MINISTRY OF EDUCATION

MANDALAY TECHNOLOGICAL UNIVERSITY

Department of Mechatronic Engineering

2018-2019 Academic Year

Fourth Year **Second Semester Examination**

McE-42077 Control Engineering II

Date: 25.9.2019(WED) **Time: 1:00 to 4:00 pm**

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1.(a) (a)The goal of vertical takeoff and landing (VTOL) aircraft is to achieve operation from relatively small airports and yet operate as a normal aircraft in level flight. A control system using adjustable jets can control the vehicle, as shown in Figure.1 (a). (a) Determine the range of gain for which the system is stable, (b) Determine the gain K for which the system is marginally stable and the roots of the characteristic equation for this value of K. (10.Marks)

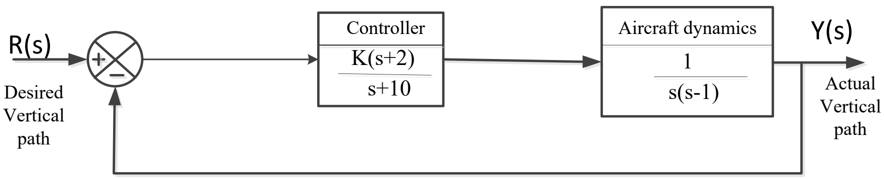


Figure.1.(a)

Solution:

$$G(s) = \frac{K(s+2)}{(s+10)} \times \frac{1}{s(s-1)} , \quad H(s) = 1$$

$$= \frac{K(s+2)}{(s+10)(s-1)s}$$

(a.) Characteristics Equation,

$1 + H(s)G(s) = 0$ 2.5

$$1 + \frac{K(s+2)}{(s+10)(s-1)s} = 0$$

$$\frac{(s+10)(s-1)s + K(s+2)}{(s+10)(s-1)s} = 0$$

$$(s+10)(s-1)s + K(s+2) = 0$$

$s^3 + 9s^2 - 10s + Ks + 2K = 0$

Using Routh-hurwitz,

S^3	1	$K-10$	2.5
S^2	9	$2K$	
S^1	$\frac{9K-90-2K}{9}$	0	
S^0	$2K$	0	

For the system to be stable, 2.5

$$\frac{7K-90}{9} > 0$$

$$7K - 90 > 0$$

$$K > \frac{90}{7}$$

$$2K > 0$$

$$K > 0$$

(b.) When $K = \frac{90}{7}$

The system is marginally stable, 2.5

S^2 row .... $q(s) = 9s^2 + 2K = 0$

$$9s^2 + 2\left(\frac{90}{7}\right) = 0$$

$$9s^2 = -\frac{180}{7}$$

$$s^2 = -\frac{180}{7} \times \frac{1}{9} = \frac{-20}{7}$$

$$s = \pm j\sqrt{\frac{20}{7}}$$

The root of the characteristic equation is,

	$S_{1,2} = \pm j\sqrt{\frac{20}{7}}$	
1.(b)	A unity feedback system has a loop transfer function $$L(s) = \frac{K}{(s+1)(s+3)(s+5)}$$ where K = 30. Find the roots of the closed-loop system's characteristic equation. **<u>Solution:</u>** $$L(s) = \frac{K}{(s+1)(s+3)(s+5)}$$ $$T(s) = \frac{L(s)}{1+L(s)H(s)}$$ $$= \frac{K}{(s+1)(s+3)(s+5)} \text{ x } \frac{(s+1)(s+3)(s+5)}{(s+1)(s+3)(s+5)+K}$$ $$= \frac{K}{(s+1)(s+3)(s+5)+K}$$ When K = 30; $$T(s) = \frac{30}{(s+1)(s+3)(s+5)+30}$$ $$= \frac{30}{(s^2+4s+3)(s+5)+30}$$ $$= \frac{30}{(s^2+4s+3)(s+5)+30}$$ $$= \frac{30}{(s^3+9s^2+23s+15)+30}$$ $$= \frac{30}{(s^3+9s^2+23s+45)}$$ $$s^3 + 9s^2+23s+45 = 0$$ X_1 = -6.53 X_2 = -1.23 + j2.32 X_3 = -1.23 – j 22.32 The roots of the closed loop system; s_1 = -6.53	**(10.Marks)** 2 2 2 2

<table>
<tr><td></td><td>$s_2 = -1.23 + j2.32$
$s_3 = -1.23 - j\,2.32$###</td><td>2</td></tr>
<tr><td>2.</td><td>A unity feedback system has the loop transfer function
$$L(s) = KG(s) = \frac{K(s+4)}{s(s+2)}$$
(a) Find the breakaway and entry points on the real axis.
(b) Find the gain and the roots when the real part of the complex roots is located at - 2 .
(c) Sketch the locus.

<u>Solution:</u>
(a) Breakaway and entry points on the real axis.
$$L(s) = KG(s) = \frac{K(s+4)}{s(s+2)}$$
$1 + KG(s) = 0$
$1 + \frac{K(s+4)}{s(s+2)} = 0$
$K = \left| -\frac{s(s+2)}{(s+4)} \right|$
$\frac{dK}{ds} = 0$
$\frac{d}{ds}\frac{s(s+2)}{(s+4)} = 0$
$\frac{(s+4)(2s+2) - s(s+2)}{(s+4)2} = 0$
$2s^2 + 2s + 8s + 8 - s^2 - 2s = 0$
$s^2 + 8s + 8 = 0$
s= -1.17 and s= -6.83
Breakaway point , s= -1.17
Entry point, s = -6.83

(b.)Let the two roots be
$S = -2 + aj$
$S = -2 - aj$</td><td>(20.Marks)
1
1
2
1
1
1</td></tr>
</table>

$(s+2-aj)\,(s+2+aj) = 0$

1

$(s+2)^2 - (aj)^2 = 0$

$s^2 + 4s + 4 + a^2 = 0$

$s^2 + 4s + (4 + a^2) = 0$ 1

Compare with $s^2 + (2 + K)\,s + 4K = 0$

$2 + K = 4$

$K = 2$, 1

$4K = 4 + a^2$

$4(2) = 4 + a^2$

$a^2 = \pm 2$

So, the roots are (-2 – 2j) and (- 2 + 2j).

The gain, K = 2.

2

(c.) The characteristic equation ,

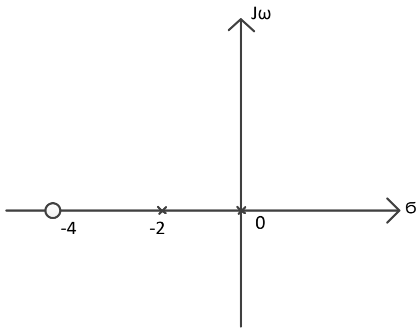
$$1 + \frac{K(s+4)}{s(s+2)} = 0$$

When K=0, poles at s=0 and s= -2

$\therefore$ np = 2

When K=∞, zeros at s=-4

$\therefore$ nz = 1 1



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Number of separate loci SL=n=2, M=1

The root loci are symmetric with the horizontal axis.

Segment of root loci at s=0 and s= -2

1

Center of Asymptotes,

$$б_A = \frac{\sum(-p) - \sum(-z)}{n_p - n_z}$$

$$= \frac{0-2-(-4)}{2-1} = 2$$ 1

Angle of Asymptotes,

$$\Phi_A = \frac{2q+1}{n_p - n_z} \times 180 \qquad q = n - M - 1 = 2 - 1 - 1 = 0$$

1

$$\Phi_A = \frac{2(0)+1}{2-1} \times 180 \quad = 180$$

Imaginary crossing point,

s (s+2) + k (s+4) = 0

$s^2 + 2s + Ks + 4K = 0$

$s^2 + (2 + K)s + 4K = 0$

1

Routh array is

s^2	1	4K
s^1	2+K	0
s^0	4K	0

1

The system to be stable,

2 + K = 0 , 4K = 0

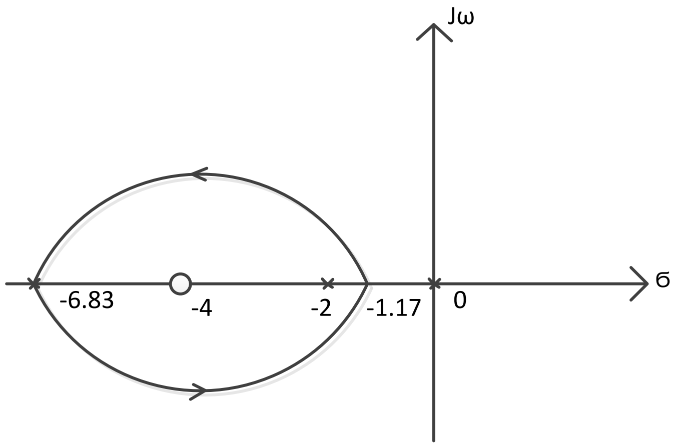
K = - 2 K = 0

K = - 2 is impossible.

∴ K = 0

------------ Asymptotes

__________Root Locus



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3.(a)	(a) A specific closed-loop control system is to be designed for an underdamped response to a step input. The specifications for the system are as follows: 20% < percent overshoot < 30%, Settling time < 0.7 s. (a) Identify the desired area for the dominant roots of the system, (b) Determine the smallest value of a third root; if the complex conjugate roots are to represent the dominant response.	**(15 marks)**

Solution

(a) Identify the desired area for the dominant roots of the system,

100* e(-$\pi\zeta$ / $\sqrt{1-\zeta^2}$) < 30%

e(-$\pi\zeta$ / $\sqrt{1-\zeta^2}$) < 0.3

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} < \ln 0.3$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} < -1.204$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} > 1.204$$

Squaring on both sides,

$$\frac{\pi^2\zeta^2}{1-\zeta^2} > 1.45$$

$$\frac{\pi^2\zeta^2}{1.45} > 1-\zeta^2$$

$6.807\,\zeta^2 > 1-\zeta^2$

$7.807\,\zeta^2 > 1$

$\zeta^2 > 0.128$

$\zeta > 0.358$

$\zeta = \cos\Theta$

$\Theta = 69^\circ$

$\cos\Theta > 0.46$

$\Theta > 69^\circ$

100* e($-\pi\zeta / \sqrt{1-\zeta^2}$) > 20%

e($-\pi\zeta / \sqrt{1-\zeta^2}$) > 0.2

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} > \ln 0.2$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} > -1.609$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} < 1.609$$

Squaring on both sides,

$$\frac{\pi^2\zeta^2}{1-\zeta^2} < 2.589$$

$$\frac{\pi^2\zeta^2}{1.45} < 1-\zeta^2$$

$3.812\,\zeta^2 < 1-\zeta^2$

$4.812\,\zeta^2 < 1$

$\zeta^2 < 0.208$

$\zeta < 0.456$

$\zeta = \cos\Theta$

$\Theta = 63^\circ$

$\cos\Theta < 0.456$

$\Theta < 63^\circ$

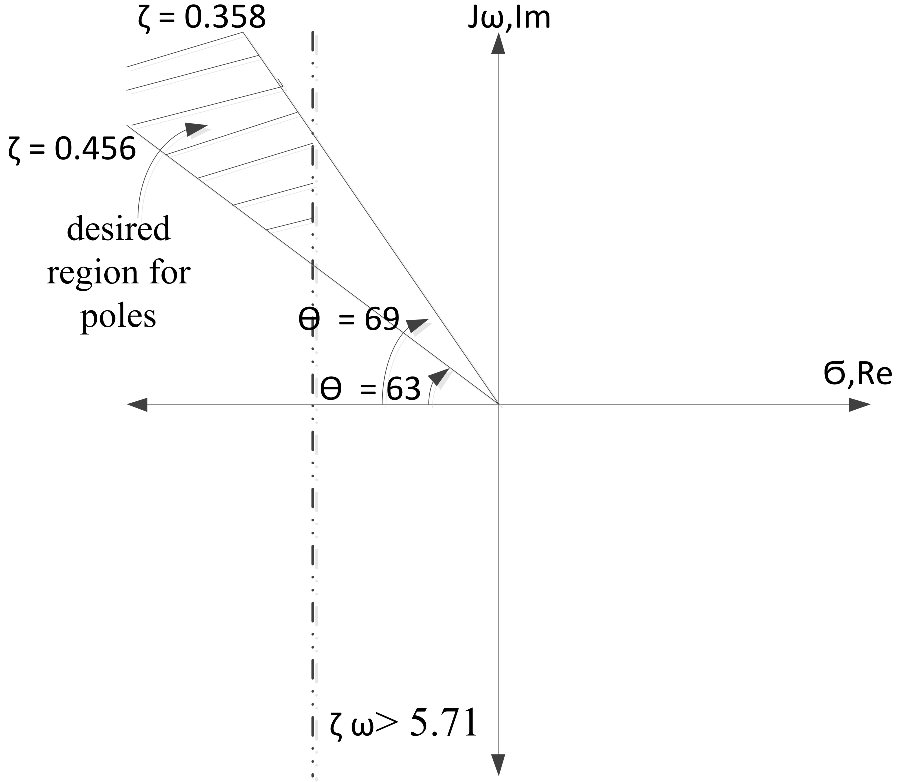
$T_s < 0.7$ sec

$$\frac{4}{\zeta\omega} < 0.7$$

$\zeta\omega > 5.71$

$\zeta < 0.456 \leftrightarrow \omega = 12.5$

$\zeta < 0.358 \leftrightarrow \omega = 15.9$



(b.) The third root should be at least 10 times farther in the left half-plane,so

$$|r_3| \geq 10\,|\zeta\,\omega|$$

$$|r_3| \geq 10 \times 5.71$$

$$r_3 \geq 57.1$$

$$r_3 = -\ 57.1 \quad (\text{ for stable})$$

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3.(b) A unity feedback control system shown in Figure.3.(b) has the process below, design a PID controller by **using Ziegler-Nichols tuning method**. **(15.Marks)**

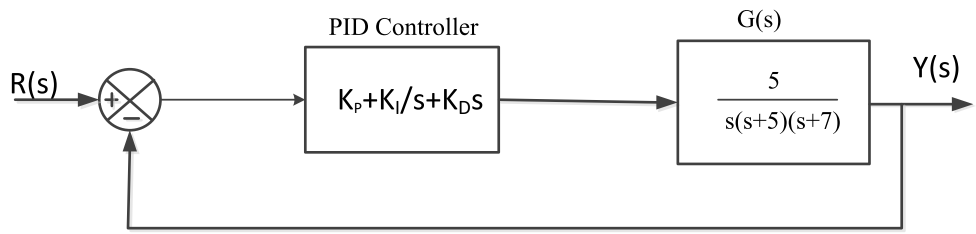

Figure.3.(b) A Unity Feedback Control System

Solution

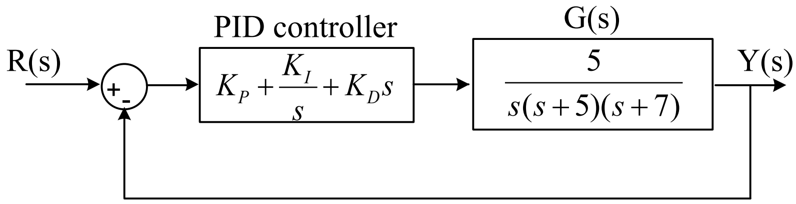

Figure-1 : A unity feedback control system

The characteristic equation of the system is $1 + K_U \dfrac{5}{s(s+5)(s+7)} = 0$

$$s^3 + 12s^2 + 35s + 5K_U = 0$$

Routh array

$$\begin{array}{c|cc} s^3 & 1 & 35 \\ s^2 & 12 & 5\mathrm{K}_U \\ s^1 & \dfrac{12\times 35 - 5\mathrm{K}_U}{12} & 0 \\ s^0 & 5\mathrm{K}_U & \end{array}$$

$$\frac{12\times 35 - 5\mathrm{K}_U}{12} \geq 0 \qquad , \qquad 5\mathrm{K}_U \geq 0 \quad \therefore \mathrm{K}_U \geq 0$$

$$\mathrm{K}_U \leq 84$$

$$\therefore \mathrm{K}_U = 84$$ 5

Auxiliary equation is $12\mathrm{s}^2 + 5\mathrm{K}_U = 0$

$$12\mathrm{s}^2 = 420$$

$$s = \pm j\sqrt{35} = \pm j\omega$$

$$\therefore \omega = \sqrt{35}$$

$$\omega T_U = 2\pi$$ 3

$$T_U = \frac{2\pi}{\sqrt{35}} = 1.06 \text{ seconds}$$

2

The gains of PID Controller by Ziegler-Nichols method are

	$K_P = 0.6K_U = 0.6\times 84 = 50.4$ $K_I = \frac{1.2K_U}{T_U} = \frac{1.2\times 84}{1.06} = 95.094$ $K_D = \frac{1.2K_U T_U}{8} = \frac{1.2\times 84\times 1.06}{8} = 13.356$ $\underline{\text{Ans: the gains of PID Controller, } K_P = 50.4, K_I = 95.094, K_D = 13.356}$	5
4.	Design the **lower order transfer function** of the given system. Determine the **percent overshoot and the peak time** of the lower order transfer function. The overall transfer function of the system is $G_H(s) = \frac{7}{s^3+6s^2+11s+7}$ **Solution:** . $G_H(s) = \frac{7}{s^3 + 6s^2 + 11s + 7}$ $G_H(s) = \frac{7}{1 + \frac{11}{7}s + \frac{6}{7}s^2 + \frac{1}{7}s^3}$ Using second-order model, $G_L(s) = \frac{1}{1 + d_1 s + d_2 s^2}$ $M(s) = 1 + d_1 s + d_2 s^2 = 1 + 11/7 s + 6/7 s^2 + 1/7 s^3$ $M^{(0)}(s) = 1 + d_1 s + d_2 s^2 = M^{(0)}(0) = 1$ $M^{(1)}(s) = d_1 + 2 d_2 s = M^{(1)}(0) = d1$ $M^{(2)}(s) = 2 d_2 = M^{(2)}(0) = 2 d_2$ $M^{(3)}(s) = 0 = M^{(3)}(0) = 0$ $\Delta^{(0)}(s) = 1 + 11/7 s + 6/7 s^2 + 1/7 s^3$, $\Delta^{(0)}(s) = 1$ $\Delta^{(1)}(s) = 11/7 + 12/7 s + 3/7 s^2$, $\Delta^{(1)}(s) = 11/7$ $\Delta^{(2)}(s) = 12/7 + 6/7 s$, $\Delta^{(1)}(s) = 12/7$ $\Delta^{(2)}(s) = 6/7$, $\Delta^{(1)}(s) = 6/7$	**(20.Marks)**

$$M_{2q} = \sum_{k=0}^{2q} \frac{(-1)^{k+q} M^{k}(0) M^{(2q-k)}(0)}{k!(2q-k)!}$$

for q = 1,

$$\text{M2} = \frac{\mathrm{M}^{(0)}(0)\mathrm{M}^{(2)}(0)}{2} + \frac{\mathrm{M}^{(1)}(0)\mathrm{M}^{(1)}(0)}{1} - \frac{\mathrm{M}^{(2)}(0)\mathrm{M}^{(0)}(0)}{2}$$

$$= -\frac{2d_2}{2} + \frac{d_1 d_2}{1} - \frac{2d_2.1}{2}$$

$$= -2\mathrm{d}_2 + d_1^2$$

for q = 2,

$$\text{M2} = \frac{\mathrm{M}^{(2)}(0)\mathrm{M}^{(2)}(0)}{4}$$

$$M_2 = d_2^2$$

$$\Delta_{2q} = \sum_{k=0}^{2q} \frac{(-1)^{k+q} \Delta^{k}(0) \Delta^{(2q-k)}(0)}{k!(2q-k)!}$$

for q = 1,

$$\Delta_2 = (-1) \frac{\Delta^{(0)}(0)\Delta^{(2)}(0)}{2} + (-1)^2 \frac{\Delta^{(1)}(0)\Delta^{(1)}(0)}{1} - (-1)^3 \quad \frac{\Delta^{(2)}(0)\Delta^{(0)}(0)}{2}$$

$$= \frac{-12}{7} + \frac{121}{49} = \frac{37}{49}$$

for q = 2,

$$\Delta_4 = (-1)^3 \frac{\Delta^{(1)}(0)\Delta^{(3)}(0)}{3\text{x}2} + (-1)^4 \frac{\Delta^{(2)}(0)\Delta^{(2)}(0)}{2 \text{ x } 2} - (-1)^5 \quad \frac{\Delta^{(3)}(0)\Delta^{(1)}(0)}{3 \text{ x } 2}$$

$$= \frac{-11}{49} - \frac{11}{49} + \frac{144}{196} = \frac{2}{7}$$

$$M_2 = \Delta_2 \quad (\text{if } q = 1)$$

$$2\mathrm{d}_2 + d_1^2 = \frac{37}{49}$$

$M_4 = \Delta_4 \quad (\text{if } q = 2)$

$d_2^2 = \frac{2}{7}$

$d_2 = 0.5345$

$d_1 = 1.35$

Second-order transfer function is,

$G_L(s) = 1/(1 + 1.35\, s + 0.5345\, s^2)$

$G_L(s) = 1.35/(s^2 + 0.5345\, s + 1.35)$

Compare with the general second order system, $s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n^2 = 1.87$

$\omega_n = 1.367$

$2\zeta\,\omega_n = 2.526$

$\zeta = 0.924$

$P.O = 100e^{\frac{-\xi\pi}{\sqrt{\zeta^2-1}}} = 0.0504\%$

The peak time, $t_p = \frac{\pi}{\omega_n\sqrt{\zeta^2-1}} = 6.01$ sec

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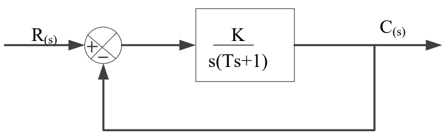
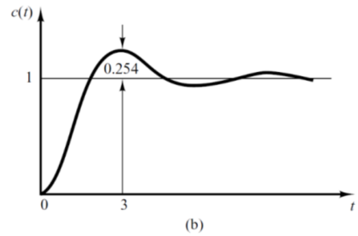
5.

When the system shown in Figure.5(a).i. is subjected to a unit-step input, the system output responds as shown in Figure.5. Determine the values of K and T from the response curve.

(10.Marks)



(i)



(ii)

Figure. 5

Solution

The maximum overshoot of 25.4% corresponds to $\zeta = 0.4$.

From the response curve, $t_p = 3$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 3$$

$$\omega_n = 1.14$$

From the block diagram,

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K}$$

$$= \frac{K/T}{s^2+1/Ts+K/T}$$

$$\omega_n = \sqrt{\frac{K}{T}}\ ,\ 2\zeta\,\omega_n = \frac{1}{T}$$

$$T = \frac{1}{2\zeta\,\omega_n} = \frac{1}{2\times0.4\times1.14} = 1.09$$

$$K = \omega_n{}^2 \times T = (1.14)^2 \times 1.09 = 1.42$$ ###

-------------------------------**End of the Questions**----------------------------