BRIEF CONTENTS

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CHAPTER 1

Introduction

1.1 POWER ELECTRONICS

Power electronics circuits convert electric power from one form to another using electronic devices. Power electronics circuits function by using semiconductor devices as switches, thereby controlling or modifying a voltage or current. Applications of power electronics range from high-power conversion equipment such as dc power transmission to everyday appliances, such as cordless screwdrivers, power supplies for computers, cell phone chargers, and hybrid automobiles. Power electronics includes applications in which circuits process milliwatts or megawatts. Typical applications of power electronics include conversion of ac to dc, conversion of dc to ac, conversion of an unregulated dc voltage to a regulated dc voltage, and conversion of an ac power source from one amplitude and frequency to another amplitude and frequency.

The design of power conversion equipment includes many disciplines from electrical engineering. Power electronics includes applications of circuit theory, control theory, electronics, electromagnetics, microprocessors (for control), and heat transfer. Advances in semiconductor switching capability combined with the desire to improve the efficiency and performance of electrical devices have made power electronics an important and fast-growing area in electrical engineering.

1.2 CONVERTER CLASSIFICATION

The objective of a power electronics circuit is to match the voltage and current requirements of the load to those of the source. Power electronics circuits convert one type or level of a voltage or current waveform to another and are hence called *converters*. Converters serve as an interface between the source and load (Fig. 1-1).

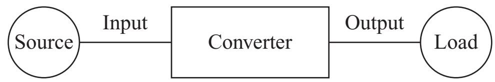


Figure 1-1 A source and load interfaced by a power electronics converter.

Converters are classified by the relationship between input and output:

ac input/dc output

The ac-dc converter produces a dc output from an ac input. Average power is transferred from an ac source to a dc load. The ac-dc converter is specifically classified as a *rectifier*. For example, an ac-dc converter enables integrated circuits to operate from a 60-Hz ac line voltage by converting the ac signal to a dc signal of the appropriate voltage.

dc input/ac output

The dc-ac converter is specifically classified as an *inverter*. In the inverter, average power flows from the dc side to the ac side. Examples of inverter applications include producing a 120-V rms 60-Hz voltage from a 12-V battery and interfacing an alternative energy source such as an array of solar cells to an electric utility.

dc input/dc output

The dc-dc converter is useful when a load requires a specified (often regulated) dc voltage or current but the source is at a different or unregulated dc value. For example, 5 V may be obtained from a 12-V source via a dc-dc converter.

ac input/ac output

The ac-ac converter may be used to change the level and/or frequency of an ac signal. Examples include a common light-dimmer circuit and speed control of an induction motor.

Some converter circuits can operate in different modes, depending on circuit and control parameters. For example, some rectifier circuits can be operated as inverters by modifying the control on the semiconductor devices. In such cases, it is the direction of average power flow that determines the converter classification. In Fig. 1-2, if the battery is charged from the ac power source, the converter is classified as a rectifier. If the operating parameters of the converter are changed and the battery acts as a source supplying power to the ac system, the converter is then classified as an inverter.

Power conversion can be a multistep process involving more than one type of converter. For example, an ac-dc-ac conversion can be used to modify an ac source by first converting it to direct current and then converting the dc signal to an ac signal that has an amplitude and frequency different from those of the original ac source, as illustrated in Fig. 1-3.

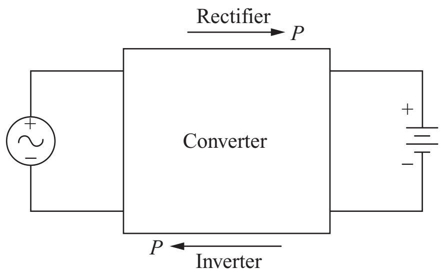


Figure 1-2 A converter can operate as a rectifier or an inverter, depending on the direction of average power *P*.

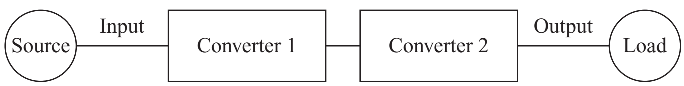


Figure 1-3 Two converters are used in a multistep process.

1.3 POWER ELECTRONICS CONCEPTS

To illustrate some concepts in power electronics, consider the design problem of creating a 3-V dc voltage level from a 9-V battery. The purpose is to supply 3 V to a load resistance. One simple solution is to use a voltage divider, as shown in Fig. 1-4. For a load resistor R_L, inserting a series resistance of $2R_L$ results in 3 V across R_L. A problem with this solution is that the power absorbed by the $2R_L$ resistor is twice as much as delivered to the load and is lost as heat, making the circuit only 33.3 percent efficient. Another problem is that if the value of the load resistance changes, the output voltage will change unless the $2R_L$ resistance changes proportionally. A solution to that problem could be to use a transistor in place of the $2R_L$ resistance. The transistor would be controlled such that the voltage across it is maintained at 6 V, thus regulating the output at 3 V. However, the same low-efficiency problem is encountered with this solution.

To arrive at a more desirable design solution, consider the circuit in Fig. 1-5*a*. In that circuit, a switch is opened and closed periodically. The switch is a short circuit when it is closed and an open circuit when it is open, making the voltage

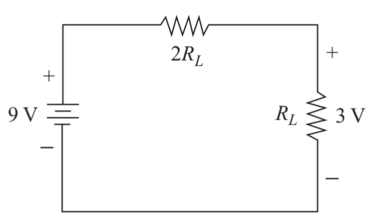


Figure 1-4 A simple voltage divider for creating 3 V from a 9-V source.

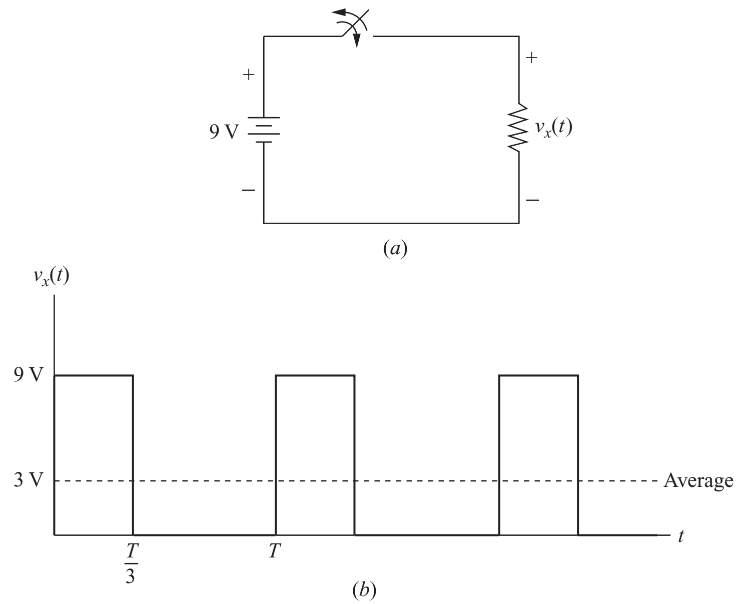


Figure 1-5 (*a*) A switched circuit; (*b*) a pulsed voltage waveform.

across R_L equal to 9 V when the switch is closed and 0 V when the switch is open. The resulting voltage across R_L will be like that of Fig. 1-5*b*. This voltage is obviously not a constant dc voltage, but if the switch is closed for one-third of the period, the average value of v_x (denoted as V_x) is one-third of the source voltage. Average value is computed from the equation

$$\operatorname{avg}(v_x) \;=\; V_x = \frac{1}{T}\int_0^T v_x(t)\,dt = \frac{1}{T}\int_0^{T/3} 9\,dt \;+\; \frac{1}{T}\int_{T/3}^{T} 0\,dt \;=\; 3\text{ V} \qquad (1\text{-}1)$$

Considering efficiency of the circuit, instantaneous power (see Chap. 2) absorbed by the switch is the product of voltage and current. When the switch is open, power absorbed by it is zero because the current in it is zero. When the switch is closed, power absorbed by it is zero because the voltage across it is zero. Since power absorbed by the switch is zero for both open and closed conditions, all power supplied by the 9-V source is delivered to R_L, making the circuit 100 percent efficient.

The circuit so far does not accomplish the design object of creating a dc voltage of 3 V. However, the voltage waveform v_x can be expressed as a Fourier series containing a dc term (the average value) plus sinusoidal terms at frequencies that are multiples of the pulse frequency. To create a 3-V dc voltage, v_x is applied to a low-pass filter. An ideal low-pass filter allows the dc component of voltage to pass through to the output while removing the ac terms, thus creating the desired dc output. If the filter is lossless, the converter will be 100 percent efficient.

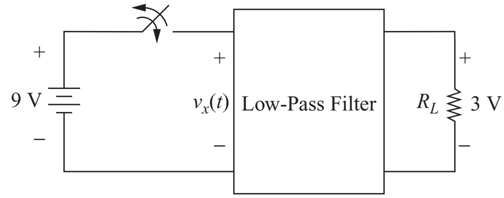


Figure 1-6 A low-pass filter allows just the average value of v_x to pass through to the load.

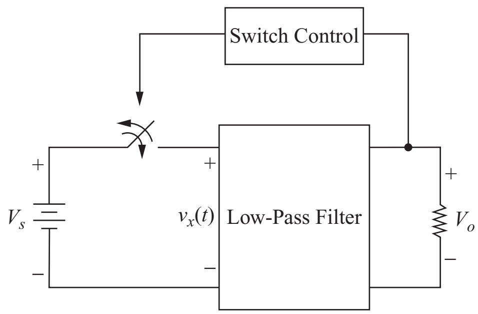


Figure 1-7 Feedback is used to control the switch and maintain the desired output voltage.

In practice, the filter will have some losses and will absorb some power. Additionally, the electronic device used for the switch will not be perfect and will have losses. However, the efficiency of the converter can still be quite high (more than 90 percent). The required values of the filter components can be made smaller with higher switching frequencies, making large switching frequencies desirable. Chaps. 6 and 7 describe the dc-dc conversion process in detail. The "switch" in this example will be some electronic device such as a metal-oxide field-effect transistors (MOSFET), or it may be comprised of more than one electronic device.

The power conversion process usually involves system control. Converter output quantities such as voltage and current are measured, and operating parameters are adjusted to maintain the desired output. For example, if the 9-V battery in the example in Fig. 1-6 decreased to 6 V, the switch would have to be closed 50 percent of the time to maintain an average value of 3 V for v_x. A feedback control system would detect if the output voltage were not 3 V and adjust the closing and opening of the switch accordingly, as illustrated in Fig. 1-7.

1.4 ELECTRONIC SWITCHES

An electronic switch is characterized by having the two states *on* and *off,* ideally being either a short circuit or an open circuit. Applications using switching devices are desirable because of the relatively small power loss in the device. If the switch is ideal, either the switch voltage or the switch current is zero, making the power absorbed by it zero. Real devices absorb some power when in the on state and when making transitions between the on and off states, but circuit efficiencies can still be quite high. Some electronic devices such as transistors can also operate in the active range where both voltage and current are nonzero, but it is desirable to use these devices as switches when processing power.

The emphasis of this textbook is on basic circuit operation rather than on device performance. The particular switching device used in a power electronics circuit depends on the existing state of device technology. The behaviors of power electronics circuits are often not affected significantly by the actual device used for switching, particularly if voltage drops across a conducting switch are small compared to other circuit voltages. Therefore, semiconductor devices are usually modeled as ideal switches so that circuit behavior can be emphasized. Switches are modeled as short circuits when on and open circuits when off. Transitions between states are usually assumed to be instantaneous, but the effects of nonideal switching are discussed where appropriate. A brief discussion of semiconductor switches is given in this section, and additional information relating to drive and snubber circuits is provided in Chap. 10. Electronic switch technology is continually changing, and thorough treatments of state-of-the-art devices can be found in the literature.

The Diode

A diode is the simplest electronic switch. It is uncontrollable in that the on and off conditions are determined by voltages and currents in the circuit. The diode is forward-biased (on) when the current i_d (Fig. 1-8*a*) is positive and reverse-biased (off) when v_d is negative. In the ideal case, the diode is a short circuit

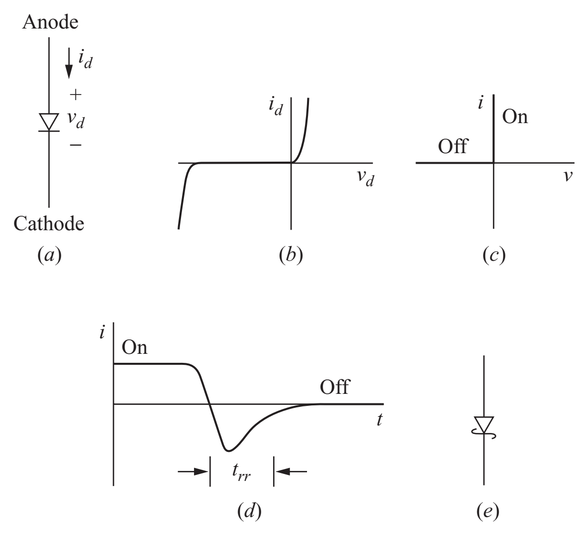


Figure 1-8 (*a*) Rectifier diode; (*b*) *i-v* characteristic; (c) idealized *i-v* characteristic; (*d*) reverse recovery time t_{rr}; (*e*) Schottky diode.

when it is forward-biased and is an open circuit when reverse-biased. The actual and idealized current-voltage characteristics are shown in Fig. 1-8*b* and *c*. The idealized characteristic is used in most analyses in this text.

An important dynamic characteristic of a nonideal diode is reverse recovery current. When a diode turns off, the current in it decreases and momentarily becomes negative before becoming zero, as shown in Fig. 1-8*d*. The time t_{rr} is the reverse recovery time, which is usually less than 1 μs. This phenomenon may become important in high-frequency applications. Fast-recovery diodes are designed to have a smaller t_{rr} than diodes designed for line-frequency applications. Silicon carbide (SiC) diodes have very little reverse recovery, resulting in more efficient circuits, especially in high-power applications.

Schottky diodes (Fig. 1-8*e*) have a metal-to-silicon barrier rather than a P-N junction. Schottky diodes have a forward voltage drop of typically 0.3 V. These are often used in low-voltage applications where diode drops are significant relative to other circuit voltages. The reverse voltage for a Schottky diode is limited to about 100 V. The metal-silicon barrier in a Schottky diode is not subject to recovery transients and turn-on and off faster than P-N junction diodes.

Thyristors

Thyristors are electronic switches used in some power electronic circuits where control of switch turn-on is required. The term *thyristor* often refers to a family of three-terminal devices that includes the silicon-controlled rectifier (SCR), the triac, the gate turnoff thyristor (GTO), the MOS-controlled thyristor (MCT), and others. *Thyristor* and *SCR* are terms that are sometimes used synonymously. The SCR is the device used in this textbook to illustrate controlled turn-on devices in the thyristor family. Thyristors are capable of large currents and large blocking voltages for use in high-power applications, but switching frequencies cannot be as high as when using other devices such as MOSFETs.

The three terminals of the SCR are the anode, cathode, and gate (Fig.1-9*a*). For the SCR to begin to conduct, it must have a gate current applied while it has a positive anode-to-cathode voltage. After conduction is established, the gate signal is no longer required to maintain anode current. The SCR will continue to conduct as long as the anode current remains positive and above a minimum value called the holding level. Figs. 1-9*a* and *b* show the SCR circuit symbol and the idealized current-voltage characteristic.

The gate turnoff thyristor (GTO) of Fig. 1-9*c*, like the SCR, is turned on by a short-duration gate current if the anode-to-cathode voltage is positive. However, unlike the SCR, the GTO can be turned off with a negative gate current. The GTO is therefore suitable for some applications where control of both turn-on and turnoff of a switch is required. The negative gate turnoff current can be of brief duration (a few microseconds), but its magnitude must be very large compared to the turn-on current. Typically, gate turnoff current is one-third the on-state anode current. The idealized *i-v* characteristic is like that of Fig. 1-9*b* for the SCR.

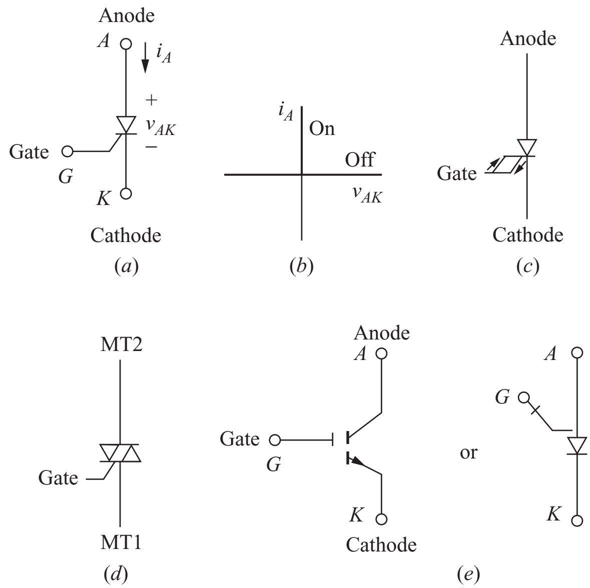


Figure 1-9 Thyristor devices: (*a*) silicon-controlled rectifier (SCR); (*b*) SCR idealized *i-v* characteristic; (*c*) gate turnoff (GTO) thyristor; (*d*) triac; (*e*) MOS-controlled thyristor (MCT).

The triac (Fig. 1-9*d*) is a thyristor that is capable of conducting current in either direction. The triac is functionally equivalent to two antiparallel SCRs (in parallel but in opposite directions). Common incandescent light-dimmer circuits use a triac to modify both the positive and negative half cycles of the input sine wave.

The MOS-controlled thyristor (MCT) in Fig. 1-9*e* is a device functionally equivalent to the GTO but without the high turnoff gate current requirement. The MCT has an SCR and two MOSFETs integrated into one device. One MOSFET turns the SCR on, and one MOSFET turns the SCR off. The MCT is turned on and off by establishing the proper voltage from gate to cathode, as opposed to establishing a gate current in the GTO.

Thyristors were historically the power electronics switch of choice because of high voltage and current ratings available. Thyristors are still used, especially in high-power applications, but ratings of power transistors have increased greatly, making the transistor more desirable in many applications.

Transistors

Transistors are operated as switches in power electronics circuits. Transistor drive circuits are designed to have the transistor either in the fully on or fully off state. This differs from other transistor applications such as in a linear amplifier circuit where the transistor operates in the region having simultaneously high voltage and current.

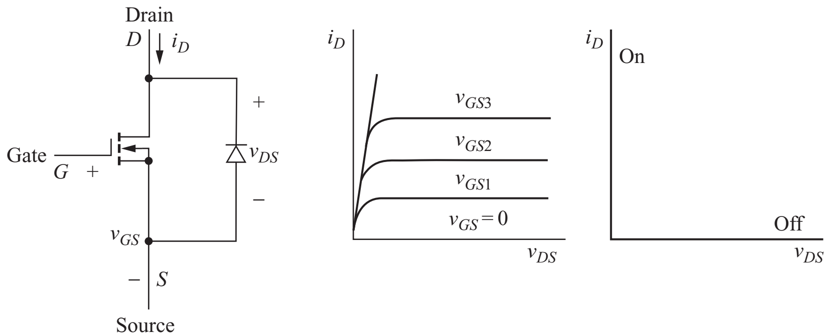


Figure 1-10 (*a*) MOSFET (N-channel) with body diode; (*b*) MOSFET characteristics; (*c*) idealized MOSFET characteristics.

Unlike the diode, turn-on and turnoff of a transistor are controllable. Types of transistors used in power electronics circuits include MOSFETs, bipolar junction transistors (BJTs), and hybrid devices such as insulated-gate bipolar junction transistors (IGBTs). Figs. 1-10 to 1-12 show the circuit symbols and the current-voltage characteristics.

The MOSFET (Fig. 1-10*a*) is a voltage-controlled device with characteristics as shown in Fig. 1-10*b*. MOSFET construction produces a parasitic (body) diode, as shown, which can sometimes be used to an advantage in power electronics circuits. Power MOSFETs are of the enhancement type rather than the depletion type. A sufficiently large gate-to-source voltage will turn the device on,

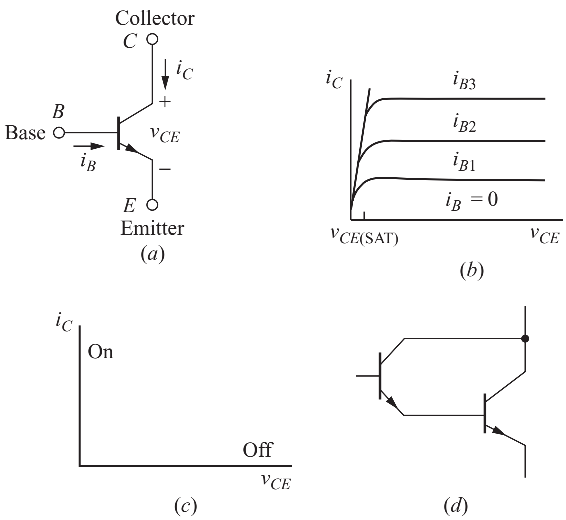


Figure 1-11 (*a*) BJT (NPN); (*b*) BJT characteristics; (*c*) idealized BJT characteristics; (*d*) Darlington configuration.

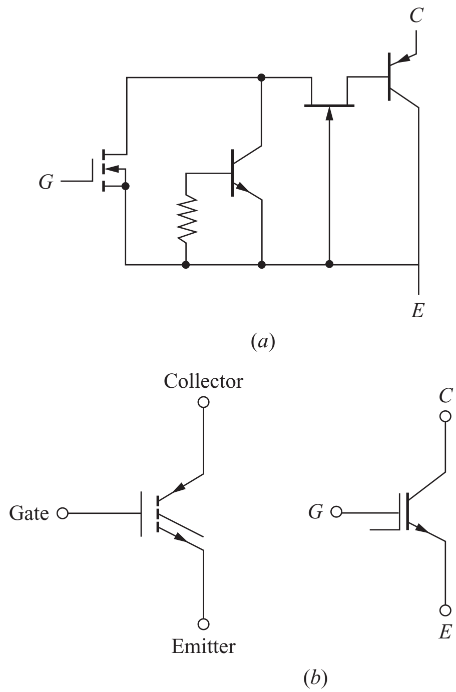


Figure 1-12 IGBT: (*a*) Equivalent circuit; (*b*) circuit symbols.

resulting in a small drain-to-source voltage. In the on state, the change in v_{DS} is linearly proportional to the change in i_D. Therefore, the on MOSFET can be modeled as an on-state resistance called $R_{DS(\text{on})}$. MOSFETs have on-state resistances as low as a few milliohms. For a first approximation, the MOSFET can be modeled as an ideal switch with a characteristic shown in Fig. 1-10*c*. Ratings are to 1500 V and more than 600 A (although not simultaneously). MOSFET switching speeds are greater than those of BJTs and are used in converters operating into the megahertz range.

Typical BJT characteristics are shown in Fig. 1-11*b*. The on state for the transistor is achieved by providing sufficient base current to drive the BJT into saturation. The collector-emitter saturation voltage is typically 1 to 2 V for a power BJT. Zero base current results in an off transistor. The idealized *i*-*v* characteristic for the BJT is shown in Fig. 1-11*c*. The BJT is a current-controlled device, and power BJTs typically have low h_{FE} values, sometimes lower than 20. If a power BJT with $h_{FE} = 20$ is to carry a collector current of 60 A, for example, the base current would need to be more than 3 A to put the transistor into saturation. The drive circuit to provide a high base current is a significant power circuit in itself. Darlington configurations have two BJTs connected as shown in Fig. 1-11*d*. The effective current gain of the combination is approximately the product of individual gains and can thus reduce the current required from the drive circuit. The Darlington configuration can be constructed from two discrete transistors or can be obtained as a single integrated device. Power BJTs are rarely used in new applications, being surpassed by MOSFETs and IGBTs.

The IGBT of Fig. 1-12 is an integrated connection of a MOSFET and a BJT. The drive circuit for the IGBT is like that of the MOSFET, while the on-state characteristics are like those of the BJT. IGBTs have replaced BJTs in many applications.

1.5 SWITCH SELECTION

The selection of a power device for a particular application depends not only on the required voltage and current levels but also on its switching characteristics. Transistors and GTOs provide control of both turn-on and turnoff, SCRs of turn-on but not turnoff, and diodes of neither.

Switching speeds and the associated power losses are very important in power electronics circuits. The BJT is a minority carrier device, whereas the MOSFET is a majority carrier device that does not have minority carrier storage delays, giving the MOSFET an advantage in switching speeds. BJT switching times may be a magnitude larger than those for the MOSFET. Therefore, the MOSFET generally has lower switching losses and is preferred over the BJT.

When selecting a suitable switching device, the first consideration is the required operating point and turn-on and turnoff characteristics. Example 1-1 outlines the selection procedure.

EXAMPLE 1-1

Switch Selection

The circuit of Fig. 1-13a has two switches. Switch S_1 is on and connects the voltage source (V_s = 24 V) to the current source (I_o = 2 A). It is desired to open switch S_1 to disconnect V_s from the current source. This requires that a second switch S_2 close to provide a path for current I_o, as in Fig. 1-13b. At a later time, S_1 must reclose and S_2 must open to restore the circuit to its original condition. The cycle is to repeat at a frequency of 200 kHz. Determine the type of device required for each switch and the maximum voltage and current requirements of each.

■ **Solution**

The type of device is chosen from the turn-on and turnoff requirements, the voltage and current requirements of the switch for the on and off states, and the required switching speed.

The steady-state operating points for S_1 are at $(v_1, i_1) = (0, I_o)$ for S_1 closed and $(V_s, 0)$ for the switch open (Fig. 1-13c). The operating points are on the positive i and v axes, and S_1 must turn off when $i_1 = I_o > 0$ and must turn on when $v_1 = V_s > 0$. The device used for S_1 must therefore provide control of both turn-on and turnoff. The MOSFET characteristic

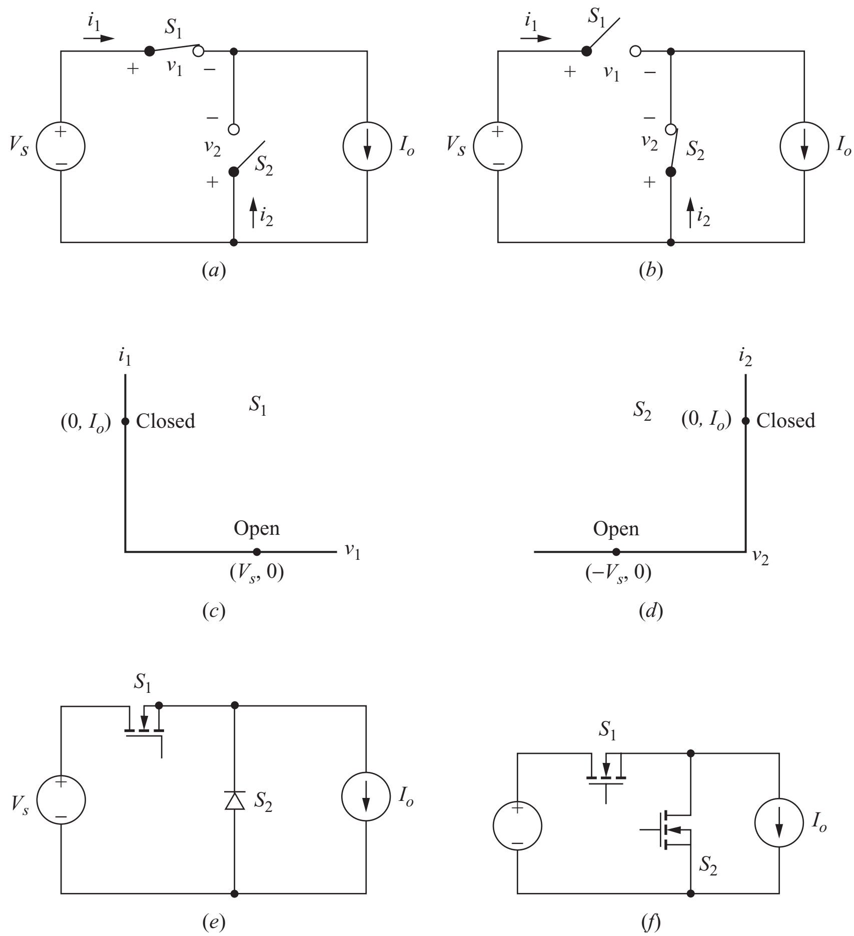


Figure 1-13 Circuit for Example 1-1. (*a*) S_1 closed, S_2 open; (*b*) S_1 open, S_2 closed; (*c*) operating points for S_1; (*d*) operating points for S_2; (*e*) switch implementation using a MOSFET and diode; (*f*) switch implementation using two MOSFETs (synchronous rectification).

of Fig. 1-10*d* or the BJT characteristic of Fig. 1-11*c* matches the requirement. A MOSFET would be a good choice because of the required switching frequency, simple gate-drive requirements, and relatively low voltage and current requirement (24 V and 2 A).

The steady-state operating points for S_2 are at $(v_2, i_2) = (-V_s, 0)$ in Fig. 1-13*a* and $(0, I_o)$ in Fig. 1-13*b*, as shown in Fig. 1-13*d*. The operating points are on the positive current axis and negative voltage axis. Therefore, a positive current in S_2 is the requirement to turn S_2 on, and a negative voltage exists when S_2 must turn off. Since the operating points match the diode (Fig. 1-8*c*) and no other control is needed for the device, a diode is an appropriate choice for S_2. Figure 1-13*e* shows the implementation of the switching circuit. Maximum current is 2 A, and maximum voltage in the blocking state is 24 V.

Although a diode is a sufficient and appropriate device for the switch S_2, a MOSFET would also work in this position, as shown in Fig. 1-13*f*. When S_2 is on and S_1 is off, current flows upward out of the drain of S_2. The advantage of using a MOSFET is that it has a much lower voltage drop across it when conducting compared to a diode, resulting in lower power loss and a higher circuit efficiency. The disadvantage is that a more complex control circuit is required to turn on S_2 when S_1 is turned off. However, several control circuits are available to do this. This control scheme is known as synchronous rectification or synchronous switching.

In a power electronics application, the current source in this circuit could represent an inductor that has a nearly constant current in it.

1.6 SPICE, PSPICE, AND CAPTURE

Computer simulation is a valuable analysis and design tool that is emphasized throughout this text. SPICE is a circuit simulation program developed in the Department of Electrical Engineering and Computer Science at the University of California at Berkeley. PSpice is a commercially available adaptation of SPICE that was developed for the personal computer. Capture is a graphical interface program that enables a simulation to be done from a graphical representation of a circuit diagram. Cadence provides a product called OrCAD Capture, and a demonstration version at no cost.[1] Nearly all simulations described in this textbook can be run using the demonstration version.

Simulation can take on various levels of device and component modeling, depending on the objective of the simulation. Most of the simulation examples and exercises use idealized or default component models, making the results first-order approximations, much the same as the analytical work done in the first discussion of a subject in any textbook. After understanding the fundamental operation of a power electronics circuit, the engineer can include detailed device models to predict more accurately the behavior of an actual circuit.

Probe, the graphics postprocessor program that accompanies PSpice, is especially useful. In Probe, the waveform of any current or voltage in a circuit can be shown graphically. This gives the student a look at circuit behavior that is not possible with pencil-and-paper analysis. Moreover, Probe is capable of mathematical computations involving currents and/or voltages, including numerical determination of rms and average values. Examples of PSpice analysis and design for power electronics circuits are an integral part of this textbook.

The PSpice circuit files listed in this text were developed using version 16.0. Continuous revision of software necessitates updates in simulation techniques.

[1] https://www.cadence.com/products/orcad/pages/downloads.aspx#demo

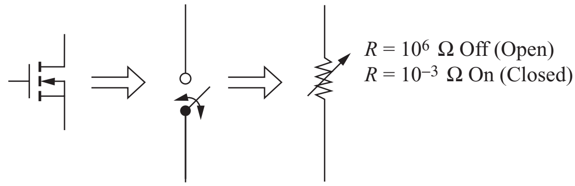


Figure 1-14 Implementing a switch with a resistance in PSpice.

1.7 SWITCHES IN PSPICE

The Voltage-Controlled Switch

The voltage-controlled switch Sbreak in PSpice can be used as an idealized model for most electronic devices. The voltage-controlled switch is a resistance that has a value established by a controlling voltage. Fig. 1-14 illustrates the concept of using a controlled resistance as a switch for PSpice simulation of power electronics circuits. A MOSFET or other switching device is ideally an open or closed switch. A large resistance approximates an open switch, and a small resistance approximates a closed switch. Switch model parameters are as follows:

Parameter	Description	Default Value
RON	"On" resistance	1 (reduce this to 0.001 or 0.01 Ω)
ROFF	"Off" resistance	10^6 Ω
VON	Control voltage for on state	1.0 V
VOFF	Control voltage for off state	0 V

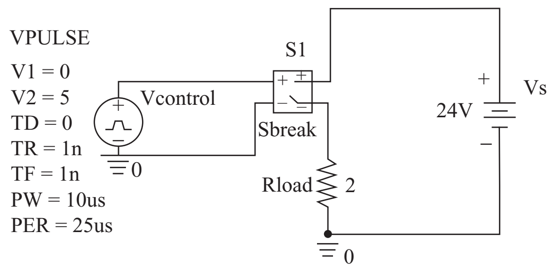
The resistance is changed from large to small by the controlling voltage. The default off resistance is 1 MΩ, which is a good approximation for an open circuit in power electronics applications. The default on resistance of 1 Ω is usually too large. If the switch is to be ideal, the on resistance in the switch model should be changed to something much lower, such as 0.001 or 0.01 Ω.

EXAMPLE 1-2

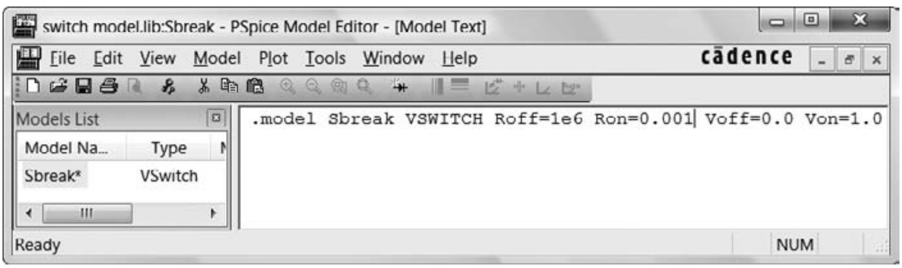
A Voltage-Controlled Switch in PSpice

The Capture diagram of a switching circuit is shown in Fig. 1-15*a*. The switch is implemented with the voltage-controlled switch Sbreak, located in the Breakout library of devices. The control voltage is VPULSE and uses the characteristics shown. The rise and fall times, TR and TF, are made small compared to the pulse width and period, PW and PER. V1 and V2 must span the on and off voltage levels for the switch, 0 and 1 V by default. The switching period is 25 ms, corresponding to a frequency of 40 kHz.

The PSpice model for Sbreak is accessed by clicking *edit*, then *PSpice model*. The model editor window is shown in Fig 1-15*b*. The on resistance Ron is changed to 0.001 Ω



(*a*)



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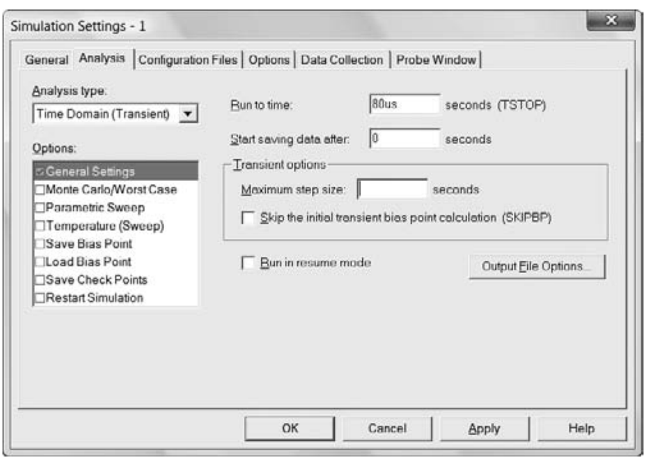
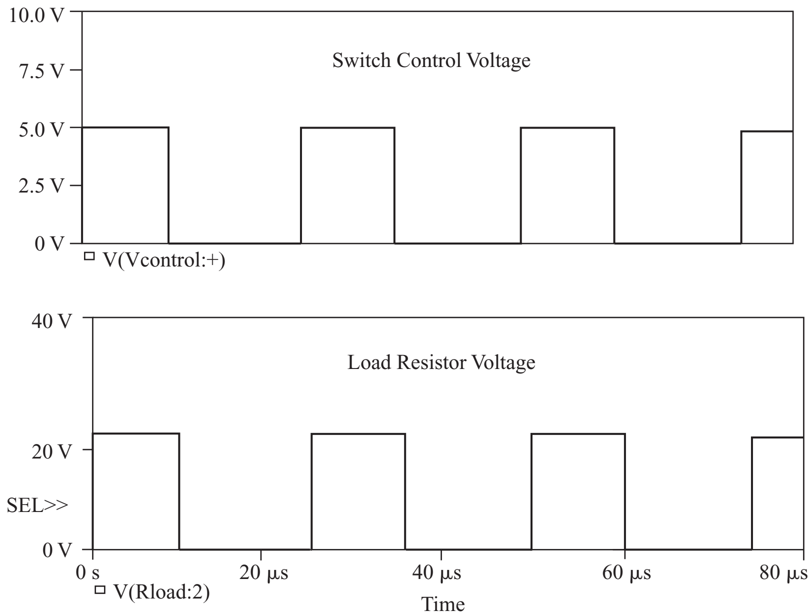


(*c*)

Figure 1-15 (*a*) Circuit for Example 1-2; (*b*) editing the PSpice Sbreak switch model to make Ron = 0.001Ω; (*c*) the transient analysis setup; (*d*) the Probe output.



(*d*)

Figure 1-15 (*continued*)

to approximate an ideal switch. The Transient Analysis menu is accessed from Simulation Settings. This simulation has a run time of 80 μs, as shown in Fig. 1-15*c*.

Probe output showing the switch control voltage and the load resistor voltage waveforms is seen in Fig. 1-15*d*.

Transistors

Transistors used as switches in power electronics circuits can be idealized for simulation by using the voltage-controlled switch. As in Example 1-2, an ideal transistor can be modeled as very small on resistance. An on resistance matching the MOSFET characteristics can be used to simulate the conducting resistance $R_{DS(\mathrm{ON})}$ of a MOSFET to determine the behavior of a circuit with nonideal components. If an accurate representation of a transistor is required, a model may be available in the PSpice library of devices or from the manufacturer's website. The IRF150 and IRF9140 models for power MOSFETs are in the demonstration version library. The default MOSFET MbreakN or MbreakN3 model must have parameters for the threshold voltage VTO and the constant KP added to the PSpice device model for a meaningful simulation. Manufacturer's websites, such as International Rectifier at www.irf.com, have SPICE models available for their

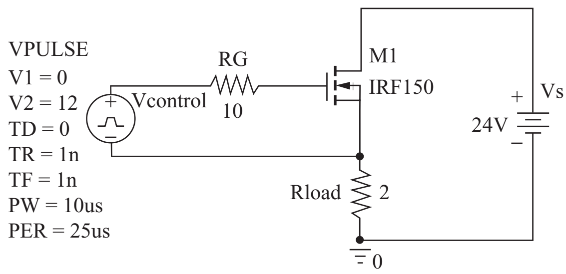


Figure 1-16 An idealized MOSFET drive circuit in PSpice.

products. The default BJT QbreakN can be used instead of a detailed transistor model for a rudimentary simulation.

Transistors in PSpice must have drive circuits, which can be idealized if the behavior of a specific drive circuit is not required. Simulations with MOSFETs can have drive circuits like that in Fig. 1-16. The voltage source VPULSE establishes the gate-to-source voltage of the MOSFET to turn it on and off. The gate resistor may not be necessary, but it sometimes eliminates numerical convergence problems.

Diodes

An ideal diode is assumed when one is developing the equations that describe a power electronics circuit, which is reasonable if the circuit voltages are much larger than the normal forward voltage drop across a conducting diode. The diode current is related to diode voltage by

$$i_d = I_S e^{v_d/nV_T} - 1 \tag{1-2}$$

where n is the emission coefficient which has a default value of 1 in PSpice. An ideal diode can be approximated in PSpice by setting n to a small number such as 0.001 or 0.01. The nearly ideal diode is modeled with the part Dbreak with PSpice model

model Dbreak D n = 0.001

With the ideal diode model, simulation results will match the analytical results from the describing equations. A PSpice diode model that more accurately predicts diode behavior can be obtained from a device library. Simulations with a detailed diode model will produce more realistic results than the idealized case. However, if the circuit voltages are large, the difference between using an ideal diode and an accurate diode model will not affect the results in any significant way. The default diode model for Dbreak can be used as a compromise between the ideal and actual cases, often with little difference in the result.

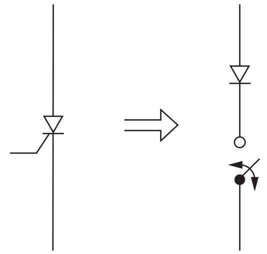


Figure 1-17 Simplified thyristor (SCR) model for PSpice.

Thyristors (SCRs)

An SCR model is available in the PSpice demonstration version part library and can be used in simulating SCR circuits. However, the model contains a relatively large number of components which imposes a size limit for the PSpice demonstration version. A simple SCR model that is used in several circuits in this text is a switch in series with a diode, as shown in Fig. 1-17. Closing the voltage-controlled switch is equivalent to applying a gate current to the SCR, and the diode prevents reverse current in the model. This simple SCR model has the significant disadvantage of requiring the voltage-controlled switch to remain closed during the entire on time of the SCR, thus requiring some prior knowledge of the behavior of a circuit that uses the device. Further explanation is included with the PSpice examples in later chapters.

Convergence Problems in PSpice

Some of the PSpice simulations in this book are subject to numerical convergence problems because of the switching that takes place in circuits with inductors and capacitors. All the PSpice files presented in this text have been designed to avoid convergence problems. However, sometimes changing a circuit parameter will cause a failure to converge in the transient analysis. In the event that there is a problem with PSpice convergence, the following remedies may be useful:

- Increase the iteration limit ITL4 from 10 to 100 or larger. This is an option accessed from the Simulation Profile Options, as shown in Fig. 1-18.
- Change the relative tolerance RELTOL to something other than the default value of 0.001.
- Change the device models to something that is less than ideal. For example, change the on resistance of a voltage-controlled switch to a larger value, or use a controlling voltage source that does not change as rapidly. An ideal diode could be made less ideal by increasing the value of n in the model. Generally, idealized device models will introduce more convergence problems than real device models.

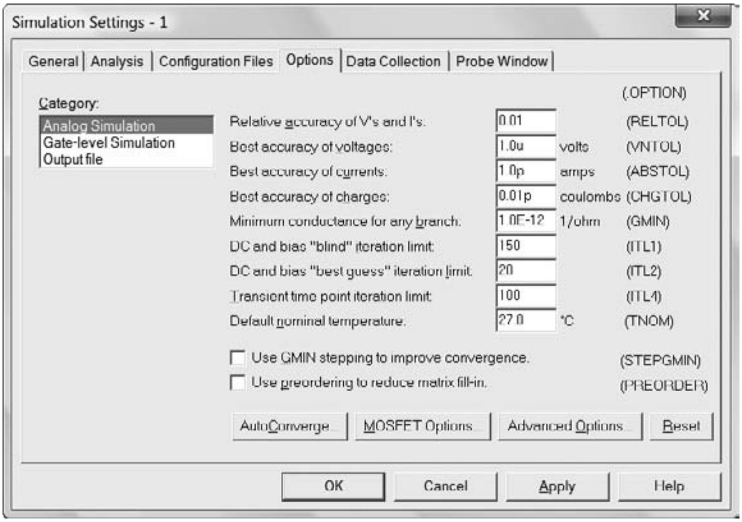


Figure 1-18 The Options menu for settings that can solve convergence problems. RELTOL and ITL4 have been changed here.

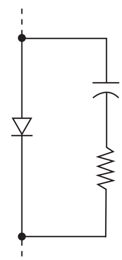


Figure 1-19 RC circuit to aid in PSpice convergence.

- Add an RC "snubber" circuit. A series resistance and capacitance with a small time constant can be placed across switches to prevent voltages from changing too rapidly. For example, placing a series combination of a 1-kΩ resistor and a 1-nF capacitor in parallel with a diode (Fig. 1-19) may improve convergence without affecting the simulation results.

1.8 BIBLIOGRAPHY

M. E. Balci and M. H. Hocaoglu, "Comparison of Power Definitions for Reactive Power Compensation in Nonsinusoidal Circuits," *International Conference on Harmonics and Quality of Power*, Lake Placid, N.Y. 2004.

L. S. Czarnecki, "Considerations on the Reactive Power in Nonsinusoidal Situations," *International Conference on Harmonics in Power Systems*, Worcester Polytechnic Institute, Worcester, Mass., 1984, pp 231–237.

A. E. Emanuel, "Powers in Nonsinusoidal Situations, A Review of Definitions and Physical Meaning," *IEEE Transactions on Power Delivery*, vol. 5, no. 3, July 1990.

G. T. Heydt, *Electric Power Quality*, Stars in a Circle Publications, West Lafayette, Ind., 1991.

W. Sheperd and P. Zand, *Energy Flow and Power Factor in Nonsinusoidal Circuits*, Cambridge University Press, 1979.

Problems

1-1. The current source in Example 1-1 is reversed so that positive current is upward. The current source is to be connected to the voltage source by alternately closing S_1 and S_2. Draw a circuit that has a MOSFET and a diode to accomplish this switching.

1-2. Simulate the circuit in Example 1-1 using PSpice. Use the voltage-controlled switch Sbreak for S_1 and the diode Dbreak for S_2. (*a*) Edit the PSpice models to idealize the circuit by using RON = 0.001 Ω for the switch and $n = 0.001$ for the diode. Display the voltage across the current source in Probe. (*b*) Use RON = 0.1 Ω in Sbreak and $n = 1$ (the default value) for the diode. How do the results of parts *a* and *b* differ?

1-3. The IRF150 power MOSFET model is in the EVAL library that accompanies the demonstration version of PSpice. Simulate the circuit in Example 1-1, using the IRF150 for the MOSFET and the default diode model Dbreak for S_2. Use an idealized gate drive circuit similar to that of Fig. 1-16. Display the voltage across the current source in Probe. How do the results differ from those using ideal switches?

1-4. Use PSpice to simulate the circuit of Example 1-1. Use the PSpice default BJT QbreakN for switch S_1. Use an idealized base drive circuit similar to that of the gate drive circuit for the MOSFET in Fig. 1-9. Choose an appropriate base resistance to ensure that the transistor turns on for a transistor h_{FE} of 100. Use the PSpice default diode Dbreak for switch S_2. Display the voltage across the current source. How do the results differ from those using ideal switches?

CHAPTER 2

Power Computations

2.1 INTRODUCTION

Power computations are essential in analyzing and designing power electronics circuits. Basic power concepts are reviewed in this chapter, with particular emphasis on power calculations for circuits with nonsinusoidal voltages and currents. Extra treatment is given to some special cases that are encountered frequently in power electronics. Power computations using the circuit simulation program PSpice are demonstrated.

2.2 POWER AND ENERGY

Instantaneous Power

The instantaneous power for any device is computed from the voltage across it and the current in it. *Instantaneous power* is

$$p(t) = v(t)i(t) \tag{2-1}$$

This relationship is valid for any device or circuit. Instantaneous power is generally a time-varying quantity. If the passive sign convention illustrated in Fig. 2-1*a* is observed, the device is absorbing power if $p(t)$ is positive at a specified value of time t. The device is supplying power if $p(t)$ is negative. Sources frequently have an assumed current direction consistent with supplying power. With the convention of Fig. 2-1*b*, a positive $p(t)$ indicates the source is supplying power.

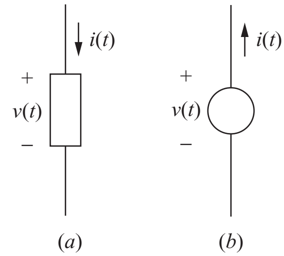


Figure 2-1 (*a*) Passive sign convention: $p(t) > 0$ indicates power is being absorbed; (*b*) $p(t) > 0$ indicates power is being supplied by the source.

Energy

Energy, or work, is the integral of instantaneous power. Observing the passive sign convention, energy absorbed by a component in the time interval from t_1 to t_2 is

$$\boxed{W = \int_{t_1}^{t_2} p(t)\, dt} \qquad (2\text{-}2)$$

If $v(t)$ is in volts and $i(t)$ is in amperes, power has units of watts and energy has units of joules.

Average Power

Periodic voltage and current functions produce a periodic instantaneous power function. Average power is the time average of $p(t)$ over one or more periods. *Average power P* is computed from

$$\boxed{P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\, dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\, dt} \qquad (2\text{-}3)$$

where T is the period of the power waveform. Combining Eqs. (2-3) and (2-2), power is also computed from energy per period.

$$\boxed{P = \frac{W}{T}} \qquad (2\text{-}4)$$

Average power is sometimes called *real power* or *active power*, especially in ac circuits. The term *power* usually means average power. The total average power absorbed in a circuit equals the total average power supplied.

EXAMPLE 2-1

Power and Energy

Voltage and current, consistent with the passive sign convention, for a device are shown in Fig. 2-2*a* and *b*. (*a*) Determine the instantaneous power $p(t)$ absorbed by the device. (*b*) Determine the energy absorbed by the device in one period. (*c*) Determine the average power absorbed by the device.

■ Solution

(*a*) The instantaneous power is computed from Eq. (2-1). The voltage and current are expressed as

$$v(t) = \begin{cases} 20\text{ V} & 0 < t < 10\text{ ms} \\ 0 & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

$$i(t) = \begin{cases} 20\text{ V} & 0 < t < 6\text{ ms} \\ -15\text{ A} & 6\text{ ms} < t < 20\text{ ms} \end{cases}$$

Instantaneous power, shown in Fig. 2-2*c*, is the product of voltage and current and is expressed as

$$p(t) = \begin{cases} 400\text{ W} & 0 < t < 6\text{ ms} \\ -300\text{ W} & 6\text{ ms} < t < 10\text{ ms} \\ 0 & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

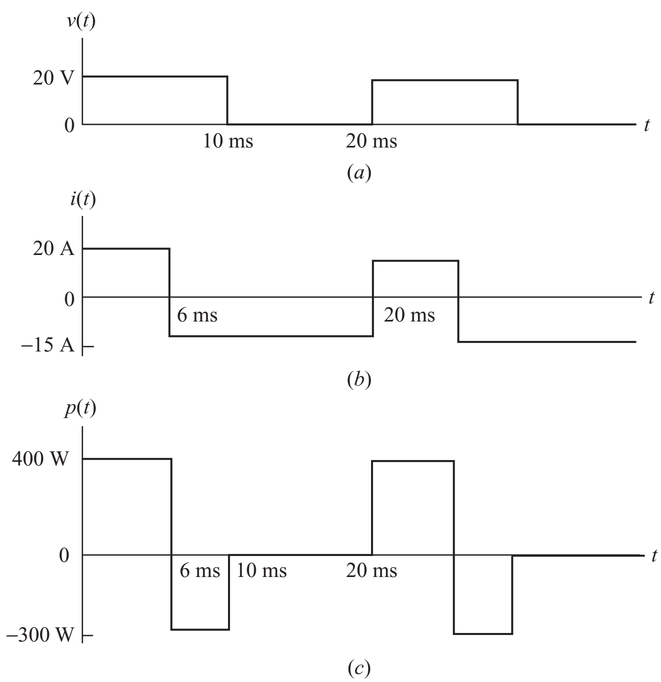


Figure 2-2 Voltage, current, and instantaneous power for Example 2-1.

(*b*) Energy absorbed by the device in one period is determined from Eq. (2-2).

$$W = \int_0^T p(t)\,dt = \int_0^{0.006} 400\,dt + \int_{0.006}^{0.010} -300\,dt + \int_{0.010}^{0.020} 0\,dt = 2.4 - 1.2 = 1.2 \text{ J}$$

(*c*) Average power is determined from Eq. (2-3).

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{0.020}\left(\int_0^{0.006} 400\,dt + \int_{0.006}^{0.010} -300\,dt + \int_{0.010}^{0.020} 0\,dt\right)$$

$$= \frac{2.4 - 1.2 - 0}{0.020} = 60 \text{ W}$$

Average power could also be computed from Eq. (2-4) by using the energy per period from part (*b*).

$$P = \frac{W}{T} = \frac{1.2 \text{ J}}{0.020 \text{ s}} = 60 \text{ W}$$

A special case that is frequently encountered in power electronics is the power absorbed or supplied by a dc source. Applications include battery-charging circuits and dc power supplies. The average power absorbed by a dc voltage source $v(t) = V_{dc}$ that has a periodic current $i(t)$ is derived from the basic definition of average power in Eq. (2-3):

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\, i(t)\,dt$$

Bringing the constant V_{dc} outside of the integral gives

$$P_{dc} = V_{dc}\left[\frac{1}{T}\int_{t_0}^{t_0+T} i(t)\,dt\right]$$

The term in brackets is the average of the current waveform. Therefore, *average power absorbed by a dc voltage source is the product of the voltage and the average current.*

$$\boxed{P_{dc} = V_{dc} I_{avg}} \tag{2-5}$$

Similarly, average power absorbed by a dc source $i(t) = I_{dc}$ is

$$P_{dc} = I_{dc} V_{avg} \tag{2-6}$$

2.3 INDUCTORS AND CAPACITORS

Inductors and capacitors have some particular characteristics that are important in power electronics applications. For periodic currents and voltages,

$$\begin{aligned} i(t + T) &= i(t) \\ v(t + T) &= v(t) \end{aligned} \tag{2-7}$$

For an inductor, the stored energy is

$$w(t) = \frac{1}{2} Li^2(t) \tag{2-8}$$

If the inductor current is periodic, the stored energy at the end of one period is the same as at the beginning. No net energy transfer indicates that *the average power absorbed by an inductor is zero for steady-state periodic operation.*

$$\boxed{P_L = 0} \tag{2-9}$$

Instantaneous power is not necessarily zero because power may be absorbed during part of the period and returned to the circuit during another part of the period.

Furthermore, from the voltage-current relationship for the inductor

$$i(t_0 + T) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\, dt + i(t_0) \tag{2-10}$$

Rearranging and recognizing that the starting and ending values are the same for periodic currents, we have

$$i(t_0 + T) - i(t_0) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\, dt = 0 \tag{2-11}$$

Multiplying by L/T yields an expression equivalent to the average voltage across the inductor over one period.

$$\boxed{\text{avg}[v_L(t)] = V_L = \frac{1}{T}\int_{t_0}^{t_0+T} v_L(t)\, dt = 0} \tag{2-12}$$

Therefore, *for periodic currents, the average voltage across an inductor is zero.* This is very important and will be used in the analysis of many circuits, including dc-dc converters and dc power supplies.

For a capacitor, stored energy is

$$w(t) = \frac{1}{2} Cv^2(t) \tag{2-13}$$

If the capacitor voltage is periodic, the stored energy is the same at the end of a period as at the beginning. Therefore, *the average power absorbed by the capacitor is zero for steady-state periodic operation.*

$$\boxed{P_C = 0} \tag{2-14}$$

From the voltage-current relationship for the capacitor,

$$v(t_0 + T) = \frac{1}{C} \int_{t_0}^{t_0+T} i_C(t)\, dt + v(t_0) \tag{2-15}$$

Rearranging the preceding equation and recognizing that the starting and ending values are the same for periodic voltages, we get

$$v(t_0 + T) - v(t_0) = \frac{1}{C} \int_{t_0}^{t_0+T} i_C(t)\, dt = 0 \tag{2-16}$$

Multiplying by C/T yields an expression for average current in the capacitor over one period.

$$\boxed{\text{avg}\,[i_C(t)] = I_C = \frac{1}{T} \int_{t_0}^{t_0+T} i_C(t)\, dt = 0} \tag{2-17}$$

Therefore, *for periodic voltages, the average current in a capacitor is zero.*

EXAMPLE 2-2

Power and Voltage for an Inductor

The current in a 5-mH inductor of Fig. 2-3*a* is the periodic triangular wave shown in Fig. 2-3*b*. Determine the voltage, instantaneous power, and average power for the inductor.

■ Solution

The voltage across the inductor is computed from $v(t) = L(di/dt)$ and is shown in Fig. 2-3*c*. The average inductor voltage is zero, as can be determined from Fig. 2-3*c* by inspection. The instantaneous power in the inductor is determined from $p(t) = v(t)i(t)$ and is shown in Fig. 2-3*d*. When $p(t)$ is positive, the inductor is absorbing power, and when $p(t)$ is negative, the inductor is supplying power. The average inductor power is zero.

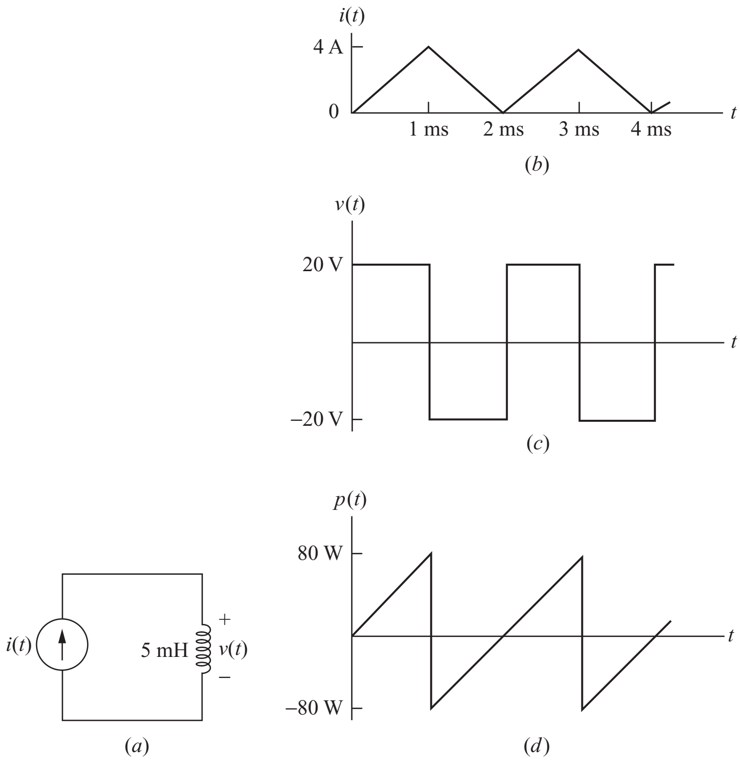


Figure 2.3 (*a*) Circuit for Example 2-2; (*b*) inductor current; (*c*) inductor voltage; (*d*) inductor instantaneous power.

2.4 ENERGY RECOVERY

Inductors and capacitors must be energized and deenergized in several applications of power electronics. For example, a fuel injector solenoid in an automobile is energized for a set time interval by a transistor switch. Energy is stored in the solenoid's inductance when current is established. The circuit must be designed to remove the stored energy in the inductor while preventing damage to the transistor when it is turned off. Circuit efficiency can be improved if stored energy can be transferred to the load or to the source rather than dissipated in circuit resistance. The concept of recovering stored energy is illustrated by the circuits described in this section.

Fig. 2-4*a* shows an inductor that is energized by turning on a transistor switch. The resistance associated with the inductance is assumed to be negligible, and the transistor switch and diode are assumed to be ideal. The diode-resistor path provides a means of opening the switch and removing the stored

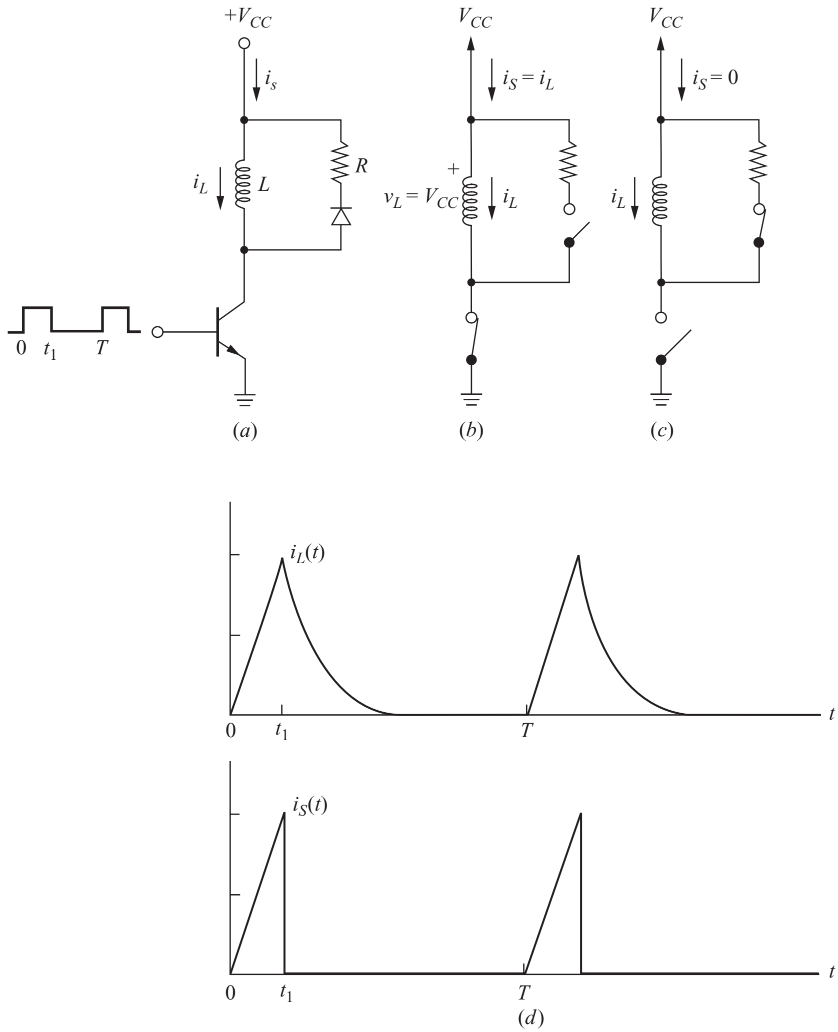


Figure 2-4 (*a*) A circuit to energize an inductance and then transfer the stored energy to a resistor; (*b*) Equivalent circuit when the transistor is on; (*c*) Equivalent circuit when the transistor is off and the diode is on; (*d*) Inductor and source currents.

energy in the inductor when the transistor turns off. Without the diode-resistor path, the transistor could be destroyed when it is turned off because a rapid decrease in inductor current would result in excessively high inductor and transistor voltages.

Assume that the transistor switch turns on at $t = 0$ and turns off at $t = t_1$. The circuit is analyzed first for the transistor switch on and then for the switch off.

Transistor on: $0 < t < t_1$

The voltage across the inductor is V_{CC}, and the diode is reverse-biased when the transistor is on (Fig. 2-4*b*).

$$v_L = V_{CC} \tag{2-18}$$

An expression for inductor current is obtained from the voltage-current relationship:

$$i_L(t) = \frac{1}{L}\int_0^t v_L(\lambda)\,d\lambda + i_L(0) = \frac{1}{L}\int_0^t V_{CC}\,d\lambda + 0 = \frac{V_{CC}t}{L} \tag{2-19}$$

Source current is the same as inductor current.

$$i_s(t) = i_L(t) \tag{2-20}$$

Inductor and source currents thus increase linearly when the transistor is on.

The circuit is next analyzed for the transistor switch off.

Transistor off: $t_1 < t < T$

In the interval $t_1 < t < T$, the transistor switch is off and the diode is on (Fig. 2-4*c*). The current in the source is zero, and the current in the inductor and resistor is a decaying exponential with time constant L/R. The initial condition for inductor current is determined from Eq. (2-19):

$$i_L(t_1) = \frac{V_{CC}t_1}{L} \tag{2-21}$$

Inductor current is then expressed as

$$i_L(t) = i_L(t_1)e^{-(t-t_1)/\tau} = \left(\frac{V_{CC}t_1}{L}\right)e^{-(t-t_1)/\tau} \qquad t_1 < t < T \tag{2-22}$$

where $\tau = L/R$. Source current is zero when the transistor is off.

$$i_S = 0 \tag{2-23}$$

Average power supplied by the dc source during the switching period is determined from the product of voltage and average current [Eq. (2-5)].

$$\begin{aligned} P_S = V_S I_S &= V_{CC}\left[\frac{1}{T}\int_0^T i_s(t)\,dt\right] \\ &= V_{CC}\left[\frac{1}{T}\int_0^{t_1}\frac{V_{CC}t}{L}\,dt + \frac{1}{T}\int_{t_1}^T 0\,dt\right] = \frac{(V_{CC}t_1)^2}{2LT} \end{aligned} \tag{2-24}$$

Average power absorbed by the resistor could be determined by integrating an expression for instantaneous resistor power, but an examination of the circuit reveals an easier way. The average power absorbed by the inductor is zero, and power absorbed by the ideal transistor and diode is zero. Therefore, all power supplied by the source must be absorbed by the resistor:

$$P_R = P_S = \frac{(V_{CC}\,t_1)^2}{2LT} \tag{2-25}$$

Another way to approach the problem is to determine the peak energy stored in the inductor,

$$W = \frac{1}{2}Li^2(t_1) = \frac{1}{2}L\left(\frac{V_{CC}t_1}{L}\right)^2 = \frac{(V_{CC}\,t_1)^2}{2L} \tag{2-26}$$

The energy stored in the inductor is transferred to the resistor while the transistor switch is open. Power absorbed by the resistor can be determined from Eq. (2-4).

$$P_R = \frac{W}{T} = \frac{(V_{CC}\,t_1)^2}{2LT} \tag{2-27}$$

which must also be the power supplied by the source. The function of the resistor in this circuit of Fig. 2-4*a* is to absorb the stored energy in the inductance and protect the transistor. This energy is converted to heat and represents a power loss in the circuit.

Another way to remove the stored energy in the inductor is shown in Fig. 2-5*a*. Two transistor switches are turned on and off simultaneously. The diodes provide a means of returning energy stored in the inductor back to the source. Assume that the transistors turn on at $t = 0$ and turn off at $t = t_1$. The analysis of the circuit of Fig. 2-5*a* begins with the transistors on.

Transistors on: $0 < t < t_1$

When the transistors are on, the diodes are reverse-biased, and the voltage across the inductor is V_{CC}. The inductor voltage is the same as the source when the transistors are on (Fig. 2-5*b*):

$$v_L = V_{CC} \tag{2-28}$$

Inductor current is the function

$$i_L(t) = \frac{1}{L}\int_0^t v_L(\lambda)\,d\lambda + i_L(0) = \frac{1}{L}\int_0^t V_{CC}\,d\lambda + 0 = \frac{V_{CC}t}{L} \tag{2-29}$$

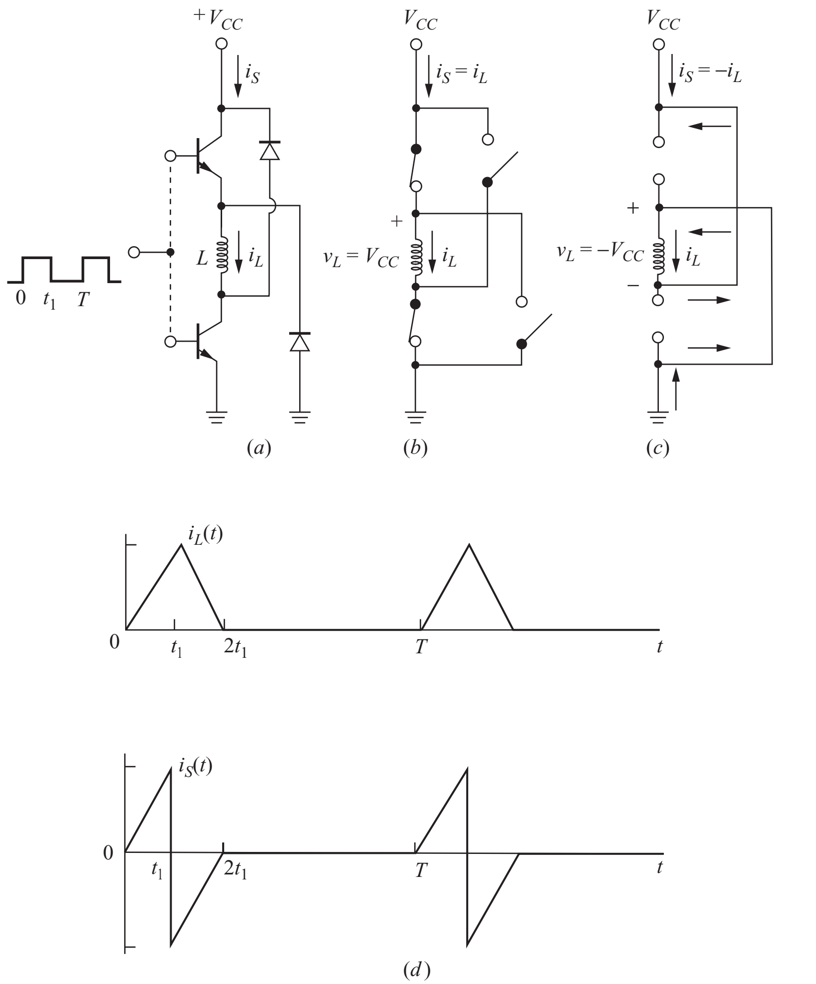


Figure 2-5 (*a*) A circuit to energize an inductance and recover the stored energy by transferring it back to the source; (*b*) Equivalent circuit when the transistors are on; (*c*) Equivalent circuit when the transistors are off and the diodes are on; (*d*) Inductor and source currents.

Source current is the same as inductor current.

$$i_S(t) = i_L(t) \tag{2-30}$$

From the preceding equations, inductor and source currents increase linearly while the transistor switches are on, as was the case for the circuit of Fig. 2-4*a*.

The circuit is next analyzed for the transistors off.

Transistors off: $t_1 < t < T$

When the transistors are turned off, the diodes become forward-biased to provide a path for the inductor current (Fig. 2-5*c*). The voltage across the inductor then becomes the opposite of the source voltage:

$$v_L = -V_{CC} \tag{2-31}$$

An expression for inductor current is obtained from the voltage-current relationship.

$$i_L(t) = \frac{1}{L}\int_{t_1}^{t} v_L(\lambda)\,d\lambda + i_L(t_1) = \frac{1}{L}\int_{t_1}^{t} (-V_{CC})\,d\lambda + \frac{V_{CC}t_1}{L}$$
$$= \left(\frac{V_{CC}}{L}\right)[(t_1 - t) + t_1]$$

or,

$$i_L(t) = \left(\frac{V_{CC}}{L}\right)(2t_1 - t) \qquad t_1 < t < 2t_1 \tag{2-32}$$

Inductor current decreases and becomes zero at $t = 2t_1$, at which time the diodes turn off. Inductor current remains at zero until the transistors turn on again.

Source current is the opposite of inductor current when the transistors are off and the diodes are on:

$$i_S(t) = -i_L(t) \tag{2-33}$$

The source is absorbing power when the source current is negative. Average source current is zero, resulting in an average source power of zero.

The source supplies power while the transistors are on, and the source absorbs power while the transistors are off and the diodes are on. Therefore, the energy stored in the inductor is recovered by transferring it back to the source. Practical solenoids or other magnetic devices have equivalent resistances that represent losses or energy absorbed to do work, so not all energy will be returned to the source. The circuit of Fig. 2-5*a* has no energy losses inherent to the design and is therefore more efficient than that of Fig. 2-4*a*.

EXAMPLE 2-3

Energy Recovery

The circuit of Fig. 2-4*a* has V_{CC} = 90 V, L = 200 mH, R = 20 Ω, t_1 = 10 ms, and T = 100 ms. Determine (*a*) the peak current and peak energy storage in the inductor, (*b*) the average power absorbed by the resistor, and (*c*) the peak and average power supplied by the source. (*d*) Compare the results with what would happen if the inductor were energized using the circuit of Fig. 2-5*a*.

■ **Solution**

(*a*) From Eq. (2-19), when the transistor switch is on, inductor current is

$$i_L(t) = \left(\frac{V_{CC}}{L}\right)t = \left(\frac{90}{0.2}\right)t = 450t \text{ A} \qquad 0 < t < 10 \text{ ms}$$

Peak inductor current and stored energy are

$$i_L(t_1) = 450(0.01) = 4.5 \text{ A}$$

$$W_L = \frac{1}{2}Li^2(t_1) = \frac{1}{2}(0.2)(4.5)^2 = 2.025 \text{ J}$$

(*b*) The time constant for the current when the switch is open is $L/R = 200 \text{ mH}/20\,\Omega = 10$ ms. The switch is open for 90 ms, which is 10 time constants, so essentially all stored energy in the inductor is transferred to the resistor:

$$W_R = W_L = 2.025 \text{ J}$$

Average power absorbed by the resistor is determined from Eq. (2-4):

$$P_R = \frac{W_R}{T} = \frac{2.025 \text{ J}}{0.1 \text{ s}} = 20.25 \text{ W}$$

(*c*) The source current is the same as the inductor current when the switch is closed and is zero when the switch is open. Instantaneous power supplied by the source is

$$p_S(t) = v_S(t)i_S(t) = \begin{cases} (90\,\text{V})(450t\text{ A}) = 40{,}500t \text{ W} & 0 < t < 10 \text{ ms} \\ 0 & 10 \text{ ms} < t < 100 \text{ ms} \end{cases}$$

which has a maximum value of 405 W at $t = 10$ ms. Average power supplied by the source can be determined from Eq. (2-3):

$$P_S = \frac{1}{T}\int_0^T p_S(t)\,dt = \frac{1}{0.1}\left(\int_0^{0.01} 40{,}500t\,dt + \int_{0.01}^{0.1} 0\,dt\right) = 20.25 \text{ W}$$

Average source power also can be determined from Eq. (2-5). Average of the triangular source current waveform over one period is

$$I_S = \frac{1}{2}\left[\frac{(0.01\text{ s})(4.5\text{ A})}{0.1\text{ s}}\right] = 0.225 \text{ A}$$

and average source power is then

$$P_S = V_{CC}I_S = (90 \text{ V})(0.225 \text{ A}) = 20.25 \text{ W}$$

Still another computation of average source power comes from recognizing that the power absorbed by the resistor is the same as that supplied by the source.

$$P_S = P_R = 20.25 \text{ W}$$

(See Example 2-13 at the end of this chapter for the PSpice simulation of this circuit.)

(*d*) When the inductor is energized from the circuit of Fig. 2-5*a*, the inductor current is described by Eqs. (2-29) and (2-32).

$$i_L(t) = \begin{cases} 450t \text{ A} & 0 < t < 10 \text{ ms} \\ 9 - 450t \text{ A} & 10 \text{ ms} < t < 20 \text{ ms} \\ 0 & 20 \text{ ms} < t < 100 \text{ ms} \end{cases}$$

The peak current and peak energy storage are the same as for the circuit of Fig. 2-4*a*. The source current has the form shown in Fig. 2-5*d* and is expressed as

$$i_S(t) = \begin{cases} 450t \text{ A} & 0 < t < 10 \text{ ms} \\ 450t - 9 \text{ A} & 10 \text{ ms} < t < 20 \text{ ms} \\ 0 & 20 \text{ ms} < t < 100 \text{ ms} \end{cases}$$

Instantaneous power supplied by the source is

$$p_S(t) = 90 i_S(t) = \begin{cases} 40{,}500t \text{ W} & 0 < t < 10 \text{ ms} \\ 40{,}500t - 810 \text{ W} & 10 \text{ ms} < t < 20 \text{ ms} \\ 0 & 20 \text{ ms} < t < 100 \text{ ms} \end{cases}$$

Average source current is zero, and average source power is zero. Peak source power is peak current times voltage, which is 405 W as in part (*c*).

2.5 EFFECTIVE VALUES: RMS

The effective value of a voltage or current is also known as the root-mean-square (rms) value. The effective value of a periodic voltage waveform is based on the average power delivered to a resistor. For a dc voltage across a resistor,

$$P = \frac{V_{dc}^2}{R} \tag{2-34}$$

For a periodic voltage across a resistor, *effective voltage* is defined as the voltage that is as effective as the dc voltage in supplying average power. Effective voltage can be computed using the equation

$$P = \frac{V_{\text{eff}}^2}{R} \tag{2-35}$$

Computing average resistor power from Eq. (2-3) gives

$$\begin{aligned} P &= \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T v(t)i(t)\,dt = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt \\ &= \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2(t)\,dt\right] \end{aligned} \tag{2-36}$$

Equating the expressions for average power in Eqs. (2-35) and (2-36) gives

$$P = \frac{V_{\text{eff}}^2}{R} = \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]$$

or

$$V_{\text{eff}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$$

resulting in the expression for effective or rms voltage

$$\boxed{V_{\text{eff}} = V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}} \tag{2-37}$$

The effective value is the square *root* of the *mean* of the *square* of the voltage—hence the term *root mean square*.

Similarly, rms current is developed from $P = I_{\text{rms}}^2$ as

$$\boxed{I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}} \tag{2-38}$$

The usefulness of the rms value of voltages and currents lies in the computing power absorbed by resistances. Additionally, ac power system voltages and currents are invariably given in rms values. Ratings of devices such as transformers are often specified in terms of rms voltage and current.

EXAMPLE 2-4

RMS Value of a Pulse Waveform

Determine the rms value of the periodic pulse waveform that has a duty ratio of D as shown in Fig. 2-6.

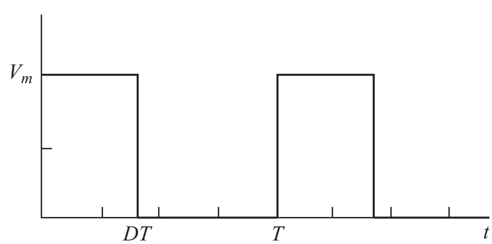


Figure 2-6 Pulse waveform for Example 2-4.

■ **Solution**

The voltage is expressed as

$$v(t) = \begin{cases} V_m & 0 < t < DT \\ 0 & DT < t < T \end{cases}$$

Using Eq. (2-37) to determine the rms value of the waveform gives

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\left(\int_0^{DT} V_m^2\,dt + \int_{DT}^{T} 0^2\,dt\right)} = \sqrt{\frac{1}{T}(V_m^2\,DT)}$$

yielding

$$V_{\text{rms}} = V_m\sqrt{D}$$

EXAMPLE 2-5

RMS Values of Sinusoids

Determine the rms values of (*a*) a sinusoidal voltage of $v(t) = V_m \sin(\omega t)$, (*b*) a full-wave rectified sine wave of $v(t) = |V_m \sin(\omega t)|$, and (*c*) a half-wave rectified sine wave of $v(t) = V_m \sin(\omega t)$ for $0 < t < T/2$ and zero otherwise.

■ **Solution**

(*a*) The rms value of the sinusoidal voltage is computed from Eq. (2-37):

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \sin^2(\omega t)\,dt} \qquad \text{where } T = \frac{2\pi}{\omega}$$

An equivalent expression uses ωt as the variable of integration. Without showing the details of the integration, the result is

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\,d(\omega t)} = \frac{V_m}{\sqrt{2}}$$

Note that the rms value is independent of the frequency.

(*b*) Equation (2-37) can be applied to the full-wave rectified sinusoid, but the results of part (*a*) can also be used to advantage. The rms formula uses the integral of the square of the function. The square of the sine wave is identical to the square of the full-wave rectified sine wave, so the rms values of the two waveforms are identical:

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

(*c*) Equation (2-37) can be applied to the half-wave rectified sinusoid.

$$V_{\mathrm{rms}} = \sqrt{\frac{1}{2\pi}\left(\int_0^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t) + \int_{\pi}^{2\pi} 0^2\, d(\omega t)\right)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

The result of part (*a*) will again be used to evaluate this expression. The square of the function has one-half the area of that of the functions in (*a*) and (*b*). That is,

$$V_{\mathrm{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)} = \sqrt{\left(\frac{1}{2}\right)\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

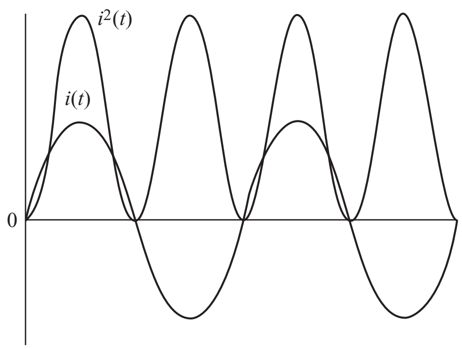
Taking the $1/2$ outside of the square root gives

$$V_{\mathrm{rms}} = \left(\sqrt{\frac{1}{2}}\right)\sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

The last term on the right is the rms value of a sine wave which is known to be $V_m/\sqrt{2}$, so the rms value of a half-wave rectified sine wave is

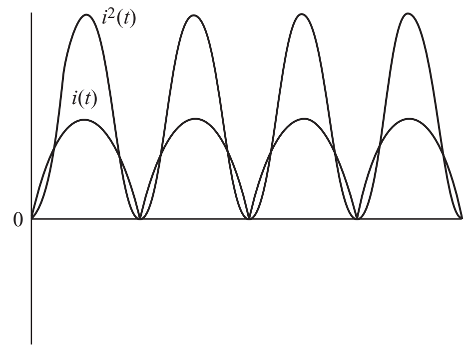
$$V_{\mathrm{rms}} = \sqrt{\frac{1}{2}}\,\frac{V_m}{\sqrt{2}} = \frac{V_m}{2}$$

Figure 2-7 shows the waveforms.

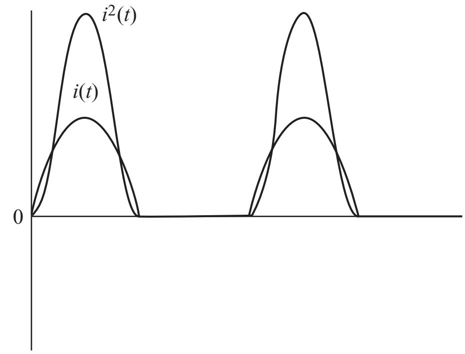


(*a*)

Figure 2-7 Waveforms and their squares for Example 2-5 (*a*) Sine wave; (*b*) full-wave rectified sine wave; (*c*) half-wave rectified sine wave.



(*b*)



(*c*)

Figure 2-7 (*continued*)

EXAMPLE 2-6

Neutral Conductor Current in a Three-Phase System

An office complex is supplied from a three-phase four-wire voltage source (Fig. 2-8*a*). The load is highly nonlinear as a result of the rectifiers in the power supplies of the equipment, and the current in each of the three phases is shown in Fig. 2-8*b*. The neutral current is the sum of the phase currents. If the rms current in each phase conductor is known to be 20 A, determine the rms current in the neutral conductor.

■ Solution

Equation (2-38) may be applied to this case. Noting by inspection that the area of the square of the current function in the neutral i_n, is 3 times that of each of the phases i_a (Fig. 2-8*c*)

$$I_{n,\,\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i_n^2(t)\,d(t)} = \sqrt{3\left(\frac{1}{T}\int_0^T i_a^2(t)\,d(t)\right)} = \sqrt{3}\,I_{a,\,\text{rms}}$$

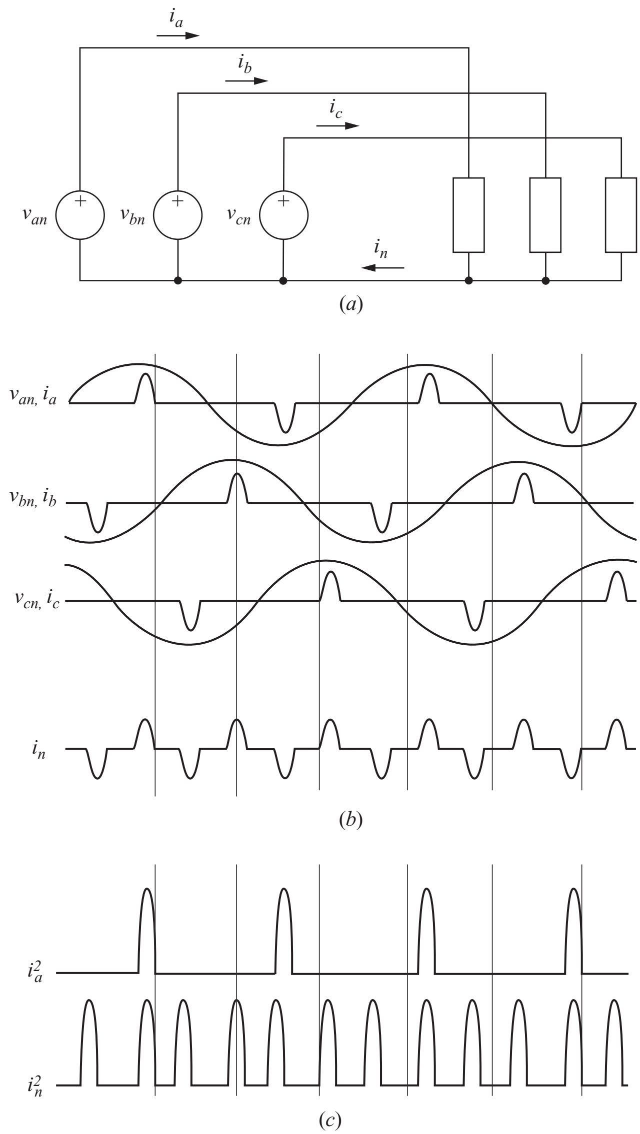


Figure 2-8 (*a*) Three-phase source supplying a balanced nonlinear three-phase load for Example 2-8; (*b*) phase and neutral currents; (*c*) squares of i_a and i_n.

The rms current in the neutral is therefore

$$I_{n,\,\mathrm{rms}} = \sqrt{3}(20) = 34.6\ \mathrm{A}$$

Note that the rms neutral current is larger than the phase currents for this situation. This is much different from that for balanced linear loads where the line currents are sinusoids which are displaced by 120° and sum to zero. Three-phase distribution systems supplying highly nonlinear loads should have a neutral conductor capable of carrying $\sqrt{3}$ times as much current as the line conductor.

If a periodic voltage is the sum of two periodic voltage waveforms, $v(t) = v_1(t) + v_2(t)$, the rms value of $v(t)$ is determined from Eq. (2-37) as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T (v_1 + v_2)^2\,dt = \frac{1}{T}\int_0^T \left(v_1^2 + 2v_1v_2 + v_2^2\right) dt$$

or

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v_1^2\,dt + \frac{1}{T}\int_0^T 2v_1v_2\,dt + \frac{1}{T}\int_0^T v_2^2\,dt$$

The term containing the product v_1v_2 in the above equation is zero if the functions v_1 and v_2 are orthogonal. A condition that satisfies that requirement occurs when v_1 and v_2 are sinusoids of different frequencies. For orthogonal functions,

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v_1^2(t)\,dt + \frac{1}{T}\int_0^T v_2^2(t)\,dt$$

Noting that

$$\frac{1}{T}\int_0^T v_1^2(t)\,dt = V_{1,\text{rms}}^2 \quad \text{and} \quad \frac{1}{T}\int_0^T v_2^2(t)\,dt = V_{2,\text{rms}}^2$$

then

$$V_{\text{rms}} = \sqrt{V_{1,\text{rms}}^2 + V_{2,\text{rms}}^2}$$

If a voltage is the sum of more than two periodic voltages, all orthogonal, the rms value is

$$\boxed{V_{\text{rms}} = \sqrt{V_{1,\text{rms}}^2 + V_{2,\text{rms}}^2 + V_{3,\text{rms}}^2 + \ldots} = \sqrt{\sum_{n=1}^{N} V_{n,\text{rms}}^2}} \qquad (2\text{-}39)$$

Similarly,

$$\boxed{I_{\text{rms}} = \sqrt{I_{1,\text{rms}}^2 + I_{2,\text{rms}}^2 + I_{3,\text{rms}}^2 + \ldots} = \sqrt{\sum_{n=1}^{N} I_{\text{n},\text{rms}}^2}} \qquad (2\text{-}40)$$

Note that Eq. (2-40) can be applied to Example 2-6 to obtain the rms value of the neutral current.

EXAMPLE 2-7

RMS Value of the Sum of Waveforms

Determine the effective (rms) value of $v(t) = 4 + 8\sin(\omega_1 t + 10°) + 5\sin(\omega_2 t + 50°)$ for (*a*) $\omega_2 = 2\omega_1$ and (*b*) $\omega_2 = \omega_1$.

■ Solution

(*a*) The rms value of a single sinusoid is $V_m/\sqrt{2}$, and the rms value of a constant is the constant. When the sinusoids are of different frequencies, the terms are orthogonal and Eq. (2-39) applies.

$$V_{\text{rms}} = \sqrt{V_{1,\text{rms}}^2 + V_{2,\text{rms}}^2 + V_{3,\text{rms}}^2} = \sqrt{4^2 + \left(\frac{8}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = 7.78 \text{ V}$$

(*b*) For sinusoids of the same frequency, Eq. (2-39) does not apply because the integral of the cross product over one period is not zero. First combine the sinusoids using phasor addition:

$$8\angle 10° + 5\angle 50° = 12.3\angle 25.2°$$

The voltage function is then expressed as

$$v(t) = 4 + 12.3 \sin(\omega_1 t + 25.2°) \text{ V}$$

The rms value of this voltage is determined from Eq. (2-39) as

$$V_{\text{rms}} = \sqrt{4^2 + \left(\frac{12.3}{\sqrt{2}}\right)^2} = 9.57 \text{ V}$$

EXAMPLE 2-8

RMS Value of Triangular Waveforms

(*a*) A triangular current waveform like that shown in Fig. 2-9a is commonly encountered in dc power supply circuits. Determine the rms value of this current.

(*b*) Determine the rms value of the offset triangular waveform in Fig. 2-9*b*.

■ Solution

(*a*) The current is expressed as

$$i(t) = \begin{cases} \dfrac{2I_m}{t_1} t - I_m & 0 < t < t_1 \\[2ex] \dfrac{-2I_m}{T - t_1} t + \dfrac{I_m(T + t_1)}{T - t_1} & t_1 < t < T \end{cases}$$

The rms value is determined from Eq. (2-38).

$$I_{\text{rms}}^2 = \frac{1}{T}\left[\int_0^{t_1} \left(\frac{2I_m}{t_1} t - I_m\right)^2 dt + \int_{t_1}^{T} \left(\frac{-2I_m}{T - t_1} t + \frac{I_m(T + t_1)}{T - t_1}\right)^2 dt\right]$$

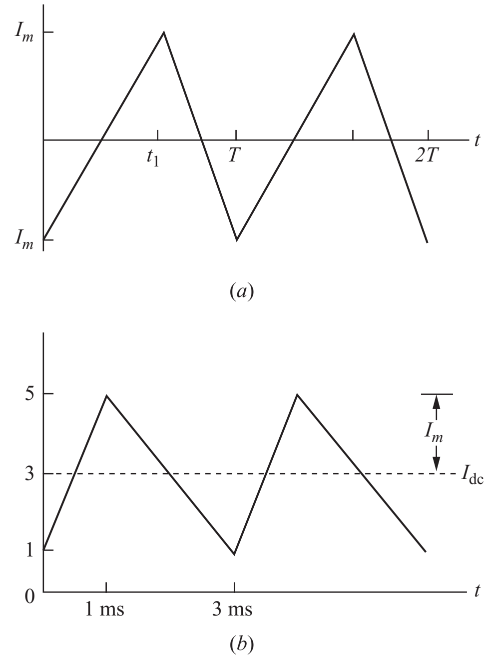


Figure 2-9 (*a*) Triangular waveform for Example 2-8; (*b*) offset triangular waveform.

The details of the integration are quite long, but the result is simple: The rms value of a triangular current waveform is

$$I_{\text{rms}} = \frac{I_m}{\sqrt{3}}$$

(*b*) The rms value of the offset triangular waveform can be determined by using the result of part (*a*). Since the triangular waveform of part (*a*) contains no dc component, the dc signal and the triangular waveform are orthogonal, and Eq. (2-40) applies.

$$I_{\text{rms}} = \sqrt{I_{1,\text{rms}}^2 + I_{2,\text{rms}}^2} = \sqrt{\left(\frac{I_m}{\sqrt{3}}\right)^2 + I_{\text{dc}}^2} = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 3^2} = 3.22 \text{ A}$$

2.6 APPARENT POWER AND POWER FACTOR

Apparent Power *S*

Apparent power is the product of rms voltage and rms current magnitudes and is often used in specifying the rating of power equipment such as transformers. Apparent power is expressed as

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}} \tag{2-41}$$

In ac circuits (linear circuits with sinusoidal sources), apparent power is the magnitude of complex power.

Power Factor

The *power factor* of a load is defined as the ratio of average power to apparent power:

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}} \tag{2-42}$$

In sinusoidal ac circuits, the above calculation results in pf $= \cos\theta$ where θ is the phase angle between the voltage and current sinusoids. However, that is a special case and should be used only when both voltage and current are sinusoids. In general, power factor must be computed from Eq. (2-42).

2.7 POWER COMPUTATIONS FOR SINUSOIDAL AC CIRCUITS

In general, voltages and/or currents in power electronics circuits are not sinusoidal. However, a nonsinusoidal periodic waveform can be represented by a Fourier series of sinusoids. It is therefore important to understand thoroughly power computations for the sinusoidal case. The following discussion is a review of power computations for ac circuits.

For linear circuits that have sinusoidal sources, all steady-state voltages and currents are sinusoids. Instantaneous power and average power for ac circuits are computed using Eqs. (2-1) and (2-3) as follows:

For any element in an ac circuit, let

$$\begin{aligned} v(t) &= V_m \cos(\omega t + \theta) \\ i(t) &= I_m \cos(\omega t + \phi) \end{aligned} \tag{2-43}$$

Then instantaneous power is

$$p(t) = v(t)i(t) = [V_m \cos(\omega t + \theta)][I_m \cos(\omega t + \phi)] \tag{2-44}$$

Using the trigonometric identity gives

$$(\cos A)(\cos B) = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \tag{2-45}$$

$$p(t) = \left(\frac{V_m I_m}{2}\right)[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)] \tag{2-46}$$

Average power is

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \left(\frac{V_m I_m}{2}\right)\int_0^T [\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)]\,dt \tag{2-47}$$

The result of the above integration can be obtained by inspection. Since the first term in the integration is a cosine function, the integral over one period is zero because of equal areas above and below the time axis. The second term in the integration is the constant $\cos(\theta - \phi)$, which has an average value of $\cos(\theta - \phi)$. Therefore, the average power in any element in an ac circuit is

$$P = \left(\frac{V_m I_m}{2}\right)\cos(\theta - \phi) \tag{2-48}$$

This equation is frequently expressed as

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta - \phi) \tag{2-49}$$

where $V_{\text{rms}} = V_m/\sqrt{2}$, $I_{\text{rms}} = I_m/\sqrt{2}$, and $\theta - \phi$ is the phase angle between voltage and current. The power factor is determined to be $\cos(\theta - \phi)$ by using Eq. (2-42).

In the steady state, no net power is absorbed by an inductor or a capacitor. The term *reactive power* is commonly used in conjunction with voltages and currents for inductors and capacitors. Reactive power is characterized by energy storage during one-half of the cycle and energy retrieval during the other half. Reactive power is computed with a relationship similar to Eq. (2-49):

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta - \phi) \tag{2-50}$$

By convention, inductors absorb positive reactive power and capacitors absorb negative reactive power.

Complex power combines real and reactive powers for ac circuits:

$$\mathbf{S} = P + jQ = (\mathbf{V}_{\text{rms}})(\mathbf{I}_{\text{rms}})^* \tag{2-51}$$

In the above equation, $\mathbf{V}_{\text{rms}}$ and $\mathbf{I}_{\text{rms}}$ are complex quantities often expressed as phasors (magnitude and angle), and $(\mathbf{I}_{\text{rms}})^*$ is the complex conjugate of phasor current, which gives results consistent with the convention that inductance, or lagging current, absorbs reactive power. Apparent power in ac circuits is the magnitude of complex power:

$$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} \tag{2-52}$$

It is important to note that the complex power in Eq. (2-52) and power factor of $\cos(\theta - \phi)$ for sinusoidal ac circuits are special cases and are not applicable to nonsinusoidal voltages and currents.

2.8 POWER COMPUTATIONS FOR NONSINUSOIDAL PERIODIC WAVEFORMS

Power electronics circuits typically have voltages and/or currents that are periodic but not sinusoidal. For the general case, the basic definitions for the power terms described at the beginning of this chapter must be applied. A common error that is made when doing power computations is to attempt to apply some special relationships for sinusoids to waveforms that are not sinusoids.

The Fourier series can be used to describe nonsinusoidal periodic waveforms in terms of a series of sinusoids. The power relationships for these circuits can be expressed in terms of the components of the Fourier series.

Fourier Series

A nonsinusoidal periodic waveform that meets certain conditions can be described by a Fourier series of sinusoids. The Fourier series for a periodic function $f(t)$ can be expressed in trigonometric form as

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \tag{2-53}$$

where

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega_0 t)\,dt \tag{2-54}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega_0 t)\,dt$$

Sines and cosines of the same frequency can be combined into one sinusoid, resulting in an alternative expression for a Fourier series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

where (2-55)

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

or

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \theta_n)$$

where (2-56)

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

The term a_0 is a constant that is the average value of $f(t)$ and represents a dc voltage or current in electrical applications. The coefficient C_1 is the amplitude of the term at the fundamental frequency ω_0. Coefficients $C_2, C_3, \ldots$ are the amplitudes of the harmonics that have frequencies $2\omega_0, 3\omega_0, \ldots$.

The rms value of $f(t)$ can be computed from the Fourier series:

$$F_{\text{rms}} = \sqrt{\sum_{n=0}^{\infty} F_{n,\text{rms}}^2} = \sqrt{a_0^2 + \sum_{n=1}^{\infty}\left(\frac{C_n}{\sqrt{2}}\right)^2} \tag{2-57}$$

Average Power

If periodic voltage and current waveforms represented by the Fourier series

$$\begin{aligned} v(t) &= V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n) \\ i(t) &= I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \phi_n) \end{aligned} \tag{2-58}$$

exist for a device or circuit, then average power is computed from Eq. (2-3).

$$P = \frac{1}{T}\int_0^T v(t)i(t)\,dt$$

The average of the products of the dc terms is $V_0 I_0$. The average of voltage and current products at the same frequency is described by Eq. (2-49), and the average of voltage and current products of different frequencies is zero. Consequently, average power for nonsinusoidal periodic voltage and current waveforms is

$$\boxed{\begin{aligned} P &= \sum_{n=0}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,\text{rms}} I_{n,\text{rms}} \cos(\theta_n - \phi_n) \\ &\text{or} \\ P &= V_0 I_0 + \sum_{n=1}^{\infty}\left(\frac{V_{n,\max} I_{n,\max}}{2}\right)\cos(\theta_n - \phi_n) \end{aligned}} \tag{2-59}$$

Note that total average power is the sum of the powers at the frequencies in the Fourier series.

Nonsinusoidal Source and Linear Load

If a nonsinusoidal periodic voltage is applied to a load that is a combination of linear elements, the power absorbed by the load can be determined by using superposition. A nonsinusoidal periodic voltage is equivalent to the series combination of the Fourier series voltages, as illustrated in Fig. 2-10. The current in the load can be determined using superposition, and Eq. (2-59) can be applied to compute average power. Recall that superposition for power is not valid when the sources are of the same frequency. The technique is demonstrated in Example 2-9.

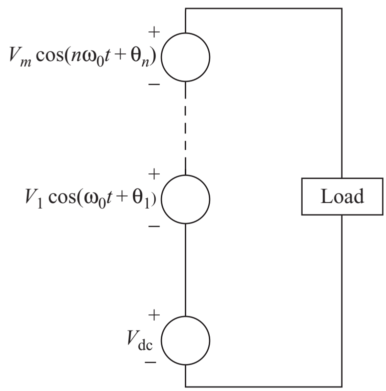


Figure 2.10 Equivalent circuit for Fourier analysis.

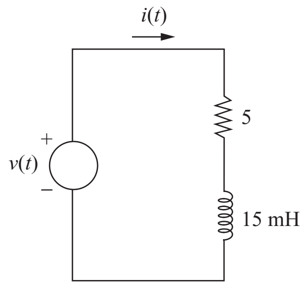


Figure 2.11 Circuit for Example 2-9.

EXAMPLE 2-9

Nonsinusoidal Source and Linear Load

A nonsinusoidal periodic voltage has a Fourier series of $v(t) = 10 + 20\cos(2\pi 60t - 25°) + 30\cos(4\pi 60t + 20°)$ V. This voltage is connected to a load that is a 5-Ω resistor and a 15-mH inductor connected in series as in Fig. 2-11. Determine the power absorbed by the load.

■ Solution

Current at each source frequency is computed separately. The dc current term is

$$I_0 = \frac{V_0}{R} = \frac{10}{5} = 2 \text{ A}$$

The amplitudes of the ac current terms are computed from phasor analysis:

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{R + j\omega_1 L} = \frac{20\angle(-25°)}{5 + j(2\pi 60)(0.015)} = 2.65\angle(-73.5°) \text{ A}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_2}{R + j\omega_2 L} = \frac{30\angle 20°}{5 + j(4\pi 60)(0.015)} = 2.43\angle(-46.2°) \text{ A}$$

Load current can then be expressed as

$$i(t) = 2 + 2.65\cos(2\pi 60t - 73.5°) + 2.43\cos(4\pi 60t - 46.2°) \text{ A}$$

Power at each frequency in the Fourier series is determined from Eq. (2-59):

$$\text{dc term:} \qquad P_0 = (10 \text{ V})(2 \text{ A}) = 20 \text{ W}$$

$$\omega = 2\pi 60: \qquad P_1 = \frac{(20)(2.65)}{2}\cos(-25° + 73.5°) = 17.4 \text{ W}$$

$$\omega = 4\pi 60: \qquad P_2 = \frac{(30)(2.43)}{2}\cos(20° + 46°) = 14.8 \text{ W}$$

Total power is then

$$P = 20 + 17.4 + 14.8 = 52.2 \text{ W}$$

Power absorbed by the load can also be computed from $I^2_{\text{rms}}R$ in this circuit because the average power in the inductor is zero.

$$P = I^2_{\text{rms}}R = \left[2^2 + \left(\frac{2.65}{\sqrt{2}}\right)^2 + \left(\frac{2.43}{\sqrt{2}}\right)^2\right]5 = 52.2 \text{ W}$$

Sinusoidal Source and Nonlinear Load

If a sinusoidal voltage source is applied to a nonlinear load, the current waveform will not be sinusoidal but can be represented as a Fourier series. If voltage is the sinusoid

$$v(t) = V_1 \sin(\omega_0 t + \theta_1) \tag{2-60}$$

and current is represented by the Fourier series

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sin(n\omega_0 t + \phi_n) \tag{2-61}$$

then average power absorbed by the load (or supplied by the source) is computed from Eq. (2-59) as

$$\begin{aligned} P &= V_0 I_0 + \sum_{n=1}^{\infty}\left(\frac{V_{n,\max} I_{n,\max}}{2}\right)\cos(\theta_n - \phi_n) \\ &= (0)(I_0) + \left(\frac{V_1 I_1}{2}\right)\cos(\theta_1 - \phi_1) + \sum_{n=2}^{\infty}\frac{(0)(I_{n,\max})}{2}\cos(\theta_n - \phi_n) \\ &= \left(\frac{V_1 I_1}{2}\right)\cos(\theta_1 - \phi_1) = V_{1,\text{rms}} I_{1,\text{rms}} \cos(\theta_1 - \phi_1) \end{aligned} \tag{2-62}$$

Note that the only nonzero power term is at the frequency of the applied voltage. The power factor of the load is computed from Eq. (2-42).

$$\begin{aligned} \text{pf} &= \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}} \\ \text{pf} &= \frac{V_{1,\text{rms}} I_{1,\text{rms}} \cos(\theta_1 - \phi_1)}{V_{1,\text{rms}} I_{\text{rms}}} = \left(\frac{I_{1,\text{rms}}}{I_{\text{rms}}}\right)\cos(\theta_1 - \phi_1) \end{aligned} \tag{2-63}$$

where rms current is computed from

$$I_{\text{rms}} = \sqrt{\sum_{n=0}^{\infty} I^2_{n,\text{rms}}} = \sqrt{I_0^2 + \sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2} \tag{2-64}$$

Note also that for a sinusoidal voltage and a sinusoidal current, $\text{pf} = \cos(\theta_1 - \phi_1)$, which is the power factor term commonly used in linear circuits and is called the *displacement power factor*. The ratio of the rms value of the fundamental frequency to the total rms value, $I_{1,\text{rms}}/I_{\text{rms}}$ in Eq. (2-63), is the *distortion factor* (DF).

$$\text{DF} = \frac{I_{1,\text{rms}}}{I_{\text{rms}}} \tag{2-65}$$

The distortion factor represents the reduction in power factor due to the nonsinusoidal property of the current. Power factor is also expressed as

$$\text{pf} = [\cos(\theta_1 - \phi_1)]\,\text{DF} \tag{2-66}$$

Total harmonic distortion (THD) is another term used to quantify the nonsinusoidal property of a waveform. THD is the ratio of the rms value of all the nonfundamental frequency terms to the rms value of the fundamental frequency term.

$$\text{THD} = \sqrt{\frac{\sum\limits_{n\neq 1} I^2_{n,\text{rms}}}{I^2_{1,\text{rms}}}} = \frac{\sqrt{\sum\limits_{n\neq 1} I^2_{n,\text{rms}}}}{I_{1,\text{rms}}} \tag{2-67}$$

THD is equivalently expressed as

$$\text{THD} = \sqrt{\frac{I^2_{\text{rms}} - I^2_{1,\text{rms}}}{I^2_{1,\text{rms}}}} \tag{2-68}$$

Total harmonic distortion is often applied in situations where the dc term is zero, in which case THD may be expressed as

$$\text{THD} = \frac{\sqrt{\sum\limits_{n=2}^{\infty} I^2_n}}{I_1} \tag{2-69}$$

Another way to express the distortion factor is

$$\text{DF} = \sqrt{\frac{1}{1 + (\text{THD})^2}} \tag{2-70}$$

Reactive power for a sinusoidal voltage and a nonsinusoidal current can be expressed as in Eq. (2-50). The only nonzero term for reactive power is at the voltage frequency:

$$Q = \frac{V_1 I_1}{2} \sin(\theta_1 - \phi_1) \tag{2-71}$$

With P and Q defined for the nonsinusoidal case, apparent power S must include a term to account for the current at frequencies which are different from the voltage frequency. The term *distortion volt-amps D* is traditionally used in the computation of S,

$$S = \sqrt{P^2 + Q^2 + D^2} \tag{2-72}$$

where

$$D = V_{1,\,\text{rms}}\sqrt{\sum_{n\neq 1}^{\infty} I_{n,\,\text{rms}}^2} = \frac{V_1}{2}\sqrt{\sum_{n\neq 1}^{\infty} I_n} \tag{2-73}$$

Other terms that are sometimes used for nonsinusoidal current (or voltages) are *form factor* and *crest factor*.

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{avg}}} \tag{2-74}$$

$$\text{Crest factor} = \frac{I_{\text{peak}}}{I_{\text{rms}}} \tag{2-75}$$

EXAMPLE 2-10

Sinusoidal Source and a Nonlinear Load

A sinusoidal voltage source of $v(t) = 100\cos(377t)$ V is applied to a nonlinear load, resulting in a nonsinusoidal current which is expressed in Fourier series form as

$$i(t) = 8 + 15\cos(377t + 30°) + 6\cos[2(377)t + 45°] + 2\cos[3(377)t + 60°]$$

Determine (*a*) the power absorbed by the load, (*b*) the power factor of the load, (*c*) the distortion factor of the load current, (*d*) the total harmonic distortion of the load current.

■ Solution

(*a*) The power absorbed by the load is determined by computing the power absorbed at each frequency in the Fourier series [Eq. (2-59)].

$$P = (0)(8) + \left(\frac{100}{\sqrt{2}}\right)\left(\frac{15}{\sqrt{2}}\right)\cos 30° + (0)\left(\frac{6}{\sqrt{2}}\right)\cos 45° + (0)\left(\frac{2}{\sqrt{2}}\right)\cos 60°$$

$$P = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{15}{\sqrt{2}}\right)\cos 30° = 650\text{ W}$$

(*b*) The rms voltage is

$$V_{\text{rms}} = \frac{100}{\sqrt{2}} = 70.7\text{ V}$$

and the rms current is computed from Eq. (2-64):

$$I_{\text{rms}} = \sqrt{8^2 + \left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 14.0 \text{ A}$$

The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}} = \frac{650}{(70.7)(14.0)} = 0.66$$

Alternatively, power factor can be computed from Eq. (2-63):

$$\text{pf} = \frac{I_{1,\,\text{rms}} \cos(\theta_1 - \phi_1)}{I_{\text{rms}}} = \frac{\left(\frac{15}{\sqrt{2}}\right)\cos(0 - 30°)}{14.0} = 0.66$$

(*c*) The distortion factor is computed from Eq. (2-65) as

$$\text{DF} = \frac{I_{1,\,\text{rms}}}{I_{\text{rms}}} = \frac{\frac{15}{\sqrt{2}}}{14.0} = 0.76$$

(*d*) The total harmonic distortion of the load current is obtained from Eq. (2-68).

$$\text{THD} = \sqrt{\frac{I_{\text{rms}}^2 - I_{1,\text{rms}}^2}{I_{1,\text{rms}}^2}} = \sqrt{\frac{14^2 - \left(\frac{15}{\sqrt{2}}\right)^2}{\left(\frac{15}{\sqrt{2}}\right)^2}} = 0.86 = 86\%.$$

2.9 POWER COMPUTATIONS USING PSPICE

PSpice can be used to simulate power electronics circuits to determine voltages, currents, and power quantities. A convenient method is to use the numerical analysis capabilities of the accompanying graphics postprocessor program Probe to obtain power quantities directly. Probe is capable of

- Displaying voltage and current waveforms $(v)(t)$ and $i(t)$
- Displaying instantaneous power $p(t)$
- Computing energy absorbed by a device
- Computing average power P
- Computing average voltage and current
- Computing rms voltages and currents
- Determining the Fourier series of a periodic waveform

The examples that follow illustrate the use of PSpice to do power computations.

EXAMPLE 2-11

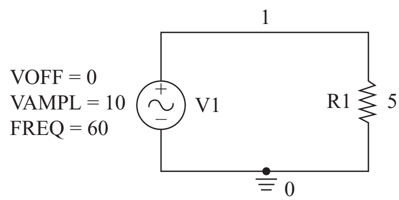
Instantaneous Power, Energy, and Average Power Using PSpice

PSpice can be used to display instantaneous power and to compute energy. A simple example is a sinusoidal voltage across a resistor. The voltage source has amplitude $V_m = 10$ V and frequency 60 Hz, and the resistor is 5 Ω. Use VSIN for the source, and select Time Domain (Transient) in the Simulation Setup. Enter a Run Time (Time to Stop) of 16.67 ms for one period of the source.

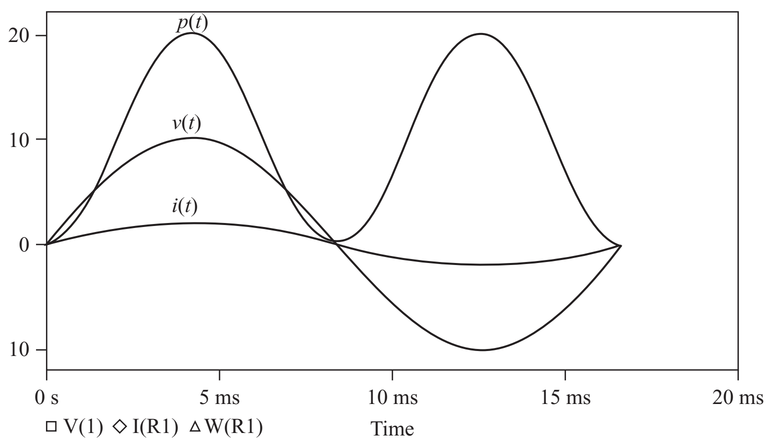
The circuit is shown in Fig. 2-12*a*. The top node is labeled as 1. When placing the resistor, rotate it 3 times so that the first node is upward. After running the simulation, the Netlist should look like this:

```
*source EXAMPLE 2-11
V_V1      1 0
+SIN 0 10 60 0 0 0
R_R1      1 0 5
```

When the simulation is completed, the Probe screen appears. The waveforms of voltage and current for the resistor are obtained by entering V(1) and I(R1). Instantaneous

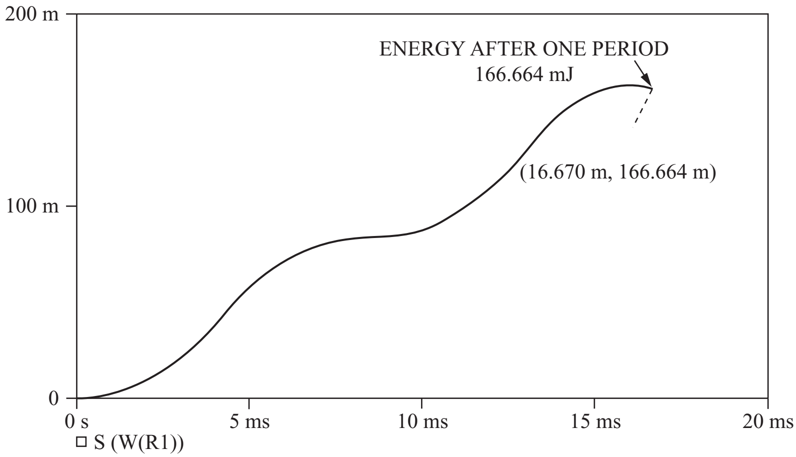


(*a*)

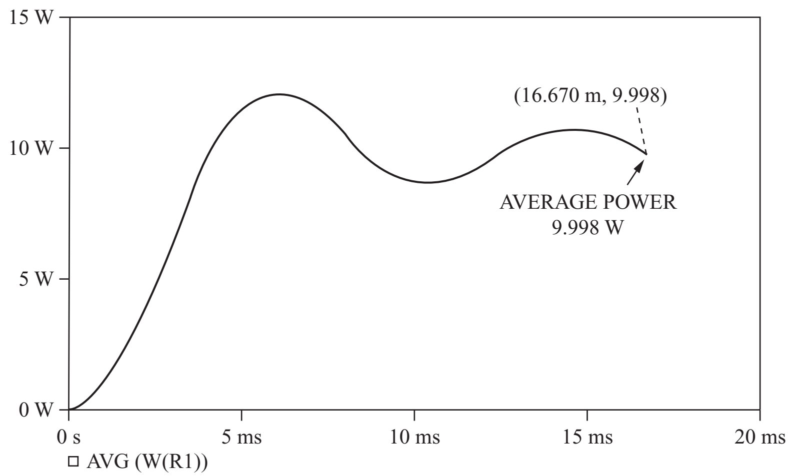


(*b*)

Figure 2.12 (*a*) PSpice circuit for Example 2-11; (*b*) voltage, current, and instantaneous power for the resistor; (*c*) energy absorbed by the resistor; (*d*) average power absorbed by the resistor.



(*c*)



(*d*)

Figure 2.12 (*continued*)

power $p(t) = v(t)i(t)$ absorbed by the resistor is obtained from Probe by entering the expression V(1)*I(R1) or by selecting W(R1). The resulting display showing $V(1)$, I(R1), and $p(t)$ is in Fig. 2-12*b*.

Energy can be computed using the definition of Eq. (2-2). When in Probe, enter the expression S(V(1)*I(R1)) or S(W(R1)), which computes the integral of instantaneous power. The result is a trace that shows that the energy absorbed increases with time. The energy absorbed by the resistor after one period of the source is determined by placing the cursor at the end of the trace, revealing $W_R = 166.66$ mJ (Fig. 2-12*c*).

The Probe feature of PSpice can also be used to determine the average value of power directly. For the circuit in the above example, average power is obtained by entering the expression AVG(V(1)*I(R1)) or AVG(W(R1)). The result is a "running" value of average power as computed in Eq. (2-3). Therefore, the average value of the power waveform must be obtained *at the end* of one or more periods of the waveform. Figure. 2-12*d* shows the output from Probe. The cursor option is used to obtain a precise value of average power. This output shows 9.998 W, very slightly different from the theoretical value of 10 W. Keep in mind that the integration is done numerically from discrete data points.

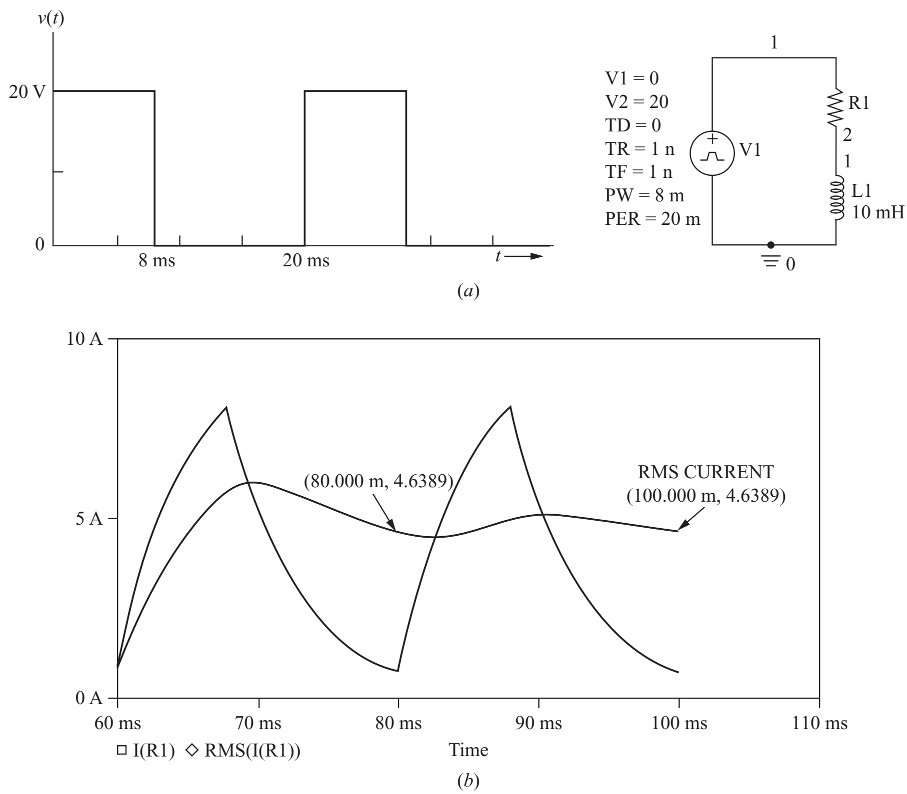
PSpice can also be used to determine power in an ac circuit containing an inductor or capacitor, but *the simulation must represent steady-state response* to be valid for steady-state operation of the circuit.

EXAMPLE 2-12

RMS and Fourier Analysis Using PSpice

Fig. 2-13*a* shows a periodic pulse voltage that is connected to a series R-L circuit with $R = 10\ \Omega$ and $L = 10$ mH. PSpice is used to determine the steady-state rms current and the Fourier components of the current.

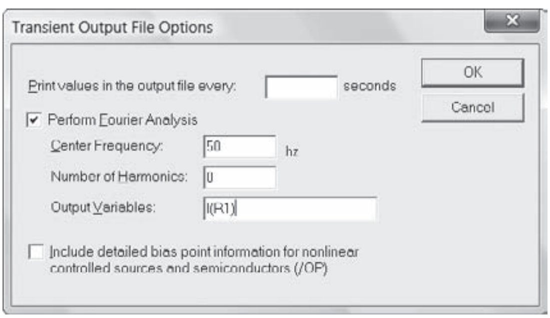
Figure 2-13 (*a*) A pulse waveform voltage source is applied to a series *R-L* circuit; (*b*) Probe output showing the steady-state current and the rms value.



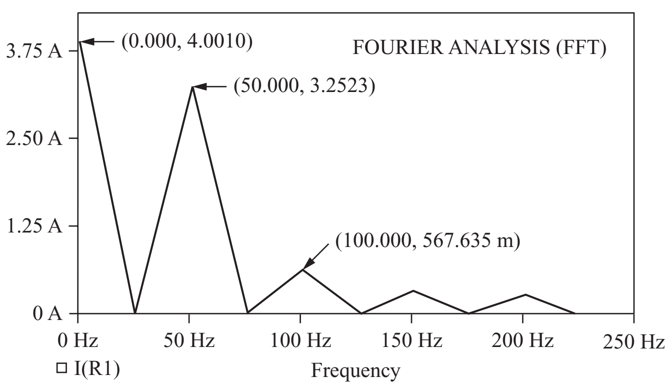
In PSpice power calculations, *it is extremely important that the output being analyzed represent steady-state voltages and currents*. Steady-state current is reached after several periods of the pulse waveform. Therefore, the Simulation Settings have the Run Time (Time to Stop) at 100 ms and the Start Saving Data set at 60 ms. The 60-ms delay allows for the current to reach steady state. A maximum step size is set at 10 μs to produce a smooth waveform.

Current is displayed in Probe by entering I(R1), and steady state is verified by noting that the starting and ending values are the same for each period. The rms current is obtained by entering the expression RMS(I(R1)). The value of rms current, 4.6389 A, is obtained at the end of any period of the current waveform. Fig. 2-13*b* shows the Probe output.

The Fourier series of a waveform can be determined using PSpice. Fourier analysis is entered under Output File Options in the Transient Analysis menu. The Fast Fourier Transform (FFT) on the waveforms of the source voltage and



(*a*)



(*b*)

Figure 2-14 (*a*) Fourier analysis setup; (*b*) Fourier Series Spectrum from Probe using FFT.

the load current will appear in the output file. The fundamental frequency (Center Frequency) of the Fourier series is 50 Hz (1/20 mS). In this example, five periods of the waveform are simulated to ensure steady-state current for this L/R time constant.

A portion of the output file showing the Fourier components of source voltage and resistor current is as follows:

```
FOURIER COMPONENTS OF TRANSIENT RESPONSE I(R_R1)

DC COMPONENT = 4.000000E+00
```

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E+01	3.252E+00	1.000E+00	−3.952E+01	0.000E+00
2	1.000E+02	5.675E−01	1.745E−01	−1.263E+02	−4.731E+01
3	1.500E+02	2.589E−01	7.963E−02	−2.402E+01	9.454E+01
4	2.000E+02	2.379E−01	7.317E−02	−9.896E+01	5.912E+01
5	2.500E+02	1.391E−07	4.277E−08	5.269E+00	2.029E+02
6	3.000E+02	1.065E−01	3.275E−02	−6.594E+01	1.712E+02
7	3.500E+02	4.842E−02	1.489E−02	−1.388E+02	1.378E+02
8	4.000E+02	3.711E−02	1.141E−02	−3.145E+01	2.847E+02
9	4.500E+02	4.747E−02	1.460E−02	−1.040E+02	2.517E+02

```
TOTAL HARMONIC DISTORTION = 2.092715E+01 PERCENT
```

When you use PSpice output for the Fourier series, remember that the values are listed as amplitudes (zero-to-peak), and conversion to rms by dividing by $\sqrt{2}$ is required for power computations. The phase angles are referenced to the sine rather than the cosine. The numerically computed Fourier components in PSpice may not be exactly the same as analytically computed values. Total harmonic distortion (THD) is listed at the end of the Fourier output. [*Note*: The THD computed in PSpice uses Eq. (2-69) and assumes that the dc component of the waveform is zero, which is not true in this case.]

The rms value of the load current can be computed from the Fourier series in the output file from Eq. (2-43).

$$I_{\text{rms}} = \sqrt{(4.0)^2 + \left(\frac{3.252}{\sqrt{2}}\right)^2 + \left(\frac{0.5675}{\sqrt{2}}\right)^2 + \cdots} \approx 4.63 \text{ A}$$

A graphical representation of the Fourier series can be produced in Probe. To display the Fourier series of a waveform, click the FFT button on the toolbar. Upon entering the variable to be displayed, the spectrum of the Fourier series will appear. It will be desirable to adjust the range of frequencies to obtain a useful graph. Fig. 2-14*b* shows the result for this example. Fourier component magnitudes are represented by the peaks of the graph, which can be determined precisely by using the cursor option.

EXAMPLE 2-13

PSpice Solution of Example 2-3

Use PSpice to simulate the inductor circuit of Fig. 2-4*a* with the parameters of Example 2-3.

■ Solution

Fig. 2-15 shows the circuit used in the PSpice simulation. The transistor is used as a switch, so a voltage-controlled switch (Sbreak) can be used in the PSpice circuit. The switch is idealized by setting the on resistance to $R_{on} = 0.001\ \Omega$. The control for the switch is a pulse voltage source which has a pulse width of 10 ms and period of 100 ms. The diode Dbreak is used.

Some of the possible results that can be obtained from the Probe output are listed below. All traces except the maximum inductor current and the stored inductor energy are read at the end of the Probe trace, which is after one complete period. Note the agreement between the results of Example 2-3 and the PSpice results.

Desired Quantity	Probe Entry	Result
Inductor current	I(L1)	max = 4.5 A
Energy stored in inductor	0.5*0.2*I(L1)*I(L1)	max = 2.025 J
Average switch power	AVG(W(S1))	0.010 W
Average source power (absorbed)	AVG(W(VCC))	−20.3 W
Average diode power	AVG(W(D1))	0.464 W
Average inductor power	AVG(W(L1))	≈ 0
Average inductor voltage	AVG(V(1,2))	≈ 0
Average resistor power	AVG(W(R1))	19.9 W
Energy absorbed by resistor	S(W(R1))	1.99 J
Energy absorbed by diode	S(W(D1))	0.046 J
Energy absorbed by inductor	S(W(L1))	≈ 0
RMS resistor current	RMS(I(R1))	0.998 A

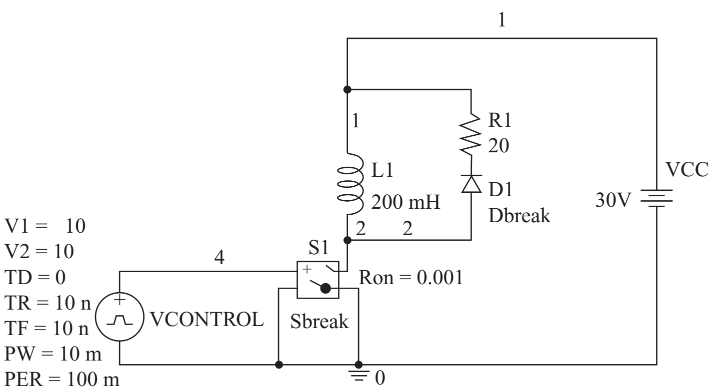


Figure 2-15 Circuit for Example 2-13, a PSpice simulation of the circuit in Example 2-4.

2.10 Summary

- Instantaneous power is the product of voltage and current at a particular time:

$$p(t) = v(t)i(t)$$

  Using the passive sign convention, the device is absorbing power if $(p)(t)$ is positive, and the device is supplying power if $(p)(t)$ is negative.

- *Power* usually refers to average power, which is the time average of periodic instantaneous power:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt$$

- The rms value is the root-mean-square or effective value of a voltage or current waveform.

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

- Apparent power is the product of rms voltage and current.

$$S = V_{\text{rms}}I_{\text{rms}}$$

- Power factor is the ratio of average power to apparent power.

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}}I_{\text{rms}}}$$

- For inductors and capacitors that have periodic voltages and currents, the average power is zero. Instantaneous power is generally not zero because the device stores energy and then returns energy to the circuit.
- For periodic currents, the average voltage across an inductor is zero.
- For periodic voltages, the average current in a capacitor is zero.
- For nonsinusoidal periodic waveforms, average power may be computed from the basic definition, or the Fourier series method may be used. The Fourier series method treats each frequency in the series separately and uses superposition to compute total power.

$$P = \sum_{n=0}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,\text{ rms}} I_{n,\text{ rms}} \cos(\theta_n - \phi_n)$$

- A simulation using the program PSpice may be used to obtain not only voltage and current waveforms but also instantaneous power, energy, rms values, and average power by using the numerical capabilities of the graphic postprocessor program

Probe. For numerical computations in Probe to be accurate, the simulation must represent steady-state voltages and currents.

- Fourier series terms are available in PSpice by using the Fourier Analysis in the Simulation Settings or by using the FFT option in Probe.

2.11 Bibliography

M. E. Balci and M. H. Hocaoglu, "Comparison of Power Definitions for Reactive Power Compensation in Nonsinusoidal Circuits*," International Conference on Harmonics and Quality of Power*, Lake Placid, New York, 2004.

L. S. Czarnecki, "Considerations on the Reactive Power in Nonsinusoidal Situations," *International Conference on Harmonics in Power Systems*, Worcester Polytechnic Institute, Worcester, Mass., 1984, pp. 231–237.

A. E. Emanuel, "Powers in Nonsinusoidal Situations, A Review of Definitions and Physical Meaning," *IEEE Transactions on Power Delivery*, vol. 5, no. 3, July 1990.

G. T. Heydt, *Electric Power Quality*, Stars in a Circle Publications, West Lafayette, Ind., 1991.

W. Sheperd and P. Zand, *Energy Flow and Power Factor in Nonsinusoidal Circuits*, Cambridge University Press, 1979.

Problems

Instantaneous and Average Power

2-1. Average power generally is *not* the product of average voltage and average current. Give an example of periodic waveforms for $v(t)$ and $i(t)$ that have zero average values and average power absorbed by the device is not zero. Sketch $v(t)$, $i(t)$, and $p(t)$.

2-2. The voltage across a 10-Ω resistor is $v(t) = 170 \sin(377t)$ V. Determine (*a*) an expression for instantaneous power absorbed by the resistor, (*b*) the peak power, and (*c*) the average power.

2-3. The voltage across an element is $v(t) = 5 \sin(2\pi t)$ V. Use graphing software to graph instantaneous power absorbed by the element, and determine the average power if the current, using the passive sign convention, is (*a*) $i(t) = 4 \sin(2\pi t)$ A and (*b*) $i(t) = 3 \sin(4\pi t)$ A.

2-4. The voltage and current for a device (using the passive sign convention) are periodic functions with $T = 100$ ms described by

$$v(t) = \begin{cases} 10\text{ V} & 0 < t < 70\text{ ms} \\ 0 & 70\text{ ms} < t < 100\text{ ms} \end{cases}$$

$$i(t) = \begin{cases} 0 & 0 < t < 50\text{ ms} \\ 4\text{ A} & 50\text{ ms} < t < 100\text{ ms} \end{cases}$$

Determine (*a*) the instantaneous power, (*b*) the average power, and (*c*) the energy absorbed by the device in each period.

2-5. The voltage and current for a device (using the passive sign convention) are periodic functions with $T = 20$ ms described by

$$v(t) = \begin{cases} 10\text{ V} & 0 < t < 14\text{ ms} \\ 0 & 14\text{ ms} < t < 20\text{ ms} \end{cases}$$

$$i(t) = \begin{cases} 7\text{ A} & 0 < t < 6\text{ ms} \\ -5\text{ A} & 6\text{ ms} < t < 10\text{ ms} \\ 4\text{ A} & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

Determine (*a*) the instantaneous power, (*b*) the average power, and (*c*) the energy absorbed by the device in each period.

2-6. Determine the average power absorbed by a 12-V dc source when the current into the positive terminal of the source is that given in (*a*) Prob. 2-4 and (*b*) Prob. 2-5.

2-7. A current of 5 sin $(2\pi 60t)$ A enters an element. Sketch the instantaneous power and determine the average power absorbed by the load element when the element is (*a*) a 5-Ω resistor, (*b*) a 10-mH inductor, and (*c*) a 12-V source (current into the positive terminal).

2-8. A current source of $i(t) = 2 + 6\sin(2\pi 60t)$ A is connected to a load that is a series combination of a resistor, an inductor, and a dc voltage source (current into the positive terminal). If $R = 4\ \Omega$, $L = 15$ mH, and $V_{dc} = 6$ V, determine the average power absorbed by each element.

2-9. An electric resistance space heater rated at 1500 W for a voltage source of $v(t) = 120\sqrt{2}\sin(2\pi 60t)$ V has a thermostatically controlled switch. The heater periodically switches on for 5 min and off for 7 min. Determine (*a*) the maximum in stantaneous power, (*b*) the average power over the 12-min cycle, and (*c*) the electric energy converted to heat in each 12-min cycle.

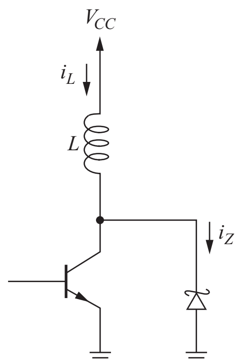
Energy Recovery

2-10. An inductor is energized as in the circuit of Fig. 2-4*a*. The circuit has $L = 100$ mH, $R = 20\ \Omega$, $V_{CC} = 90$ V, $t_1 = 4$ ms, and $T = 40$ ms. Assuming the transistor and diode are ideal, determine (*a*) the peak energy stored in the inductor, (*b*) the energy absorbed by the resistor in each switching period, and (*c*) the average power supplied by the source. (*d*) If the resistor is changed to 40 Ω, what is the average power supplied by the source?

2-11. An inductor is energized as in the circuit of Fig. 2-4*a*. The circuit has $L = 10$ mH and $V_{CC} = 14$ V. (*a*) Determine the required on time of the switch such that the peak energy stored in the inductor is 1.2 J. (*b*) Select a value for R such that the switching cycle can be repeated every 20 ms. Assume the switch and the diode are ideal.

2-12. An inductor is energized as in the circuit of Fig. 2-5*a*. The circuit has $L = 50$ mH, $V_{CC} = 90$ V, $t_1 = 4$ ms, and $T = 50$ ms. (*a*) Determine the peak energy stored in the inductor. (*b*) Graph the inductor current, source current, inductor instantaneous power, and source instantaneous power versus time. Assume the transistors are ideal.

2-13. An alternative circuit for energizing an inductor and removing the stored energy without damaging a transistor is shown in Fig. P2-13. Here $V_{CC} = 12$ V, $L = 75$ mH, and the zener breakdown voltage is $V_Z = 20$ V. The transistor switch opens and closes periodically with $t_{on} = 20$ ms and $t_{off} = 50$ ms. (*a*) Explain how the zener diode allows the switch to open. (*b*) Determine and sketch the inductor current $i_L(t)$ and the zener diode current $i_Z(t)$ for one switching period. (*c*) Sketch $(p)(t)$ for the inductor and the zener diode. (*d*) Determine the average power absorbed by the inductor and by the zener diode.



2-14. Repeat Prob. 2-13 with $V_{CC} = 20\ V$, $L = 50$ mH, $V_Z = 30$ V, $t_{on} = 15$ ms, and $t_{off} = 60$ ms.

Effective Values: RMS

2-15. The rms value of a sinusoid is the peak value divided by $\sqrt{2}$. Give two examples to show that this is generally not the case for other periodic waveforms.

2-16. A three-phase distribution system is connected to a nonlinear load that has line and neutral currents like those of Fig. 2-8. The rms current in each phase is 12 A, and the resistance in each of the line and neutral conductors is 0.5 Ω. Determine the total power absorbed by the conductors. What should the resistance of the neutral conductor be such that it absorbs the same power as one of the phase conductors?

2-17. Determine the rms values of the voltage and current waveforms in Prob. 2-4.

2-18. Determine the rms values of the voltage and current waveforms in Prob. 2-5.

Nonsinusoidal Waveforms

2-19. The voltage and current for a circuit element are $v(t) = 2 + 5\cos(2\pi 60t) - 3\cos(4\pi 60t + 45°)$ V and $i(t) = 1.5 + 2\cos(2\pi 60t + 20°) + 1.1\cos(4\pi 60t - 20°)$ A. (*a*) Determine the rms values of voltage and current. (*b*) Determine the power absorbed by the element.

2-20. A current source $i(t) = 3 + 4\cos(2\pi 60t) + 6\cos(4\pi 60t)$ A is connected to a parallel RC load with $R = 100\ \Omega$ and $C = 50\ \mu$F. Determine the average power absorbed by the load.

2-21. In Fig. P2-21, $R = 4\ \Omega$, $L = 10$ mH, $V_{dc} = 12$ V, and $v_s(t) = 50 + 30\cos(4\pi 60t) + 10\cos(8\pi 60t)$ V. Determine the power absorbed by each component.

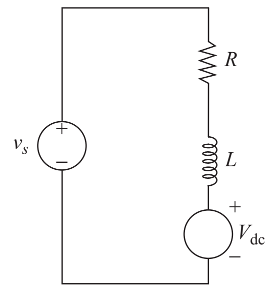


Figure P2-21

2-22. A nonsinusoidal periodic voltage has a Fourier series of $v(t) = 6 + 5\cos(2\pi 60t) + 3\cos(6\pi 60t)$. This voltage is connected to a load that is a 16-Ω resistor in series with a 25-mH inductor as in Fig. 2-11. Determine the power absorbed by the load.

2-23. Voltage and current for a device (using the passive sign convention) are

$$v(t) = 20 + \sum_{n=1}^{\infty}\left(\frac{20}{n}\right)\cos(n\pi t)\ \text{V}$$

$$i(t) = 5 + \sum_{n=1}^{\infty}\left(\frac{5}{n^2}\right)\cos(n\pi t)\ \text{A}$$

Determine the average power based on the terms through $n = 4$.

2-24. Voltage and current for a device (using the passive sign convention) are

$$v(t) = 50 + \sum_{n=1}^{\infty}\left(\frac{50}{n}\right)\cos(n\pi t)\ \text{V}$$

$$i(t) = 10 + \sum_{n=1}^{\infty}\left(\frac{10}{n^2}\right)\cos\left(n\pi t - \tan^{-1}n/2\right)$$

Determine the average power based on the terms through $n = 4$.

2-25. In Fig. P2-21, $R = 20\ \Omega$, $L = 25$ mH, and $V_{dc} = 36$ V. The source is a periodic voltage that has the Fourier series

$$v_s(t) = 50 + \sum_{n=1}^{\infty}\left(\frac{400}{n\pi}\right)\sin\left(200n\pi t\right)$$

Using the Fourier series method, determine the average power absorbed by R, L, and V_{dc} when the circuit is operating in the steady state. Use as many terms in the Fourier series as necessary to obtain a reasonable estimate of power.

2-26. A sinusoidal current of 10 A rms at a 60-Hz fundamental frequency is contaminated with a ninth harmonic current. The current is expressed as

$$i(t) = 10\sqrt{2}\,\sin(2\pi 60t) + I_9\sqrt{2}\,\sin(18\pi 60t)\ \text{A}$$

Determine the value of the ninth harmonic rms current I_9 if the THD is (*a*) 5 percent, (*b*) 10 percent, (*c*) 20 percent, and (*d*) 40 percent. Use graphing software or PSpice to show $i(t)$ for each case.

2-27. A sinusoidal voltage source of $v(t) = 170\cos(2\pi 60t)$ V is applied to a nonlinear load, resulting in a nonsinusoidal current that is expressed in Fourier series form as $i(t) = 10\cos(2\pi 60t + 30°) + 6\cos(4\pi 60t + 45°) + 3\cos(8\pi 60t + 20°)$ A. Determine (*a*) the power absorbed by the load, (*b*) the power factor of the load, (*c*) the distortion factor, and (*d*) the total harmonic distortion of the load current.

2-28. Repeat Prob. 2-27 with $i(t) = 12\cos(2\pi 60t - 40°) + 5\sin(4\pi 60t) + 4\cos(8\pi 60t)$ A.

2-29. A sinusoidal voltage source of $v(t) = 240\sqrt{2}\,\sin(2\pi 60t)$ V is applied to a nonlinear load, resulting in a current $i(t) = 8\sin(2\pi 60t) + 4\sin(4\pi 60t)$ A. Determine (*a*) the power absorbed by the load, (*b*) the power factor of the load, (*c*) the THD of the load current, (*d*) the distortion factor of the load current, and (*e*) the crest factor of the load current.

2-30. Repeat Prob. 2-29 with $i(t) = 12\sin(2\pi 60t) + 9\sin(4\pi 60t)$ A.

2-31. A voltage source of $v(t) = 5 + 25\cos(1000t) + 10\cos(2000t)$ V is connected to a series combination of a 2-Ω resistor, a 1-mH inductor, and a 1000-μF capacitor. Determine the rms current in the circuit, and determine the power absorbed by each component.

PSpice

2-32. Use PSpice to simulate the circuit of Example 2-1. Define voltage and current with PULSE sources. Determine instantaneous power, energy absorbed in one period, and average power.

2-33. Use PSpice to determine the instantaneous and average power in the circuit elements of Prob. 2-7.

2-34. Use PSpice to determine the rms values of the voltage and current waveforms in (*a*) Prob. 2-5 and (*b*) Prob. 2-6.

2-35. Use PSpice to simulate the circuit of Prob. 2-10. (*a*) Idealize the circuit by using a voltage-controlled switch that has $R_{on} = 0.001\ \Omega$ and a diode with $n = 0.001$. (*b*) Use $R_{on} = 0.5\ \Omega$ and use the default diode.

2-36. Use PSpice to simulate the circuit of Fig. 2-5*a*. The circuit has $V_{CC} = 75$ V, $t_0 = 40$ ms, and $T = 100$ ms. The inductance is 100 mH and has an internal resistance of 20 Ω. Use a voltage-controlled switch with $R_{on} = 1\ \Omega$ for the transistors, and use the PSpice default diode model. Determine the average power absorbed by each circuit element. Discuss the differences between the behavior of this circuit and that of the ideal circuit.

2-37. Use PSpice to simulate the circuit of Prob. 2-13. Use $R_{on} = 0.001\ \Omega$ for the switch model and use $n = 0.001$, BV = 20 V for the breakdown voltage and $I_{BV} = 10$ A for the current at breakdown for the zener diode model. (*a*) Display $i_L(t)$ and $i_Z(t)$. Determine the average power in the inductor and in the zener diode. (*b*) Repeat part (*a*) but include a 1.5-Ω series resistance with the inductor and use $R_{on} = 0.5\ \Omega$ for the switch.

2-38. Repeat Prob. 2-37, using the circuit of Prob. 2-14.

2-39. Use PSpice to determine the power absorbed by the load in Example 2-10. Model the system as a voltage source and four current sources in parallel.

2-40. Modify the switch model so $R_{on} = 1\ \Omega$ in the PSpice circuit file in Example 2-13. Determine the effect on each of the quantities obtained from Probe in the example.

2-41. Demonstrate with PSpice that a triangular waveform like that of Fig. 2-9*a* has an rms value of $V_m/\sqrt{3}$. Choose an arbitrary period *T,* and use at least three values of t_1. Use a VPULSE source with the rise and fall times representing the triangular wave.

CHAPTER 3

Half-Wave Rectifiers

The Basics of Analysis

3.1 INTRODUCTION

A rectifier converts ac to dc. The purpose of a rectifier may be to produce an output that is purely dc, or the purpose may be to produce a voltage or current waveform that has a specified dc component.

In practice, the half-wave rectifier is used most often in low-power applications because the average current in the supply will not be zero, and nonzero average current may cause problems in transformer performance. While practical applications of this circuit are limited, it is very worthwhile to analyze the half-wave rectifier in detail. A thorough understanding of the half-wave rectifier circuit will enable the student to advance to the analysis of more complicated circuits with a minimum of effort.

The objectives of this chapter are to introduce general analysis techniques for power electronics circuits, to apply the power computation concepts of the previous chapter, and to illustrate PSpice solutions.

3.2 RESISTIVE LOAD

Creating a DC Component Using an Electronic Switch

A basic half-wave rectifier with a resistive load is shown in Fig. 3-1*a*. The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only. For the positive half-cycle of the source in this circuit, the diode is on (forward-biased).

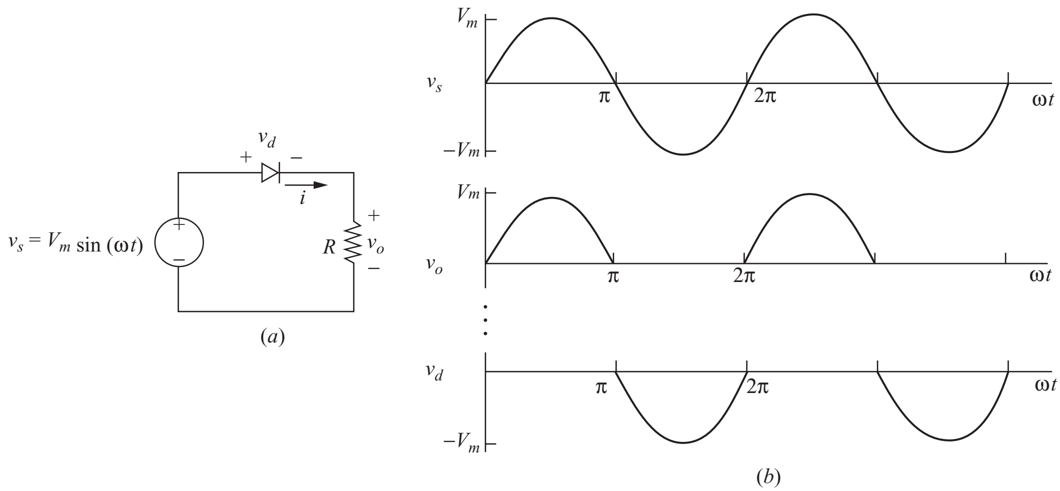


Figure 3-1 (*a*) Half-wave rectifier with resistive load; (*b*) Voltage waveforms.

Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive.

For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.

The voltage waveforms across the source, load, and diode are shown in Fig. 3-1*b*. Note that the units on the horizontal axis are in terms of angle (ωt). This representation is useful because the values are independent of frequency. The dc component V_o of the output voltage is the average value of a half-wave rectified sinusoid

$$V_o = V_{\text{avg}} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi} \tag{3-1}$$

The dc component of the current for the purely resistive load is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \tag{3-2}$$

Average power absorbed by the resistor in Fig. 3-1*a* can be computed from $P = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$. When the voltage and current are half-wave rectified sine waves,

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{\text{rms}} = \frac{V_m}{2R} \tag{3-3}$$

In the preceding discussion, the diode was assumed to be ideal. For a real diode, the diode voltage drop will cause the load voltage and current to be reduced, but not appreciably if V_m is large. For circuits that have voltages much larger than the typical diode drop, the improved diode model may have only second-order effects on the load voltage and current computations.

EXAMPLE 3-1

Half-Wave Rectifier with Resistive Load

For the half-wave rectifier of Fig. 3-1*a*, the source is a sinusoid of 120 V rms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (*a*) the average load current, (*b*) the average power absorbed by the load and (*c*) the power factor of the circuit.

■ **Solution**

(*a*) The voltage across the resistor is a half-wave rectified sine wave with peak value $V_m = 120\sqrt{2} = 169.7$ V. From Eq. (3-2), the average voltage is V_m/π, and average current is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{5\pi} = 10.8 \text{ A}$$

(*b*) From Eq. (3-3), the rms voltage across the resistor for a half-wave rectified sinusoid is

$$V_{\text{rms}} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9 \text{ V}$$

The power absorbed by the resistor is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{84.9^2}{4} = 1440 \text{ W}$$

The rms current in the resistor is $V_m/(2R) = 17.0$ A, and the power could also be calculated from $I_{\text{rms}}^2 R = (17.0)^2(5) = 1440$ W.

(*c*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1440}{(120)(17)} = 0.707$$

3.3 RESISTIVE-INDUCTIVE LOAD

Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit of Fig. 3-2*a*, the diode becomes forward-biased. The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} \tag{3-4}$$

The solution can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = i_f(t) + i_n(t) \tag{3-5}$$

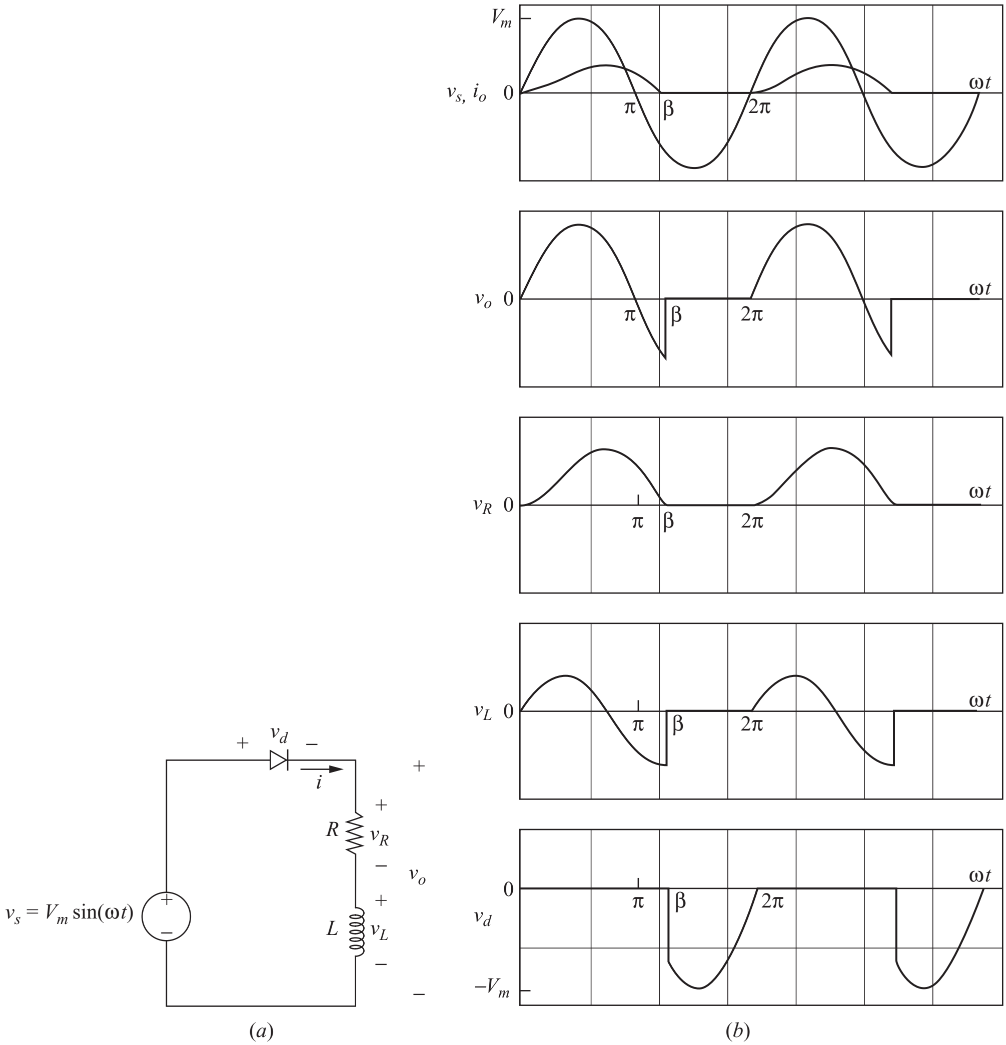


Figure 3-2 (*a*) Half-wave rectifier with an *RL* load; (*b*) Waveforms.

The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present. This steady-state current can be found from phasor analysis, resulting in

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) \tag{3-6}$$

$$\text{where} \qquad Z = \sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode:

$$R\,i(t) + L\frac{di(t)}{dt} = 0 \tag{3-7}$$

For this first-order circuit, the natural response has the form

$$i_n(t) = Ae^{-t/\tau} \tag{3-8}$$

where τ is the time constant L/R and A is a constant that is determined from the initial condition. Adding the forced and natural responses gets the complete solution.

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau} \tag{3-9}$$

The constant A is evaluated by using the initial condition for current. The initial condition of current in the inductor is zero because it was zero before the diode started conducting and it cannot change instantaneously.

Using the initial condition and Eq. (3-9) to evaluate A yields

$$\begin{aligned} i(0) &= \frac{V_m}{Z}\sin(0 - \theta) + Ae^{0} = 0 \\ A &= -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin\theta \end{aligned} \tag{3-10}$$

Substituting for A in Eq. (3-9) gives

$$\begin{aligned} i(t) &= \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)\,e^{-t/\tau} \\ &= \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\,e^{-t/\tau}\right] \end{aligned} \tag{3-11}$$

It is often convenient to write the function in terms of the angle ωt rather than time. This merely requires ωt to be the variable instead of t. To write the above equation in terms of angle, t in the exponential must be written as ωt, which requires τ to be multiplied by ω also. The result is

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\,e^{-\omega t/\omega\tau}\right] \tag{3-12}$$

A typical graph of circuit current is shown in Fig. 3-2*b*. Equation (3-12) is valid for positive currents only because of the diode in the circuit, so current is zero when the function in Eq. (3-12) is negative. When the source voltage again becomes positive, the diode turns on, and the positive part of the waveform in Fig. 3-2*b* is repeated. This occurs at every positive half-cycle of the source. The voltage waveforms for each element are shown in Fig. 3-2*b*.

Note that the diode remains forward-biased longer than π rad and that the source is negative for the last part of the conduction interval. This may seem unusual, but an examination of the voltages reveals that Kirchhoff's voltage law is satisfied and there is no contradiction. Also note that the inductor voltage is negative when the current is decreasing ($v_L = L\,di/dt$).

The point when the current reaches zero in Eq. (3-12) occurs when the diode turns off. The first positive value of ωt in Eq. (3-12) that results in zero current is called the extinction angle β.

Substituting $\omega t = \beta$ in Eq. (3-12), the equation that must be solved is

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}\right] = 0 \tag{3-13}$$

which reduces to

$$\boxed{\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau} = 0} \tag{3-14}$$

There is no closed-form solution for β, and some numerical method is required. To summarize, the current in the half-wave rectifier circuit with RL load (Fig. 3-2) is expressed as

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases} \tag{3-15}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The average power absorbed by the load is $I_{\text{rms}}^2 R$, since the average power absorbed by the inductor is zero. The rms value of the current is determined from the current function of Eq. (3-15).

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d(\omega t)} \tag{3-16}$$

Average current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d(\omega t) \tag{3-17}$$

EXAMPLE 3-2

Half-Wave Rectifier with *RL* Load

For the half-wave rectifier of Fig. 3-2*a*, $R = 100\ \Omega$, $L = 0.1$ H, $\omega = 377$ rad/s, and $V_m = 100$ V. Determine (*a*) an expression for the current in this circuit, (*b*) the average current, (*c*) the rms current, (*d*) the power absorbed by the RL load, and (*e*) the power factor.

■ Solution

For the parameters given,

$$Z = [R^2 + (\omega L)^2]^{0.5} = 106.9\,\Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 20.7° = 0.361 \text{ rad}$$
$$\omega t = \omega L/R = 0.377 \text{ rad}$$

(*a*) Equation (3-15) for current becomes

$$i(\omega t) = 0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377} \quad \text{A} \qquad \text{for } 0 \le \omega t \le \beta$$

Beta is found from Eq. (3-14).

$$\sin(\beta - 0.361) + \sin(0.361)e^{-\beta/0.377} = 0$$

Using a numerical root-finding program, β is found to be 3.50 rad, or 201°

(*b*) Average current is determined from Eq. (3-17).

$$I_o = \frac{1}{2\pi}\int_0^{3.50}\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]d(\omega t) = 0.308 \text{ A}$$

(A numerical integration program is recommended.)

(*c*) The rms current is found from Eq. (3-16) to be

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{3.50}\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]^2 d(\omega t)} = 0.474 \text{ A}$$

(*d*) The power absorbed by the resistor is

$$P = I_{\text{rms}}^2 R = (0.474)^2(100) = 22.4 \text{ W}$$

The average power absorbed by the inductor is zero. Also P can be computed from the definition of average power:

$$P = \frac{1}{2\pi}\int_0^{2\pi} p(\omega t)\,d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t)i(\omega t)\,d(\omega t)$$
$$= \frac{1}{2\pi}\int_0^{3.50} [100\sin(\omega t)]\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]d(\omega t)$$
$$= 22.4 \text{ W}$$

(*e*) The power factor is computed from the definition pf = P/S, and P is power supplied by the source, which must be the same as that absorbed by the load.

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}}I_{\text{rms}}} = \frac{22.4}{\left(100/\sqrt{2}\right)0.474} = 0.67$$

Note that the power factor is *not* cos θ.

3.4 PSPICE SIMULATION

Using Simulation Software for Numerical Computations

A computer simulation of the half-wave rectifier can be performed using PSpice. PSpice offers the advantage of having the postprocessor program Probe which can display the voltage and current waveforms in the circuit and perform numerical computations. Quantities such as the rms and average currents, average power absorbed by the load, and power factor can be determined directly with PSpice. Harmonic content can be determined from the PSpice output.

A transient analysis produces the desired voltages and currents. One complete period is a sufficient time interval for the transient analysis.

EXAMPLE 3-3

PSpice Analysis

Use PSpice to analyze the circuit of Example 3-2.

■ **Solution**

The circuit of Fig. 3-2*a* is created using VSIN for the source and Dbreak for the diode. In the simulation settings, choose Time Domain (transient) for the analysis type, and set the Run Time to 16.67 ms for one period of the source. Set the Maximum Step Size to 10 μs to get adequate sampling of the waveforms. A transient analysis with a run time of 16.67 ms (one period for 60 Hz) and a maximum step size of 10 μs is used for the simulation settings.

If a diode model that approximates an ideal diode is desired for the purpose of comparing the simulation with analytical results, editing the PSpice model and using $n = 0.001$ will make the voltage drop across the forward-biased diode close to zero. Alternatively, a model for a power diode may be used to obtain a better representation of a real rectifier circuit. For many circuits, voltages and currents will not be affected significantly when different diode models are used. Therefore, it may be convenient to use the Dbreak diode model for a preliminary analysis.

When the transient analysis is performed and the Probe screen appears, display the current waveform by entering the expression I(R1). A method to display angle instead of time on the x axis is to use the x-variable option within the x-axis menu, entering TIME*60*360. The factor of 60 converts the axis to periods ($f = 60$ Hz), and the factor 360 converts the axis to degrees. Entering TIME*60*2*3.14 for the x variable converts the x axis to radians. Figure 3-3*a* shows the result. The extinction angle β is found to be 200° using the cursor option. Note that using the default diode model in PSpice resulted in a value of β very close to the 201° in Example 3-2.

Probe can be used to determine numerically the rms value of a waveform. While in Probe, enter the expression RMS(I(R1)) to obtain the rms value of the resistor current. Probe displays a "running" value of the integration in Eq. (3-16), so the appropriate value is *at the end of one or more complete periods* of the waveform. Figure 3-3*b* shows how to obtain the rms current. The rms current is read as approximately 468 mA. This compares

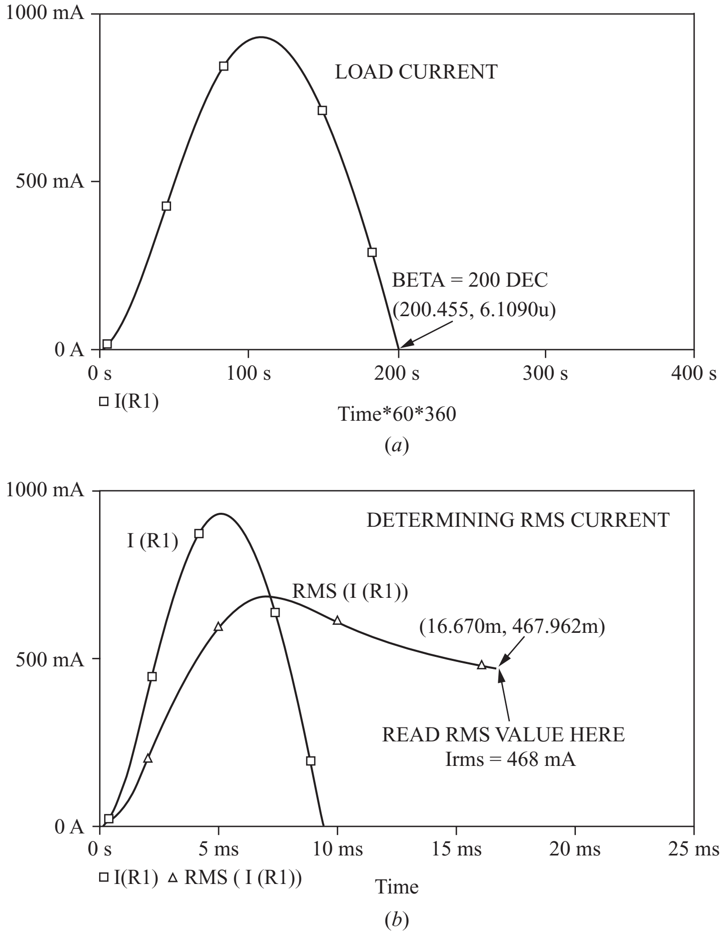


Figure 3-3 (*a*) Determining the extinction angle β in Probe. The time axis is changed to angle using the x-variable option and entering Time*60*360; (*b*) Determining the rms value of current in Probe.

very well with the 474 mA calculated in Example 3-2. Remember that the default diode model is used in PSpice and an ideal diode was used in Example 3-2. The average current is found by entering AVG(I(R1)), resulting in $I_o = 304$ mA.

PSpice is also useful in the design process. For example, the objective may be to design a half-wave rectifier circuit to produce a specified value of average current by selecting the proper value of L in an RL load. Since there is no closed-form solution, a trial-and-error iterative method must be used. A PSpice simulation that includes a parametric sweep is used to try several values of L. Example 3-4 illustrates this method.

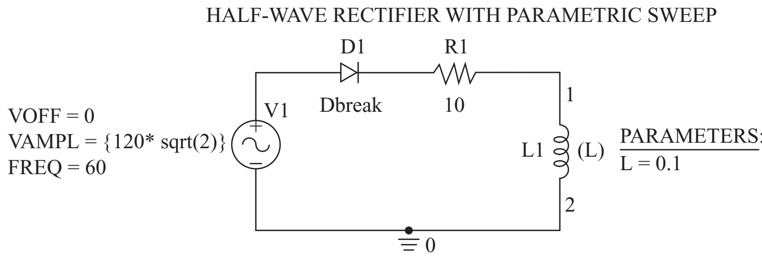
EXAMPLE 3-4

Half-Wave Rectifier Design Using PSpice

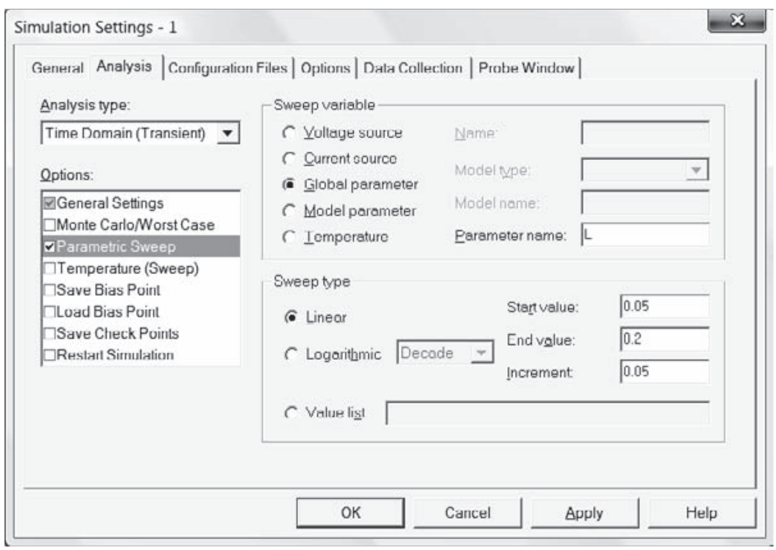
Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.

■ Solution

A half-wave rectifier is one circuit that can be used for this application. If a simple half-wave rectifier with the 10-Ω resistance were used, the average current would be $(120\sqrt{2}/\pi)/8 = 6.5$ A. Some means must be found to reduce the average current to the specified 2 A. A series resistance could be added to the load, but resistances absorb power. An added series inductance will reduce the current without adding losses, so an



(*a*)



(*b*)

Figure 3-4 (*a*) PSpice circuit for Example 3-4; (*b*) A parametric sweep is established in the Simulation Settings box; (*c*) $L = 0.15$ H for an average current of approximately 2 A.

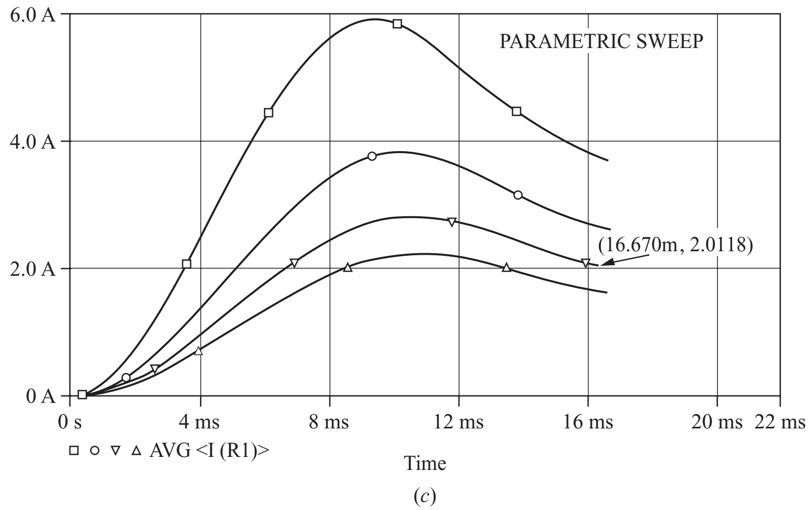


Figure 3-4 (*continued*)

inductor is chosen. Equations (3-15) and (3-17) describe the current function and its average for *RL* loads. There is no closed-form solution for *L*. A trial-and-error technique in PSpice uses the parameter (PARAM) part and a parametric sweep to try a series of values for *L*. The PSpice circuit and the Simulation Settings box are shown in Fig. 3-4.

Average current in the resistor is found by entering AVG(I(R1)) in Probe, yielding a family of curves for different inductance values (Fig. 3-4*c*). The third inductance in the sweep (0.15 H) results in an average current of 2.0118 A in the resistor, which is very close to the design objective. If further precision is necessary, subsequent simulations can be performed, narrowing the range of *L*.

3.5 *RL*-SOURCE LOAD

Supplying Power to a DC Source from an AC Source

Another variation of the half-wave rectifier is shown in Fig. 3-5*a*. The load consists of a resistance, an inductance, and a dc voltage. Starting the analysis at $\omega t = 0$ and assuming the initial current is zero, recognize that the diode will remain off as long as the voltage of the ac source is less than the dc voltage. Letting α be the value of ωt that causes the source voltage to be equal to V_{dc},

$$V_m \sin \alpha = V_{dc}$$

or

$$\boxed{\alpha = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)} \tag{3-18}$$

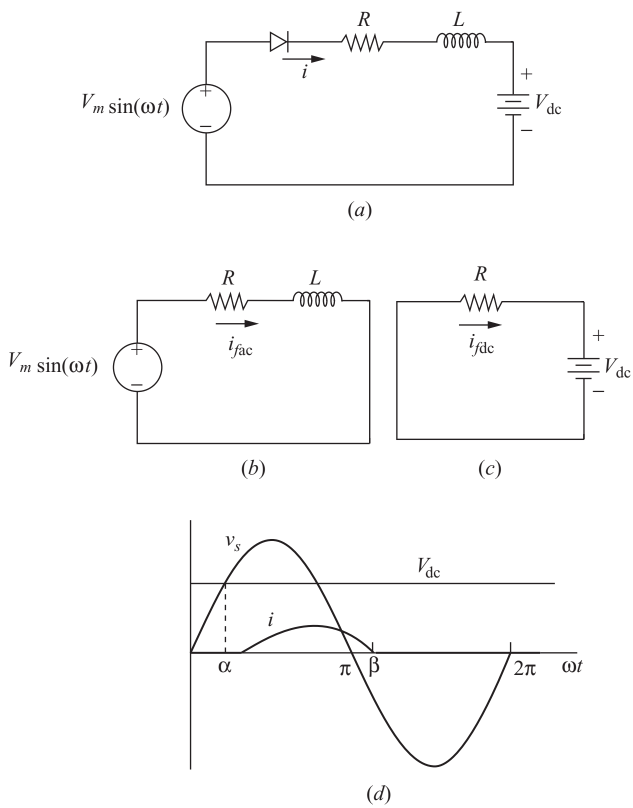


Figure 3-5 (*a*) Half-wave rectifier with *RL* source load; (*b*) Circuit for forced responce from ac source; (*c*) Circuit for forced responce from dc source; (*d*) Waveforms.

The diode starts to conduct at $\omega t = \alpha$. With the diode conducting, Kirchhoff's voltage law for the circuit yields the equation

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} + V_{dc} \tag{3-19}$$

Total current is determined by summing the forced and natural responses:

$$i(t) = i_f(t) + i_n(t)$$

The current $i_f(t)$ is determined using superposition for the two sources. The forced response from the ac source (Fig. 3-5*b*) is $(V_m/Z)\sin(\omega t - \theta)$. The forced response due to the dc source (Fig. 3-5*c*) is $-V_{dc}/R$. The entire forced response is

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{V_{dc}}{R} \tag{3-20}$$

The natural response is

$$i_n(t) = Ae^{-t/\tau} \tag{3-21}$$

Adding the forced and natural responses gives the complete response.

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-22}$$

The extinction angle β is defined as the angle at which the current reaches zero, as was done earlier in Eq. (3-15). Using the initial condition of $i(\alpha) = 0$ and solving for A,

$$\boxed{A = \left[-\frac{V_m}{Z}\sin(\alpha - \beta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}} \tag{3-23}$$

Figure 3-5*d* shows voltage and current waveforms for a half-wave rectifier with *RL*-source load.

The average power absorbed by the resistor is $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} \tag{3-24}$$

The average power absorbed by the dc source is

$$P_{\text{dc}} = I_o V_{\text{dc}} \tag{3-25}$$

where I_o is the average current, that is,

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) \tag{3-26}$$

Assuming the diode and the inductor to be ideal, there is no average power absorbed by either. The power supplied by the ac source is equal to the sum of the power absorbed by the resistor and the dc source

$$P_{\text{ac}} = I_{\text{rms}}^2 R + I_o V_{\text{dc}} \tag{3-27}$$

or it can be computed from

$$P_{\text{ac}} = \frac{1}{2\pi}\int_{0}^{2\pi} v(\omega t)\, i(\omega t)\, d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\beta} (V_m \sin \omega t)\, i(\omega t)\, d(\omega t) \tag{3-28}$$

EXAMPLE 3-5

Half-Wave Rectifier with *RL*-Source Load

For the circuit of Fig. 3-5*a*, $R = 2\ \Omega$, $L = 20$ mH, and $V_{dc} = 100$ V. The ac source is 120 V rms at 60 Hz. Determine (*a*) an expression for the current in the circuit, (*b*) the power absorbed by the resistor, (*c*) the power absorbed by the dc source, and (*d*) the power supplied by the ac source and the power factor of the circuit.

■ Solution

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 1.31 \text{ rad}$$
$$\alpha = \sin^{-1}(100/169.7) = 36.1° = 0.630 \text{ rad}$$
$$\omega\tau = 377(0.02/2) = 3.77 \text{ rad}$$

(*a*) Using Eq. (3-22),

$$i(\omega t) = 21.8 \sin(\omega t - 1.31) - 50 + 75.3e^{-\omega t/3.77} \quad \text{A}$$

The extinction angle β is found from the solution of

$$i(\beta) = 21.8 \sin(\beta - 1.31) - 50 + 75.3e^{-\beta/3.77} = 0$$

which results in $\beta = 3.37$ rad (193°) using root-finding software.

(*b*) Using the preceding expression for $i(\omega t)$ in Eq. (3-24) and using a numerical integration program, the rms current is

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{0.63}^{3.37} i^2(\omega t)\,d(\omega t)} = 3.98 \text{ A}$$

resulting in

$$P_R = I_{rms}^2 R = 3.98^2(2) = 31.7 \text{ W}$$

(*c*) The power absorbed by the dc source is $I_o V_{dc}$. Using Eq. (3-26),

$$I_o = \frac{1}{2\pi}\int_{0.63}^{3.37} i(\omega t)\,d(\omega t) = 2.25 \text{ A}$$

yielding

$$P_{dc} = I_o V_{dc} = (2.25)(100) = 225 \text{ W}$$

(*d*) The power supplied by the ac source is the sum of the powers absorbed by the load.

$$P_s = P_R + P_{dc} = 31.2 + 225 = 256 \text{ W}$$

The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\,rms} I_{rms}} = \frac{256}{(120)(3.98)} = 0.54$$

■ **PSpice Solution**

The power quantities in this example can be determined from a PSpice simulation of this circuit. The circuit of Fig. 3-5*a* is created using VSIN, Dbreak, *R*, and *L*. In the simulation settings, choose Time Domain (transient) for the analysis type, and set the Run Time to 16.67 ms for one period of the source. Set the Maximum Step Size to 10 μs to get adequate sampling of the waveforms. A transient analysis with a run time of 16.67 ms (one period for 60 Hz) and a maximum step size of 10 μs is used for the simulation settings.

Average power absorbed by the 2-Ω resistor can be computed in Probe from the basic definition of the average of $p(t)$ by entering AVG(W(R1)), resulting in 29.7 W, or from $I^2_{\text{rms}}R$ by entering RMS(I(R1))*RMS(I(R1))*2. The average power absorbed by the dc source is computed from the Probe expression AVG(W(Vdc)), yielding 217 W.

The PSpice values differ slightly from the values obtained analytically because of the diode model. However, the default diode is more realistic than the ideal diode in predicting actual circuit performance.

3.6 INDUCTOR-SOURCE LOAD

Using Inductance to Limit Current

Another variation of the half-wave rectifier circuit has a load that consists of an inductor and a dc source, as shown in Fig. 3-6. Although a practical implementation of this circuit would contain some resistance, the resistance may be negligible compared to other circuit parameters.

Starting at $\omega t = 0$ and assuming zero initial current in the inductor, the diode remains reverse-biased until the ac source voltage reaches the dc voltage. The value of ωt at which the diode starts to conduct is α, calculated using Eq. (3-18). With the diode conducting, Kirchhoff's voltage law for the circuit is

$$V_m \sin(\omega t) = L\frac{di(t)}{dt} + V_{\text{dc}} \tag{3-29}$$

or

$$V_m \sin(\omega t) = \frac{L}{\omega}\frac{di(\omega t)}{dt} + V_{\text{dc}} \tag{3-30}$$

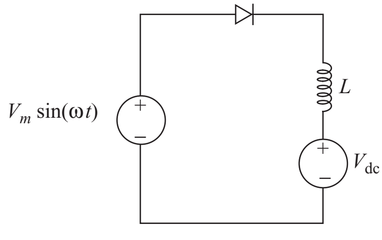


Figure 3-6 Half-wave rectifier with inductor source load.

Rearranging gives

$$\frac{di(\omega t)}{dt} = \frac{V_m \sin(\omega t) - V_{dc}}{\omega L} \tag{3-31}$$

Solving for $i(\omega t)$,

$$i(\omega t) = \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_m \sin\lambda\, d(\lambda) - \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_{dc}\, d(\lambda) \tag{3-32}$$

Performing the integration,

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{\omega L}(\cos\alpha - \cos\omega t) + \dfrac{V_{dc}}{\omega L}(\alpha - \omega t) & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-33}$$

A distinct feature of this circuit is that the power supplied by the source is the same as that absorbed by the dc source, less any losses associated with a nonideal diode and inductor. If the objective is to transfer power from the ac source to the dc source, losses are kept to a minimum by using this circuit.

EXAMPLE 3-6

Half-Wave Rectifier with Inductor-Source Load

For the circuit of Fig. 3-6, the ac source is 120 V rms at 60 Hz, $L = 50$ mH, and $V_{dc} = 72$ V. Determine (*a*) an expression for the current, (*b*) the power absorbed by the dc source, and (*c*) the power factor.

■ Solution

For the parameters given,

$$\alpha = \sin^{-1}\left(\frac{72}{120\sqrt{2}}\right) = 25.1° = 0.438 \text{ rad}$$

(*a*) The equation for current is found from Eq. (3-33).

$$i(\omega t) = 9.83 - 9.00\cos(\omega t) - 3.82\,\omega t \qquad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

where β is found to be 4.04 rad from the numerical solution of $9.83 - 9.00\cos\beta - 3.82\beta = 0$.

(*b*) The power absorbed by the dc source is $I_o V_{dc}$, where

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_{0.438}^{4.04} [9.83 - 9.00\cos(\omega t) - 3.82\,\omega t]\, d(\omega t) = 2.46 \text{ A}$$

resulting in

$$P_{dc} = V_{dc} I_o = (2.46)(72) = 177 \text{ W}$$

(*c*) The rms current is found from

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} = 3.81 \text{ A}$$

Therefore,

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}} = \frac{177}{(120)(3.81)} = 0.388$$

3.7 THE FREEWHEELING DIODE

Creating a DC Current

A freewheeling diode, D_2 in Fig. 3-7*a*, can be connected across an *RL* load as shown. The behavior of this circuit is somewhat different from that of the half-wave rectifier of Fig. 3-2. The key to the analysis of this circuit is to determine when each diode conducts. First, it is observed that both diodes cannot be forward-biased at the same time. Kirchhoff's voltage law around the path containing the source and the two diodes shows that one diode must be reverse-biased. Diode D_1 will be on when the source is positive, and diode D_2 will be on when the source is negative.

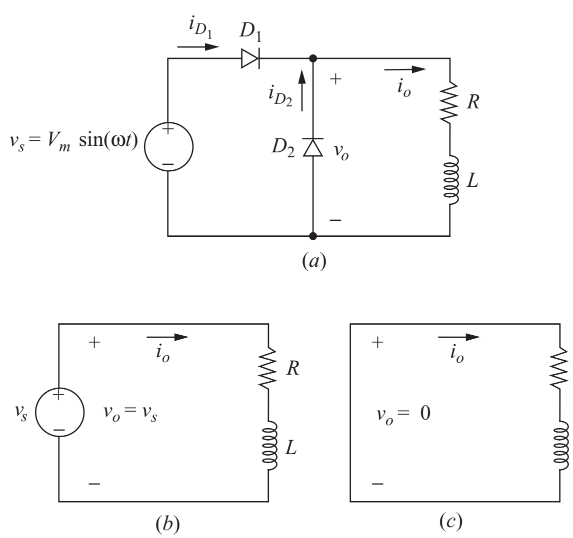


Figure 3-7 (*a*) Half-wave rectifier with freewheeling diode; (*b*) Equivalent circuit for $v_s > 0$; (*c*) Equivalent circuit for $v_s < 0$.

For a positive source voltage,

- D_1 is on.
- D_2 is off.
- The equivalent circuit is the same as that of Fig. 3-2, shown again in Fig. 3-7*b*.
- The voltage across the *RL* load is the same as the source.

For a negative source voltage,

- D_1 is off.
- D_2 is on.
- The equivalent circuit is the same at that of Fig. 3-7*c*.
- The voltage across the *RL* load is zero.

Since the voltage across the *RL* load is the same as the source voltage when the source is positive and is zero when the source is negative, the load voltage is a half-wave rectified sine wave.

When the circuit is first energized, the load current is zero and cannot change instantaneously. The current reaches periodic steady state after a few periods (depending on the L/R time constant), which means that the current at the end of a period is the same as the current at the beginning of the period, as shown in Fig. 3-8. The steady-state current is usually of greater interest than the transient that occurs when the circuit is first energized. Steady-state load, source, and diode currents are shown in Fig. 3-9.

The Fourier series for the half-wave rectified sine wave for the voltage across the load is

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t) \tag{3-34}$$

The current in the load can be expressed as a Fourier series by using superposition, taking each frequency separately. The Fourier series method is illustrated in Example 3-7.

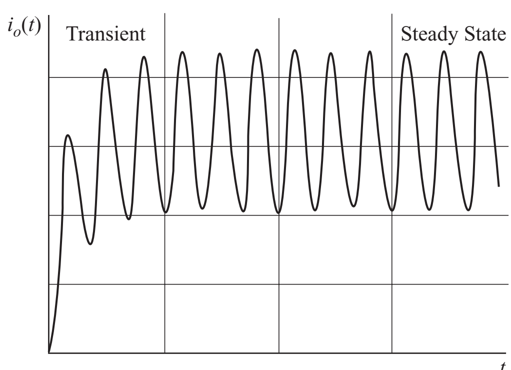


Figure 3-8 Load current reaching steady state after the circuit is energized.

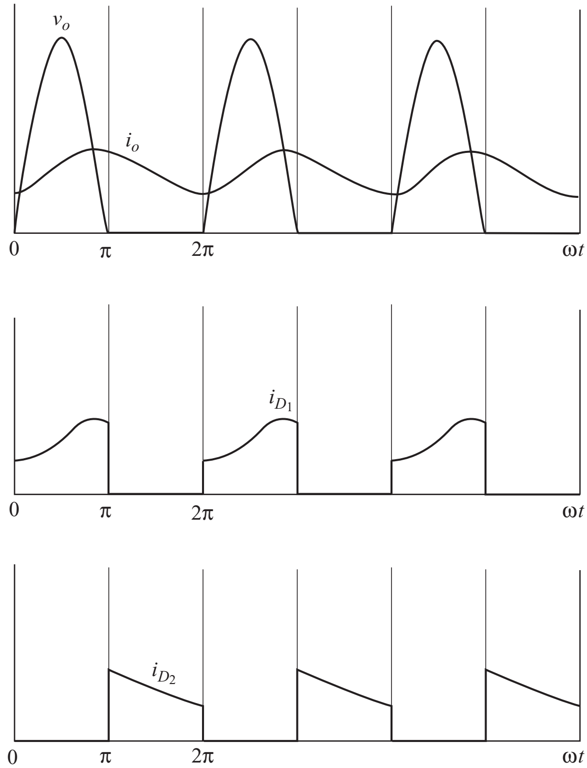


Figure 3-9 Steady-state load voltage and current waveforms with freewheeling diode.

EXAMPLE 3-7

Half-Wave Rectifier with Freewheeling Diode

Determine the average load voltage and current, and determine the power absorbed by the resistor in the circuit of Fig. 3-7*a*, where $R = 2\ \Omega$ and $L = 25$ mH, V_m is 100 V, and the frequency is 60 Hz.

■ Solution

The Fourier series for this half-wave rectified voltage that appears across the load is obtained from Eq. (3-34). The average load voltage is the dc term in the Fourier series:

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.8 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{31.8}{2} = 15.9 \text{ A}$$

Load power can be determined from $I_{\text{rms}}^2 R$, and rms current is determined from the Fourier components of current. The amplitudes of the ac current components are determined from phasor analysis:

$$I_n = \frac{V_n}{Z_n}$$

where $$Z_n = |R + jn\omega_0 L| = |2 + jn377(0.025)|$$

The ac voltage amplitudes are determined from Eq. (3-34), resulting in

$$V_1 = \frac{V_m}{2} = \frac{100}{2} = 50 \text{ V}$$

$$V_2 = \frac{2V_m}{(2^2 - 1)\pi} = 21.2 \text{ V}$$

$$V_4 = \frac{2V_m}{(4^2 - 1)\pi} = 4.24 \text{ V}$$

$$V_6 = \frac{2V_m}{(6^2 - 1)\pi} = 1.82 \text{ V}$$

The resulting Fourier terms are as follows:

n	V_n (V)	Z_n (Ω)	I_n (A)
0	31.8	2.00	15.9
1	50.0	9.63	5.19
2	21.2	18.96	1.12
4	4.24	37.75	0.11
6	1.82	56.58	0.03

The rms current is obtained using Eq. (2-64).

$$I_{\text{rms}} = \sqrt{\sum_{k=0}^{\infty} I_{k,\text{rms}}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34 \text{ A}$$

Notice that the contribution to rms current from the harmonics decreases as n increases, and higher-order terms are not significant. Power in the resistor is $I_{\text{rms}}^2 R = (16.34)^2 2 = 534$ W.

■ PSpice Solution

The circuit of Fig. 3-7*a* is created using VSIN, Dbreak, R, and L. The PSpice model for Dbreak is changed to make $n = 0.001$ to approximate an ideal diode. A transient analysis is run with a run time of 150 ms with data saved after 100 ms to eliminate the start-up transient from the data. A maximum step size of 10 μs gives a smooth waveform.

A portion of the output file is as follows:

```
****     FOURIER ANALYSIS       TEMPERATURE =  27.000 DEG C

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(OUT)

DC COMPONENT =      3.183002E+01
```

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	6.000E+01	5.000E+01	1.000E+00	-5.804E-05	0.000E+00
2	1.200E+02	2.122E+01	4.244E-01	-9.000E+01	-9.000E+01
3	1.800E+02	5.651E-05	1.130E-06	-8.831E+01	-8.831E+01
4	2.400E+02	4.244E+00	8.488E-02	-9.000E+01	-9.000E+01
5	3.000E+02	5.699E-05	1.140E-06	-9.064E+01	-9.064E+01
6	3.600E+02	1.819E+00	3.638E-02	-9.000E+01	-9.000E+01
7	4.200E+02	5.691E-05	1.138E-06	-9.111E+01	-9.110E+01
8	4.800E+02	1.011E+00	2.021E-02	-9.000E+01	-9.000E+01
9	5.400E+02	5.687E-05	1.137E-06	-9.080E+01	-9.079E+01

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(R_R1)

DC COMPONENT = 1.591512E+01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	6.000E+01	5.189E+00	1.000E+00	-7.802E+01	0.000E+00
2	1.200E+02	1.120E+00	2.158E-01	-1.739E+02	-1.788E+01
3	1.800E+02	1.963E-04	3.782E-05	-3.719E+01	1.969E+02
4	2.400E+02	1.123E-01	2.164E-02	-1.770E+02	1.351E+02
5	3.000E+02	7.524E-05	1.450E-05	6.226E+01	4.524E+02
6	3.600E+02	3.217E-02	6.200E-03	-1.781E+02	2.900E+02
7	4.200E+02	8.331E-05	1.605E-05	1.693E+02	7.154E+02
8	4.800E+02	1.345E-02	2.592E-03	-1.783E+02	4.458E+02
9	5.400E+02	5.435E-05	1.047E-05	-1.074E+02	5.948E+02

Note the close agreement between the analytically obtained Fourier terms and the PSpice output. Average current can be obtained in Probe by entering AVG(I(R1)), yielding 15.9 A. Average power in the resistor can be obtained by entering AVG(W(R1)), yielding $P = 535$ W. It is important that the simulation represent steady-state periodic current for the results to be valid.

Reducing Load Current Harmonics

The average current in the *RL* load is a function of the applied voltage and the resistance but not the inductance. The inductance affects only the ac terms in the Fourier series. If the inductance is infinitely large, the impedance of the load to ac terms in the Fourier series is infinite, and the load current is purely dc. The load current is then

$$\boxed{i_o(t) \approx I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \qquad \frac{L}{R} \to \infty} \tag{3-35}$$

A large inductor ($L/R \gg T$) with a freewheeling diode provides a means of establishing a nearly constant load current. Zero-to-peak fluctuation in load current can be estimated as being equal to the amplitude of the first ac term in the Fourier series. The peak-to-peak ripple is then

$$\Delta I_o \approx 2I_1 \tag{3-36}$$

EXAMPLE 3-8

Half-Wave Rectifier with Freewheeling Diode: $L/R \to \infty$

For the half-wave rectifier with a freewheeling diode and *RL* load as shown in Fig. 3-7*a*, the source is 240 V rms at 60 Hz and $R = 8\ \Omega$. (*a*) Assume L is infinitely large. Determine the power absorbed by the load and the power factor as seen by the source. Sketch v_o, i_{D_1}, and i_{D_2}. (*b*) Determine the average current in each diode. (*c*) For a finite inductance, determine L such that the peak-to-peak current is no more than 10 percent of the average current.

■ **Solution**

(*a*) The voltage across the *RL* load is a half-wave rectified sine wave, which has an average value of V_m/π. The load current is

$$i(\omega t) = I_o = \frac{V_o}{R} = \frac{V_m/\pi}{R} = \frac{\left(240\sqrt{2}\right)/\pi}{8} = 13.5\text{ A} \approx I_{\text{rms}}$$

Power in the resistor is

$$P = (I_{\text{rms}})^2 R = (13.5)^2 8 = 1459\text{ W}$$

Source rms current is computed from

$$I_{s,\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} (13.5)^2\, d(\omega t)} = 9.55\text{ A}$$

The power factor is

$$\text{pf} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1459}{(240)(9.55)} = 0.637$$

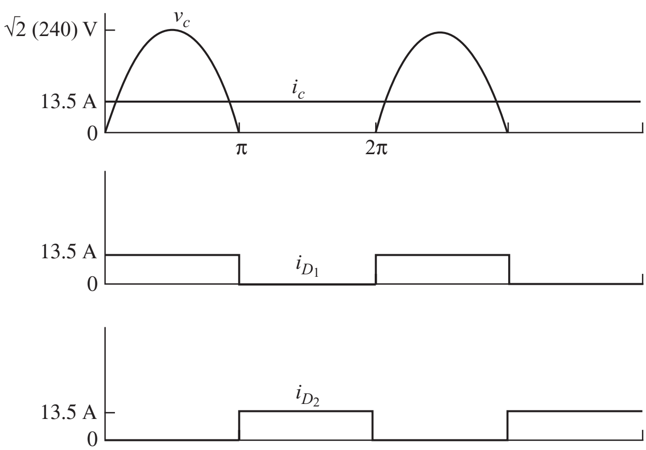


Figure 3-10 Waveforms for the half-wave rectifier with freewheeling diode of Example 3-8 with $L/R \to \infty$.

Voltage and current waveforms are shown in Fig. 3-10.

(*b*) Each diode conducts for one-half of the time. Average current for each diode is $I_o/2 = 13.5/2 = 6.75$ A.

(*c*) The value of inductance required to limit the variation in load current to 10 percent can be approximated from the fundamental frequency of the Fourier series. The voltage input to the load for $n = 1$ in Eq. (3-34) has amplitude $V_m/2 = \sqrt{2}(240)/2 = 170$ V the peak-to-peak current must be limited to

$$\Delta I_o = (0.10)(I_o) = (0.10)(13.5) = 1.35 \text{ A}$$

which corresponds to an amplitude of $1.35/2 = 0.675$ A. The load impedance at the fundamental frequency must then be

$$Z_1 = \frac{V_1}{I_1} = \frac{170}{0.675} = 251\ \Omega$$

The load impedance is

$$Z_1 = 251 = |R + j\omega L| = |8 + j377L|$$

Since the 8-Ω resistance is negligible compared to the total impedance, the inductance can be approximated as

$$L \approx \frac{Z_1}{\omega} = \frac{251}{377} = 0.67 \text{ H}$$

The inductance will have to be slightly larger than 0.67 H because Fourier terms higher than $n = 1$ were neglected in this estimate.

3.8 HALF-WAVE RECTIFIER WITH A CAPACITOR FILTER

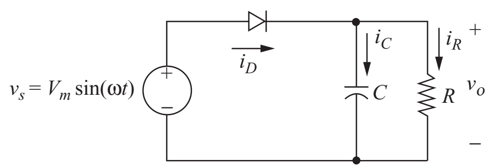
Creating a DC Voltage from an AC Source

A common application of rectifier circuits is to convert an ac voltage input to a dc voltage output. The half-wave rectifier of Fig. 3-11*a* has a parallel *RC* load. The purpose of the capacitor is to reduce the variation in the output voltage, making it more like dc. The resistance may represent an external load, and the capacitor may be a filter which is part of the rectifier circuit.

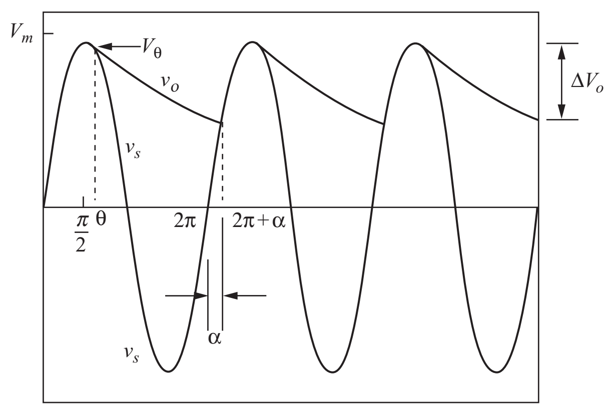
Assuming the capacitor is initially uncharged and the circuit is energized at $\omega t = 0$, the diode becomes forward-biased as the source becomes positive. With the diode on, the output voltage is the same as the source voltage, and the capacitor charges. The capacitor is charged to V_m when the input voltage reaches its positive peak at $\omega t = \pi/2$.

As the source decreases after $\omega t = \pi/2$, the capacitor discharges into the load resistor. At some point, the voltage of the source becomes less than the output voltage, reverse-biasing the diode and isolating the load from the source. The output voltage is a decaying exponential with time constant *RC* while the diode is off.

The point when the diode turns off is determined by comparing the rates of change of the source and the capacitor voltages. The diode turns off when the



(*a*)



(*b*)

Figure 3-11 (*a*) Half-wave rectifier with *RC* load; (*b*) Input and output voltages.

downward rate of change of the source exceeds that permitted by the time constant of the RC load. The angle $\omega t = \theta$ is the point when the diode turns off in Fig. 3-11b. The output voltage is described by

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases} \tag{3-37}$$

where
$$V_\theta = V_m \sin \theta \tag{3-38}$$

The slopes of these functions are

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t) \tag{3-39}$$

and

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta\left(-\frac{1}{\omega RC}\right)e^{-(\omega t - \theta)/\omega RC} \tag{3-40}$$

At $\omega t = \theta$, the slopes of the voltage functions are equal:

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right)e^{-(\theta - \theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

Solving for θ and expressing θ so it is in the proper quadrant, we have

$$\boxed{\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi} \tag{3-41}$$

In practical circuits where the time constant is large,

$$\boxed{\theta \approx \frac{\pi}{2} \qquad \text{and} \qquad V_m \sin\theta \approx V_m} \tag{3-42}$$

When the source voltage comes back up to the value of the output voltage in the next period, the diode becomes forward-biased, and the output again is the same as the source voltage. The angle at which the diode turns on in the second period, $\omega t = 2\pi + \alpha$, is the point when the sinusoidal source reaches the same value as the decaying exponential output:

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta)e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\boxed{\sin\alpha - (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC} = 0} \tag{3-43}$$

Equation (3-43) must be solved numerically for α.

The current in the resistor is calculated from $i_R = v_o/R$. The current in the capacitor is calculated from

$$i_C(t) = C\frac{dv_o(t)}{dt}$$

which can also be expressed, using ωt as the variable, as

$$i_C(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

Using v_o from Eq. (3-37),

$$\boxed{i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m \sin\theta}{R}\right)e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}} \tag{3-44}$$

The source current, which is the same as the diode current, is

$$i_S = i_D = i_R + i_C \tag{3-45}$$

The average capacitor current is zero, so the average diode current is the same as the average load current. Since the diode is on for a short time in each cycle, the peak diode current is generally much larger than the average diode current. Peak capacitor current occurs when the diode turns on at $\omega t = 2\pi + \alpha$. From Eq. (3-44),

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha \tag{3-46}$$

Resistor current at $\omega t = 2\pi + \alpha$ is obtained from Eq. (3-37).

$$i_R(2\omega t + \alpha) = \frac{V_m \sin(2\omega t + \alpha)}{R} = \frac{V_m \sin\alpha}{R} \tag{3-47}$$

Peak diode current is

$$\boxed{I_{D,\text{ peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)} \tag{3-48}$$

The effectiveness of the capacitor filter is determined by the variation in output voltage. This may be expressed as the difference between the maximum and minimum output voltage, which is the peak-to-peak ripple voltage. For the half-wave rectifier of Fig. 3-11a, the maximum output voltage is V_m. The minimum output voltage occurs at $\omega t = 2\pi + \alpha$, which can be computed from $V_m \sin \alpha$. The peak-to-peak ripple for the circuit of Fig. 3-11*a* is expressed as

$$\boxed{\Delta V_o = V_m - V_m \sin \alpha = V_m(1 - \sin \alpha)} \tag{3-49}$$

In circuits where the capacitor is selected to provide for a nearly constant dc output voltage, the *RC* time constant is large compared to the period of the sine wave, and Eq. (3-42) applies. Moreover, the diode turns on close to the peak of the sine wave when $\alpha \approx \pi/2$. The change in output voltage when the diode is off is described in Eq. (3-37). In Eq. (3-37), if $V_\theta \approx \mathrm{V_m}$ and $\theta \approx \pi/2$, then Eq. (3-37) evaluated at $\alpha = \pi/2$ is

$$v_o(2\pi + \alpha) = V_m e^{-(2\pi + \pi/2 - \pi/2)\omega RC} = V_m e^{-2\pi/\omega RC}$$

The ripple voltage can then be approximated as

$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1 - e^{-2\pi/\omega RC}\right) \tag{3-50}$$

Furthermore, the exponential in the above equation can be approximated by the series expansion:

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

Substituting for the exponential in Eq. (3-50), the peak-to-peak ripple is approximately

$$\boxed{\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}} \tag{3-51}$$

The output voltage ripple is reduced by increasing the filter capacitor *C*. As *C* increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

EXAMPLE 3-9

Half-Wave Rectifier with *RC* Load

The half-wave rectifier of Fig. 3-11*a* has a 120-V rms source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu\mathrm{F}$. Determine (*a*) an expression for output voltage, (*b*) the peak-to-peak voltage variation on the output, (*c*) an expression for capacitor current, (*d*) the peak diode current, and (*e*) the value of *C* such that ΔV_o is 1 percent of V_m.

■ Solution

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7\ \mathrm{V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85\ \mathrm{rad}$$

The angle θ is determined from Eq. (3-41).

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62\,\text{rad} = 93°$$
$$V_m \sin\theta = 169.5 \text{ V}$$

The angle α is determined from the numerical solution of Eq. (3-43).

$$\sin\alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62/18.85)} = 0$$

yielding

$$\alpha = 0.843 \text{ rad} = 48°$$

(*a*) Output voltage is expressed from Eq. (3-37).

$$v_o(\omega t) = \begin{cases} 169.7\sin(\omega t) & 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ 169.5e^{-(\omega t - 1.62)/18.85} & \theta \le \omega t \le 2\pi + \alpha \end{cases}$$

(*b*) Peak-to-peak output voltage is described by Eq. (3-49).

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

(*c*) The capacitor current is determined from Eq. (3-44).

$$i_C(\omega t) = \begin{cases} -0.339e^{-(\omega t - 1.62)/18.85} \quad \text{A} & \theta \le \omega t \le 2\pi + \alpha \\ 6.4\cos(\omega t) \quad \text{A} & 2\pi + \alpha \le \omega t \le 2\pi + \theta \end{cases}$$

(*d*) Peak diode current is determined from Eq. (3-48).

$$I_{D,\text{peak}} = \sqrt{2}(120)\left[377(10)^{-4}\cos 0.843 + \frac{\sin 8.43}{500}\right]$$
$$= 4.26 + 0.34 = 4.50 \text{ A}$$

(*e*) For $\Delta V_o = 0.01V_m$, Eq. (3-51) can be used.

$$C \approx \frac{V_m}{fR(\Delta V_o)} = \frac{V_m}{(60)(500)(0.01V_m)} = \frac{1}{300}\text{F} = 3333\ \mu\text{F}$$

Note that peak diode current can be determined from Eq. (3-48) using an estimate of α from Eq. (3-49).

$$\alpha \approx \sin^{-1}\left(1 - \frac{\Delta V_o}{V_m}\right) = \sin^{-1}\left(1 - \frac{1}{fRC}\right) = 81.9°$$

From Eq. (3-48), peak diode current is 30.4 A.

■ PSpice Solution

A PSpice circuit is created for Fig. 3-11*a* using VSIN, Dbreak, *R*, and *C*. The diode Dbreak used in this analysis causes the results to differ slightly from the analytic solution based on the ideal diode. The diode drop causes the maximum output voltage to be slightly less than that of the source.

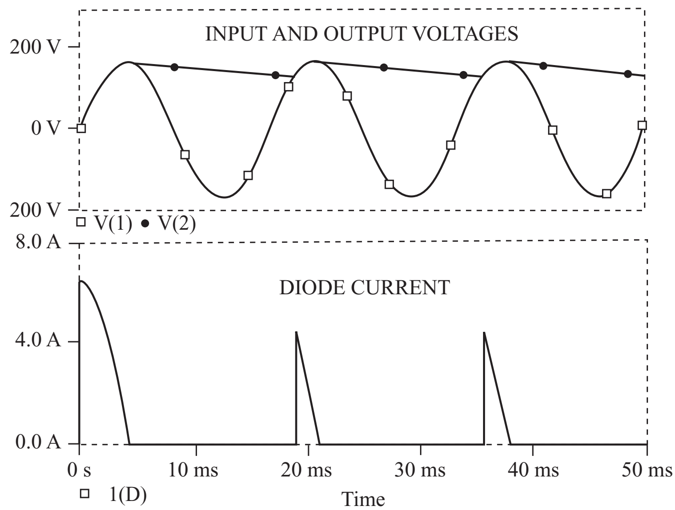


Figure 3-12 Probe output for Example 3-9.

The Probe output is shown in Fig. 3-12. Angles θ and α are determined directly by first modifying the x-variable to indicate degrees (x-variable = time*60*360) and then using the cursor option. The restrict data option is used to compute quantities based on steady-state values (16.67 to 50 ms). Steady state is characterized by waveforms beginning and ending a period at the same values. Note that the peak diode current is largest in the first period because the capacitor is initially uncharged.

■ Results from the Probe Cursor

Quantity	Result
$\alpha + 360°$	408° ($\alpha = 48°$)
θ	98.5°
V_o max	168.9 V
V_o min	126 V
ΔV_o	42.9 V
$I_{D,\text{peak}}$	4.42 A steady state; 6.36 A first period
$I_{C,\text{peak}}$	4.12 A steady state; 6.39 A first period

■ Results after Restricting the Data to Steady State

Quantity	Probe Expression	Result
$I_{D,\text{avg}}$	AVG(I(D1))	0.295 A
$I_{C,\text{rms}}$	RMS(I(C1))	0.905 A
$I_{R,\text{avg}}$	AVG(W(R1))	43.8 W
P_s	AVG(W(Vs))	−44.1 W
P_D	AVG(W(D1))	345 mW

In this example, the ripple, or variation in output voltage, is very large, and the capacitor is not an effective filter. In many applications, it is desirable to produce an output that is closer to dc. This requires the time constant *RC* to be large compared to the period of the input voltage, resulting in little decay of the output voltage. For an effective filter capacitor, the output voltage is essentially the same as the peak voltage of the input.

3.9 THE CONTROLLED HALF-WAVE RECTIFIER

The half-wave rectifiers analyzed previously in this chapter are classified as uncontrolled rectifiers. Once the source and load parameters are established, the dc level of the output and the power transferred to the load are fixed quantities.

A way to control the output of a half-wave rectifier is to use an SCR[1] instead of a diode. Figure 3-13*a* shows a basic controlled half-wave rectifier with a resistive load. Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.

Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.

Resistive Load

Figure 3-13*b* shows the voltage waveforms for a controlled half-wave rectifier with a resistive load. A gate signal is applied to the SCR at $\omega t = \alpha$, where α is the delay angle. The average (dc) voltage across the load resistor in Fig. 3-13*a* is

$$\boxed{V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)} \tag{3-52}$$

The power absorbed by the resistor is V_{rms}^2/R, where the rms voltage across the resistor is computed from

$$\begin{aligned} V_{\text{rms}} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(\omega t)\, d(\omega t)} \\ &= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)} \\ &= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} \end{aligned} \tag{3-53}$$

[1] Switching with other controlled turn-on devices such as transistors or IGBTs can be used to control the output of a converter.

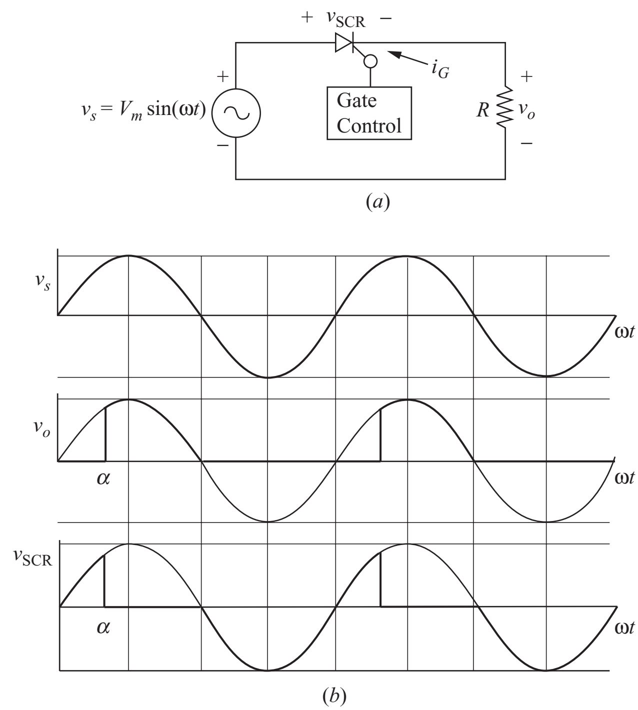


Figure 3-13 (*a*) A basic controlled rectifier; (*b*) Voltage waveforms.

EXAMPLE 3-10

Controlled Half-Wave Rectifier with Resistive Load

Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

■ Solution

If an uncontrolled half-wave rectifier is used, the average voltage will be $V_m/\pi = 120\sqrt{2}/\pi = 54$ V. Some means of reducing the average resistor voltage to the design specification of 40 V must be found. A series resistance or inductance could be added to an uncontrolled rectifier, or a controlled rectifier could be used. The controlled rectifier of Fig. 3-13*a* has the advantage of not altering the load or introducing losses, so it is selected for this application.

Equation (3-52) is rearranged to determine the required delay angle:

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

Equation (3-53) gives

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is

$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

RL Load

A controlled half-wave rectifier with an *RL* load is shown in Fig. 3-14*a*. The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses, and Eq. (3-9) applies:

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition $i(\alpha) = 0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] = e^{\alpha/\omega\tau} \tag{3-54}$$

Substituting for A and simplifying,

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-55}$$

The *extinction angle* β is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When $\omega t = \beta$,

$$\boxed{i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}\right]} \tag{3-56}$$

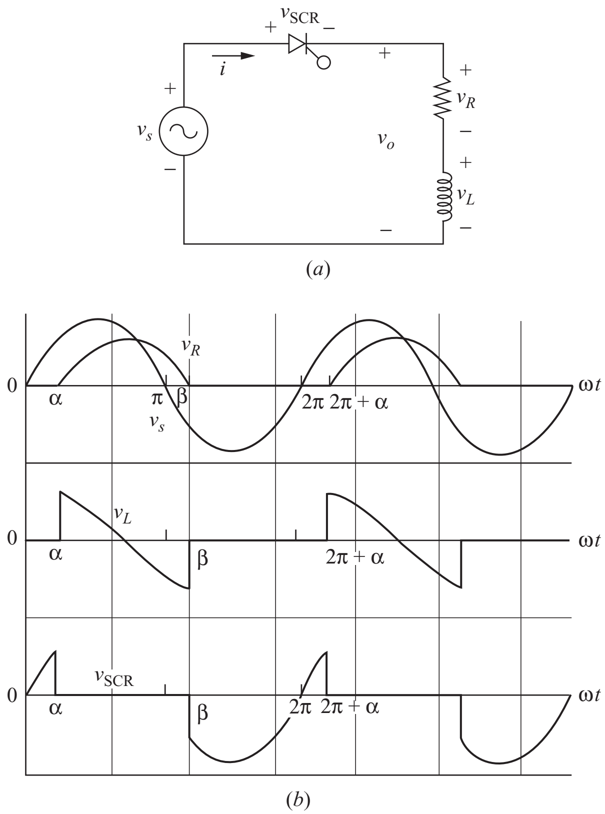


Figure 3-14 (*a*) Controlled half-wave rectifier with *RL* load; (*b*) Voltage waveforms.

which must be solved numerically for β. The angle $\beta - \alpha$ is called the *conduction angle* γ. Figure 3-14*b* shows the voltage waveforms.

The average (dc) output voltage is

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) \tag{3-57}$$

The average current is computed from

$$I_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) \tag{3-58}$$

where $i(\omega t)$ is defined in Eq. (3-55). Power absorbed by the load is $I_{rms}^2 R$, where the rms current is computed from

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} \tag{3-59}$$

EXAMPLE 3-11

Controlled Half-Wave Rectifier with *RL* Load

For the circuit of Fig. 3-14*a*, the source is 120 V rms at 60 Hz, $R = 20\ \Omega$, $L = 0.04$ H, and the delay angle is 45°. Determine (*a*) an expression for $i(\omega t)$, (*b*) the average current, (*c*) the power absorbed by the load, and (*d*) the power factor.

■ Solution

(*a*) From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04/20) = 0.646 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$
$$\alpha = 45° = 0.785 \text{ rad}$$

Substituting the preceding quantities into Eq. (3-55), current is expressed as

$$i(\omega t) = 6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754} \quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

The preceding equation is valid from α to β, where β is found numerically by setting the equation to zero and solving for ωt, with the result $\beta = 3.79$ rad (217°). The conduction angle is $\gamma = \beta - \alpha = 3.79 - 0.785 = 3.01$ rad $= 172°$.

(*b*) Average current is determined from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]d(\omega t) = 2.19 \text{ A}$$

(*c*) The power absorbed by the load is computed from $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]^2 d(\omega t)} = 3.26 \text{ A}$$

yielding

$$P = I_{\text{rms}}^2 R = (3.26)^2(20) = 213 \text{ W}$$

(*d*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{213}{(120)(3.26)} = 0.54$$

RL-Source Load

A controlled rectifier with a series resistance, inductance, and dc source is shown in Fig. 3-15. The analysis of this circuit is very similar to that of the uncontrolled half-wave rectifier discussed earlier in this chapter. The major difference is that for the uncontrolled rectifier, conduction begins as soon as the source voltage

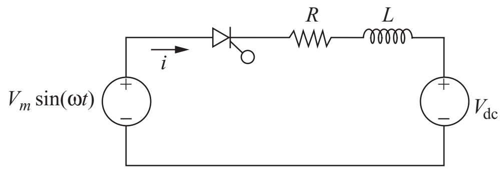


Figure 3-15 Controlled rectifier with *RL*-source load.

reaches the level of the dc voltage. For the controlled rectifier, conduction begins when a gate signal is applied to the SCR, provided the SCR is forward-biased. Thus, the gate signal may be applied at any time that the ac source is larger than the dc source:

$$\alpha_{\min} = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \tag{3-60}$$

Current is expressed as in Eq. (3-22), with α specified within the allowable range:

$$\boxed{i(\omega t)\begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-61}$$

where A is determined from Eq. (3-61):

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

EXAMPLE 3-12

Controlled Rectifier with *RL*-Source Load

The controlled half-wave rectifier of Fig. 3-15 has an ac input of 120 V rms at 60 Hz, $R = 2\ \Omega$, $L = 20$ mH, and $V_{dc} = 100$ V. The delay angle α is 45°. Determine (*a*) an expression for the current, (*b*) the power absorbed by the resistor, and (*c*) the power absorbed by the dc source in the load.

■ **Solution:**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [2^2 + (377*0.02)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.02)/2) = 1.312 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.02/2 = 3.77$$
$$\alpha = 45° = 0.785 \text{ rad}$$

(*a*) First, use Eq. (3-60) to determine if $\alpha = 45°$ is allowable. The minimum delay angle is

$$\alpha_{\min} = \sin^{-1}\left(\frac{100}{120\sqrt{2}}\right) = 36°$$

which indicates that 45° is allowable. Equation (3-61) becomes

$$i(\omega t) = 21.8 \sin(\omega t - 1.312) - 50 + 75.0e^{-\omega t/3.77} \text{ A} \quad \text{for } 0.785 \leq \omega t \leq 3.37 \text{ rad}$$

where the extinction angle β is found numerically to be 3.37 rad from the equation $i(\beta) = 0$.

(*b*) Power absorbed by the resistor is $I^2_{\text{rms}}R$, where I_{rms} is computed from Eq. (3-59) using the preceding expression for $i(\omega t)$.

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} = 3.90 \text{ A}$$

$$P = (3.90)^2(2) = 30.4 \text{ W}$$

(*c*) Power absorbed by the dc source is $I_o V_{\text{dc}}$, where I_o is computed from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) = 2.19 \text{ A}$$

$$P_{\text{dc}} = I_o V_{\text{dc}} = (2.19)(100) = 219 \text{ W}$$

3.10 PSPICE SOLUTIONS FOR CONTROLLED RECTIFIERS

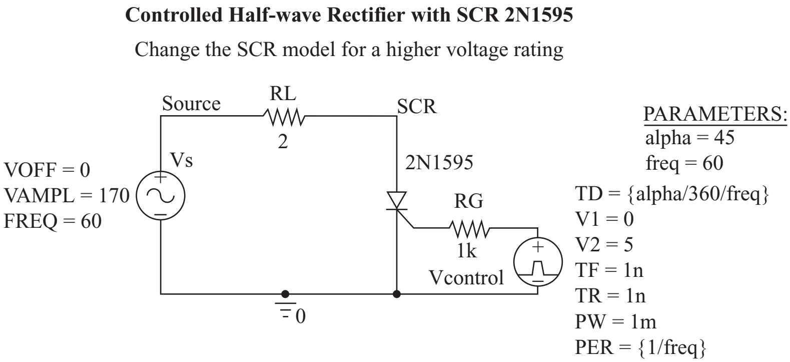
Modeling the SCR in PSpice

To simulate the controlled half-wave rectifier in PSpice, a model for the SCR must be selected. An SCR model available in a device library can be utilized in the simulation of a controlled half-wave rectifier. A circuit for Example 3-10 using the 2N1595 SCR in the PSpice demo version library of devices is shown in Fig. 3-16*a*. An alternative model for the SCR is a voltage-controlled switch and a diode as described in Chap. 1. The switch controls when the SCR begins to conduct, and the diode allows current in only one direction. The switch must be closed for at least the conduction angle of the current. An advantage of using this SCR model is that the device can be made ideal. The major disadvantage of the model is that the switch control must keep the switch closed for the entire conduction period and open the switch before the source becomes positive again. A circuit for the circuit in Example 3-11 is shown in Fig. 3-16*b*.

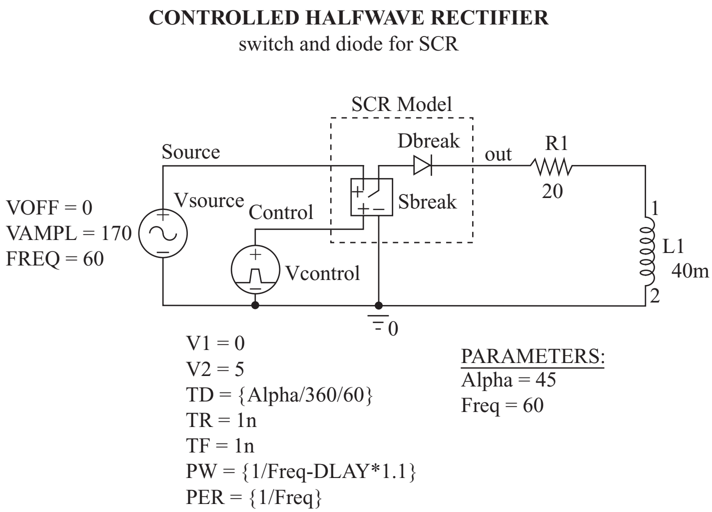
EXAMPLE 3-13

Controlled Half-Wave Rectifier Design Using PSpice

A load consists of a series-connected resistance, inductance, and dc voltage source with $R = 2\ \Omega$, $L = 20$ mH, and $V_{\text{dc}} = 100$ V. Design a circuit that will deliver 150 W to the dc voltage source from a 120-V rms 60-Hz ac source.



(*a*)



(*b*)

Figure 3-16 (*a*) A controlled half-wave rectifier using an SCR from the library of devices; (*b*) An SCR model using a voltage-controlled switch and a diode.

■ Solution

Power in the dc source of 150 W requires an average load current of 150 W/100 V = 1.5 A. An uncontrolled rectifier with this source and load will have an average current of 2.25 A and an average power in the dc source of 225 W, as was computed in Example 3-5 previously. A means of limiting the average current to 1.5 A must be found. Options include the addition of series resistance or inductance. Another option that is chosen for this application is the controlled half-wave rectifier of Fig. 3-15. The power delivered to the load components is determined by the delay angle α. Since there is no closed-form solution for α, a trial-and-error iterative method must be used. A PSpice simulation that includes a parametric sweep is used to try several values of alpha. The parametric sweep

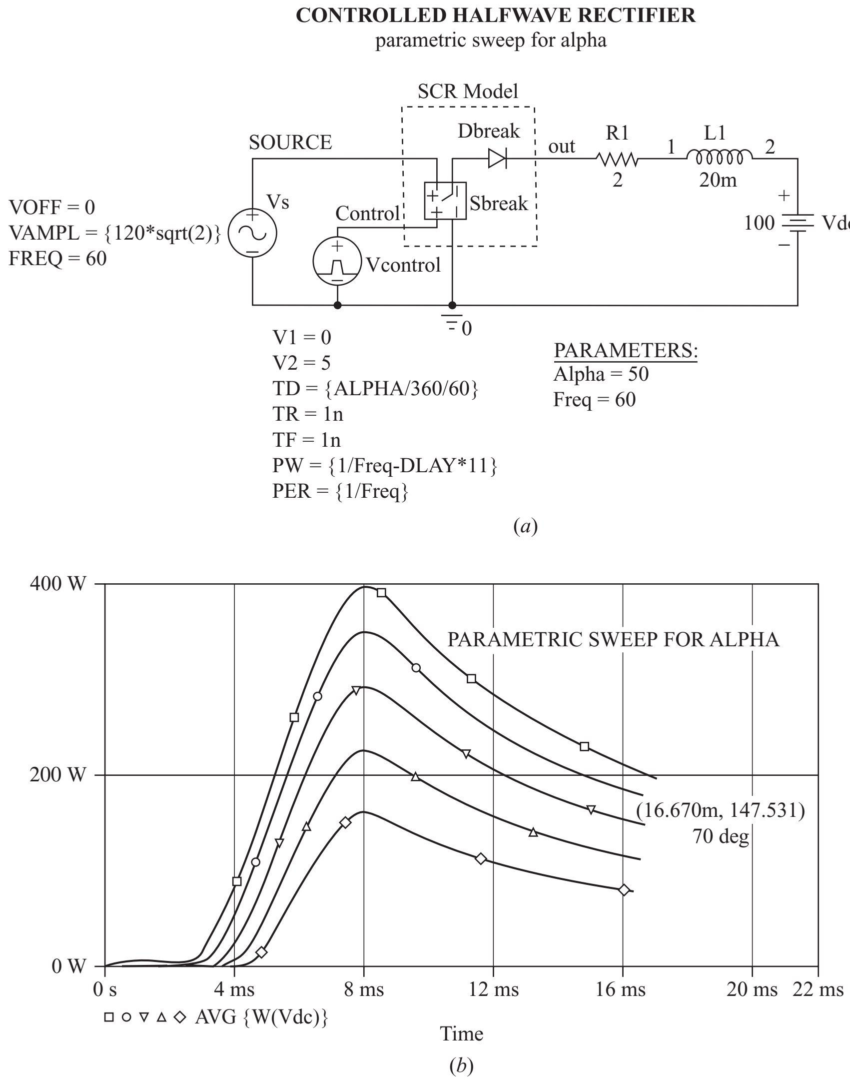


Figure 3-17 (*a*) PSpice circuit for Example 3-13; (*b*) Probe output for showing a family of curves for different delay angles.

is established in the Simulation Setting menu (see Example 3-4). A PSpice circuit is shown in Fig. 3-17*a*.

When the expression AVG(W(Vdc)) is entered, Probe produces a family of curves representing the results for a number of values of α, as shown in Fig. 3-17*b*. An α of 70°, which results in about 148 W delivered to the load, is the approximate solution.

The following results are obtained from Probe for $\alpha = 70°$:

Quantity	Expression	Result
DC source power	AVG(W(Vdc))	148 W (design objective of 150 W)
RMS current	RMS(I(R1))	2.87 A
Resistor power	AVG(W(R1))	16.5 W
Source apparent power	RMS(V(SOURCE))*RMS(I(Vs))	344 VA
Source average power	AVG(W(Vs))	166 W
Power factor (P/S)	166/344	0.48

3.11 COMMUTATION

The Effect of Source Inductance

The preceding discussion on half-wave rectifiers assumed an ideal source. In practical circuits, the source has an equivalent impedance which is predominantly inductive reactance. For the single-diode half-wave rectifiers of Figs. 3-1 and 3-2, the nonideal circuit is analyzed by including the source inductance with the load elements. However, the source inductance causes a fundamental change in circuit behavior for circuits like the half-wave rectifier with a freewheeling diode.

A half-wave rectifier with a freewheeling diode and source inductance L_s is shown in Fig. 3-18*a*. Assume that the load inductance is very large, making the load current constant. At $t = 0^-$, the load current is I_L, D_1 is off, and D_2 is on. As the source voltage becomes positive, D_1 turns on, but the source current does not instantly equal the load current because of L_s. Consequently, D_2 must remain on while the current in L_s and D_1 increases to that of the load. The interval when both D_1 and D_2 are on is called the commutation time or commutation angle. *Commutation is the process of turning off an electronic switch, which usually involves transferring the load current from one switch to another.*[2]

When both D_1 and D_2 are on, the voltage across L_s is

$$v_{Ls} = V_m \sin(\omega t) \tag{3-62}$$

and current in L_s and the source is

$$\begin{aligned} i_s &= \frac{1}{\omega L_s}\int_0^{\omega t} v_{Ls}\, d(\omega t) + i_s(0) = \frac{1}{\omega L_s}\int_0^{\omega t} V_m \sin(\omega t)\, d(\omega t) + 0 \\ i_s &= \frac{V_m}{\omega L_s}(1 - \cos\omega t) \end{aligned} \tag{3-63}$$

[2] Commutation in this case is an example of *natural commutation* or *line commutation,* where the change in instantaneous line voltage results in a device turning off. Other applications may use *forced commutation,* where current in a device such as a thyristor is forced to zero by additional circuitry. *Load commutation* makes use of inherent oscillating currents produced by the load to turn a device off.

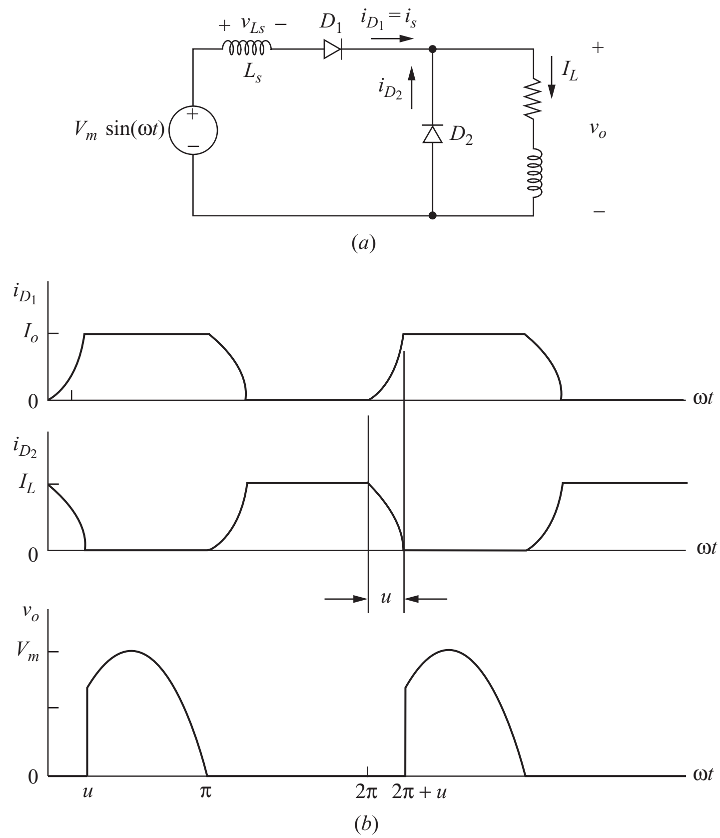


Figure 3-18 (*a*) Half-wave rectifier with freewheeling diode and source inductance; (*b*) Diode currents and load voltage showing the effects of Commutation.

Current in D_2 is

$$i_{D_2} = I_L - i_s = I_L - \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

The current in D_2 starts at I_L and decreases to zero. Letting the angle at which the current reaches zero be $\omega t = u$,

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

Solving for u,

$$\boxed{u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)} \qquad (3\text{-}64)$$

where $X_s = \omega L_s$ is the reactance of the source. Figure 3-18*b* shows the effect of the source reactance on the diode currents. The commutation from D_1 to D_2 is analyzed similarly, yielding an identical result for the commutation angle u.

The commutation angle affects the voltage across the load. Since the voltage across the load is zero when D_2 is conducting, the load voltage remains at zero through the commutation angle, as shown in Fig. 3-17*b*. Recall that the load voltage is a half-wave rectified sinusoid when the source is ideal.

Average load voltage is

$$V_o = \frac{1}{2\pi}\int_u^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{2\pi}[-\cos(\omega t)]\Big|_u^{\pi} = \frac{V_m}{2\pi}(1 + \cos u)$$

Using u from Eq. (3-64),

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)} \tag{3-65}$$

Recall that the average of a half-wave rectified sine wave is V_m/π. Source reactance thus reduces average load voltage.

3.12 Summary

- A rectifier converts ac to dc. Power transfer is from the ac source to the dc load.
- The half-wave rectifier with a resistive load has an average load voltage of V_m/π and an average load current of $V_m/\pi R$.
- The current in a half-wave rectifier with an *RL* load contains a natural and a forced response, resulting in

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2}, \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

The diode remains on as long as the current is positive. Power in the *RL* load is $I_{\text{rms}}^2 R$.

- A half-wave rectifier with an *RL*-source load does not begin to conduct until the ac source reaches the dc voltage in the load. Power in the resistance is $I_{\text{rms}}^2 R$, and power absorbed by the dc source is $I_o V_{\text{dc}}$, where I_o is the average load current. The load current is expressed as

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \beta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

- A freewheeling diode forces the voltage across an *RL* load to be a half-wave rectified sine wave. The load current can be analyzed using Fourier analysis. A large load inductance results in a nearly constant load current.
- A large filter capacitor across a resistive load makes the load voltage nearly constant. Average diode current must be the same as average load current, making the peak diode current large.
- An SCR in place of the diode in a half-wave rectifier provides a means of controlling output current and voltage.
- PSpice simulation is an effective way of analyzing circuit performance. The parametric sweep in PSpice allows several values of a circuit parameter to be tried and is an aid in circuit design.

3.13 Bibliography

S. B. Dewan and A. Straughen, *Power Semiconductor Circuits*, Wiley, New York, 1975.

Y.-S. Lee and M. H. L. Chow, *Power Electronics Handbook*, edited by M. H. Rashid, Academic Press, New York, 2001, Chapter 10.

N. Mohan, T. M. Undeland, and W. P. Robbins, *Power Electronics: Converters, Applications, and Design,* 3d ed., Wiley, New York, 2003.

M. H. Rashid, *Power Electronics: Circuits, Devices, and Systems,* 3d ed., Prentice-Hall, Upper Saddle River, NJ., 2004.

R. Shaffer, *Fundamentals of Power Electronics with MATLAB,* Charles River Media, Boston, Mass., 2007.

B. Wu, *High-Power Converters and AC Drives,* Wiley, New York, 2006.

Problems

Half-Wave Rectifier with Resistive Load

3-1. The half-wave rectifier circuit of Fig. 3-1*a* has $v_s(t) = 170 \sin(377t)$ V and a load resistance $R = 15\ \Omega$. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the power absorbed by the load, (*d*) the apparent power supplied by the source, and (*e*) the power factor of the circuit.

3-2. The half-wave rectifier circuit of Fig. 3-1*a* has a transformer inserted between the source and the remainder of the circuit. The source is 240 V rms at 60 Hz, and the load resistor is 20 Ω. (*a*) Determine the required turns ratio of the transformer such that the average load current is 12 A. (*b*) Determine the average current in the primary winding of the transformer.

3-3. For a half-wave rectifier with a resistive load, (*a*) show that the power factor is $1/\sqrt{2}$ and (*b*) determine the displacement power factor and the distortion factor as defined in Chap. 2. The Fourier series for the half-wave rectified voltage is given in Eq. (3-34).

Half-Wave Rectifier with *RL* Load

3-4. A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 12\ \Omega$ and $L = 12$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

3-5. A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 10\ \Omega$ and $L = 15$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

3-6. A half-wave rectifier has a source of 240 V rms at 60 Hz and an *RL* load with $R = 15\ \Omega$ and $L = 80$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor. (*e*) Use PSpice to simulate the circuit. Use the default diode model and compare your PSpice results with analytical results.

3-7. The inductor in Fig. 3-2*a* represents an electromagnet modeled as a 0.1-H inductance. The source is 240 V at 60 Hz. Use PSpice to determine the value of a series resistance such that the average current is 2.0 A.

Half-Wave Rectifier with *RL*-Source Load

3-8. A half-wave rectifier of Fig. 3-5*a* has a 240 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 75$ mH, $R = 10\ \Omega$, and $V_{dc} = 100$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-9. A half-wave rectifier of Fig. 3-5*a* has a 120 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 120$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-10. A half-wave rectifier of Fig. 3-6 has a 120 V rms, 60 Hz ac source. The load is a series inductance and dc voltage with $L = 100$ mH and $V_{dc} = 48$ V. Determine the power absorbed by the dc voltage source.

3-11. A half-wave rectifier with a series inductor-source load has an ac source of 240 V rms, 60 Hz. The dc source is 96 V. Use PSpice to determine the value of inductance which results in 150 W absorbed by the dc source. Use the default diode model.

3-12. A half-wave rectifier with a series inductor and dc source has an ac source of 120 V rms, 60 Hz. The dc source is 24 V. Use PSpice to determine the value of inductance which results in 50 W absorbed by the dc source. Use the default diode.

Freewheeling Diode

3-13. The half-wave rectifier with a freewheeling diode (Fig. 3-7*a*) has $R = 12\ \Omega$ and $L = 60$ mH. The source is 120 V rms at 60 Hz. (*a*) From the Fourier series of the half-wave rectified sine wave that appears across the load, determine the dc component of the current. (*b*) Determine the amplitudes of the first four nonzero ac terms in the Fourier series. Comment on the results.

3-14. In Example 3-8, the inductance required to limit the peak-to-peak ripple in load current was estimated by using the first ac term in the Fourier series. Use PSpice to determine the peak-to-peak ripple with this inductance, and compare it to the estimate. Use the ideal diode model ($n = 0.001$).

3-15. The half-wave rectifier with a freewheeling diode (Fig. 3-7*a*) has $R = 4\ \Omega$ and a source with $V_m = 50$ V at 60 Hz. (*a*) Determine a value of L such that the amplitude of the first ac current term in the Fourier series is less than 5 percent of the dc current. (*b*) Verify your results with PSpice, and determine the peak-to-peak current.

3-16. The circuit of Fig. P3-16 is similar to the circuit of Fig. 3-7*a* except that a dc source has been added to the load. The circuit has $v_s(t) = 170 \sin(377t)$ V, $R = 10\ \Omega$, and $V_{dc} = 24$ V. From the Fourier series, (*a*) determine the value of L such that the peak-to-peak variation in load current is no more than 1 A. (*b*) Determine the power absorbed by the dc source. (*c*) Determine the power absorbed by the resistor.

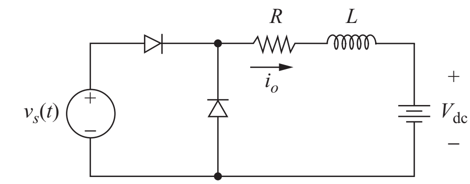


Figure P3-16

Half-Wave Rectifier with a Filter Capacitor

3-17. A half-wave rectifier with a capacitor filter has $V_m = 200$ V, $R = 1$ kω, $C = 1000\ \mu$F, and $\omega = 377$. (*a*) Determine the ratio of the RC time constant to the period of the input sine wave. What is the significance of this ratio? (*b*) Determine the peak-to-peak ripple voltage using the exact equations. (*c*) Determine the ripple using the approximate formula in Eq. (3-51).

3-18. Repeat Prob. 3-17 with (*a*) $R = 100\ \Omega$ and (*b*) $R = 10\ \Omega$. Comment on the results.

3-19. A half-wave rectifier with a 1-kΩ load has a parallel capacitor. The source is 120 V rms, 60 Hz. Determine the peak-to-peak ripple of the output voltage when the capacitor is (*a*) 4000 μF and (*b*) 20 μF. Is the approximation of Eq. (3-51) reasonable in each case?

3-20. Repeat Prob. 3-19 with $R = 500\ \Omega$.

3-21. A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 750 Ω. Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V. Determine the average and peak values of diode current.

3-22. A half-wave rectifier has a 120 V rms 60 Hz ac source. The load is 50 W. (*a*) Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 1.5 V. (*b*) Determine the average and peak values of diode current.

Controlled Half-Wave Rectifier

3-23. Show that the controlled half-wave rectifier with a resistive load in Fig. 3-13*a* has a power factor of

$$\text{pf} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

3-24. For the controlled half-wave rectifier with resistive load, the source is 120 V rms at 60 Hz. The resistance is 100 Ω, and the delay angle α is 45°. (*a*) Determine the average voltage across the resistor. (*b*) Determine the power absorbed by the resistor. (*c*) Determine the power factor as seen by the source.

3-25. A controlled half-wave rectifier has an ac source of 240 V rms at 60 Hz. The load is a 30-Ω resistor. (*a*) Determine the delay angle such that the average load current is 2.5 A. (*b*) Determine the power absorbed by the load. (*c*) Determine the power factor.

3-26. A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series RL load has $R = 25\ \omega$ and $L = 50$ mH. The delay angle is 30°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

3-27. A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series RL load has $R = 40\ \Omega$ and $L = 75$ mH. The delay angle is 60°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

3-28. A controlled half-wave rectifier has an RL load with $R = 20\ \Omega$ and $L = 40$ mH. The source is 120 V rms at 60 Hz. Use PSpice to determine the delay angle required to produce an average current of 2.0 A in the load. Use the default diode in the simulation.

3-29. A controlled half-wave rectifier has an RL load with $R = 16\ \Omega$ and $L = 60$ mH. The source is 120 V rms at 60 Hz. Use PSpice to determine the delay angle required to produce an average current of 1.8 A in the load. Use the default diode in the simulation.

3-30. A controlled half-wave rectifier has a 120 V, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 100$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. The delay angle is 50°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-31. A controlled half-wave rectifier has a 240 V rms 60 Hz ac source. The load is a series resistance, inductance, and dc source with $R = 100\ \Omega$, $L = 150$ mH, and $V_{dc} = 96$ V. The delay angle is 60°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-32. Use PSpice to determine the delay angle required such that the dc source in Prob. 3-31 absorbs 35 W.

3-33. A controlled half-wave rectifier has a series resistance, inductance, and dc voltage source with $R = 2\ \Omega$, $L = 75$ mH, and $V_{dc} = 48$ V. The source is 120 V rms at 60 Hz. The delay angle is 50°. Determine (*a*) an expression for load current, (*b*) the power absorbed by the dc voltage source, and (*c*) the power absorbed by the resistor.

3-34. Use PSpice to determine the delay angle required such that the dc source in Prob. 3-33 absorbs 50 W.

3-35. Develop an expression for current in a controlled half-wave rectifier circuit that has a load consisting of a series inductance L and dc voltage V_{dc}. The source is $v_s = V_m \sin \omega t$, and the delay angle is α. (*a*) Determine the average current if $V_m = 100$ V, $L = 35$ mH, $V_{dc} = 24$ V, $\omega = 2\pi 60$ rad/s, and $\alpha = 75°$. (*b*) Verify your result with PSpice.

3-36. A controlled half-wave rectifier has an RL load. A freewheeling diode is placed in parallel with the load. The inductance is large enough to consider the load current to be constant. Determine the load current as a function of the delay angle alpha. Sketch the current in the SCR and the freewheeling diode. Sketch the voltage across the load.

Commutation

3-37. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source that has an inductance of 1.5 mH. The load current is a constant 5 A. Determine the commutation angle and the average output voltage. Use PSpice to verify your results. Use ideal diodes in the simulation. Verify that the commutation angle for D_1 to D_2 is the same as for D_2 to D_1.

3-38. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source which has an inductance of 10 mH. The load is a series resistance-inductance with $R = 20\ \Omega$ and $L = 500$ mH. Use PSpice to determine (*a*) the steady-state average load current, (*b*) the average load voltage, and (*c*) the commutation angle. Use the default diode in the simulation. Comment on the results.

3-39. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source which has an inductance of 5 mH. The load is a series resistance-inductance with $R = 15\ \Omega$ and $L = 500$ mH. Use PSpice to determine (*a*) the steady-state average load current, (*b*) the average load voltage, and (*c*) the commutation angle. Use the default diode in the simulation.

3-40. The commutation angle given in Eq. (3-64) for the half-wave rectifier with a freewheeling diode was developed for commutation of load current from D_2 to D_1. Show that the commutation angle is the same for commutation from D_1 to D_2.

3-41. Diode D_1 in Fig. 3-18*a* is replaced with an SCR to make a controlled half-wave rectifier. Show that the angle for commutation from the diode to the SCR is

$$u = \cos^{-1}\left(\cos\alpha - \frac{I_L X_s}{V_m}\right) - \alpha$$

where α is the delay angle of the SCR.

Design Problems

3-42. A certain situation requires that either 160 or 75 W be supplied to a 48 V battery from a 120 V rms 60 Hz ac source. There is a two-position switch on a control panel set at either 160 or 75. Design a single circuit to deliver both values of power, and specify what the control switch will do. Specify the values of all the components in your circuit. The internal resistance of the battery is 0.1 Ω.

3-43. Design a circuit to produce an average current of 2 A in an inductance of 100 mH. The ac source available is 120 V rms at 60 Hz. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.

3-44. Design a circuit that will deliver 100 W to a 48 V dc source from a 120 V rms 60 Hz ac source. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.

3-45. Design a circuit which will deliver 150 W to a 100 V dc source from a 120 V rms 60 Hz ac source. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.

CHAPTER 4

Full-Wave Rectifiers

Converting ac to dc

4.1 INTRODUCTION

The objective of a full-wave rectifier is to produce a voltage or current that is purely dc or has some specified dc component. While the purpose of the full-wave rectifier is basically the same as that of the half-wave rectifier, full-wave rectifiers have some fundamental advantages. The average current in the ac source is zero in the full-wave rectifier, thus avoiding problems associated with nonzero average source currents, particularly in transformers. The output of the full-wave rectifier has inherently less ripple than the half-wave rectifier.

In this chapter, uncontrolled and controlled single-phase and three-phase full-wave converters used as rectifiers are analyzed for various types of loads. Also included are examples of controlled converters operating as inverters, where power flow is from the dc side to the ac side.

4.2 SINGLE-PHASE FULL-WAVE RECTIFIERS

The bridge rectifier and the center-tapped transformer rectifier of Figs. 4-1 and 4-2 are two basic single-phase full-wave rectifiers.

The Bridge Rectifier

For the bridge rectifier of Fig. 4-1, these are some basic observations:

1. Diodes D_1 and D_2 conduct together, and D_3 and D_4 conduct together. Kirchhoff's voltage law around the loop containing the source, D_1, and D_3

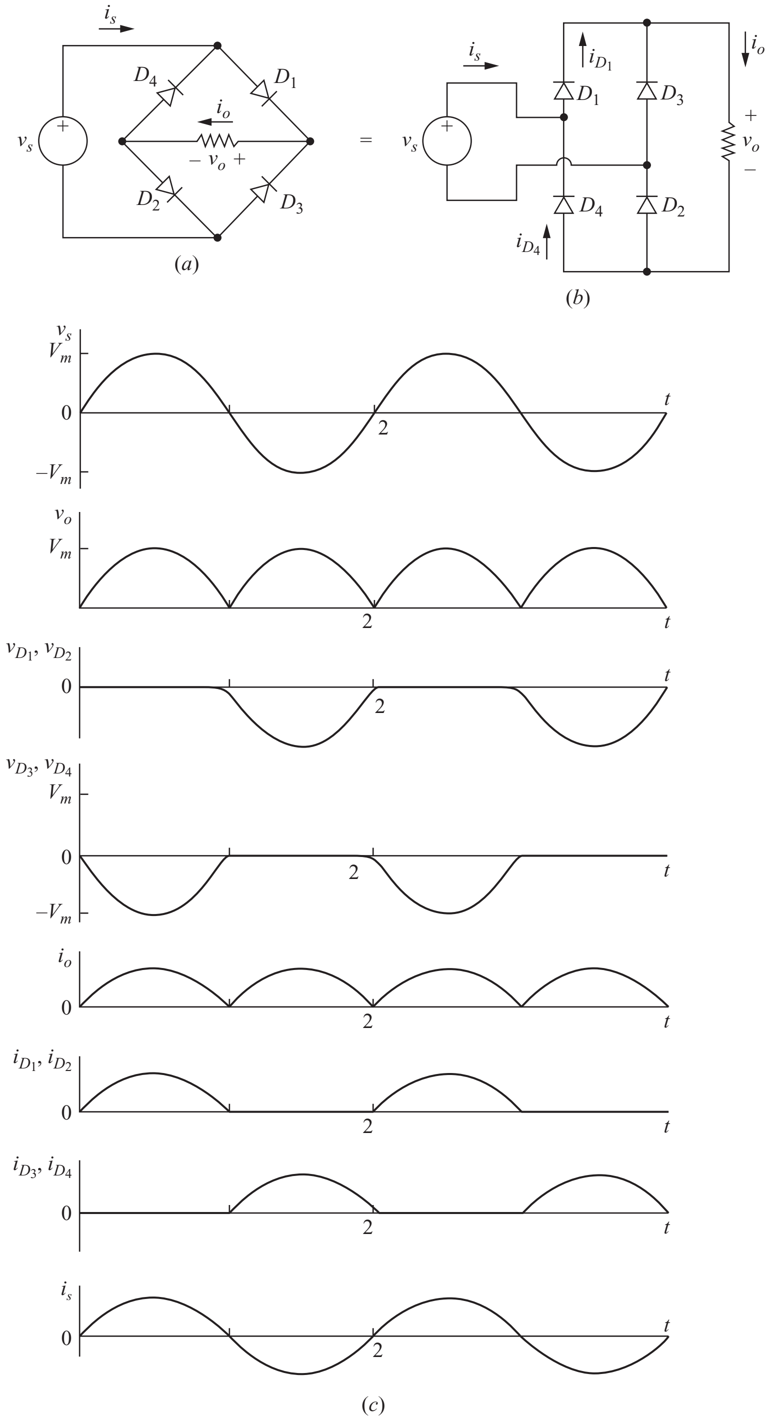


Figure 4-1 Full-wave bridge rectifier. (*a*) Circuit diagram. (*b*) Alternative representation. (*c*) Voltages and currents.

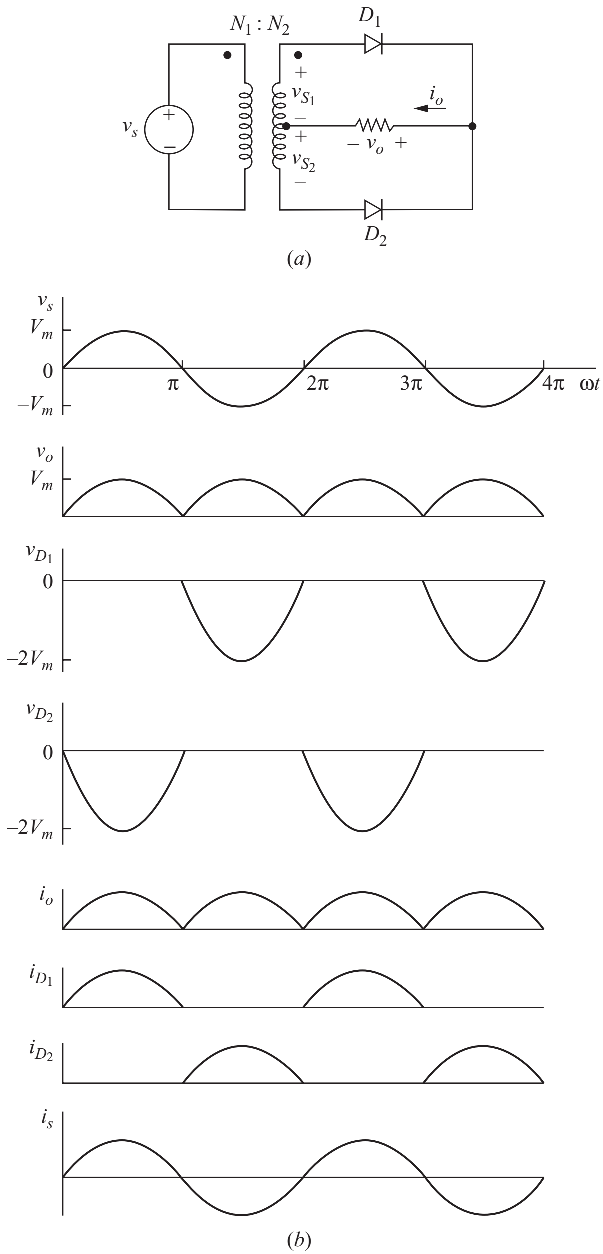


Figure 4-2 Full-wave center-tapped rectifier (*a*) circuit; (*b*) voltages and currents.

shows that D_1 and D_3 cannot be on at the same time. Similarly, D_2 and D_4 cannot conduct simultaneously. The load current can be positive or zero but can never be negative.

2. The voltage across the load is $+v_s$ when D_1 and D_2 are on. The voltage across the load is $-v_s$ when D_3 and D_4 are on.
3. The maximum voltage across a reverse-biased diode is the peak value of the source. This can be shown by Kirchhoff's voltage law around the loop containing the source, D_1, and D_3. With D_1 on, the voltage across D_3 is $-v_s$.
4. The current entering the bridge from the source is $i_{D_1} - i_{D_4}$, which is symmetric about zero. Therefore, the average source current is zero.
5. The rms source current is the same as the rms load current. The source current is the same as the load current for one-half of the source period and is the negative of the load current for the other half. The squares of the load and source currents are the same, so the rms currents are equal.
6. The fundamental frequency of the output voltage is 2ω, where ω is the frequency of the ac input since two periods of the output occur for every period of the input. The Fourier series of the output consists of a dc term and the even harmonics of the source frequency.

The Center-Tapped Transformer Rectifier

The voltage waveforms for a resistive load for the rectifier using the center-tapped transformer are shown in Fig. 4-2. Some basic observations for this circuit are as follows:

1. Kirchhoff's voltage law shows that only one diode can conduct at a time. Load current can be positive or zero but never negative.
2. The output voltage is $+v_{s_1}$ when D_1 conducts and is $-v_{s_2}$ when D_2 conducts. The transformer secondary voltages are related to the source voltage by $v_{s_1} = v_{s_2} = v_s(N_2/2N_1)$.
3. Kirchhoff's voltage law around the transformer secondary windings, D_1, and D_2 shows that the maximum voltage across a reverse-biased diode is *twice* the peak value of the load voltage.
4. Current in each half of the transformer secondary is reflected to the primary, resulting in an average source current of zero.
5. The transformer provides electrical isolation between the source and the load.
6. The fundamental frequency of the output voltage is 2ω since two periods of the output occur for every period of the input.

The lower peak diode voltage in the bridge rectifier makes it more suitable for high-voltage applications. The center-tapped transformer rectifier, in addition to including electrical isolation, has only one diode voltage drop between the source and load, making it desirable for low-voltage, high-current applications.

The following discussion focuses on the full-wave bridge rectifier but generally applies to the center-tapped circuit as well.

Resistive Load

The voltage across a resistive load for the bridge rectifier of Fig. 4-1 is expressed as

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } 0 \leq \omega t \leq \pi \\ -V_m \sin \omega t & \text{for } \pi \leq \omega t \leq 2\pi \end{cases} \tag{4-1}$$

The dc component of the output voltage is the average value, and load current is simply the resistor voltage divided by resistance.

$$\boxed{\begin{gathered} V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi} \\ I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \end{gathered}} \tag{4-2}$$

Power absorbed by the load resistor can be determined from $I_{\text{rms}}^2 R$, where I_{rms} for the full-wave rectified current waveform is the same as for an unrectified sine wave,

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \tag{4-3}$$

The source current for the full-wave rectifier with a resistive load is a sinusoid that is in phase with the voltage, so the power factor is 1.

RL Load

For an *RL* series-connected load (Fig. 4-3*a*), the method of analysis is similar to that for the half-wave rectifier with the freewheeling diode discussed in Chap. 3. After a transient that occurs during start-up, the load current i_o reaches a periodic steady-state condition similar to that in Fig. 4-3*b*.

For the bridge circuit, current is transferred from one pair of diodes to the other pair when the source changes polarity. The voltage across the *RL* load is a full-wave rectified sinusoid, as it was for the resistive load. The full-wave rectified sinusoidal voltage across the load can be expressed as a Fourier series consisting of a dc term and the even harmonics

$$v_o(t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

where (4-4)

$$V_o = \frac{2V_m}{\pi} \qquad \text{and} \qquad V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

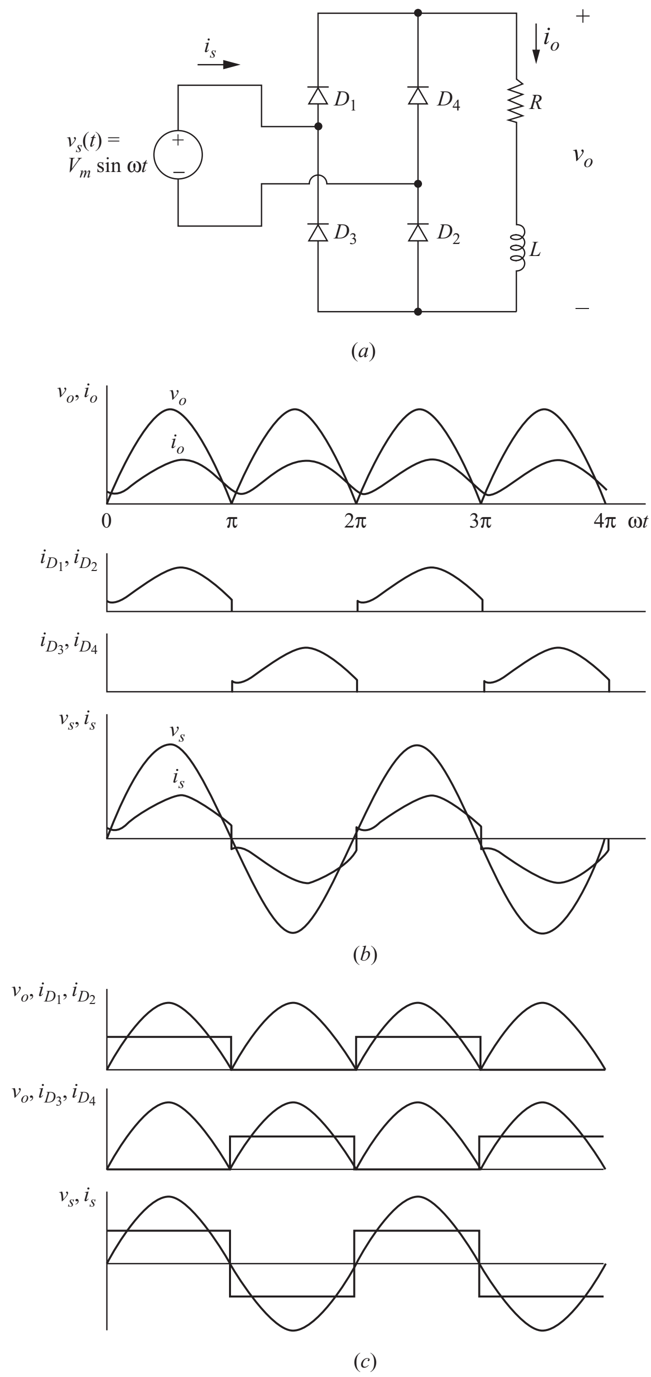


Figure 4-3 (*a*) Bridge rectifier with an *RL* load; (*b*) Voltages and currents; (*c*) Diode and source currents when the inductance is large and the current is nearly constant.

The current in the RL load is then computed using superposition, taking each frequency separately and combining the results. The dc current and current amplitude at each frequency are computed from

$$\boxed{\begin{aligned} I_0 &= \frac{V_0}{R} \\ I_n &= \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|} \end{aligned}} \tag{4-5}$$

Note that as the harmonic number n increases in Eq.(4-4), the voltage amplitude decreases. For an RL load, the impedance Z_n increases as n increases. The combination of decreasing V_n and increasing Z_n makes I_n decrease rapidly for increasing harmonic number. Therefore, the dc term and only a few, if any, of the ac terms are usually necessary to describe current in an RL load.

EXAMPLE 4-1

Full-Wave Rectifier with *RL* Load

The bridge rectifier circuit of Fig. 4-3*a* has an ac source with $V_m = 100$ V at 60 Hz and a series RL load with $R = 10\ \Omega$ and $L = 10$ mH. (*a*) Determine the average current in the load. (*b*) Estimate the peak-to-peak variation in load current based on the first ac term in the Fourier series. (*c*) Determine the power absorbed by the load and the power factor of the circuit. (*d*) Determine the average and rms currents in the diodes.

■ Solution

(*a*) The average load current is determined from the dc term in the Fourier series. The voltage across the load is a full-wave rectified sine wave that has the Fourier series determined from Eq. (4-4). Average output voltage is

$$V_0 = \frac{2V_m}{\pi} = \frac{2(200)}{\pi} = 63.7 \text{ V}$$

and average load current is

$$I_0 = \frac{V_0}{R} = \frac{63.7 \text{ V}}{10\ \Omega} = 6.37 \text{ A}$$

(*b*) Amplitudes of the ac voltage terms are determined from Eq. (4-4). For $n = 2$ and 4,

$$V_2 = \frac{2(100)}{\pi}\left(\frac{1}{1} - \frac{1}{3}\right) = 42.4 \text{ V}$$

$$V_4 = \frac{2(100)}{\pi}\left(\frac{1}{3} - \frac{1}{5}\right) = 8.49 \text{ V}$$

The amplitudes of first two ac current terms in the current Fourier series are computed from Eq. (4-5).

$$I_2 = \frac{42.4}{|10 + j(2)(377)(0.01)|} = \frac{42.4 \text{ V}}{12.5\ \Omega} = 3.39 \text{ A}$$

$$I_4 = \frac{8.49}{|10 + j(4)(377)(0.01)|} = \frac{8.49 \text{ V}}{18.1\ \Omega} = 0.47 \text{ A}$$

The current I_2 is much larger than I_4 and higher-order harmonics, so I_2 can be used to estimate the peak-to-peak variation in load current $\Delta i_o \approx 2(3.39) = 6.78$ A. Actual variation in i_o will be larger because of the higher-order terms.

(*c*) The power absorbed by the load is determined from I^2_{rms}. The rms current is then determined from Eq. (2-43) as

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\Sigma I^2_{n,\text{rms}}} \\ &= \sqrt{(6.37)^2 + \left(\frac{3.39}{\sqrt{2}}\right)^2 + \left(\frac{0.47}{\sqrt{2}}\right)^2 + \cdots} \approx 6.81 \text{ A} \end{aligned}$$

Adding more terms in the series would not be useful because they are small and have little effect on the result. Power in the load is

$$P = I^2_{\text{rms}} R = (6.81)^2(10) = 464 \text{ W}$$

The rms source current is the same as the rms load current. Power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{464}{\left(\frac{100}{\sqrt{2}}\right)(6.81)} = 0.964$$

(*d*) Each diode conducts for one-half of the time, so

$$I_{D,\text{avg}} = \frac{I_o}{2} = \frac{6.37}{2} = 3.19 \text{ A}$$

and

$$I_{D,\text{rms}} = \frac{I_{\text{rms}}}{\sqrt{2}} = \frac{6.81}{\sqrt{2}} = 4.82 \text{ A}$$

In some applications, the load inductance may be relatively large or made large by adding external inductance. If the inductive impedance for the ac terms in the Fourier series effectively eliminates the ac current terms in the load, the load current is essentially dc. If $\omega L \gg R$,

$$\begin{aligned} i(\omega t) &\approx I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \qquad \text{for } \omega L \gg R \\ I_{\text{rms}} &\approx I_o \end{aligned} \tag{4-6}$$

Load and source voltages and currents are shown in Fig. 4-3*c*.

Source Harmonics

Nonsinusoidal source current is a concern in power systems. Source currents like that of Fig. 4-3 have a fundamental frequency equal to that of the source but are rich in the odd-numbered harmonics. Measures such as total harmonic distortion (THD) and distortion factor (DF) as presented in Chap. 2 describe the nonsinusoidal property of the source current. Where harmonics are of concern, filters can be added to the input of the rectifier.

PSpice Simulation

A PSpice simulation will give the output voltage, current, and power for full-wave rectifier circuits. Fourier analysis from the FOUR command or from Probe will give the harmonic content of voltages and currents in the load and source. The default diode model will give results that differ from the analytical results that assume an ideal diode. For the full-wave rectifier, two diodes will conduct at a time, resulting in two diode voltage drops. In some applications, the reduced voltage at the output may be significant. Since voltage drops across the diodes exist in real circuits, PSpice results are a better indicator of circuit performance than results that assume ideal diodes. (To simulate an ideal circuit in PSpice, a diode model with $n = 0.001$ will produce forward voltage drops in the microvolt range, approximating an ideal diode.)

EXAMPLE 4-2

PSpice Simulation of a Full-Wave Rectifier

For the full-wave bridge rectifier in Example 4-1, obtain the rms current and power absorbed by the load from a PSpice simulation.

■ **Solution**

The PSpice circuit for Fig. 4-3 is created using VSIN for the source, Dbreak for the diodes, and R and L for the load. A transient analysis is performed using a run time of 50 ms and data saved after 33.33 ms to obtain steady-state current.

The Probe output is used to determine the operating characteristics of the rectifier using the same techniques as presented in Chaps. 2 and 3. To obtain the average value of the load current, enter AVG(I(R1)). Using the cursor to identify the point at the end of the resulting trace, the average current is approximately 6.07 A. The Probe output is shown in Fig. 4-4.

Entering RMS(I(R1)) shows that the rms current is approximately 6.52 A. Power absorbed by the resistor can be computed from $I^2_{\text{rms}}R$, or average power in the load can be computed directly from Probe by entering AVG(W(R1)), which yields 425.4 W. This is significantly less than the 464 W obtained in Example 4-1 when assuming ideal diodes.

The power supplied by the ac source is computed from AVG(W(V1)) as 444.6 W. When ideal diodes were assumed, power supplied by the ac source was identical to the power absorbed by the load, but this analysis reveals that power absorbed by the diodes in the bridge is $444.6 - 425.4 = 19.2$ W. Another way to determine power absorbed by the bridge is to enter AVG(W(D1)) to obtain the power absorbed by diode D_1, which is 4.8 W. Total power for the diodes is 4 times 4.8, or 19.2 W. Better models for power diodes would yield a more accurate estimate of power dissipation in the diodes.

Comparing the results of the simulation to the results based on ideal diodes shows how more realistic diode models reduce the current and power in the load.

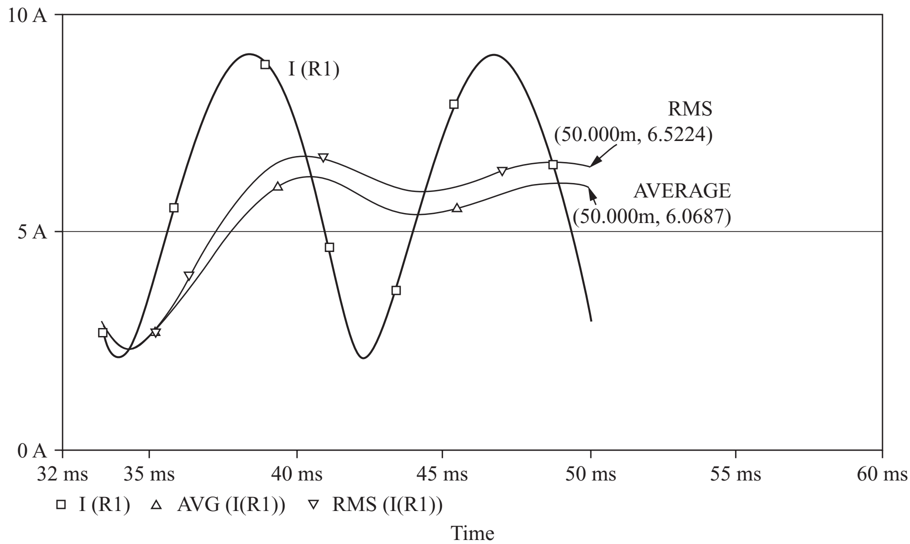


Figure 4-4 PSpice output for Example 4-2.

RL-Source Load

Another general industrial load may be modeled as a series resistance, inductance, and a dc voltage source, as shown in Fig. 4-5*a*. A dc motor drive circuit and a battery charger are applications for this model. There are two possible modes of operation for this circuit, the continuous-current mode and the discontinuous-current mode. In the continuous-current mode, the load current is always positive for steady-state operation (Fig. 4-5*b*). Discontinuous load current is characterized by current returning to zero during every period (Fig. 4-5*c*).

For continuous-current operation, one pair of diodes is always conducting, and the voltage across the load is a full-wave rectified sine wave. The only modification to the analysis that was done for an *RL* load is in the dc term of the Fourier series. The dc (average) component of current in this circuit is

$$\boxed{I_o = \frac{V_o - V_{dc}}{R} = \frac{\dfrac{2V_m}{\pi} - V_{dc}}{R}} \tag{4-7}$$

The sinusoidal terms in the Fourier analysis are unchanged by the dc source provided that the current is continuous.

Discontinuous current is analyzed like the half-wave rectifier of Sec. 3.5. The load voltage is not a full-wave rectified sine wave for this case, so the Fourier series of Eq. (4-4) does not apply.

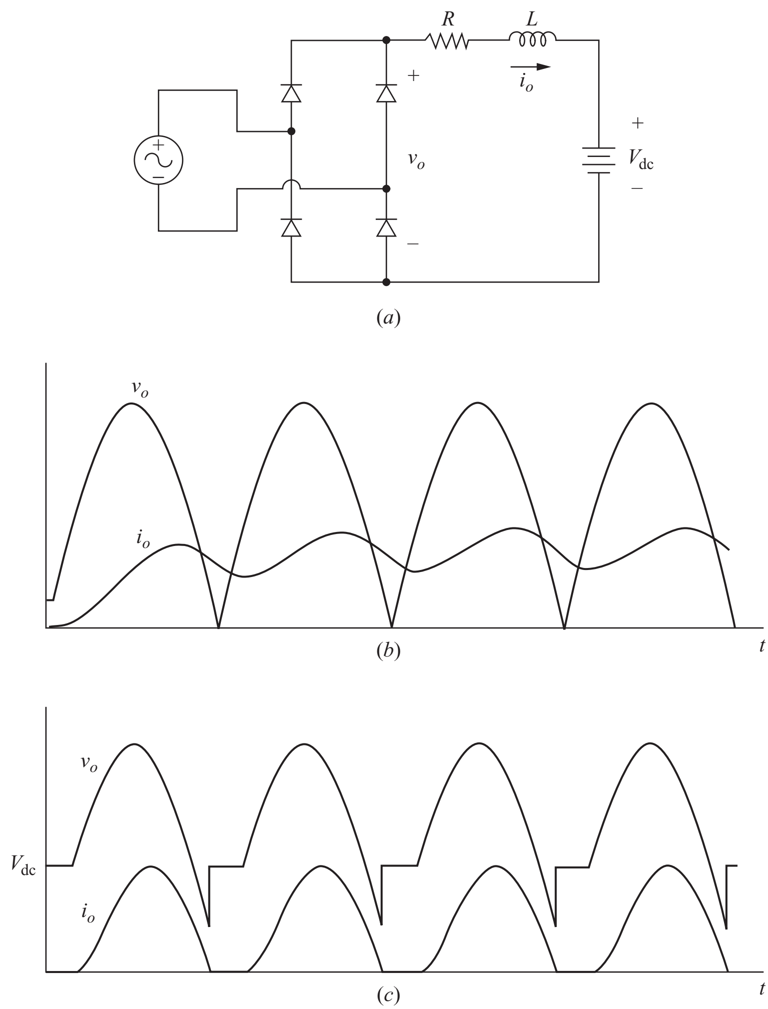


Figure 4-5 (*a*) Rectifier with *RL*-source load; (*b*) Continuous current: when the circuit is energized, the load current reaches the steady-state after a few periods; (*c*) Discontinuous current: the load current returns to zero during every period.

EXAMPLE 4-3

Full-Wave Rectifier with *RL*-Source Load—Continuous Current

For the full-wave bridge rectifier circuit of Fig. 4-5*a*, the ac source is 120 V rms at 60 Hz, $R = 2\ \Omega$, $L = 10$ mH, and $V_{dc} = 80$ V. Determine the power absorbed by the dc voltage source and the power absorbed by the load resistor.

■ **Solution**

For continuous current, the voltage across the load is a full-wave rectified sine wave which has the Fourier series given by Eq. (4-4). Equation (4-7) is used to compute the average current, which is used to compute power absorbed by the dc source,

$$I_0 = \frac{\dfrac{2V_m}{\pi} - V_{dc}}{R} = \frac{\dfrac{2\sqrt{2}(120)}{\pi} - 80}{2} = 14.0 \text{ A}$$

$$P_{dc} = I_0 V_{dc} = (14)(80) = 1120 \text{ W}$$

The first few terms of the Fourier series using Eqs. (4-4) and (4-5) are shown in Table 4-1.

Table 4-1 Fourier series components

n	V_n	Z_n	I_n
0	108	2.0	14.0
2	72.0	7.80	9.23
4	14.4	15.2	0.90

The rms current is computed from Eq. (2-43).

$$I_{rms} = \sqrt{14^2 + \left(\frac{9.23}{\sqrt{2}}\right)^2 + \left(\frac{0.90}{\sqrt{2}}\right)^2 + \cdots} \approx 15.46 \text{ A}$$

Power absorbed by the resistor is

$$P_R = I_{rms}^2 R = (15.46)^2(2) = 478 \text{ W}$$

PSpice Solution

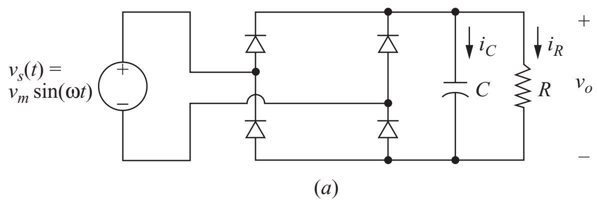
PSpice simulation of the circuit of Fig 4-5*a* using the default diode model yields these results from Probe:

Quantity	Expression Entered	Result
I_o	AVG(I(R1))	11.9 A
I_{rms}	RMS(I(R1))	13.6 A
P_{ac}	AVG(W(Vs))	1383 W
P_{D1}	AVG(W(D1))	14.6 W
P_{dc}	AVG(W(VDC))	955 W
P_R	AVG(W(R))	370 W

Note that the simulation verifies the assumption of continuous load current.

Capacitance Output Filter

Placing a large capacitor in parallel with a resistive load can produce an output voltage that is essentially dc (Fig. 4-6). The analysis is very much like that of the half-wave rectifier with a capacitance filter in Chap. 3. In the full-wave circuit, the time that the capacitor discharges is smaller than that for the half-wave circuit



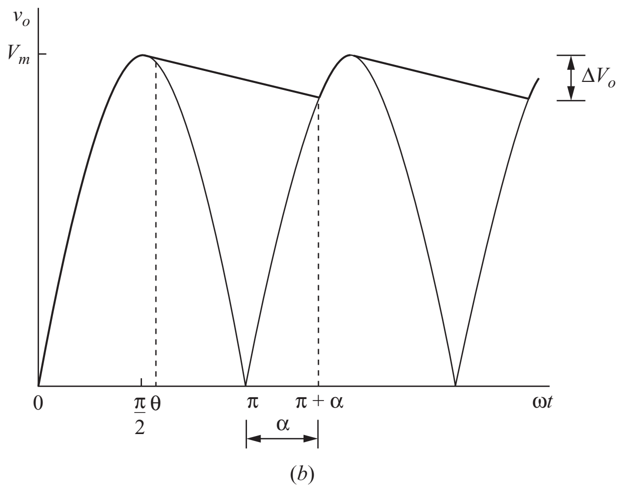


Figure 4-6 (*a*) Full-wave rectifier with capacitance filter; (*b*) Source and output voltage.

because of the rectified sine wave in the second half of each period. The output voltage ripple for the full-wave rectifier is approximately one-half that of the half-wave rectifier. The peak output voltage will be less in the full-wave circuit because there are two diode voltage drops rather than one.

The analysis proceeds exactly as for the half-wave rectifier. The output voltage is a positive sine function when one of the diode pairs is conducting and is a decaying exponential otherwise. Assuming ideal diodes,

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta)e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases} \tag{4-8}$$

where θ is the angle where the diodes become reverse biased, which is the same as that for the half-wave rectifier and is found using Eq. (3-41).

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi \tag{4-9}$$

The maximum output voltage is V_m, and the minimum output voltage is determined by evaluating v_o at the angle at which the second pair of diodes turns on, which is at $\omega t = \pi + \alpha$. At that boundary point,

$$(V_m \sin \theta)e^{-(\pi + \alpha - \theta)/\omega RC} = -V_m \sin(\pi + \alpha)$$

or

$$(\sin\theta)e^{-(\pi+\alpha-\theta)/\omega RC} - \sin\alpha = 0 \tag{4-10}$$

which must be solved numerically for α.

The peak-to-peak voltage variation, or ripple, is the difference between maximum and minimum voltages.

$$\boxed{\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin\alpha)} \tag{4-11}$$

This is the same as Eq. (3-49) for voltage variation in the half-wave rectifier, but α is larger for the full-wave rectifier and the ripple is smaller for a given load. Capacitor current is described by the same equations as for the half-wave rectifier.

In practical circuits where $\omega RC \gg \pi$.

$$\theta \approx \pi/2 \qquad \alpha \approx \pi/2 \tag{4-12}$$

The minimum output voltage is then approximated from Eq. (4-9) for the diodes off evaluated at $\omega t = \pi$.

$$v_o(\pi + \alpha) = V_m e^{-(\pi+\pi/2-\pi/2)/\omega RC} = V_m e^{-\pi/\omega RC}$$

The ripple voltage for the full-wave rectifier with a capacitor filter can then be approximated as

$$\Delta V_o \approx V_m(1 - e^{-\pi/\omega RC})$$

Furthermore, the exponential in the above equation can be approximated by the series expansion

$$e^{-\pi/\omega RC} \approx 1 - \frac{\pi}{\omega RC}$$

Substituting for the exponential in the approximation, the peak-to-peak ripple is

$$\boxed{\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}} \tag{4-13}$$

Note that the approximate peak-to-peak ripple voltage for the full-wave rectifier is one-half that of the half-wave rectifier from Eq. (3-51). As for the half-wave rectifier, the peak diode current is much larger than the average diode current and Eq. (3-48) applies. The average source current is zero.

EXAMPLE 4-4

Full-Wave Rectifier with Capacitance Filter

The full-wave rectifier of Fig. 4-6*a* has a 120 V source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu$F. (*a*) Determine the peak-to-peak voltage variation of the output. (*b*) Determine the value of capacitance that would reduce the output voltage ripple to 1percent of the dc value.

■ **Solution**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85$$

The angle θ is determined from Eq. (4-9).

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

The angle α is determined by the numerical solution of Eq. (4-10).

$$\sin(1.62)e^{-(\pi+\alpha-1.62)/18.85} - \sin\alpha = 0$$

$$\alpha = 1.06 \text{ rad} = 60.6°$$

(*a*) Peak-to-peak output voltage is described by Eq. (4-11).

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7[1 - \sin(1.06)] = 22 \text{ V}$$

Note that this is the same load and source as for the half-wave rectifier of Example 3-9 where $\Delta V_o = 43$ V.

(*b*) With the ripple limited to 1 percent, the output voltage will be held close to V_m and the approximation of Eq. (4-13) applies.

$$\frac{\Delta V_o}{V_m} = 0.01 \approx \frac{1}{2fRC}$$

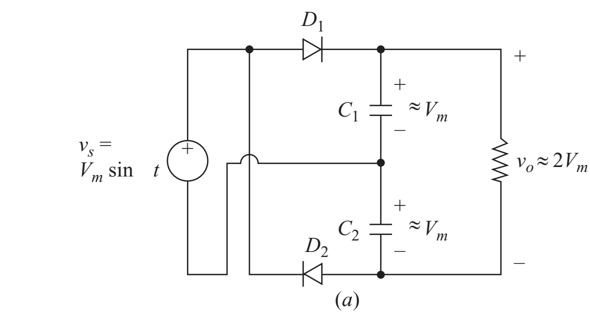
Solving for C,

$$C \approx \frac{1}{2fR(\Delta V_o/V_m)} = \frac{1}{(2)(60)(500)(0.01)} = 1670 \ \mu\text{F}$$

Voltage Doublers

The rectifier circuit of Fig. 4-7*a* serves as a simple voltage doubler, having an output of twice the peak value of the source. For ideal diodes, C_1 charges to V_m through D_1 when the source is positive; C_2 charges to V_m through D_2 when the source is negative. The voltage across the load resistor is the sum of the capacitor voltages $2V_m$. This circuit is useful when the output voltage of a rectifier must be larger than the peak input voltage. Voltage doubler circuits avoid using a transformer to step up the voltage, saving expense, volume, and weight.

The full-wave rectifier with a capacitive output filter can be combined with the voltage doubler, as shown in Fig. 4-7*b*. When the switch is open, the circuit is similar to the full-wave rectifier of Fig. 4-6*a*, with output at approximately V_m when the capacitors are large. When the switch is closed, the circuit acts as the voltage doubler of Fig. 4-7*a*. Capacitor C_1 charges to V_m through D_1 when



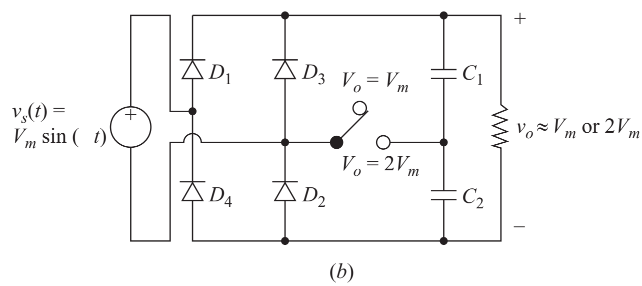


Figure 4-7 (*a*) Voltage doubler. (*b*) Dual-voltage rectifier.

the source is positive, and C_2 charges to V_m through D_4 when the source is negative. The output voltage is then $2V_m$. Diodes D_2 and D_3 remain reverse-biased in this mode.

This voltage doubler circuit is useful when equipment must be used on systems with different voltage standards. For example, a circuit could be designed to operate properly in both the United States, where the line voltage is 120 V, and places abroad where the line voltage is 240 V.

LC Filtered Output

Another full-wave rectifier configuration has an *LC* filter on the output, as shown in Fig. 4-8*a*. The purpose of the filter is to produce an output voltage that is close to purely dc. The capacitor holds the output voltage at a constant level, and the inductor smooths the current from the rectifier and reduces the peak current in the diodes from that of the current of Fig. 4-6*a*.

The circuit can operate in the continuous- or discontinuous-current mode. For continuous current, the inductor current is always positive, as illustrated in Fig. 4-8*b*. Discontinuous current is characterized by the inductor current returning to zero in each cycle, as illustrated in Fig. 4-8*c*. The continuous-current case is easier to analyze and is considered first.

Continuous Current for *LC* Filtered Output For continuous current, the voltage v_x in Fig. 4-8*a* is a full-wave rectified sine wave, which has an average

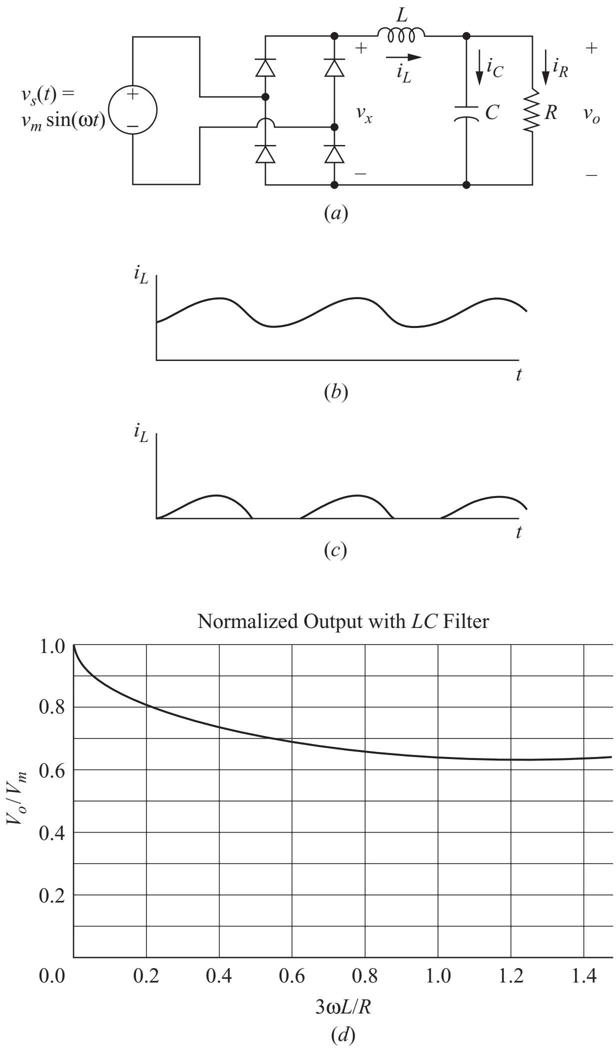


Figure 4-8 (*a*) Rectifier with *LC* filtered output; (*b*) Continuous inductor current; (*c*) Discontinuous inductor current; (*d*) Normalized output.

value of $2V_m/\pi$. Since the average voltage across the inductor in the steady state is zero, the average output voltage for continuous inductor current is

$$V_o = \frac{2V_m}{\pi} \tag{4-14}$$

Average inductor current must equal the average resistor current because the average capacitor current is zero.

$$I_L = I_R = \frac{V_o}{R} = \frac{2V_m}{\pi R} \tag{4-15}$$

The variation in inductor current can be estimated from the first ac term in the Fourier series. The first ac voltage term is obtained from Eq. (4-4) with $n = 2$. Assuming the capacitor to be a short circuit to ac terms, the harmonic voltage v_2 exists across the inductor. The amplitude of the inductor current for $n = 2$ is

$$I_2 = \frac{V_2}{Z_2} \approx \frac{V_2}{2\omega L} = \frac{4V_m/3\pi}{2\omega L} = \frac{2V_m}{3\pi\omega L} \tag{4-16}$$

For the current to always be positive, the amplitude of the ac term must be less than the dc term (average value). Using the above equations and solving for L,

$$I_2 < I_L$$
$$\frac{2V_m}{3\pi\omega L} < \frac{2V_m}{\pi R}$$
$$L > \frac{R}{3\omega}$$

or

$$\frac{3\omega L}{R} > 1 \qquad \text{for continuous current} \tag{4-17}$$

If $3\omega L/R > 1$, the current is continuous and the output voltage is $2V_m/\pi$. Otherwise, the output voltage must be determined from analysis for discontinuous current, discussed as follows.

Discontinuous Current for *LC* Filtered Output For discontinuous inductor current, the current reaches zero during each period of the current waveform (Fig. 4-8*c*). Current becomes positive again when the bridge output voltage reaches the level of the capacitor voltage, which is at $\omega t = \alpha$.

$$\alpha = \sin^{-1}\left(\frac{V_o}{V_m}\right) \tag{4-18}$$

While current is positive, the voltage across the inductor is

$$v_L = V_m \sin(\omega t) - V_o \tag{4-19}$$

where the output voltage V_o is yet to be determined. Inductor current is expressed as

$$i_L(\omega t) = \frac{1}{\omega L}\int_{\alpha}^{\omega t} \left[V_m \sin(\omega t) - \mathrm{V_o}\right] d(\omega t)$$

$$= \frac{1}{\omega L}\left[V_m\left(\cos\alpha - \cos\omega t\right)\right] - V_o(\omega t - \alpha) \tag{4-20}$$

$$\text{for } \alpha \le \omega t \le \beta \qquad \text{when } \beta < \pi$$

which is valid until the current reaches zero, at $\omega t = \beta$.

The solution for the load voltage V_o is based on the fact that the average inductor current must equal the current in the load resistor. Unfortunately, a closed-form solution is not available, and an iterative technique is required.

A procedure for determining V_o is as follows:

1. Estimate a value for V_o slightly below V_m and solve for α in Eq. (4-18).
2. Solve for β numerically in Eq. (4-20) for inductor current,

$$i_L(\beta) = 0 = V_m(\cos\alpha - \cos\beta) - V_o(\beta - \alpha)$$

3. Solve for average inductor current I_L.

$$\begin{aligned} i_L &= \frac{1}{\pi}\int_{\alpha}^{\beta} i_L(\omega t)\, d(\omega t) \\ &= \frac{1}{\pi}\int_{\alpha}^{\beta} \frac{1}{\omega L}\left[V_m(\cos\alpha - \cos\omega t) - V_o(\omega t - \alpha)\right] d(\omega t) \end{aligned} \tag{4-21}$$

4. Solve for load voltage V_o based upon the average inductor current from step 3.

$$I_R = I_L = \frac{V_o}{R}$$

or

$$V_o = I_L R \tag{4-22}$$

5. Repeat steps 1 to 4 until the computed value of V_o in step 4 equals the estimated V_o in step 1.

Output voltage for discontinuous current is larger than for continuous current. If there is no load, the capacitor charges to the peak value of the source so the maximum output is V_m. Figure 4-8*d* shows normalized output V_o/V_m as a function of $3\omega L/R$.

EXAMPLE 4-5

Full-Wave Rectifier with *LC* Filter

A full-wave rectifier has a source of $v_s(t) = 100 \sin(377t)$ V. An *LC* filter as in Fig. 4-8*a* is used, with $L = 5$ mH and $C = 10{,}000$ μF. The load resistance is (*a*) 5 Ω and (*b*) 50 Ω. Determine the output voltage for each case.

■ **Solution**

Using Eq. (4-17), continuous inductor current exists when

$$R < 3\omega L = 3(377)(0.005) = 5.7\ \Omega$$

which indicates continuous current for 5 Ω and discontinuous current for 50 Ω.

(*a*) For $R = 5\ \Omega$ with continuous current, output voltage is determined from Eq. (4-14).

$$V_o = \frac{2V_m}{\pi} = \frac{2(100)}{\pi} = 63.7\ \text{V}$$

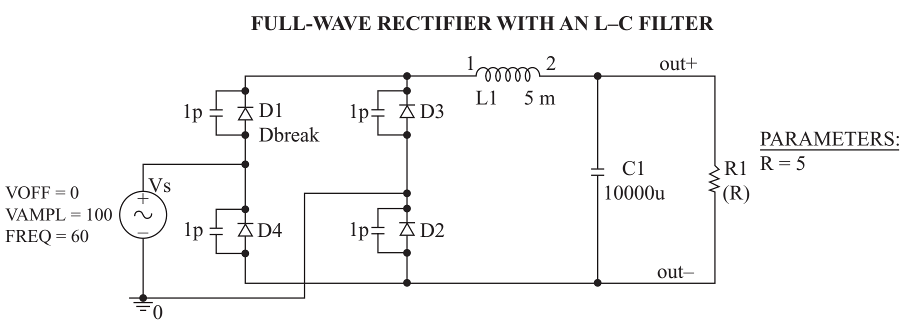
(*b*) For $R = 50\ \Omega$ with discontinuous current, the iteration method is used to determine V_o. Initially, V_o is estimated to be 90 V. The results of the iteration are as follows:

Estimated V_o	α	β	Calculated V_o	
90	1.12	2.48	38.8	(Estimate is too high)
80	0.93	2.89	159	(Estimate is too low)
85	1.12	2.70	88.2	(Estimate is slightly low)
86	1.04	2.66	76.6	(Estimate is too high)
85.3	1.02	2.69	84.6	(Approximate solution)

Therefore, V_o is approximately 85.3 V. As a practical matter, three significant figures for the load voltage may not be justified when predicting performance of a real circuit. Knowing that the output voltage is slightly above 85 V after the third iteration is probably sufficient. Output could also be estimated from the graph of Fig. 4-8*d*.

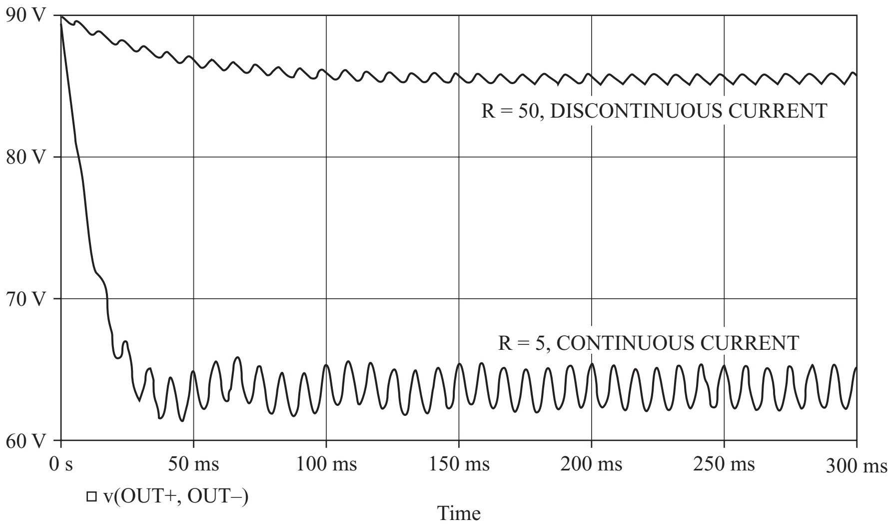
PSpice Solution

The circuit is created using VSIN for the source and Dbreak for the diodes, with the diode model modified to represent an ideal diode by using $n = 0.01$. The voltage of the filter capacitor is initialized at 90 V, and small capacitors are placed across the diodes to avoid convergence problems. Both values of R are tested in one simulation by using a parametric sweep. The transient analysis must be sufficiently long to allow a steady-state periodic output to be observed. The Probe output for both load resistors is shown in Fig. 4-9. Average output voltage for each case is obtained from Probe by entering AVG(V(out+)−V(out−)) after restricting the data to represent steady-state output (after about 250 ms), resulting in $V_o = 63.6$ V for $R = 5\ \Omega$ (continuous current) and $V_o = 84.1$ V for $R = 50\ \Omega$ (discontinuous current). These values match very well with those of the analytical solution.



(*a*)

Figure 4-9 PSpice output for Example 4-6. (*a*) Full-wave rectifier with an *LC* filter. The small capacitors across the diodes help with convergence; (*b*) The output voltage for continuous and discontinuous inductor current.



(*b*)

Figure 4-9 (*continued*)

4.3 CONTROLLED FULL-WAVE RECTIFIERS

A versatile method of controlling the output of a full-wave rectifier is to substitute controlled switches such as thyristors (SCRs) for the diodes. Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range.

Controlled full-wave rectifiers are shown in Fig. 4-10. For the bridge rectifier, SCRs S_1 and S_2 will become forward-biased when the source becomes positive but will not conduct until gate signals are applied. Similarly, S_3 and S_4 will become forward-biased when the source becomes negative but will not conduct until they receive gate signals. For the center-tapped transformer rectifier, S_1 is forward-biased when v_s is positive, and S_2 is forward-biased when v_s is negative, but each will not conduct until it receives a gate signal.

The delay angle α is the angle interval between the forward biasing of the SCR and the gate signal application. If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes. The discussion that follows generally applies to both bridge and center-tapped rectifiers.

Resistive Load

The output voltage waveform for a controlled full-wave rectifier with a resistive load is shown in Fig. 4-10*c*. The average component of this waveform is determined from

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1 + \cos\alpha)} \tag{4-23}$$

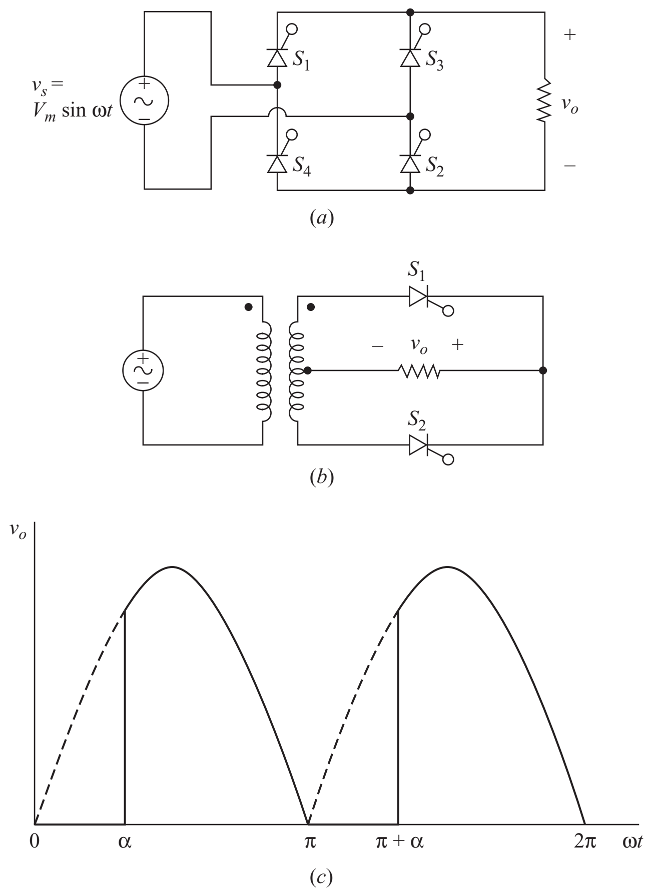


Figure 4-10 (*a*) Controlled full-wave bridge rectifier; (*b*) Controlled full-wave center-tapped transformer rectifier; (*c*) Output for a resistive load.

Average output current is then

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos \alpha) \tag{4-24}$$

The power delivered to the load is a function of the input voltage, the delay angle, and the load components; $P = I_{\text{rms}}^2 R$ is used to determine the power in a resistive load, where

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin \omega t\right)^2 d(\omega t)} \\ &= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}} \end{aligned} \tag{4-25}$$

The rms current in the source is the same as the rms current in the load.

EXAMPLE 4-6

Controlled Full-Wave Rectifier with Resistive Load

The full-wave controlled bridge rectifier of Fig. 4-10*a* has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

■ Solution

The average output voltage is determined from Eq. (4-23).

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current from Eq. (4-24), remembering to use α in radians.

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2\,(20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

RL Load, Discontinuous Current

Load current for a controlled full-wave rectifier with an *RL* load (Fig. 4-11*a*) can be either continuous or discontinuous, and a separate analysis is required for each. Starting the analysis at $\omega t = 0$ with zero load current, SCRs S_1 and S_2 in the bridge rectifier will be forward-biased and S_3 and S_4 will be reverse-biased as the source voltage becomes positive. Gate signals are applied to S_1 and S_2 at $\omega t = \alpha$, turning S_1 and S_2 on. With S_1 and S_2 on, the load voltage is equal to the source voltage. For this condition, the circuit is identical to that of the controlled half-wave rectifier of Chap. 3, having a current function

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

where (4-26)

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

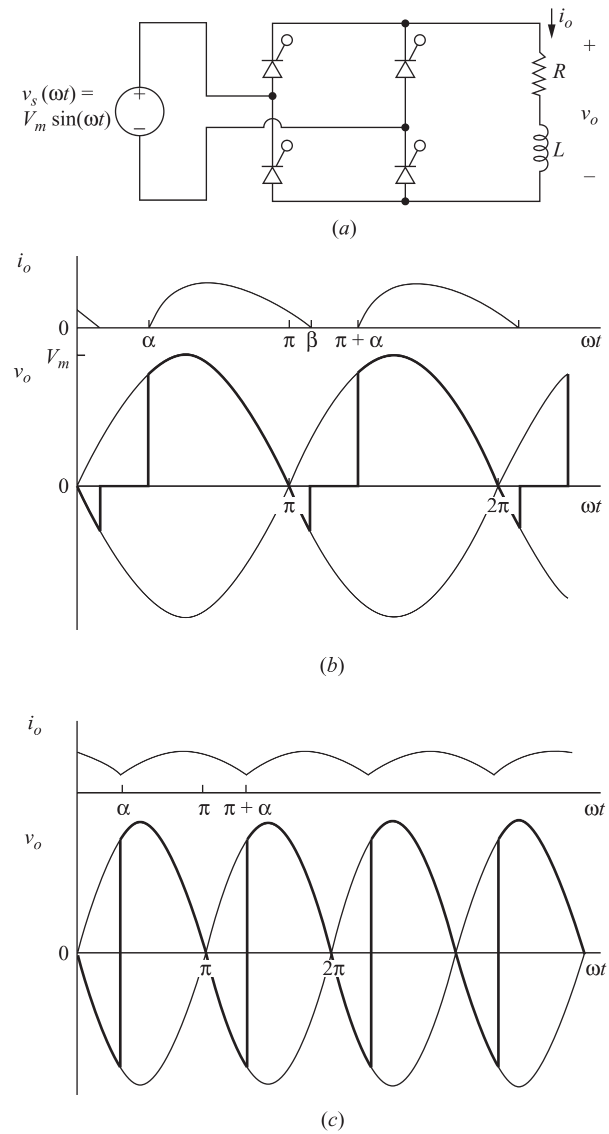


Figure 4-11 (*a*) Controlled rectifier with *RL* load; (*b*) Discontinuous current; (*c*) Continuous current.

The above current function becomes zero at $\omega t = \beta$. If $\beta < \pi + \alpha$, the current remains at zero until $\omega t = \pi + \alpha$ when gate signals are applied to S_3 and S_4 which are then forward-biased and begin to conduct. This mode of operation is called *discontinuous current*, which is illustrated in Fig. 4-11*b*.

$$\beta < \alpha + \pi \quad \rightarrow \quad \text{discontinuous current} \tag{4-27}$$

Analysis of the controlled full-wave rectifier operating in the discontinuous-current mode is identical to that of the controlled half-wave rectifier except that the period for the output current is π rather than 2π rad.

EXAMPLE 4-7

Controlled Full-Wave Rectifier, Discontinuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, $R = 10\ \Omega$, $L = 20$ mH, and $\alpha = 60°$. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

■ Solution

From the parameters given,

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

(*a*) Substituting into Eq. (4-26),

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

(*b*) Average load current is determined from the numerical integration of

$$I_o = \frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) = 7.05 \text{ A}$$

(*c*) Power absorbed by the load occurs in the resistor and is computed from $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)} = 8.35 \text{ A}$$

$$P = (8.35)^2(10) = 697 \text{ W}$$

RL Load, Continuous Current

If the load current is still positive at $\omega t = \pi + \alpha$ when gate signals are applied to S_3 and S_4 in the above analysis, S_3 and S_4 are turned on and S_1 and S_2 are forced off. Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat. Equation (4-26) is not valid in the steady state for continuous current. For an *RL* load with continuous current, the steady-state current and voltage waveforms are generally as shown in Fig. 4-11*c*.

The boundary between continuous and discontinuous current occurs when β for Eq. (4-26) is $\pi + \alpha$. The current at $\omega t = \pi + \alpha$ must be greater than zero for continuous-current operation.

$$i(\pi + \alpha) \geq 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi + \alpha - \alpha)/\omega\tau} \geq 0$$

Using

$$\sin(\pi + \alpha - \theta) = \sin(\theta - \alpha)$$

$$\sin(\theta - \alpha)\left(1 - e^{-(\pi/\omega\tau)}\right) \geq 0$$

Solving for α,

$$\alpha \leq \theta$$

Using

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{for continuous current} \tag{4-28}$$

Either Eq. (4-27) or Eq. (4-28) can be used to check whether the load current is continuous or discontinuous.

A method for determining the output voltage and current for the continuous-current case is to use the Fourier series. The Fourier series for the voltage waveform for continuous-current case shown in Fig. 4-11*c* is expressed in general form as

$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n) \tag{4-29}$$

The dc (average) value is

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha} \tag{4-30}$$

The amplitudes of the ac terms are calculated from

$$V_n = \sqrt{a_n^2 + b_n^2} \tag{4-31}$$

where

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \tag{4-32}$$

$$n = 2, 4, 6, \ldots$$

Figure 4-12 shows the relationship between normalized harmonic content of the output voltage and delay angle.

The Fourier series for current is determined by superposition as was done for the uncontrolled rectifier earlier in this chapter. The current amplitude at each frequency is determined from Eq. (4-5). The rms current is determined by combining the rms currents at each frequency. From Eq. (2-43),

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}$$

where

$$I_o = \frac{V_o}{R} \qquad \text{and} \qquad I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_0 L|} \tag{4-33}$$

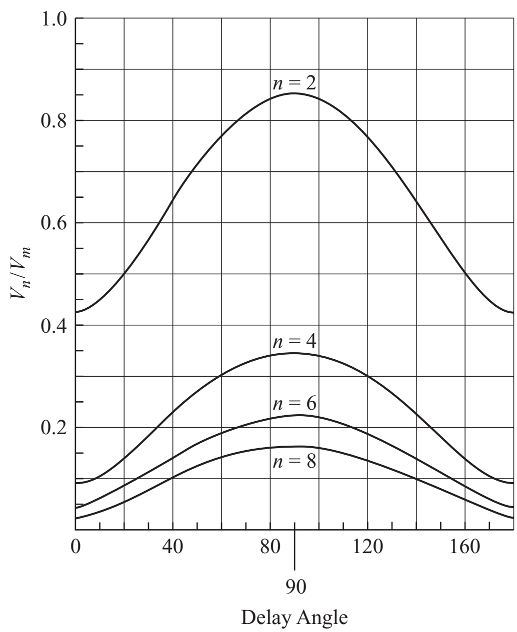


Figure 4-12 Output harmonic voltages as a function of delay angle for a single-phase controlled rectifier.

As the harmonic number increases, the impedance for the inductance increases. Therefore, it may be necessary to solve for only a few terms of the series to be able to calculate the rms current. If the inductor is large, the ac terms will become small, and the current is essentially dc.

EXAMPLE 4-8

Controlled Full-Wave Rectifier with *RL* Load, Continuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, an *RL* load where $R = 10\ \Omega$ and $L = 100$ mH. The delay angle $\alpha = 60°$ (same as Example 4-7 except L is larger). (*a*) Verify that the load current is continuous. (*b*) Determine the dc (average) component of the current. (*c*) Determine the power absorbed by the load.

■ Solution

(*a*) Equation (4-28) is used to verify that the current is continuous.

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

(*b*) The voltage across the load is expressed in terms of the Fourier series of Eq. (4-29). The dc term is computed from Eq. (4-30).

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60°) = 54.0\text{ V}$$

(*c*) The amplitudes of the ac terms are computed from Eqs. (4-31) and (4-32) and are summarized in the following table where, $Z_n = |R + j\omega L|$ and $I_n = V_n/Z_n$.

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

The rms current is computed from Eq. (4-33).

$$I_{\text{rms}} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54\text{ A}$$

Power is computed from $I_{\text{rms}}^2 R$.

$$P = (5.54)^2(10) = 307\text{ W}$$

Note that the rms current could be approximated accurately from the dc term and one ac term ($n = 2$). Higher-frequency terms are very small and contribute little to the power in the load.

PSpice Simulation of Controlled Full-Wave Rectifiers

To simulate the controlled full-wave rectifier in PSpice, a suitable SCR model must be chosen. As with the controlled half-wave rectifier of Chap. 3, a simple switch and diode can be used to represent the SCR, as shown in Fig. 4-13*a*. This circuit requires the full version of PSpice.

EXAMPLE 4-9

PSpice Simulation of a Controlled Full-Wave Rectifier

Use PSpice to determine the solution of the controlled full-wave rectifier in Example 4-8.

■ Solution

A PSpice circuit that uses the controlled-switch model for the SCRs is shown in Fig. 4-13*a*. (This circuit is too large for the demo version and requires the full production version of PSpice.)

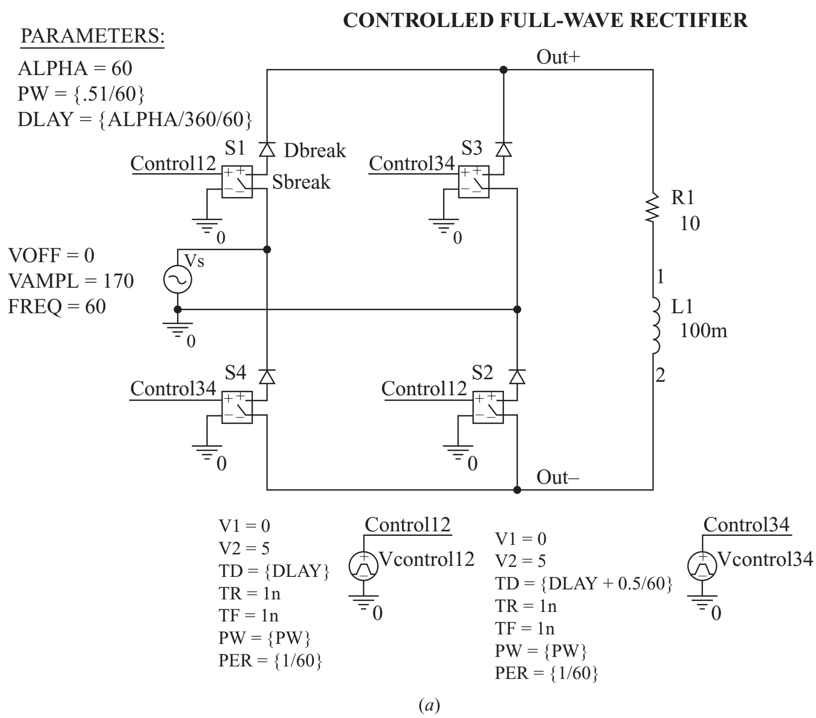


Figure 4-13 (*a*) PSpice circuit for a controlled full-wave rectifier of Example 4-8; (*b*) Probe output showing load voltage and current.

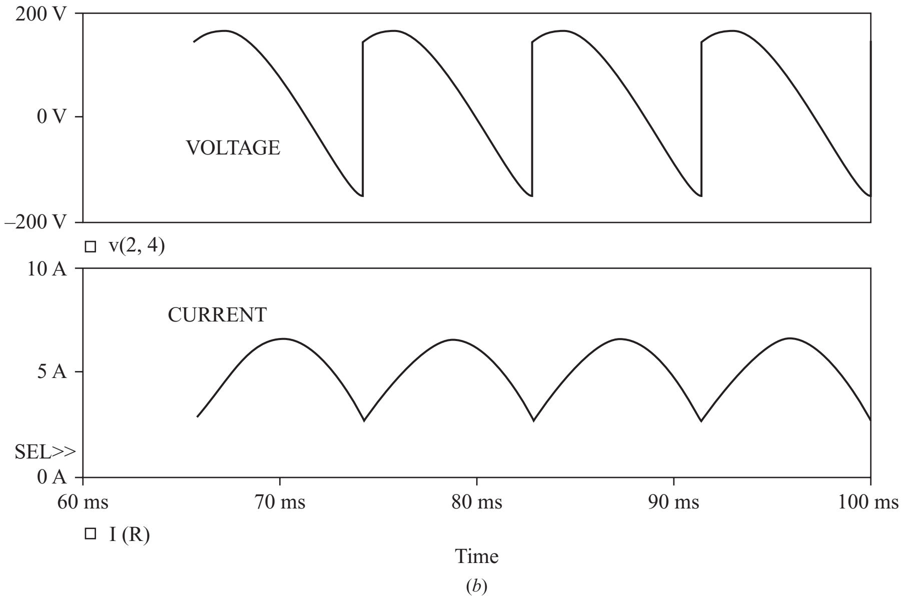


Figure 4-13 (*continued*)

Controlled Rectifier with *RL*-Source Load

The controlled rectifier with a load that is a series resistance, inductance, and dc voltage (Fig. 4-14) is analyzed much like the uncontrolled rectifier of Fig. 4-5*a* discussed earlier in this chapter. For the controlled rectifier, the SCRs may be turned on at any time that they are forward-biased, which is at an angle

$$\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \tag{4-34}$$

For the continuous-current case, the bridge output voltage is the same as in Fig. 4-11*c*. The average bridge output voltage is

$$V_o = \frac{2V_m}{\pi}\cos\alpha \tag{4-35}$$

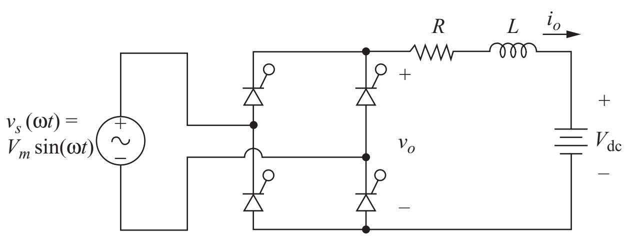


Figure 4-14 Controlled rectifier with *RL*-source load.

The average load current is

$$I_o = \frac{V_o - V_{dc}}{R} \tag{4-36}$$

The ac voltage terms are unchanged from the controlled rectifier with an *RL* load in Fig. 4-11*a* and are described by Eqs. (4-29) to (4-32). The ac current terms are determined from the circuit of Fig. 4-14*c*. Power absorbed by the dc voltage is

$$P_{dc} = I_o V_{dc} \tag{4-37}$$

Power absorbed by the resistor in the load is $I_{rms}^2 R$. If the inductance is large and the load current has little ripple, power absorbed by the resistor is approximately $I_o^2 R$.

EXAMPLE 4-10

Controlled Rectifier with *RL*-Source Load

The controlled rectifier of Fig. 4-14 has an ac source of 240 V rms at 60 Hz, $V_{dc} = 100$ V, $R = 5\ \Omega$, and an inductor large enough to cause continuous current. (*a*) Determine the delay angle α such that the power absorbed by the dc source is 1000 W. (*b*) Determine the value of inductance that will limit the peak-to-peak load current variation to 2 A.

■ Solution

(*a*) For the power in the 100-V dc source to be 1000 W, the current in it must be 10 A. The required output voltage is determined from Eq. (4-36) as

$$V_o = V_{dc} + I_o R = 100 + (10)(5) = 150 \text{ V}$$

The delay angle which will produce a 150 V dc output from the rectifier is determined from Eq. (4-35).

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{(150)(\pi)}{2\sqrt{2}(240)}\right] = 46°$$

(*b*) Variation in load current is due to the ac terms in the Fourier series. The load current amplitude for each of the ac terms is

$$I_n = \frac{V_n}{Z_n}$$

where V_n is described by Eqs. (4-31) and (4-32) or can be estimated from the graph of Fig. 4-12. The impedance for the ac terms is

$$Z_n = |R + jn\omega_0 L|$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to diminishing ac currents as n increases, the peak-to-peak current variation will be estimated from the first ac term. For $n = 2$, V_n/V_m is estimated from Fig. 4-12 as 0.68 for $\alpha = 46°$, making $V_2 = 0.68V_m = 0.68\ (240\sqrt{2}) = 230$ V. The peak-to-peak variation of 2 A corresponds to a 1-A zero-to-peak amplitude. The required load impedance for $n = 2$ is then

$$Z_2 = \frac{V_2}{I_2} = \frac{230 \text{ V}}{1 \text{ A}} = 230\ \Omega$$

The 5-Ω resistor is insignificant compared to the total 230-Ω required impedance, so $Z_n \approx n\omega L$. Solving for L,

$$L \approx \frac{Z_2}{2\omega} = \frac{230}{2(377)} = 0.31 \text{ H}$$

A slightly larger inductance should be chosen to allow for the effect of higher-order ac terms.

Controlled Single-Phase Converter Operating as an Inverter

The above discussion focused on circuits operating as rectifiers, which means that the power flow is from the ac source to the load. It is also possible for power to flow from the load to the ac source, which classifies the circuit as an inverter.

For inverter operation of the converter in Fig. 4-14, power is supplied by the dc source, and power is absorbed by the bridge and is transferred to the ac system. The load current must be in the direction shown because of the SCRs in the bridge. For power to be supplied by the dc source, V_{dc} must be negative. For power to be absorbed by the bridge and transferred to the ac system, the bridge output voltage V_o must also be negative. Equation (4-35) applies, so a delay angle larger than 90° will result in a negative output voltage.

$$\begin{aligned} 0 < \alpha < 90^\circ &\rightarrow V_o > 0 \qquad \text{rectifier operation} \\ 90^\circ < \alpha < 180^\circ &\rightarrow V_o < 0 \qquad \text{inverter operation} \end{aligned} \tag{4-38}$$

The voltage waveform for $\alpha = 150^\circ$ and continuous inductor current is shown in Fig. 4-15. Equations (4-36) to (4-38) apply. If the inductor is large enough to

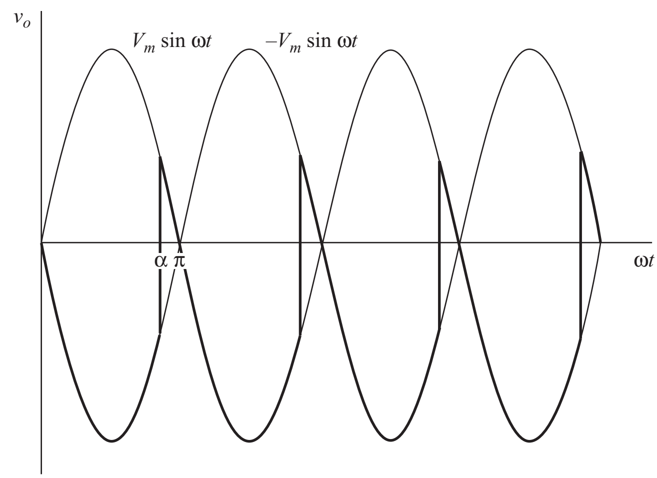


Figure 4-15 Output voltage for the controlled single-phase converter of Fig. 4-14 operating as an inverter, $\alpha = 150^\circ$ and $V_{dc} < 0$.

effectively eliminate the ac current terms and the bridge is lossless, the power absorbed by the bridge and transferred to the ac system is

$$P_{\text{bridge}} = P_{\text{ac}} = -I_o V_o \tag{4-39}$$

EXAMPLE 4-11

Single-Phase Bridge Operating as an Inverter

The dc voltage in Fig. 4-14 represents the voltage generated by an array of solar cells and has a value of 110 V, connected such that $V_{\text{dc}} = -110$ V. The solar cells are capable of producing 1000 W. The ac source is 120 V rms, $R = 0.5\ \Omega$, and L is large enough to cause the load current to be essentially dc. Determine the delay angle α such that 1000 W is supplied by the solar cell array. Determine the power transferred to the ac system and the losses in the resistance. Assume ideal SCRs.

■ Solution

For the solar cell array to supply 1000 W, the average current must be

$$I_o = \frac{P_{\text{dc}}}{V_{\text{dc}}} = \frac{1000}{110} = 9.09\ \text{A}$$

The average output voltage of the bridge is determined from Eq. (4-36).

$$V_o = I_o R + V_{\text{dc}} = (9.09)(0.5) + (-110) = -105.5\ \text{V}$$

The required delay angle is determined from Eq. (4-35).

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{-105.5\pi}{2\sqrt{2}(120)}\right] = 165.5°$$

Power absorbed by the bridge and transferred to the ac system is determined from Eq. (4-39).

$$P_{\text{ac}} = -V_o I_o = (-9.09)(-105.5) = 959\ \text{W}$$

Power absorbed by the resistor is

$$P_R = I_{\text{rms}}^2 R \approx I_o^2 R = (9.09)^2(0.5) = 41\ \text{W}$$

Note that the load current and power will be sensitive to the delay angle and the voltage drops across the SCRs because bridge output voltage is close to the dc source voltage. For example, assume that the voltage across a conducting SCR is 1 V. Two SCRs conduct at all times, so the average bridge output voltage is reduced to

$$V_o = -105.5 - 2 = -107.5\ \text{V}$$

Average load current is then

$$I_o = \frac{-107.5 - (-110)}{0.5} = 5.0\ \text{A}$$

Power delivered to the bridge is then reduced to

$$P_{\text{bridge}} = (107.5)(5.0) = 537.5 \text{ W}$$

Average current in each SCR is one-half the average load current. Power absorbed by each SCR is approximately

$$P_{\text{SCR}} = I_{\text{SCR}}V_{\text{SCR}} = \frac{1}{2}I_oV_{\text{SCR}} = \frac{1}{2}(5)(1) = 2.5 \text{ W}$$

Total power loss in the bridge is then $4(2.5) = 10$ W, and power delivered to the ac source is $537.5 - 10 = 527.5$ W.

4.4 THREE-PHASE RECTIFIERS

Three-phase rectifiers are commonly used in industry to produce a dc voltage and current for large loads. The three-phase full-bridge rectifier is shown in Fig. 4-16*a*. The three-phase voltage source is balanced and has phase sequence *a-b-c*. The source and the diodes are assumed to be ideal in the initial analysis of the circuit.

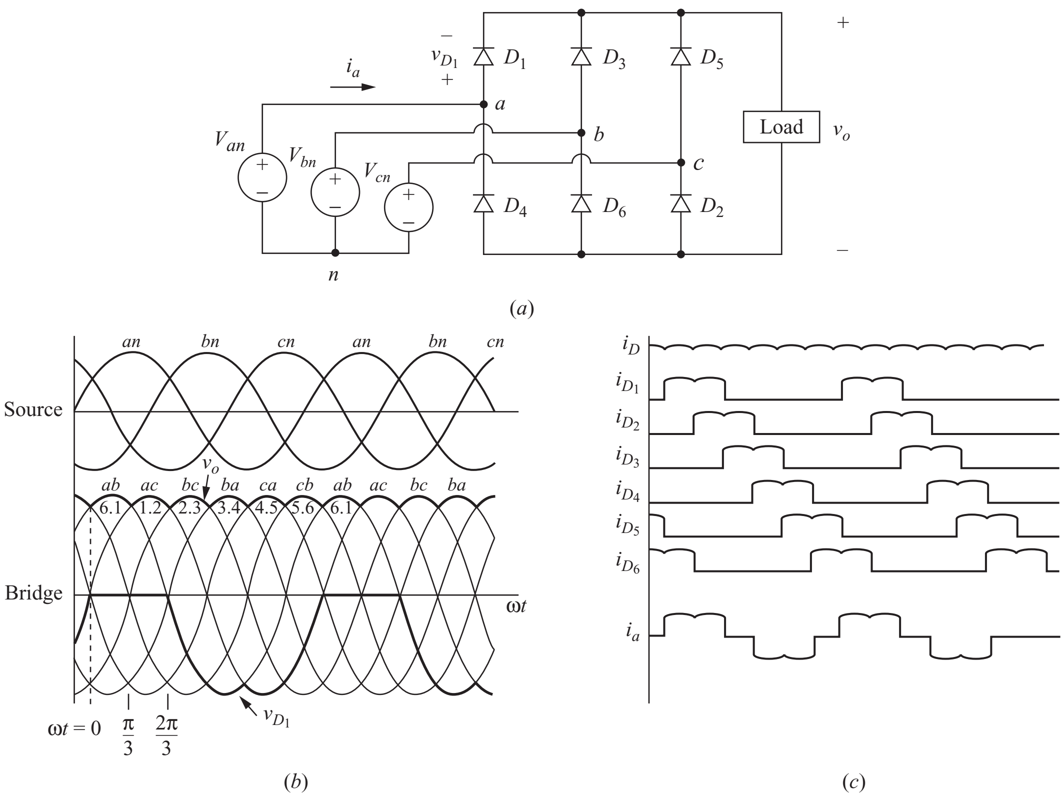


Figure 4-16 (*a*) Three-phase full-bridge rectifier; (*b*) Source and output voltages; (*c*) Currents for a resistive load.

Some basic observations about the circuit are as follows:

1. Kirchhoff's voltage law around any path shows that only one diode in the top half of the bridge may conduct at one time (D_1, D_3, or D_5). The diode that is conducting will have its anode connected to the phase voltage that is highest at that instant.
2. Kirchhoff's voltage law also shows that only one diode in the bottom half of the bridge may conduct at one time (D_2, D_4, or D_6). The diode that is conducting will have its cathode connected to the phase voltage that is lowest at that instant.
3. As a consequence of items 1 and 2 above, D_1 and D_4 cannot conduct at the same time. Similarly, D_3 and D_6 cannot conduct simultaneously, nor can D_5 and D_2.
4. The output voltage across the load is one of the line-to-line voltages of the source. For example, when D_1 and D_2 are on, the output voltage is v_{ac}. Furthermore, the diodes that are on are determined by which line-to-line voltage is the highest at that instant. For example, when v_{ac} is the highest line-to-line voltage, the output is v_{ac}.
5. There are six combinations of line-to-line voltages (three phases taken two at a time). Considering one period of the source to be 360°, a transition of the highest line-to-line voltage must take place every 360°/6 = 60°. Because of the six transitions that occur for each period of the source voltage, the circuit is called a *six-pulse rectifier*.
6. The fundamental frequency of the output voltage is 6ω, where ω is the frequency of the three-phase source.

Figure 4-16*b* shows the phase voltages and the resulting combinations of line-to-line voltages from a balanced three-phase source. The current in each of the bridge diodes for a resistive load is shown in Fig. 4-16*c*. The diodes conduct in pairs (6,1), (1,2), (2,3), (3,4), (4,5), (5,6), (6,1), ... Diodes turn on in the sequence 1, 2, 3, 4, 5, 6, 1, ...

The current in a conducting diode is the same as the load current. To determine the current in each phase of the source, Kirchhoff's current law is applied at nodes *a, b,* and *c,*

$$\begin{aligned} i_a &= i_{D_1} - i_{D_4} \\ i_b &= i_{D_3} - i_{D_6} \\ i_c &= i_{D_5} - i_{D_2} \end{aligned} \tag{4-40}$$

Since each diode conducts one-third of the time, resulting in

$$\boxed{\begin{aligned} I_{D,\text{avg}} &= \frac{1}{3} I_{o,\text{avg}} \\ I_{D,\text{rms}} &= \frac{1}{\sqrt{3}} I_{o,\text{rms}} \\ I_{s,\text{rms}} &= \sqrt{\frac{2}{3}}\, I_{o,\text{rms}} \end{aligned}} \tag{4-41}$$

The apparent power from the three-phase source is

$$S = \sqrt{3}\, V_{L-L,\text{rms}}\, I_{S,\text{rms}} \tag{4-42}$$

The maximum reverse voltage across a diode is the peak line-to-line voltage. The voltage waveform across diode D_1 is shown in Fig. 4-16*b*. When D_1 conducts, the voltage across it is zero. When D_1 is off, the output voltage is v_{ab} when D_3 is on and is v_{ac} when D_5 is on.

The periodic output voltage is defined as $v_o(\omega t) = V_{m,L-L}\sin(\omega t)$ for $\pi/3 \le \omega t \le 2\pi/3$ with period $\pi/3$ for the purpose of determining the Fourier series coefficients. The coefficients for the sine terms are zero from symmetry, enabling the Fourier series for the output voltage to be expressed as

$$v_o(t) = V_o + \sum_{n=6,12,18\,\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi) \tag{4-43}$$

The average or dc value of the output voltage is

$$\boxed{V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{m,L-L}\sin(\omega t)\,d(\omega t) = \frac{3V_{m,L-L}}{\pi} = 0.955\,V_{m,L-L}} \tag{4-44}$$

where $V_{m,L-L}$ is the peak line-to-line voltage of the three-phase source, which is $\sqrt{2}V_{L-L,\text{rms}}$. The amplitudes of the ac voltage terms are

$$V_n = \frac{6V_{m,L-L}}{\pi(n^2-1)} \qquad n = 6, 12, 18, \ldots \tag{4-45}$$

Since the output voltage is periodic with period one-sixth of the ac supply voltage, the harmonics in the output are of order $6k\omega$, $k = 1, 2, 3 \ldots$ An advantage of the three-phase rectifier over the single-phase rectifier is that the output is inherently like a dc voltage, and the high-frequency low-amplitude harmonics enable filters to be effective.

In many applications, a load with series inductance results in a load current that is essentially dc. For a dc load current, the diode and ac line currents are shown in Fig. 4-17. The Fourier series of the currents in phase *a* of the ac line is

$$i_a(t) = \frac{2\sqrt{3}}{\pi}I_o\left(\cos\omega_0 t - \frac{1}{5}\cos 5\omega_0 t + \frac{1}{7}\cos 7\omega_0 t - \frac{1}{11}\cos 11\omega_0 t + \frac{1}{13}\cos 13\omega_0 t - \cdots\right) \tag{4-46}$$

which consists of terms at the fundamental frequency of the ac system and harmonics of order $6k \pm 1$, $k = 1, 2, 3, \ldots$.

Because these harmonic currents may present problems in the ac system, filters are frequently necessary to prevent these harmonics from entering the ac system. A typical filtering scheme is shown in Fig. 4-18. Resonant filters are used to provide a path to ground for the fifth and seventh harmonics, which are the two lowest and are the strongest in amplitude. Higher-order harmonics are reduced with the high-pass filter. These filters prevent the harmonic currents from propagating through the ac power system. Filter components are chosen such that the impedance to the power system frequency is large.

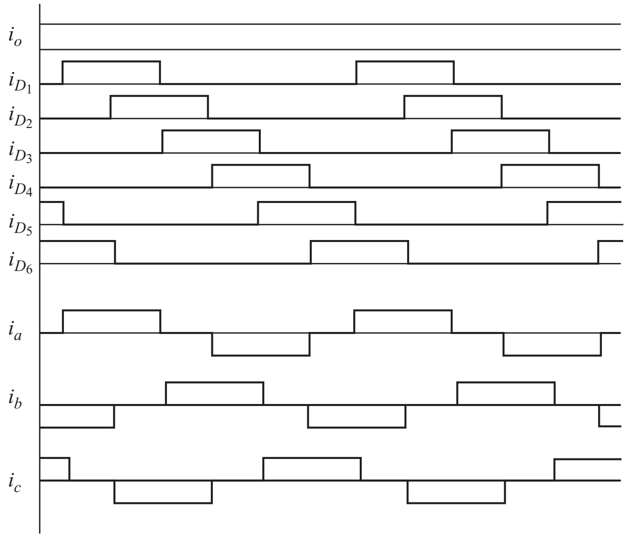


Figure 4-17 Three-phase rectifier currents when the output is filtered.

EXAMPLE 4-12

Three-Phase Rectifier

The three-phase rectifier of Fig. 4-16*a* has a three-phase source of 480 V rms line-to-line, and the load is a 25-Ω resistance in series with a 50-mH inductance. Determine (*a*) the dc level of the output voltage, (*b*) the dc and first ac term of the load current, (*c*) the average and rms current in the diodes, (*d*) the rms current in the source, and (*e*) the apparent power from the source.

■ Solution

(*a*) The dc output voltage of the bridge is obtained from Eq. (4-44).

$$V_o = \frac{3V_{m,L-L}}{\pi} = \frac{3\sqrt{2}\,(480)}{\pi} = 648 \text{ V}$$

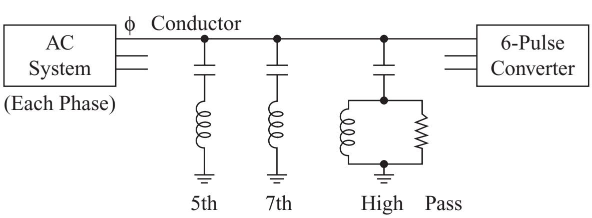


Figure 4-18 Filters for ac line harmonics.

(*b*) The average load current is

$$I_o = \frac{V_o}{R} = \frac{648}{25} = 25.9 \text{ A}$$

The first ac voltage term is obtained from Eq. (4-45) with $n = 6$, and current is

$$I_6 = \frac{V_6}{Z_6} = \frac{0.0546V_m}{\sqrt{R^2 + (6\omega L)^2}} = \frac{0.0546\sqrt{2}(480)}{\sqrt{25^2 + [6(377)(0.05)]^2}} = \frac{37.0 \text{ V}}{115.8 \ \Omega} = 0.32 \text{ A}$$

$$I_{6,\text{rms}} = \frac{0.32}{\sqrt{2}} = 0.23 \text{ A}$$

This and other ac terms are much smaller than the dc term and can be neglected.

(*c*) Average and rms diode currents are obtained from Eq. (4-41). The rms load current is approximately the same as average current since the ac terms are small.

$$I_{D,\text{avg}} = \frac{I_o}{3} = \frac{25.9}{3} = 8.63 \text{ A}$$

$$I_{D,\text{rms}} = \frac{I_{o,\text{rms}}}{\sqrt{3}} \approx \frac{25.9}{\sqrt{3}} = 15.0 \text{ A}$$

(*d*) The rms source current is also obtained from Eq. (4-41).

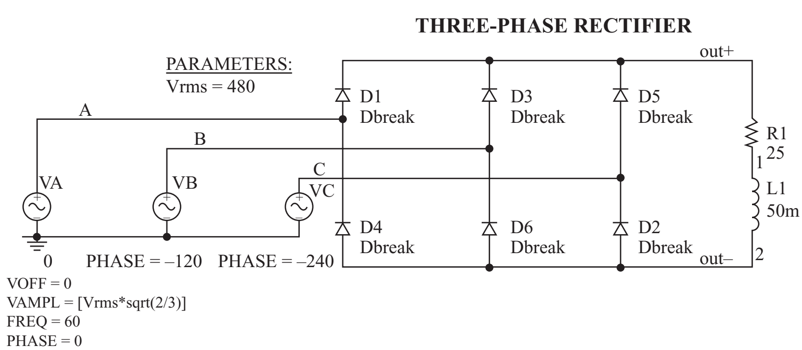
$$I_{s,\text{rms}} = \left(\sqrt{\frac{2}{3}}\right) I_{o,\text{rms}} \approx \left(\sqrt{\frac{2}{3}}\right) 25.9 = 21.2 \text{ A}$$

(*e*) The apparent power from the source is determined from Eq. (4-42).

$$S = \sqrt{3}(V_{L-L,\text{rms}})(I_{s,\text{rms}}) = \sqrt{3}\,(480)(21.2) = 17.6 \text{ kVA}$$

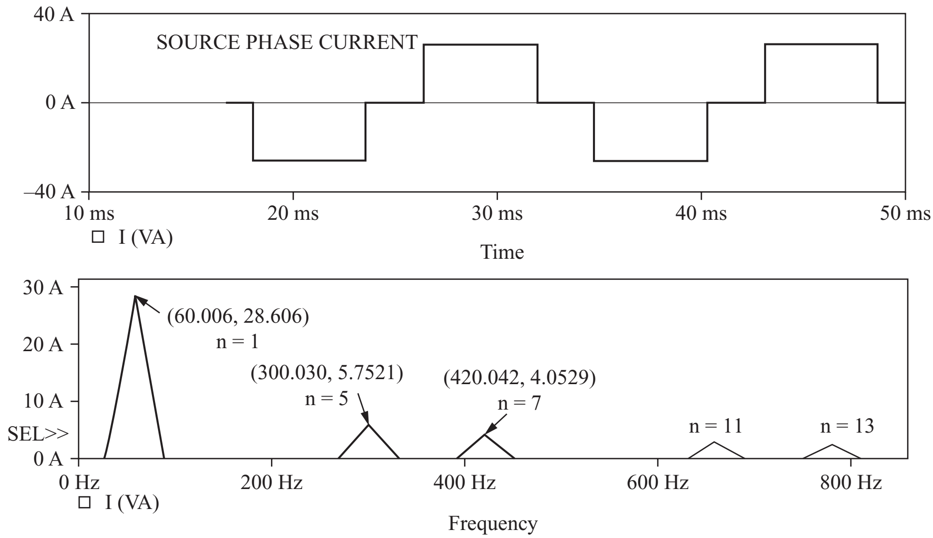
PSpice Solution

A circuit for this example is shown in Fig. 4-19*a*. VSIN is used for each of the sources. Dbreak, with the model changed to make $n = 0.01$, approximates an ideal diode. A transient analysis starting at 16.67 ms and ending at 50 ms represents steady-state currents.



(*a*)

Figure 4-19 (*a*) PSpice circuit for a three-phase rectifier; (*b*) Probe output showing the current waveform and the Fourier analysis in one phase of the source.



(*b*)

Figure 4-19 (*continued*)

All the circuit currents as calculated above can be verified. The Probe output in Fig. 4-19*b* shows the current and Fourier (FFT) components in one of the sources. Note that the harmonics correspond to those in Eq. (4-46).

4.5 CONTROLLED THREE-PHASE RECTIFIERS

The output of the three-phase rectifier can be controlled by substituting SCRs for diodes. Figure 4-20*a* shows a controlled six-pulse three-phase rectifier. With SCRs, conduction does not begin until a gate signal is applied while the SCR is forward-biased. Thus, the transition of the output voltage to the maximum instantaneous line-to-line source voltage can be delayed. The delay angle α is referenced from where the SCR would begin to conduct if it were a diode. The delay angle is the interval between when the SCR becomes forward-biased and when the gate signal is applied. Figure 4-20*b* shows the output of the controlled rectifier for a delay angle of 45°.

The average output voltage is

$$\boxed{V_o = \frac{1}{\pi/3}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_{m,L-L}\sin(\omega t)\, d(\omega t) = \frac{3V_{m,L-L}}{\pi}\cos\alpha} \tag{4-47}$$

Equation (4-47) shows that the average output voltage is reduced as the delay angle α increases.

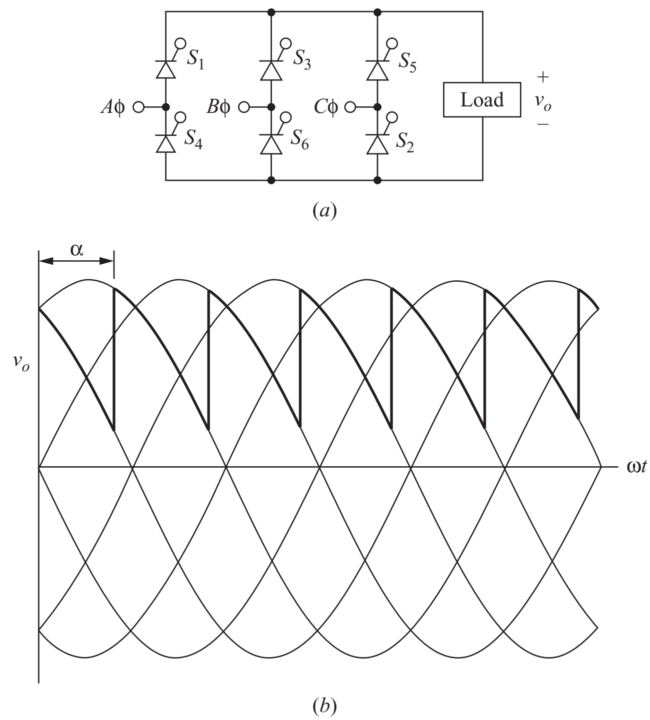


Figure 4-20 (*a*) A controlled three-phase rectifier; (*b*) Output voltage for $\alpha = 45°$.

Harmonics for the output voltage remain of order $6k$, but the amplitudes are functions of α. Figure 4-21 shows the first three normalized harmonic amplitudes.

EXAMPLE 4-13

A Controlled Three-Phase Rectifier

A three-phase controlled rectifier has an input voltage which is 480 V rms at 60 Hz. The load is modeled as a series resistance and inductance with $R = 10\ \Omega$ and $L = 50$ mH. (*a*) Determine the delay angle required to produce an average current of 50 A in the load. (*b*) Determine the amplitude of harmonics $n = 6$ and $n = 12$.

■ Solution

(*a*) The required dc component in the bridge output voltage is

$$V_o = I_o R = (50)(10) = 500 \text{ V}$$

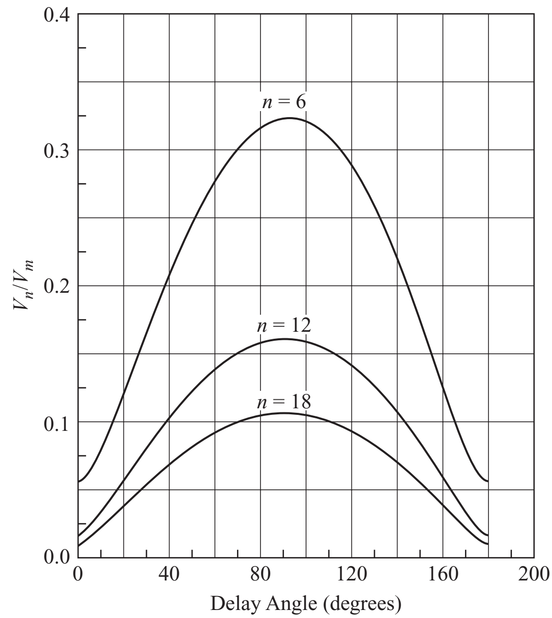


Figure 4-21 Normalized output voltage harmonics as a function of delay angle for a three-phase rectifier.

Equation (4-47) is used to determine the required delay angle:

$$\alpha = \cos^{-1}\left(\frac{V_o\pi}{3V_{m,L-L}}\right) = \cos^{-1}\left(\frac{500\pi}{3\sqrt{2}(480)}\right) = 39.5°$$

(*b*) Amplitudes of harmonic voltages are estimated from the graph in Fig. 4-21. For $\alpha = 39.5°$, normalized harmonic voltages are $V_6/V_m \approx 0.21$ and $V_{12}/V_m \approx 0.10$. Using $V_m = \sqrt{2}(480)$, $V_6 = 143$ V, and $V_{12} = 68$ V, harmonic currents are then

$$I_6 = \frac{V_6}{Z_6} = \frac{143}{\sqrt{10^2 + [6(377)(0.05)]^2}} = 1.26 \text{ A}$$

$$I_{12} = \frac{V_{12}}{Z_{12}} = \frac{68}{\sqrt{10^2 + [12(377)(0.05)]^2}} = 0.30 \text{ A}$$

Twelve-Pulse Rectifiers

The three-phase six-pulse bridge rectifier shows a marked improvement in the quality of the dc output over that of the single-phase rectifier. Harmonics of the output voltage are small and at frequencies that are multiples of 6 times the source frequency. Further reduction in output harmonics can be accomplished by

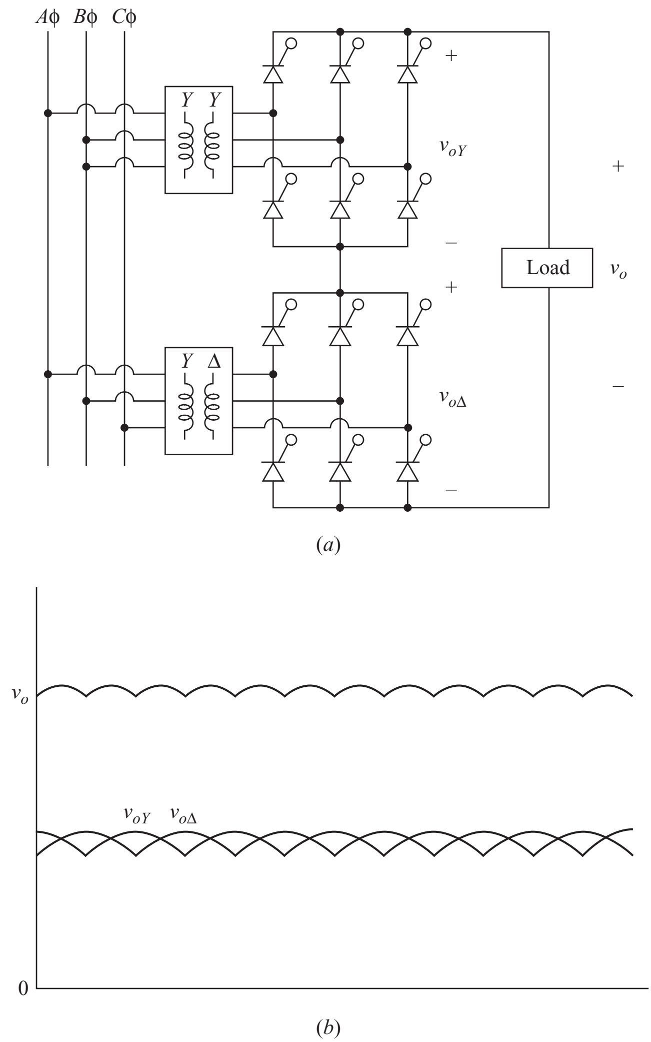


Figure 4-22 (*a*) A 12-pulse three-phase rectifier; (*b*) Output voltage for $\alpha = 0$.

using two six-pulse bridges as shown in Fig. 4-22*a*. This configuration is called a 12-pulse converter.

One of the bridges is supplied through a *Y*-*Y* connected transformer, and the other is supplied through a *Y*-Δ (or Δ-*Y*) transformer as shown. The purpose of the *Y*-Δ transformer connection is to introduce a 30° phase shift between the source and the bridge. This results in inputs to the two bridges which are 30° apart. The two bridge outputs are similar, but also shifted by 30°. The overall output voltage is the sum of the two bridge outputs. The delay angles for the bridges are typically the same. The dc output is the sum of the dc output of each bridge

$$V_o = V_{o,Y} + V_{o,\Delta} = \frac{3V_{m,L-L}}{\pi}\cos\alpha + \frac{3V_{m,L-L}}{\pi}\cos\alpha = \frac{6V_{m,L-L}}{\pi}\cos\alpha \quad \text{(4-48)}$$

The peak output of the 12-pulse converter occurs midway between alternate peaks of the 6-pulse converters. Adding the voltages at that point for $\alpha = 0$ gives

$$V_{o,\text{peak}} = 2V_{m,L-L}\cos(15°) = 1.932\ V_{m,L-L} \quad \text{(4-49)}$$

Figure 4-22*b* shows the voltages for $\alpha = 0$.

Since a transition between conducting thyristors occurs every 30°, there are a total of 12 such transitions for each period of the ac source. The output has harmonic frequencies that are multiples of 12 times the source frequency ($12k$, $k = 1, 2, 3, \ldots$). Filtering to produce a relatively pure dc output is less costly than that required for the 6-pulse rectifier.

Another advantage of using a 12-pulse converter rather than a 6-pulse converter is the reduced harmonics that occur in the ac system. The current in the ac lines supplying the *Y*-*Y* transformer is represented by the Fourier series

$$\begin{aligned} i_Y(t) = \frac{2\sqrt{3}}{\pi}I_o\bigg(&\cos\omega_0 t - \frac{1}{5}\cos 5\omega_0 t + \frac{1}{7}\cos 7\omega_0 t \\ &- \frac{1}{11}\cos 11\omega_0 t + \frac{1}{13}\cos 13\omega_0 t - \cdots\bigg) \end{aligned} \quad \text{(4-50)}$$

The current in the ac lines supplying the *Y*-Δ transformer is represented by the Fourier series

$$\begin{aligned} i_\Delta(t) = \frac{2\sqrt{3}}{\pi}I_o\bigg(&\cos\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \frac{1}{7}\cos 7\omega_0 t \\ &- \frac{1}{11}\cos 11\omega_0 t + \frac{1}{13}\cos 13\omega_0 t + \cdots\bigg) \end{aligned} \quad \text{(4-51)}$$

The Fourier series for the two currents are similar, but some terms have opposite algebraic signs. The ac system current, which is the sum of those transformer currents, has the Fourier series

$$\begin{aligned} i_{\text{ac}}(t) &= i_Y(t) + i_\Delta(t) \\ &= \frac{4\sqrt{3}}{\pi}I_o\left(\cos\omega_0 t - \frac{1}{11}\cos 11\omega_0 t + \frac{1}{13}\cos 13\,\omega_0 t \cdots\right) \end{aligned} \quad \text{(4-52)}$$

Thus, some of the harmonics on the ac side are canceled by using the 12-pulse scheme rather than the 6-pulse scheme. The harmonics that remain in the ac system are of order $12k \pm 1$. Cancellation of harmonics $6(2n-1) \pm 1$ has resulted from this transformer and converter configuration.

This principle can be expanded to arrangements of higher pulse numbers by incorporating increased numbers of 6-pulse converters with transformers that have the appropriate phase shifts. The characteristic ac harmonics of a p-pulse converter will be $pk \pm 1$, $k = 1, 2, 3, \ldots$. Power system converters have a practical limitation of 12 pulses because of the large expense of producing high-voltage transformers with the appropriate phase shifts. However, lower-voltage industrial systems commonly have converters with up to 48 pulses.

The Three-Phase Converter Operating as an Inverter

The above discussion focused on circuits operating as rectifiers, meaning that the power flow is from the ac side of the converter to the dc side. It is also possible for the three-phase bridge to operate as an inverter, having power flow from the dc side to the ac side. A circuit that enables the converter to operate as an inverter is shown in Fig. 4-23*a*. Power is supplied by the dc source, and power is

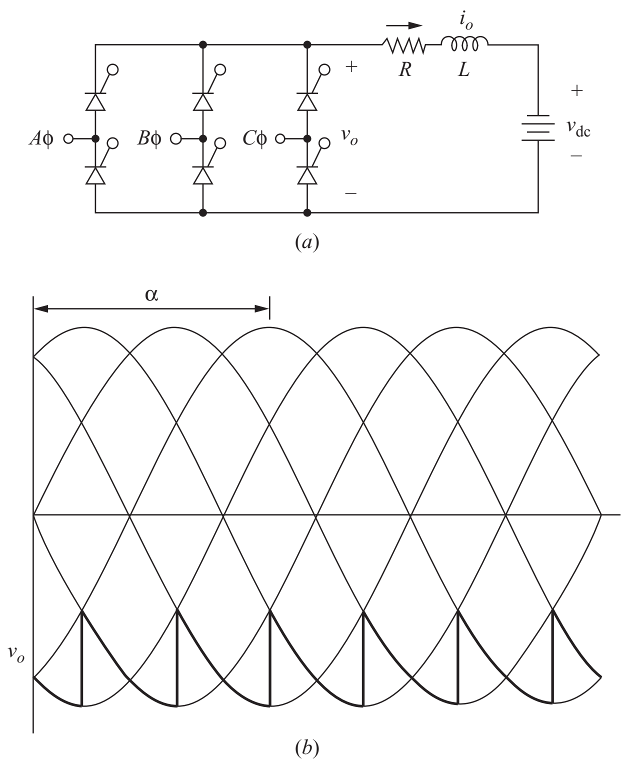


Figure 4-23 (*a*) Six-pulse three-phase converter operating as an inverter; (*b*) Bridge output voltage for $\alpha = 150°$.

absorbed by the converter and transferred to the ac system. The analysis of the three-phase inverter is similar to that of the single-phase case.

The dc current must be in the direction shown because of the SCRs in the bridge. For power to be absorbed by the bridge and transferred to the ac system, the bridge output voltage must be negative. Equation (4-47) applies, so a delay angle larger than 90° results in a negative bridge output voltage.

$$\begin{aligned} &0 < \alpha < 90^\circ \quad && V_o > 0 \quad \rightarrow \quad \text{rectifier operation} \\ &90^\circ < \alpha < 180^\circ \quad && V_o < 0 \quad \rightarrow \quad \text{inverter operation} \end{aligned} \tag{4-53}$$

The output voltage waveform for $\alpha = 150^\circ$ and continuous load current is shown in Fig. 4-23*b*.

EXAMPLE 4-14

Three-Phase Bridge Operating as an Inverter

The six-pulse converter of Fig. 4-23*a* has a delay angle $\alpha = 120^\circ$. The three-phase ac system is 4160 V rms line-to-line. The dc source is 3000 V, $R = 2\ \Omega$, and L is large enough to consider the current to be purely dc. (*a*) Determine the power transferred to the ac source from the dc source. (*b*) Determine the value of L such that the peak-to-peak variation in load current is 10 percent of the average load current.

■ Solution

(*a*) The dc output voltage of the bridge is computed from Eq. (4-47) as

$$V_o = \frac{3V_{m,L-L}}{\pi}\cos\alpha = \frac{3\sqrt{2}(4160)}{\pi}\cos(120^\circ) = -2809\ \text{V}$$

The average output current is

$$I_o = \frac{V_o + V_{dc}}{R} = \frac{-2809 + 3000}{2} = 95.5\ \text{A}$$

The power absorbed by the bridge and transferred back to the ac system is

$$P_{ac} = -I_o V_o = (-95.5)(-2809) = 268.3\ \text{kW}$$

Power supplied by the dc source is

$$P_{dc} = I_o V_{dc} = (95.5)(3000) = 286.5\ \text{kW}$$

Power absorbed by the resistance is

$$P_R = I_{rms}^2 R \approx I_o^2 R = (95.5)^2(2) = 18.2\ \text{kW}$$

(*b*) Variation in load current is due to the ac terms in the Fourier series. The load current amplitudes for each of the ac terms is

$$I_n = \frac{V_n}{Z_n}$$

where V_n can be estimated from the graph of Fig. 4-21 and

$$Z_n = |R + jn\omega_0 L|$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to diminishing ac currents as n increases, the peak-to-peak current variation will be estimated from the first ac term. For $n = 6$, V_n/V_m is estimated from Fig. 4-21 as 0.28, making $V_6 = 0.28(4160\sqrt{2}) = 1650$ V. The peak-to-peak variation of 10 percent corresponds to a zero-to-peak amplitude of (0.05)(95.5) = 4.8 A. The required load impedance for $n = 6$ is then

$$Z_6 = \frac{V_6}{I_6} = \frac{1650 \text{ V}}{4.8 \text{ A}} = 343 \ \Omega$$

The 2-Ω resistor is insignificant compared to the total 343-Ω required impedance, so $Z_6 \approx 6\omega_0 L$. Solving for L,

$$L \approx \frac{Z_6}{6\omega_0} = \frac{343}{6(377)} = 0.15 \text{ H}$$

4.6 DC POWER TRANSMISSION

The controlled 12-pulse converter of Fig. 4-22*a* is the basic element for dc power transmission. DC transmission lines are commonly used for transmission of electric power over very long distances. Examples include the Pacific Intertie; the Square Butte Project from Center, North Dakota, to Duluth, Minnesota; and the Cross Channel Link under the English Channel between England and France. Modern dc lines use SCRs in the converters, while very old converters used mercury-arc rectifiers.

Advantages of dc power transmission include the following:

1. The inductance of the transmission line has zero impedance to dc, whereas the inductive impedance for lines in an ac system is relatively large.
2. The capacitance that exists between conductors is an open circuit for dc. For ac transmission lines, the capacitive reactance provides a path for current, resulting in additional I^2R losses in the line. In applications where the conductors are close together, the capacitive reactance can be a significant problem for ac transmission lines, whereas it has no effect on dc lines.
3. There are two conductors required for dc transmission rather than three for conventional three-phase power transmission. (There will likely be an additional ground conductor in both dc and ac systems.)
4. Transmission towers are smaller for dc than ac because of only two conductors, and right-of-way requirements are less.
5. Power flow in a dc transmission line is controllable by adjustment of the delay angles at the terminals. In an ac system, power flow over a given transmission line is not controllable, being a function of system generation and load.

6. Power flow can be modulated during disturbances on one of the ac systems, resulting in increased system stability.
7. The two ac systems that are connected by the dc line do not need to be in synchronization. Furthermore, the two ac systems do not need to be of the same frequency. A 50-Hz system can be connected to a 60-Hz system via a dc link.

The disadvantage of dc power transmission is that a costly ac-dc converter, filters, and control system are required at each end of the line to interface with the ac system.

Figure 4-24*a* shows a simplified scheme for dc power transmission using six-pulse converters at each terminal. The two ac systems each have their own generators, and the purpose of the dc line is to enable power to be interchanged between the ac systems. The directions of the SCRs are such that current i_o will be positive as shown in the line.

In this scheme, one converter operates as a rectifier (power flow from ac to dc), and the other terminal operates as an inverter (power flow from dc to ac). Either terminal can operate as a rectifier or inverter, with the delay angle determining the mode of operation. By adjusting the delay angle at each terminal, power flow is controlled between the two ac systems via the dc link.

The inductance in the dc line is the line inductance plus an extra series inductor to filter harmonic currents. The resistance is that of the dc line conductors. For analysis purposes, the current in the dc line may be considered to be a ripple-free dc current.

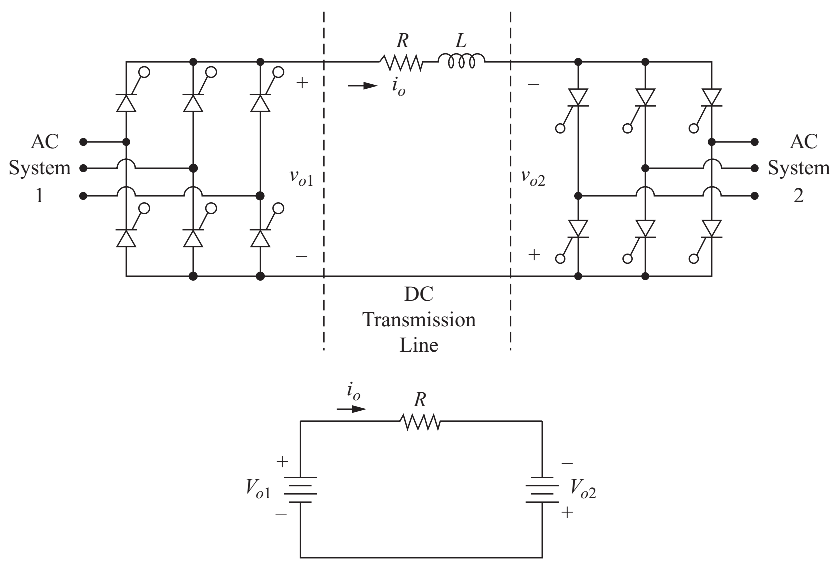


Figure 4-24 (*a*) An elementary dc transmission system; (*b*) Equivalent circuit.

Voltages at the terminals of the converters V_{o_1} and V_{o_2} are positive as shown for α between 0 and 90° and negative for α between 90 and 180°. The converter supplying power will operate with a positive voltage while the converter absorbing power will have a negative voltage.

With converter 1 in Fig. 4-24*a* operating as a rectifier and converter 2 operating as an inverter, the equivalent circuit for power computations is shown in Fig. 4-24*b*. The current is assumed to be ripple-free, enabling only the dc component of the Fourier series to be relevant. The dc current is

$$I_o = \frac{V_{o1} + V_{o2}}{R} \tag{4-54}$$

where

$$\begin{aligned} V_{o1} &= \frac{3V_{m1,L-L}}{\pi} \cos \alpha_1 \\ V_{o2} &= \frac{3V_{m2,L-L}}{\pi} \cos \alpha_2 \end{aligned} \tag{4-55}$$

Power supplied by the converter at terminal 1 is

$$P_1 = V_{o1} I_o \tag{4-56}$$

Power supplied by the converter at terminal 2 is

$$P_2 = V_{o2} I_o \tag{4-57}$$

EXAMPLE 4-15

DC Power Transmission

For the elementary dc transmission line represented in Fig. 4-24*a*, the ac voltage to each of the bridges is 230 kV rms line to line. The total line resistance is 10 Ω, and the inductance is large enough to consider the dc current to be ripple-free. The objective is to transmit 100 MW to ac system 2 from ac system 1 over the dc line. Design a set of operating parameters to accomplish this objective. Determine the required current-carrying capacity of the dc line, and compute the power loss in the line.

■ **Solution**

The relationships that are required are from Eqs. (4-54) to (4-57), where

$$P_2 = I_o V_{o2} = -100 \text{ MW} \qquad (100 \text{ MW absorbed})$$

The maximum dc voltage that is obtainable from each converter is, for $\alpha = 0$ in Eq. (4-47),

$$V_{o,\max} = \frac{3V_{m,L-L}}{\pi} = \frac{3\sqrt{2}\,(230 \text{ kV})}{\pi} = 310.6 \text{ kV}$$

The dc output voltages of the converters must have magnitudes less than 310.6 kV, so a voltage of −200 kV is arbitrarily selected for converter 2. This voltage must be negative because power must be absorbed at converter 2. The delay angle at converter 2 is then computed from Eq. (4-47).

$$V_{o2} = \frac{3V_{m,L-L}}{\pi} \cos \alpha_2 = (310.6 \text{ kV}) \cos \alpha_2 = -200 \text{ kV}$$

Solving for α_2,

$$\alpha_2 = \cos^{-1}\left(\frac{-200 \text{ kV}}{310.6 \text{ kV}}\right) = 130°$$

The dc current required to deliver 100 MW to converter 2 is then

$$I_o = \frac{100 \text{ MW}}{200 \text{ kV}} = 500 \text{ A}$$

which is the required current-carrying capacity of the line.

The required dc output voltage at converter 1 is computed as

$$V_{o1} = -V_{o2} + I_o R = 200 \text{ kV} + (500)(10) = 205 \text{ kV}$$

The required delay angle at converter 1 is computed from Eq. (4-47).

$$\alpha_1 = \cos^{-1} \frac{205 \text{ kV}}{310.6 \text{ kV}} = 48.7°$$

Power loss in the line is $I^2_{\text{rms}}R$, where $I_{\text{rms}} \approx I_{\text{o}}$ because the ac components of line current are filtered by the inductor. Line loss is

$$P_{\text{loss}} = I^2_{\text{rms}}R \approx (500)^2(10) = 2.5 \text{ MW}$$

Note that the power supplied at converter 1 is

$$P_1 = V_{\text{dc}1}I_o = (205 \text{ kV})(500 \text{ A}) = 102.5 \text{ MW}$$

which is the total power absorbed by the other converter and the line resistance.

Certainly other combinations of voltages and current will meet the design objectives, as long as the dc voltages are less than the maximum possible output voltage and the line and converter equipment can carry the current. A better design might have higher voltages and a lower current to reduce power loss in the line. That is one reason for using 12-pulse converters and bipolar operation, as discussed next.

A more common dc transmission line has a 12-pulse converter at each terminal. This suppresses some of the harmonics and reduces filtering requirements. Moreover, a pair of 12-pulse converters at each terminal provides bipolar operation. One of the lines is energized at $+V_{\text{dc}}$ and the other is energized at $-V_{\text{dc}}$. In emergency situations, one pole of the line can operate without the other pole, with current returning through the ground path. Figure 4-25 shows a bipolar scheme for dc power transmission.

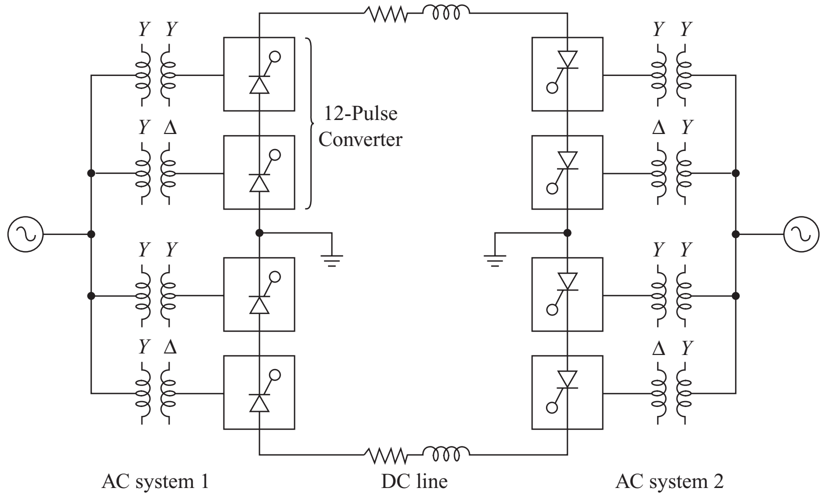


Figure 4-25 A dc transmission system with two 12-pulse converters at each terminal.

4.7 COMMUTATION: THE EFFECT OF SOURCE INDUCTANCE

Single-Phase Bridge Rectifier

An uncontrolled single-phase bridge rectifier with a source inductance L_s and an inductive load is shown in Fig. 4-26*a*. When the source changes polarity, source current cannot change instantaneously, and current must be transferred gradually from one diode pair to the other over a commutation interval u, as shown in Fig. 4-26*b*. Recall from Chap. 3 that commutation is the process of transferring the load current from one diode to another or, in this case, one diode pair to the other. (See Sec. 3.11.) During commutation, all four diodes are on, and the voltage across L_s is the source voltage $V_m \sin(\omega t)$.

Assume that the load current is a constant I_o. The current in L_s and the source during the commutation from D_1-D_2 to D_3-D_4 starts at $+I_o$ and goes to $-I_o$. This commutation interval starts when the source changes polarity at $\omega t = \pi$ as is expressed in

$$i_s(\omega t) = \frac{1}{\omega L_s}\int_{\pi}^{\omega t} V_m \sin(\omega t)\, d(\omega t) + I_o$$

Evaluating,

$$i_s(\omega t) = -\frac{V_m}{\omega L_s}(1 + \cos \omega t) + I_o \tag{4-58}$$

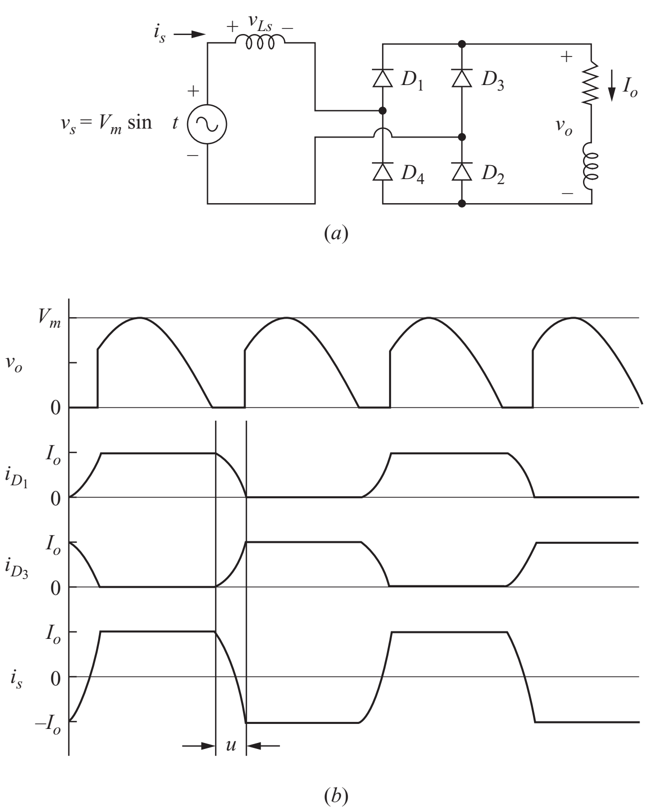


Figure 4-26 Commutation for the single-phase rectifier (a) circuit with source inductance L_s; (b) voltage and current waveforms.

When commutation is complete at $\omega t = \pi + u$,

$$i(\pi + u) = -I_o = -\frac{V_m}{\omega L_S}[1 + \cos(\pi + u)] + I_o \tag{4-59}$$

Solving for the commutation angle u,

$$u = \cos^{-1}\left(1 - \frac{2I_o\omega L_S}{V_m}\right) = \cos^{-1}\left(1 - \frac{2I_oX_S}{V_m}\right) \tag{4-60}$$

where $X_s = \omega L_s$ is the reactance of the source. Figure 4-26b shows the effect of the source reactance on the load current and voltage.

Average load voltage is

$$V_o = \frac{1}{\pi}\int_u^\pi V_m \sin(\omega t)\,d(\omega t) = \frac{V_m}{\pi}(1 + \cos u)$$

Using u from Eq. (4-60),

$$\boxed{V_o = \frac{2V_m}{\pi}\left(1 - \frac{I_o X_s}{V_m}\right)} \tag{4-61}$$

Thus, source inductance lowers the average output voltage of full-wave rectifiers.

Three-Phase Rectifier

For the uncontrolled three-phase bridge rectifier with source reactance (Fig. 4-27*a*), assume that diodes D_1 and D_2 are conducting and the load current is a constant I_o. The next transition has load current transferred from D_1 to D_3 in the top half of the bridge. The equivalent circuit during commutation from D_1 to D_3 is shown in Fig. 4-27*b*. The voltage across L_a is

$$v_{La} = \frac{v_{AB}}{2} = \frac{V_{m,L-L}}{2}\sin(\omega t) \tag{4-62}$$

Current in L_a starts at I_o and decreases to zero in the commutation interval,

$$i_{La}(\pi + u) = 0 = \frac{1}{\omega L_a}\int_{\pi}^{\pi+u} \frac{V_{m,L-L}}{2}\sin(\omega t)\,d(\omega t) + I_o \tag{4-63}$$

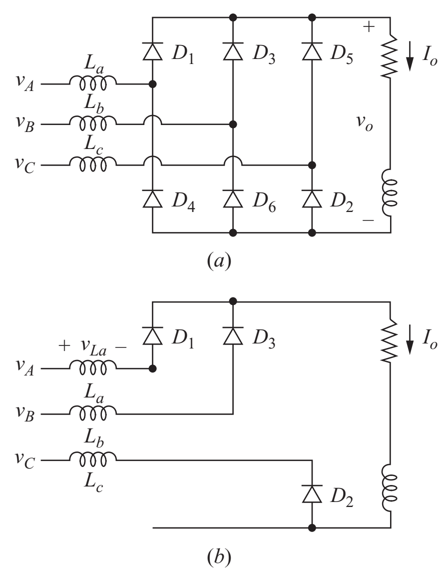


Figure 4-27 Commutation for the three-phase rectifier. (*a*) Circuit; (*b*) Circuit during commutation from D_1 to D_3; (*c*) Output voltage and diode currents.

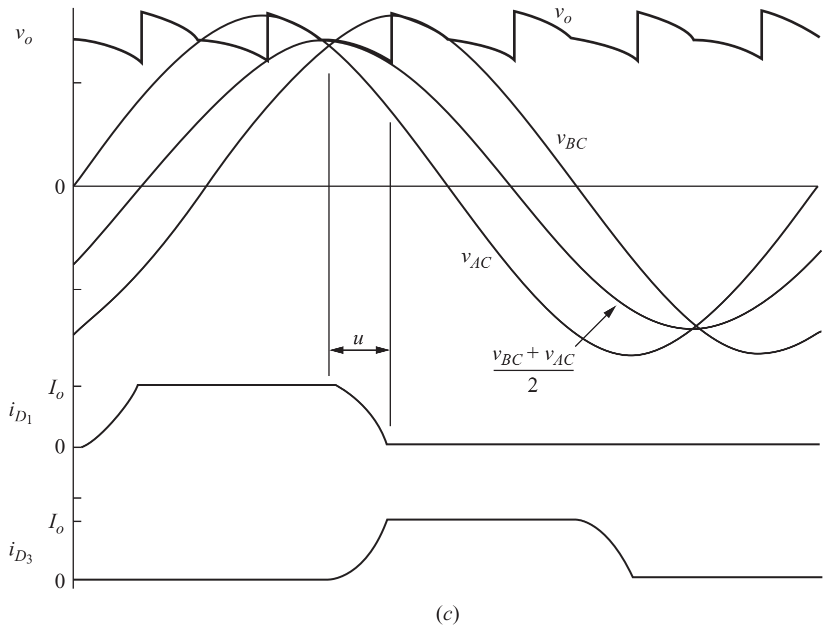


Figure 4-27 (*continued*)

Solving for u,

$$u = \cos^{-1}\left(1 - \frac{2\omega L_a I_o}{V_{m,L-L}}\right) = \cos^{-1}\left(1 - \frac{2X_s I_o}{V_{m,L-L}}\right) \tag{4-64}$$

During the commutation interval from D_1 to D_3, the converter output voltage is

$$v_o = \frac{v_{bc} + v_{ac}}{2} \tag{4-65}$$

Output voltage and diode currents are shown in Fig. 4-27*c*. Average output voltage for the three-phase converter with a nonideal source is

$$\boxed{V_o = \frac{3V_{m,L-L}}{\pi}\left(1 - \frac{X_s I_o}{V_{m,L-L}}\right)} \tag{4-66}$$

Therefore, *source inductance lowers the average output voltage of three-phase rectifiers*.

4.8 Summary

- Single-phase full-wave rectifiers can be of the bridge or center-tapped transformer types.
- The average source current for single-phase full-wave rectifiers is zero.
- The Fourier series method can be used to analyze load currents.

- A large inductor in series with a load resistor produces a load current that is essentially dc.
- A filter capacitor on the output of a rectifier can produce an output voltage that is nearly dc. An *LC* output filter can further improve the quality of the dc output and reduce the peak current in the diodes.
- Switches such as SCRs can be used to control the output of a single-phase or three-phase rectifier.
- Under certain circumstances, controlled converters can be operated as inverters.
- The 6-pulse three-phase rectifiers have 6 diodes or SCRs, and 12-pulse rectifiers have 12 diodes or SCRs.
- Three-phase bridge rectifiers produce an output that is inherently like dc.
- DC power transmission has a three-phase converter at each end of a dc line. One converter is operated as a rectifier and the other is operated as a converter.
- Source inductance reduces the dc output of a single-phase or three-phase rectifier.

4.9 Bibliography

S. B. Dewan and A. Straughen, *Power Semiconductor Circuits*, Wiley, New York, 1975.

J. Dixon, *Power Electronics Handbook*, edited by M. H. Rashid, Academic Press, San Diego, 2001, Chapter 12.

E. W. Kimbark, *Direct Current Transmission*, Wiley-Interscience, New York, 1971.

P. T. Krein, *Elements of Power Electronics*, Oxford University Press, 1998.

Y.-S. Lee and M. H. L. Chow, *Power Electronics Handbook*, edited by M. H. Rashid, Academic Press, San Diego, 2001, Chapter 10.

N. Mohan, T. M. Undeland, and W. P. Robbins, *Power Electronics: Converters, Applications, and Design,* 3d ed., Wiley, New York, 2003.

M. H. Rashid, *Power Electronics: Circuits, Devices, and Systems,* 3d ed., Prentice-Hall, Upper Saddle River, N.J., 2004.

B. Wu, *High-Power Converters and AC Drives*, Wiley, New York, 2006.

Problems

Uncontrolled Single-Phase Rectifiers

4-1. A single-phase full-wave bridge rectifier has a resistive load of 18 Ω and an ac source of 120-V rms. Determine the average, peak, and rms currents in the load and in each diode.

4-2. A single-phase rectifier has a resistive load of 25 Ω. Determine the average current and peak reverse voltage across each of the diodes for (*a*) a bridge rectifier with an ac source of 120 V rms and 60 Hz and (*b*) a center-tapped transformer rectifier with 120 V rms on each half of the secondary winding.

4-3. A single-phase bridge rectifier has an *RL* load with $R = 15\ \Omega$ and $L = 60$ mH. The ac source is $v_s = 100 \sin(377t)$ V. Determine the average and rms currents in the load and in each diode.

4-4. A single-phase bridge rectifier has an *RL* load with $R = 10\ \Omega$ and $L = 25$ mH. The ac source is $v_s = 170 \sin(377t)$ V. Determine the average and rms currents in the load and in each diode.