Four Mecanum Wheeled Mobile Robot along X axis

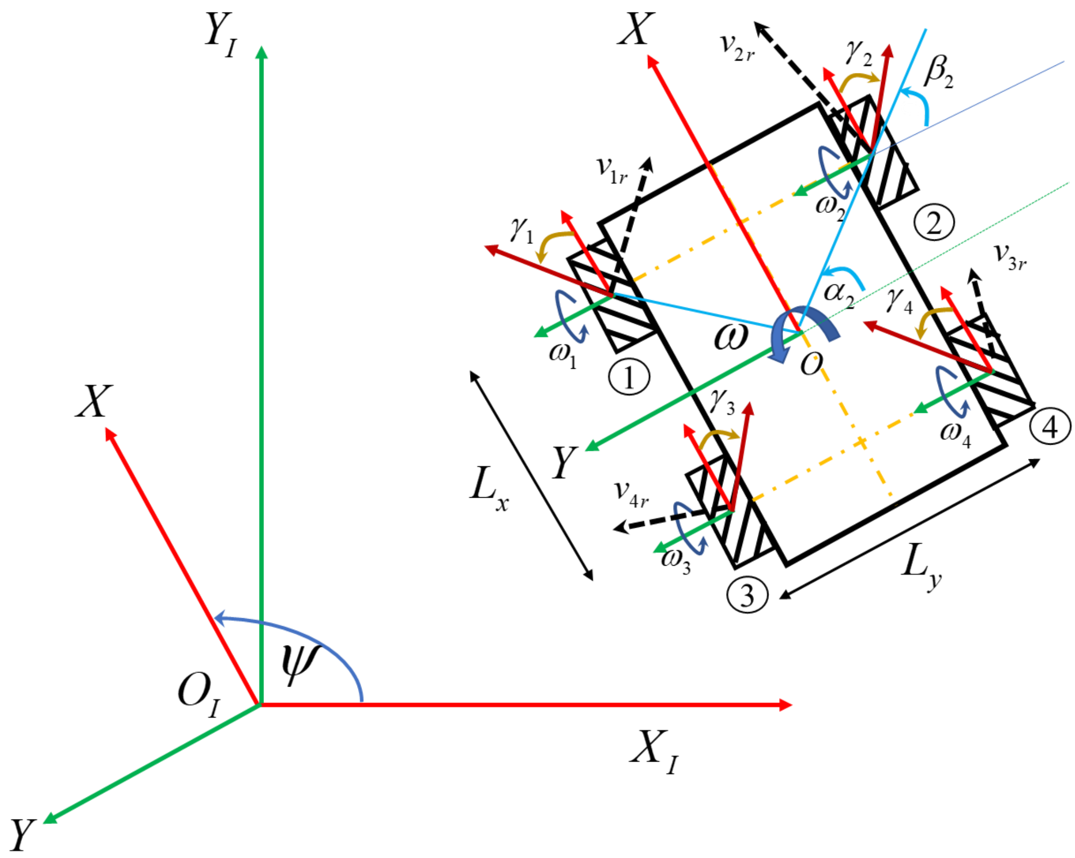


Fig. 1. Four Mecanum Wheeled Mobile Robot

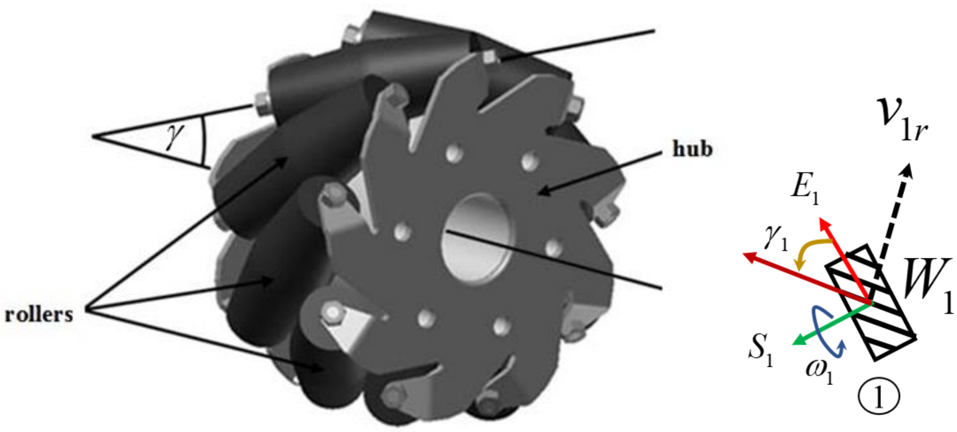


Fig. 2. Mecanum Wheel

Since the roller is free to roll in the direction indicated by the green line, normal to the roller's axis, there is potentially arbitrary velocity in that direction. A desired velocity v can be resolve into two components:

1. One parallel to the direction of wheel motion, and
2. One parallel to the rolling direction.

Calculate the velocity of the wheel *i*.

Consider the rolling wheel with a point of contact *P* with the ground,

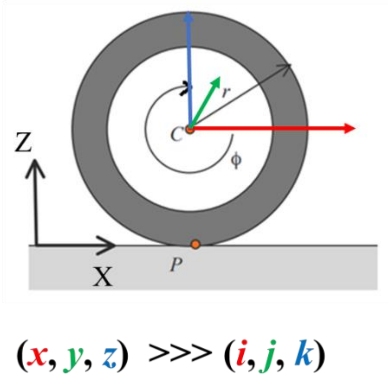
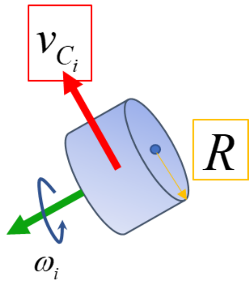


Fig. 3. Wheel Frame

Since it is rolling, the velocity of the point P is zero ($v_P = 0$).

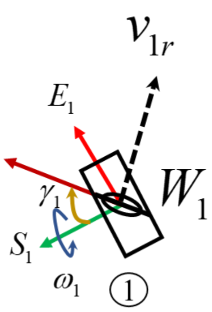


The velocity of the point C of the wheel can be obtained from the vector equation:

$$v_{C_i} = v_{P_i} + \omega_i \hat{j} \times R\hat{k} = R\omega_i \hat{i}$$

and the tangential velocity of the free roller attached to the wheel touching the floor:

$$v_{ir} = \frac{v_{C_i}}{\cos \gamma_i}$$



Tangential velocity of the roller:

$$v_{E_i} = v_{C_i} + v_{ir} \cos \gamma_i = R\omega_i + v_{ir} \cos \gamma_i$$

Lateral velocity of the roller:

$$v_{S_i} = v_{ir} \sin \gamma_i$$

From these two equations,

$$v_{E_i} = v_{C_i} + v_{ir} \cos \gamma_i = R\omega_i + v_{ir} \cos \gamma_i$$

$$v_{S_i} = v_{ir} \sin \gamma_i$$

In matrix form,

$$\begin{bmatrix} v_{E_i} \\ v_{S_i} \end{bmatrix} = \begin{bmatrix} R & \cos\gamma_i \\ 0 & \sin\gamma_i \end{bmatrix} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix}$$

The transformation matrix from velocities of the wheel to its center is

$$T_P^W = \begin{bmatrix} R & \cos\gamma_i \\ 0 & \sin\gamma_i \end{bmatrix}$$

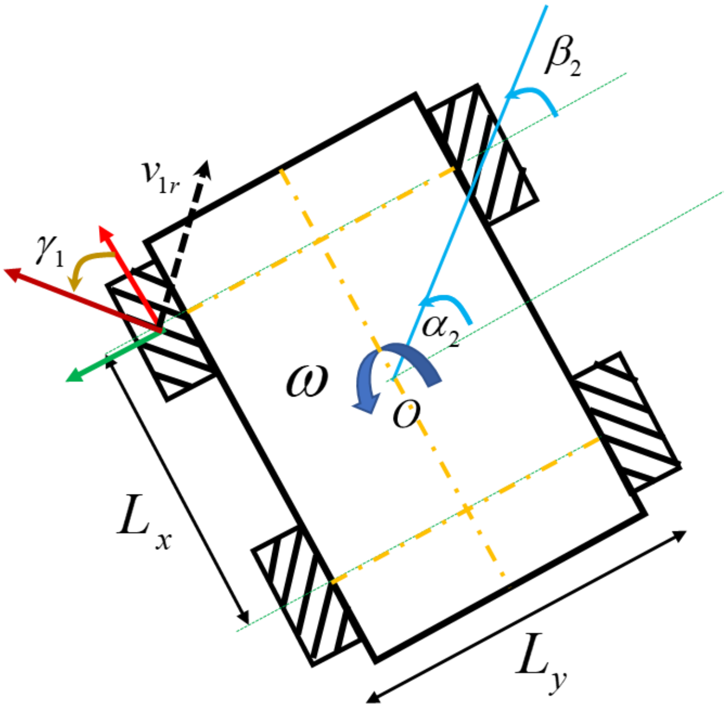


Fig. 4. Frames of the robot

$$\alpha_i = \begin{bmatrix} 45^\circ & -45^\circ & 135^\circ & -135^\circ \end{bmatrix}$$

$$L_{ix} = \frac{L_x}{2} sign(\cos\alpha_i)$$

$$L_{iy} = \frac{L_y}{2} sign(\sin\alpha_i)$$

$$v_O = 0$$

$$v_1 = v_O + \omega\hat{k} \times \begin{bmatrix} L_{ix} & L_{iy} & 0 \end{bmatrix}$$

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -L_{iy} \\ 0 & 1 & L_{ix} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

The transformation matrix from velocities of the wheel to its center is

$$T_W^O = \begin{bmatrix} 1 & 0 & -L_{iy} \\ 0 & 1 & L_{ix} \end{bmatrix}$$

$$\begin{bmatrix} v_{E_i} \\ v_{S_i} \end{bmatrix} = \begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix}$$

$$\begin{bmatrix} R & \cos\gamma_i \\ 0 & \sin\gamma_i \end{bmatrix} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -L_{iy} \\ 0 & 1 & L_{ix} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = \begin{bmatrix} R & \cos\gamma_i \\ 0 & \sin\gamma_i \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -L_{iy} \\ 0 & 1 & L_{ix} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = \begin{bmatrix} R & \cos\gamma_i \\ 0 & \sin\gamma_i \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -L_{iy} \\ 0 & 1 & L_{ix} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & -\frac{\cos\gamma_i}{R\sin\gamma_i} & -L_{iy} \\ 0 & \frac{1}{\sin\gamma_i} & L_{ix} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

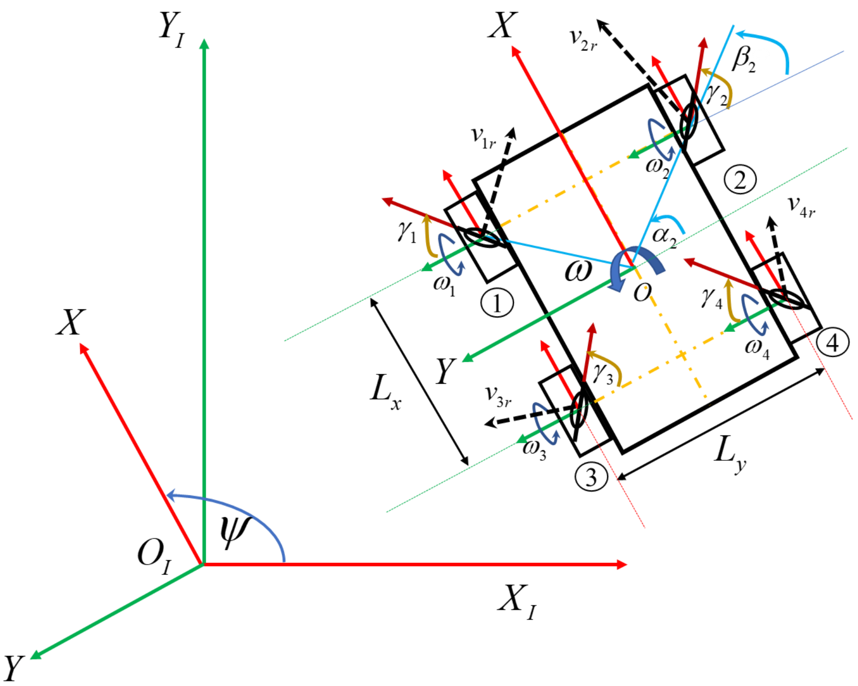
Because the rollers rotate passively, the velocity of roller is an uncontrollable variable, which is usually not taken into account. Therefore, the robot's base velocity (at point O) related to the rotational velocity of the i^{th} wheel is

$$\omega_i = \frac{1}{R} v_x - \frac{\cos\gamma_i}{R\sin\gamma_i} v_y - \left(\frac{1}{R} L_{iy} + \frac{\cos\gamma_i}{R\sin\gamma_i} L_{ix} \right) \omega$$

In matrix form,

$$\omega_i = \frac{1}{R} \begin{bmatrix} 1 & -\cot\gamma_i & -\left(L_{iy} + \cot\gamma_i L_{ix} \right) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

A single mecanum wheel does not allow any control in the rolling direction but for three or more mecanum wheels, suitably arranged, the motion in the rolling direction of any one wheel will be driven by the other wheels.



Wheel assignments are

$$\gamma_i = \begin{bmatrix} 45^\circ & -45^\circ & -45^\circ & 45^\circ \end{bmatrix}$$

$$\alpha_i = \begin{bmatrix} 45^\circ & -45^\circ & 135^\circ & -135^\circ \end{bmatrix}$$

$$L_{ix} = \begin{bmatrix} \frac{L_x}{2} & \frac{L_x}{2} & -\frac{L_x}{2} & -\frac{L_x}{2} \end{bmatrix}$$

$$L_{iy} = \begin{bmatrix} \frac{L_y}{2} & -\frac{L_y}{2} & \frac{L_y}{2} & -\frac{L_y}{2} \end{bmatrix}$$

Calculating the inverse Jacobian matrix combining all four mecanum wheels,

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & -\cot\gamma_1 & -\left(L_{1y} + \cot\gamma_1 L_{1x}\right) \\ 1 & -\cot\gamma_2 & -\left(L_{2y} + \cot\gamma_2 L_{2x}\right) \\ 1 & -\cot\gamma_3 & -\left(L_{3y} + \cot\gamma_3 L_{3x}\right) \\ 1 & -\cot\gamma_4 & -\left(L_{4y} + \cot\gamma_4 L_{4x}\right) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & -\cot\gamma_1 & -\left(L_{1y} + \cot\left(45^\circ\right) L_{1x}\right) \\ 1 & -\cot\gamma_2 & -\left(L_{2y} + \cot\left(-45^\circ\right) L_{2x}\right) \\ 1 & -\cot\gamma_3 & -\left(L_{3y} + \cot\left(-45^\circ\right) L_{3x}\right) \\ 1 & -\cot\gamma_4 & -\left(L_{4y} + \cot\left(45^\circ\right) L_{4x}\right) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & -1 & -\frac{L_x + L_y}{2} \\ 1 & 1 & \frac{L_x + L_y}{2} \\ 1 & 1 & -\frac{L_x + L_y}{2} \\ 1 & -1 & \frac{L_x + L_y}{2} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix}$$

Finally, the inverse Jacobian matrix of a four-Mecanum-Wheeled mobile robot is

$$J_{IK} = \frac{1}{R} \begin{bmatrix} 1 & -1 & -\frac{L_x + L_y}{2} \\ 1 & 1 & \frac{L_x + L_y}{2} \\ 1 & 1 & -\frac{L_x + L_y}{2} \\ 1 & -1 & \frac{L_x + L_y}{2} \end{bmatrix}$$

Then, the forward Jacobian matrix is the pseudo inverse of J_{IK} ,

$$J_{IK} = \frac{1}{R} \begin{bmatrix} 1 & -1 & -\frac{L_x + L_y}{2} \\ 1 & 1 & \frac{L_x + L_y}{2} \\ 1 & 1 & -\frac{L_x + L_y}{2} \\ 1 & -1 & \frac{L_x + L_y}{2} \end{bmatrix}$$

$$J_{IK}{}^{T} = \frac{1}{R} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{L_x + L_y}{2} & \frac{L_x + L_y}{2} & -\frac{L_x + L_y}{2} & \frac{L_x + L_y}{2} \end{bmatrix}$$

$$J_{IK}{}^{T} \times J_{IK} = \frac{4}{R^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\left(L_x + L_y\right)^2}{4} \end{bmatrix}$$

$$\left(J_{IK}{}^{T}\times J_{IK}\right)^{-1}=\frac{R^{2}}{4}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \dfrac{4}{\left(L_{x}+L_{y}\right)^{2}}\end{bmatrix}$$

$$J_{FK}=\left(J_{IK}{}^{T}\times J_{IK}\right)^{-1}\times J_{IK}{}^{T}=\frac{R^{2}}{4}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \dfrac{4}{\left(L_{x}+L_{y}\right)^{2}}\end{bmatrix}\times\frac{1}{R}\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & 1 & -1\\ -\dfrac{L_{x}+L_{y}}{2} & \dfrac{L_{x}+L_{y}}{2} & -\dfrac{L_{x}+L_{y}}{2} & \dfrac{L_{x}+L_{y}}{2}\end{bmatrix}$$

$$J_{FK}=\frac{R}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & 1 & -1\\ -\dfrac{2}{L_{x}+L_{y}} & \dfrac{2}{L_{x}+L_{y}} & -\dfrac{2}{L_{x}+L_{y}} & \dfrac{2}{L_{x}+L_{y}}\end{bmatrix}$$