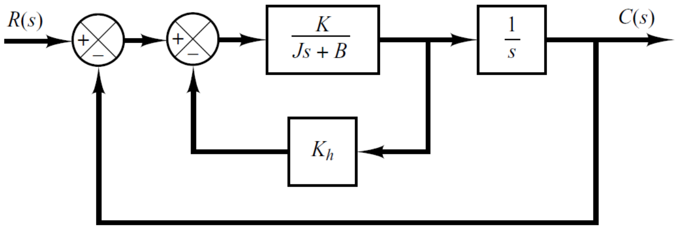
$$e_{ss}=\lim_{s\to 0} sE(s)$$

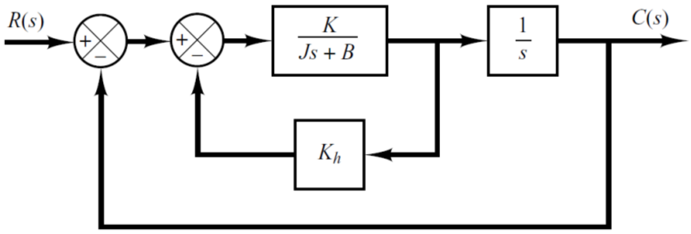
$$0.01=\lim_{s\to 0} s \times \frac{20}{s^2}\left[\frac{s(\tau s+1)}{s(\tau s+1)+10K}\right]$$

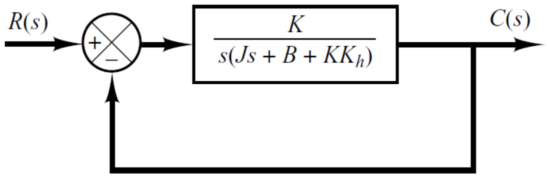
$$0.01=\frac{1}{K}$$

$K = 100$ ###

5. ---







Since J = 1 kgm^2, $\frac{C(s)}{R(s)}=\frac{K}{Js^2+(B+KK_h)s+K}$

$$\frac{C(s)}{R(s)}=\frac{K}{s^2+(1+KK_h)s+K}$$

$$\frac{C(s)}{R(s)}=\frac{K}{Js^2+(B+KK_h)s+K}$$

Compare with $\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n=\sqrt{K}$ $\qquad \zeta=\frac{(1+KK_h)}{2\sqrt{K}}$

Maximum overshoot in the unit-step response is 0.2

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

$$\ln(e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}) = \ln(0.2)$$

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

$\zeta = 0.456$,

The peak time is 1 sec,

$$t_p = \frac{\pi}{\omega_d}$$

$$1 = \frac{3.141}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = \frac{3.141}{\sqrt{1-0.456^2}}$$

$$\omega_n = 3.53$$

$$\omega_n = \sqrt{K}$$

$$3.53 = \sqrt{K}$$

$$3.53^2 = K$$

$$K = 12.5$$

$$\zeta = \frac{(1+KK_h)}{2\sqrt{K}}$$

$$0.456 \times 2\sqrt{12.5} = (1+12.5K_h)$$

$$K_h = 0.178$$

--End--