Second Semester

Third Year

MANDALAY TECHNOLOGICAL UNIVERSITY

DEPARTMENT OF MECHATRONIC ENGINEERING

ELECTRONIC CIRCUIT ANALYSIS II

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Motto: Creative, Innovative, Mechatronics

10

CAPACITORS AND CAPACITANCE

CHAPTER PREVIEW

A capacitor is a circuit component designed to store electrical charge. If you connect a dc voltage source to a capacitor, for example, the capacitor will "charge" to the voltage of the source. If you then disconnect the source, the capacitor will remain charged, that is, its voltage will remain constant at the value that it attained while connected to the source (assuming no leakage). Because of this tendency to hold voltage, *a capacitor opposes changes in voltage.* It is this characteristic that gives capacitors their unique properties.

Capacitors are widely used in electrical and electronic applications. They are used in electronic systems for signal conditioning and timing, for example, in speaker system crossover networks to separate high- and low-frequency signals, in cameras to store the charge that fires the photoflash, on pump and refrigeration motors to increase starting torque, in electric power systems to increase operating efficiency, and so on. A photo of some typical capacitors is shown on the right.

Capacitance is the electrical property of capacitors: it is a measure of how much charge a capacitor can hold. In this chapter, we look at capacitance and its basic properties. In Chapter 11, we look at capacitors in dc and pulse circuits; in later chapters, we look at capacitors in ac applications. ■

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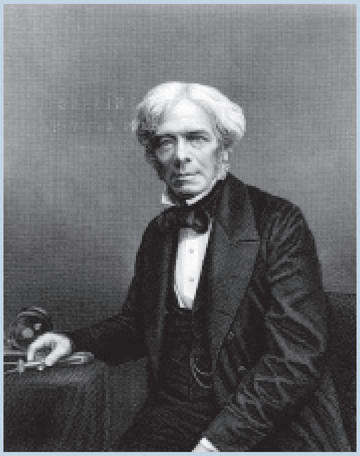
Typical capacitors.

Putting It in Perspective

Michael Faraday and the Field Concept



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THE UNIT OF CAPACITANCE, the farad, is named after Michael Faraday (1791–1867). Born in England to a working class family, Faraday received limited education. Nonetheless, he was responsible for many of the fundamental discoveries of electricity and magnetism. Lacking mathematical skills, he used his intuitive ability rather than mathematical models to develop conceptual pictures of basic phenomena. It was his development of the field concept, for example, that made it possible to map out the fields that exist around magnetic poles and electrical charges.

To get at this idea, recall from Chapter 2 that unlike charges attract and like charges repel, that is, a force exists between electrical charges. We call the region where this force acts an electric field. To visualize this field, we use Faraday's field concept and draw lines of force (or flux lines) that show at every point in space the magnitude and direction of the force. Now, rather than supposing that one charge exerts a force on another, we instead visualize that the original charges create a field in space and that other charges introduced into this field experience a force due to the field. This concept is helpful in studying certain aspects of capacitors, as you will see in this chapter.

The development of the field concept had a significant impact on science. We now picture several important phenomena in terms of fields, including electric fields, gravitation, and magnetism. When Faraday published his theory in 1844, however, it was not taken seriously, much like Ohm's work two decades earlier. It is also interesting to note that the development of the field concept grew out of Faraday's research into magnetism, not electric charge. ■

10.1 Capacitance

A **capacitor** consists of two conductors separated by an insulator. One of its basic forms is the parallel-plate capacitor shown in Figure 10–1. It consists of two metal plates separated by a nonconducting material (i.e., an insulator) called a **dielectric.** The dielectric may be air, oil, mica, plastic, ceramic, or other suitable insulating material.

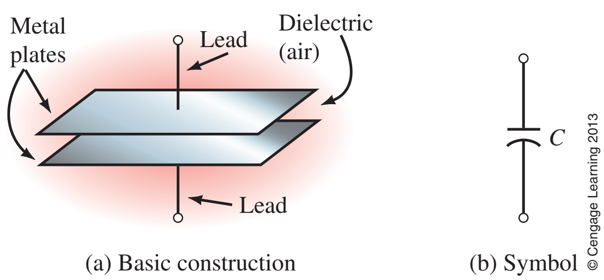


FIGURE 10–1 Parallel-plate capacitor.

Since the plates of the capacitor are metal, they contain huge numbers of free electrons. In their normal state, however, they are uncharged, that is, there is no excess or deficiency of electrons on either plate. If a dc source is now connected (Figure 10–2), electrons are pulled from the top plate by the positive potential of the battery and the same number deposited on the bottom plate. This leaves the top plate with a deficiency of electrons (i.e., positive charge) and the bottom plate with an excess (i.e., negative charge). In this state, the capacitor is said to be **charged.** (Note that no current can pass through the dielectric between the plates—thus, the movement of electrons illustrated in Figure 10–2 will cease when the capacitor reaches full charge.)

If Q coulombs of electrons are moved during the charging process (leaving the top plate with a deficiency of Q electrons and the bottom with an excess of Q), we say that the capacitor has a charge of Q.

If we now disconnect the source (Figure 10–3), the excess electrons that were moved to the bottom plate remain trapped as they have no way to return to the top plate. The capacitor therefore remains charged with voltage E across it even though no source is present. Because of this, we say that *a capacitor can store charge.* Capacitors with little leakage (Section 10.5) can hold their charge for a considerable time—sometimes for years.

Large-value capacitors charged to high voltages contain a great deal of energy and can store a potentially lethal charge; you must therefore be very careful with them. Even high-value, low-voltage units can contain enough energy to vaporize a screwdriver and spray metal in your eyes if you attempt to short the leads. If you intend to handle such capacitors, or work on them, always discharge them after power has been removed. You can do this by connecting a high-wattage resistor (sometimes referred to as a bleeder resistor) of about 50 Ω/V of the rated voltage of the capacitor across the capacitor's terminals until it is discharged, but be careful to protect yourself from shock. Note also that some voltage may spring back due to dielectric absorption after the discharge resistor is removed—see Section 10.5.

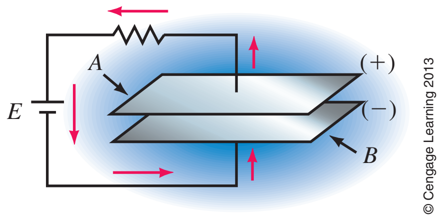


FIGURE 10–2 Capacitor during charging. At the instant that you connect the source, there is a momentary surge of current as electrons are pulled from plate A and an equal number deposited on plate B. This leaves the top plate positively charged and the bottom plate negatively charged. When charging is complete, there is no further movement of electrons, and the current is thus zero.

CircuitSim 10-1

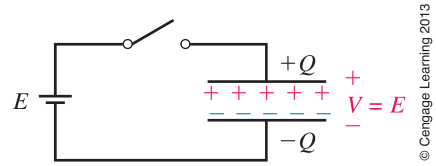


FIGURE 10–3 Capacitor after charging. When you disconnect the source, electrons are trapped on the bottom plate and cannot return—thus, charge is stored.

Definition of Capacitance

The amount of charge Q that a capacitor can store depends on the applied voltage. Experiments show that for a given capacitor, Q is proportional to voltage. Let the constant of proportionality be C. Then,

$$Q = CV \tag{10–1}$$

Rearranging terms yields

$$C = \frac{Q}{V} \quad \text{(farads, F)} \tag{10–2}$$

The term C is defined as the capacitance of the capacitor. As indicated, its unit is the **farad.** By definition, *the **capacitance** of a capacitor is one farad if it stores one coulomb of charge when the voltage across its terminals is one volt.* The farad, however, is a very large unit. Most practical capacitors (except supercapacitors, Section 10.6) range in size from picofarads (pF or 10^{-12} F) to microfarads (μF or 10^{-6} F). The larger the value of C, the more charge that the capacitor can hold for a given voltage.

EXAMPLE 10–1

a. How much charge is stored on a 10-μF capacitor when it is connected to a 24-volt source?

b. The charge on a 20-nF capacitor is 1.7 μC. What is its voltage?

Solution

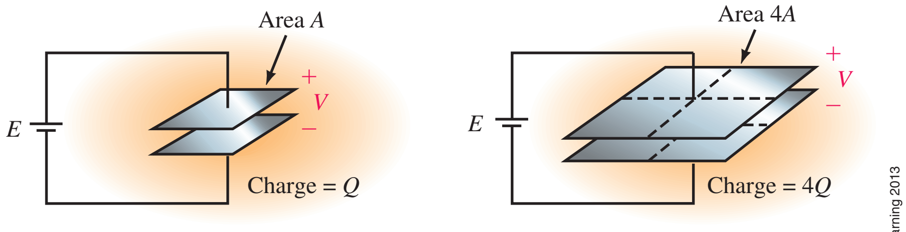
a. From Equation 10–1, $Q = CV$. Thus, $Q = (10 \times 10^{-6}\text{ F})(24\text{ V}) = 240\ \mu\text{C}$.

b. Rearranging Equation 10–1, $V = Q/C = (1.7 \times 10^{-6}\text{ C})/(20 \times 10^{-9}\text{ F}) = 85\text{ V}$.

10.2 Factors Affecting Capacitance

Effect of Area

As shown by Equation 10–2, capacitance is directly proportional to charge. This means that the more charge you can put on a capacitor's plates for a given voltage, the greater will be its capacitance. Consider Figure 10–4. The capacitor of (b) has four times the area of (a). Since it has the same number of free electrons per unit area, it has four times the total charge and hence four times the capacitance. This turns out to be true in general, that is, *capacitance is directly proportional to plate area.*



(a) Capacitor with area A and charge Q

(b) Plates with four times the area have four times the charge and therefore, four times the capacitance

FIGURE 10–4 For a fixed separation, capacitance is proportional to plate area.

Effect of Spacing

Now consider Figure 10–5. Since the top plate has a deficiency of electrons and the bottom plate an excess, a force of attraction exists across the gap. For a fixed spacing as in (a), the charges are in equilibrium. Now move the plates closer together as in (b). As spacing decreases, the force of attraction increases, pulling more electrons from within the bulk material of plate B to its top surface. This creates a deficiency of electrons throughout B. To replenish these, the source moves additional electrons around the circuit, leaving A with an even greater deficiency and B with an even greater excess. The charge on the plates therefore increases and hence, according to Equation 10–2, so does the capacitance. We therefore conclude that decreasing spacing increases capacitance, and vice versa. In fact, as we will show later, *capacitance is inversely proportional to plate spacing*.

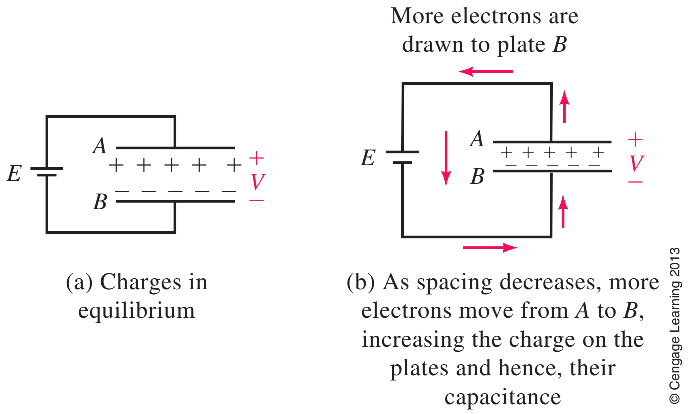


FIGURE 10–5 Decreasing spacing increases capacitance.

Effect of Dielectric

Capacitance also depends on the dielectric. Consider Figure 10–6(a), which shows an air-dielectric capacitor. If you substitute different materials for air, the capacitance increases. Table 10–1 shows the factor by which capacitance increases for a number of different materials. For example, if Teflon® is used instead of air, capacitance is increased by a factor of 2.1. This factor is called the **relative permittivity** of the material (or in older terminology, its **dielectric constant**). (**Permittivity** is a measure of how easy it is to establish electric flux in a material. We revisit it below.) Note that high-permittivity ceramic increases capacitance by as much as 7500, as indicated in Figure 10–6(b).

TABLE 10–1 Relative Permittivity of Various Materials

Material	$\in_r$ (Nominal Values)
Vacuum	1
Air	1.0006
Ceramic	30–7500
Mica	5.5
Mylar (Note 3)	3
Oil	4
Paper (dry)	2.2
Polystyrene	2.6
Teflon (Note 3)	2.1

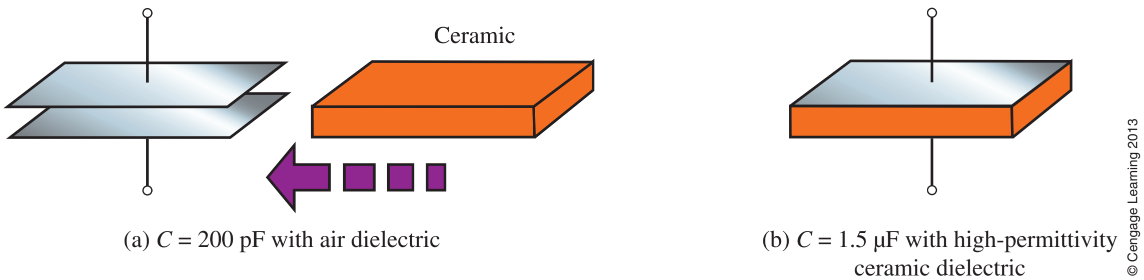


FIGURE 10–6 The factor by which a dielectric causes capacitance to increase is termed its *relative permittivity.* The ceramic used here has a value of 7500.

Capacitance of a Parallel-Plate Capacitor

From the preceding observations, we see that capacitance is directly proportional to plate area, inversely proportional to plate separation, and dependent on the dielectric. In equation form,

$$C = \in \frac{A}{d} \quad \text{(F)} \tag{10–3}$$

where area A is in square meters and spacing d is in meters—see Note 1.

Permittivity (Revisited)

The constant $\in$ in Equation 10–3 is the absolute permittivity of the insulating material. Its units are farads per meter (F/m). For air or vacuum, $\in$ has a value

NOTES...

1. Actually, Equation 10–3 is true in general, that is, it holds for other shapes as well as for the parallel-plate configuration. It is just harder to determine the effective area and spacing for other geometries.
2. In modern terminology, $\in_0$ (the absolute permittivity of free space) is also called the **electric constant.**
3. Teflon is a trademark of DuPont. Mylar is a trademark of Dupont Teijin Films.

of $\in_o = 8.854 \times 10^{-12}$ F/m—see Note 2. For other materials, $\in$ is expressed as the product of the relative permittivity, $\in_r$ (shown in Table 10–1), times $\in_o$. That is,

$$\in = \in_r \in_o \tag{10–4}$$

Consider, again, Equation 10–3: $C = \in A/d = \in_r \in_o A/d$. Note that $\in_o A/d$ is the capacitance of a vacuum- (or air-) dielectric capacitor. Denote it by C_o. Then, for any other dielectric,

$$C = \in_r C_o \tag{10–5}$$

EXAMPLE 10–2

Compute the capacitance of a parallel-plate capacitor with plates 10 cm by 20 cm, separation of 5 mm, and

a. an air dielectric,

b. a ceramic dielectric with a relative permittivity of 7500.

Solution Convert all dimensions to meters. Thus, A = (0.1 m)(0.2 m) = 0.02 m^2, and $d = 5 \times 10^{-3}$ m.

a. For air, $C = \in_o A/d = (8.854 \times 10^{-12})(2 \times 10^{-2})/(5 \times 10^{-3}) = 35.4 \times 10^{-12}$ F = 35.4 pF.

b. For ceramic with $\in_r = 7500$, $C = 7500(35.4\text{ pF}) = 0.266\ \mu\text{F}$.

EXAMPLE 10–3

A parallel-plate capacitor with air dielectric has a value of $C = 12$ pF. What is the capacitance of a capacitor that has the following:

a. The same separation and dielectric but five times the plate area?

b. The same dielectric but four times the area and one-fifth the plate spacing?

c. A dry paper dielectric, six times the plate area, and twice the plate spacing?

Solution

a. Since the plate area has increased by a factor of five and everything else remains the same, C increases by a factor of five. Thus, $C = 5(12\text{ pF}) = 60$ pF.

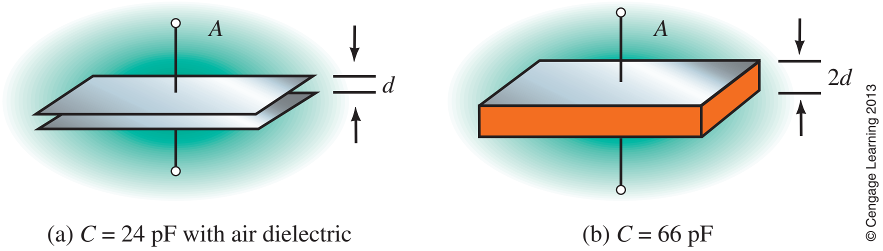
b. With four times the plate area, C increases by a factor of four. With one-fifth the plate spacing, C increases by a factor of five. Thus, $C = (4)(5)(12\text{ pF}) = 240$ pF.

c. Dry paper increases C by a factor of 2.2. The increase in plate area increases C by a factor of six. Doubling the plate spacing reduces C by one-half. Thus, $C = (2.2)(6)(\frac{1}{2})(12\text{ pF}) = 79.2$ pF.

IN-PROCESS LEARNING CHECK 1

(Answers are at the end of the chapter.)

1. A capacitor with plates 7.5 cm × 8 cm and plate separation of 0.1 mm has an oil dielectric:
   a. Compute its capacitance;
   b. If the charge on this capacitor is 0.424 μC, what is the voltage across its plates?
2. For a parallel-plate capacitor, if you triple the plate area and halve the plate spacing, how does capacitance change?
3. For the capacitor of Figure 10–6, if you use mica instead of ceramic, what will be the capacitance?
4. What is the dielectric for the capacitor of Figure 10–7(b)?



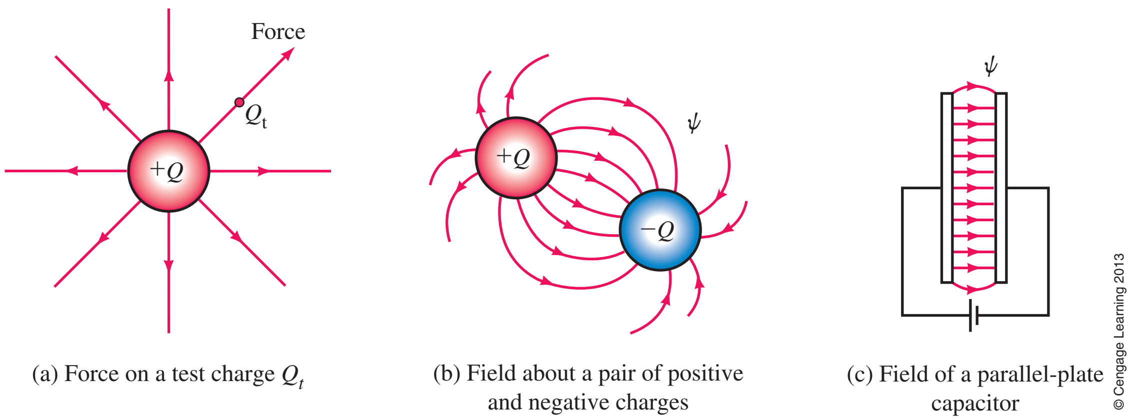
(a) C = 24 pF with air dielectric (b) C = 66 pF

FIGURE 10–7

10.3 Electric Fields

Electric Flux

In Chapter 2, we learned that unlike charges attract while like charges repel; that is, a force exists between them. The region where this force exists is called an **electric field.** To visualize this field, Faraday introduced the idea of using a small positive test charge Q_t to map the forces. He defined the direction of a field as the direction of force on the test charge, and the strength of the field as the force per unit charge (considered below). He then mapped the field as lines of force where the direction of the lines represented the direction of the force, and the density of lines represented its strength. To illustrate, consider Figure 10–8(a). Because like charges repel, the force on the positive test charge is directed outward, resulting in a radial field as indicated. Now consider (b).



(a) Force on a test charge Q_t (b) Field about a pair of positive and negative charges (c) Field of a parallel-plate capacitor

FIGURE 10–8 Electric fields.

The test charge will be repelled by charge $+Q$ and attracted by charge $-Q$, resulting in the curved field shown. Faraday determined that lines never cross and that the stronger the field, the more dense the lines. Figure 10–8(c) shows another example: the field of a parallel-plate capacitor. In this case, the field is uniform across the gap with some fringing near its edges. Electric field lines (also called **flux** lines) are denoted by the Greek letter ψ (psi).

Electric Field Intensity $\mathscr{E}$

As noted previously, the strength of an electric field (more usually called its intensity) is defined as the force per unit charge that the field exerts on a small positive test charge Q_t. Let this **electric field intensity** be denoted by the script letter $\mathscr{E}$. Then by definition,

$$\mathscr{E} = F/Q_t \quad \text{(newtons/coulomb, N/C)} \tag{10–6}$$

To illustrate, let us determine the field about the point charge Q of Figure 10–8(a). When the test charge is placed near Q, it experiences a force of $F = kQQ_t/r^2$ (Coulomb's law, Chapter 2). The constant in Coulomb's law is actually equal to $1/4\pi\epsilon$. Thus, $F = QQ_t/4\pi\epsilon r^2$, and from Equation 10–6,

$$\mathscr{E} = \frac{F}{Q_t} = \frac{Q}{4\pi\epsilon r^2} \quad \text{(N/C)} \tag{10–7}$$

Electric Flux Density

Because of the presence of ϵ in Equation 10–7, the electric field intensity depends on the medium in which the charge is located. Let us define a new quantity, D, that is independent of the medium. Let

$$D = \epsilon\mathscr{E} \tag{10–8}$$

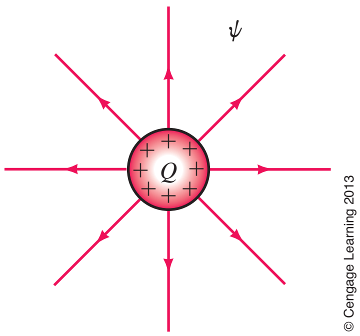


FIGURE 10–9 In the SI system, total flux ψ equals charge Q.

D is known as **electric flux density.** As is shown in physics, D represents the density of flux lines in space, that is,

$$D = \frac{\text{total flux}}{\text{area}} = \frac{\psi}{A} \tag{10–9}$$

where ψ is the flux passing through area A.

Electric Flux (Revisited)

Consider Figure 10–9. Flux ψ is due to the charge Q. Again, as shown in physics, in the SI system the number of flux lines emanating from a charge Q is equal to the charge itself, that is,

$$\psi = Q \quad \text{(C)} \tag{10–10}$$

An easy way to visualize this is to think of one flux line as emanating from each positive charge on the body as shown in Figure 10–9. Then, as indicated, the total number of lines is equal to the total number of charges.

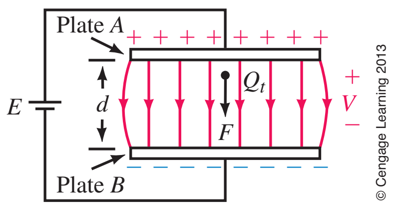


FIGURE 10–10 Work moving test charge Q_t is force times distance.

Field of a Parallel-Plate Capacitor

Now consider a parallel-plate capacitor (Figure 10–10). The field here is created by the charge distributed over its plates. Since plate A has a deficiency of electrons, it looks like a sheet of positive charge, while plate B looks like a sheet of negative charge. A positive test charge Q_t between these sheets is therefore repelled by the positive sheet and attracted by the negative sheet.

Now move the test charge from plate B to plate A. The work W required to move the charge against the force F is force times distance. Thus,

$$W = Fd \quad \text{(J)} \tag{10–11}$$

In Chapter 2, we defined voltage as work divided by charge, that is, $V = W/Q$. Since the charge here is the test charge, Q_t, the voltage between plates A and B is

$$V = W/Q_t = (Fd)/Q_t \quad \text{(V)} \tag{10–12}$$

Now divide both sides by d. This yields $V/d = F/Q_t$. But $F/Q_t = \mathscr{E}$, from Equation 10–6. Thus,

$$\mathscr{E} = V/d \quad \text{(V/m)} \tag{10–13}$$

Equation 10–13 shows that the electric field intensity between capacitor plates is equal to the voltage across the plates divided by the distance between them. Thus, if $V = 30$ V and $d = 10$ mm, $\mathscr{E} = 3000$ V/m.

EXAMPLE 10–4

Suppose that the electric field intensity between the plates of a capacitor is 50 000 V/m when 80 V is applied:

a. What is the plate spacing if the dielectric is air? If the dielectric is ceramic?

b. What is $\mathscr{E}$ if the plate spacing is halved?

Solution

a. $\mathscr{E} = V/d$, independent of dielectric. Thus,

$$d = \frac{V}{\mathscr{E}} = \frac{80 \text{ V}}{50 \times 10^3 \text{ V/m}} = 1.6 \times 10^{-3} \text{ m}$$

b. Since $\mathscr{E} = V/d$, $\mathscr{E}$ will double to 100 000 V/m.

PRACTICE PROBLEMS 1

1. What happens to the electric field intensity of a capacitor if you do the following:
   a. Double the applied voltage?
   b. Triple the applied voltage and double the plate spacing?
2. If the electric field intensity of a capacitor with polystyrene dielectric and plate size 2 cm by 4 cm is 100 kV/m when 50 V is applied, what is its capacitance?

Answers

1. a. Doubles, b. Increases by a factor of 1.5; 2. 36.8 pF

Capacitance (Revisited)

With the preceding background, we can examine capacitance a bit more rigorously. Recall, $C = Q/V$. Using the above relationships yields

$$C = \frac{Q}{V} = \frac{\psi}{V} = \frac{AD}{\mathscr{E}d} = \frac{D}{\mathscr{E}}\left(\frac{A}{d}\right) = \in\frac{A}{d}$$

This confirms Equation 10–3 that we developed intuitively in Section 10.2.

10.4 Dielectrics

As you saw in Figure 10–6, a dielectric increases capacitance. We now examine why. Consider Figure 10–11. For a charged capacitor, electron orbits (which are normally nearly circular) become elliptical as electrons are attracted toward the positive (+) plate and repelled from the negative (−) plate. This makes the end of the atom nearest the positive plate appear negative while its other end appears positive. Such atoms are **polarized.** Throughout the bulk of the dielectric, the negative end of a polarized atom is adjacent to the positive end of another polarized atom, and the effects cancel. However, at the surfaces of the dielectric, there are no atoms to cancel, and the net effect is as if a layer of negative charge exists on the surface of the dielectric at the positive plate and a layer of positive charge at the negative plate. This makes the plates appear closer together, thus increasing capacitance. Materials for which the effect is largest result in the greatest increase in capacitance.

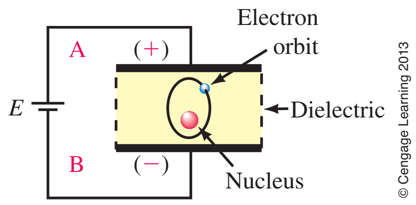


FIGURE 10–11 Effect of the capacitor's electric field on an atom of its dielectric.

Dielectric Breakdown

If the voltage of Figure 10–11 is increased beyond a critical value, the force on the electrons is so great that they are literally torn from orbit. This is called **dielectric breakdown,** and the electric field intensity (Equation 10–13) at breakdown is called the **dielectric strength** of the material. For air, breakdown occurs when the voltage gradient reaches 3 kV/mm. The breakdown strengths for other materials are shown in Table 10–2. Since the quality of a dielectric depends on many factors, dielectric strength varies from sample to sample.

Breakdown is not limited to capacitors; it can occur with any type of electrical apparatus whose insulation is stressed beyond safe limits. (For example, air breaks down and flashovers occur on high-voltage transmission lines when they are struck by lightning.) The shape of conductors also affects breakdown voltage. Breakdown occurs at lower voltages at sharp points than at blunt points. This effect is made use of in lightning arresters.

TABLE 10–2 Dielectric Strength*

Material	kV/mm
Air	3
Ceramic (high $\in_r$)	3
Mica	40
Mylar	16
Oil	15
Polystyrene	24
Rubber	18
Teflon®	60

*Values depend on the composition of the material. These are the values we use in this book.

EXAMPLE 10–5

A capacitor with plate dimensions of 2.5 cm by 2.5 cm and a ceramic dielectric with $\in_r = 7500$ experiences breakdown at 2400 V. What is C?

Solution From Table 10–2 dielectric strength = 3 kV/mm. Thus, $d = 2400\text{ V}/3000\text{ V/mm} = 0.8\text{ mm} = 8 \times 10^{-4}$ m. So

$$\begin{aligned} C &= \in_r \in_o A/d \\ &= (7500)(8.854 \times 10^{-12})(0.025\text{ m})^2/(8 \times 10^{-4}\text{ m}) \\ &= 51.9\text{ nF} \end{aligned}$$

PRACTICAL NOTES...

Breakdown in solid dielectrics is generally destructive. Typically, a carbonized pinhole results, and since this is a conductive path, the dielectric is no longer a useful insulator. Air, vacuum, and insulating liquids like oil, on the other hand, recover from the flashover once it is extinguished.

PRACTICE PROBLEMS 2

1. At what voltage will breakdown occur for a Mylar dielectric capacitor with plate spacing of 0.25 cm?
2. An air-dielectric capacitor breaks down at 500 V. If the plate spacing is doubled and the capacitor is filled with oil, at what voltage will breakdown occur?

Answers
1. 40 kV; 2. 5 kV

Capacitor Voltage Rating

Because of dielectric breakdown, capacitors are rated for maximum operating voltage (called **working voltage**) by their manufacturer (indicated on the capacitor as WVDC or working voltage dc). If you operate a capacitor beyond its working voltage, you may damage it.

10.5 Nonideal Effects

So far, we have assumed ideal capacitors. However, real capacitors have several nonideal characteristics.

Leakage Current

When a charged capacitor is disconnected from its source, it will eventually discharge. This is because no insulator is perfect, and a small amount of charge "leaks" through the dielectric. Similarly, a small leakage current will pass through its dielectric when a capacitor is connected to a source.

The effect of **leakage** is modeled by a resistor in Figure 10–12. Except for electrolytic capacitors (see Section 10.6), leakage is very small and R is very large, typically hundreds of megohms. The larger R is, the longer a capacitor can hold its charge. For most applications, leakage can be neglected, and you can treat the capacitor as ideal.

CircuitSim 10-2

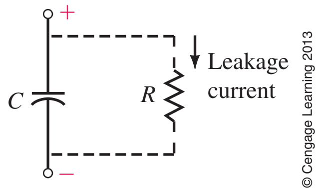


FIGURE 10–12 Leakage current.

Equivalent Series Resistance (ESR)

Real capacitors have a small amount of resistance due to the resistance of their leads, the contact connections between leads and plates, and ac losses in the dielectric. This resistance, known as **equivalent series resistance** or **ESR,** can be modeled as a resistance in series with the capacitor. Note that ESR is of concern only with ac operation—it does not affect *dc* operation. For many types of capacitors, ESR is so small that it can be neglected, but in others, ESR is a serious concern, largely because it is responsible for heat generation inside the capacitor, and heat is often the cause of capacitor failure. (ESR is mainly a problem with electrolytic capacitors.) Typically, ESR values are less than 0.1 Ω for high-frequency capacitors, but they may be 10 ohms or more for electrolytic capacitors. Note that ESR cannot be measured with an ohmmeter, but it must be measured using specialized test instruments of the type depicted in Section 10.10.

Dielectric Absorption

When you discharge a capacitor by temporarily placing a resistor across its terminals, it should have zero (or nearly zero) volts across itself after the resistor is removed. However, considerable residual voltage may spring back. To see why, recall polarization, Figure 10–11. In some dielectrics, when source voltage is removed, the relaxation of atoms to their nonpolarized state may take considerable time, and it is the residual charge from these remaining polarized atoms that creates the residual voltage. This effect is known as **dielectric absorption.** In electronic circuits, the voltage due to dielectric absorption can upset circuit voltage levels; in old CRT-type TV tubes and with electrical power apparatus, it can result in large and potentially dangerous voltages. You may have to put the shorting resistor back on to complete the discharge.

Temperature Coefficient

Because dielectrics are affected by temperature, capacitance may change with temperature. If capacitance increases with increasing temperature, the capacitor is said to have a positive temperature coefficient; if it decreases, the capacitor has a negative temperature coefficient; if it remains essentially constant, the capacitor has a zero temperature coefficient.

The **temperature coefficient** is specified as a change in capacitance in parts per million (ppm) per degree Celsius. Consider a 1-μF capacitor. Since 1 μF = 1 million pF, 1 ppm is 1 pF. Thus, a 1-μF capacitor with a temperature coefficient of 200 ppm/° C could change as much as 200 pF per degree Celsius. If the circuit you are designing is sensitive to a capacitor's value, you may have to use a capacitor with a small temperature coefficient.

10.6 Types of Capacitors

Since no single capacitor type suits all applications, capacitors are made in a variety of types and sizes. Among these are fixed and variable types with differing dielectrics and recommended areas of application. (In the following, we talk in general about recommended areas of application, but many manufacturers typically recommend where each type that they make should be used.)

Fixed Capacitors

Fixed capacitors are often identified by their dielectric. Common dielectric materials include ceramic, plastic, and mica, plus, for electrolytic capacitors, aluminum and tantalum oxide. Design variations include tubular and interleaved plates. The interleave design (Figure 10–13) uses multiple plates to increase effective plate area. A layer of insulation separates plates, and alternate plates are connected together. The tubular design (Figure 10–14) uses sheets of metal foil separated by an insulator such as plastic film. Fixed capacitors are encapsulated

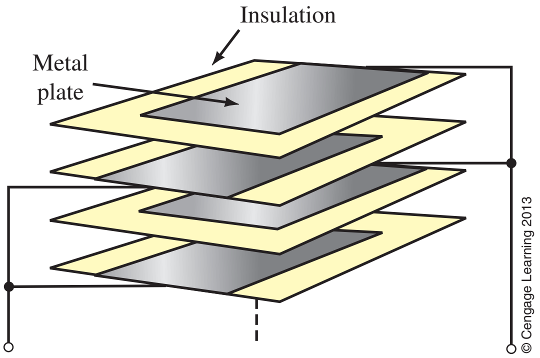


FIGURE 10–13 Stacked capacitor construction. The stack is compressed, leads attached, and the unit coated with epoxy resin or other insulating material.

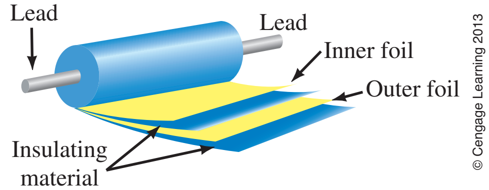


FIGURE 10–14 Tubular capacitor with axial leads.

in plastic, epoxy resin, or other insulating material and identified with value, tolerance, and other appropriate data either via body markings or color coding. Electrical characteristics and physical size depend on the dielectric used.

Ceramic Capacitors

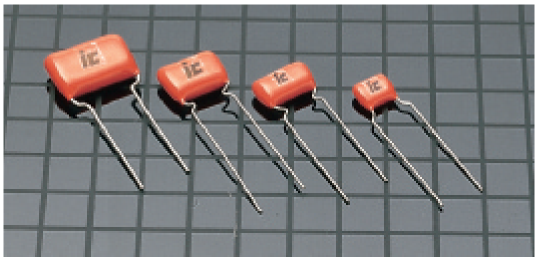
First, consider ceramic. The permittivity of ceramic varies widely (as indicated in Table 10–1). At one end are ceramics with extremely high permittivity. These permit packaging a great deal of capacitance in a small space, but they yield capacitors whose characteristics vary widely with temperature and operating voltage. However, they are popular in applications where temperature variations are modest and small size and cost are important. At the other end are ceramics with highly stable characteristics. They yield capacitors whose values change little with temperature, voltage, or aging. However, since their dielectric constants are relatively low (typically 30 to 80), these capacitors are physically larger than equivalent capacitors made using high-permittivity ceramic. Many surface mount capacitors (considered later in this section) use ceramic dielectrics, as do disc capacitors of the type illustrated in Figure 10–15. Capacitors as in Figure 10–15 are used in bypass, filtering, and dc blocking applications (tasks that you will learn about in your electronics courses).

Courtesy of Illinois Capacitor, Inc.

FIGURE 10–15 Ceramic disc capacitor.

Plastic Film Capacitors

Plastic-film capacitors are of two basic types: film/foil or metallized film. Film/foil capacitors use metal foil separated by plastic film as in Figure 10–14, while metallized-film capacitors have their foil material vacuum-deposited directly onto plastic film. Film/foil capacitors are generally larger than metallized-foil units, but have better capacitance stability and higher insulation resistance. Typical film materials are polyester, Mylar, polypropylene, and polycarbonate. Figure 10–16 shows a selection of plastic-film capacitors.



Courtesy of Illinois Capacitor, Inc.

FIGURE 10–16 Radial lead film capacitors.

Metallized-film capacitors are self-healing. Thus, if voltage stress at an imperfection exceeds breakdown, an arc occurs that evaporates the metallized area around the fault, isolating the defect. (Film/foil capacitors are not self-healing.)

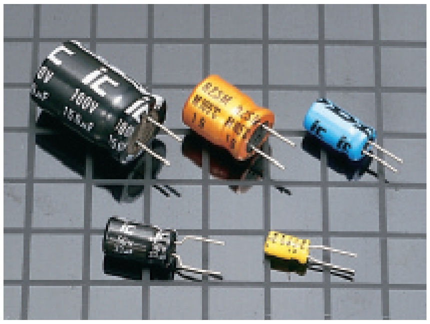
Silver Mica Capacitors

Silver mica capacitors provide high accuracy, low leakage, and good stability. Available values range from a few picofarads to about 0.1 μF.

Electrolytic Capacitors

Electrolytic capacitors provide relatively large capacitance (i.e., up to several thousand microfarads) at a relatively low cost. (Their capacitance is large because they have a very thin layer of oxide as their dielectric.) However, their leakage is relatively high and breakdown voltage relatively low. Electrolytics have either aluminum or tantalum as their plate material. Tantalum devices are smaller than aluminum devices, have less leakage, and are more stable.

The basic aluminum electrolytic capacitor construction is similar to that of Figure 10–14, with strips of aluminum foil separated by gauze saturated with an electrolyte. During manufacture, chemical action creates a thin oxide layer that acts as the dielectric. This layer must be maintained during use. For this reason, electrolytic capacitors are polarized (marked with a + and − sign), and the plus (+) terminal must always be kept positive with respect to the minus (−) terminal. Aluminum electrolytic capacitors have a shelf life; that is, if they are not used for an extended period, they may fail when powered up again. Figure 10–17(a) shows a selection of typical electrolytic capacitors, while Figure 10–17(b) shows their graphic symbol as used on circuit schematics. These capacitors are typically used in power supplies to smooth rectified waveforms.



(a) Radial lead aluminium electrolytic capacitors

Photo courtesy of Illinois Capacitor Inc.

(b) Schematic symbol for an electrolytic capacitor

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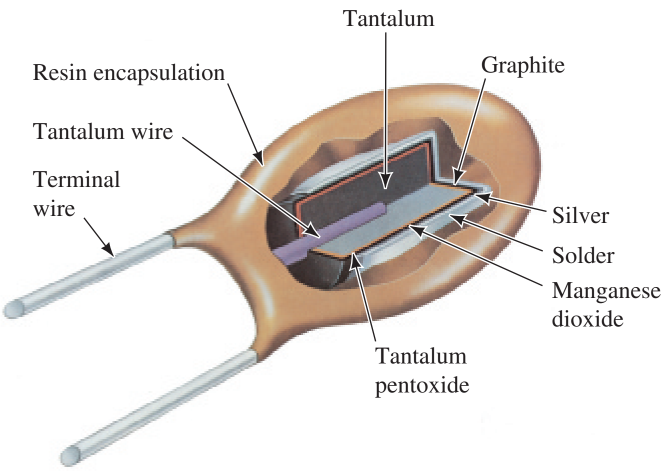
FIGURE 10–17 Electrolytic capacitors.

Tantalum capacitors come in two basic types: wet slug and solid dielectric. Figure 10–18 shows a cutaway view of a solid tantalum unit. The slug, made from powdered tantalum, is highly porous and provides a large internal surface area that is coated with an oxide to form the dielectric. Tantalum capacitors are polarized and must be inserted into a circuit properly.

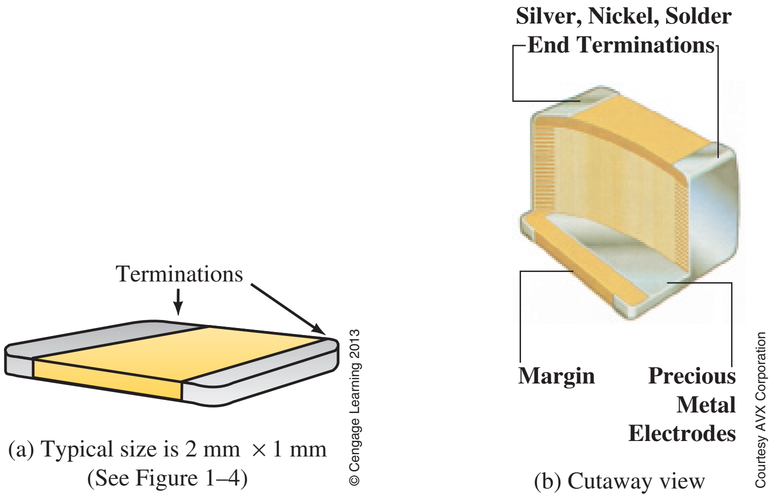


Courtesy AVX Corporation

FIGURE 10–18 Cutaway view of a solid tantalum capacitor.

Surface Mount Capacitors

Most electronic products now use **surface mount devices** (SMDs). (SMDs do not have connection leads, but are soldered directly onto printed circuit boards by automated assembly machines.) Figure 10–19 shows a surface mount, ceramic chip capacitor. Such devices are extremely small and provide high packaging density as can be seen in Figure 10–30 later in this chapter.



(a) Typical size is 2 mm × 1 mm (See Figure 1–4)

(b) Cutaway view

FIGURE 10–19 Surface mount, ceramic chip capacitor.

Variable Capacitors

Several types of variable capacitors are still in use. One type (used for many years in radios to tune in stations) has a set of stationary plates and a set of movable plates that are ganged together and mounted on a shaft. As the user rotates the shaft, the movable plates mesh with the stationary plates, changing the effective surface area (and hence the capacitance).

Another adjustable type is the trimmer or padder capacitor, which is used for fine adjustments, usually over a very small range. A trimmer is usually set to its required value, then never touched again.

The symbol for a variable capacitor may be found in the table of schematic symbols, Table 1–7 of Chapter 1.

Standard Capacitor Values

Capacitors are manufactured only in specific standard value sizes, specifically 0.1 μF, 0.22 μF, 0.47 μF, and so on, plus multiples and submultiples thereof such as 0.1 pF, 0.22 pF, and 0.47 pF.

Supercapacitors

A relatively recent entry among capacitor offerings are **supercapacitors.** These devices, also known as ultracapacitors, are available with enormous capacitance values—values that extend up into the hundreds of farads range and beyond. (By comparison, the largest electrolytic capacitors usually do not exceed a few thousand microfarads.) The areas of application of supercapacitors are somewhat different than ordinary capacitors, however—typically, they are used in power source roles (roles normally serviced by batteries)—in GPS tracking systems, automotive regenerative braking systems, medical equipment, PDAs, and alarm and security systems. A common setup utilizes a supercapacitor in conjunction with a battery, wherein the battery supplies the steady bulk power demands of the load while the supercapacitor supplies the high-energy bursts that are required from time to time.

Most major capacitor manufacturers now have supercapacitors in their product lines. As indicated in Figure 10–20, from the outside, the capacitor looks like an ordinary capacitor. It achieves its massive capacitance through its internal construction, which creates an enormous amount of surface area with a very small separation between "plates." However, their voltage rating is quite low, typically only a few volts.

FIGURE 10–20 Supercapacitors. Most supercapacitors look pretty much like ordinary capacitors, but don't let that fool you—their storage capacity is immense. Here we show a pair of flat pack supercapacitors.

PRACTICE PROBLEMS 3

A 2.5-μF capacitor has a tolerance of +80% and −20%. Determine what its maximum and minimum values could be.

Answer
4.5 μF and 2 μF

10.7 Capacitors in Parallel and Series

Capacitors in Parallel

For capacitors in parallel, the effective plate area is the sum of the individual plate areas; thus, the total capacitance is the sum of the individual capacitances. This is easily shown mathematically. Consider Figure 10–21. The charge on each capacitor is given by Equation 10–1, that is, $Q_1 = C_1V$ and $Q_2 = C_2V$. Thus, $Q_T = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)\,V$. But $Q_T = C_TV$. Thus, $C_T = C_1 + C_2$. For more than two capacitors,

$$C_T = C_1 + C_2 + \cdots + C_N \qquad \textbf{(10–14)}$$

That is, *the total capacitance of capacitors in parallel is the sum of their individual capacitances.*

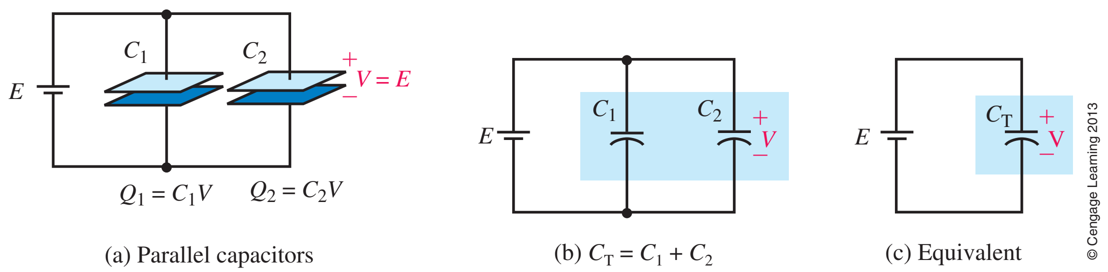


FIGURE 10–21 Capacitors in parallel. Total capacitance is the sum of the individual capacitances.

EXAMPLE 10–6

A 10-μF, a 15-μF, and a 100-μF capacitor are connected in parallel across a 50-V source. Determine the following:

a. Total capacitance.
b. Total charge stored.
c. Charge on each capacitor.

Solution

a. $C_T = C_1 + C_2 + C_3 = 10\ \mu F + 15\ \mu F + 100\ \mu F = 125\ \mu F$

b. $Q_T = C_T V = (125\ \mu F)(50\ V) = 6.25\ mC$

c. $Q_1 = C_1 V = (10\ \mu F)(50\ V) = 0.5\ mC$

$Q_2 = C_2 V = (15\ \mu F)(50\ V) = 0.75\ mC$

$Q_3 = C_3 V = (100\ \mu F)(50\ V) = 5.0\ mC$

Check: $Q_T = Q_1 + Q_2 + Q_3 = (0.5 + 0.75 + 5.0)\ mC = 6.25\ mC$.

PRACTICE PROBLEMS 4

1. Three capacitors are connected in parallel. If $C_1 = 20\ \mu F$, $C_2 = 10\ \mu F$, and $C_T = 32.2\ \mu F$, what is C_3?
2. Three capacitors are paralleled across an 80-V source, with $Q_T = 0.12$ C. If $C_1 = 200\ \mu F$ and $C_2 = 300\ \mu F$, what is C_3?
3. Three capacitors are paralleled. If the value of the second capacitor is twice that of the first and the value of the third is one-quarter that of the second and the total capacitance is 70 μF, what are the values of each capacitor?

Answers
1. 2.2 μF; 2. 1000 μF; 3. 20 μF, 40 μF, and 10 μF

Capacitors in Series

For capacitors in series (Figure 10–22), the same charge appears on each. Thus, $Q = C_1V_1$, $Q = C_2V_2$, and so on. Solving for voltages yields $V_1 = Q/C_1$, $V_2 = Q/C_2$, and so on. Applying KVL, we get $V = V_1 + V_2 + \cdots + V_N$. Therefore,

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_N} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)$$

But $V = Q/C_T$. Equating this with the right side and cancelling Q yields

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \qquad \textbf{(10–15)}$$

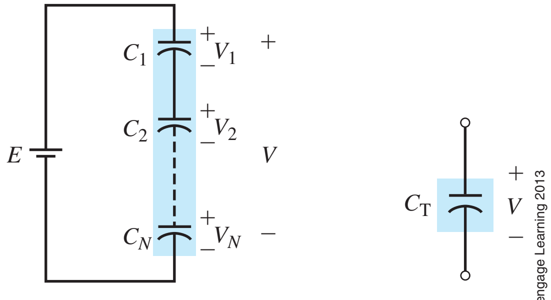
For two capacitors in series, this reduces to

$$C_T = \frac{C_1C_2}{C_1 + C_2} \qquad \textbf{(10–16)}$$

NOTES...

1. For capacitors in parallel, total capacitance is always larger than the largest capacitance, while for capacitors in series, total capacitance is always smaller than the smallest capacitance.
2. The formula for capacitors in parallel is similar to the formula for resistors in series, while the formula for capacitors in series is similar to the formula for resistors in parallel.

For N equal capacitors in series, Equation 10–15 yields $C_T = C/N$.



(a) Series connection (b) Equivalent

FIGURE 10–22 Capacitors in series: $\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots + \dfrac{1}{C_N}$.

EXAMPLE 10–7

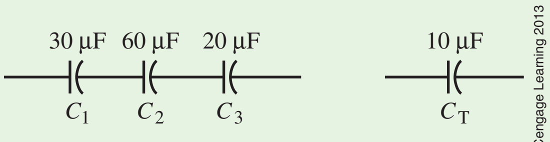
> **Calculator Hint:** You can also use your calculator to evaluate Equation 10–15 directly. For the capacitors of Example 10–7, the solution for C_T would look similar to the following, depending on the make and model of your calculator.
>
> ```
> 30⁻¹+60⁻¹+20⁻¹
>                .1
> Ans⁻¹
>                10
> ```
>
> Thus, C_T = 10 μF.

Refer to Figure 10–23(a):

a. Determine C_T.

b. If 50 V is applied across the capacitors, determine Q.

c. Determine the voltage on each capacitor.



(a) (b)

FIGURE 10–23

Solution

a. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{30\ \mu\text{F}} + \frac{1}{60\ \mu\text{F}} + \frac{1}{20\ \mu\text{F}}$$

$$= 0.0333 \times 10^6 + 0.0167 \times 10^6 + 0.05 \times 10^6 = 0.1 \times 10^6$$

Therefore as indicated in (b),

$$C_T = \frac{1}{0.1 \times 10^6} = 10\ \mu\text{F}$$

b. $Q = C_T V = (10 \times 10^{-6}\ \text{F})(50\ \text{V}) = 0.5\ \text{mC}$

c. $V_1 = Q/C_1 = (0.5 \times 10^{-3}\ \text{C})/(30 \times 10^{-6}\ \text{F}) = 16.7\ \text{V}$

$V_2 = Q/C_2 = (0.5 \times 10^{-3}\ \text{C})/(60 \times 10^{-6}\ \text{F}) = 8.3\ \text{V}$

$V_3 = Q/C_3 = (0.5 \times 10^{-3}\ \text{C})/(20 \times 10^{-6}\ \text{F}) = 25.0\ \text{V}$

Check: $V_1 + V_2 + V_3 = 16.7 + 8.3 + 25 = 50$ V.

EXAMPLE 10–8

For the circuit of Figure 10–24(a), determine C_T.

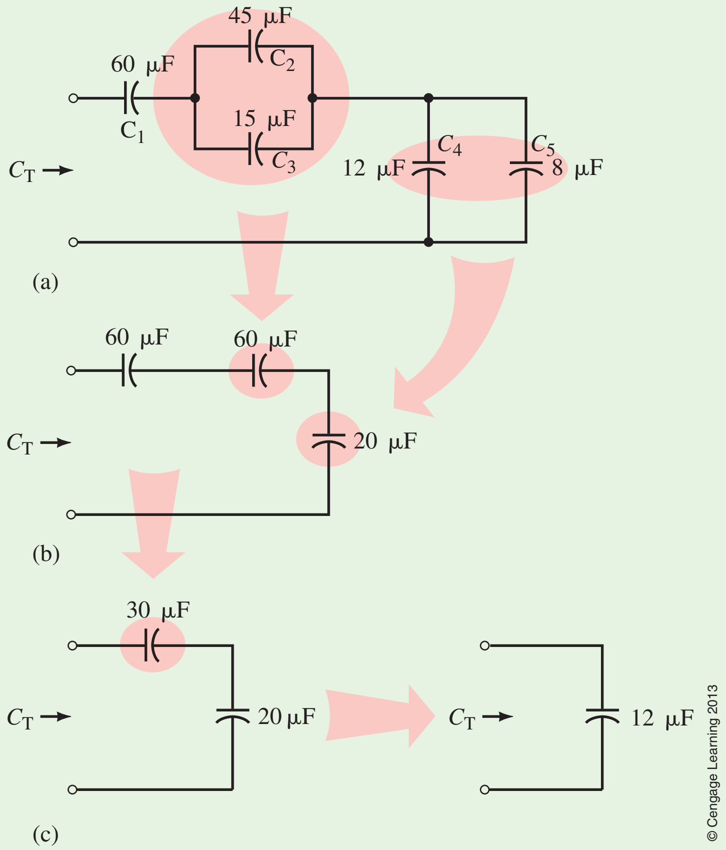


FIGURE 10–24 Systematic reduction.

Solution The problem is easily solved through step-by-step reduction. C_2 and C_3 in parallel yield 45 μF + 15 μF = 60 μF. C_4 and C_5 in parallel total 20 μF. The reduced circuit is shown in (b). The two 60-μF capacitances in series reduce to 30 μF. The series combination of 30 μF and 20 μF can be found from Equation 10–16. Thus,

$$C_T = \frac{30\ \mu\text{F} \times 20\ \mu\text{F}}{30\ \mu\text{F} + 20\ \mu\text{F}} = 12\ \mu\text{F}$$

Alternately, use your calculator to reduce (b) as described in Example 10–7.

Voltage Divider Rule for Series Capacitors

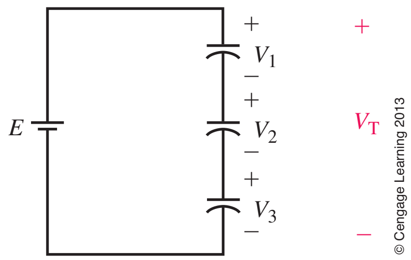


FIGURE 10–25 Capacitive voltage divider.

For capacitors in series (Figure 10–25) a simple voltage divider rule can be developed. Recall, for individual capacitors, $Q_1 = C_1V_1$, $Q_2 = C_2V_2$, and so on, and for the complete string, $Q_T = C_TV_T$. As noted earlier, $Q_1 = Q_2 = \cdots = Q_T$. Thus, $C_1V_1 = C_TV_T$. Solving for V_1 yields

$$V_1 = \left(\frac{C_T}{C_1}\right)V_T$$

This type of relationship holds for all capacitors. Thus,

$$V_x = \left(\frac{C_T}{C_x}\right)V_T \quad \text{(10–17)}$$

From this, you can see that the voltage across a capacitor is inversely proportional to its capacitance, that is, the smaller the capacitance, the larger the voltage, and vice versa. Other useful variations are

$$V_1 = \left(\frac{C_2}{C_1}\right)V_2, \quad V_1 = \left(\frac{C_3}{C_1}\right)V_3, \quad V_2 = \left(\frac{C_3}{C_2}\right)V_3, \quad \text{and so on.}$$

PRACTICE PROBLEMS 5

1. Verify the voltages of Example 10–7 using the voltage divider rule for capacitors.
2. Determine the voltage across each capacitor of Figure 10–24 if the voltage across C_5 is 30 V.

Answers
1. $V_1 = 16.7$ V, $V_2 = 8.3$ V, $V_3 = 25.0$ V; 2. $V_1 = 10$ V, $V_2 = V_3 = 10$ V, $V_4 = V_5 = 30$ V

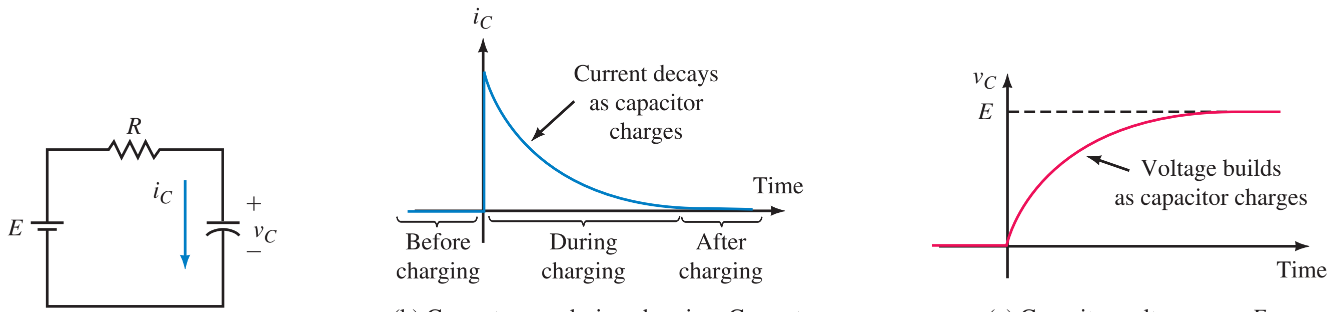
10.8 Capacitor Current and Voltage During Charging

As noted earlier (Figure 10–2), during charging, electrons are moved from one plate of a capacitor to the other plate. Several points should be noted.

1. This movement of electrons constitutes a current.
2. This current lasts only long enough for the capacitor to charge. When the capacitor is fully charged, current is zero.
3. Current in the circuit during charging is due solely to the movement of electrons from one plate to the other around the external circuit through the battery; no current passes through the dielectric between the plates.
4. As charge is deposited on the plates, the capacitor voltage builds. However, this voltage does not jump to full value immediately since it takes time to move electrons from one plate to the other. (Billions of electrons must be moved.)
5. Since voltage builds up as charging progresses, the difference in voltage between the source and the capacitor decreases and hence the rate of movement of electrons (i.e., the current) decreases as the capacitor approaches full charge.

Figure 10–26 shows what the voltage and current look like during the charging process. As indicated, the current starts out with an initial surge, then decays to zero while the capacitor voltage gradually climbs from zero to full voltage. The charging time typically ranges from nanoseconds to milliseconds, depending on the resistance and capacitance of the circuit. (We study these relationships in detail in Chapter 11.) A similar surge (but in the opposite direction) occurs during discharge.

As Figure 10–26 indicates, current exists only while the capacitor voltage is changing. This observation turns out to be true in general, that is, *current in a capacitor exists only while capacitor voltage is changing*. The reason is not



(a)

(b) Current surge during charging. Current is 0 when fully charged.

(c) Capacitor voltage. $v_C = E$ when fully charged.

FIGURE 10–26 The capacitor does not charge instantaneously, as a finite amount of time is required to move electrons around the circuit.

CircuitSim 10-3

hard to understand. As you saw before, a capacitor's dielectric is an insulator, and consequently no current can pass through it (assuming zero leakage). The only charges that can move, therefore, are the free electrons that exist on the capacitor's plates. When capacitor voltage is constant, these charges are in equilibrium, no net movement of charge occurs, and the current is thus zero. However, if the source voltage is increased, additional electrons are pulled from the positive plate; inversely, if the source voltage is decreased, excess electrons on the negative plate are returned to the positive plate. Thus, in both cases, capacitor current results when capacitor voltage is changed. As we show next, this current is proportional to the rate of change of voltage. Before we do this, however, we need to look at symbols.

Symbols for Time-Varying Voltages and Currents

Quantities that vary with time are called **instantaneous quantities.** *Standard industry practice requires that we use lowercase letters for time-varying quantities, rather than capital letters as for dc.* Thus, we use v_C and i_C to represent changing capacitor voltage and current rather than V_C and I_C. (Often we drop the subscripts and just use v and i.) Since these quantities are functions of time, they may also be shown as $v_C(t)$ and $i_C(t)$.

Capacitor *v-i* Relationship

The relationship between charge and voltage for a capacitor is given by Equation 10–1. For the time-varying case, it is

$$q = Cv_C \tag{10–18}$$

But current is the rate of movement of charge. In calculus notation, this is $i_C = dq/dt$. Differentiating Equation 10–18 yields

$$i_C = \frac{dq}{dt} = \frac{d}{dt}(Cv_C) \tag{10–19}$$

Since C is constant, we get

$$i_C = C\frac{dv_C}{dt} \quad \text{(A)} \tag{10–20}$$

Equation 10–20 shows that *current through a capacitor is equal to C times the rate of change of voltage across it.* This means that the faster the voltage

NOTES...

Calculus is introduced at this point to aid in the development of ideas and to help explain concepts. However, not everyone who uses this book knows calculus. Therefore, the material is presented in such a manner that it never relies entirely on mathematics; thus, where calculus is used, intuitive explanations accompany it. However, to provide the enrichment that calculus offers, optional derivations and problems are included, but they are marked with a ∫ icon so that they may be omitted if desired.

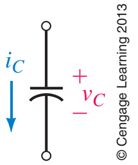


FIGURE 10–27 The + sign for v_C goes at the tail of the current arrow.

changes, the larger the current, and vice versa. It also means that if the voltage is constant, the current is zero (as we noted earlier).

Reference conventions for voltage and current are shown in Figure 10–27. As usual, the plus sign goes at the tail of the current arrow. If the voltage is increasing, dv_C/dt is positive and the current is in the direction of the reference arrow; if the voltage is decreasing, dv_C/dt is negative and the current is opposite to the arrow.

The derivative dv_C/dt of Equation 10–20 is the slope of the capacitor voltage versus time curve. When capacitor voltage varies linearly with time (i.e., the relationship is a straight line as in Figure 10–28), Equation 10–20 reduces to

$$i_C = C\frac{\Delta v_C}{\Delta t} = C\frac{\text{rise}}{\text{run}} = C \times \text{slope of the line} \qquad \textbf{(10–21)}$$

EXAMPLE 10–9

A signal generator applies voltage to a 5-μF capacitor with a waveform as in Figure 10–28(a). The voltage rises linearly from 0 V to 10 V in 1 ms, falls linearly to −10 V at $t = 3$ ms, remains constant until $t = 4$ ms, rises to 10 V at $t = 5$ ms, and remains constant thereafter.

a. Determine the slope of v_C in each time interval.

b. Determine the current and sketch its graph.

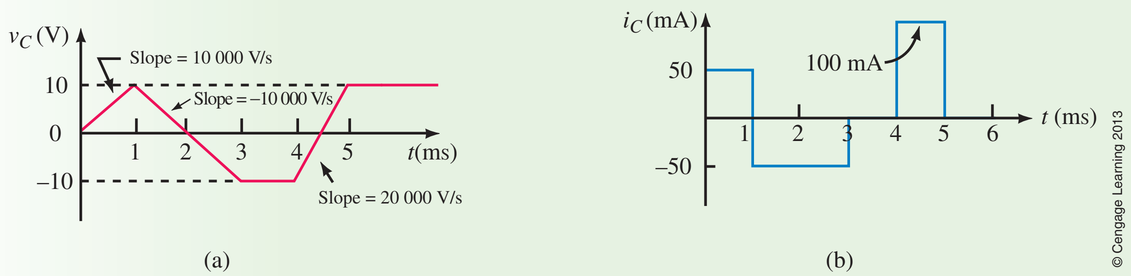


FIGURE 10–28

Solution

a. We need the slope of v_C during each time interval where slope = rise/run = $\Delta v/\Delta t$.

0 ms to 1 ms: $\Delta v = 10$ V; $\Delta t = 1$ ms; therefore, slope $= 10$ V/1 ms $=$ 10 000 V/s.

1 ms to 3 ms: Slope $= -20$ V/2 ms $= -10\,000$ V/s.

3 ms to 4 ms: Slope $= 0$ V/s.

4 ms to 5 ms: Slope $= 20$ V/1 ms $= 20\,000$ V/s.

CircuitSim 10-4

b. $i_C = Cdv_C/dt = C$ times slope. Thus,

0 ms to 1 ms: $i = (5 \times 10^{-6}\text{ F})(10\,000\text{ V/s}) = 50$ mA.

1 ms to 3 ms: $i = -(5 \times 10^{-6}\text{ F})(10\,000\text{ V/s}) = -50$ mA.

3 ms to 4 ms: $i = (5 \times 10^{-6}\text{ F})(0\text{ V/s}) = 0$ A.

4 ms to 5 ms: $i = (5 \times 10^{-6}\text{ F})(20\,000\text{ V/s}) = 100$ mA.

The current is plotted in Figure 10–28(b).

EXAMPLE 10–10

The voltage across a 20-μF capacitor is $v_C = 100\,t\,e^{-t}$ V. Determine current i_C.

CircuitSim 10-5

Solution Differentiation by parts using $\frac{d(uv)}{dt} = u\frac{dv}{dt} + v\frac{du}{dt}$ with $u = 100\,t$ and $v = e^{-t}$ yields

$$i_C = C\frac{d}{dt}(100\ \ t\,e^{-t}) = 100\,C\frac{d}{dt}(t\,e^{-t}) = 100\,C\left(t\frac{d}{dt}(e^{-t}) + e^{-t}\frac{dt}{dt}\right)$$
$$= 2000 \times 10^{-6}(-t\,e^{-t} + e^{-t})\text{ A} = 2.0\,(1 - t)e^{-t}\text{ mA}$$

CircuitSim 10-6

10.9 Energy Stored by a Capacitor

An ideal capacitor does not dissipate power. When power is transferred to a capacitor, all of it is stored as energy in the capacitor's electric field. When the capacitor is discharged, this stored energy is returned to the circuit. You will find that this is an important observation when you study ac in Chapter 15.

To determine the stored energy, consider Figure 10–29. Power is given by $p = vi$ watts. Using calculus (see), it can be shown that the stored energy is given by

$$W = \frac{1}{2}CV^2 \quad \text{(J)} \tag{10–22}$$

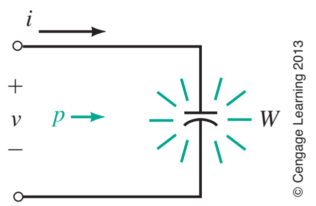


FIGURE 10–29 Storing energy in a capacitor.

where V is the voltage across the capacitor. This means that the energy at any time depends on the value of the capacitor's voltage at that time. We will leave this topic at this time, but return to it again in Chapter 17 and later chapters where you encounter it in an ac context.

Deriving Equation 10–22

Power to the capacitor (Figure 10–29) is given by $p = vi$, where $i = Cdv/dt$. Therefore, $p = Cvdv/dt$. However, $p = dW/dt$. Equate the two values of p, then after a bit of manipulation, you can integrate. Thus,

$$W = \int_0^t pdt = C\int_0^t v\frac{dv}{dt}\,dt = C\int_0^V vdv = \frac{1}{2}CV^2$$

10.10 Capacitor Failures and Troubleshooting

Although capacitors are quite reliable, they may fail because of misapplication, excessive voltage, current, temperature, or simply because they age. They can short internally, leads may become open, dielectrics may become excessively leaky, and they may fail catastrophically due to incorrect use. (If an electrolytic capacitor is connected with its polarity reversed, for example, it may explode.) Capacitors should be used well within their rating limits. Excessive voltage can lead to dielectric puncture creating pinholes that short the plates together. High temperatures may cause an increase in leakage and/or a permanent shift in capacitance. High temperatures may be caused by inadequate heat removal, excessive current, lossy dielectrics, or an operating frequency beyond the capacitor's rated limit. Generally, capacitors are so inexpensive that you simply replace them if you suspect they are faulty. To help locate faulty capacitors, you can sometimes use an ohmmeter or a capacitor tester, as described next.

Basic Testing with an Ohmmeter

Some basic (out-of-circuit) tests can be made with an analog ohmmeter. The ohmmeter can detect opens and shorts and, to a certain extent, leaky dielectrics. First, ensure that the capacitor is discharged, then set the ohmmeter to its highest range and connect it to the capacitor. (For electrolytic devices, ensure that the plus [+] side of the ohmmeter is connected to the plus [+] side of the capacitor.)

Initially, the ohmmeter reading should be low, then for a good capacitor gradually increase to infinity as the capacitor charges through the ohmmeter circuit. (Or at least a very high value, since most good capacitors, except electrolytics, have a resistance of hundreds of megohms.) (For small capacitors, this test is probably useless, as the time for charging may be too short to yield useful results.)

Faulty capacitors respond differently. If a capacitor is shorted, the meter resistance reading will stay low. If it is leaky, the reading will be lower than normal. If it is open circuited, the meter will indicate infinity immediately, without dipping to zero when first connected.

NOTES...

Quality factor and dissipation factor as applied to capacitor quality are specialized topics that are beyond the scope of this book, and thus they are not dealt with here. Manuals (such as that for the 889B as found on the manufacturer's Web site) are often good sources of information. However, wait until you have studied Chapter 21 of this book before you try to understand the concepts.

Capacitor Testers

Ohmmeter testing is of limited usefulness as it tells you nothing about capacitor quality issues such as ESR. For investigating these, more sophisticated tester/analyzers are needed. Figure 10–30 shows two. The *Smart Tweezers™* device of (a) is designed primarily for testing surface mount components (such as that shown in Figure 10–19). It is a fully featured LCR tester that can test capacitors, resistors, and inductors. For capacitor testing, it determines capacitance, ESR, and dissipation factor, and also checks for shorts. As indicated, values are displayed directly in standard units. The *BK Precision 889B* bench mount instrument of (b) is an LCR/ESR component tester designed to test inductors, capacitors, and resistors. In terms of capacitor testing, it measures capacitance, determines leakage, dielectric absorption, ESR, quality factor Q and more—see Note.

(a) These Smart Tweezers™ measure capacitance, inductance, and resistance plus secondary parameters such as capacitor ESR.

Photo courtesy of Advance Devices, Inc.

(b) This LCR/ESR tester/analyzer features a USB interface for connection to a computer.

Courtesy of B & K Precision

FIGURE 10–30 Instruments for capacitor testing and troubleshooting.

Problems

10.1 Capacitance

1. For Figure 10–31, determine the charge on the capacitor, its capacitance, or the voltage across it as applicable for each of the following.
   a. $E = 40$ V, $C = 20$ μF
   b. $V = 500$ V, $Q = 1000$ μC
   c. $V = 200$ V, $C = 500$ nF
   d. $Q = 3 \times 10^{-4}$ C, $C = 10 \times 10^{-6}$ F
   e. $Q = 6$ mC, $C = 40$ μF
   f. $V = 1200$ V, $Q = 1.8$ mC
2. Repeat Question 1 for the following:
   a. $V = 2.5$ kV, $Q = 375$ μC
   b. $V = 1.5$ kV, $C = 0.04 \times 10^{-4}$ F
   c. $V = 150$ V, $Q = 6 \times 10^{-5}$ C
   d. $Q = 10$ μC, $C = 400$ nF
   e. $V = 150$ V, $C = 40 \times 10^{-5}$ F
   f. $Q = 6 \times 10^{-9}$ C, $C = 800$ pF
3. The charge on a 50-μF capacitor is 10×10^{-3} C. What is the potential difference between its terminals?
4. When 10 μC of charge is placed on a capacitor, its voltage is 25 V. What is the capacitance?
5. You charge a 5-μF capacitor to 150 V. Your lab partner then momentarily places a resistor across its terminals and bleeds off enough charge that its voltage falls to 84 V. What is the final charge on the capacitor?

FIGURE 10–31

10.2 Factors Affecting Capacitance

6. A capacitor with circular plates 0.1 m in diameter and an air dielectric has 0.1 mm spacing between its plates. What is its capacitance?
7. A parallel-plate capacitor with a mica dielectric has dimensions of 1 cm × 1.5 cm and separation of 0.1 mm. What is its capacitance?
8. For the capacitor of Problem 7, if the mica is removed, what is its new capacitance?
9. The capacitance of an oil-filled capacitor is 200 pF. If the separation between its plates is 0.1 mm, what is the area of its plates?
10. A 0.01-μF capacitor has ceramic with a dielectric constant of 7500. If the ceramic is removed, the plate separation doubled, and the spacing between plates filled with oil, what is the new value for *C?*
11. A capacitor with a Teflon dielectric has a capacitance of 33 μF. A second capacitor with identical physical dimensions but with a Mylar dielectric carries a charge of 55×10^{-4} C. What is its voltage?
12. The plate area of a capacitor is 4.5 in.^{2} and the plate separation is 5 mils. If the relative permittivity of the dielectric is 80, what is *C?*

10.3 Electric Fields

13. a. What is the electric field strength $\mathscr{E}$ at a distance of 1 cm from a 100-mC charge in transformer oil?
    b. What is $\mathscr{E}$ at twice the distance?
14. Suppose that 150 V is applied across a 100-pF parallel-plate capacitor whose plates are separated by 1 mm. What is the electric field intensity $\mathscr{E}$ between the plates?

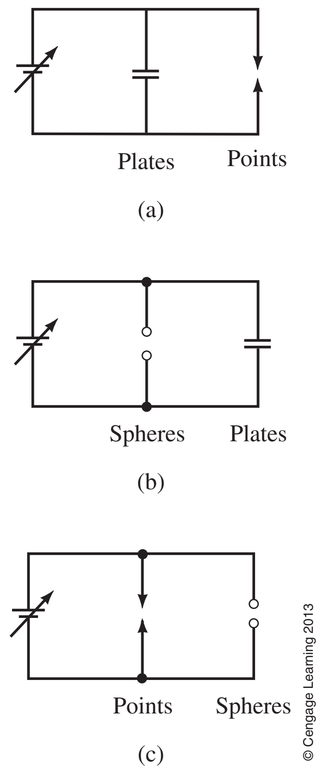


FIGURE 10–32 Source voltage is increased until one of the gaps breaks down. (The source has high internal resistance to limit current following breakdown.)

10.4 Dielectrics

15. An air-dielectric capacitor has plate spacing of 1.5 mm. How much voltage can be applied before breakdown occurs?
16. Repeat Problem 15 if the dielectric is mica and the spacing is 2 mils.
17. A mica-dielectric capacitor breaks down when E volts is applied. The mica is removed and the spacing between plates doubled. If breakdown now occurs at 500 V, what is E?
18. Determine at what voltage the dielectric of a 200 nF Mylar capacitor with a plate area of 0.625 m^2 will break down.
19. Figure 10–32 shows several gaps, including a parallel-plate capacitor, a set of small spherical points, and a pair of sharp points. The spacing is the same for each. As the voltage is increased, which gap breaks down for each case?
20. If you continue to increase the source voltage of Figures 10–32(a), (b), and (c) after a gap breaks down, will the second gap also break down? Justify your answer.

10.5 Nonideal Effects

21. A 25-μF capacitor has a negative temperature coefficient of 175 ppm/° C. By how much and in what direction might it vary if the temperature rises by 50° C? What would be its new value?
22. If a 4.7-μF capacitor changes to 4.8 μF when the temperature rises 40° C, what is its temperature coefficient?

10.7 Capacitors in Parallel and Series

23. What is the equivalent capacitance of 10 μF, 12 μF, 22 μF, and 33 μF connected in parallel?
24. What is the equivalent capacitance of 0.10 μF, 220 nF, and 4.7×10^{-7} F connected in parallel?
25. Repeat Problem 23 if the capacitors are connected in series.
26. Repeat Problem 24 if the capacitors are connected in series.
27. Determine C_T for each circuit of Figure 10–33.
28. Determine total capacitance looking at the terminals for each circuit of Figure 10–34.
29. A 30-μF capacitor is connected in parallel with a 60-μF capacitor, and a 10-μF capacitor is connected in series with the parallel combination. What is C_T?
30. For Figure 10–35, determine C_x.
31. For Figure 10–36, determine C_3 and C_4.
32. For Figure 10–37, determine C_T.
33. You have capacitors of 22 μF, 47 μF, 2.2 μF, and 10 μF. Connecting these any way you want, what is the largest equivalent capacitance you can get? The smallest?
34. A 10-μF and a 4.7-μF capacitor are connected in parallel. After a third capacitor is added to the circuit, $C_T = 2.695$ μF. What is the value of the third capacitor? How is it connected?
35. Consider capacitors of 1 μF, 1.5 μF, and 10 μF. If $C_T = 10.6$ μF, how are the capacitors connected?

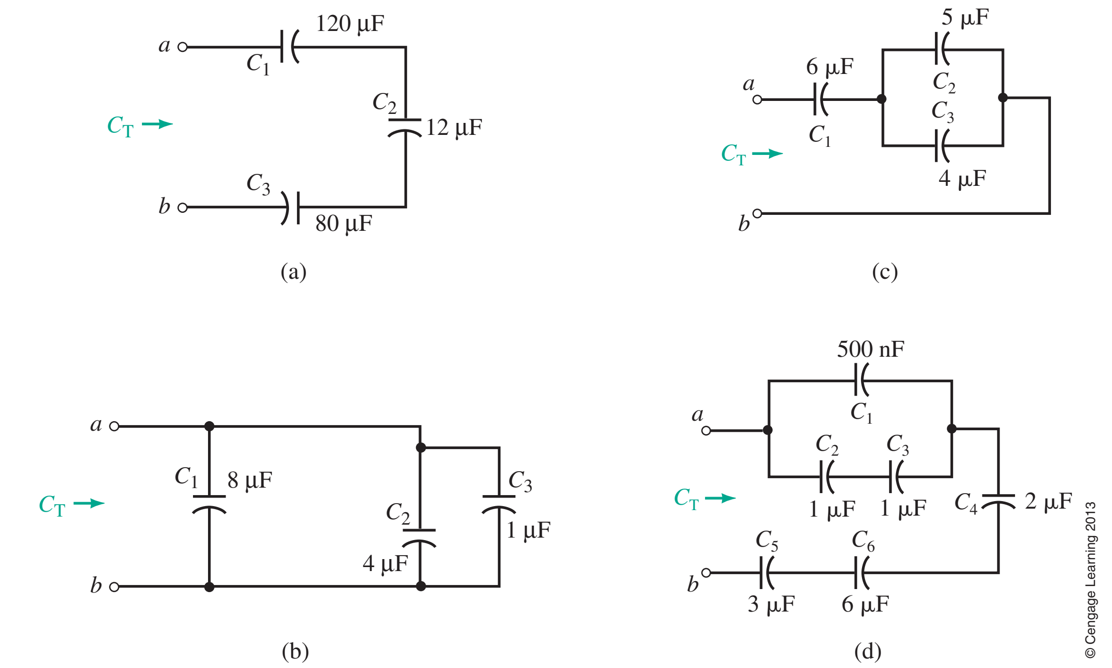


FIGURE 10–33

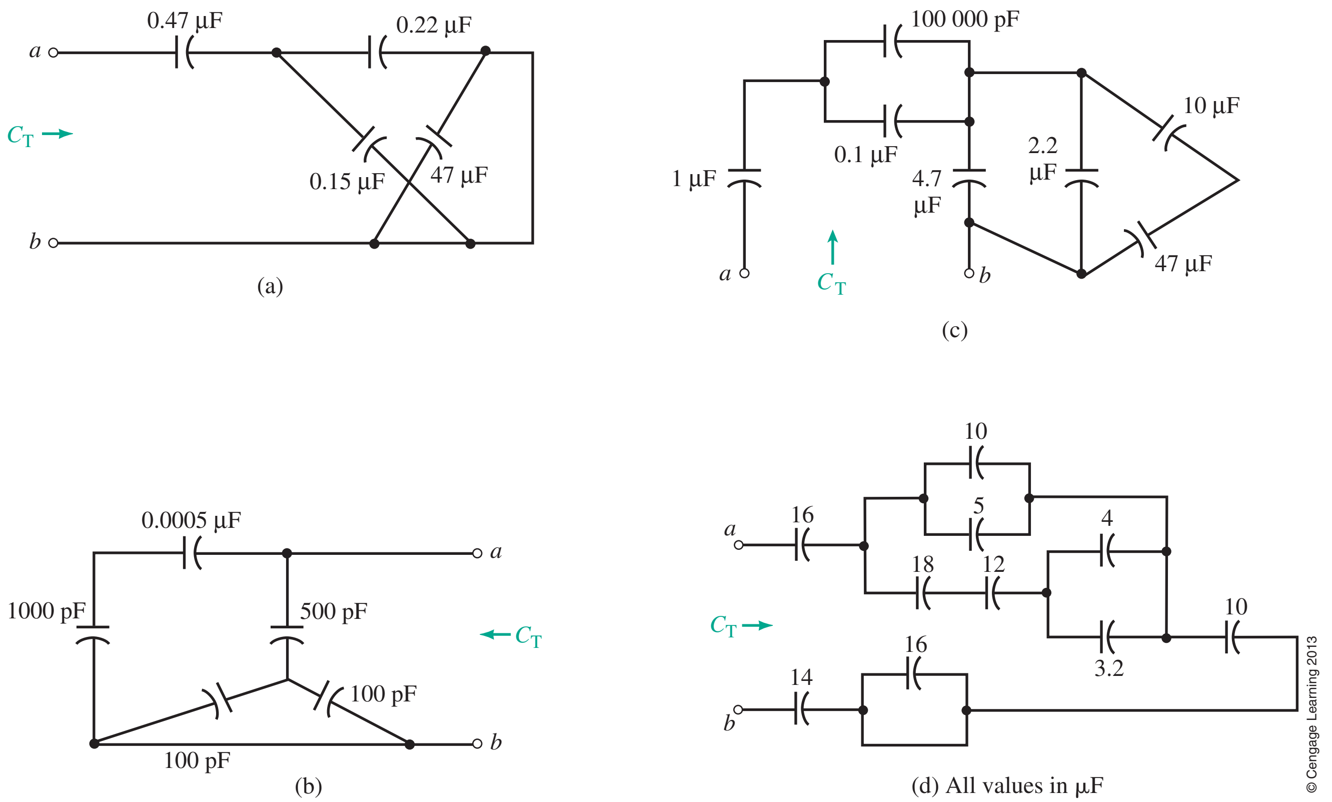


FIGURE 10–34

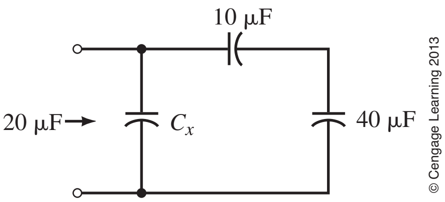


FIGURE 10–35

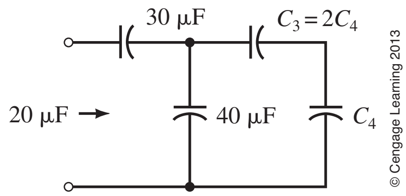


FIGURE 10–36

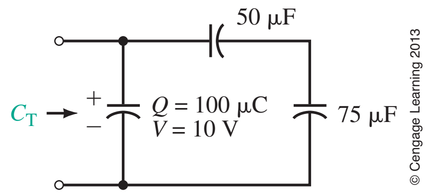


FIGURE 10–37

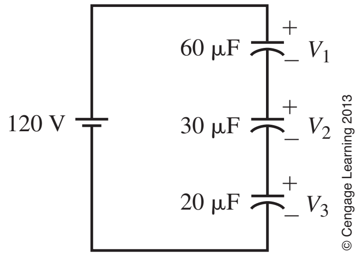


FIGURE 10–38

36. For the capacitors of Problem 35, if $C_T = 2.304$ μF, how are the capacitors connected?
37. For Figures 10–33(c) and (d), find the voltage on each capacitor if 100 V is applied to terminals *a–b*.
38. Use the voltage divider rule to find the voltage across each capacitor of Figure 10–38.
39. Repeat Problem 38 for the circuit of Figure 10–39.
40. For Figure 10–40, $V_x = 50$ V. Determine C_x and C_T.
41. For Figure 10–41, determine C_x.
42. A dc source is connected to terminals *a–b* of Figure 10–35. If C_x is 12 μF and the voltage across the 40-μF capacitor is 80 V,
    a. What is the source voltage?
    b. What is the total charge on the capacitors?
    c. What is the charge on each individual capacitor?

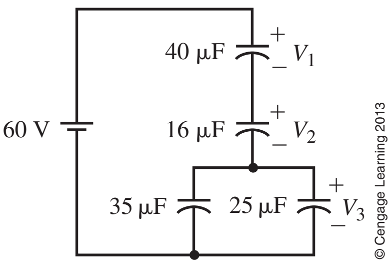


FIGURE 10–39

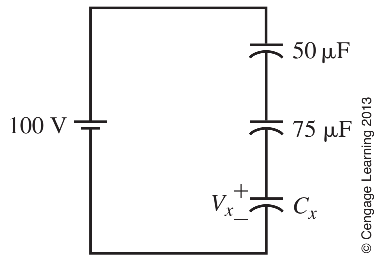


FIGURE 10–40

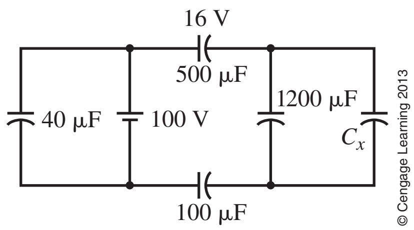


FIGURE 10–41

10.8 Capacitor Current and Voltage During Charging

43. The voltage across the capacitor of Figure 10–42(a) is shown in (b). Sketch current i_C scaled with numerical values.
44. The current through a 1-μF capacitor is shown in Figure 10–43. Sketch voltage v_C scaled with numerical values. Voltage at $t = 0$ s is 0 V.
45. If the voltage across a 4.7-μF capacitor is $v_C = 100e^{-0.05t}$ V, what is i_C?
46. Current through a 0.1-μF capacitor is $i_C = t\,e^{-t}$ μA. If the capacitor voltage is 0 V at $t = 0$, determine the equation for v_C. Hint: Integration of Equation 10–20 is required.

10.9 Energy Stored by a Capacitor

47. For Figure 10–42, determine the capacitor's energy at each of the following times: $t = 0$, 1 ms, 4 ms, 5 ms, 7 ms, and 9 ms.
48. For the circuit of Figure 10–38, determine the energy stored in each capacitor.

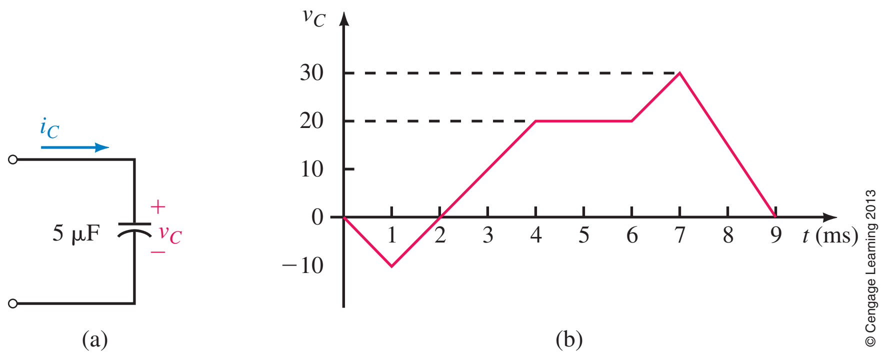


FIGURE 10–42

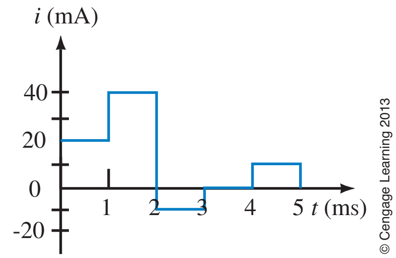


FIGURE 10–43

10.10 Capacitor Failures and Troubleshooting

49. The capacitor tester in Figure 10–44 measures and displays capacitance. For each case, determine the likely fault.

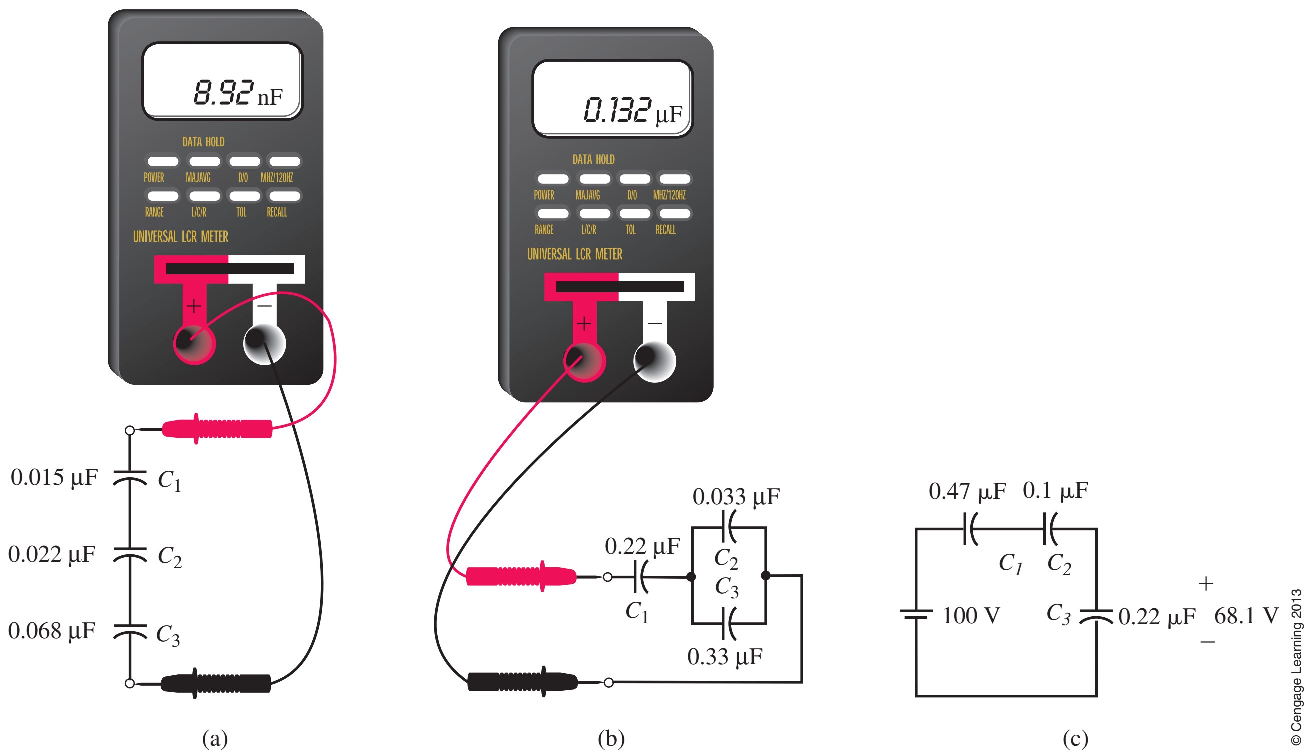


FIGURE 10–44

ANSWERS TO IN-PROCESS LEARNING CHECKS

IN-PROCESS LEARNING CHECK 1

1. a. 2.12 nF
   b. 200 V
2. It becomes six times larger.
3. 1.1 nF
4. Mica

KEY TERMS

Capacitive Loading
Capacitive Transients
Continuous
Discontinuous
Duty Cycle
Exponential Functions
Final Condition Circuit
Initial Condition
Period
Pulse
Pulse Repetition Frequency (PRF)
Pulse Repetition Rate (PRR)
Pulse Train
Pulse Width
Rise and Fall Times
Square Wave
Steady State
Stray Capacitance
Time Constant
Transient
Transient Duration
Universal Time Constant Curve

OUTLINE

Introduction
Capacitor Charging Equations
Capacitor with an Initial Voltage
Capacitor Discharging Equations
More Complex Circuits
An *RC* Timing Application
Pulse Response of *RC* Circuits
Transient Analysis Using Computers

OBJECTIVES

After studying this chapter, you will be able to

- explain why transients occur in *RC* circuits,
- explain why an uncharged capacitor looks like a short circuit when first energized,
- describe why a capacitor looks like an open circuit to steady state dc,
- describe charging and discharging of simple *RC* circuits with dc excitation,
- determine voltages and currents in simple *RC* circuits during charging and discharging,
- plot voltage and current transients,
- understand the part that time constants play in determining the duration of transients,
- compute time constants,
- describe the use of charging and discharging waveforms in simple timing applications,
- calculate the pulse response of simple *RC* circuits,
- solve simple *RC* transient problems using PSpice and Multisim.

11

CAPACITOR CHARGING, DISCHARGING, AND SIMPLE WAVESHAPING CIRCUITS

CHAPTER PREVIEW

As you saw in Chapter 10, Figure 10–26, a capacitor does not charge instantaneously; instead, voltages and currents take time to reach their new values. As we show in this chapter, this time depends on the capacitance of the circuit and the resistance through which it charges—the larger the resistance and capacitance, the longer it takes. (Similar comments hold for discharge.) Since the voltages and currents that exist during charging and discharging are transitory in nature, they are called **transients.** Transients do not last very long, typically only a fraction of a second. However, they are important to us for a number of reasons, some of which you will learn in this chapter.

Transients occur in both capacitive and inductive circuits. In capacitive circuits, they occur because capacitor voltage cannot change instantaneously; in inductive circuits, they occur because inductor current cannot change instantaneously. In this chapter, we look at **capacitive transients;** in Chapter 14, we look at inductive transients. As you will see, many of the basic principles are the same. ■

Putting It in Perspective

Desirable and Undesirable Transients

TRANSIENTS OCCUR IN CAPACITIVE AND inductive circuits whenever circuit conditions are changed, for example, by the sudden application of a voltage, the switching in or out of a circuit element, or the malfunctioning of a circuit component. Some transients are desirable and useful; others occur under abnormal conditions and are potentially destructive in nature.

An example of the latter is the transient that results when lightning strikes a power line. Following a strike, the line voltage, which may have been only a few thousand volts before the strike, momentarily rises to many hundreds of thousands of volts or higher, then rapidly decays, while the current, which may have been only a few hundred amps, suddenly rises to many times its normal value. Although these transients do not last very long, they can cause serious damage. While this is a rather severe example of a transient, it nonetheless illustrates that during transient conditions, many of a circuit's or system's most difficult problems may arise.

Some transient effects, on the other hand, are useful. For example, many electronic devices and circuits (such as oscillators and timers) utilize transient effects due to capacitor charging and discharging as the basis for their operation. You will see an example of such a usage in Section 11.6 of this chapter. ■

11.1 Introduction

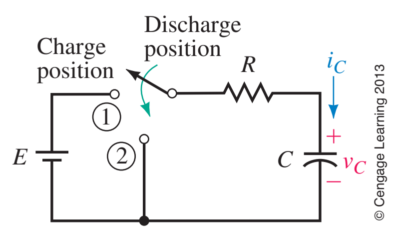


FIGURE 11–1 Circuit for studying capacitor charging and discharging. Transient voltages and currents result when the circuit is switched.

Capacitor Charging

Capacitor charging and discharging may be studied using the simple circuit of Figure 11–1. We will begin with charging. First, assume the capacitor is uncharged and that the switch is open—see Note 1. Now move the switch to the charge position, Figure 11–2(a). At the instant the switch is closed the current jumps to E/R amps, then decays to zero, while the voltage, which is zero at the instant the switch is closed, gradually climbs to E volts. This is shown in (b) and (c). The shapes of these curves are easily explained.

First, consider voltage. In order to change capacitor voltage, electrons must be moved from one plate to the other. Even for a relatively small capacitor, billions of electrons must be moved. This takes time. Consequently, *capacitor voltage cannot*

NOTES...

1. In textbooks, it is traditional to use switches to initiate transients. Although transients in real life result from a variety of causes, the principles that govern their behavior are the same no matter how they are created. In a learning environment, the simplest way to create a transient is by means of a switch—so that is what we do here.
2. For those with a calculus background, recall,

$$i_C = C\frac{dv_C}{dt}$$

If voltage is not continuous, the derivative yields an infinite current spike. Since infinite current spikes are not possible, we conclude that voltage cannot be discontinuous.

change instantaneously, that is, it cannot jump abruptly from one value to another. Instead, it climbs gradually and smoothly as illustrated in Figure 11–2(b). Stated another way, capacitor voltage must be **continuous** at all times—see Note 2.

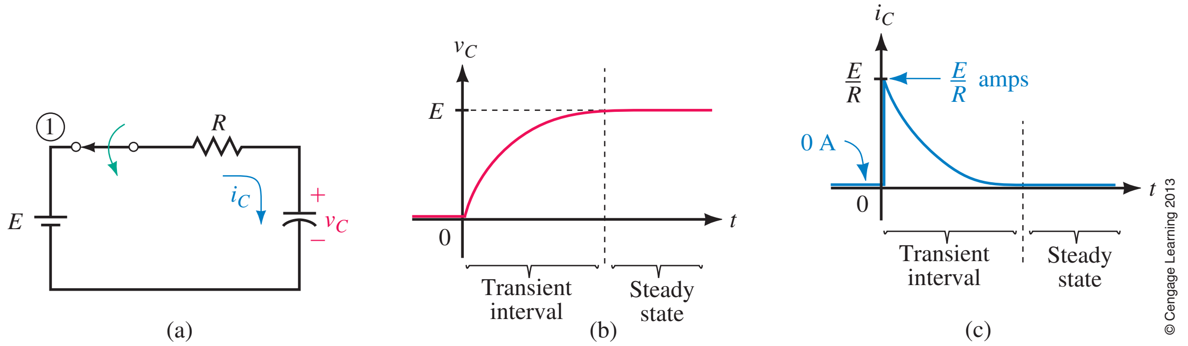
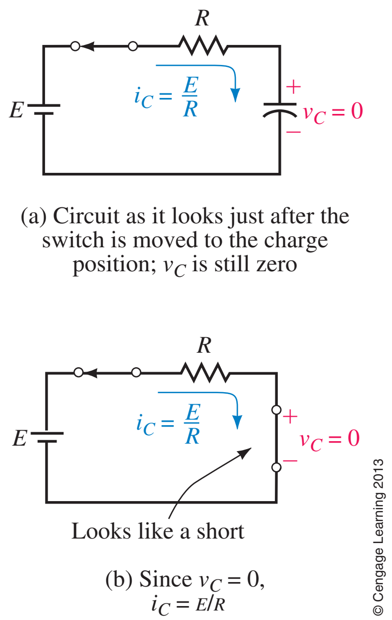


FIGURE 11–2 Capacitor voltage and current during charging. Time $t = 0$ s is defined as the instant the switch is moved to the charge position. The capacitor is initially uncharged.

CircuitSim 11-1

Now consider current. The movement of electrons noted previously is a current. As indicated in Figure 11–2(c), this current jumps abruptly from 0 to E/R amps, that is, the current is **discontinuous.** To understand why, consider Figure 11–3(a). Since capacitor voltage cannot change instantaneously, its value just after the switch is closed will be the same as it was just before the switch is closed, namely 0 V. Since the voltage across the capacitor just after the switch is closed is zero (even though there is current through it), *the capacitor looks momentarily like a short circuit.* This is indicated in (b). This is an important observation and is true in general, that is, *an uncharged capacitor looks like a short circuit at the instant of switching*. Applying Ohm's law yields $i_C = E/R$ amps. This agrees with what we indicated in Figure 11–2(c).

Finally, note the trailing end of the current curve Figure 11–2(c). Since the dielectric between the capacitor plates is an insulator, no current can pass through it. This means that the current in the circuit, which is due entirely to the movement of electrons from one plate to the other through the battery, must decay to zero as the capacitor charges.



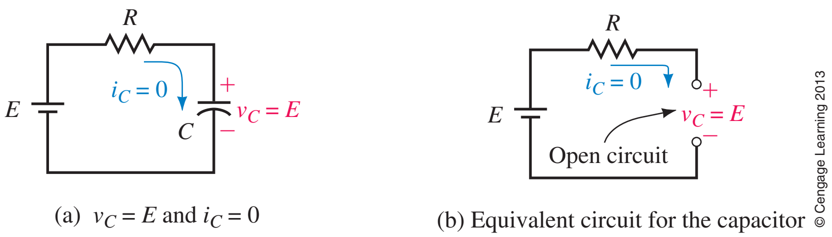
(a) Circuit as it looks just after the switch is moved to the charge position; v_C is still zero

(b) Since $v_C = 0$, $i_C = E/R$

FIGURE 11–3 An uncharged capacitor looks like a short circuit at the instant of switching.

Steady State Conditions

When the capacitor voltage and current reach their final values and stop changing [Figure 11–2(b) and (c)], the circuit is said to be in **steady state.** Figure 11–4(a) shows the circuit after it has reached steady state. Note that $v_C = E$ and $i_C = 0$. Since the capacitor has voltage across it but no current through it, it looks like an open circuit as indicated in (b). This is also an important observation and one that is true in general, that is, *a capacitor looks like an open circuit while in steady state dc.* [The circuit of Figure 11–4(b) is referred to as a **final condition circuit**.]



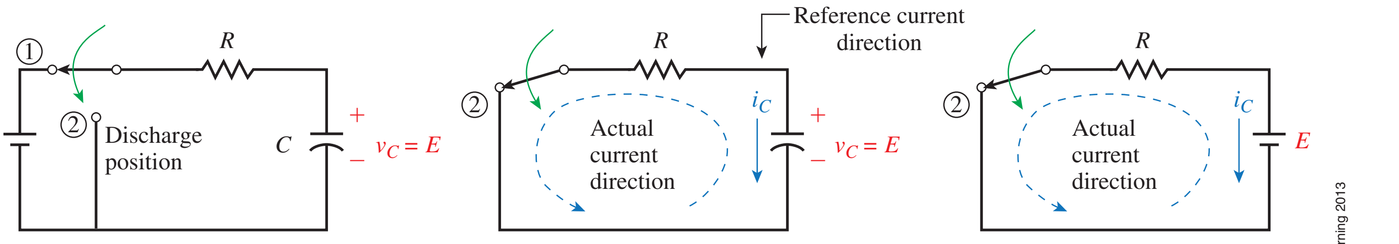
(a) $v_C = E$ and $i_C = 0$

(b) Equivalent circuit for the capacitor

FIGURE 11–4 Charging circuit after it has reached steady state. Since the capacitor has voltage across it but no current, it looks like an open circuit in steady state dc.

Capacitor Discharging

Now consider the discharge case, Figures 11–5 and 11–6. First, assume the capacitor is charged to E volts prior to switching as indicated in Figure 11–5(a). Now move the switch to discharge. Since the capacitor has E volts across it just before the switch is moved, and since its voltage cannot change instantaneously, it will still have E volts across it just after as well. This is indicated in (b). The capacitor therefore looks momentarily like a voltage source [Figure 11–5(c)], and the current thus jumps immediately to $-E/R$ amps. (Note that the current is negative since it is opposite in direction to the reference arrow.) The voltage and current then decay to zero as indicated in Figure 11–6.



(a) Voltage $v_C = E$ just before the switch is moved to discharge position.

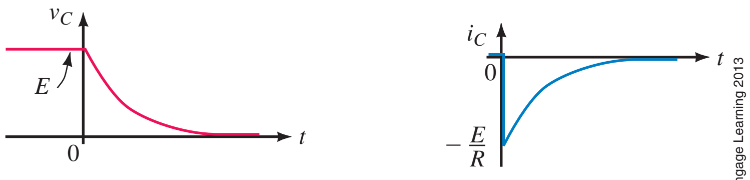
(b) Immediately after the switch is moved, v_C still equals E.

(c) The capacitor therefore momentarily looks like a voltage source. Ohm's law yields $i_C = -E/R$.

FIGURE 11–5 According to the voltage/current reference convention that you learned in Chapter 4, the current reference arrow must be drawn such that the + sign for v_C is at its tail end. Since the polarity of v_C is already known from (a), this establishes the reference direction for i_C as in (b). Since the actual current is opposite in direction to this reference, i_C is negative as indicated in (c).

CircuitSim 11-2

CircuitSim 11-3



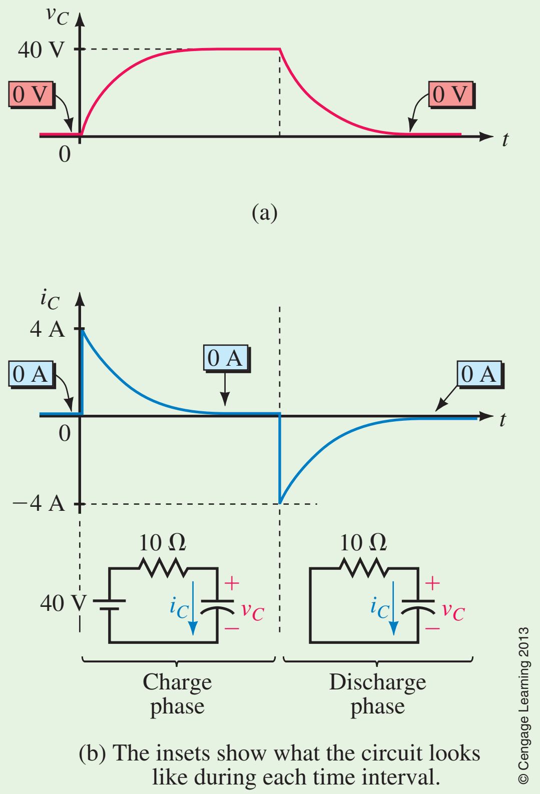
(a) (b)

FIGURE 11–6 Voltage and current during discharge. Time $t = 0$ s is defined as the instant the switch is moved to the discharge position.

EXAMPLE 11–1

For Figure 11–1, $E = 40$ V, $R = 10\ \Omega$, and the capacitor is initially uncharged. The switch is moved to the charge position and the capacitor allowed to charge fully. Then the switch is moved to the discharge position and the capacitor allowed to discharge fully. Sketch the voltages and currents and determine the values at switching and in steady state.

Solution The current and voltage curves are shown in Figure 11–7. Initially, $i = 0$ A since the switch is open. Immediately after it is moved to the charge position, the current jumps to $E/R = 40\text{ V}/10\ \Omega = 4$ A; then it decays to zero. At the same time, v_C starts at 0 V and climbs to 40 V. When the switch is moved to the discharge position, the capacitor looks momentarily like a 40-V source and the current jumps to a negative value of $-40\text{ V}/10\ \Omega = -4$ A; then it decays to zero. At the same time, v_C also decays to zero.



CircuitSim 11-4

CircuitSim 11-5

FIGURE 11–7 A charge/discharge example.

The Meaning of Time in Transient Analysis

Time t used in transient analysis is measured from the instant of switching. Thus, $t = 0$ in Figure 11–2 is defined as the instant the switch is moved to charge, while in Figure 11–6, it is defined as the instant the switch is moved to discharge. Following switching, voltages and currents may be represented by functions of time as $v_C(t)$ and $i_C(t)$. For example, the voltage across a capacitor at $t = 0$ s may be denoted as $v_C(0)$, while the voltage at $t = 10$ ms may be denoted as $v_C(10 \text{ ms})$, and so on. If you know numeric values, they can be included. Thus, if a capacitor has a voltage of 9 V at $t = 5$ ms and a current of 3 mA, these can be represented as $v_C(5 \text{ ms}) = 9$ V and $i_C(5 \text{ ms}) = 3$ mA, respectively.

A problem seems to arise when a quantity is discontinuous as is the current of Figure 11–2(c). Since its value is changing at $t = 0$ s, $i_C(0)$ cannot be defined. To get around this problem, we define two values for 0 s. We define $t = 0^-$ s as $t = 0$ s just prior to switching and $t = 0^+$ s as $t = 0$ s just after switching. In Figure 11–2(c), therefore, $i_C(0^-) = 0$ A while $i_C(0^+) = E/R$ amps. For Figure 11–6, $i_C(0^-) = 0$ A and $i_C(0^+) = -E/R$ amps. Note that capacitor voltage is continuous—thus, $v_C(0^+) = v_C(0^-)$—see Note.

NOTES...

That $v_C(0^+)$ equals $v_C(0^-)$ is actually a statement of continuity for capacitor voltage. If switching occurs at some time other than $t = 0$ s, a similar statement holds. For example, if switching occurs at $t = 20$ s, and voltage has a value of 40 volts at that instant, then the statement of continuity becomes $v_C(20^+) = v_C(20^-) = 40$ V.

Exponential Functions

As we will soon show, the waveforms of Figures 11–2 and 11–6 are exponential and vary according to e^{-x} or $(1 - e^{-x})$, where e is the base of the natural logarithm. Fortunately, **exponential functions** are easy to evaluate with

TABLE 11–1 Table of Exponentials

x	e^{-x}	$1 - e^{-x}$
0	1	0
1	0.3679	0.6321
2	0.1353	0.8647
3	0.0498	0.9502
4	0.0183	0.9817
5	0.0067	0.9933

modern calculators using their e^x function. You need to be able to evaluate both e^{-x} and $(1 - e^{-x})$ for any value of x. Table 11–1 shows a tabulation of values for both cases. Note that as x gets larger, e^{-x} gets smaller and approaches zero, while $(1 - e^{-x})$ gets larger and approaches 1. These observations will be important to you in what follows.

PRACTICE PROBLEMS 1

1. Use your calculator and verify the entries in Table 11–1. Be sure to change the sign of x before using the e^x function. Note that $e^{-0} = e^0 = 1$ since any quantity raised to the zeroth power is one. Use your calculator to confirm this.
2. Plot the computed values on graph paper and verify that they yield curves that look like those shown in Figure 11–2(b) and (c).
3. Assume that the discharge portion of the curves of Figure 11–7 begin at $t = 10$ ms. What are $i_C(10\text{ ms}^-)$ and $i_C(10\text{ ms}^+)$? What is the statement of continuity for v_C at $t = 10$ ms?

Answers

3. $i_C(10\text{ ms}^-) = 0$ A; $i_C(10\text{ ms}^+) = -4$ A; $v_C(10\text{ ms}^+) = v_C(10\text{ ms}^-) = 40$ V

11.2 Capacitor Charging Equations

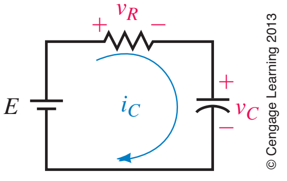


FIGURE 11–8 Circuit for the charging case. Capacitor is initially uncharged.

We will now develop equations for voltages and current during charging. Consider Figure 11–8. Kirchhoff's voltage law (KVL) yields

$$v_R + v_C = E \tag{11–1}$$

But $v_R = Ri_C$ and $i_C = Cdv_C/dt$ (Equation 10–20). Thus, $v_R = RCdv_C/dt$. Substituting this into Equation 11–1 yields

$$RC\frac{dv_C}{dt} + v_C = E \tag{11–2}$$

Equation 11–2 can be solved for v_C using basic calculus (see ∫). The result is

$$v_C = E(1 - e^{-t/RC}) \tag{11–3}$$

where R is in ohms, C is in farads, t is in seconds, and $e^{-t/RC}$ is the exponential function discussed earlier. The product RC has units of seconds. (This is left as an exercise for the student to show.)

∫ Solving Equation 11–2 (Optional Derivation—See Notes)

First, rearrange Equation 11–2:

$$\frac{dv_C}{dt} = \frac{1}{RC}(E - v_C)$$

Rearrange again:

$$\frac{dv_C}{E - v_C} = \frac{dt}{RC}$$

Now multiply both sides by -1 and integrate.

$$\int_0^{v_C} \frac{dv_C}{v_C - E} = -\frac{1}{RC}\int_0^t dt$$

$$\Big[\ln(v_C - E)\Big]_0^{v_C} = \Big[-\frac{t}{RC}\Big]_0^t$$

Next, substitute integration limits,

$$\ln(v_C - E) - \ln(-E) = -\frac{t}{RC}$$

$$\ln\left(\frac{v_C - E}{-E}\right) = -\frac{t}{RC}$$

Finally, take the inverse log of both sides. Thus,

$$\frac{v_C - E}{-E} = e^{-t/RC}$$

When you rearrange this, you get Equation 11–3. That is,

$$v_C = E(1 - e^{-t/RC})$$

Now consider the resistor voltage. From Equation 11–1, $v_R = E - v_C$. Substituting v_C from Equation 11–3 yields $v_R = E - E(1 - e^{-t/RC}) = E - E + Ee^{-t/RC}$. After cancellation, you get

$$v_R = Ee^{-t/RC} \qquad \textbf{(11–4)}$$

Now divide both sides by R. Since $i_C = i_R = v_R/R$, this yields

$$i_C = \frac{E}{R}e^{-t/RC} \qquad \textbf{(11–5)}$$

The waveforms are shown in Figure 11–9. Values at any time may be determined by substitution.

NOTES...

Optional problems and derivations using calculus are marked by a ∫ icon. These may be omitted without loss of continuity by those who do not require calculus.

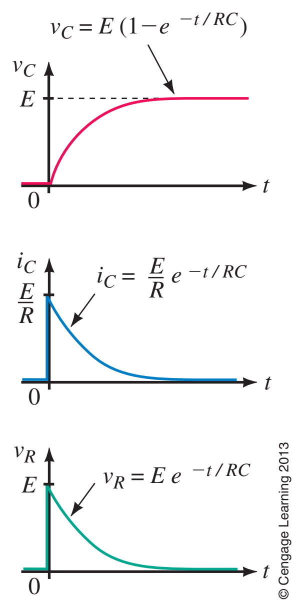


FIGURE 11–9 Curves for the circuit of Figure 11–8.

EXAMPLE 11–2

Suppose $E = 100$ V, $R = 10$ kΩ, and $C = 10$ μF:

a. Determine the expression for v_C.
b. Determine the expression for i_C.
c. Compute the capacitor voltage at $t = 150$ ms.
d. Compute the capacitor current at $t = 150$ ms.
e. Locate the computed points on the curves.

Solution

a. $RC = (10 \times 10^3\ \Omega)(10 \times 10^{-6}\ \text{F}) = 0.1$ s. From Equation 11–3, $v_C = E(1 - e^{-t/RC}) = 100(1 - e^{-t/0.1}) = 100(1 - e^{-10t})$ V.
b. From Equation 11–5, $i_C = (E/R)e^{-t/RC} = (100\ \text{V}/10\ \text{k}\Omega)e^{-10t} = 10e^{-10t}$ mA.
c. At $t = 0.15$ s, $v_C = 100(1 - e^{-10t}) = 100(1 - e^{-10(0.15)}) = 100(1 - e^{-1.5}) = 100(1 - 0.223) = 77.7$ V.
d. $i_C = 10e^{-10t}$ mA $= 10e^{-10(0.15)}$ mA $= 10e^{-1.5}$ mA $= 2.23$ mA.
e. The corresponding points are shown in Figure 11–10.

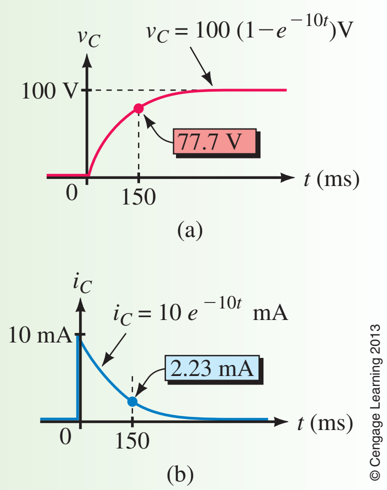


FIGURE 11–10 The computed points plotted on the v_C and i_C curves.

CircuitSim 11-6

In the preceding example, we expressed voltage as $v_C = 100(1 - e^{-t/0.1})$ and as $100(1 - e^{-10t})$ V. Similarly, current can be expressed as $i_C = 10e^{-t/0.1}$ or as $10e^{-10t}$ mA. Although some people prefer one notation over the other, both are correct and you can use them interchangeably.

PRACTICE PROBLEMS 2

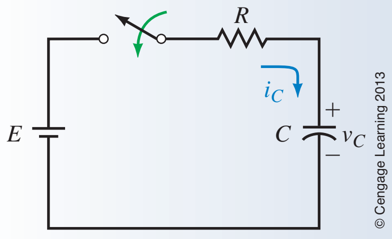


FIGURE 11–11

1. Determine additional voltage and current points for Figure 11–10 by computing values of v_C and i_C at values of time from $t = 0$ s to $t = 500$ ms at 100-ms intervals. Plot the results.
2. The switch of Figure 11–11 is closed at $t = 0$ s. If $E = 80$ V, $R = 4$ kΩ, and $C = 5$ μF, determine expressions for v_C and i_C. Plot the results from $t = 0$ s to $t = 100$ ms at 20-ms intervals. Note that charging takes less time here than for Problem 1.

CircuitSim 11-7

Answers

1.

t(ms)	v_C(V)	i_C(mA)
0	0	10
100	63.2	3.68
200	86.5	1.35
300	95.0	0.498
400	98.2	0.183
500	99.3	0.067

2. $80(1 - e^{-50t})$V $20e^{-50t}$ mA

t(ms)	v_C(V)	i_c(mA)
0	0	20
20	50.6	7.36
40	69.2	2.70
60	76.0	0.996
80	78.6	0.366
100	79.4	0.135

EXAMPLE 11–3

For the circuit of Figure 11–11, $E = 60$ V, $R = 2$ kΩ, and $C = 25$ μF. The switch is closed at $t = 0$ s, opened 40 ms later and left open. Determine equations for capacitor voltage and current and plot.

Solution $RC = (2\ \text{k}\Omega)(25\ \mu\text{F}) = 50$ ms. As long as the switch is closed (i.e., from $t = 0$ s to 40 ms), the following equations hold:

$$v_C = E(1 - e^{-t/RC}) = 60(1 - e^{-t/50\ \text{ms}})\ \text{V}$$

$$i_C = (E/R)e^{-t/RC} = 30e^{-t/50\ \text{ms}}\ \text{mA}$$

Voltage starts at 0 V and rises exponentially. At $t = 40$ ms, the switch is opened, interrupting charging. At this instant, $v_C = 60(1 - e^{-(40/50)}) = 60(1 - e^{-0.8}) = 33.0$ V. Since the switch is left open, the voltage remains constant at 33 V thereafter as indicated in Figure 11–12. (The dotted curve shows how the voltage would have kept rising if the switch had remained closed.)

Now consider current. The current starts at 30 mA and decays to $i_C = 30e^{-(40/50)}$ mA $= 13.5$ mA at $t = 40$ ms. At this point, the switch is opened, and the current drops instantly to zero. (The dotted line shows how the current would have decayed if the switch had not been opened.)

NOTES...

As you go through the examples and problems of this chapter, you will sometimes find that you cannot simply substitute numbers into equations to get the correct solution—you may in fact have to reason your way through several steps. This may be seen in Example 11–3, where early switching prematurely stops charging.

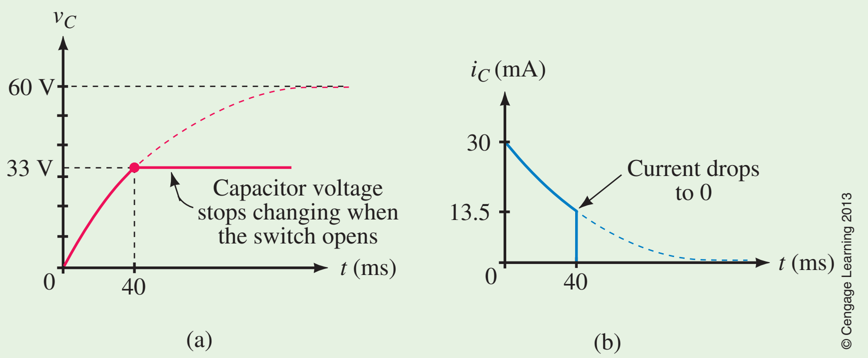


FIGURE 11–12 Interrupted charging. The switch of Figure 11–11 was opened at $t = 40$ ms, causing charging to cease.

CircuitSim 11-8

The Time Constant

The rate at which a capacitor charges depends on the product of R and C. This product is known as the **time constant** of the circuit and is given the symbol τ (the Greek letter tau). As noted earlier, RC has units of seconds. Thus,

$$\tau = RC \text{ (seconds, s)} \quad \textbf{(11–6)}$$

Using τ, Equations 11–3 to 11–5 can be written as

$$v_C = E(1 - e^{-t/\tau}) \quad \textbf{(11–7)}$$

$$i_C = \frac{E}{R}e^{-t/\tau} \quad \textbf{(11–8)}$$

and

$$v_R = Ee^{-t/\tau} \quad \textbf{(11–9)}$$

Duration of a Transient

The length of time that a transient lasts (i.e., the **transient duration**) depends on the exponential function $e^{-t/\tau}$. As t increases, $e^{-t/\tau}$ decreases, and when it reaches zero, the transient is gone. Theoretically, this takes infinite time. In practice, however, over 99% of the transition takes place during the first five time constants (i.e., transients are within 1% of their final value at $t = 5\,\tau$). This can be verified by direct substitution. At $t = 5\,\tau$, $v_C = E(1 - e^{-t/\tau}) = E(1 - e^{-5}) = E(1 - 0.0067) = 0.993E$, meaning that the transient has achieved 99.3% of its final value. Similarly, the current falls to within 1% of its final value in five time constants. Thus, *for all practical purposes, transients can be considered to last for only five time constants* (Figure 11–13). Figure 11–14 summarizes how

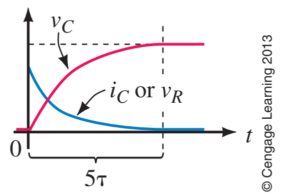


FIGURE 11–13 Transients last five time constants.

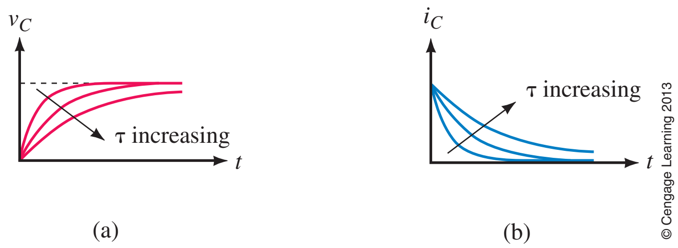


FIGURE 11–14 Illustrating how voltage and current in an RC circuit are affected by its time constant. The larger the time constant, the longer the capacitor takes to charge.

transient voltages and currents are affected by the time constant of a circuit—the larger the time constant, the longer the duration of the transient.

EXAMPLE 11–4

For the circuit of Figure 11–11, how long will it take for the capacitor to charge if $R = 2\ \text{k}\Omega$ and $C = 10\ \mu\text{F}$?

Solution $\tau = RC = (2\ \text{k}\Omega)(10\ \mu\text{F}) = 20$ ms. Therefore, the capacitor charges in $5\ \tau = 100$ ms.

EXAMPLE 11–5

The transient in a circuit with $C = 40\ \mu\text{F}$ lasts 0.5 s. What is R?

Solution $5\ \tau = 0.5$ s. Thus, $\tau = 0.1$ s and $R = \tau/C = 0.1\ \text{s}/(40 \times 10^{-6}\ \text{F}) = 2.5\ \text{k}\Omega$.

Universal Time Constant Curves

Let us now plot capacitor voltage and current with their time axes scaled as multiples of τ and their vertical axes scaled in percent. The results, Figure 11–15, are **universal time constant curves.**

Points are computed from $v_C = 100(1 - e^{-t/\tau})$ and $i_C = 100e^{-t/\tau}$. For example, at $t = \tau$, $v_C = 100(1 - e^{-t/\tau}) = 100(1 - e^{-\tau/\tau}) = 100(1 - e^{-1}) = 63.2$ V, that is, 63.2%, and $i_C = 100e^{-\tau/\tau} = 100e^{-1} = 36.8$ A, which is 36.8%, and so on. These curves provide an easy method to determine voltages and currents with a minimum of computation.

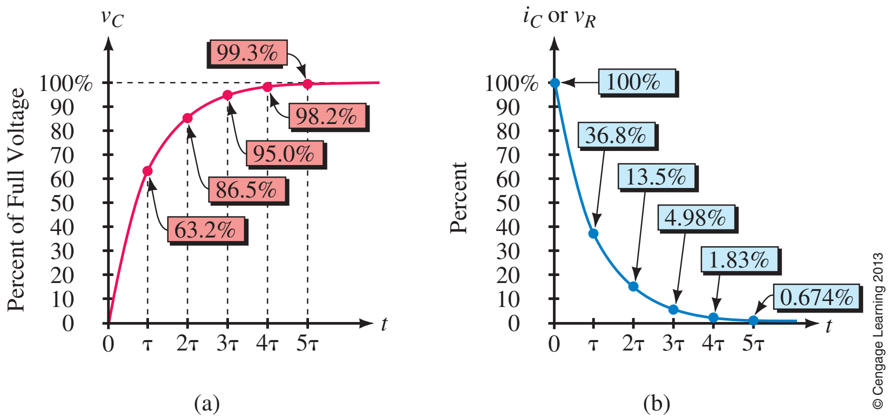


FIGURE 11–15 Universal voltage and current curves for *RC* circuits.

EXAMPLE 11–6

Using Figure 11–15, compute v_C and i_C at two time constants into charge for a circuit with $E = 25$ V, $R = 5\ \text{k}\Omega$, and $C = 4\ \mu\text{F}$. What is the corresponding value of time?

Solution At $t = 2\ \tau$, v_C equals 86.5% of E or $0.865(25\ \text{V}) = 21.6$ V. Similarly, $i_C = 0.135I_0 = 0.135\ (E/R) = 0.675$ mA. These values occur at $t = 2\ \tau = 2RC = 40$ ms.

IN-PROCESS LEARNING CHECK 1

(Answers are at the end of the chapter.)

1. If the capacitor of Figure 11–16 is uncharged, what is the current immediately after closing the switch?
2. Given $i_C = 50e^{-20t}$ mA.
   a. What is τ?
   b. Compute the current at $t = 0^+$ s, 25 ms, 50 ms, 75 ms, 100 ms, and 250 ms and sketch it. Verify answers using the universal time constant curves.
3. Given $v_C = 100(1 - e^{-50t})$ V, compute v_C at the same time points as in Problem 2 and sketch.
4. For Figure 11–16, determine expressions for v_C and i_C. Compute capacitor voltage and current at $t = 0.6$ s. Verify answers using the universal time constant curves.
5. Refer to Figure 11–10:
   a. What are $v_C(0^-)$ and $v_C(0^+)$?
   b. What are $i_C(0^-)$ and $i_C(0^+)$?
   c. What are the steady state voltage and current?
6. For the circuit of Figure 11–11, the current just after the switch is closed is 2 mA. The transient lasts 40 ms and the capacitor charges to 80 V. Determine E, R, and C.

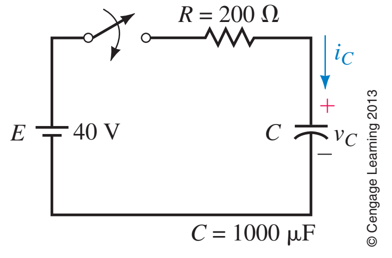


FIGURE 11–16

11.3 Capacitor with an Initial Voltage

Suppose a previously charged capacitor has not been discharged and thus still has voltage on it. Let this voltage be denoted as V_0. (V_0 is referred to as an **initial condition.**) If the capacitor is now placed in a circuit like that in Figure 11–16, the voltage and current during charging will be affected by the initial voltage. In this case, Equations 11–7 and 11–8 become

$$v_C = E + (V_0 - E)e^{-t/\tau} \tag{11–10}$$

$$i_C = \frac{E - V_0}{R}e^{-t/\tau} \tag{11–11}$$

A few comments are in order about these equations. Consider Equation 11–10. When you set $t = 0$, you get $v_C = E + (V_0 - E) = V_0$. This agrees with our assertion that the capacitor was initially charged to V_0. If you now set $t = \infty$, you get $v_C = E$, which confirms that the capacitor charges to E volts as expected. Consider Equation 11–11. When you set $t = 0$, you get $i_C = (E - V_0)/R$. Recalling that an initially charged capacitor looks like a voltage source [see Figure 11–5(c)], you can see that if you replace C in Figure 11–16 with a source V_0, the current at $t = 0$ will be $(E - V_0)/R$ as noted. Note also that these revert to Equation 11–7 and Equation 11–8 when you set $V_0 = 0$ V.

EXAMPLE 11–7

Suppose the capacitor of Figure 11–16 has 25 volts on it with polarity shown at the time the switch is closed.

a. Determine the expression for v_C.
b. Determine the expression for i_C.
c. Compute v_C and i_C at $t = 0.1$ s.
d. Sketch v_C and i_C.

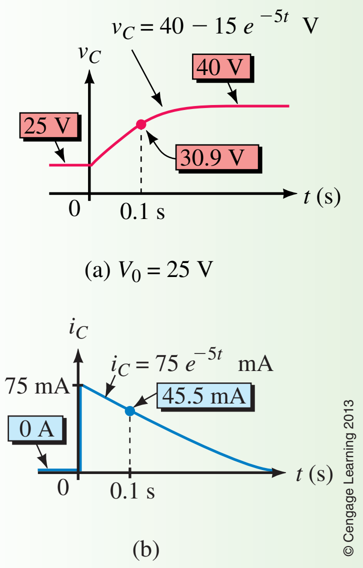


FIGURE 11–17 Capacitor with an initial voltage.

CircuitSim 11-9

Solution $\tau = RC = (200\ \Omega)(1000\ \mu\text{F}) = 0.2$ s

a. From Equation 11–10,

$$v_C = E + (V_0 - E)e^{-t/\tau} = 40 + (25 - 40)e^{-t/0.2} = 40 - 15e^{-5t}\text{ V}$$

b. From Equation 11–11,

$$i_C = \frac{E - V_0}{R}e^{-t/\tau} = \frac{40 - 25}{200}e^{-5t} = 75e^{-5t}\text{ mA}$$

c. At $t = 0.1$ s,

$$v_C = 40 - 15e^{-5t} = 40 - 15e^{-0.5} = 30.9\text{ V}$$
$$i_C = 75e^{-5t}\text{ mA} = 75e^{-0.5}\text{ mA} = 45.5\text{ mA}$$

d. The waveforms are shown in Figure 11–17 with the preceding points plotted.

PRACTICE PROBLEMS 3

1. For Example 11–7, compute voltage and current at $t = 0.25$ s.
2. Repeat Example 11–7 for the circuit of Figure 11–16 if $V_0 = -150$ V.

Answers

1. 35.7 V, 21.5 mA
2. a. $40 - 190e^{-5t}$ V
   b. $0.95e^{-5t}$ A
   c. -75.2 V; 0.576 A
   d. Curves are similar to Figure 11–17 except that v_C starts at -150 V and rises to 40 V while i_C starts at 0.95 A and decays to zero.

11.4 Capacitor Discharging Equations

To determine the discharge equations, move the switch to the discharge position (Figure 11–18). (Note carefully the reference direction for current i_C.) KVL yields $v_R + v_C = 0$. Substituting $v_R = RCdv_C/dt$ from Section 11.2 yields

$$RC\frac{dv_C}{dt} + v_C = 0 \tag{11–12}$$

This can be solved for v_C using basic calculus. The result is

$$v_C = V_0e^{-t/RC} \tag{11–13}$$

where V_0 is the voltage on the capacitor at the instant the switch is moved to discharge. Now consider the resistor voltage. Since $v_R + v_C = 0$, $v_R = -v_C$ and

$$v_R = -V_0 e^{-t/RC} \tag{11–14}$$

Now divide both sides by R. Since $i_C = i_R = v_R/R$,

$$i_C = -\frac{V_0}{R} e^{-t/RC} \tag{11–15}$$

Note that this is negative, since, during discharge, the actual current is opposite in direction to the reference arrow of Figure 11–18. Voltage v_C and current i_C are shown in Figure 11–19. As in the charging case, *discharge transients last five time constants.* You can also write these equations in terms of τ, for example, $v_C = V_0 e^{-t/\tau}$, and so forth.

In Equations 11–13 to 11–15, V_0 represents the voltage on the capacitor at the instant the switch is moved to the discharge position. If the switch has been in the charge position long enough for the capacitor to fully charge, $V_0 = E$, and Equations 11–13 and 11–15 become $v_C = Ee^{-t/RC}$ and $i_C = -(E/R)e^{-t/RC}$, respectively.

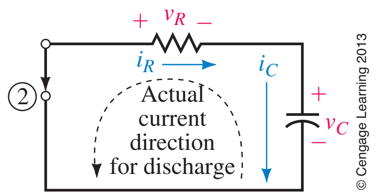


FIGURE 11–18 Discharge case. Initial capacitor voltage is V_0. Note the reference for i_C. (To conform to the standard voltage/current reference convention, i_C must be drawn in this direction so that the + sign for v_C is at the tail of the current arrow.) Since the actual current direction is opposite to the reference direction, i_C will be negative. This is indicated in Figure 11–19(b).

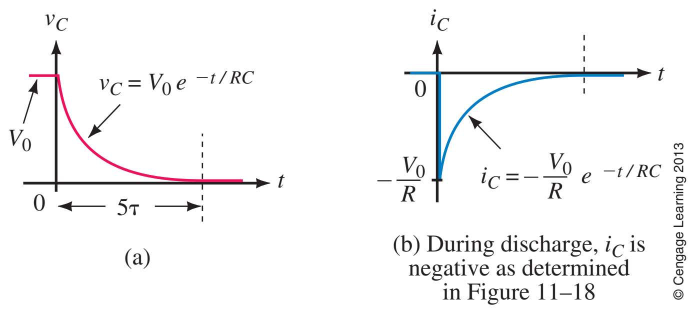


FIGURE 11–19 Capacitor voltage and current for the discharge case.

CircuitSim 11-10

EXAMPLE 11–8

For the circuit of Figure 11–18, assume the capacitor is charged to 100 V before the switch is moved to the discharge position. Suppose $R = 5\ \text{k}\Omega$ and $C = 25\ \mu\text{F}$. After the switch is moved to discharge,

a. Determine the expression for v_C.
b. Determine the expression for i_C.
c. Compute the voltage and current at 0.375 s.

Solution $RC = (5\ \text{k}\Omega)(25\ \mu\text{F}) = 0.125$ s and $V_0 = 100$ V. Therefore,

a. $v_C = V_0 e^{-t/RC} = 100e^{-t/0.125} = 100e^{-8t}$ V.
b. $i_C = -(V_0/R)e^{-t/RC} = -20e^{-8t}$ mA.
c. At $t = 0.375$ s,

$$v_C = 100e^{-8t} = 100e^{-3} = 4.98\ \text{V}$$
$$i_C = -20e^{-8t}\ \text{mA} = -20e^{-3}\ \text{mA} = -0.996\ \text{mA}$$

Let us verify the answers of Example 11–8 using the appropriate universal time constant curve. As noted, $\tau = 0.125$ s, thus $0.375\ \text{s} = 3\tau$. From Figure 11–15(b), we see that capacitor voltage has fallen to 4.98% of E at 3τ. This is (0.0498)(100 V) = 4.98 V as we computed earlier. Current can be verified similarly. I will leave this for you to do.

NOTES...

The universal time constant curve of Figure 11–15(b) may also be used for discharge problems since it has the same shape as the discharge waveforms.

11.5 More Complex Circuits

The charge and discharge equations and the universal time constant curves apply only to circuits of the forms shown in Figures 11–2 and 11–18. Fortunately, many circuits can be reduced to these forms using standard circuit reduction techniques such as series and parallel combinations, source conversions, Thévenin's theorem, and so on. Once a circuit has been reduced to its series equivalent, you can use any of the techniques that we have developed so far.

EXAMPLE 11–9

For the circuit of Figure 11–20(a), determine expressions for v_C and i_C. Capacitors are initially uncharged.

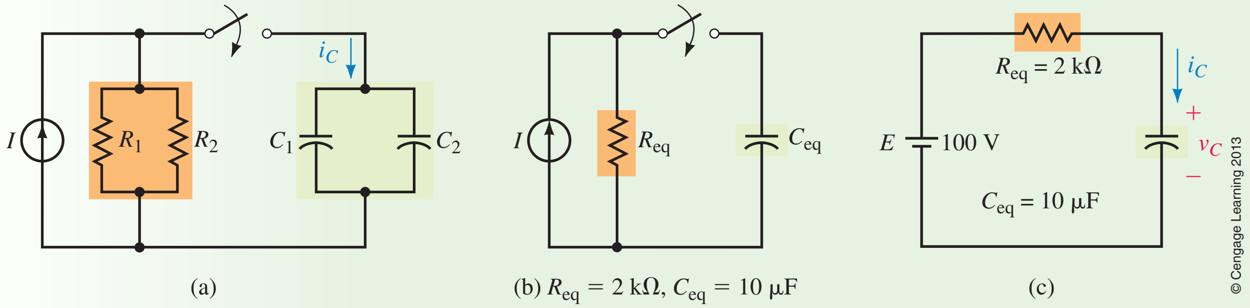


FIGURE 11–20 $I = 50$ mA, $R_1 = 3$ kΩ, $R_2 = 6$ kΩ, $C_1 = 8$ μF, $C_2 = 2$ μF.

Solution $R_{eq} = 3\text{ k}\Omega \| 6\text{ k}\Omega = 2\text{ k}\Omega$, $C_{eq} = 8\ \mu\text{F} \| 2\ \mu\text{F} = 10\ \mu\text{F}$. The reduced circuit is shown in (b). Converting to a voltage source representation yields (c).

$$R_{eq}C_{eq} = (2\text{ k}\Omega)(10 \times 10^{-6}\text{ F}) = 0.020\text{ s}$$

Thus,

$$v_C = E(1 - e^{-t/R_{eq}C_{eq}}) = 100(1 - e^{-t/0.02}) = 100(1 - e^{-50t})\text{ V}$$

$$i_C = \frac{E}{R_{eq}}e^{-t/R_{eq}C_{eq}} = \frac{100}{2000}e^{-t/0.02} = 50\,e^{-50t}\text{ mA}$$

EXAMPLE 11–10

The capacitor of Figure 11–21 is initially uncharged. Close the switch at $t = 0$ s.

a. Determine the expression for v_C.
b. Determine the expression for i_C.
c. Determine capacitor current and voltage at $t = 5$ ms and $t = 10$ ms.

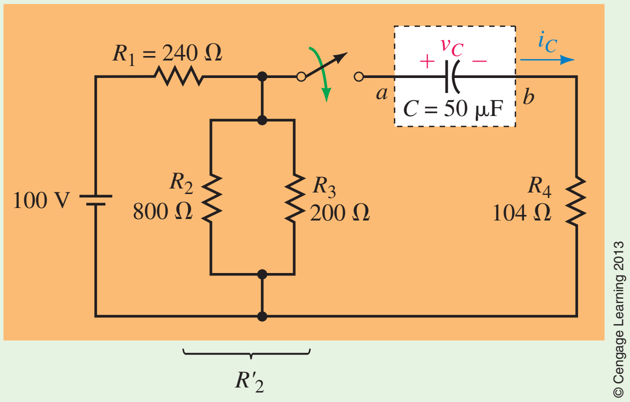


FIGURE 11–21

CircuitSim 11-11

Solution Reduce the circuit to its series equivalent using Thévenin's theorem:

$$R'_2 = R_2 \| R_3 = 160\ \Omega$$

From Figure 11–22(a),

$$R_{Th} = R_1 \| R'_2 + R_4 = 240 \| 160 + 104 = 96 + 104 = 200\ \Omega$$

From Figure 11–22(b),

$$V'_2 = \left(\frac{R'_2}{R_1 + R'_2}\right) E = \left(\frac{160}{240 + 160}\right) \times 100\text{ V} = 40\text{ V}$$

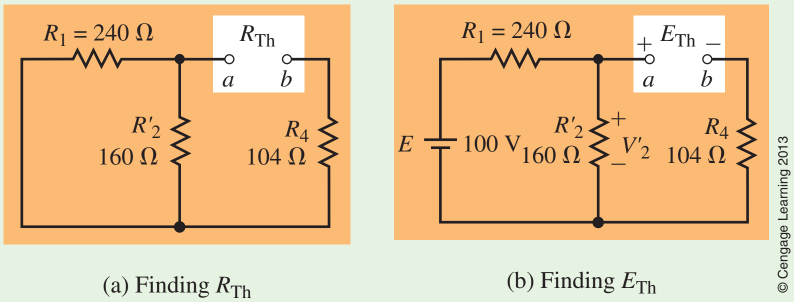


FIGURE 11–22 Determining the Thévenin equivalent of Figure 11–21 following switch closure.

From KVL, $E_{Th} = V'_2 = 40$ V. The resultant equivalent circuit is shown in Figure 11–23.

$$\tau = R_{Th}C = (200\ \Omega)(50\ \mu F) = 10\text{ ms}$$

a. $v_C = E_{Th}(1 - e^{-t/\tau}) = 40(1 - e^{-100t})$ V

b. $i_C = \dfrac{E_{Th}}{R_{Th}} e^{-t/\tau} = \dfrac{40}{200} e^{-t/0.01} = 200e^{-100t}$ mA

c. At $t = 5$ ms, $i_C = 200e^{-100(5\text{ ms})} = 121$ mA. Similarly, $v_C = 15.7$ V. Similarly, at 10 ms, $i_C = 73.6$ mA and $v_C = 25.3$ V.

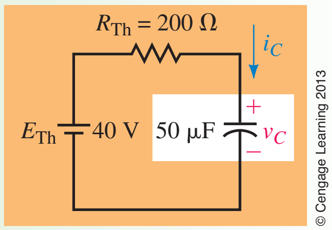


FIGURE 11–23 The Thévenin equivalent of Figure 11–21.

PRACTICE PROBLEMS 4

1. For Example 11–10, determine v_C and i_C at 5 ms and 10 ms using the universal time constant curves and compare to above. (You will have to estimate the point on the curves for $t = 5$ ms.)
2. For Figure 11–21, if $R_1 = 400\ \Omega$, $R_2 = 1200\ \Omega$, $R_3 = 300\ \Omega$, $R_4 = 50\ \Omega$, $C = 20\ \mu\text{F}$, and $E = 200$ V, determine equations for v_C and i_C.
3. Using the values shown in Figure 11–21, determine equations for v_C and i_C if the capacitor has an initial voltage of 60 V.
4. Using the values of Problem 2, determine equations for v_C and i_C if the capacitor has an initial voltage of -50 V.

Answers

1. 1.57 V, 121 mA; 25.3 V, 73.6 mA
2. $75(1 - e^{-250t})$ V; $0.375e^{-250t}$ A
3. $40 + 20e^{-100t}$ V; $-0.1e^{-100t}$ A
4. $75 - 125e^{-250t}$ V; $0.625e^{-250t}$ A

PRACTICAL NOTES...

Notes about Time References and Time Constants

1. So far, we have dealt with charging and discharging problems separately. For these, we define $t = 0$ s as the instant the switch is moved to the charge position for charging problems and to the discharge position for discharging problems.

2. When you have both charge and discharge cases in the same example, you need to establish clearly what you mean by "time." We use the following procedure:

a. Define $t = 0$ s as the instant the switch is moved to the first position, then determine corresponding expressions for v_C and i_C. These expressions and the corresponding time scale are valid until the switch is moved to its new position.

b. When the switch is moved to its new position, shift the time reference and make $t = 0$ s the time at which the switch is moved to its new position, determine new expressions for v_C and i_C, then use continuity considerations to link the two solutions together. (These new expressions are only valid from the new $t = 0$ s reference point. The old expressions are not valid on the new time scale.)

c. We now have two time scales for the same graph. However, we generally only show the first scale explicitly; the second scale is implied rather than shown. Computations based on the new equations must use the new scale.

d. Use τ_C to represent the time constant for charging and τ_d to represent the time constant for discharging. Since the equivalent resistance and capacitance for discharging may be different than for charging, the time constants may be different for the two cases.

EXAMPLE 11–11

The capacitor of Figure 11–24(a) is uncharged. The switch is moved to position 1 for 10 ms, then to position 2, where it remains.

a. Determine v_C during charge.
b. Determine i_C during charge.
c. Determine v_C during discharge.
d. Determine i_C during discharge.
e. Sketch the charge and discharge waveforms.

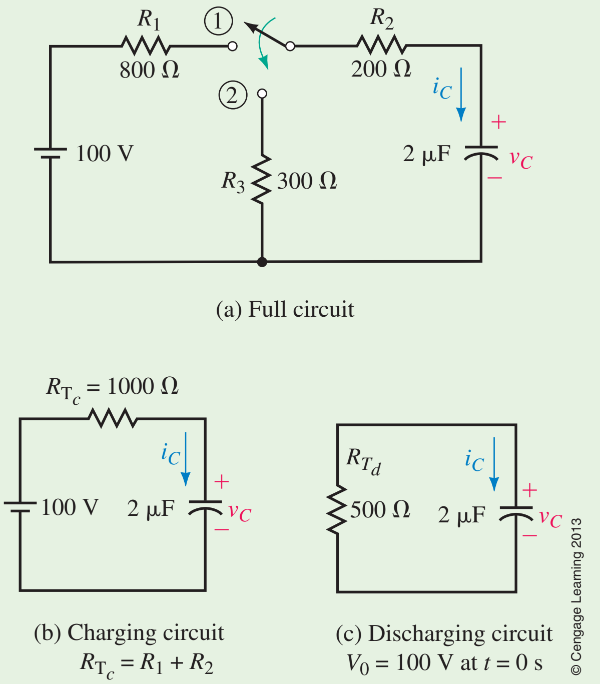


FIGURE 11–24 R_{T_c} is the total resistance of the charge circuit, while R_{T_d} is the total resistance of the discharge circuit.

Solution Figure 11–24(b) shows the equivalent charging circuit. Here,

$$\tau_C = (R_1 + R_2)C = (1\text{ k}\Omega)(2\ \mu\text{F}) = 2.0\text{ ms}.$$

a. $v_C = E(1 - e^{-t/\tau_c}) = 100(1 - e^{-500t})$ V

b. $i_C = \dfrac{E}{R_{T_c}}e^{-t/\tau_c} = \dfrac{100}{1000}e^{-500t} = 100e^{-500t}$ mA

Since $5\tau_c = 10$ ms, charging is complete by the time the switch is moved to discharge. Thus, $V_0 = 100$ V when discharging begins. (This is the continuity linkage.)

c. Figure 11–24(c) shows the equivalent discharge circuit.

$$\tau_d = (500\ \Omega)(2\ \mu\text{F}) = 1.0\text{ ms}$$
$$v_C = V_0e^{-t/\tau_d} = 100e^{-1000t}\text{ V}$$

where $t = 0$ s has been redefined for discharge as noted earlier.

d. $i_C = -\dfrac{V_0}{R_2 + R_3}e^{-t/\tau_d} = -\dfrac{100}{500}e^{-1000t} = -200e^{-1000t}$ mA

e. See Figure 11–25. Note that discharge is more rapid than charge since $\tau_d < \tau_c$.

NOTES...

When solving a transient problem with multiple switch operations, you should always draw the circuit as it looks during each time interval of interest. (It doesn't take long to do this, and it helps clarify just what it is you need to look at for each part of the solution.) This is illustrated in Example 11–11. Here, we have drawn the circuit in Figure 11–24(b) as it looks during charging and in (c) as it looks during discharging. These diagrams make clear which components are relevant to the charging phase and which are relevant to the discharging phase.

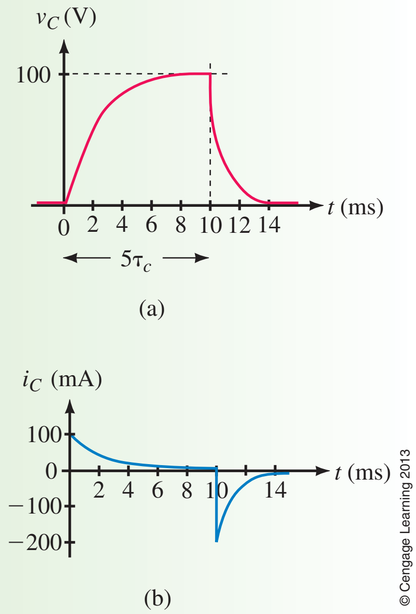


FIGURE 11–25 Waveforms for the circuit of Figure 11–24. Note that only the first time scale is shown explicitly.

CircuitSim 11-12

EXAMPLE 11–12

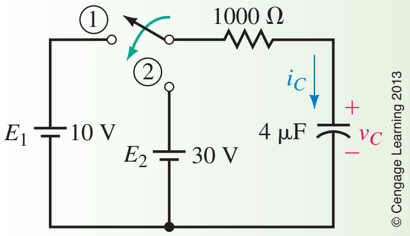


FIGURE 11–26

The capacitor of Figure 11–26 is uncharged. The switch is moved to position 1 for 5 ms, then to position 2 and left there.

a. Determine v_C while the switch is in position 1.
b. Determine i_C while the switch is in position 1.
c. Compute v_C and i_C at $t = 5$ ms.
d. Determine v_C while the switch is in position 2.
e. Determine i_C while the switch is in position 2.
f. Sketch the voltage and current waveforms.
g. Determine v_C and i_C at $t = 10$ ms.

CircuitSim 11-13

Solution

$$\tau_c = \tau_d = RC = (1\ \text{k}\Omega)(4\ \mu\text{F}) = 4\ \text{ms}$$

a. $v_C = E_1(1 - e^{-t/\tau_c}) = 10(1 - e^{-250t})$ V

b. $i_C = \dfrac{E_1}{R}e^{-t/\tau_c} = \dfrac{10}{1000}e^{-250t} = 10e^{-250t}$ mA

c. At $t = 5$ ms,

$$v_C = 10(1 - e^{-250 \times 0.005}) = 7.14\ \text{V}$$
$$i_C = 10e^{-250 \times 0.005}\ \text{mA} = 2.87\ \text{mA}$$

d. In position 2, $E_2 = 30$ V, and $V_0 = 7.14$ V. (V_0 is the continuity linkage.) Use Equation 11–10:

$$v_C = E_2 + (V_0 - E_2)e^{-t/\tau_d} = 30 + (7.14 - 30)e^{-250t}$$
$$= 30 - 22.86e^{-250t}\ \text{V}$$

where $t = 0$ s has been redefined for position 2.

e. $i_C = \dfrac{E_2 - V_0}{R}e^{-t/\tau_d} = \dfrac{30 - 7.14}{1000}e^{-250t} = 22.86e^{-250t}$ mA

f. See Figure 11–27.

g. $t = 10$ ms is 5 ms into the new time scale. Thus, $v_C = 30 - 22.86e^{-250(5\ \text{ms})} = 23.5$ V and $i_C = 22.86e^{-250(5\ \text{ms})} = 6.55$ mA. Values are plotted on the graph.

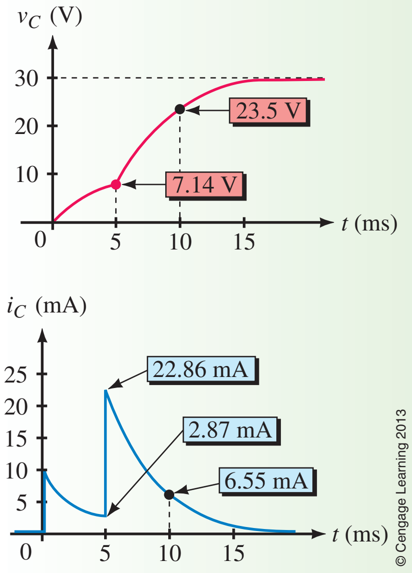


FIGURE 11–27 Capacitor voltage and current for the circuit of Figure 11–26.

EXAMPLE 11–13

In Figure 11–28(a), the capacitor is initially uncharged. The switch is moved to the charge position, then to the discharge position, yielding the current shown in (b). The capacitor takes 1.75 ms to discharge. Determine the following:

a. E b. R_1 c. C

Solution

a. Since the capacitor charges fully, it has a value of E volts when switched to discharge. The discharge current spike is therefore

$$-\frac{E}{10\ \Omega + 25\ \Omega} = -3\ \text{A}$$

Thus, $E = 105$ V.

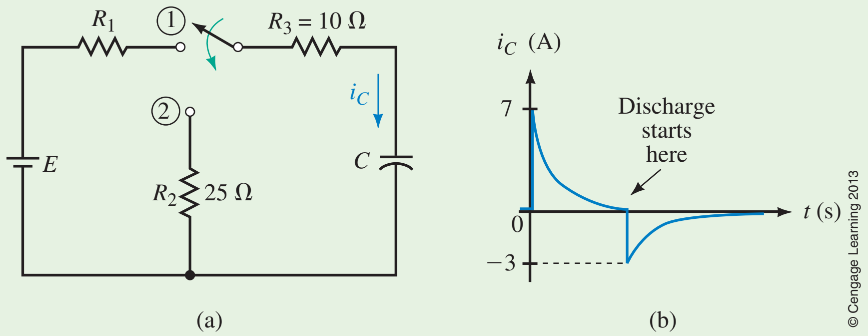


FIGURE 11–28

b. The charging current spike has a value of

$$\frac{E}{10\ \Omega + R_1} = 7\ \text{A}$$

Since $E = 105$ V, this yields $R_1 = 5\ \Omega$.

c. $5\tau_d = 1.75$ ms. Therefore, $\tau_d = 350\ \mu$s. But $\tau_d = (R_2 + R_3)C$. Thus, $C = 350\ \mu\text{s}/35\ \Omega = 10\ \mu$F.

RC Circuits in Steady State dc

When an *RC* circuit reaches steady state dc, its capacitors look like open circuits and a transient analysis is not needed—see Note.

EXAMPLE 11–14

The circuit of Figure 11–29(a) has reached steady state. Determine the capacitor voltages.

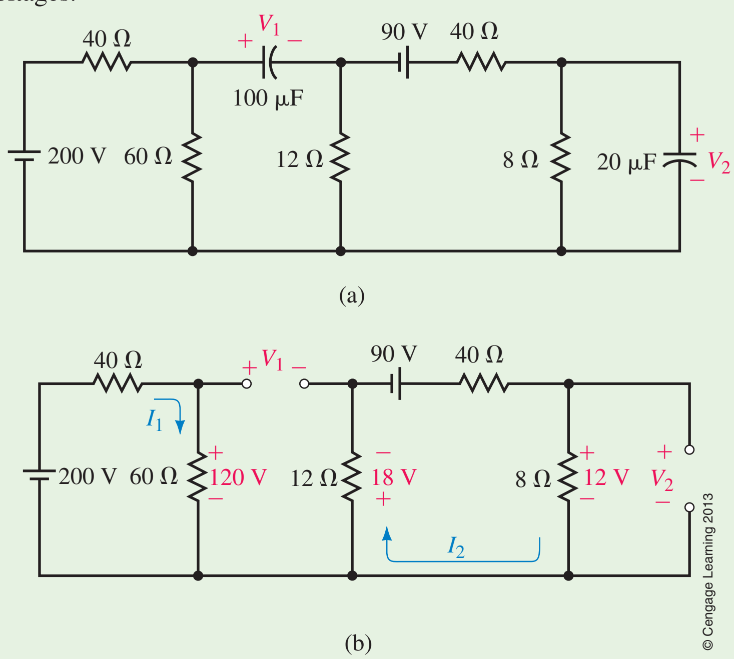


FIGURE 11–29

NOTES...

Since a capacitor consists of conducting plates separated by an insulator, there is no conductive path from terminal to terminal through the capacitor. Therefore, when a capacitor is placed across a dc source, except for a brief transient surge, its current is zero. Thus, as we concluded earlier, a capacitor looks like an open circuit to steady state dc.

CircuitSim 11-14

Solution Replace all capacitors with open circuits as in (b). Thus,

$$I_1 = \frac{200\text{ V}}{40\ \Omega + 60\ \Omega} = 2\text{ A}, \qquad I_2 = \frac{90\text{ V}}{40\ \Omega + 8\ \Omega + 12\ \Omega} = 1.5\text{ A}$$

KVL: $V_1 - 120 - 18 = 0$. Therefore, $V_1 = 138$ V. Further,

$$V_2 = (8\ \Omega)(1.5\text{ A}) = 12\text{ V}$$

PRACTICE PROBLEMS 5

1. The capacitor of Figure 11–30(a) is initially uncharged. At $t = 0$ s, the switch is moved to position 1 and 100 ms later, to position 2. Determine expressions for v_C and i_C for position 2.
2. Repeat for Figure 11–30(b). Hint: Use Thévenin's theorem.

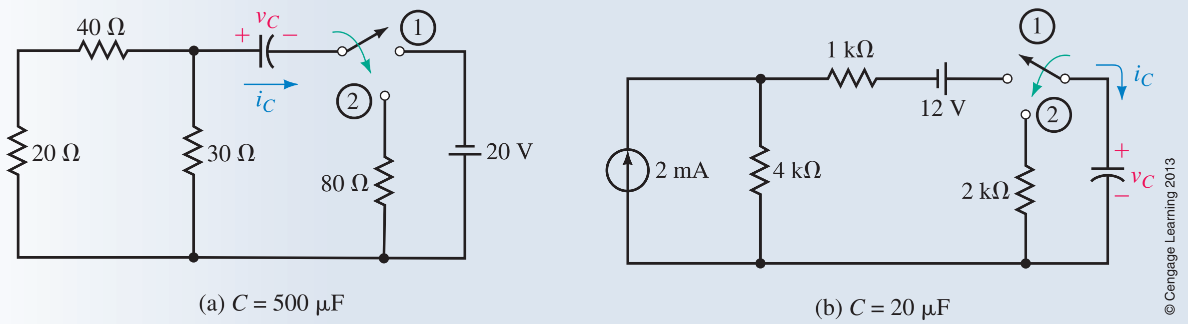


FIGURE 11–30

3. The circuit of Figure 11–31 has reached steady state. Determine source currents I_1 and I_2.

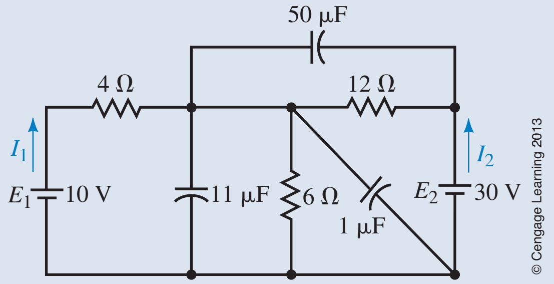


FIGURE 11–31

Answers
1. $20e^{-20t}$ V; $-0.2e^{-20t}$ A
2. $12.6e^{-25t}$ V; $-6.3e^{-25t}$ mA
3. 0 A; 1.67 A

11.6 An *RC* Timing Application

RC circuits are used to create delays for alarm, motor control, and timing applications. Figure 11–32 shows an alarm application. The alarm unit contains a threshold detector, and when the input to this detector exceeds a preset value, the alarm is turned on.

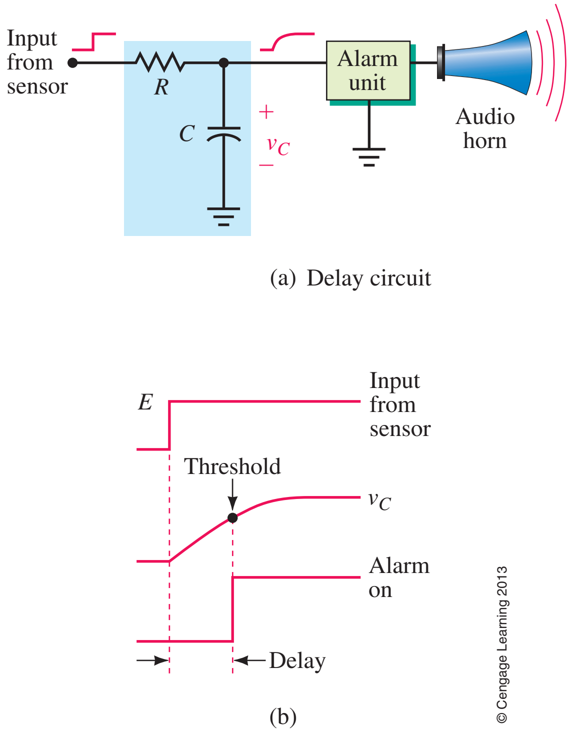


FIGURE 11–32 Creating a time delay with an *RC* circuit. Assume the alarm unit does not load the *RC* circuit.

CircuitSim 11-15

EXAMPLE 11–15

The circuit of Figure 11–32 is part of a building security system. When an armed door is opened, you have a specified number of seconds to disarm the system before the alarm goes off. If $E = 20$ V, $C = 40\ \mu$F, the alarm is activated when v_C reaches 16 V, and you want a delay of at least 25 s, what value of R is needed?

Solution $v_C = E(1 - e^{-t/RC})$. After a bit of manipulation, you get

$$e^{-t/RC} = \frac{E - v_C}{E}$$

Taking the natural log of both sides yields (see Notes)

$$-\frac{t}{RC} = \ln\left(\frac{E - v_C}{E}\right)$$

At $t = 25$ s, $v_C = 16$ V. Thus,

$$-\frac{t}{RC} = \ln\left(\frac{20 - 16}{20}\right) = \ln 0.2 = -1.6094$$

Substituting $t = 25$ s and $C = 40\ \mu$F yields

$$R = \frac{t}{1.6094C} = \frac{25\text{ s}}{1.6094 \times 40 \times 10^{-6}} = 388\text{ k}\Omega$$

Choose the next higher standard value, namely 390 kΩ.

NOTES...

The relationship between the exponential function and natural logs is discussed briefly in Appendix B.

PRACTICE PROBLEMS 6

1. Suppose you want to increase the disarm time of Example 11–15 to at least 35 s. Compute the new value of R.
2. If, in Example 11–15, the threshold is 15 V and $R = 1\ \text{M}\Omega$, what is the disarm time?

Answers

1. 544 kΩ. Use 560 kΩ.
2. 55.5 s

IN-PROCESS LEARNING CHECK 2

(Answers are at the end of the chapter.)

1. Refer to Figure 11–16:
   a. Determine the expression for v_C when $V_0 = 80$ V. Sketch v_C.
   b. Repeat (a) if $V_0 = 40$ V. Why is there no transient?
   c. Repeat (a) if $V_0 = -60$ V.
2. For part (c) of Question 1, v_C starts at -60 V and climbs to $+40$ V. Determine at what time v_C passes through 0 V, using the technique of Example 11–15.
3. For the circuit of Figure 11–18, suppose $R = 10\ \text{k}\Omega$ and $C = 10\ \mu\text{F}$:
   a. Determine the expressions for v_C and i_C when $V_0 = 100$ V. Sketch v_C and i_C.
   b. Repeat (a) if $V_0 = -100$ V.
4. Repeat Example 11–12 if voltage source 2 is reversed, i.e., $E_2 = -30$ V.
5. The switch of Figure 11–33(a) is closed at $t = 0$ s. The Norton equivalent of the circuit in the box is shown in (b). Determine expressions for v_C and i_C. The capacitor is initially uncharged.

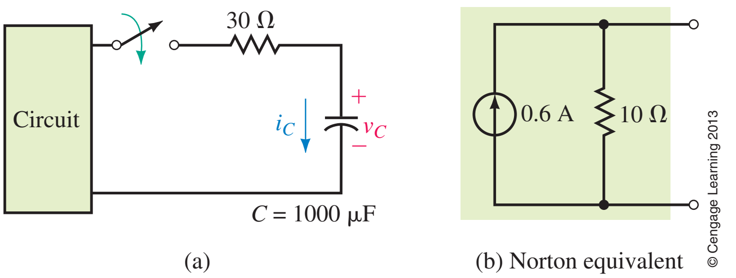


FIGURE 11–33 Hint: Use a source transformation.

11.7 Pulse Response of *RC* Circuits

In previous sections, we looked at the response of *RC* circuits to switched dc inputs. In this section, we consider the effect that *RC* circuits have on pulse waveforms. Since many electronic devices and systems utilize pulse or rectangular waveforms, including computers, communications systems, and motor control circuits, these are important considerations.

Pulse Basics

A **pulse** is a voltage or current that changes from one level to the other and back again as in Figure 11–34(a) and (b). A **pulse train** is a repetitive stream of pulses, as in (c). If a waveform's high time equals its low time, as in (d), it is called a **square wave.**

The length of each cycle of a pulse train is termed its **period,** T, and the number of pulses per second is defined as its **pulse repetition rate (PRR)** or **pulse repetition frequency (PRF).** For example, in (e), there are two complete cycles in 1 second; therefore, the PRR = 2 pulses/s. With two cycles every second, the time for one cycle is $T = \frac{1}{2}$ s. Note that this is 1/PRR. This is true in general. That is,

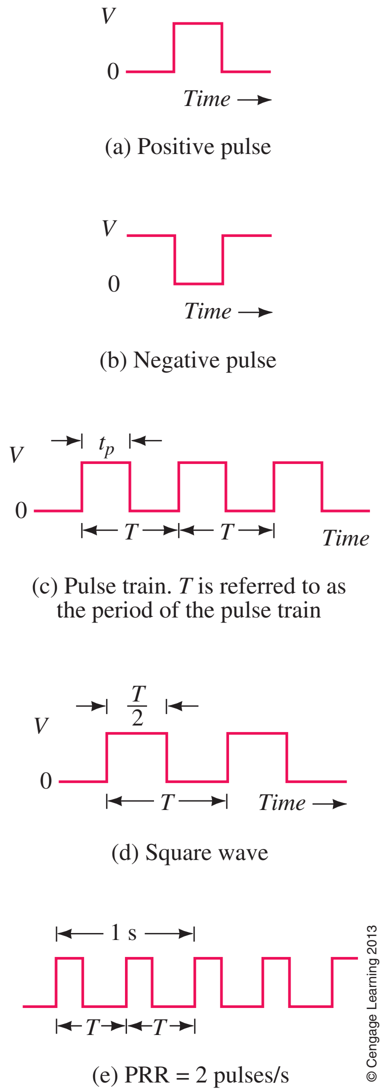
$$T = \frac{1}{\text{PRR}} \quad \text{s} \tag{11–16}$$

The width, t_p, of a pulse relative to its period [Figure 11–34(c)] is its **duty cycle.** Thus,

$$\text{percent duty cycle} = \frac{t_p}{T} \times 100\% \tag{11–17}$$

A square wave [Figure 11–34(d)] therefore has a 50% duty cycle, while a waveform with $t_p = 1.5\ \mu\text{s}$ and a period of 10 μs has a duty cycle of 15%.

In practice, waveforms are not ideal, that is, they do not change from low to high or high to low instantaneously. Instead, they have finite **rise** and **fall times.** Rise and fall times are denoted as t_r and t_f and are measured between the 10% and 90% points as indicated in Figure 11–35(a). **Pulse width** is measured at the 50% point. The difference between a real waveform and an ideal waveform is often slight. For example, rise and fall times of real pulses may be only a few nanoseconds and when viewed on an oscilloscope, as in Figure 11–35(b),



(a) Positive pulse

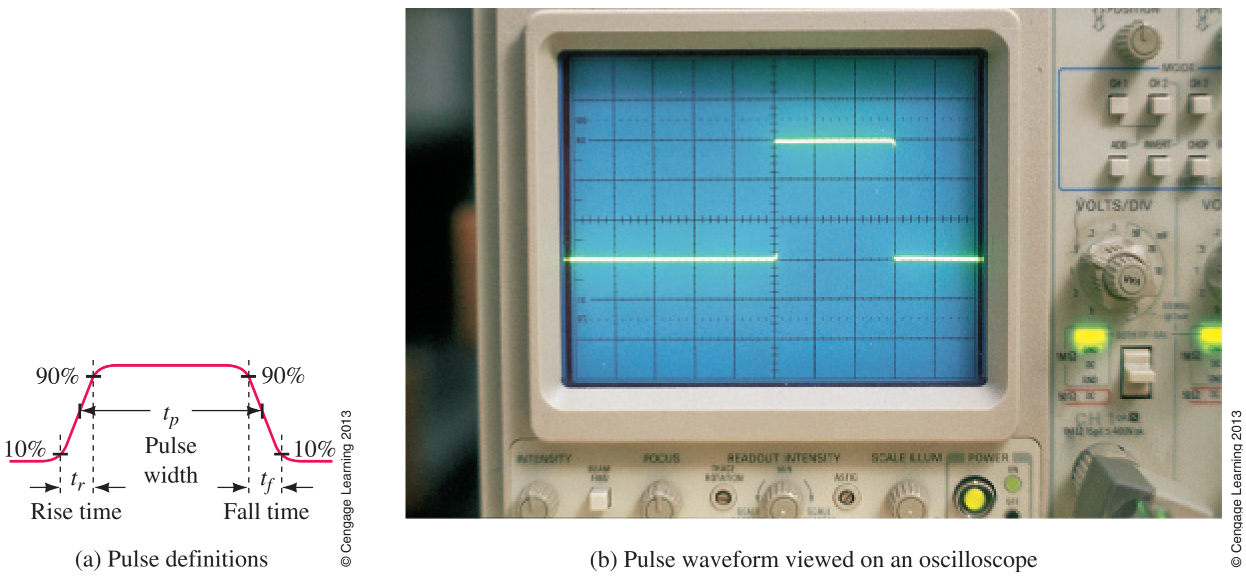
(b) Negative pulse

(c) Pulse train. T is referred to as the period of the pulse train

(d) Square wave

(e) PRR = 2 pulses/s

FIGURE 11–34 Ideal pulses and pulse waveforms.



(a) Pulse definitions

(b) Pulse waveform viewed on an oscilloscope

FIGURE 11–35 Practical pulse waveforms.

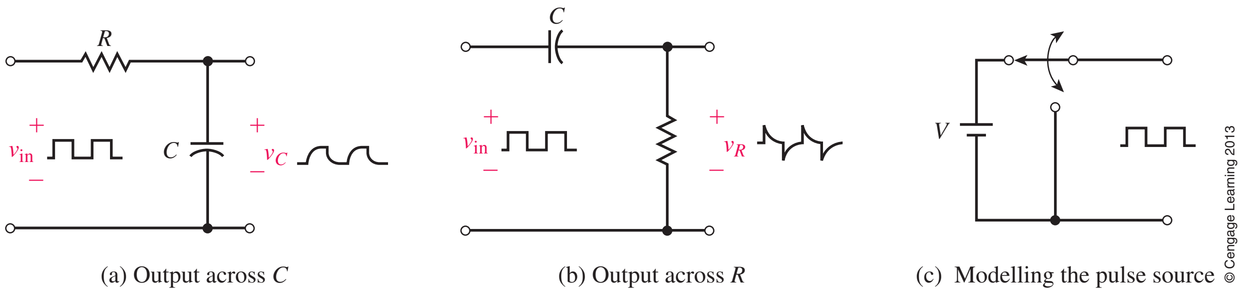


FIGURE 11–36 *RC* circuits with pulse input. Although we have modelled the source here as a battery and a switch, in practice, pulses are usually created by electronic circuits.

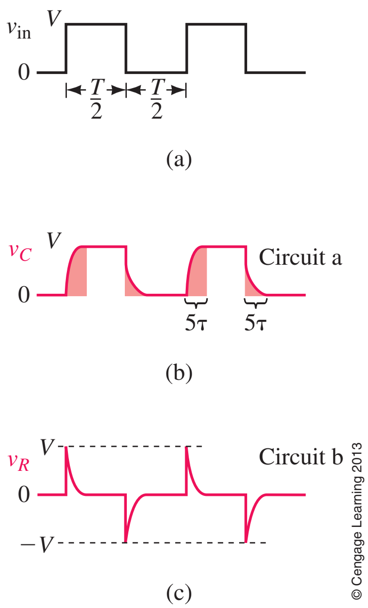


FIGURE 11–37 Pulse width much greater than 5τ. Note that the shaded areas indicate where the capacitor is charging and discharging. Spikes occur on the input voltage transitions.

appear to be ideal. While in what follows, we assume ideal waveforms, in more advanced electronics courses, you often have to take rise and fall times into account.

The Effect of Pulse Width

The width of a pulse relative to a circuit's time constant determines how it is affected by an *RC* circuit. Consider Figure 11–36. In (a), the circuit has been drawn to focus on the voltage across *C;* in (b), it has been drawn to focus on the voltage across *R.* (Otherwise, the circuits are identical.) An easy way to visualize the operation of these circuits is to assume that the pulse is generated by a switch that is moved rapidly back and forth between *V* and common as in (c). This alternately creates a charge and discharge circuit, and thus all of the ideas developed in this chapter apply directly.

Pulse Width $t_p \gg 5\,\tau$

First, consider the output of circuit (a). When the pulse width and time between pulses are very long compared with the circuit time constant, the capacitor charges and discharges fully, Figure 11–37(b). (This case is similar to what we have already seen in this chapter.) Note that charging and discharging occur at the transitions of the pulse. The transients therefore increase the rise and fall times of the output. In high-speed circuits, this may be a problem. (You will learn more about this in your digital electronics courses.)

EXAMPLE 11–16

A square wave is applied to the input of Figure 11–36(a). If $R = 1\text{ k}\Omega$ and $C = 100\text{ pF}$, estimate the rise and fall time of the output signal using the universal time constant curve of Figure 11–15(a).

Solution Here, $\tau = RC = (1 \times 10^3)(100 \times 10^{-12}) = 100\text{ ns}$. From Figure 11–15(a), note that v_C reaches the 10% point at about 0.1 τ, which is (0.1)(100 ns) = 10 ns. The 90% point is reached at about 2.3 τ, which is (2.3)(100 ns) = 230 ns. The rise time is therefore approximately 230 ns – 10 ns = 220 ns. The fall time will be the same. (A theoretical analysis yields 2.197 *RC*. Use this relationship and compare results.)

CircuitSim 11-16

Now consider the circuit in Figure 11–36(b). Here, current i_C will be similar to that of Figure 11–7(b), except that the pulse widths will be narrower. Since voltage $v_R = Ri_C$, the output will be a series of short, sharp spikes that occur at input transitions as in Figure 11–37(c). Under the conditions here (i.e., pulse width much greater than the circuit time constant), v_R is an approximation to the derivative of v_{in} and the circuit is called a differentiator circuit. Such circuits have important practical uses.

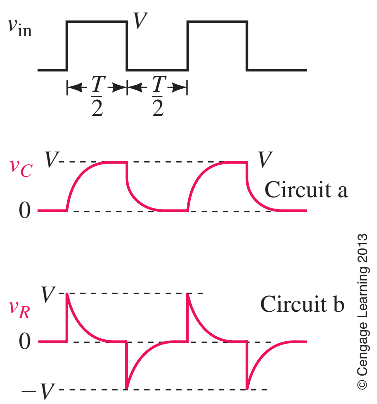


FIGURE 11–38 Pulse width equal to 5τ. These waveforms are the same as Figure 11–37, except that the transitions last relatively longer.

Pulse Width $t_p = 5\tau$

These waveforms are shown in Figure 11–38. Since the pulse width is 5τ, the capacitor fully charges and discharges during each pulse. Thus, waveforms here are again similar to what we have seen throughout this chapter.

Pulse Width $t_p \ll 5\tau$

This case differs from what we have seen so far in this chapter only in that the capacitor does not have time to charge and discharge significantly between pulses. The result is that switching occurs on the early (nearly straight line) part of the charging and discharging curves and thus, v_C is roughly triangular in shape, Figure 11–39(a). It has an average value of $V/2$. Under the conditions here, v_C is the approximate integral of v_{in} and the circuit is called an integrator circuit.

It should be noted that v_C does not reach the steady state shown in Figure 11–39 immediately. Instead, it works its way up over a period of five time constants (Figure 11–40). To illustrate, assume an input square wave of 5 V with a pulse width of 0.1 s and $\tau = 0.1$ s. Proceed as follows:

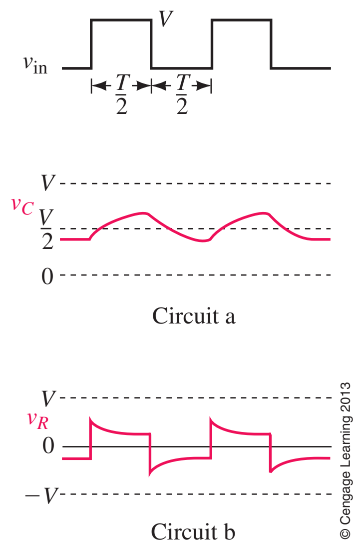


FIGURE 11–39 Pulse width much less than 5τ. The circuit does not have time to charge or discharge substantially.

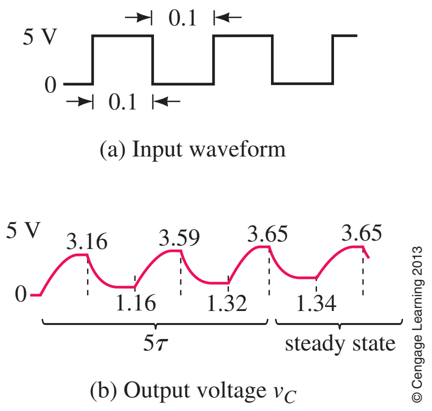


FIGURE 11–40 Notice how the solution takes five time constants to reach its steady state, after which it cycles between 1.34 and 3.65 V. You can easily see this using Multisim or PSpice.

CircuitSim 11-17

Pulse 1 $v_C = E(1 - e^{-t/\tau})$. At the end of the first pulse ($t = 0.1$ s), v_C has climbed to $v_C = 5(1 - e^{-0.1/0.1}) = 5(1 - e^{-1}) = 3.16$ V. From the end of pulse 1 to the beginning of pulse 2 (i.e., over an interval of 0.1 s), v_C decays from 3.16 V to $3.16e^{-0.1/0.1} = 3.16e^{-1} = 1.16$ V.

Pulse 2 v_C starts at 1.16 V and 0.1 s later has a value of $v_C = E + (V_0 - E)e^{-t/\tau} = 5 + (1.16 - 5)e^{-0.1/0.1} = 5 - 3.84e^{-1} = 3.59$ V. It then decays to $3.59e^{-1} = 1.32$ V over the next 0.1 s.

Continuing in this manner, the remaining values for Figure 11–40(b) are determined. After 5τ, v_C cycles between 1.34 and 3.65 V, with an average of $(1.34 + 3.65)/2 = 2.5$ V, or half the input pulse amplitude.

PRACTICE PROBLEMS 7

Verify the remaining points of Figure 11–40(b).

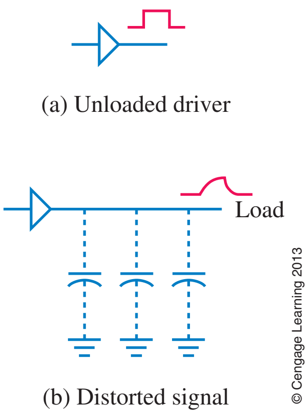


FIGURE 11–41 Distortion caused by capacitive loading.

Capacitive Loading

Capacitance occurs whenever conductors are separated by insulating material. This means that capacitance exists between wires in cables, between traces on printed circuit boards, and so on. In general, this capacitance is undesirable, but it cannot be avoided. It is called **stray capacitance.** Fortunately, stray capacitance is often so small that it can be neglected. However, in high-speed circuits, it may cause problems.

To illustrate, consider Figure 11–41. The electronic driver of (a) produces square pulses. However, when it drives a long line as in (b), stray capacitance loads it and increases the signal's rise and fall times (since capacitance takes time to charge and discharge). This effect is referred to as **capacitive loading.** If the rise and fall times become excessively long, the signal reaching the load may be so degraded that the system malfunctions. (Capacitive loading is a serious issue, but we will leave it for future courses to deal with.)

11.8 Transient Analysis Using Computers

Multisim and PSpice are well suited for studying transients as they both incorporate easy-to-use graphing facilities. When plotting transients, you must specify the time duration for your plot—that is, the length of time that you expect the transient to last. (This time is designated *TSTOP* by both Multisim and PSpice.) A good value to use is 5τ where τ is the time constant of the circuit. (For complex circuits, if you do not know τ, make an estimate, run a simulation, adjust the time scale, and repeat until you get an acceptable plot.)

Multisim

As a first example, use Multisim to compute and plot capacitor voltage and current waveforms for the circuit of Example 11–2, Section 11.2. Read the Multisim notes and then, if you are a legacy software user, go to our Web site for instructions; otherwise, proceed as follows:

- Create the circuit of Figure 11–42 on your screen. (Use a virtual capacitor and rotate it once so that its "1" end is at the top as described in Appendix A.)
- Click *Options/Sheet Properties* and under *Net Names*, click *Show All*, then *OK*. Node numbers will appear on your schematic—see Note 5. In what follows, we use the node numbers shown in Figure 11–42 in our discussion.
- Click *Simulate/Analysis* and select *Transient Analysis.* Compute the duration of the transient (it is 0.5 s) and enter this value as *TSTOP*. From the *Initial Conditions* box, choose *Set to zero,* then click the *Output* tab.
- Click *V*(2) to highlight it, then click *Add*. (This is capacitor voltage.)
- Click *I*(C1) to highlight it, then click *Add*. (This is capacitor current.)
- Click *Simulate*.

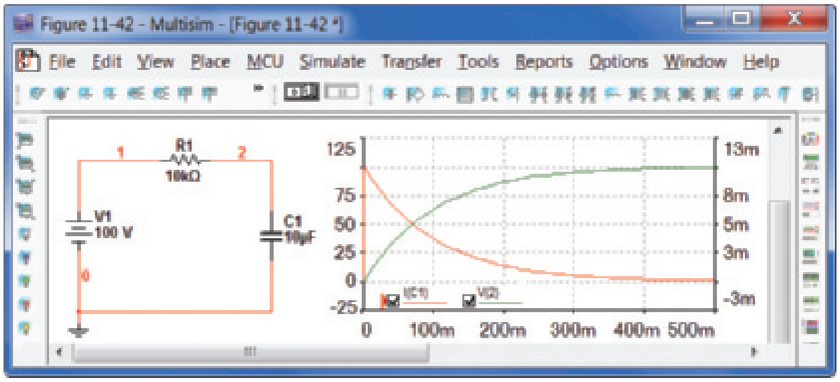


FIGURE 11–42 Multisim solution of Example 11–2. (The left-hand graph scale is for the voltage waveform.) The transient is initiated by software, so no switch is needed in the simulation setup.

- *Grapher View* opens. Click *View* and ensure that *Toolbar* is selected. Find the icon that identifies itself as *Black Background*, then click. Now find the *Show Grid* icon and click. Right-click the screen, find *Show Select Marks*, and click.

Note that the current waveform appears as a flat line along the zero axis. You need to expand it so that you can see its detail. Proceed as follows (or see Appendix A, *Displaying Multiple Traces*).

- On *Grapher,* click *Trace/Trace Properties*. In the dialog box that opens, select the *Traces* tab at the top of the box. The item marked *Trace Label* should indicate *I*(C1). If it doesn't, use the *Trace ID* up/down arrows to change it.
- Near the bottom of the box (under *Y-vertical axis*), click the *Right axis* button, then click *Apply*. Select the *Right Axis* tab at the top of the box, click the *Axis Enabled box,* click *Auto-range,* click *Apply* then *OK*. (You should have the voltage and current waveforms of Figure 11–42 on your screen, although trace colors may not match.)
- Activate the cursors by clicking the *Show Cursors* icon.
- With your mouse pointer, drag #1 cursor into the display area. Place your mouse pointer on it, right click, select *Set X Value,* type in **150m,** then click *OK*. The cursor moves to the 150 ms point and the *Cursor* display box shows the results. Your display should show current = 2.23 mA and voltage = 77.69 volts at t = 150 ms. Compare these to the answers indicated in Figure 11–10, Example 11–2.

Use your cursor to measure values at 100 ms intervals, then compare to the answers from Practice Problems 2, Question 1.

Initial Conditions in Multisim

Let us change the preceding problem to include an initial voltage of 150 V on the capacitor. Get the circuit of Figure 11–42 back on the screen, double-click the capacitor symbol, and in the dialog box under *Initial conditions*, type in **150,** ensure that units are set to V, and click *OK*. Select *Transient Analysis;* from the *Initial Conditions* box, choose *User-defined.* Run the simulation and note that the transient starts at 150 V and decays to its steady state value of 100 V in five time constants as expected.

Now expand the current trace scale as we did above. You should have the waveforms of Figure 11–43 on your screen. With your cursor, measure values at t = 100 ms and verify using Equations 11–10 and 11–11.

NOTES...

Multisim Operational Notes

1. If you are a Multisim 9 or 10 user, go to our Web site at cengagebrain.com and follow the links to *Special Instructions for Multisim 9 and 10 Users.*
2. You don't need a switch to initiate a transient. Simply build the circuit without a switch and select transient analysis. Multisim then performs the transient simulation and plots the results.
3. These examples were prepared using Multisim 11, the version current at the time of writing. Since software is updated regularly, this may not be the version that you are using.
4. To place your cursor at an exact time point on a Multisim waveform, for example,at t = 150 ms, place your mouse pointer on the cursor and right-click. Select *Set X Value,* type in **150m,** then click OK.
5. Multisim assigns node numbers according to the order in which you wire your circuit. Thus, your node numbers may be different than the ones we show.
6. Multisim computes node voltages with respect to ground. Thus, in Figure 11–42, the voltage at Node 2 is the voltage across the capacitor. Multisim 11 refers to it as V(2).

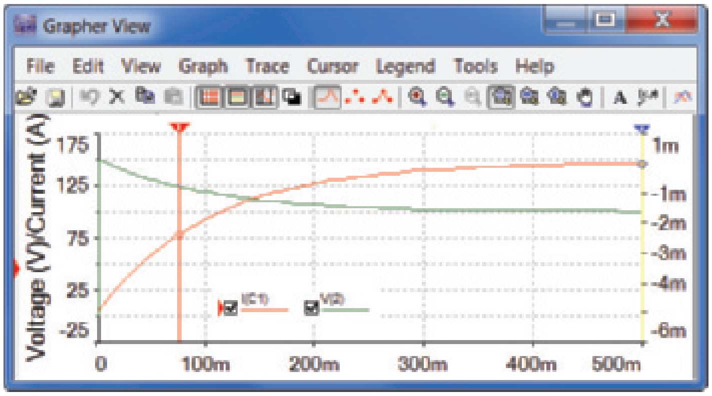


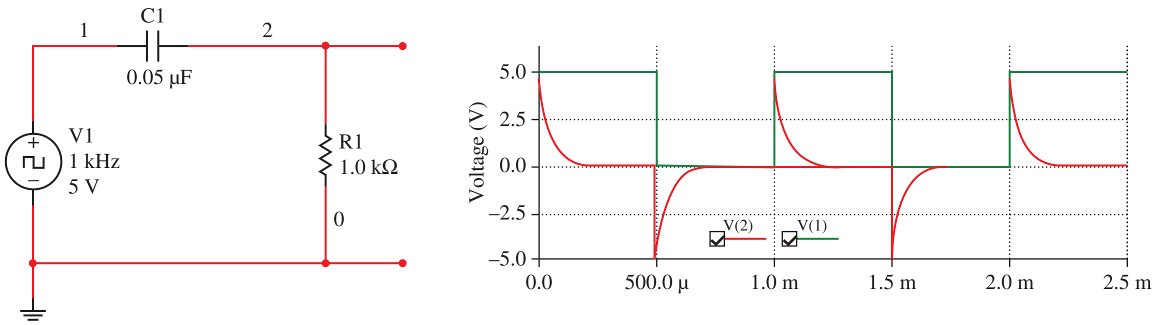
FIGURE 11–43 Solution for the circuit of Figure 11–42 with $V_0 = 150$ V.

PRACTICE PROBLEMS 8

For the circuit of Figure 11–42, set the capacitor initial condition to –50 V, then repeat the preceding simulation. Verify answers using Equations 11–10 and 11–11.

Another Example

Right-click *View* and ensure that the Signal Source Components toolbar is selected. In its toolbin, locate and click *Place Clock Voltage Source,* then build the circuit of Figure 11–44(a) on your screen. (The clock, with its default settings, produces a square wave that cycles between 0 V and 5 V with a cycle length of $T = 1$ ms.) This means that its *on* time t_p is $T/2 = 500$ μs. Since the time constant of Figure 11–44 is $\tau = RC = 50$ μs, t_p is greater than 5τ, and a waveform similar to that of Figure 11–37(c) should result. To verify, follow the procedure of the previous example, except set *End Time (TSTOP)* to 0.0025 in the *Analysis/Transient* dialog box, choose *Initial Conditions set to zero,* then select V(1) and V(2) for display. (Legacy users, select nodes 1 and 2 instead.) Click *Simulate* and the waveforms of Figure 11–44(b) appear. Note that output spikes occur on the transitions of the input waveform as predicted.



(a)

(b) The red waveform is the output voltage.

FIGURE 11–44 Multisim analysis of the circuit of Figure 11–36(b). Compare (b) to Figure 11–37(c).

PRACTICE PROBLEMS 9

For the circuit of Figure 11–44, run simulations for $R = 500\ \Omega$, $2\ \text{k}\Omega$, and $20\ \text{k}\Omega$. For each case, compute τ, then note how the resulting waveform fits with the predicted waveforms of Figures 11–37 to 11–39.

PSpice

NOTES...

PSpice Operational Notes

1. Do not use a space between a value and its unit. Thus, use 50ms, not 50 ms, and so forth.
2. When instructed to enter data via a Property editor (e.g., initial conditions), first click the Parts tab at the bottom of the screen; scroll right until you find the cell that you want, type in its value, then click *Apply*. In the warning box, click Yes, then click the Windows close button (the x beside the word *cadence*) to close the Property Editor. (Don't click the large red boxed close button above it because that will close PSpice.)
3. For transient problems you must specify an initial condition (IC) for each capacitor and inductor, even if they are zero. The procedure is described in the examples.
4. For most simple transient analysis problems, you don't need a switch—simply specify transient analysis at run time.
5. To select the waveform that you want the cursor to apply to, click the applicable symbol at the bottom of the screen.

As a first example, consider Figure 11–2 with $R = 200\ \Omega$, $C = 50\ \mu\text{F}$, and $E = 40$ V. Let the capacitor be initially uncharged (i.e., $V_0 = 0$ V). First, read the PSpice Operational Notes, then proceed as follows:

- Create the circuit on the screen as in Figure 11–45—see Note 4. Remember to rotate the capacitor as discussed in Appendix A, then set its initial condition

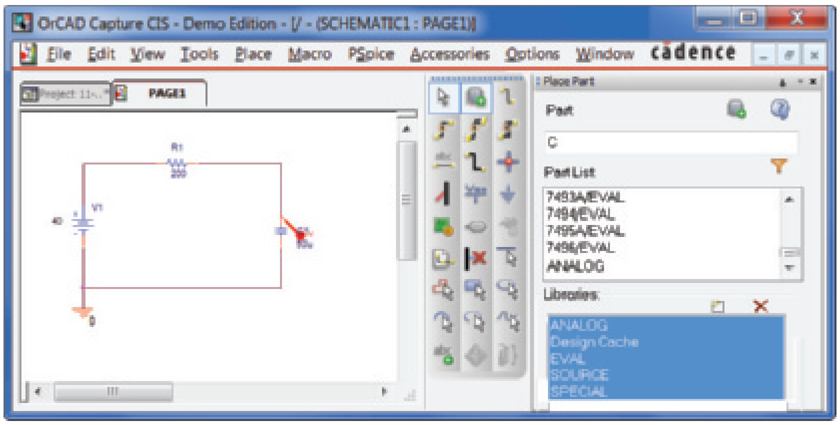


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FIGURE 11–45 PSpice example. The voltage marker displays voltage with respect to ground, which, in this case, is the voltage across C_1.

(IC) to zero. To do this, double-click the capacitor symbol, type **0V** into the Property editor cell labeled IC—see Note 2—click *Apply*, then close the editor. Click the *New Simulation Profile* icon, enter a name (e.g., **Figure 11–45**) then click *Create*. In the *Simulation Settings* box, click the *Analysis* tab, select *Time Domain (Transient)* and in *Options,* select *General Settings*. Set the duration of the transient *(TSTOP)* to 50ms, then click *OK*. Find the voltage marker on the toolbar and place as shown. (It will be gray, but will change color after the simulation is run.)

- Click the *Run* icon. A trace of capacitor voltage appears. Click *Plot,* then *Add Y Axis*. Click *Trace/Add Trace,* then double-click *I(C1)*. This adds the current trace, Figure 11–46.

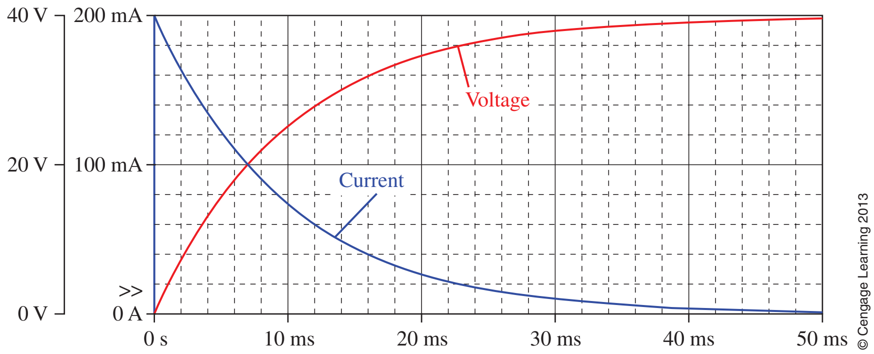


FIGURE 11–46 Waveforms for the circuits of Figures 11–45 and 11–47.

Analysis of Results

Click the *Toggle cursor* icon, then use the cursor to determine values from the screen. For example, at t = 5 ms, you should find v_C = 15.7 V and i_C = 121 mA as determined earlier in Example 11–10, part (c).

PRACTICE PROBLEMS 10

For the circuit of Figure 11–45, set the capacitor initial condition to 100 V, then repeat the preceding simulation. Verify answers using Equations 11–10 and 11–11.

As a second example, consider the circuit of Figure 11–21 (shown as Figure 11–47). Create the circuit using the same general procedure as in the previous example, except do not rotate the capacitor. Be sure to set V_0 (the initial capacitor voltage) to zero. In the *Simulation Profile* box, set *TSTOP* to 50ms. Place differential voltage markers (found on the toolbar at the top of the screen) across C (with marker V^+ on the left) to graph the capacitor voltage. Run the analysis, create a second axis, then add the current plot. You should get the same graph (i.e., Figure 11–46) as you got for the previous example, since its circuit is the Thévenin equivalent of this one.

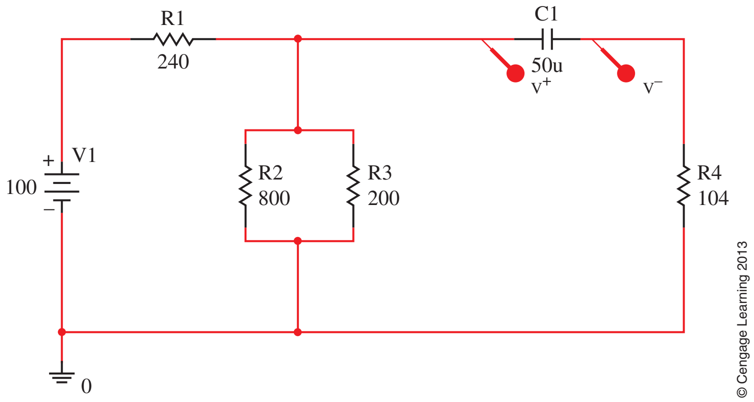


FIGURE 11–47 The differential markers display the voltage across C_1.

As a final example, consider Figure 11–48(a), which shows double switching action.

EXAMPLE 11–17

The capacitor of Figure 11–48(a) has an initial voltage of −10 V. The switch is moved to the charge position for 1 s, then to the discharge position where it remains. Determine curves for v_C and i_C.

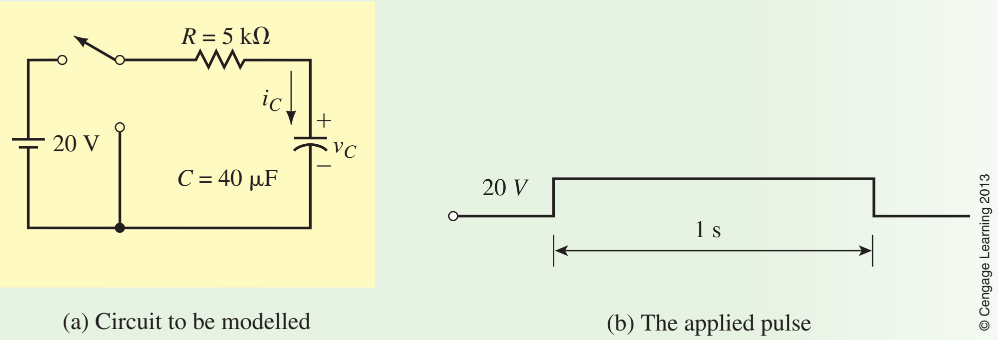


FIGURE 11–48 Creating a charge/discharge waveform using PSpice.

Solution PSpice has no switch that implements the switching sequence of Figure 11–48. However, moving the switch first to charge then to discharge is equivalent to placing 20 V across the *RC* combination for the charge time, then 0 V thereafter as indicated in (b). You can do this with a pulse source (VPULSE) as indicated in Figure 11–49(a). Note the parameters listed beside the symbol.

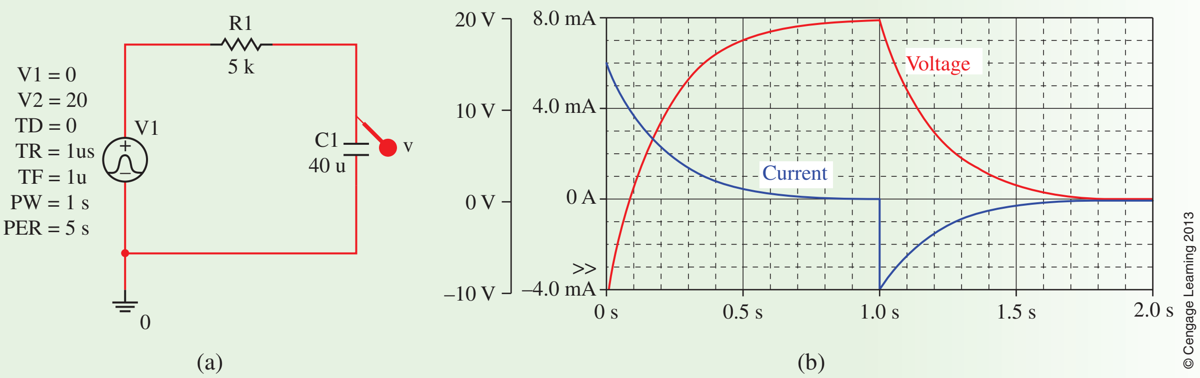


FIGURE 11–49 Modelling switching action using a pulse source.

Click each in turn and set as indicated; for example, click *V*1 and when its parameter box opens, type **0V**. (This defines a pulse with a period of 5 s, a width of 1 s, rise and fall times of 1 μs, amplitude of 20 V, and an initial value of 0 V.) Double-click the capacitor symbol and set *IC* to –10V in the Property Editor; click *Apply* then close. Click the *New Simulation Profile* icon and set *TSTOP* to 2s. Place a Voltage Marker as shown, then click *Run*. Add the second axis and the current trace as described in the previous examples. You should have the curves of (b) on your screen.

Note that voltage starts at −10 V and climbs to 20 V while current starts at $(E - V_0)/R = 30\text{ V}/5\text{ k}\Omega = 6$ mA and decays to zero. When the switch is turned to the discharge position, the current drops from 0 A to $-20\text{ V}/5\text{ k}\Omega = -4$ mA and then decays to zero while the voltage decays from 20 V to zero. Thus, the solution checks.

Putting It into Practice

An electronic device employs a timer circuit of the kind shown in Figure 11–32(a), that is, an *RC* charging circuit and a threshold detector. [Its timing waveforms are thus identical to those of Figure 11–32(b).] The input to the *RC* circuit is a 0 V to 5 V ±4% step, $R = 680\text{ k}\Omega \pm 10\%$, $C = 0.22\ \mu\text{F} \pm 10\%$, the threshold detector activates at $v_C = 1.8\text{ V} \pm 0.05\text{ V}$, and the required delay is 67 ms ±18 ms. You test a number of units as they come off the production line and find that some do not meet the timing spec. Perform a design review and determine the cause. Redesign the timing portion of the circuit in the most economical way possible.

Problems

11.1 Introduction

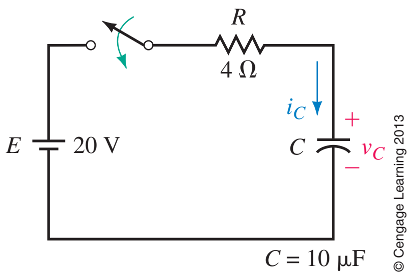


FIGURE 11–50

1. The capacitor of Figure 11–50 is uncharged.
   a. What are the capacitor voltage and current just after the switch is closed?
   b. What are the capacitor voltage and current after the capacitor is fully charged?
2. Repeat Problem 1 if the 20-V source is replaced by a −60-V source.
3. a. What does an uncharged capacitor look like at the instant of switching?
   b. What does a charged capacitor look like at the instant of switching?
   c. What does a capacitor look like to steady state dc?
   d. What do we mean by $i(0^-)$? By $i(0^+)$?

4. For a charging circuit, $E = 25$ V, $R = 2.2$ kΩ, and the capacitor is initially uncharged. The switch is closed at $t = 0$. What is $i(0^+)$?
5. For a charging circuit, $R = 5.6$ kΩ and $v_C(0^-) = 0$ V. If $i(0^+) = 2.7$ mA, what is E?

11.2 Capacitor Charging Equations

6. The switch of Figure 11–50 is closed at $t = 0$ s. The capacitor is initially uncharged.
   a. Determine the equation for charging voltage v_C.
   b. Determine the equation for charging current i_C.
   c. By direct substitution, compute v_C and i_C at $t = 0^+$ s, 40 μs, 80 μs, 120 μs, 160 μs, and 200 μs.
   d. Plot v_C and i_C on graph paper using the results of (c). Hint: See Example 11–2.
7. Repeat Problem 6 if $R = 500$ Ω, $C = 25$ μF, and $E = 45$ V, except compute and plot values at $t = 0^+$ s, 20 ms, 40 ms, 60 ms, 80 ms, and 100 ms.
8. The switch of Figure 11–51 is closed at $t = 0$ s. Determine the equations for capacitor voltage and current. Compute v_C and i_C at $t = 50$ ms.
9. Repeat Problem 8 for the circuit of Figure 11–52.
10. The capacitor of Figure 11–2 is uncharged at the instant the switch is closed. If $E = 80$ V, $C = 10$ μF, and $i_C(0^+) = 20$ mA, determine the equations for v_C and i_C.
11. Determine the time constant for the circuit of Figure 11–50. How long (in seconds) will it take for the capacitor to charge?
12. A capacitor takes 200 ms to charge. If $R = 5$ kΩ, what is C?
13. For Figure 11–50, the capacitor voltage with the switch open is 0 V. Close the switch at $t = 0$ and determine capacitor voltage and current at $t = 0^+$, 40 μs, 80 μs, 120 μs, 160 μs, and 200 μs using the universal time constant curves.
14. If $i_C = 25e^{-40t}$ A, what is the time constant τ and how long will the transient last?
15. For Figure 11–2, the current jumps to 3 mA when the switch is closed. The capacitor takes 1 s to charge. If $E = 75$ V, determine R and C.
16. For Figure 11–2, if $v_C = 100(1 - e^{-50t})$ V and $i_C = 25e^{-50t}$ mA, what are E, R, and C?
17. For Figure 11–2, determine E, R, and C if the capacitor takes 5 ms to charge, the current at 1 time constant after the switch is closed is 3.679 mA, and the capacitor charges to 45 volts steady state.
18. For Figure 11–2, $v_C(\tau) = 41.08$ V and $i_C(2\tau) = 219.4$ mA. Determine E and R.

11.3 Capacitor with an Initial Voltage

19. The capacitor of Figure 11–50 has an initial voltage. If $V_0 = 10$ V, what is the current just after the switch is closed?
20. Repeat Problem 19 if $V_0 = -10$ V.
21. For the capacitor of Figure 11–51, $V_0 = 30$ V.
   a. Determine the expression for charging voltage v_C.
   b. Determine the expression for current i_C.
   c. Sketch v_C and i_C.
22. Repeat Problem 21 if $V_0 = -5$ V.

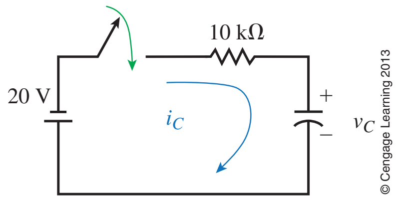


FIGURE 11–51 $V_0 = 0$ V, $C = 10$ μF.

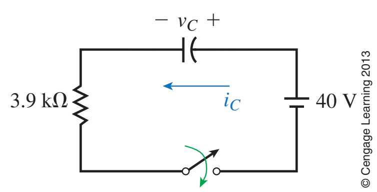


FIGURE 11–52 $C = 10$ μF, $V_0 = 0$ V.

11.4 Capacitor Discharging Equations

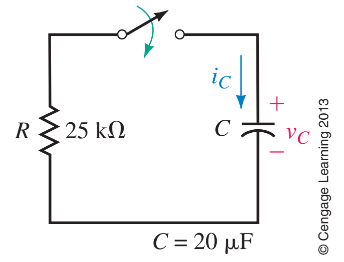


FIGURE 11–53

23. For the circuit of Figure 11–53, assume the capacitor is charged to 50 V before the switch is closed.
    a. Determine the equation for discharge voltage v_C.
    b. Determine the equation for discharge current i_C.
    c. Determine the time constant of the circuit.
    d. Compute v_C and i_C at $t = 0^+$ s, $t = \tau$, 2τ, 3τ, 4τ, and 5τ.
    e. Plot the results of (d) with the time axis scaled in seconds and time constants.
24. The initial voltage on the capacitor of Figure 11–53 is 55 V. The switch is closed at $t = 0$. Determine capacitor voltage and current at $t = 0^+$, 0.5 s, 1 s, 1.5 s, 2 s, and 2.5 s using the universal time constant curves.
25. A 4.7-μF capacitor is charged to 43 volts. If a 39-kΩ resistor is then connected across the capacitor, what is its voltage 200 ms after the resistor is connected?
26. The initial voltage on the capacitor of Figure 11–53 is 55 V. The switch is closed at $t = 0$ s and opened 1 s later. Sketch v_C. What is the capacitor's voltage at $t = 3.25$ s?
27. For Figure 11–54, let $E = 200$ V, $R_2 = 1$ kΩ, and $C = 0.5$ μF. After the capacitor has fully charged in position 1, the switch is moved to position 2.
    a. What is the capacitor voltage immediately after the switch is moved to position 2? What is its current?
    b. What is the discharge time constant?
    c. Determine discharge equations for v_C and i_C.
28. For Figure 11–54, C is fully charged before the switch is moved to discharge. Current just after it is moved is $i_C = -4$ mA and C takes 20 ms to discharge. If $E = 80$ V, what are R_2 and C?

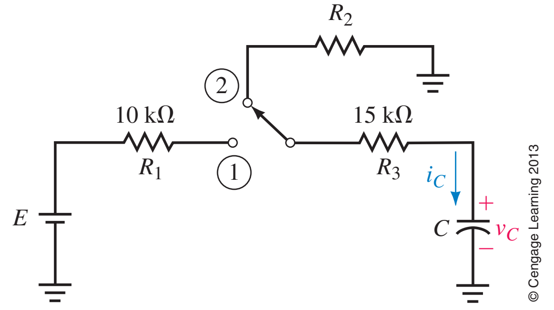


FIGURE 11–54

11.5 More Complex Circuits

29. The capacitors of Figure 11–55 are uncharged. The switch is closed at $t = 0$. Determine the equation for v_C. Compute v_C at one time constant using the equation and the universal time constant curve. Compare answers.
30. For Figure 11–56, the switch is closed at $t = 0$. Given $V_0 = 0$ V.
    a. Determine the equations for v_C and i_C.
    b. Compute the capacitor voltage at $t = 0^+$ 2, 4, 6, 8, 10, and 12 ms.
    c. Repeat (b) for the capacitor current.
    d. Why does 225 V/30 Ω also yield $i(0^+)$?

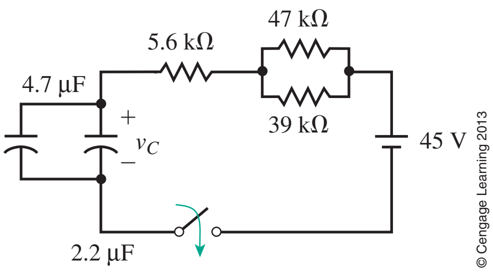


FIGURE 11–55

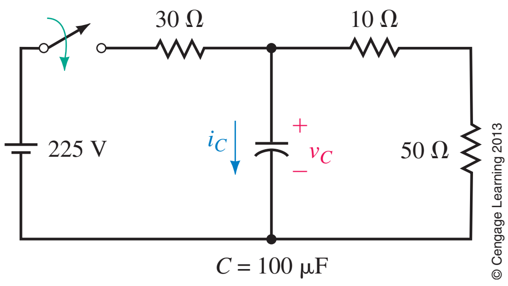


FIGURE 11–56

31. Repeat Problem 30, parts (a) to (c) for the circuit of Figure 11–57.

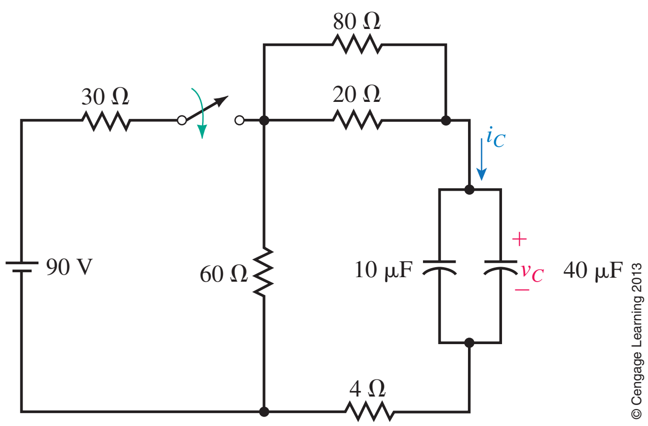


FIGURE 11–57

32. Consider again Figure 11–54. Suppose $E = 80$ V, $R_2 = 25$ kΩ, and $C = 0.5$ μF:
    a. What is the charge time constant?
    b. What is the discharge time constant?
    c. With the capacitor initially discharged, move the switch to position 1 and determine equations for v_C and i_C during charge.
    d. Move the switch to the discharge position. How long does it take for the capacitor to discharge?
    e. Sketch v_C and i_C from the time the switch is placed in charge to the time that the capacitor is fully discharged. Assume the switch is in the charge position for 80 ms.
33. For the circuit of Figure 11–54, the capacitor is initially uncharged. The switch is first moved to charge, then to discharge, yielding the current shown in Figure 11–58. The capacitor fully charges in 12.5 s. Determine E, R_2, and C.

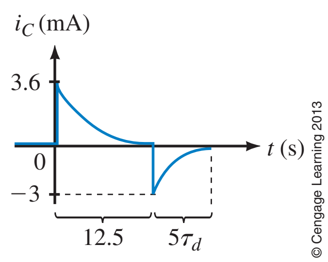


FIGURE 11–58

34. Refer to the circuit of Figure 11–59:
    a. What is the charge time constant?
    b. What is the discharge time constant?
    c. The switch is in position 2 and the capacitor is uncharged. Move the switch to position 1 and determine equations for v_C and i_C.
    d. After the capacitor has charged for two time constants, move the switch to position 2 and determine equations for v_C and i_C during discharge.
    e. Sketch v_C and i_C.

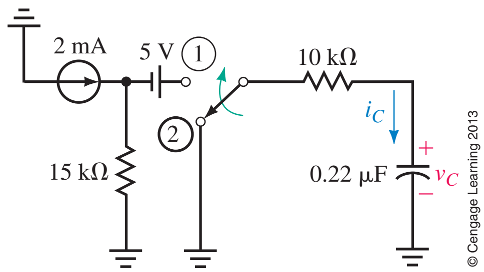


FIGURE 11–59

35. Determine the capacitor voltages and the source current for the circuit of Figure 11–60 after it has reached steady state.

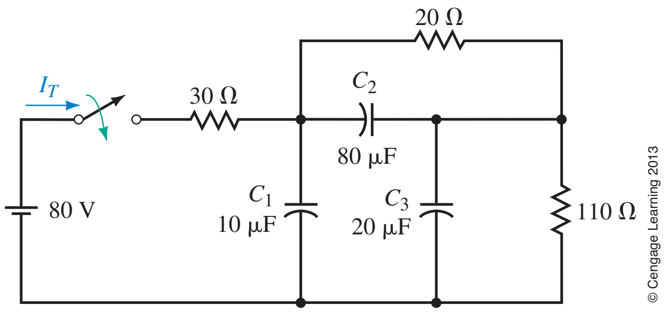


FIGURE 11–60

36. A black box containing dc sources and resistors has open-circuit voltage of 45 volts as in Figure 11–61(a). When the output is shorted as in (b), the short-circuit current is 1.5 mA. A switch and an uncharged 500-μF capacitor are connected as in (c). Determine the capacitor voltage and current 25 s after the switch is closed.

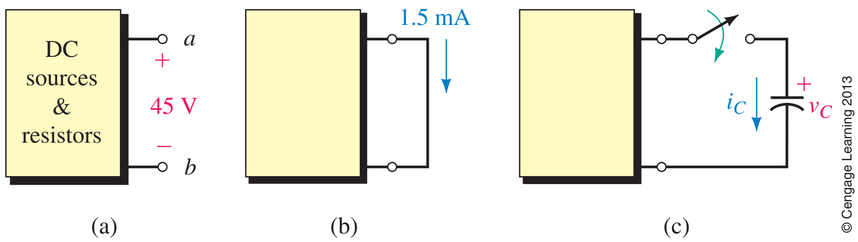


FIGURE 11–61

11.6 An *RC* Timing Application

37. For the alarm circuit of Figure 11–32, if the input from the sensor is 5 V, $R = 750$ kΩ, and the alarm is activated at 15 s when $v_C = 3.8$ V, what is *C?*
38. For the alarm circuit of Figure 11–32, the input from the sensor is 5 V, $C = 47$ μF, and the alarm is activated when $v_C = 4.2$ V. Choose the nearest standard resistor value to achieve a delay of at least 37 s.

11.7 Pulse Response of *RC* Circuits

39. Consider the waveform of Figure 11–62.
    a. What is the period?
    b. What is the duty cycle?
    c. What is the PRR?
40. Repeat Problem 39 for the waveform of Figure 11–63.

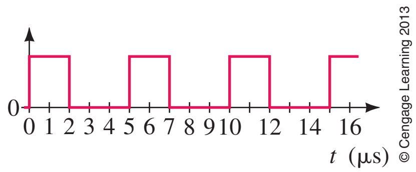


FIGURE 11–62

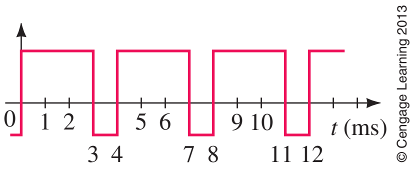


FIGURE 11–63

41. Determine the rise time, fall time, and pulse width for the pulse in Figure 11–64.
42. A single pulse is input to the circuit of Figure 11–65. Assuming that the capacitor is initially uncharged, sketch the output for each of the following sets of values:
    a. $R = 2\ \text{k}\Omega$, $C = 1\ \mu\text{F}$.
    b. $R = 2\ \text{k}\Omega$, $C = 0.1\ \mu\text{F}$.

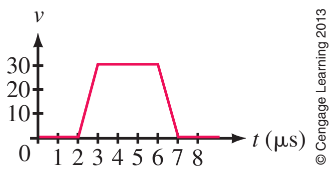


FIGURE 11–64

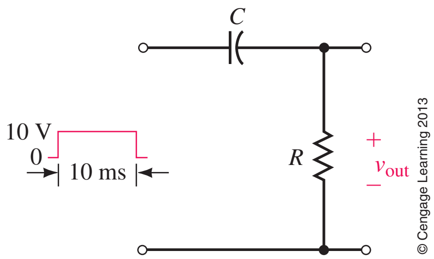


FIGURE 11–65

43. A switch (not shown) is closed to apply *E* volts to the circuit of Figure 11–66. If $R = 150\ \Omega$ and $C = 20\ \text{pF}$, estimate the rise time of the output voltage.
44. A pulse train is input to the circuit of Figure 11–66. Assuming that the capacitor is initially uncharged, sketch the output for each of the following sets of values after the circuit has reached steady state:
    a. $R = 2\ \text{k}\Omega$, $C = 0.1\ \mu\text{F}$.
    b. $R = 20\ \text{k}\Omega$, $C = 1.0\ \mu\text{F}$.

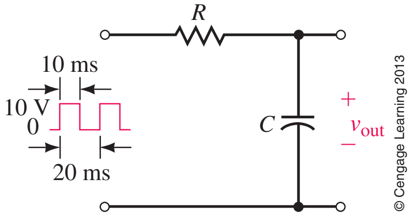


FIGURE 11–66

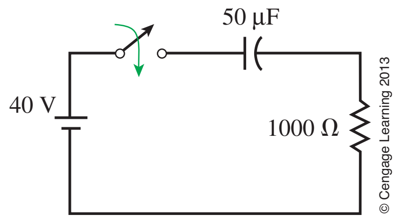


FIGURE 11–67

11.8 Transient Analysis Using Computers

Read the Notes, then use Multisim or PSpice to solve the following problems. (Note 4 applies only to Multisim users.) For Multisim current waveforms, you will need to add a new scale to expand the trace as you did in the examples of Section 11.8.

45. Graph capacitor voltage and current for the circuit of Figure 11–2 with $E = -25$ V, $R = 40\ \Omega$, $V_0 = 0$ V, and $C = 400\ \mu$F. Scale values from the plot at $t = 20$ ms intervals using the cursor. Confirm your results using Equations 11–3 and 11–5.
46. Obtain a plot of circuit current and resistor voltage for the circuit of Figure 11–67. Assume an initially uncharged capacitor. Use the cursor to read values at $t = 50$ ms and confirm analytically. Repeat if $V_0 = 100$ V.
47. For Figure 11–57, both capacitors are uncharged. Place your Multisim ground at the bottom end of the capacitors and:
    a. Plot capacitor voltage and find v_C at $t = 4$ ms.
    b. Determine the current in the 4-Ω resistor at $t = 3.5$ ms.
48. Using Multisim, construct Figure 11–68 on your screen. (The source is found in the Sources Group. Click _Place/Component_ to find it.) Repeat Example 11–17 and plot capacitor voltage and current. You should get the curves of Figure 11–49(b) as your solution.
49. Using PSpice, graph capacitor voltage and current for a charging circuit with $E = -25$ V, $R = 40\ \Omega$, $V_0 = 0$ V, and $C = 400\ \mu$F. Scale values from the plot using the cursor. Compare to the results you get using Equations 11–3 and 11–5 or the curves of Figure 11–15.
50. Repeat the problem of Question 46 using PSpice.
51. The switch of Figure 11–69 is closed at $t = 0$ s. Using PSpice, plot voltage and current waveforms. Use the cursor to determine v_C and i_C at $t = 10$ ms.

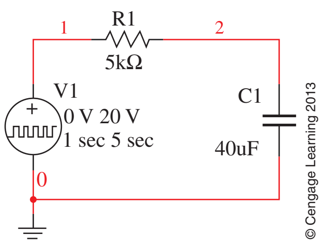


FIGURE 11–68

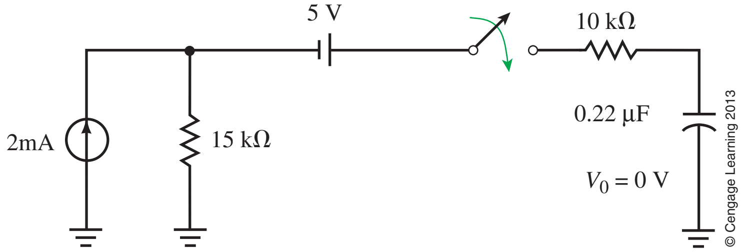


FIGURE 11–69

52. Using PSpice, redo Example 11–17 with the switch in the charge position for 0.5 s and everything else the same. With your calculator, compute v_C and i_C at 0.5 s and compare them to the PSpice plots. Repeat for i_C just after moving the switch to the discharge position.
53. For the circuit of Figure 11–60, use PSpice to determine and plot voltage and current waveforms for capacitors C_1 and C_3, then from the waveforms, determine their steady state voltages and currents. Compare to the answers of Problem 35.

ANSWERS TO IN-PROCESS LEARNING CHECKS

IN-PROCESS LEARNING CHECK 1

1. 0.2 A
2. a. 50 ms

   b.

t(ms)	i_c(mA)
0	50
25	30.3
50	18.4
75	11.2
100	6.8
250	0.337

3.

t(ms)	v_c(V)
0	0
25	71.3
50	91.8
75	97.7
100	99.3
250	100

4. $40(1 - e^{-5t})$ V, $200e^{-5t}$ mA, 38.0 V, 9.96 mA
5. a. $v_C(0^+) = v_C(0^-) = 0$; b. $i_C(0^-) = 0$; $i_C(0^+) = 10$ mA; c. 100 V, 0 A
6. 80 V, 40 kΩ, 0.2 μF

IN-PROCESS LEARNING CHECK 2

1. a. $40 + 40e^{-5t}$ V. v_C starts at 80 V and decays exponentially to 40 V.

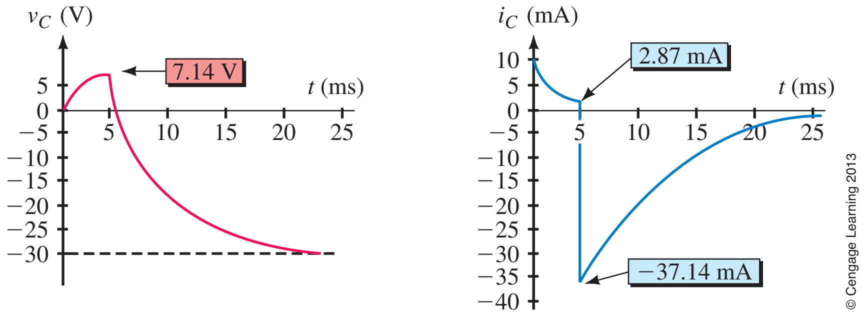
   b. There is no transient since initial value = final value.

   c. $40 - 100e^{-5t}$ V. v_C starts at −60 V and climbs exponentially to 40 V.
2. 0.1833 s
3. a. $100e^{-10t}$ V; $-10e^{-10t}$ mA; v_C starts at 100 V and decays to 0 in 0.5 s (i.e., 5 time constants); i_C starts at −10 mA and decays to 0 in 0.5 s.

   b. $-100e^{-10t}$ V; $10e^{-10t}$ mA; v_C starts at −100 V and decays to 0 in 0.5 s (i.e., 5 time constants); i_C starts at 10 mA and decays to 0 in 0.5 s.
4. a., b., and c. Same as Example 11–12.

   d. $-30 + 37.14e^{-250t}$ V e. $-37.14e^{-250t}$ mA

   f.



   g. −19.36 V; −10.64 mA
5. $6(1 - e^{-25t})$ V; $150e^{-25t}$ mA

NOTES...

1. Note the icon beside each problem, and use the corresponding simulation software, Multisim or PSpice for its solution.
2. Since current is discontinuous at the instant of switching, you get a meaningless value if you try to use a cursor to measure current at $t = 0$.
3. Switches are generally shown in the problem sets to denote transients, but since you use software (i.e., the Run button), to initiate Multisim and PSpice transients, switches are not usually needed.
4. Multisim examples in this book were created using Multisim 11 (the version current at the time of writing). If you are using legacy software, you may find it awkward to determine currents. In this case, simulate voltage waveforms only.

KEY TERMS

Ampere's Circuital Law
Ampere-Turns
B-H Curves
Domains
Ferromagnetic
Flux Density (B)
Hysteresis Loop
Magnetic Circuit
Magnetic Field
Magnetic Field Intensity (H)
Magnetic Flux
Magnetomotive Force (mmf)
Permeability
Reluctance ($\mathfrak{R}$)
Residual Magnetism
Saturation
Tesla (T)
Weber (Wb)

OUTLINE

The Nature of a Magnetic Field
Electromagnetism
Magnetic Flux and Flux Density
Magnetic Circuits and Their Applications
Air Gaps, Fringing, and Laminated Cores
Series Elements and Parallel Elements
Magnetic Circuits with dc Excitation
Magnetic Field Intensity and Magnetization Curves
Ampere's Circuital Law
Series Magnetic Circuits: Given Φ, Find NI
Series-Parallel Magnetic Circuits
Series Magnetic Circuits: Given NI, Find Φ
Force Due to an Electromagnet
Properties of Magnetic Materials
Sensing and Measuring Magnetic Fields

OBJECTIVES

After studying this chapter, you will be able to

- represent magnetic fields using Faraday's flux concept,
- describe magnetic fields quantitatively in terms of flux and flux density,
- explain what magnetic circuits are and why they are used,
- determine magnetic field intensity or magnetic flux density from a *B-H* curve,
- solve series magnetic circuits,
- solve series-parallel magnetic circuits,
- compute the attractive force of an electromagnet,
- explain the domain theory of magnetism,
- describe the demagnetization process.

12

MAGNETISM AND MAGNETIC CIRCUITS

CHAPTER PREVIEW

Many common devices rely on magnetism. Familiar examples include computer disk drives, audio system components such as headphones and speakers, transformers, generators, motors, and medical devices such as MRI machines. To understand their operation, you need a knowledge of magnetism and magnetic circuit principles. In this chapter, we look at fundamentals of magnetism, relationships between electrical and magnetic quantities, magnetic circuit concepts, and methods of analysis. In Chapter 13, we look at electromagnetic induction and inductance, and in Chapter 24, we apply magnetic principles to the study of transformers. ■

Putting It in Perspective

Magnetism and Electromagnetism

Photo Researchers/Photo Researchers/Getty Images

WHILE THE BASIC FACTS about magnetism have been known since ancient times, it was not until the early 1800s that the connection between electricity and magnetism was made and the foundations of modern electromagnetic theory laid down.

In 1819, Hans Christian Oersted, a Danish scientist, demonstrated that electricity and magnetism were related when he showed that a compass needle was deflected by a current-carrying conductor. The following year, Andre Ampere (1775–1836) showed that current-carrying conductors attract or repel each other just like magnets. However, it was Michael Faraday (recall Chapter 10) who developed our present concept of the magnetic field as a collection of flux lines in space that conceptually represent both the intensity and the direction of the field. It was this concept that led to an understanding of magnetism and the development of important practical devices such as the transformer and the electric generator.

In 1873, James Clerk Maxwell (see photo), a Scottish scientist, tied the then known theoretical and experimental concepts together and developed a unified theory of electromagnetism that predicted the existence of radio waves. Some 30 years later, Heinrich Hertz, a German physicist, showed experimentally that such waves existed, thus verifying Maxwell's theories and paving the way for modern radio and television. ■

12.1 The Nature of a Magnetic Field

Magnetism refers to the force that acts between magnets and magnetic materials. We know, for example, that magnets attract pieces of iron, deflect compass needles, attract or repel other magnets, and so on. This force acts at a distance and without the need for direct physical contact. The region where the force is felt is called the "field of the magnet" or simply, its **magnetic field.** Thus, *a magnetic field is a force field.*

NOTES...

Flux is perhaps an unfortunate name to apply to a magnetic field. Flux suggests a flow, but in a magnetic field, nothing actually flows; a magnetic field is simply a condition of space, that is, a region in which magnetic force exists. Nonetheless, the concept of flux is enormously helpful as an aid to visualizing magnetic phenomena, and we will continue to use it for that purpose.

Magnetic Flux

Faraday's flux concept (recall Putting It in Perspective, Chapter 10) helps us visualize this field. Using Faraday's representation, magnetic fields are drawn as lines in space. These lines, called **magnetic flux** lines or lines of force, show the direction and intensity of the field at all points. This is illustrated in Figure 12–1 for the field of a bar magnet. As indicated, the field is strongest at the poles of the magnet (where flux lines are most dense), its direction is from north (N) to south (S) external to the magnet, and flux lines never cross. The symbol for magnetic flux (Figure 12–1) is the Greek letter Φ (phi).

Figure 12–2 shows what happens when two magnets are brought close together. In (a), unlike poles attract, and flux lines pass from one magnet to the other. In (b), like poles repel, and the flux lines are pushed back as indicated by the flattening of the field between the two magnets.

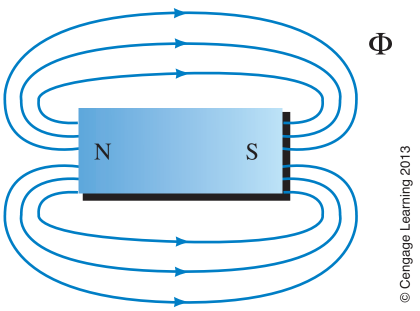


FIGURE 12–1 Field of a bar magnet. Magnetic flux is denoted by the symbol Φ.

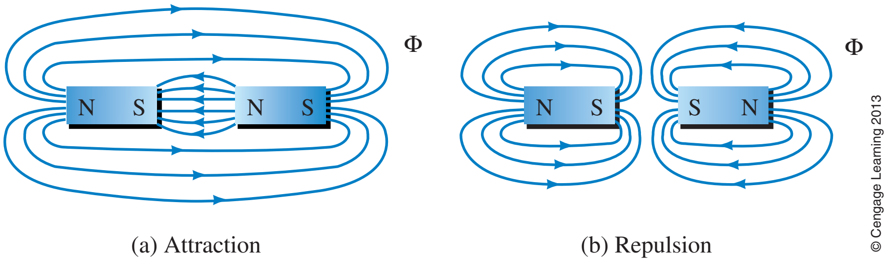


FIGURE 12–2 Field patterns due to attraction and repulsion.

Ferromagnetic Materials

Magnetic materials (materials that are attracted by magnets such as iron, nickel, cobalt, and their alloys) are called **ferromagnetic** materials. Ferromagnetic materials provide an easy path for magnetic flux. This is illustrated in Figure 12–3 where the flux lines take the longer (but easier) path through the soft iron, rather than the shorter path (of Figure 12–1) that they would normally take. Note, however, that nonmagnetic materials (plastic, wood, glass, and so on) have no effect on the field.

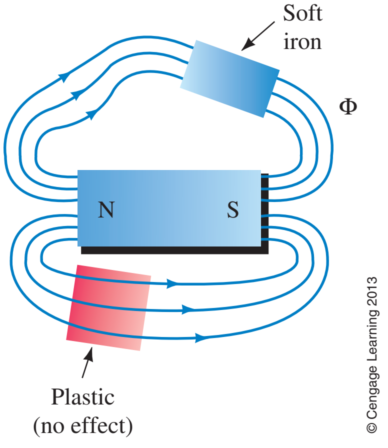
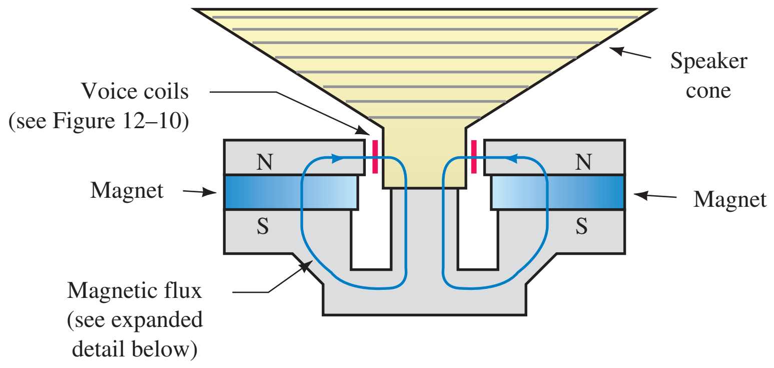
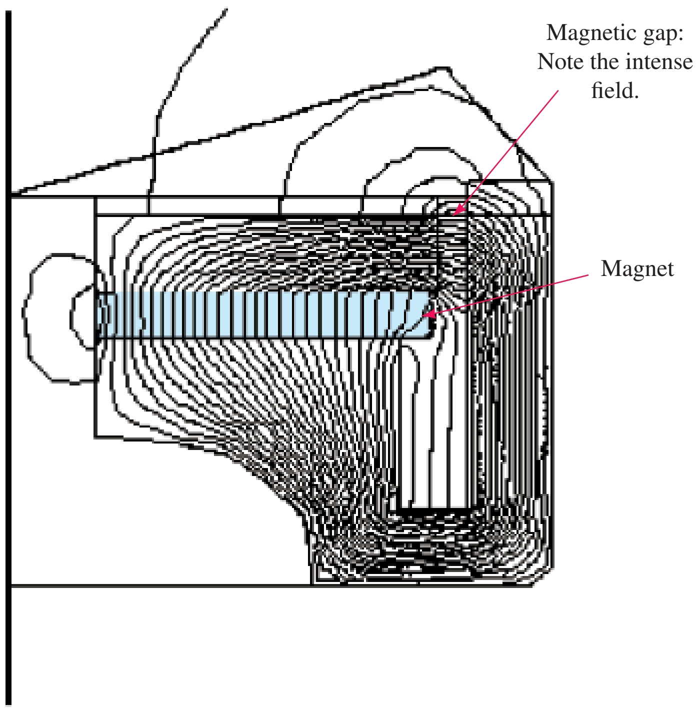


FIGURE 12–3 Magnetic field follows the longer (but easier) path through the iron. The plastic has no effect on the field.

Figure 12–4 shows an application of these principles. Part (a) shows a simplified representation of a loudspeaker, and part (b) shows expanded details of its magnetic field. The field is created by the permanent magnet, and the iron pole pieces guide the field and concentrate it in the gap where the speaker coil is placed. (For a description of how the speaker works, see Section 12.4.)



(a) Simplified representation of the magnetic field. Here, the complex field of (b) is represented symbolically by a single line.



(b) Magnetic field pattern for the loud speaker. The field is symmetrical so only half the structure is shown.

FIGURE 12–4 Magnetic circuit of a loudspeaker. The magnetic structure and voice coil are called a "speaker motor." The field is created by the permanent magnet.

Within the iron structure, the flux crowds together at sharp interior corners, spreads apart at exterior corners, and is essentially uniform elsewhere, as indicated in (b). This is characteristic of magnetic fields in iron.

12.2 Electromagnetism

Most applications of magnetism involve magnetic effects due to electric currents. We look first at some basic principles. Consider Figure 12–5. The current, I, creates a magnetic field that is concentric about the conductor, uniform

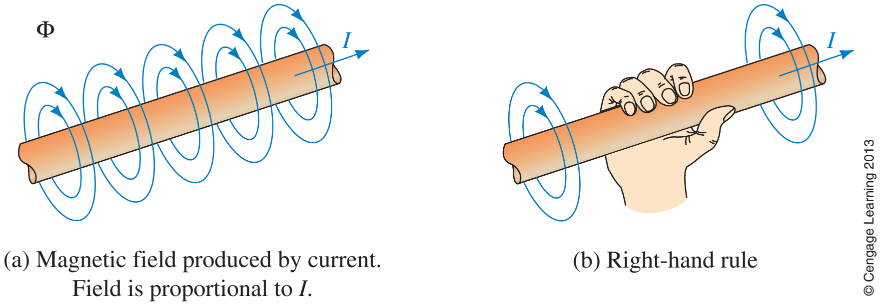


FIGURE 12–5 Field about a current-carrying conductor. If current is reversed, the field remains concentric but the direction of the flux lines reverses.

along its length, and whose strength is directly proportional to I. Note the direction of the field. It may be remembered with the aid of the right-hand rule. As indicated in (b), imagine placing your right hand around the conductor with your thumb pointing in the direction of current. Your fingers then point in the direction of the field. If you reverse the direction of the current, the direction of the field reverses. If the conductor is wound into a coil, the fields of its individual turns combine, producing a resultant field as in Figure 12–6. The direction of the coil flux can also be remembered by means of a simple rule: curl the fingers of your right hand around the coil in the direction of the current and your thumb will point in the direction of the field. If the direction of the current is reversed, the field also reverses. Provided no ferromagnetic material is present, the strength of the coil's field is directly proportional to its current.

If the coil is wound on a ferromagnetic core as in Figure 12–7 (transformers are built this way), almost all flux is confined to the core, although a small amount (called stray or leakage flux) passes through the surrounding air. However, now that ferromagnetic material is present, the core flux is no longer proportional to current. The reason for this is discussed in Section 12.14.

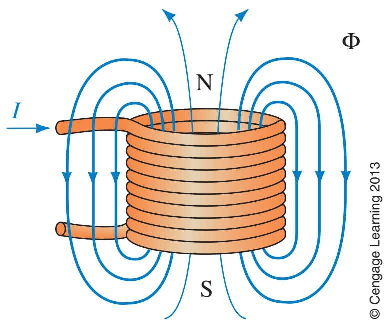


FIGURE 12–6 Field produced by a coil.

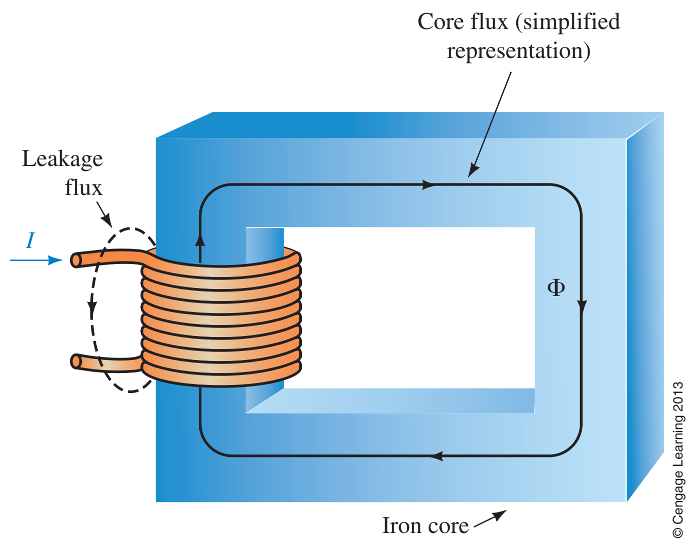


FIGURE 12–7 For ferromagnetic materials, most flux is confined to the core.

12.3 Magnetic Flux and Flux Density

As noted in Figure 12–1, magnetic flux is represented by the symbol Φ. In the SI system, the unit of flux is the **weber (Wb),** in honor of pioneer researcher Wilhelm Eduard Weber, 1804–1891. However, we are often more interested in **flux density** (***B***) (i.e., flux per unit area) than in total flux Φ. Since flux Φ is measured in Wb and area A in m^2, flux density is measured as Wb/m^2. However, to honor Nikola Tesla (another early researcher, 1856–1943) the unit of flux density is called the **tesla (T)** where $1\ T = 1\ Wb/m^2$. Flux density is found by dividing the total flux passing perpendicularly through an area by the size of the area, Figure 12–8. That is,

$$B = \frac{\Phi}{A} \quad \text{(tesla, T)} \tag{12–1}$$

NOTES...

Although the weber appears at this point to be just an abstract quantity, it can in fact be linked to the familiar electrical system of units. For example, if you pass a conductor through a magnetic field such that the conductor cuts the flux lines at the rate of 1 Wb per second, the voltage induced is 1 V.

Thus, if $\Phi = 600\ \mu Wb$ of flux pass perpendicularly through an area $A = 20 \times 10^{-4}\ m^2$, the flux density is $B = (600 \times 10^{-6}\ Wb)/(20 \times 10^{-4}\ m^2) = 0.3\ T$. The greater the flux density, the stronger the field.

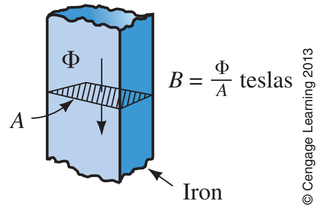


FIGURE 12–8 Concept of flux density. $1\ T = 1\ Wb/m^2$.

EXAMPLE 12–1

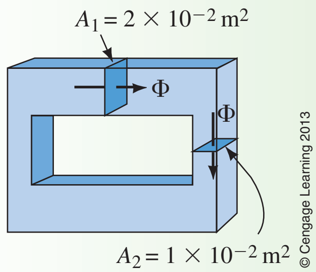


FIGURE 12–9

For the magnetic core of Figure 12–9, the flux density at cross section 1 is $B_1 = 0.4\ T$. Determine B_2.

Solution $\Phi = B_1 \times A_1 = (0.4\ T)(2 \times 10^{-2}\ m^2) = 0.8 \times 10^{-2}\ Wb$. Since all flux is confined to the core, the flux at cross section 2 is the same as at cross section 1. Therefore,

$$B_2 = \Phi/A_2 = (0.8 \times 10^{-2}\ Wb)/(1 \times 10^{-2}\ m^2) = 0.8\ T$$

PRACTICE PROBLEMS 1

1. Refer to the core of Figure 12–8:
   a. If A is 2 cm × 2.5 cm and $B = 0.4$ T, compute Φ in webers.
   b. If A is 0.5 inch by 0.8 inch and $B = 0.35$ T, compute Φ in webers.
2. In Figure 12–9, if $\Phi = 100 \times 10^{-4}$ Wb, compute B_1 and B_2.

Answers
1. a. 2×10^{-4} Wb; b. 90.3 μWb; 2. 0.5 T; 1.0 T

To gain a feeling for the size of magnetic units, note that the strength of the earth's field is approximately 50 μT near the earth's surface, the field of a large generator or motor is on the order of 1 T or 2 T, and the largest fields yet produced (using superconducting magnets) are on the order of 45 T.

Other systems of units (now largely outdated) are the CGS system and the English systems. In the CGS system, flux is measured in maxwells and flux density in gauss. In the English system, flux is measured in lines and flux density in lines per square inch. Conversion factors are given in Table 12–1. We use only the SI system in this book.

TABLE 12–1 Magnetic Units Conversion Table

System	Flux (Φ)	Flux Density (B)
SI	webers (Wb)	teslas (T)
		1 T = 1 Wb/m^2
English	lines	lines/in^2
	1 Wb = 10^8 lines	1 T = 6.452×10^4 lines/in^2
CGS	maxwells	gauss
	1 Wb = 10^8 maxwells	1 gauss = 1 maxwell/cm^2
		1 T = 10^4 gauss

IN-PROCESS LEARNING CHECK 1

(Answers are at the end of the chapter.)

1. A magnetic field is a _____ field.
2. With Faraday's flux concept, the density of lines represents the _____ of the field and their direction represents the _____ of the field.
3. Three ferromagnetic materials are _____, _____, and _____.
4. The direction of a magnetic field is from _____ to _____ outside a magnet.
5. For Figures 12–5 and 12–6, if the direction of current is reversed, sketch what the fields look like.
6. If the core shown in Figure 12–7 is plastic, sketch what the field will look like.
7. Flux density B is defined as the ratio Φ/A, where A is the area _____ [parallel, perpendicular] to Φ.
8. For Figure 12–9, if A_1 is 2 cm $\times$ 2.5 cm, B_1 is 0.5 T, and $B_2 = 0.25$ T, what is A_2?

12.4 Magnetic Circuits and Their Applications

Many practical applications of magnetism use magnetic structures to guide and shape magnetic fields by providing well-defined paths for flux. Such structures are called **magnetic circuits.** Magnetic circuits are found in motors, generators, audio speakers, transformers, and so on. While most magnetic circuit applications involve magnetic fields created by electric currents, some involve a combination of fields created by electric currents and fixed (permanent) magnets.

The speaker structure of Figure 12–4 illustrates the latter. It uses a powerful magnet to create flux and an iron structure to guide this flux to the air gap to provide the intense field in which the voice coil is suspended. Figure 12–10 illustrates how the speaker actually works. First note that a source of audio

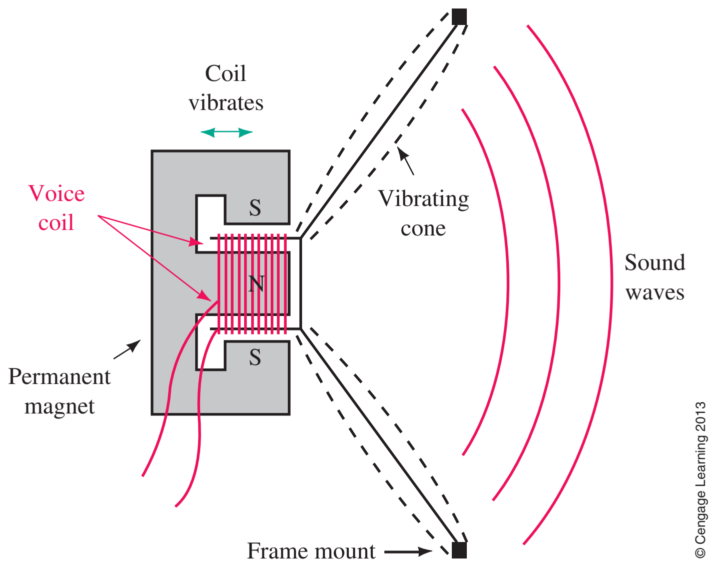


FIGURE 12–10 Audio speaker, conceptual representation.

(radio, sound reinforcement systems, etc.—recall Figure 1–5 of Chapter 1) outputs a current (proportional to the audio sound) to the speaker. The speaker utilizes a flexible cone to reproduce this sound. A coil of fine wire, wound around the cylindrical apex of this cone, is suspended in the fixed field of the air gap. Current from the amplifier passes through this coil, creating a varying magnetic field that interacts with the fixed field, creating varying forces that vibrate the cone. Since the current output by the amplifier is an amplified audio signal, the cone's vibrations correspond to this signal and thus, reproduce the original sound (in amplified form).

Figure 12–11 shows another example. Bank ATM cards, credit cards, library cards, travel reward cards, and a host of others feature a magnetic stripe that contains information encoded in the form of magnetic patterns. As the user swipes the card through the reader (past the read head), the varying magnetic field induces a voltage in the pickup winding. This tiny voltage is amplified, then sent to decoding circuitry where the data is recovered. As indicated, the pickup part of this system is a magnetic circuit.

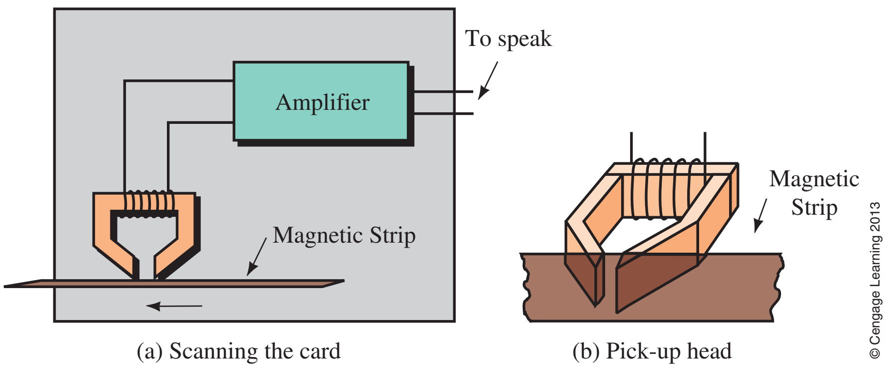
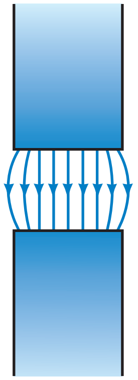


FIGURE 12–11 The read head of a magnetic stripe reader is a magnetic circuit.

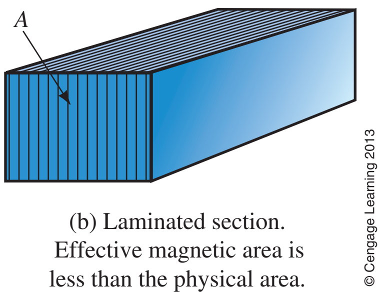
Another example of the use of magnetism is depicted by the magnetic resonance imaging (MRI) machine of Figure 1–2, Chapter 1. Utilizing superconductor coils to create an intense magnetic field into which the patient is placed, it uses radio waves that interact with this field to produce detailed images of internal organs of the body. The result is noninvasive, radiation-free body scanning. The particular MRI suite illustrated in Figure 1–2 utilizes a 3 T strength magnet as the heart of the system.

12.5 Air Gaps, Fringing, and Laminated Cores

Consider again Figures 12–10 and 12–11. Note that both structures have gaps in their magnetic flux paths. This is typical—most practical magnetic circuits have air gaps that are essential to their operation. However, at such gaps, fringing occurs. This fringing results in a slight weakening of the field in the gap as depicted in Figure 12–12(a). For short gaps, this weakening can often be neglected. Alternatively, you can estimate the effect of fringing in your calculations by increasing each cross-sectional dimension of the core by the length of the gap as illustrated in Example 12–2.



(a) Fringing at gap



(b) Laminated section. Effective magnetic area is less than the physical area.

FIGURE 12–12 Fringing and laminations.

EXAMPLE 12–2

A core with cross-sectional dimensions of 2.5 cm by 3 cm has a 0.1-mm gap. If flux density $B = 0.86$ T in the iron, what is the approximate uncorrected and corrected flux density in the gap?

Solution Neglecting fringing, gap area is the same as the core area. Thus, $B_g \approx 0.86$ T. Correcting for fringing yields

$$\Phi = BA = (0.86\text{ T})(2.5 \times 10^{-2}\text{ m})(3 \times 10^{-2}\text{ m}) = 0.645\text{ mWb}$$

$$A_g \simeq (2.51 \times 10^{-2}\text{ m})(3.01 \times 10^{-2}\text{ m}) = 7.555 \times 10^{-4}\text{ m}^2$$

Thus,

$$B_g \simeq 0.645\text{ mWb}/7.555 \times 10^{-4}\text{ m}^2 = 0.854\text{ T}$$

Now consider laminations. Many practical magnetic circuits (such as transformers) use thin sheets of stacked iron or steel as in Figure 12–12(b). Since the core is not a solid block, its effective cross-sectional area (i.e., the actual area of iron) is less than the core's physical cross-sectional area. A stacking factor, defined as the ratio of the actual area of ferrous material to the physical area of the core cross section, permits you to determine the core's effective area.

PRACTICE PROBLEMS 2

A product data sheet states that a laminated section of core has cross-sectional dimensions of 0.03 m by 0.05 m and a stacking factor of 0.9.

a. What is the effective area of the core?
b. Given $\Phi = 1.4 \times 10^{-3}$ Wb, what is the flux density, B?

Answers
a. 1.35×10^{-3} m^2; b. 1.04 T

12.6 Series Elements and Parallel Elements

Magnetic circuits may have sections of different materials. For example, the circuit of Figure 12–13 has sections of cast iron, sheet steel, and an air gap. For this circuit, flux Φ is the same in all sections. Such a circuit is called a series magnetic circuit. Although the flux is the same in all sections, the flux density in each section may vary, depending on its effective cross-sectional area as you saw earlier.

A circuit may also have elements in parallel (Figure 12–14). At each junction, the sum of fluxes entering is equal to the sum leaving. This is the counterpart of Kirchhoff's current law (KCL). Thus, for Figure 12–14, if $\Phi_1 = 25\ \mu$Wb and $\Phi_2 = 15\ \mu$Wb, then $\Phi_3 = 10\ \mu$Wb. For cores that are symmetrical about the center leg, $\Phi_2 = \Phi_3$.

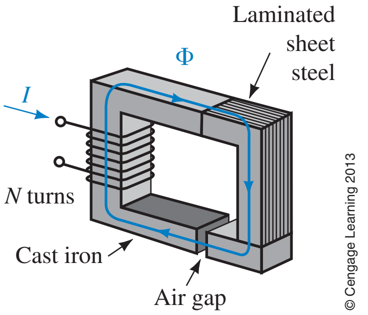


FIGURE 12–13 Series magnetic circuit. Flux Φ is the same throughout.

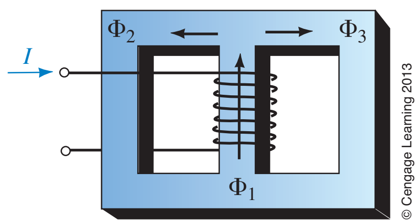


FIGURE 12–14 The sum of the flux entering a junction equals the sum leaving. Here, $\Phi_1 = \Phi_2 + \Phi_3$.

IN-PROCESS LEARNING CHECK 2

(Answers are at the end of the chapter.)

1. Why is the flux density in each section of Figure 12–13 different?
2. For Figure 12–13, $\Phi = 1.32$ mWb, the cross section of the core is 3 cm by 4 cm, the laminated section has a stacking factor of 0.8, and the gap is 1 mm. Determine the flux density in each section, taking fringing into account.
3. If the core of Figure 12–14 is symmetrical about its center leg, $B_1 = 0.4$ T, and the cross-sectional area of the center leg is 25 cm^2, what are Φ_2 and Φ_3?

12.7 Magnetic Circuits with dc Excitation

We now look at the analysis of magnetic circuits with dc excitation. There are two basic problems to consider: (1) given the flux, to determine the current required to produce it and (2) given the current, to compute the flux produced. To help visualize how to solve such problems, we first establish an analogy between magnetic circuits and electric circuits.

MMF: The Source of Magnetic Flux

Current through a coil creates magnetic flux. The greater the current or the greater the number of turns, the greater will be the flux. This flux-producing ability of a coil is called its **magnetomotive force (mmf)** and is measured in **ampere-turns.** Given the symbol $\mathscr{F}$, it is defined as

$$\mathscr{F} = NI \quad \text{(ampere-turns, At)} \tag{12–2}$$

Thus, a coil with 100 turns and 2.5 amps will have an mmf of 250 ampere-turns, while a coil with 500 turns and 4 amps will have an mmf of 2000 ampere-turns.

Reluctance, $\Re$: Opposition to Magnetic Flux

Flux in a magnetic circuit also depends on the opposition that the circuit presents to it. Termed **reluctance ($\Re$),** this opposition depends on the dimensions of the core and the material of which it is made. Like the resistance of a wire, reluctance is directly proportional to length and inversely proportional to cross-sectional area. In equation form,

$$\Re = \frac{\ell}{\mu A} \quad \text{(At/Wb)} \tag{12–3}$$

where μ is a property of the core material called its **permeability** (discussed in Section 12.8). Permeability is a measure of how easy it is to establish flux in a material. Ferromagnetic materials have high permeability and hence low $\Re$, while nonmagnetic materials have low permeability and high $\Re$.

Ohm's Law for Magnetic Circuits

The relationship between flux, mmf, and reluctance is

$$\Phi = \mathscr{F}/\Re \quad \text{(Wb)} \tag{12–4}$$

This relationship is similar to Ohm's law and is depicted symbolically in Figure 12–15. (Remember however that flux, unlike electric current, does not flow—see note in Section 12.1.)

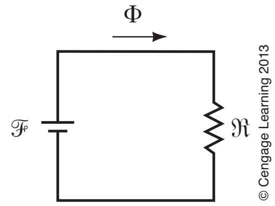


FIGURE 12–15 Electric circuit analogy of a magnetic circuit. $\Phi = \mathscr{F}/\Re$.

EXAMPLE 12–3

For Figure 12–16, if the reluctance of the magnetic circuit is $\Re = 12 \times 10^4$ At/Wb, what is the flux in the circuit?

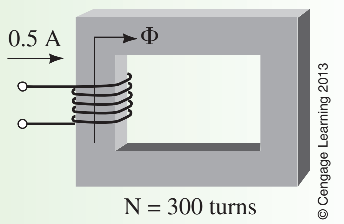


FIGURE 12–16

Solution

$$\mathscr{F} = NI = (300)(0.5\text{ A}) = 150\text{ At}$$
$$\Phi = \mathscr{F}/\Re = (150\text{ At})/(12 \times 10^4\text{ At/Wb}) = 12.5 \times 10^{-4}\text{ Wb}$$

In Example 12–3, we assumed that the reluctance of the core was constant. This is only approximately true under certain conditions. In general, it is not true, since $\Re$ is a function of flux density. Thus Equation 12–4 is not really very useful, since for ferromagnetic material, $\Re$ depends on flux, the very quantity that you are trying to find. The main use of Equations 12–3 and 12–4 is to provide an analogy between electric and magnetic circuit analysis.

12.8 Magnetic Field Intensity and Magnetization Curves

We now look at a more practical approach to analyzing magnetic circuits. First, we require a quantity called **magnetic field intensity, *H*** (also known as magnetizing force). It is a measure of the mmf per unit length of a circuit.

To get at the idea, suppose you apply the same mmf (say 600 At) to two circuits with different path lengths (Figure 12–17). In (a), you have 600 ampere-turns of mmf to "drive" flux through 0.6 m of core; in (b), you have the same mmf but it is spread across only 0.15 m of path length. Thus, the mmf per unit length in the second case is more intense. Based on this idea, one can define magnetic field intensity as the ratio of applied mmf to the length of path that it acts over. Thus,

$$H = \mathcal{F}/\ell = NI/\ell \quad \text{(At/m)} \tag{12–5}$$

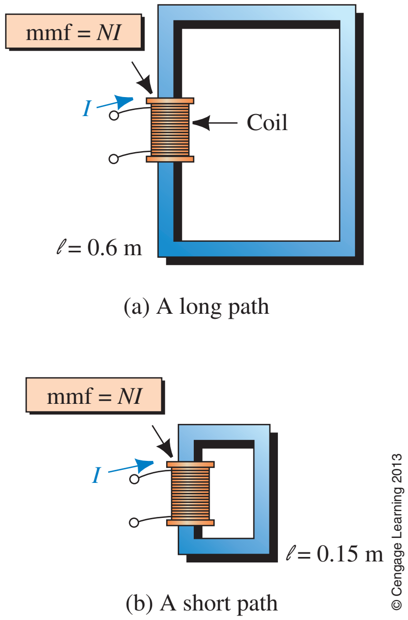


FIGURE 12–17 By definition, H = mmf/length = NI/ℓ.

For the circuit of Figure 12–17(a), H = 600 At/0.6 m = 1000 At/m, while for the circuit of (b), H = 600 At/0.15 m = 4000 At/m. Thus, in (a) you have 1000 ampere-turns of "driving force" per meter of length to establish flux in the core, whereas in (b) you have four times as much. (However, you won't get four times as much flux, since the opposition to flux varies with the density of the flux.)

Rearranging Equation 12–5 yields an important result:

$$NI = H\ell \quad \text{(At)} \tag{12–6}$$

In an analogy with electric circuits (Figure 12–18), the NI product is an mmf source, while the $H\ell$ product is an mmf drop.

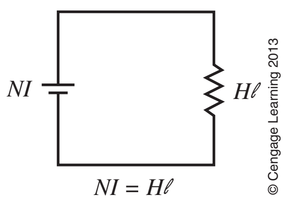


FIGURE 12–18 Circuit analogy, $H\ell$ model.

The Relationship between *B* and *H*

From Equation 12–5, you can see that magnetizing force, H, is a measure of the flux-producing ability of the coil (since it depends on NI). You also know that B is a measure of the resulting flux (since $B = \Phi/A$). Thus, B and H are related. The relationship is

$$B = \mu H \tag{12–7}$$

where μ is the permeability of the core (recall Equation 12–3).

It was stated earlier that permeability is a measure of how easy it is to establish flux in a material. To see why, note from Equation 12–7 that the larger the value of μ, the larger the flux density for a given H. However, H is proportional to current; therefore, the larger the value of μ, the larger the flux density for a given magnetizing current. From this, it follows that the larger the permeability, the more flux you get for a given magnetizing current.

In the SI system, μ has units of webers per ampere-turn-meter. The permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$. For all practical purposes, the permeability of air and other nonmagnetic materials (e.g., plastic) is the same as for a vacuum. Thus, in air gaps,

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times H_g \tag{12–8}$$

Rearranging Equation 12–8 yields

$$H_g = \frac{B_g}{4\pi \times 10^{-7}} = 7.96 \times 10^5 B_g \quad \text{(At/m)} \tag{12–9}$$

PRACTICE PROBLEMS 3

For Figure 12–16, the core cross section is 0.05 m × 0.08 m. If a gap is cut in the core and H in the gap is 3.6×10^5 At/m, what is the flux Φ in the core? Neglect fringing.

Answer
1.81 mWb

B-H Curves

For ferromagnetic materials, μ is not constant but varies with flux density, and there is no easy way to compute it. In reality, however, it isn't μ that you are interested in: What you really want to know is, given B, what is H, and vice versa. A set of curves, called ***B-H* curves,** provides this information. (These curves, also called magnetization curves, are obtained experimentally and are available in handbooks. A separate curve is required for each material.) Figure 12–19 shows typical curves for cast iron, cast steel, and sheet steel.

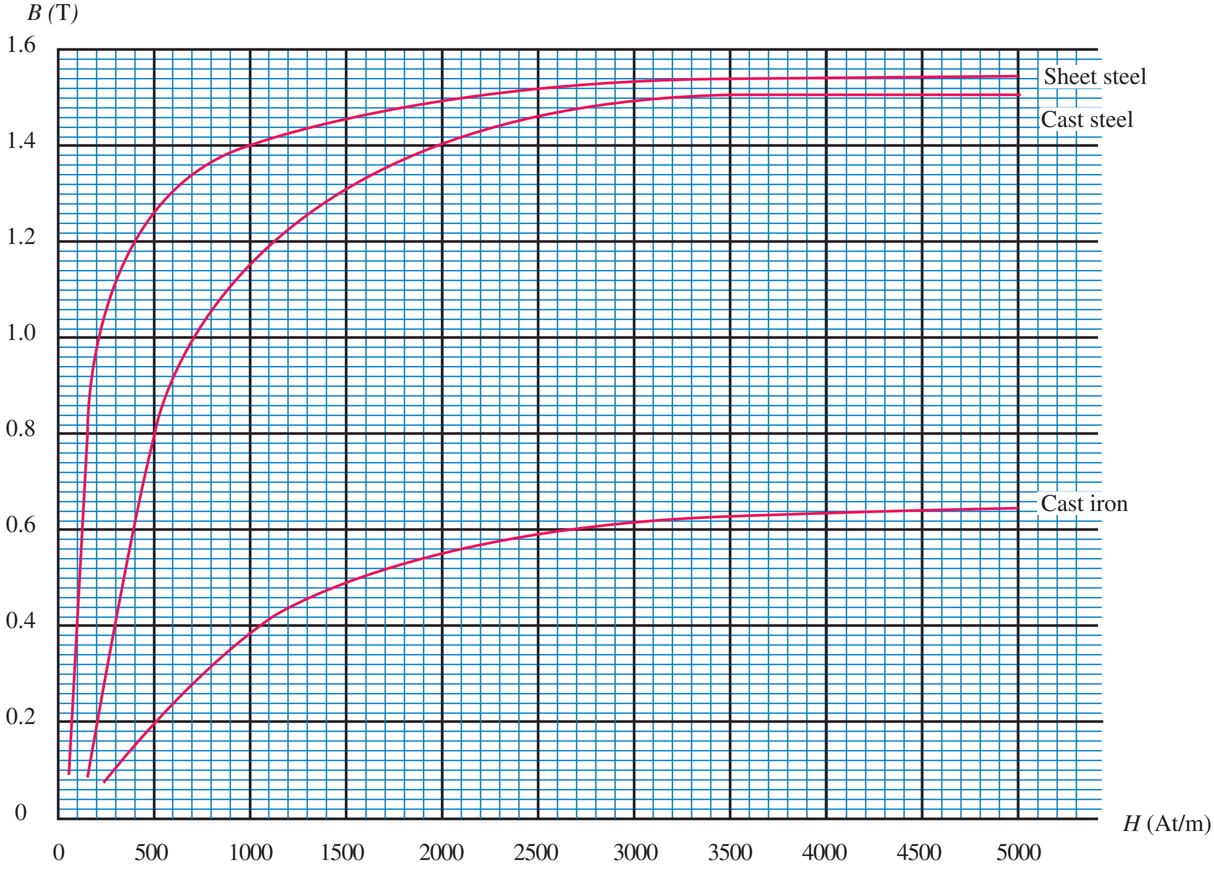


FIGURE 12–19 *B-H* curves for selected materials.

EXAMPLE 12–4

If $B = 1.4$ T for sheet steel, what is H?

Solution Enter Figure 12–19 on the axis at $B = 1.4$ T, continue across until you encounter the curve for sheet steel, then read the corresponding value for H as indicated in Figure 12–20: $H = 1000$ At/m.

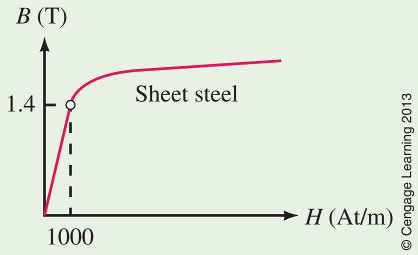


FIGURE 12–20 For sheet steel, $H = 1000$ At/m when $B = 1.4$ T.

PRACTICE PROBLEMS 4

1. The cross section of a sheet steel core is 0.1 m × 0.1 m and its stacking factor is 0.93. If $\Phi = 13.5$ mWb, what is H?
2. Sketch the B-H curves for both air and plastic.

Answers

1. 1500 At/m
2. $B = \mu H$. For air, μ is constant (recall $\mu_o = 4\pi \times 10^{-7}$). Thus, B is proportional to H, and the curve is a straight line. Choose two arbitrary points to establish it. When $H = 0$, $B = 0$; thus, it passes through the origin. When $H = 5000$, $B = (4\pi \times 10^{-7})(5000) = 6.28 \times 10^{-3}$ T. The curve for plastic is the same.

12.9 Ampere's Circuital Law

One of the key relationships in magnetic circuit theory is **Ampere's circuital law,** Equation 12–10. This law was determined experimentally and is a generalization of the relationship $\mathcal{F} = NI = H\ell$ that we developed earlier. Ampere showed that the algebraic sum of mmfs around a closed loop in a magnetic circuit is zero, regardless of the number of sections or coils. That is,

$$\sum_{\bigcirc} \mathcal{F} = 0 \qquad \textbf{(12–10)}$$

This can be rewritten as

$$\sum_{\bigcirc} NI = \sum_{\bigcirc} H\ell \quad \text{At} \qquad \textbf{(12–11)}$$

which states that the sum of applied mmfs around a closed loop equals the sum of the mmf drops. The summation is algebraic and terms are additive or subtractive, depending on the direction of flux and how the coils are wound. To illustrate, consider again Figure 12–13. Here,

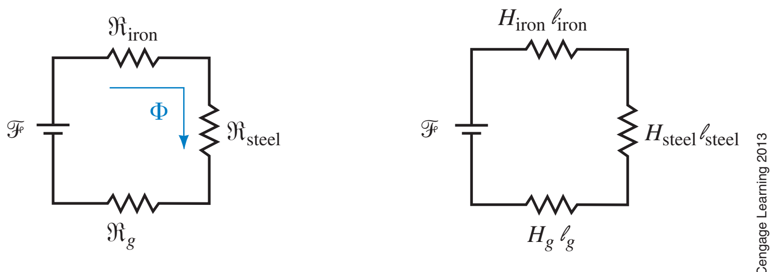
$$NI - H_{\text{iron}}\ell_{\text{iron}} - H_{\text{steel}}\ell_{\text{steel}} - H_g\ell_g = 0$$

Thus,

$$\underbrace{NI}_{\substack{\text{Applied}\\ \text{mmf}}} = \underbrace{H_{\text{iron}}\ell_{\text{iron}} + H_{\text{steel}}\ell_{\text{steel}} + H_g\ell_g}_{\text{sum of mmf drops}}$$

The path to use for the $H\ell$ terms is the mean (average) path.

You now have two magnetic circuit models (Figure 12–21). While the reluctance model (a) is not very useful for solving problems, it helps relate magnetic circuit problems to familiar electrical circuit concepts. The Ampere's law model, on the other hand, permits us to solve practical problems. We look at how to do this in the next section.



(a) Reluctance model (b) Ampere's circuital law model

FIGURE 12–21 Two models for the magnetic circuit of Figure 12–13.

IN-PROCESS LEARNING CHECK 3

(Answers are at the end of the chapter.)

1. If the mmf of a 200-turn coil is 700 At, the current in the coil is _____ amps.
2. For Figure 12–17, if $H = 3500$ At/m and $N = 1000$ turns, then for (a), I is _____ A, while for (b), I is _____ A.
3. For cast iron, if $B = 0.5$ T, then $H =$ _____ At/m.
4. A series circuit consists of one coil, a section of iron, a section of steel, and two air gaps (of different sizes). Draw the Ampere's law model.
5. Which is the correct answer for the circuit of Figure 12–22?
   a. Ampere's law around loop 1 yields ($NI = H_1\ell_1 + H_2\ell_2$, or $NI = H_1\ell_1 - H_2\ell_2$).
   b. Ampere's law around loop 2 yields ($0 = H_2\ell_2 + H_3\ell_3$, or $0 = H_2\ell_2 - H_3\ell_3$).
6. For the circuit of Figure 12–23, the length ℓ to use in Ampere's law is ____ [0.36 m, 0.32 m, 0.28 m]. Why?

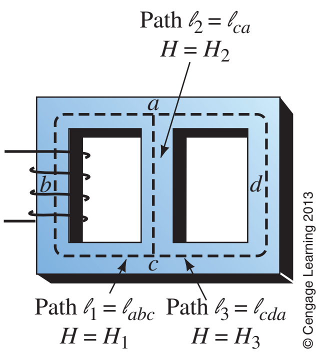


FIGURE 12–22

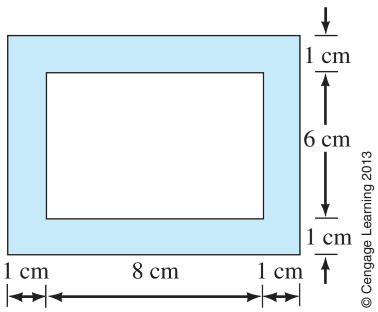


FIGURE 12–23

12.10 Series Magnetic Circuits: Given Φ, Find *NI*

You now have the tools needed to solve basic magnetic circuit problems. We will begin with series circuits where Φ is known and we want to find the excitation to produce it. Problems of this type can be solved using four basic steps:

1. Compute B for each section using $B = \Phi/A$.
2. Determine H for each magnetic section from the *B-H* curves. Use $H_g = 7.96 \times 10^5 B_g$ for air gaps.
3. Compute NI using Ampere's circuital law.
4. Use the computed NI to determine coil current or turns as required. (Circuits with more than one coil are handled as in Example 12–6.)

Be sure to use the mean path through the circuit when applying Ampere's law. Unless directed otherwise, neglect fringing.

PRACTICAL NOTE...

Magnetic circuit analysis is not as precise as electric circuit analysis because (1) the assumption of uniform flux density breaks down at sharp corners as you saw in Figure 12–4, and (2) the *B-H* curve is a mean curve and has considerable uncertainty as discussed later (Section 12–14).

Although the answers are approximate, they are adequate for many purposes.

EXAMPLE 12–5

If the core of Figure 12–24 is cast iron and $\Phi = 0.1 \times 10^{-3}$ Wb, what is the coil current?

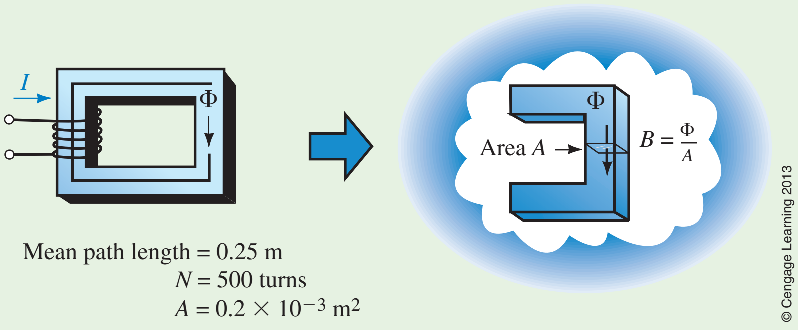


FIGURE 12–24

Solution Following the four basic steps:

1. The flux density is
$$B = \frac{\Phi}{A} = \frac{0.1 \times 10^{-3}}{0.2 \times 10^{-3}} = 0.5\text{ T}$$
2. From the *B-H* curve (cast iron), Figure 12–19, H = 1550 At/m.
3. Apply Ampere's law. There is only one coil and one core section. Length = 0.25 m. Thus,
$$H\ell = 1550 \times 0.25 = 388\text{ At} = NI$$
4. Solve for I:
$$I = H\ell/N = 388/500 = 0.78\text{ amps}$$

EXAMPLE 12–6

A second coil is added as shown in Figure 12–25. If $\Phi = 0.1 \times 10^{-3}$ Wb as before, but $I_1 = 1.5$ amps, what is I_2?

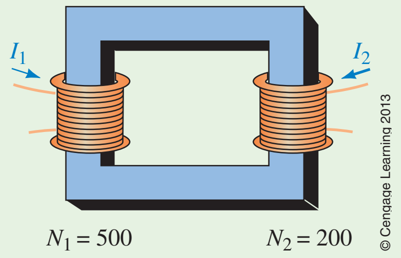


FIGURE 12–25

Solution From the previous example, you know that a current of 0.78 amps in coil 1 produces $\Phi = 0.1 \times 10^{-3}$ Wb. But you already have 1.5 amps in coil 1. Thus, coil 2 must be wound in opposition so that its mmf is subtractive. Applying Ampere's law yields $N_1I_1 - N_2I_2 = H\ell$. Hence,

$$(500)(1.5\ A) - 200I_2 = 388\ \text{At}$$

and so $I_2 = 1.8$ amps.

NOTES...

Since magnetic circuits are nonlinear, you cannot use superposition; that is, you cannot consider each coil of Figure 12–25 by itself, then sum the results. You must consider them simultaneously as we do in this example.

More Examples

If a magnetic circuit contains an air gap, add another element to the conceptual models (recall Figure 12–21). Since air represents a poor magnetic path, its reluctance will be high compared with that of iron. Recalling our analogy to electric circuits, this suggests that the mmf drop across the gap will be large compared with that of the iron. You can see this in the following example.

EXAMPLE 12–7

The core of Figure 12–24 has a 0.008-m gap cut as shown in Figure 12–26. Determine how much the current must increase to maintain the original core flux. Neglect fringing.

Solution

Iron

$\ell_{\text{iron}} = 0.25 - 0.008 = 0.242$ m. Since Φ does not change, *B* and *H* will be the same as before. Thus, $B_{\text{iron}} = 0.5$ T and $H_{\text{iron}} = 1550$ At/m.

Air Gap

B_g is the same as B_{iron}. Thus, $B_g = 0.5$ T and $H_g = 7.96 \times 10^5 B_g = 3.98 \times 10^5$ At/m.

Ampere's Law

$NI = H_{\text{iron}}\ell_{\text{iron}} + H_g\ell_g = (1550)(0.242) + (3.98 \times 10^5)(0.008) = 375 + 3184 = 3559$ At. Thus, $I = 3559/500 = 7.1$ amps. Note that the current had to increase from 0.78 amp to 7.1 amps in order to maintain the same flux, over a ninefold increase.

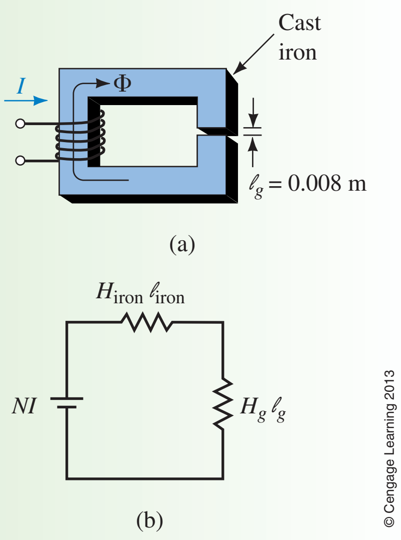


FIGURE 12–26

EXAMPLE 12–8

The laminated sheet steel section of Figure 12–27 has a stacking factor of 0.9. Compute the current required to establish a flux of $\Phi = 1.4 \times 10^{-4}$ Wb. Neglect fringing. All dimensions are shown in inches.

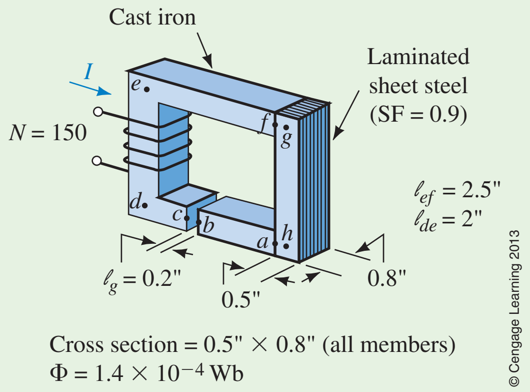


FIGURE 12–27

Solution Convert all dimensions to metric.

Cast Iron

$$\ell_{\text{iron}} = \ell_{adef} - \ell_g = 2.5 + 2 + 2.5 - 0.2 = 6.8 \text{ in} = 0.173 \text{ m}$$
$$A_{\text{iron}} = (0.5 \text{ in})(0.8 \text{ in}) = 0.4 \text{ in}^2 = 0.258 \times 10^{-3} \text{ m}^2$$
$$B_{\text{iron}} = \Phi/A_{\text{iron}} = (1.4 \times 10^{-4})/(0.258 \times 10^{-3}) = 0.54 \text{ T}$$
$$H_{\text{iron}} = 1850 \text{ At/m} \quad \text{(from Figure 12–19)}$$

Sheet Steel

$$\ell_{\text{steel}} = \ell_{fg} + \ell_{gh} + \ell_{ha} = 0.25 + 2 + 0.25 = 2.5 \text{ in} = 6.35 \times 10^{-2} \text{ m}$$
$$A_{\text{steel}} = (0.9)(0.258 \times 10^{-3}) = 0.232 \times 10^{-3} \text{ m}^2$$
$$B_{\text{steel}} = \Phi/A_{\text{steel}} = (1.4 \times 10^{-4})/(0.232 \times 10^{-3}) = 0.60 \text{ T}$$
$$H_{\text{steel}} = 125 \text{ At/m} \quad \text{(from Figure 12–19)}$$

Air Gap

$$\ell_g = 0.2 \text{ in} = 5.08 \times 10^{-3} \text{ m}$$
$$B_g = B_{\text{iron}} = 0.54 \text{ T}$$
$$H_g = (7.96 \times 10^5)(0.54) = 4.3 \times 10^5 \text{ At/m}$$

Ampere's Law

$$\begin{aligned} NI &= H_{\text{iron}}\ell_{\text{iron}} + H_{\text{steel}}\ell_{\text{steel}} + H_g\ell_g \\ &= (1850)(0.173) + (125)(6.35 \times 10^{-2}) + (4.3 \times 10^5)(5.08 \times 10^{-3}) \\ &= 320 + 7.9 + 2184 = 2512 \text{ At} \end{aligned}$$
$$I = 2512/N = 2512/150 = 16.7 \text{ amps}$$

EXAMPLE 12–9

Figure 12–28 shows a portion of a solenoid. Flux $\Phi = 4 \times 10^{-4}$ Wb when $I = 2.5$ amps. Find the number of turns on the coil.

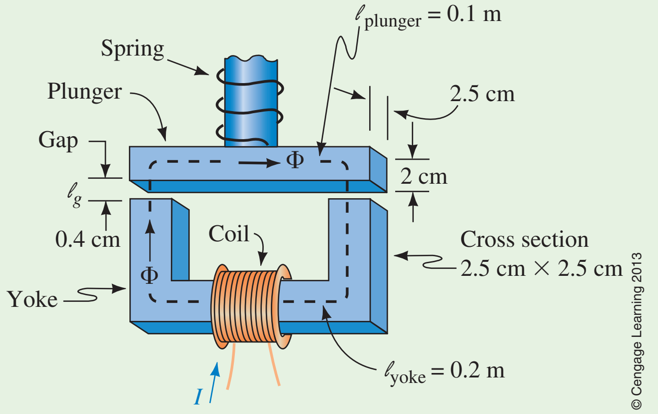


FIGURE 12–28 Solenoid. All parts are cast steel.

Solution

Yoke

$$A_{yoke} = 2.5 \text{ cm} \times 2.5 \text{ cm} = 6.25 \text{ cm}^2 = 6.25 \times 10^{-4} \text{ m}^2$$

$$B_{yoke} = \frac{\Phi}{A_{yoke}} = \frac{4 \times 10^{-4}}{6.25 \times 10^{-4}} = 0.64 \text{ T}$$

$$H_{yoke} = 410 \text{ At/m} \quad \text{(from Figure 12–19)}$$

Plunger

$$A_{plunger} = 2.0 \text{ cm} \times 2.5 \text{ cm} = 5.0 \text{ cm}^2 = 5.0 \times 10^{-4} \text{ m}^2$$

$$B_{plunger} = \frac{\Phi}{A_{plunger}} = \frac{4 \times 10^{-4}}{5.0 \times 10^{-4}} = 0.8 \text{ T}$$

$$H_{plunger} = 500 \text{ At/m} \quad \text{(from Figure 12–19)}$$

Air Gap

There are two identical gaps. For each,

$$B_g = B_{yoke} = 0.64 \text{ T}$$

Thus,

$$H_g = (7.96 \times 10^5)(0.64) = 5.09 \times 10^5 \text{ At/m}$$

The results are summarized in Table 12–2.

Ampere's Law

$$NI = H_{yoke}\ell_{yoke} + H_{plunger}\ell_{plunger} + 2H_g\ell_g = 82 + 50 + 2(2036) = 4204 \text{ At}$$

$$N = 4204/2.5 = 1682 \text{ turns}$$

TABLE 12–2

Material	Section	Length (m)	*A* (m²)	*B* (T)	*H* (At/m)	*Hℓ* (At)
Cast steel	yoke	0.2	6.25×10^{-4}	0.64	410	82
Cast steel	plunger	0.1	5×10^{-4}	0.8	500	50
Air	gap	0.4×10^{-2}	6.25×10^{-4}	0.64	5.09×10^5	2036

12.11 Series-Parallel Magnetic Circuits

Series-parallel magnetic circuits are handled using the sum of fluxes principle (Figure 12–14) and Ampere's law.

EXAMPLE 12–10

The core of Figure 12–29 is cast steel. Determine the current to establish an air-gap flux $\Phi_g = 6 \times 10^{-3}$ Wb. Neglect fringing.

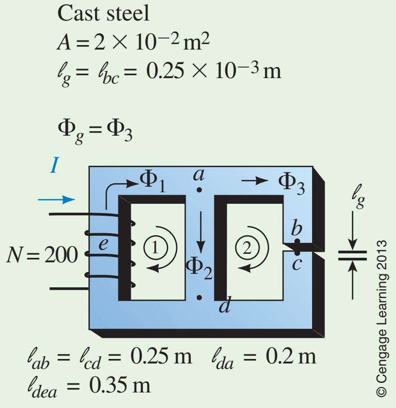


FIGURE 12–29

Solution Consider each section in turn.

Air Gap

$$B_g = \Phi_g/A_g = (6 \times 10^{-3})/(2 \times 10^{-2}) = 0.3 \text{ T}$$
$$H_g = (7.96 \times 10^5)(0.3) = 2.388 \times 10^5 \text{ At/m}$$

Sections ab and cd

$$B_{ab} = B_{cd} = B_g = 0.3 \text{ T}$$
$$H_{ab} = H_{cd} = 250 \text{ At/m (from Figure 12–19)}$$

Ampere's Law (Loop 2)

$\sum_{\bigcirc} NI = \sum_{\bigcirc} H\ell$. Since you are going opposite to flux in leg da, the corresponding term (i.e., $H_{da}\ell_{da}$) will be subtractive. Also, $NI = 0$ for loop 2. Thus,

$$0 = \sum_{\bigcirc\ \text{loop2}} H\ell$$

$$\begin{aligned} 0 &= H_{ab}\ell_{ab} + H_g\ell_g + H_{cd}\ell_{cd} - H_{da}\ell_{da} \\ &= (250)(0.25) + (2.388 \times 10^5)(0.25 \times 10^{-3}) + (250)(0.25) - 0.2H_{da} \\ &= 62.5 + 59.7 + 62.5 - 0.2H_{da} = 184.7 - 0.2H_{da} \end{aligned}$$

Thus, $0.2H_{da} = 184.7$ and $H_{da} = 925$ At/m. From Figure 12–19, $B_{da} = 1.12$ T.

$$\Phi_2 = B_{da}A = 1.12 \times 0.02 = 2.24 \times 10^{-2} \text{ Wb}$$
$$\Phi_1 = \Phi_2 + \Phi_3 = 2.84 \times 10^{-2} \text{ Wb.}$$
$$B_{dea} = \Phi_1/A = (2.84 \times 10^{-2})/0.02 = 1.42 \text{ T}$$
$$H_{dea} = 2125 \text{ At/m} \quad \text{(from Figure 12–19)}$$

Ampere's Law (Loop 1)

$$NI = H_{dea}\ell_{dea} + H_{ad}\ell_{ad} = (2125)(0.35) + 184.7 = 929 \text{ At}$$
$$I = 929/200 = 4.65 \text{ A}$$

PRACTICE PROBLEMS 5

The cast-iron core of Figure 12–30 is symmetrical. Determine current *I*. Hint: To find *NI*, you can write Ampere's law around either loop. Be sure to make use of symmetry.

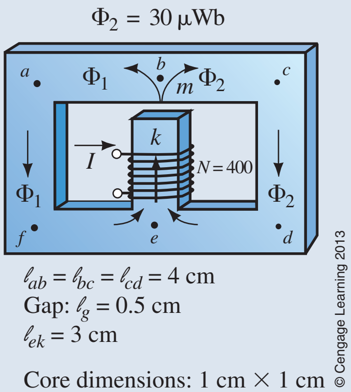


FIGURE 12–30

Answer
6.5 A

12.12 Series Magnetic Circuits: Given *NI*, Find Φ

In previous problems, you were given the flux and asked to find the current. We now look at the converse problem: given *NI*, find the resultant flux. For the special case of a core of one material and constant cross section (Example 12–11) this is straightforward. For all other cases, trial and error must be used.

EXAMPLE 12–11

For the circuit of Figure 12–31, $NI = 250$ At. Determine Φ.

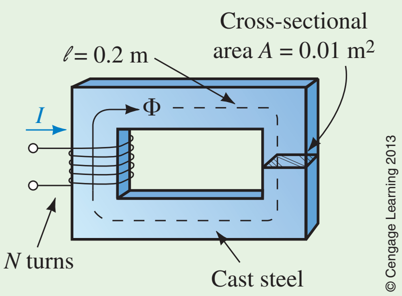
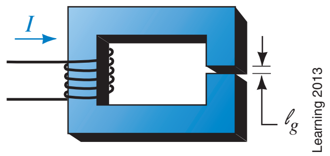


FIGURE 12–31

Solution $H\ell = NI$. Thus, $H = NI/\ell = 250/0.2 = 1250$ At/m. From the *B-H* curve of Figure 12–19, $B = 1.24$ T. Therefore, $\Phi = BA = 1.24 \times 0.01 = 1.24 \times 10^{-2}$ Wb.



$\mathscr{F} = H_{steel}\ell_{steel} + H_g\ell_g$
$\simeq H_g\ell_g$ if $H_g\ell_g >> H_{steel}\ell_{steel}$

FIGURE 12–32

For circuits with two or more sections, the process is not so simple. Before you can find H in any section, for example, you need to know the flux density. However, in order to determine flux density, you need to know H. Thus, neither Φ nor H can be found without knowing the other first.

To get around this problem, use trial and error. First, take a guess at the value for flux, compute NI using the 4-step procedure of Section 12.10, then compare the computed NI against the given NI. If they agree, the problem is solved. If they don't, adjust your guess and try again. Repeat the procedure until you are within 5% of the given NI.

The problem is how to come up with a good first guess. For circuits of the type of Figure 12–32, note that $NI = H_{steel}\ell_{steel} + H_g\ell_g$. As a first guess, assume that the reluctance of the air gap is so high that the full mmf drop appears across the gap. Thus, $NI \simeq H_g\ell_g$, and

$$H_g \simeq NI/\ell_g \tag{12–12}$$

You can now apply Ampere's law to see how close to the given NI your trial guess is (see Notes).

EXAMPLE 12–12

NOTES...

Since you know that some of the mmf drop appears across the steel, you can start at less than 100% for the gap. Some basic knowledge and a bit of experience help. The relative size of the mmf drops depends on the core material. For cast iron, the percentage drop across the iron is larger than the percentage across a similar piece of sheet steel or cast steel. This is illustrated in Examples 12–12 and 12–13.

The core of Figure 12–32 is cast steel, NI = 1100 At, the cross-sectional area everywhere is 0.0025 m^2, ℓ_g = 0.002 m, and ℓ_{steel} = 0.2 m. Determine the flux in the core.

Solution

Initial Guess

Assume that 90% of the mmf appears across the gap. The applied mmf is 1100 At. Ninety percent of this is 990 At. Thus, $H_g \simeq 0.9NI/\ell = 990/0.002 = 4.95 \times 10^5$ At/m and $B_g = \mu_0 H_g = (4\pi \times 10^{-7})(4.95 \times 10^5) = 0.62$ T.

Trial 1

Since the area of the steel is the same as that of the gap, the flux density is the same, neglecting fringing. Thus, $B_{steel} = B_g = 0.62$ T. From the B-H curve, H_{steel} = 400 At/m. Now apply Ampere's law:

$$NI = H_{steel}\ell_{steel} + H_g\ell_g = (400)(0.2) + (4.95 \times 10^5)(0.002)$$
$$= 80 + 990 = 1070 \text{ At}$$

This answer is 2.7% lower than the given NI of 1100 At and is therefore acceptable. Thus, $\Phi = BA = 0.62 \times 0.0025 = 1.55 \times 10^{-3}$ Wb.

The initial guess in Example 12–12 yielded an acceptable answer on the first trial. (You are seldom this lucky.)

EXAMPLE 12–13

If the core of Example 12–12 is cast iron instead of steel, compute Φ.

Solution Because cast iron has a larger H for a given flux density (Figure 12–19), it will have a larger $H\ell$ drop, and less will appear across the gap. Assume 75% across the gap.

Initial Guess

$H_g \simeq 0.75\ NI/\ell = (0.75)(1100)/0.002 = 4.125 \times 10^5$ At/m.

$B_g = \mu_0 H_g = (4\mu \times 10^{-7})(4.125 \times 10^5) = 0.52$ T.

Trial 1

$B_{iron} = B_g$. Thus, $B_{iron} = 0.52$ T. From the *B-H* curve, $H_{iron} = 1700$ At/m.

Ampere's Law

$$NI = H_{iron}\ell_{iron} + H_g\ell_g = (1700)(0.2) + (4.125 \times 10^5)(0.002)$$
$$= 340 + 825 = 1165 \text{ At} \quad \text{(high by 5.9\%)}$$

Trial 2

Reduce the guess by 5.9% to $B_{iron} = 0.49$ T. Thus, $H_{iron} = 1500$ At/m (from the *B-H* curve) and $H_g = 7.96 \times 10^5\ B_g = 3.90 \times 10^5$ At/m.

Ampere's Law

$$NI = H_{iron}\ell_{iron} + H_g\ell_g = (1500)(0.2) + (3.90 \times 10^5)(0.002)$$
$$= 300 + 780 = 1080 \text{ At}$$

The error is now 1.82%, which is excellent. Thus, $\Phi = BA = (0.49)(2.5 \times 10^{-3}) = 1.23 \times 10^{-3}$ Wb. If the error had been larger than 5%, a third trial would have been needed.

12.13 Force Due to an Electromagnet

Electromagnets are used in relays, door bells, lifting magnets, and so on. For an electromagnetic relay as in Figure 12–33, it can be shown that the force created by the magnetic field is

$$F = \frac{B_g^2 A_g}{2\mu_0} \qquad (12\text{–}13)$$

where B_g is flux density in the gap in teslas, A_g is gap area in square meters, and F is force in newtons.

EXAMPLE 12–14

Figure 12–33 shows a typical relay. The force due to the current-carrying coil pulls the pivoted arm against spring tension to close the contacts and energize the load. If the pole face is ¼ inch square and $\Phi = 0.5 \times 10^{-4}$ Wb, what is the pull on the armature in pounds?

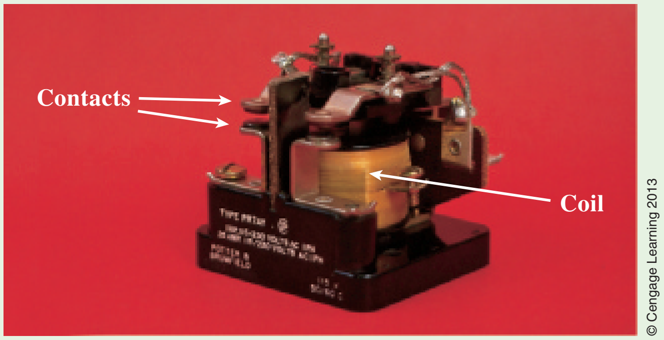


FIGURE 12–33 A typical relay.

Solution Convert to metric units.

$$A_g = (0.25\text{ in})(0.25\text{ in}) = 0.0625\text{ in}^2 = 0.403 \times 10^{-4}\text{ m}^2$$

$$B_g = \Phi/A_g = (0.5 \times 10^{-4})/(0.403 \times 10^{-4}) = 1.24\text{ T}$$

Thus,

$$F = \frac{B_g^2 A}{2\mu_0} = \frac{(1.24)^2(0.403 \times 10^{-4})}{2(4\pi \times 10^{-7})} = 24.66\text{ N} = 5.54\text{ lb}$$

Figure 12–34 shows how a relay may be used in practice. When the switch is closed, the energized coil pulls the armature down. This closes the contacts and energizes the load. When the switch is opened, the spring pulls the contacts open again. Schemes like this use relatively small currents to control large loads. In addition, they permit remote control, as the relay and load may be a considerable distance from the actuating switch.

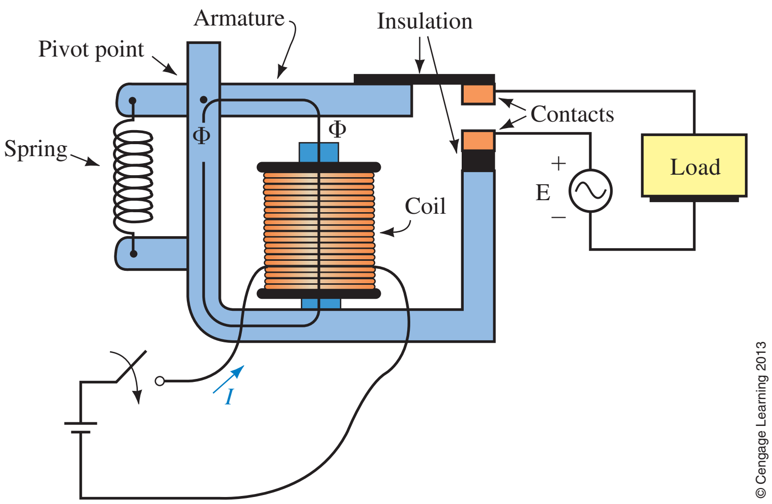


FIGURE 12–34 Controlling a load with a relay.

12.14 Properties of Magnetic Materials

Magnetic properties are related to atomic structure. Each atom of a substance, for example, produces a tiny atomic-level magnetic field because its moving (i.e., orbiting) electrons constitute an atomic-level current, and currents create magnetic fields. For nonmagnetic materials, these fields are randomly oriented and cancel. However, for ferromagnetic materials, the fields in small regions, called **domains** (Figure 12–35), do not cancel. (Domains are of microscopic size, but are large enough to hold from 10^{17} to 10^{21} atoms.) If the domain fields in a ferromagnetic material line up, the material is magnetized; if they are randomly oriented, the material is not magnetized.

Magnetizing a Specimen

A nonmagnetized specimen can be magnetized by making its domain fields line up. Figure 12–36 shows how this can be done. As current through the

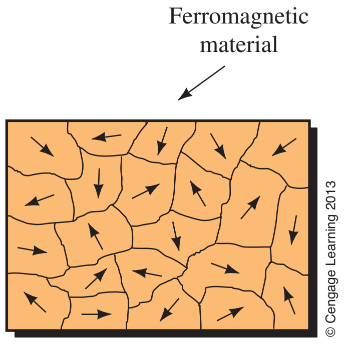
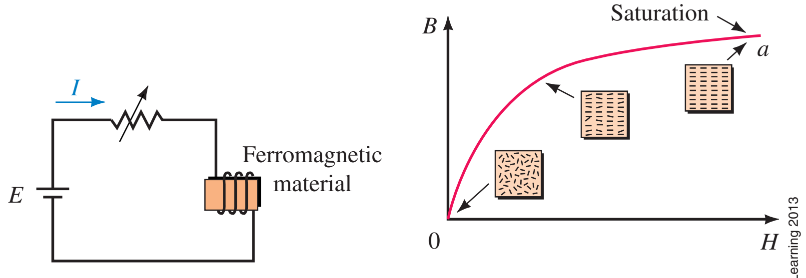


FIGURE 12–35 Random orientation of microscopic fields in a nonmagnetized ferromagnetic material. The small regions are called domains.



(a) The magnetizing circuit

(b) Progressive change in the domain orientations as the field is increased. H is proportional to current I.

FIGURE 12–36 The magnetization process.

coil is increased, the field strength increases and more and more domains align themselves in the direction of the field. If the field is made strong enough, almost all domain fields line up, and the material is said to be in **saturation** (the almost flat portion of the B-H curve). In saturation, the flux density increases slowly as magnetization intensity increases. This means that once the material is in saturation, you cannot magnetize it much further no matter how hard you try. Path 0-a of Figure 12–36 (traced from the nonmagnetized state to the saturated state) is termed the dc curve or normal magnetization curve. (This is the B-H curve that you used earlier when you solved magnetic circuit problems.)

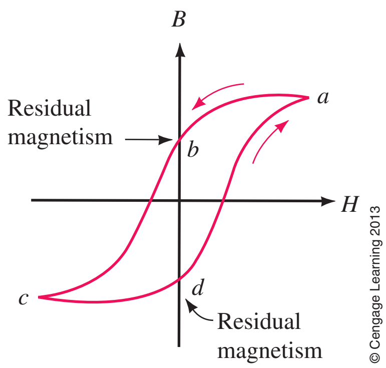


FIGURE 12–37 Hysteresis loop.

Hysteresis

If you now reduce the current to zero, you will find that the curve does not track back to zero. Instead, it follows a different path (Figure 12–37), with the result that the specimen retains some magnetism (called **residual magnetism**) as indicated by point b. If now you reverse the current, the flux reverses and you trace the path to c. If you again decrease the current to zero, you trace the path to d. Reversing the current and increasing it takes you back to a. The result is called a **hysteresis loop.** A major source of uncertainty in magnetic circuit behavior should now be apparent: As you can see, flux density depends not just on current; it also depends on which arm of the curve the sample is magnetized on, that is, it depends on the circuit's past history. For this reason, B-H curves are the average of the two arms of the hysteresis loop, as in the curve of Figure 12–36.

The Demagnetization Process

As indicated previously, simply turning the current off does not demagnetize ferromagnetic material. To demagnetize it, you must successively decrease its hysteresis loop to zero as in Figure 12–38. You can place the specimen inside a coil that is driven by a variable ac source and gradually decrease the coil current to zero, or you can use a fixed ac supply and gradually withdraw the specimen from the field. Such procedures are referred to as "degaussing." ("Degauss" is a term that dates back to the earlier use of gauss as the unit of magnetic flux density—recall Table 12–1.)

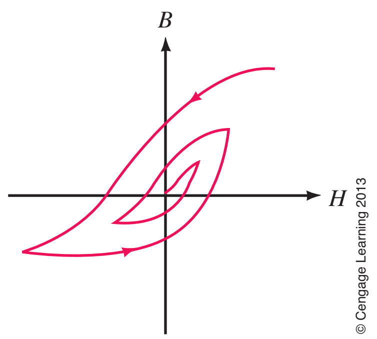


FIGURE 12–38 Demagnetization by successively shrinking the hysteresis loop.

12.15 Sensing and Measuring Magnetic Fields

One way to sense and measure magnetic field strength is to use the Hall effect (after E. H. Hall). The basic idea is illustrated in Figure 12–39. When a strip of semiconductor material such as indium arsenide is placed in a magnetic field, a small voltage, called the Hall voltage, V_H, appears across opposite edges. For a fixed current I, V_H is proportional to magnetic field strength B. Instruments using this principle are known as Hall-effect gaussmeters. To measure a magnetic field with such a meter, insert its probe into the field perpendicular to the field (Figure 12–40). The meter indicates flux density directly in teslas.

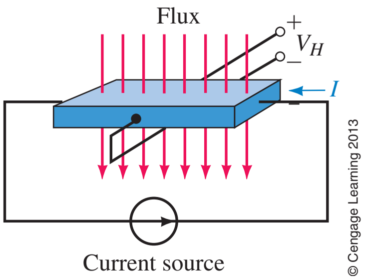


FIGURE 12–39 The Hall effect.

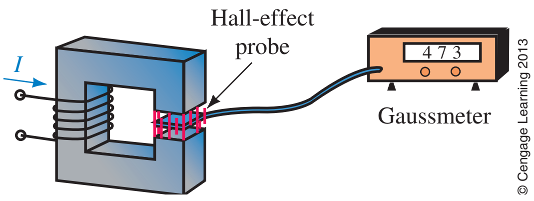


FIGURE 12–40 Magnetic field measurement.

Problems

12.3 Magnetic Flux and Flux Density

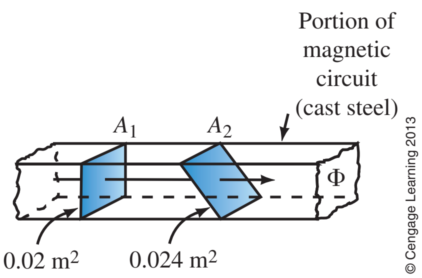


FIGURE 12–41

1. Refer to Figure 12–41:
   a. Which area, A_1 or A_2, do you use to calculate flux density?
   b. If $\Phi = 28$ mWb, what is flux density in teslas?
2. For Figure 12–41, if $\Phi = 250\ \mu$Wb, $A_1 = 1.25$ in^2, and $A_2 = 2.0$ in^2, what is the flux density in the English system of units?
3. The toroid of Figure 12–42 has a circular cross section and $\Phi = 628\ \mu$Wb. If $r_1 = 8$ cm and $r_2 = 12$ cm, what is the flux density in teslas?
4. If r_1 of Figure 12–42 is 3.5 inches and r_2 is 4.5 inches, what is the flux density in the English system of units if $\Phi = 628\ \mu$Wb?

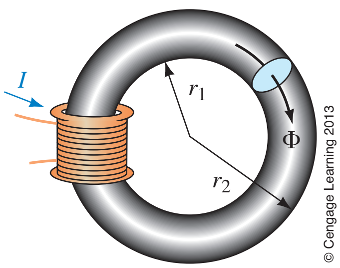


FIGURE 12–42

12.5 Air Gaps, Fringing, and Laminated Cores

5. If the section of core in Figure 12–43 is 0.025 m by 0.04 m, has a stacking factor of 0.85, and $B = 1.45$ T, what is Φ in webers?

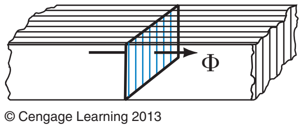


FIGURE 12–43

12.6 Series Elements and Parallel Elements

6. For the iron core of Figure 12–44, flux density $B_2 = 0.6$ T. Compute B_1 and B_3.
7. For the section of iron core of Figure 12–45, if $\Phi_1 = 12$ mWb and $\Phi_3 = 2$ mWb, what is B_2?
8. For the section of iron core of Figure 12–45, if $B_1 = 0.8$ T and $B_2 = 0.6$ T, what is B_3?

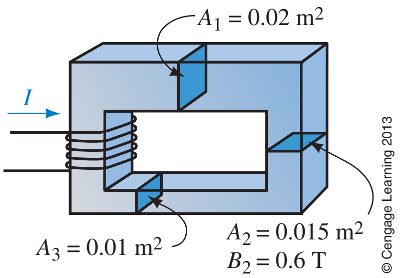


FIGURE 12–44

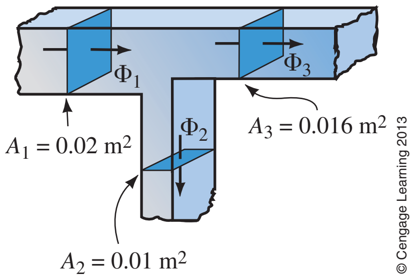


FIGURE 12–45

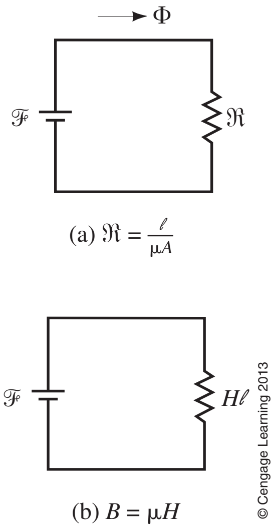


FIGURE 12–46 $\mathscr{F} = NI$.

12.8 Magnetic Field Intensity and Magnetization Curves

9. A core with dimensions 2 cm × 3 cm has a magnetic intensity of 1200 At/m. What is Φ if the core is cast iron? If it is cast steel? If it is sheet steel with stacking factor = 0.94?
10. Figure 12–46 shows the two electric circuit equivalents for magnetic circuits. Show that μ in $\Re = \ell/\mu A$ is the same as μ in $B = \mu H$.
11. Consider again Figure 12–42. If $I = 10$ A, $N = 40$ turns, $r_1 = 5$ cm, and $r_2 = 7$ cm, what is H in ampere-turns per meter?

12.9 Ampere's Circuital Law

12. Let H_1 and ℓ_1 be the magnetizing force and path length, respectively, where flux Φ_1 exists in Figure 12–47 and similarly for Φ_2 and Φ_3. Write Ampere's law around each of the windows.
13. Assume that a coil N_2 carrying current I_2 is added on leg 3 of the core shown in Figure 12–47 and that it produces flux directed upward. Assume, however, that the net flux in leg 3 is still downward. Write the Ampere's law equations for this case.
14. Repeat Problem 13 if the net flux in leg 3 is upward but the directions of Φ_1 and Φ_2 remain as in Figure 12–47.

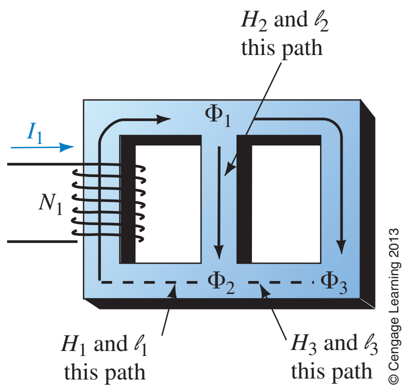


FIGURE 12–47

12.10 Series Magnetic Circuits: Given Φ, Find *NI*

15. Find the current I in Figure 12–48 if $\Phi = 0.16$ mWb.
16. Let everything be the same as in Problem 15 except that the cast steel portion is replaced with laminated sheet steel with a stacking factor of 0.85.

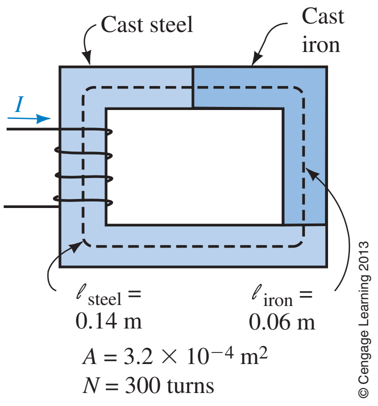


FIGURE 12–48

17. A gap of 0.5 mm is cut in the cast steel portion of the core in Figure 12–48. Find the current for $\Phi = 0.128$ mWb. Neglect fringing.
18. Two gaps, each 1 mm, are cut in the circuit of Figure 12–48, one in the cast steel portion and the other in the cast iron portion. Determine current for $\Phi = 0.128$ mWb. Neglect fringing.
19. The cast iron core of Figure 12–49 measures 1 cm × 1.5 cm, $\ell_g = 0.3$ mm, the air-gap flux density is 0.426 T, and $N = 600$ turns. The end pieces are half circles. Taking into account fringing, find current I.
20. For the circuit of Figure 12–50, $\Phi = 141\ \mu$Wb and $N = 400$ turns. The bottom member is sheet steel with a stacking factor of 0.94, while the remainder is cast steel. All pieces are 1 cm × 1 cm. The length of the cast steel path is 16 cm. Find current I.

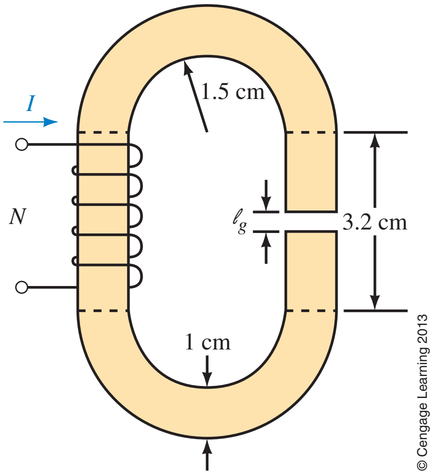


FIGURE 12–49

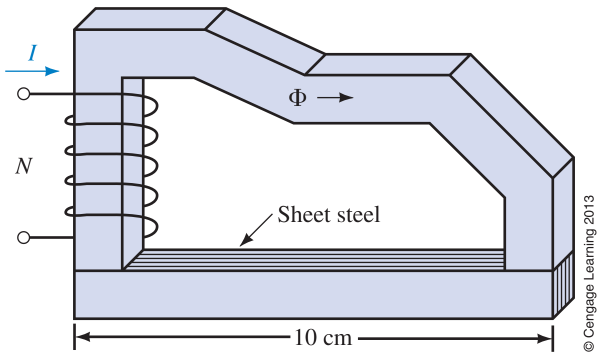


FIGURE 12–50

21. For the circuit of Figure 12–51, $\Phi = 30\ \mu$Wb and $N = 2000$ turns. Neglecting fringing, find current I.
22. For the circuit of Figure 12–52, Φ = 25,000 lines. The stacking factor for the sheet steel portion is 0.95. Find current I.
23. A second coil of 450 turns with $I_2 = 4$ amps is wound on the cast steel portion of Figure 12–52. Its flux is in opposition to the flux produced by the original coil. The resulting flux is 35,000 lines in the counterclockwise direction. Find the current I_1.

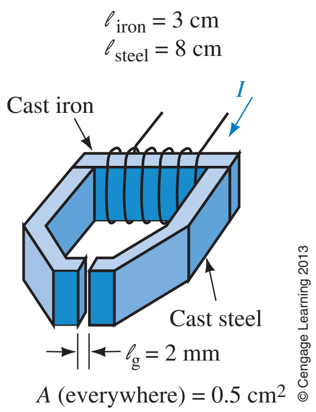


FIGURE 12–51

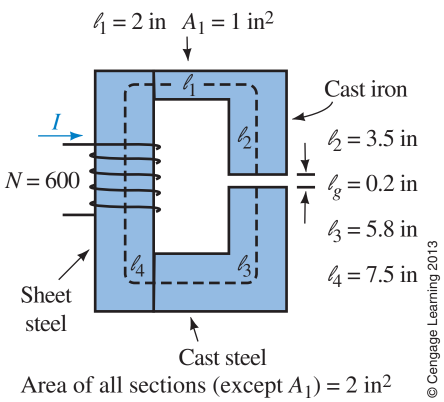


FIGURE 12–52

12.11 Series-Parallel Magnetic Circuits

24. For Figure 12–53, if $\Phi_g = 80\ \mu\text{Wb}$, find I.
25. If the circuit of Figure 12–53 has no gap and $\Phi_3 = 0.2$ mWb, find I.

12.12 Series Magnetic Circuits: Given *NI*, Find Φ

26. A cast steel magnetic circuit with $N = 2500$ turns, $I = 200$ mA, and a cross-sectional area of 0.02 m^2 has an air gap of 0.00254 m. Assuming 90% of the mmf appears across the gap, estimate the flux in the core.
27. If $NI = 644$ At for the cast steel core of Figure 12–54, find the flux, Φ.
28. A gap $\ell = 0.004$ m is cut in the core of Figure 12–54. Everything else remains the same. Find the flux, Φ.

12.13 Force Due to an Electromagnet

29. For the relay of Figure 12–34, if the pole face is 2 cm by 2.5 cm and a force of 2 pounds is required to close the gap, what flux (in webers) is needed?
30. For the solenoid of Figure 12–28, $\Phi = 4 \times 10^{-4}$ Wb. Find the force of attraction on the plunger in newtons and in pounds.

12.15 Sensing and Measuring Magnetic Fields

31. Computers store information on hard disk drives using magnetic means. This data is read back by read heads. One of the latest read-head technologies is magnetoresistance technology. The earliest versions of this technology were known as AMR (sometimes shortened to MR). A more advanced version is known as GMR. Go to the Internet and research these read-head technologies.

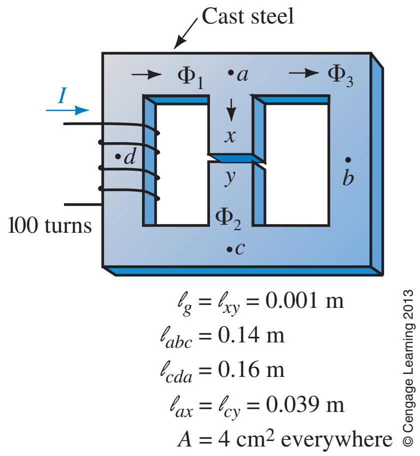


FIGURE 12–53

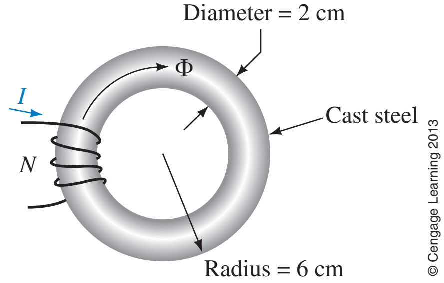


FIGURE 12–54

ANSWERS TO IN-PROCESS LEARNING CHECKS

IN-PROCESS LEARNING CHECK 1

1. Force
2. Strength, direction
3. Iron, nickel, cobalt
4. North, south
5. Same except direction of flux reversed
6. Same as Figure 12–6, since plastic does not affect the field.
7. Perpendicular
8. 10 cm^2

IN-PROCESS LEARNING CHECK 2

1. While flux is the same throughout, the effective area of each section differs.
2. $B_{\text{iron}} = 1.1$ T; $B_{\text{steel}} = 1.38$ T; $B_g = 1.04$ T
3. $\Phi_2 = \Phi_3 = 0.5$ mWb

IN-PROCESS LEARNING CHECK 3

1. 3.5 A
2. a. 2.1 A
   b. 0.525 A
3. 1550 At/m
4. Same as Figure 12–21(b) except add $H_{g_2}\ell_{g_2}$.
5. a. $NI = H_1\ell_1 + H_2\ell_2$
   b. $0 = H_2\ell_2 - H_3\ell_3$
6. 0.32 m; use the mean path length.

KEY TERMS

Choke (Reactor)

Counter EMF (Back Voltage)

Faraday's Law

Flux Linkage

Henry (H)

Induced Voltage

Inductance

Inductor

Lenz's Law

Self-Inductance

OUTLINE

Electromagnetic Induction

Induced Voltage and Inductance

Self-Inductance

Computing Induced Voltage

Inductances in Series and Parallel

Practical Considerations

Inductance and Steady State dc

Energy Stored by an Inductance

Inductor Troubleshooting Hints

OBJECTIVES

After studying this chapter, you will be able to

- describe what an inductor is and what its effect on circuit operation is,
- explain Faraday's law and Lenz's law,
- compute induced voltage using Faraday's law,
- define inductance,
- compute voltage across an inductance,
- compute inductance for series and parallel configurations,
- compute inductor voltages and currents for steady state dc excitation,
- compute energy stored in an inductance,
- describe common inductor problems and how to test for them.

13

INDUCTANCE AND INDUCTORS

CHAPTER PREVIEW

In this chapter, we look at self-inductance (usually just called inductance) and inductors. To get at the idea, recall that when current flows through a conductor, it creates a magnetic field; as you will see in this chapter, this field affects circuit operation. To describe this effect, we introduce a circuit parameter called **inductance.** Inductance is due entirely to the magnetic field created by the current, and its effect is to slow the buildup and collapse of the current and—in general—oppose its change. Thus, in a sense, inductance can be likened to inertia in a mechanical system. The advantage of using inductance in our analyses is that we can dispense with all considerations of magnetism and magnetic fields—and just concentrate on familiar circuit quantities, voltage, current, and the newly introduced circuit parameter, inductance.

A circuit element built to possess inductance is called an **inductor.** In its simplest form an inductor is simply a coil of wire, Figure 13–1(a). Ideally, inductors have only inductance. However, since they are made of wire, practical inductors also have some resistance. Initially, however, we assume that this resistance is negligible and treat inductors as ideal (i.e., we assume that they have no property other than inductance). (Coil resistance is considered in Sections 13.6 and 13.7.) In practice, inductors are also referred to as **chokes** (because they try to limit or "choke" current change) or as **reactors** (for reasons to be discussed in Chapter 16). In this chapter, we refer to them mainly as inductors.

On circuit diagrams and in equations, inductance is represented by the letter *L.* Its circuit symbol is a coil as shown in Figure 13–1(b). The unit of inductance is the henry.

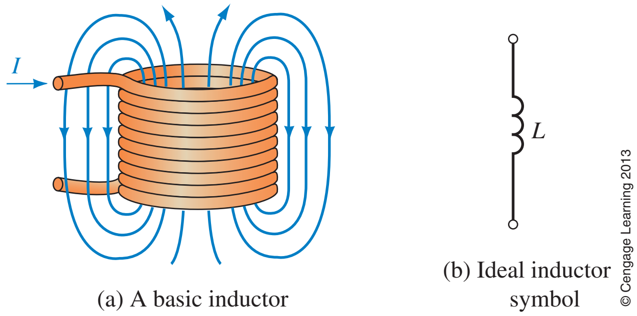


FIGURE 13–1 Inductance is due to the magnetic field created by an electric current.

Inductors are used in many places. In electronics, for example, they are used in switched-mode power supplies as energy storage devices, in electrical power systems they are used to help control short circuit currents during fault conditions, and in telecommunication systems they are used for electrical noise reduction in communication circuits. ■

Putting It in Perspective

The Discovery of Electromagnetic Induction

MOST OF OUR IDEAS CONCERNING INDUCTANCE and induced voltages are due to Michael Faraday (recall Chapter 12) and Joseph Henry (1797–1878). Working independently (Faraday in England and Henry—shown at left—in the USA), they discovered, almost simultaneously, the fundamental laws governing electromagnetic induction.

While experimenting with magnetic fields, Faraday developed the transformer. He wound two coils on an iron ring and energized one of them from a battery. As he closed the switch energizing the first coil, Faraday noticed that a momentary voltage was induced in the second coil, and when he opened the switch, he found that a momentary voltage was again induced but with opposite polarity. When the current was steady, no voltage was produced at all.

Faraday explained this effect in terms of his magnetic lines of the flux concept. When current was first turned on, he visualized the lines as springing outward into space; when it was turned off, he visualized the lines as collapsing inward. He then visualized that voltage was produced by these lines as they cut across circuit conductors. Companion experiments showed that voltage was also produced when a magnet was passed through a coil or when a conductor was moved through a magnetic field. Again, he visualized these voltages in terms of flux cutting a conductor.

Working independently in the United States, Henry discovered essentially the same results. In fact, Henry's work preceded Faraday's by a few months, but because he did not publish them first, credit was given to Faraday. However, Henry is credited with the discovery of self-induction, and in honor of his work the unit of inductance was named the henry. ■

13.1 Electromagnetic Induction

Inductance depends on **induced voltage.** Thus, we begin with a review of electromagnetic induction. First, we look at Faraday's and Henry's results. Consider Figure 13–2. In (a), a magnet is moved through a coil of wire, and this action induces a voltage in the coil. When the magnet is thrust into the coil, the meter deflects upscale; when it is withdrawn, the meter deflects downscale, indicating that polarity has changed. The voltage magnitude is proportional to how fast the magnet is moved. In (b), when the conductor is moved through the field, voltage is induced. If the conductor is moved to the right, its far end is positive; if it is moved to the left, the polarity reverses and its far end becomes negative. Again, the voltage magnitude is proportional to how fast the wire is moved. In (c), voltage is induced in coil 2 due to the magnetic field created by the current in coil 1. At the instant the switch is closed, the meter kicks upscale; at the instant it is opened, the meter kicks downscale. In (d) voltage is induced in a coil by its own current. At the instant the switch is closed, the top end of the coil becomes positive, while at the instant it is opened, the polarity reverses

NOTES...

Since we work with time-varying flux linkages in this chapter, we use ϕ rather than Φ for flux (as we did in Chapter 12). This is in keeping with the standard practice of using lowercase symbols for time-varying quantities and uppercase symbols for dc quantities.

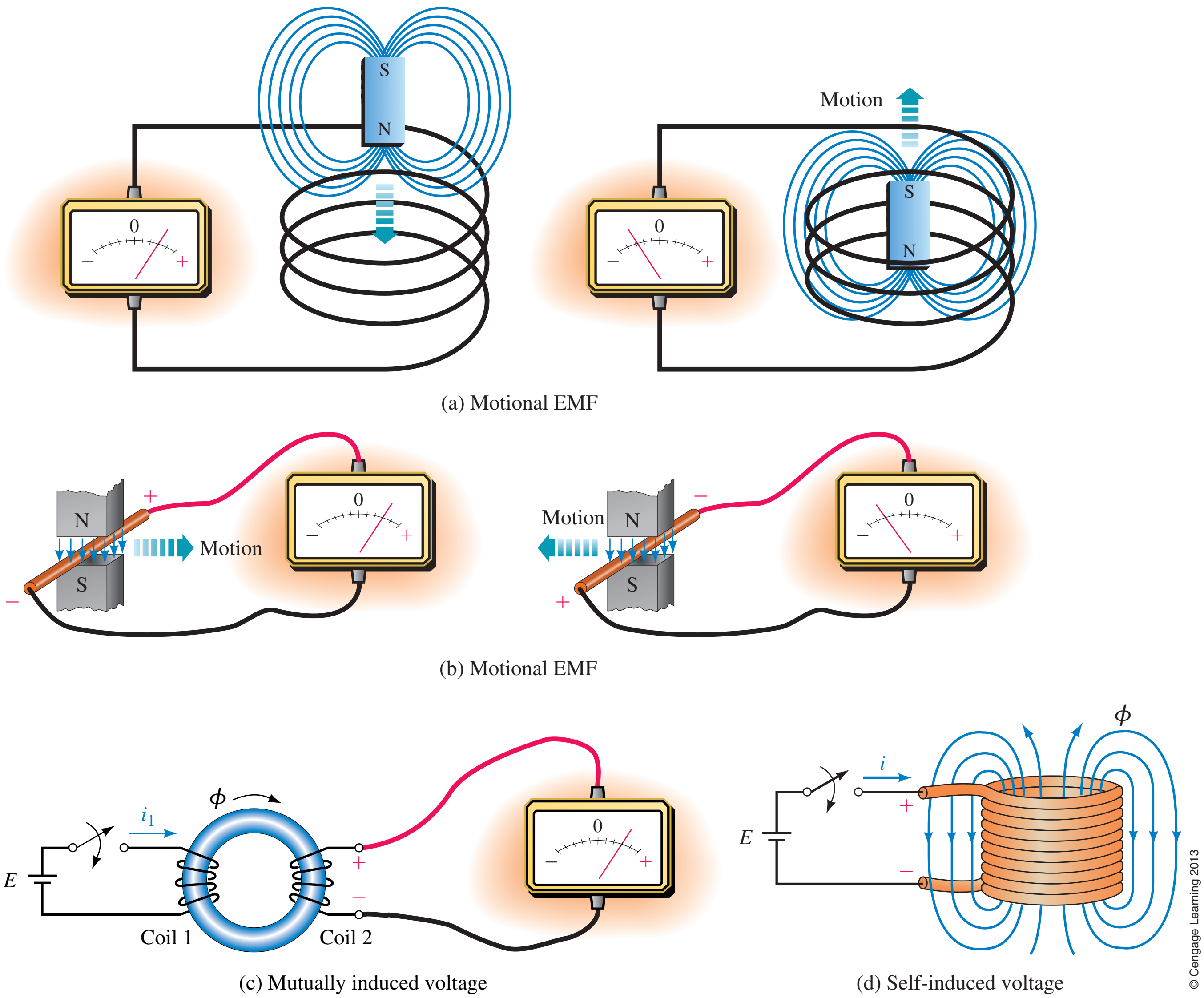


FIGURE 13–2 Illustrating Faraday's experiments. Voltage is induced only while the flux linking a circuit is changing.

and the top end becomes negative. Note that no voltage is induced in any of these cases when the flux linking the circuit is unchanging—that is, when the magnet is stationary in (a), when the wire is unmoving in (b), or when the current has reached a steady state in (c) and (d).

Faraday's Law

Based on these observations, Faraday concluded that *voltage is induced in a circuit whenever the flux linking* (i.e., passing through) *the circuit is changing and that the magnitude of the voltage is proportional to the rate of change of the flux linkages.* This result, known as **Faraday's law,** is also sometimes stated in terms of the rate of cutting flux lines. We look at this viewpoint in Chapter 15.

Lenz's Law

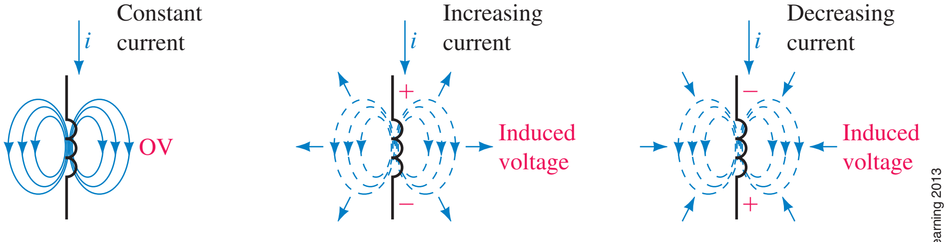
Heinrich Lenz (a Russian physicist, 1804–1865) determined a companion result. He showed that *the polarity of the induced voltage is such as to oppose the cause producing it.* This result is known as **Lenz's law.**

13.2 Induced Voltage and Inductance

We now focus on inductors, Figure 13–2(d). As noted earlier, inductance is due entirely to the magnetic field created by current. Consider Figure 13–3 (which shows the inductor at three instants of time). In (a) the current is constant, and since the magnetic field is due to this current, the magnetic field is also constant. Applying Faraday's law, we note that, because the flux linking the coil is not changing, the induced voltage is zero. Now consider (b). Here, the current (and hence the field) is increasing. According to Faraday's law, a voltage is induced that is proportional to how fast the field is changing, and according to Lenz's law, the polarity of this voltage must be such as to oppose the increase in current. Thus, the polarity of the voltage is as shown. Note that the faster the current increases, the larger the opposing voltage. Now consider (c). Since the current is decreasing, Lenz's law shows that the polarity of the induced voltage reverses, that is, the collapsing field produces a voltage that tries to keep the current going. Again, the faster the rate of change of current, the larger is this voltage.

Counter EMF

Because the induced voltage in Figure 13–3 tries to counter (i.e., opposes) changes in current, it is called a **counter EMF** or **back voltage.** Note carefully, however,



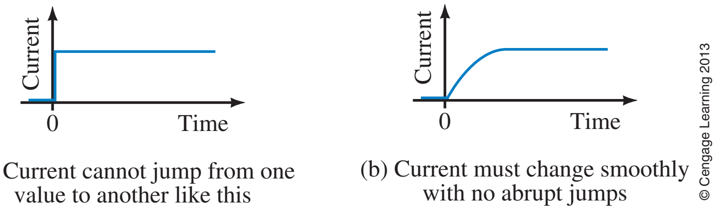
(a) Steady current: Induced voltage is zero.

(b) Increasing current: The induced voltage opposes the current buildup.

(c) Decreasing current: The induced voltage opposes the current decay.

FIGURE 13–3 Self-induced voltage due to a coil's own current. The induced voltage opposes the current change. Note carefully the polarities in (b) and (c).

that this voltage does not oppose current, it opposes only changes in current. It also does not prevent the current from changing; it only prevents it from changing abruptly. The result is that current in an inductor changes gradually and smoothly from one value to another as indicated in Figure 13–4(b). The effect of inductance is thus similar to the effect of inertia in a mechanical system. The flywheel used on an engine, for example, prevents abrupt changes in engine speed but does not prevent the engine from gradually changing from one speed to another.



(a) Current cannot jump from one value to another like this

(b) Current must change smoothly with no abrupt jumps

FIGURE 13–4 Current in inductance.

Iron-Core and Air-Core Inductors

As Faraday discovered, the voltage induced in a coil depends on flux linkages, and flux linkages depend on core materials. Coils with ferromagnetic cores (called iron-core coils) have their flux almost entirely confined to their cores, while coils wound on nonferromagnetic materials do not. (The latter are sometimes called air-core coils because all nonmagnetic core materials have the same permeability as air and thus behave magnetically the same as air.)

First, consider the iron-core case, Figure 13–5. Ideally, all flux lines are confined to the core and hence pass through (link) all turns of the winding. The product of flux times the number of turns that it passes through is defined as the **flux linkage** of the coil. For Figure 13–5, ϕ lines pass through N turns yielding a flux linkage of $N\phi$. By Faraday's law, the induced voltage is proportional to the rate of change of $N\phi$. In the SI system, the constant of proportionality is 1, and Faraday's law for this case may therefore be stated as

$$e = N \times \text{the rate of change of } \phi \tag{13–1}$$

In calculus notation,

$$e = N\frac{d\phi}{dt} \quad \text{(volts, V)} \tag{13–2}$$

where ϕ is in webers, t in seconds, and e in volts. Thus, if the flux changes at the rate of 1 Wb/s in a 1-turn coil, the voltage induced is 1 volt.

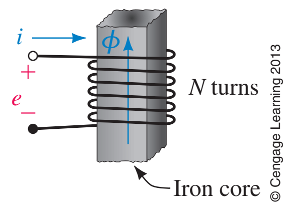


FIGURE 13–5 When flux ϕ passes through all N turns, the flux linking the coil is $N\phi$.

NOTES...

Equation 13–2 is sometimes shown with a minus sign. However, the minus sign is unnecessary. In circuit theory, we use Equation 13–2 to determine the magnitude of the induced voltage and Lenz's law to determine its polarity.

EXAMPLE 13–1

If the flux through a 200-turn coil changes steadily from 1 Wb to 4 Wb in 1 second, what is the voltage induced?

Solution The flux changes by 3 Wb in 1 second. Thus, its rate of change is 3 Wb/s.

$$\begin{aligned} e &= N \times \text{rate of change of flux} \\ &= (200 \text{ turns})(3 \text{ Wb/s}) = 600 \text{ volts} \end{aligned}$$

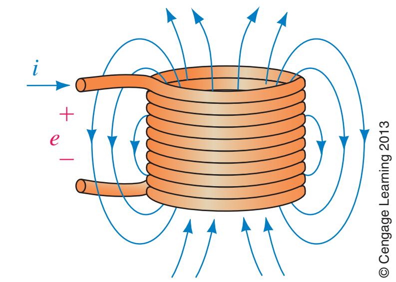


FIGURE 13–6 The flux linking the coil is proportional to current. Flux linkage is LI.

Now consider an air-core inductor (Figure 13–6). Since not all flux lines pass through all windings, you cannot determine flux linkages as above. However, (since no ferromagnetic material is present) flux is directly proportional to current. In this case, then, since induced voltage is proportional to the rate of change of flux, and since flux is proportional to current, induced voltage will be proportional to the rate of change of current. Let the constant of proportionality be L. Thus,

$$e = L \times \text{rate of change of current} \qquad \textbf{(13–3)}$$

In calculus notation, this can be written as

$$e = L\frac{di}{dt} \quad \text{(volts, V)} \qquad \textbf{(13–4)}$$

L is called the **self-inductance** (or usually, just inductance) of the coil, and in the SI system its unit is the henry. (This is discussed in more detail in Section 13.3.)

We now have two equations for coil voltage. Equation 13–4 is the more useful form for this chapter, while Equation 13–2 is needed for some of the circuits of Chapter 24. We look at Equation 13–4 in the next section.

IN-PROCESS LEARNING CHECK 1

(Answers are at the end of the chapter.)

1. Which of the current graphs shown in Figure 13–7 cannot be the current in an inductor? Why?

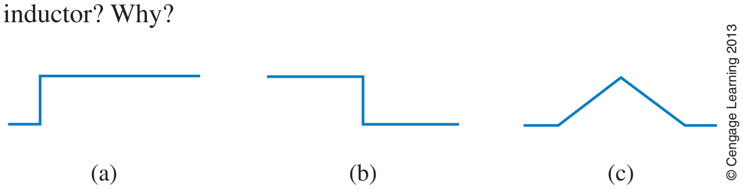


FIGURE 13–7

2. Compute the flux linkage for the coil of Figure 13–5, given $\phi = 500$ mWb and $N = 1200$ turns.
3. If the flux ϕ of Question 2 changes steadily from 500 mWb to 525 mWb in 1 s, what is the voltage induced in the coil?
4. If the flux ϕ of Question 2 changes steadily from 500 mWb to 475 mWb in 100 ms, what is the voltage induced?
5. If the flux for the 200-turn coil of Figure 13–5 is given by $\phi = 25\, t\, e^{-t}$ mWb, what is the equation for its induced voltage?

13.3 Self-Inductance

In the preceding section, we showed that the voltage induced in a coil is $e = Ldi/dt$, where L is the self-inductance of the coil (usually referred to simply as inductance as noted previously) and di/dt is the rate of change of its current. In the SI system, L is measured in henries. As can be seen from Equation 13–4, it is the ratio of voltage induced in a coil to the rate of change of current producing it. From this, we get the definition of the **henry (H)**. By definition, *the inductance of a coil is one henry if the voltage created by its changing current is one volt when its current changes at the rate of one ampere per second.*

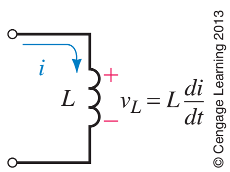


FIGURE 13–8 Voltage-current reference convention. As usual, the plus sign for voltage goes at the tail of the current arrow.

In practice, the voltage across an inductance is usually denoted by v_L rather than by e (Figure 13–8). Thus,

$$v_L = L\frac{di}{dt} \quad \text{(V)} \qquad \textbf{(13–5)}$$

EXAMPLE 13–2

If the current through a 5-mH inductance changes at the rate of 1000 A/s, what is the voltage induced?

Solution

$$v_L = L \times \text{rate of change of current}$$
$$= (5 \times 10^{-3}\ \text{H})(1000\ \text{A/s}) = 5\ \text{volts}$$

PRACTICE PROBLEMS 1

1. The voltage across an inductance is 250 V when its current changes at the rate of 10 mA/μs. What is L?
2. If the voltage across a 2-mH inductance is 50 volts, how fast is the current changing?

Answers
1. 25 mH; 2. 25×10^3 A/s

Inductance Formulas

Inductance for some simple shapes can be determined using the principles of Chapter 12. For example, the approximate inductance of the air core coil of Figure 13–9 can be shown to be

$$L = \frac{\mu N^2 A}{\ell} \quad \text{(H)} \qquad \textbf{(13–6)}$$

where ℓ is in meters, A is in square meters, N is the number of turns, and μ is the permeability of the core. (Details can be found in many physics books.)

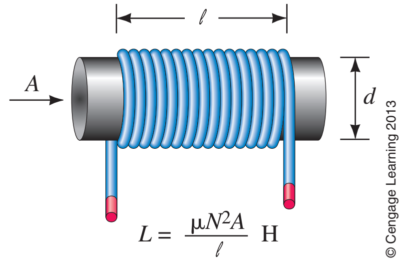


FIGURE 13–9 Approximate inductance formula for a single-layer air coil.

EXAMPLE 13–3

A 0.15-m-long air-core coil has a radius of 0.006 m and 120 turns. Compute its inductance.

Solution

$$A = \pi r^2 = 1.131 \times 10^{-4}\ \text{m}^2$$
$$\mu = \mu_0 = 4\pi \times 10^{-7}$$

Thus,

$$L = 4\pi \times 10^{-7}\ (120)^2\ (1.131 \times 10^{-4})/0.15 = 13.6\ \mu\text{H}$$

The accuracy of Equation 13–6 breaks down for small ℓ/d ratios. (If ℓ/d is greater than 10, the error is less than 4%.) Improved formulas may be found on the Internet or in design handbooks, such as the *Radio Amateur's Handbook* published by the American Radio Relay League (ARRL).

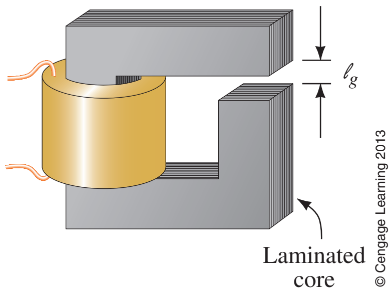


FIGURE 13–10 Iron-core coil with air gap. The gap keeps the core from going into saturation.

To provide greater inductance in smaller spaces, iron cores are sometimes used. Unless the core flux is kept below saturation, however, permeability varies and inductance is not constant. To get relatively constant inductance, an air gap may be used (Figure 13–10). If the gap is wide enough to dominate, coil inductance is approximately

$$L \approx \frac{\mu_0 N^2 A_g}{\ell_g} \text{ (H)} \tag{13–7}$$

where μ_0 is the permeability of air, A_g is the area of the air gap, and ℓ_g is its length. (See end-of-chapter Problem 11.) Another way to increase inductance is to use a ferrite core (Section 13.6).

EXAMPLE 13–4

The inductor of Figure 13–10 has 1000 turns, a 5-mm gap, and a cross-sectional area at the gap of 5×10^{-4} m^2. What is its inductance?

Solution

$$L \approx (4\pi \times 10^{-7})(1000)^2(5 \times 10^{-4})/(5 \times 10^{-3}) = 0.126 \text{ H}$$

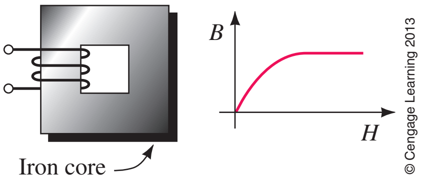


FIGURE 13–11 This coil does not have a fixed inductance because its flux is not proportional to its current.

PRACTICAL NOTES...

1. Since inductance is due to a conductor's magnetic field, it depends on the same factors that the magnetic field depends on. The stronger the field for a given current, the greater the inductance. Thus, a coil of many turns will have more inductance than a coil of a few turns (L is proportional to N^2) and a coil wound on a magnetic core will have greater inductance than a coil wound on a nonmagnetic form.
2. However, if a coil is wound on a magnetic core, the core's permeability μ may change with flux density. Since flux density depends on current, L becomes a function of current. For example, the inductor of Figure 13–11 has a nonlinear inductance due to core saturation. All inductors encountered in this book are assumed to be linear, that is, of constant value.

IN-PROCESS LEARNING CHECK 2

(Answers are at the end of the chapter.)

1. The voltage across an inductance whose current changes uniformly by 10 mA in 4 μs is 70 volts. What is its inductance?
2. If you triple the number of turns in the inductor of Figure 13–10, but everything else remains the same, by what factor does the inductance increase?

13.4 Computing Induced Voltage

Earlier, we determined that the voltage across an inductance is given by $v_L = L\,di/dt$, where the voltage and current references are shown in Figure 13–8. Note that the polarity of v_L depends on whether the current is increasing or decreasing. For example, if the current is increasing, di/dt is positive and so v_L is positive, while if the current is decreasing, di/dt is negative and v_L is negative.

To compute voltage, we need to determine di/dt. In general, this requires calculus. However, since di/dt is slope, you can determine voltage without calculus for currents that can be described by straight lines, as in Figure 13–12. For any Δt segment, slope $= \Delta i/\Delta t$, where Δi is the amount that the current changes during time interval Δt.

EXAMPLE 13–5

Figure 13–12 is the current through a 10-mH inductance. Determine voltage v_L and sketch it.

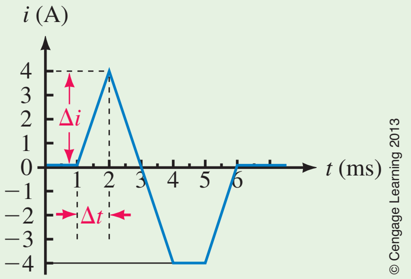


FIGURE 13–12

Solution Break the problem into intervals over which the slope is constant, determine the slope for each segment, then compute voltage using $v_L = L \times$ slope for that interval:

0 to 1 ms:	Slope = 0. Thus, $v_L = 0$ V.
1 ms to 2 ms:	Slope $= \Delta i/\Delta t = 4\ \text{A}/(1 \times 10^{-3}\ \text{s}) = 4 \times 10^3$ A/s. Thus, $v_L = L\Delta i/\Delta t = (0.010\ \text{H})(4 \times 10^3\ \text{A/s}) = 40$ V.
2 ms to 4 ms:	Slope $= \Delta i/\Delta t = -8\ \text{A}/(2 \times 10^{-3}\ \text{s}) = -4 \times 10^3$ A/s. Thus, $v_L = L\Delta i/\Delta t = (0.010\ \text{H})(-4 \times 10^3\ \text{A/s}) = -40$ V.
4 ms to 5 ms:	Slope = 0. Thus, $v_L = 0$ V.
5 ms to 6 ms:	Same slope as from 1 ms to 2 ms. Thus, $v_L = 40$ V.

The voltage waveform is shown in Figure 13–13.

CircuitSim 13-1

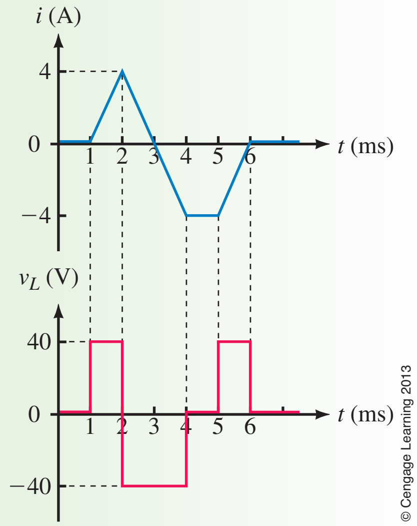


FIGURE 13–13

For currents that are not linear functions of time, you need to use calculus as illustrated in the following example.

EXAMPLE 13–6

What is the equation for the voltage across a 12.5-H inductance whose current is $i = te^{-t}$ amps?

Solution Differentiate by parts using

$$\frac{d(uv)}{dt} = u\frac{dv}{dt} + v\frac{du}{dt} \text{ with } u = t \text{ and } v = e^{-t}$$

Thus,

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}(te^{-t}) = L[t(-e^{-t}) + e^{-t}] = 12.5e^{-t}(1 - t) \text{ volts}$$

PRACTICE PROBLEMS 2

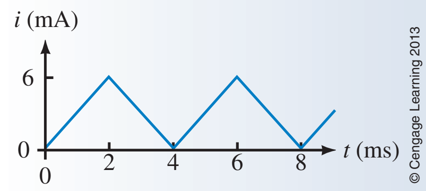


FIGURE 13–14

1. Figure 13–14 shows the current through a 5-H inductance. Determine voltage v_L and sketch it.
2. If the current of Figure 13–12 is applied to an unknown inductance and the voltage from 1 ms to 2 ms is 28 volts, what is L?
3. The current in a 4-H inductance is $i = t^2e^{-5t}$ A. What is voltage v_L?

Answers

1. v_L is a square wave. Between 0 ms and 2 ms, its value is 15 V; between 2 ms and 4 ms, its value is -15 V, and so on.
2. 7 mH; 3. $4e^{-5t}(2t - 5t^2)$ V

IN-PROCESS LEARNING CHECK 3

(Answers are at the end of the chapter.)

1. An inductance L_1 of 50 mH is in series with an inductance L_2 of 35 mH. If the voltage across L_1 at some instant is 125 volts, what is the voltage across L_2 at that instant? Hint: Since the same current passes through both inductances, the rate of change of current is the same for both.

CircuitSim 13-2

2. Current through a 5-H inductance changes linearly from 10 A to 12 A in 0.5 s. Suppose now the current changes linearly from 2 mA to 6 mA in 1 ms. Although the currents are significantly different, the induced voltage is the same in both cases. Why? Compute the voltage.
3. If the current for a 5-H coil is given by $i = 4\,t^3\,e^{-2t} + 4$ A, what is the equation for voltage v_L?
4. Repeat Question 3 if $i = 50 \sin 1000\,t$ mA.

13.5 Inductances in Series and Parallel

For inductances in series or parallel, the equivalent inductance is found by using the same rules that you used for resistance. For the series case (Figure 13–15) the total inductance is the sum of the individual inductances—see Practice Problems 3, Question 2. Thus,

$$L_T = L_1 + L_2 + L_3 + \cdots + L_N \tag{13–8}$$

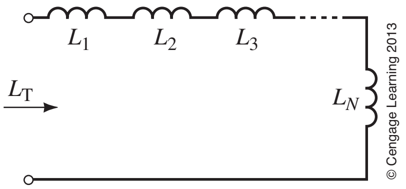


FIGURE 13–15 $L_T = L_1 + L_2 + \cdots + L_N$.

For the parallel case (Figure 13–16),

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \tag{13–9}$$

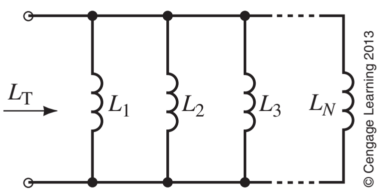


FIGURE 13–16 $\dfrac{1}{L_T} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \cdots + \dfrac{1}{L_N}$.

For two inductances, Equation 13–9 reduces to

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \qquad \textbf{(13–10)}$$

EXAMPLE 13–7

Find L_T for the circuit of Figure 13–17.

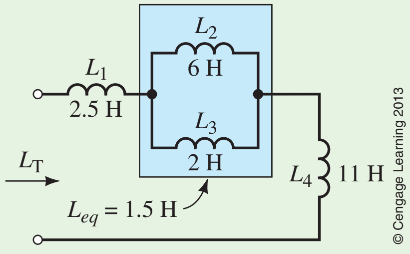


FIGURE 13–17

Solution The parallel combination of L_2 and L_3 is

$$L_{eq} = \frac{L_2 L_3}{L_2 + L_3} = \frac{6 \times 2}{6 + 2} = 1.5 \text{ H}$$

This is in series with L_1 and L_4. Thus, $L_T = 2.5 + 1.5 + 11 = 15$ H.

Calculator Hint: Using the inverse key on your calculator, you can determine the parallel equivalent of inductances using the same procedure you used for parallel resistances. Typically, your calculator steps would look something like the following (depending on the particular calculator you are using):

PRACTICE PROBLEMS 3

1. For Figure 13–18, $L_T = 2.25$ H. Determine L_x.
2. For Figure 13–15, the current is the same in each inductance and $v_1 = L_1 di/dt$, $v_2 = L_2 di/dt$, and so on. Apply Kirchhoff's voltage law (KVL) and show that $L_T = L_1 + L_2 + L_3 + \cdots + L_N$.

Answer
1. 3 H

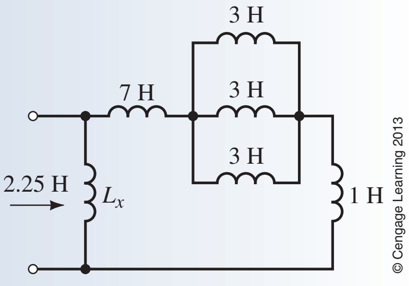


FIGURE 13–18

13.6 Practical Considerations

Core Types

The type of core used in an inductor depends to a great extent on its intended use and frequency range. (Although you have not studied frequency yet, you can get a feel for frequency by noting that the electrical power system operates at low frequency [60 cycles per second, called 60 hertz], while radio and TV systems operate at high frequency [hundreds of megahertz].) Inductors used in audio or power supply applications generally have laminated iron cores (because they need large inductance values). (Laminations reduce core loss—see Chapter 23, Section 23.6.) Inductors used for radio-frequency circuits generally have air or ferrite (or similar) cores. (Ferrite is a mixture of iron oxide in

NOTES...

Plastic or similar screwdrivers should be used so as not to disturb the coil's magnetic field and upset its value while making the adjustments.

a ceramic binder. It has characteristics that make it suitable for high-frequency work.) Iron cores cannot be used in high-frequency applications, however, since they have large power losses at high frequencies (for reasons discussed in Chapter 17).

Variable Inductors

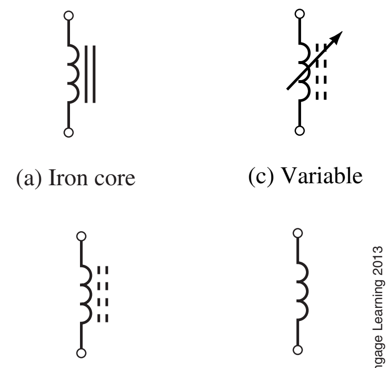
Inductors can be made so that their inductance is variable. In one approach, inductance is varied by changing coil spacing with a screwdriver adjustment—see Note. In another approach, a threaded core slug is screwed in or out of the coil to vary its inductance. Inductors of the latter type are shown in Figure 13–19. The units illustrated are low-profile tunable inductors designed for mounting on printed circuit boards.

Photo courtesy of Coilcraft Inc.

FIGURE 13–19 A selection of color-coded, tunable, low-profile, high-frequency inductors. A threaded core slug is used to adjust inductance.

Circuit Symbols

Figure 13–20 shows inductor symbols. Iron cores are identified by double solid lines, while dashed lines denote a ferrite core. Air-core inductors have no core symbol. An arrow indicates a variable inductor.



(a) Iron core (c) Variable

(b) Ferrite core (d) Air core

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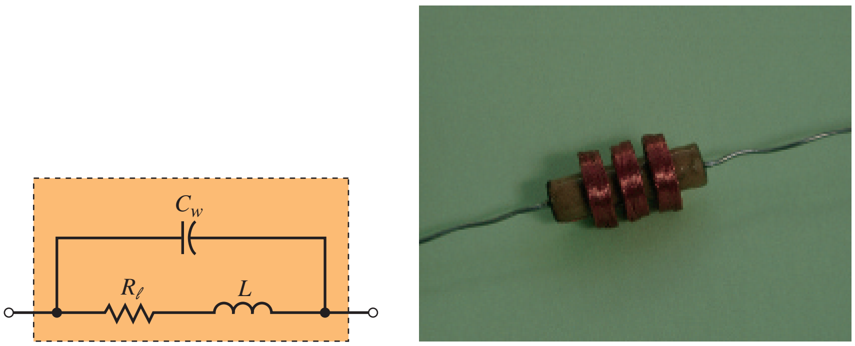
FIGURE 13–20 Circuit symbols for inductors.

Coil Resistance

Ideally, inductors have only inductance. However, since inductors are made of imperfect conductors (e.g., copper wire), they also have resistance. [We can view this resistance as being in series with the coil's inductance as indicated in Figure 13–21(a). Also shown is stray capacitance, considered next.] Although coil resistance is generally small, it cannot always be ignored, and thus it must



(a) Real inductors have stray capacitance and winding resistance

(b) Separating coil into sections helps reduce stray capacitance

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FIGURE 13–21 A ferrite-core choke.

sometimes be included in the analysis of a circuit. In Section 13.7, we show how this resistance is taken into account in dc analysis; in later chapters, you will learn how to take it into account in ac analysis.

Stray Capacitance

Because the turns of an inductor are separated from each other by insulation, a small amount of capacitance exists from winding to winding. This capacitance is called stray or parasitic capacitance. Although this capacitance is distributed from turn to turn, its effect can be approximated by lumping it as in Figure 13–21(a). The effect of stray capacitance depends on frequency. At low frequencies, it can usually be neglected; at high frequencies, it may have to be taken into account as you will see in later courses. Some coils are wound in multiple sections, as in Figure 13–21(b), to reduce stray capacitance.

Stray Inductance

Because inductance is due entirely to the magnetic effects of electric current, all current-carrying conductors have inductance. This means that leads on circuit components such as resistors, capacitors, transistors, and so on, all have inductance, as do traces on printed circuit boards and wires in cables. We call this inductance "stray inductance." Fortunately, in many cases, the stray inductance is so small that it can be neglected (see Practical Notes).

PRACTICAL NOTES...

Although stray inductance is small, it is not always negligible. In general, stray inductance will not be a problem for short wires at low to moderate frequencies. However, even a short piece of wire can be a problem at high frequencies, or a long piece of wire at low frequencies. For example, the inductance of just a few centimeters of conductor in a high-speed logic system may be nonnegligible because the current through it changes at such a high rate.

13.7 Inductance and Steady State dc

We now look at inductive circuits with constant dc current. Consider Figure 13–22. The voltage across an ideal inductance with constant dc current is zero because the rate of change of current is zero. This is indicated in (a). Since the inductor has current through it but no voltage across it, it looks like a short circuit, (b). This is true in general, that is, *an ideal inductor looks like a short circuit in steady state dc.* (This should not be surprising since it is just a piece of wire to dc.) For a nonideal inductor, its dc equivalent is its coil resistance (Figure 13–23). For steady state dc, problems can be solved using simple dc circuit analysis techniques.

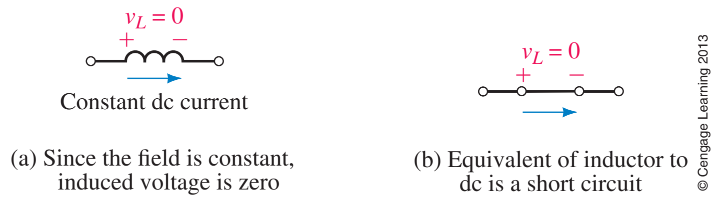


FIGURE 13–22 Inductance looks like a short circuit to steady state dc.

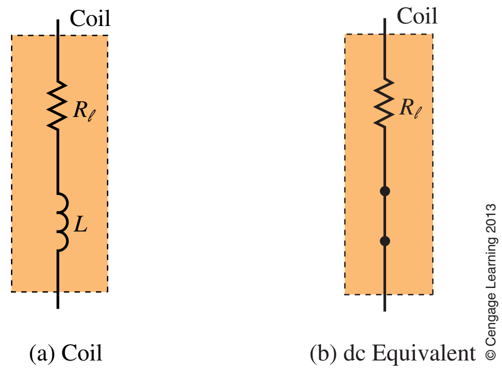


FIGURE 13–23 Steady state dc equivalent of a coil with winding resistance.

EXAMPLE 13–8

In Figure 13–24(a), the coil resistance is 14.4 Ω. What is the steady state current I?

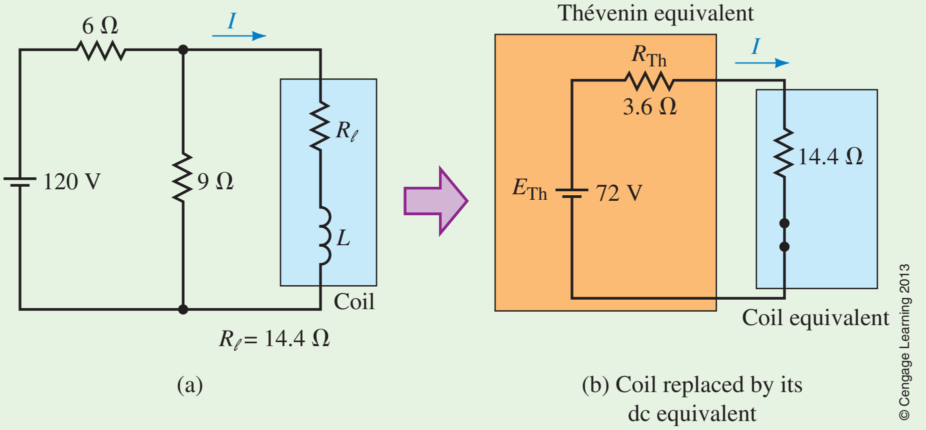


FIGURE 13–24

Solution Reduce the circuit as in (b).

$$E_{Th} = (9/15)(120) = 72 \text{ V}$$
$$R_{Th} = 6 \ \Omega \| 9 \ \Omega = 3.6 \ \Omega$$

Now replace the coil with its dc equivalent circuit as in (b). Thus,

$$I = E_{Th}/R_T = 72/(3.6 + 14.4) = 4\text{A}$$

EXAMPLE 13–9

The resistance of coil 1 in Figure 13–25(a) is 30 Ω and that of coil 2 is 15 Ω. Find the voltage across the capacitor assuming steady state dc.

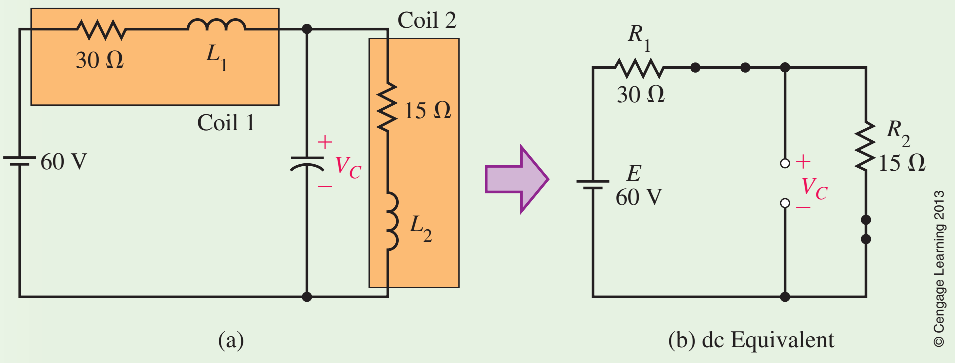


FIGURE 13–25

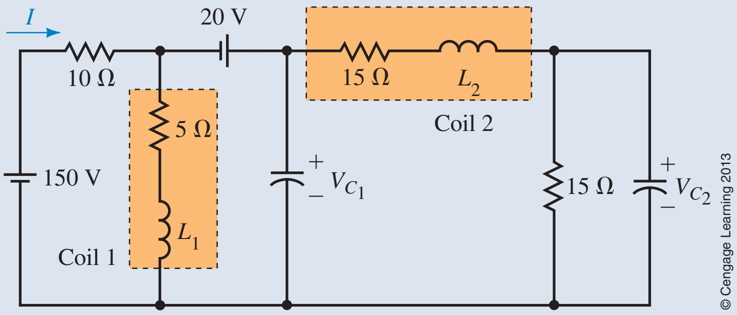
Solution Replace each coil inductance with a short circuit and the capacitor with an open circuit. As you can see from (b), the voltage across C is the same as the voltage across R_2. Thus,

CircuitSim 13-3

$$V_C = \frac{R_2}{R_1 + R_2}E = \left(\frac{15 \ \Omega}{45 \ \Omega}\right)(60 \text{ V}) = 20 \text{ V}$$

PRACTICE PROBLEMS 4

For Figure 13–26, find I, V_{C_1}, and V_{C_2} in the steady state.



CircuitSim 13-4

FIGURE 13–26

Answers
10.7 A; 63 V; 31.5 V

13.8 Energy Stored by an Inductance

When power flows into an inductor, energy is stored in its magnetic field. When the field collapses, this energy is returned to the circuit. For an ideal inductor, $R_\ell = 0$ ohm and hence no power is dissipated; thus, an ideal inductor has zero power loss.

To determine the energy stored by an ideal inductor, consider Figure 13–27. Power to the inductor is given by $p = v_L i$ watts, where $v_L = L di/dt$. By summing this power (see next ∫), the energy is found to be

$$W = \frac{1}{2}Li^2 \quad \text{(J)} \tag{13–11}$$

where i is the instantaneous value of current. When current reaches its steady state value I, $W = ½LI^2$ J. This energy remains stored in the field as long as the current continues. When the current goes to zero, the field collapses and the energy is returned to the circuit. (We look at this idea in more detail in Chapter 17 when we consider ac.)

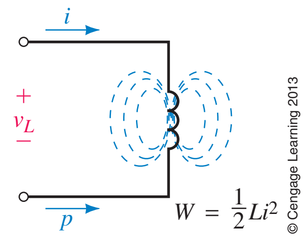
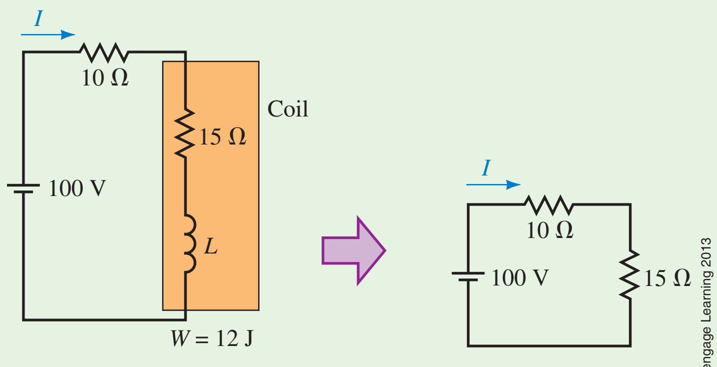


FIGURE 13–27 Energy is stored in the magnetic field of an inductor.

EXAMPLE 13–10

The coil of Figure 13–28(a) has a resistance of 15 Ω. When the current reaches its steady state value, the energy stored is 12 J. What is the inductance of the coil?



(a) (b) dc Equivalent

FIGURE 13–28

Solution From (b),

$$I = 100\text{ V}/25\ \Omega = 4\text{ A}$$

$$W = \frac{1}{2}LI^2\text{ J}$$

$$12\text{ J} = \frac{1}{2}L(4\text{ A})^2$$

Thus,

$$L = 2(12)/4^2 = 1.5\text{ H}$$

Deriving Equation 13–11

The power to the inductor in Figure 13–27 is given by $p = v_L i$, where $v_L = Ldi/dt$. Therefore, $p = Lidi/dt$. However, $p = dW/dt$. Equating the two expressions for power, then integrating both sides yields

$$W = \int_0^t pdt = \int_0^t Li\frac{di}{dt}dt = L\int_0^i idi = \frac{1}{2}Li^2$$

13.9 Inductor Troubleshooting Hints

Inductors may fail by either opening or shorting. Failures may be caused by misuse, defects in manufacturing, or faulty installation.

Open Coil

Opens can be the result of poor solder joints or broken connections. First, make a visual inspection. If nothing wrong is found, disconnect the inductor and check it with an ohmmeter. An open-circuited coil has infinite resistance.

Shorts

Shorts can occur between windings or between the coil and its core (for an iron-core unit). A short may result in excessive current and overheating. Again, check visually. Look for burned insulation, discolored components, an acrid odor, and other evidence of overheating. An ohmmeter can be used to check for shorts between windings and the core. However, checking coil resistance for shorted turns is often of little value, especially if only a few turns are shorted. This is because the shorting of a few windings may not change the overall resistance enough to be measurable. The LCR testers of Chapter 10 may also prove useful. Sometimes, however, the only conclusive test is to substitute a known good inductor for the suspected bad one.

Problems

Unless otherwise indicated, assume ideal inductors and coils.

13.2 Induced Voltage and Inductance

1. If the flux linking a 75-turn coil (Figure 13–29) changes at the rate of 3 Wb/s, what is the voltage across the coil?
2. If 80 volts is induced when the flux linking a coil changes at a uniform rate from 3.5 mWb to 4.5 mWb in 0.5 ms, how many turns does the coil have?

3. Flux changing at a uniform rate for 1 ms induces 60 V in a coil. What is the induced voltage if the same flux change takes place in 0.01 s?

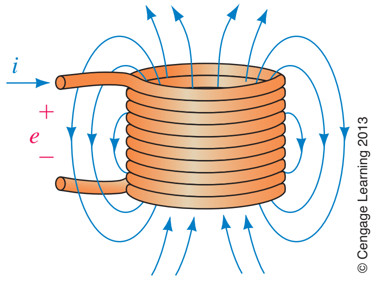


FIGURE 13–29

13.3 Self-Inductance

4. The current in a 0.4-H inductor (Figure 13–30) is changing at the rate of 200 A/s. What is the voltage across it?
5. The current in a 75-mH inductor (Figure 13–30) changes uniformly by 200 μA in 0.1 ms. What is the voltage across it?
6. The voltage across an inductance is 25 volts when the current changes at 5 A/s. What is L?
7. The voltage induced when current changes uniformly from 3 amps to 5 amps in a 10-H inductor is 180 volts. How long did it take for the current to change from 3 amps to 5 amps?
8. Current changing at a uniform rate for 1 ms induces 45 V in a coil. What is the induced voltage if the same current change takes place in 100 μs?
9. Compute the inductance of the air-core coil of Figure 13–31, given $\ell = 20$ cm, $N = 200$ turns, and $d = 2$ cm.
10. The iron-core inductor of Figure 13–32 has 2000 turns, a cross section of 1.5×1.2 inches, and an air gap of 0.2 inch. Compute its inductance.

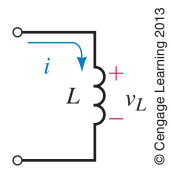


FIGURE 13–30

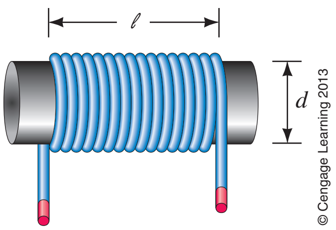


FIGURE 13–31

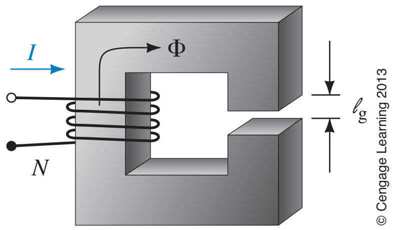


FIGURE 13–32

11. The iron-core inductor of Figure 13–32 has a high-permeability core. Therefore, by Ampere's law, $NI \simeq H_g \ell_g$. Because the air gap dominates, saturation does not occur, and the core flux is proportional to the current, that is, the flux linkage equals LI. In addition, since all flux passes through the coil, the flux linkage equals $N\Phi$. By equating the two values of flux linkage and using ideas from Chapter 12, show that the inductance of the coil is

$$L = \frac{\mu_0 N^2 A_g}{\ell_g}$$

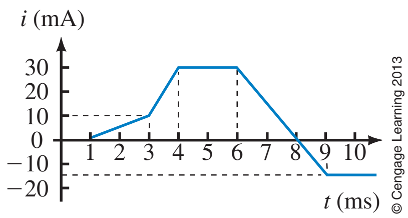


FIGURE 13–33

13.4 Computing Induced Voltage

12. Figure 13–33 shows the current in a 0.75-H inductor. Determine v_L and plot its waveform.
13. Figure 13–34 shows the current in a coil. If the voltage from 0 ms to 2 ms is 100 volts, what is L?
14. Why is Figure 13–35 not a valid inductor current? Sketch the voltage across L to show why. Pay particular attention to $t = 10$ ms.
15. Figure 13–36 shows the graph of the voltage across an inductance. The current changes from 4 A to 5 A during the time interval from 4 s to 5 s.
    a. What is L?
    b. Determine the current waveform and plot it.
    c. What is the current at $t = 10$ s?

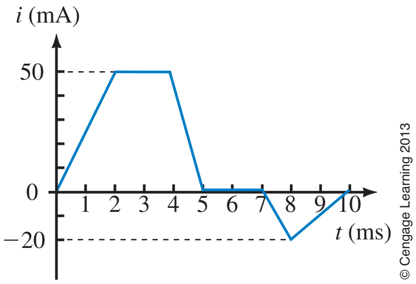


FIGURE 13–34

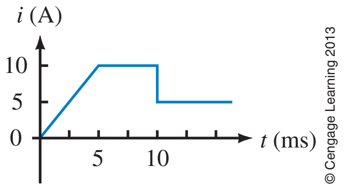


FIGURE 13–35

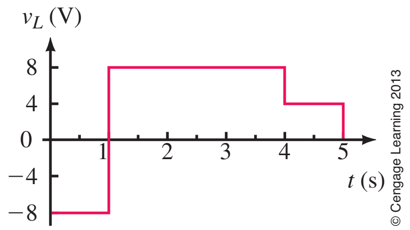


FIGURE 13–36

16. If the current in a 25-H inductance is $i_L = 20e^{-12t}$ mA, what is v_L?

13.5 Inductances in Series and Parallel

17. What is the equivalent inductance of 12 mH, 14 mH, 22 mH, and 36 mH connected in series?
18. What is the equivalent inductance of 0.010 H, 22 mH, 86×10^{-3} H, and 12 000 μH connected in series?
19. Repeat Problem 17 if the inductances are connected in parallel.
20. Repeat Problem 18 if the inductances are connected in parallel.
21. Determine L_T for the circuits of Figure 13–37.

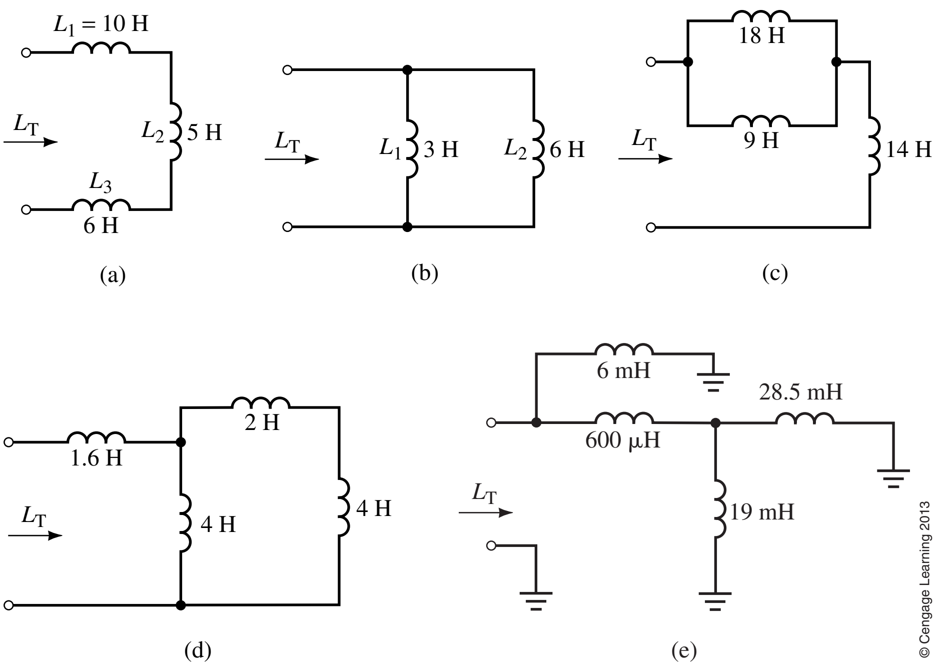


FIGURE 13–37

22. Determine L_T for the circuits of Figure 13–38.
23. A 30-μH inductance is connected in series with a 60-μH inductance, and a 10-μH inductance is connected in parallel with the series combination. What is L_T?

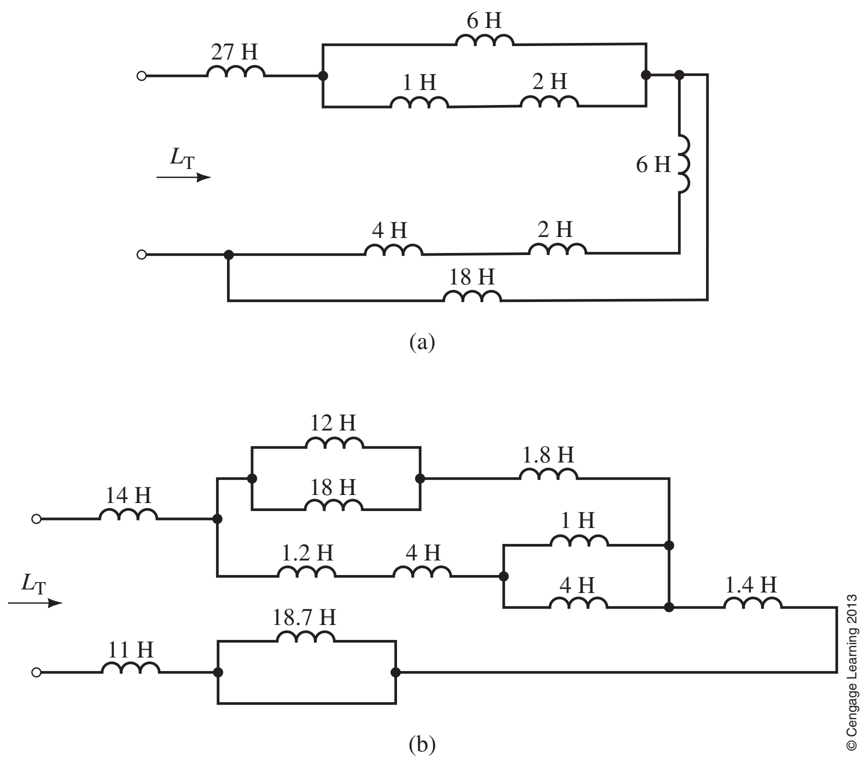


FIGURE 13–38

24. For Figure 13–39, determine L_x.
25. For the circuits of Figure 13–40, determine L_3 and L_4.
26. You have inductances of 24 mH, 36 mH, 22 mH, and 10 mH. Connecting these any way you want, what is the largest equivalent inductance you can get? The smallest?

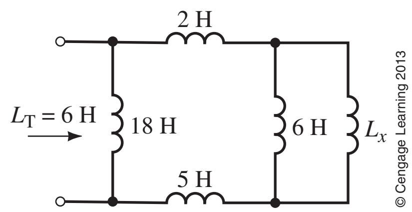


FIGURE 13–39

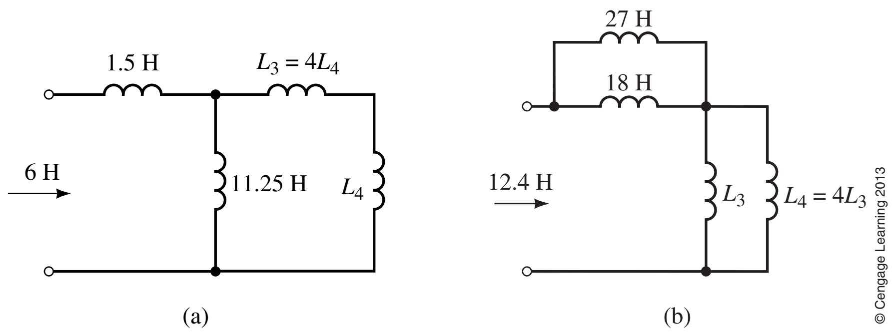


FIGURE 13–40

27. A 6-H and a 4-H inductance are connected in parallel. After a third inductance is added, $L_T = 4$ H. What is the value of the third inductance and how was it connected?
28. Inductances of 2 H, 4 H, and 9 H are connected in a circuit. If $L_T = 3.6$ H, how are the inductors connected?
29. Inductances of 8 H, 12 H, and 1.2 H are connected in a circuit. If $L_T = 6$ H, how are the inductors connected?
30. For inductors in parallel (Figure 13–41), the same voltage appears across each. Thus, $v = L_1 di_1/dt$, $v = L_2 di_2/dt$, and so on. Apply KCL and show that $1/L_T = 1/L_1 + 1/L_2 + \cdots + 1/L_N$.

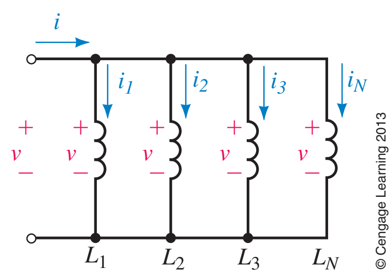


FIGURE 13–41

31. By combining elements, reduce each of the circuits of Figure 13–42 to their simplest form.

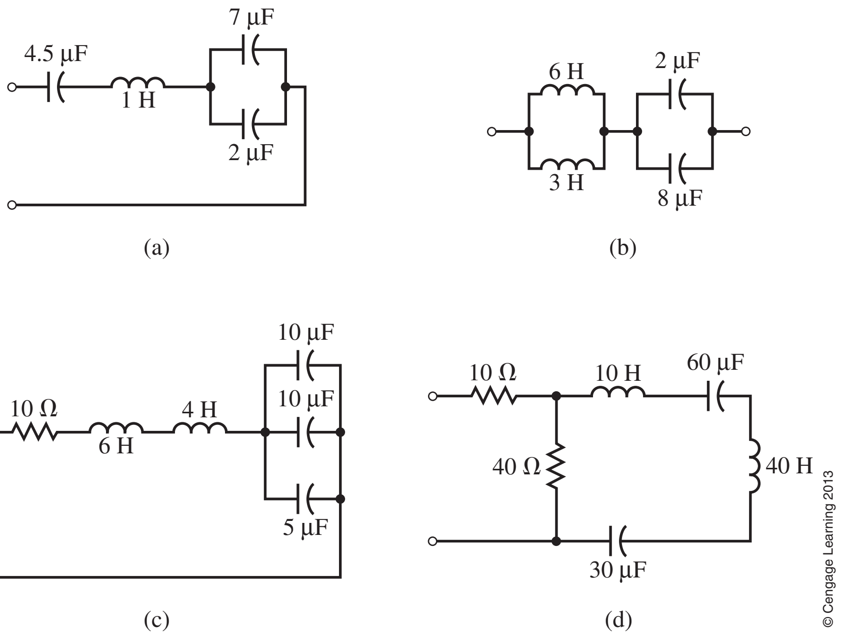


FIGURE 13–42

13.7 Inductance and Steady State dc

32. For each of the circuits of Figure 13–43, the voltages and currents have reached their final (steady state) values. Solve for the quantities indicated.

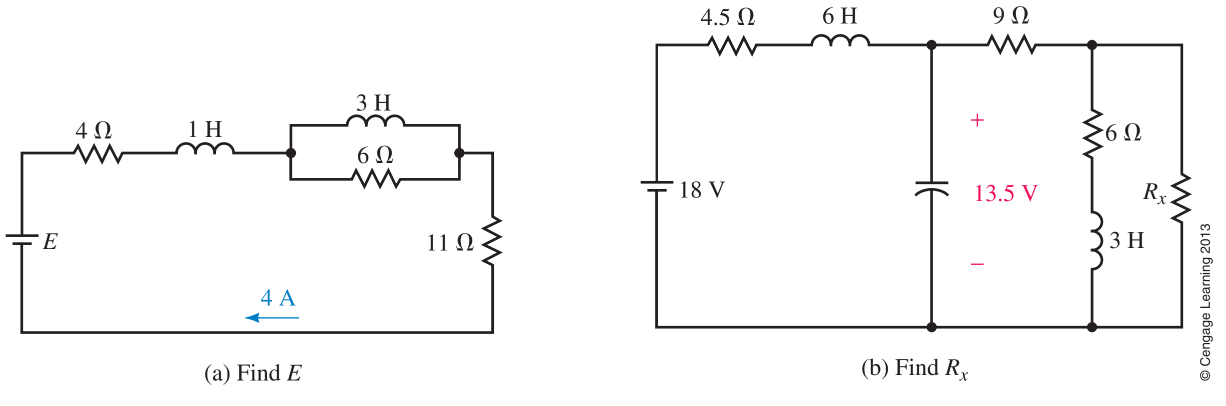


FIGURE 13–43

13.8 Energy Stored by an Inductance

33. Find the energy stored in the inductor of Figure 13–44.

34. In Figure 13–45, $L_1 = 2L_2$. The total energy stored is $W_T = 75$ J. Find L_1 and L_2.

13.9 Inductor Troubleshooting Hints

35. In Figure 13–46, an inductance meter measures 7 H. What is the likely fault?

36. Referring to Figure 13–47, an inductance meter measures $L_T = 8$ mH. What is the likely fault?

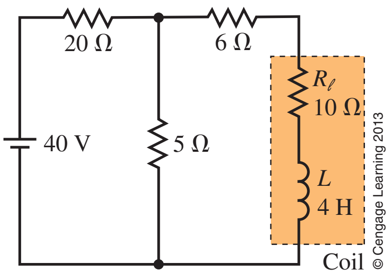


FIGURE 13–44

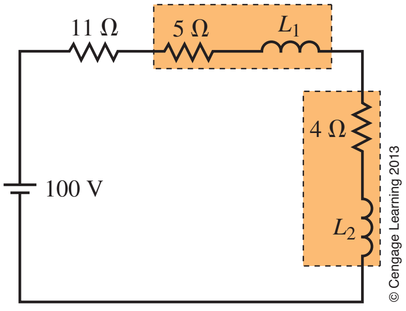


FIGURE 13–45

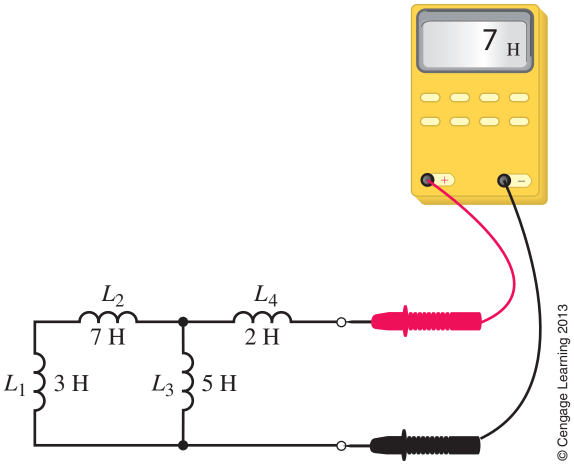


FIGURE 13–46

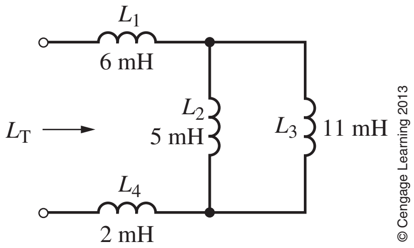


FIGURE 13–47

ANSWERS TO IN-PROCESS LEARNING CHECKS

IN-PROCESS LEARNING CHECK 1

1. Both a and b. Current cannot change instantaneously.
2. 600 Wb turns
3. 30 V
4. −300 V
5. $5e^{-t}(1 - t)$ V

IN-PROCESS LEARNING CHECK 2

1. 28 mH
2. 9 times

IN-PROCESS LEARNING CHECK 3

1. 87.5 V
2. Rate of change of current is the same; 20 V
3. $20t^2 e^{-2t}(3 - 2t)$ V
4. $250 \cos 1000t$

KEY TERMS

Continuity of Current for Inductance

De-Energizing Transient

Inductive Kick

Inductive Transients

Initial Condition Circuit

OUTLINE

OBJECTIVES

After studying this chapter you will be able to

- explain why transients occur in *RL* circuits,
- explain why an inductor with zero initial conditions looks like an open circuit when first energized,
- compute time constants for *RL* circuits,
- compute voltage and current transients in *RL* circuits during the current buildup phase,
- compute voltage and current transients in *RL* circuits during the current decay phase,
- explain why an inductor with nonzero initial conditions looks like a current source when disturbed,
- solve moderately complex *RL* transient problems using circuit simplification techniques,
- solve *RL* transient problems using Multisim and PSpice.

14

INDUCTIVE TRANSIENTS

CHAPTER PREVIEW

In Chapter 11, you learned that transients occur in capacitive circuits because capacitor voltage cannot change instantaneously. In this chapter, you will learn that transients occur in inductive circuits because inductor current cannot change instantaneously. Although the details differ, you will find that many of the basic ideas are the same.

Inductive transients (also called *RL* transients) result when circuits containing inductance are disturbed. More so than capacitive transients, inductive transients are potentially destructive and dangerous. For example, if you break the current in an inductive circuit, a voltage spike of a few hundred or more volts may result. Such a spike can easily injure personnel or damage sensitive electronic components if proper precautions are not taken. (Current turn-on transients do not create such spikes.)

In this chapter, you study basic *RL* transients. You look at transients during current buildup and decay and learn how to calculate the voltages and currents that result. In later studies, you will learn how to both control and safeguard circuits from the effects of those transients that might be potentially destructive. ■

Putting It in Perspective

Inductance, the Dual of Capacitance

INDUCTANCE IS THE DUAL of capacitance. This means that the effect that inductance has on circuit operation is identical with that of capacitance if you interchange the term current for voltage, open circuit for short circuit, and so on. For example, for simple dc transients, current in an *RL* circuit has the same form as voltage in an *RC* circuit: they both rise to their final value in an exponential fashion according to $1 - e^{-t/\tau}$. Similarly, voltage across inductance decays in the same manner as current through capacitance, that is, according to $e^{-t/\tau}$. In fact, as you will see, there is complete duality between all the equations that describe transient voltage and current behavior in capacitive and inductive circuits.

Duality applies to steady state and initial condition representations as well. To steady state dc, for example, a capacitor looks like an open circuit, while an inductor looks like a short circuit. Similarly, the dual of a capacitor that looks like a short circuit at the instant of switching is an inductor that looks like an open circuit. Finally, the dual of a capacitor that has an initial condition of V_0 volts is an inductance with an initial condition of I_0 amps.

The principle of duality is helpful in circuit analysis as it lets you transfer the principles and concepts learned in one area directly into another. You will find, for example, that many of the ideas you learned in Chapter 11 reappear here in their dual form. ■

14.1 Introduction

As you saw in Chapter 11, when a circuit containing capacitance is disturbed, voltages and currents do not change to their new values immediately, but instead pass through a transitional phase as the circuit capacitance charges or discharges. The voltages and currents during this transitional interval are called transients. In a dual fashion, transients occur when circuits containing inductances are disturbed. In this case, however, transients occur because current in inductance cannot change instantaneously.

To get at the idea, consider Figure 14–1. In (a), we see a purely resistive circuit. At the instant the switch is closed, current jumps from 0 to *E*/*R* as required by Ohm's law. Thus, no transient (i.e., transitional phase) occurs because current reaches its final value immediately. Now consider (b). Here, we have added inductance. At the instant the switch is closed, a counter EMF appears across the inductance. This voltage attempts to stop the current from changing and consequently slows its rise. Current thus does not jump to *E*/*R* immediately as in (a), but instead climbs gradually and smoothly as in (b). The larger the inductance, the longer the transition takes.

NOTES...

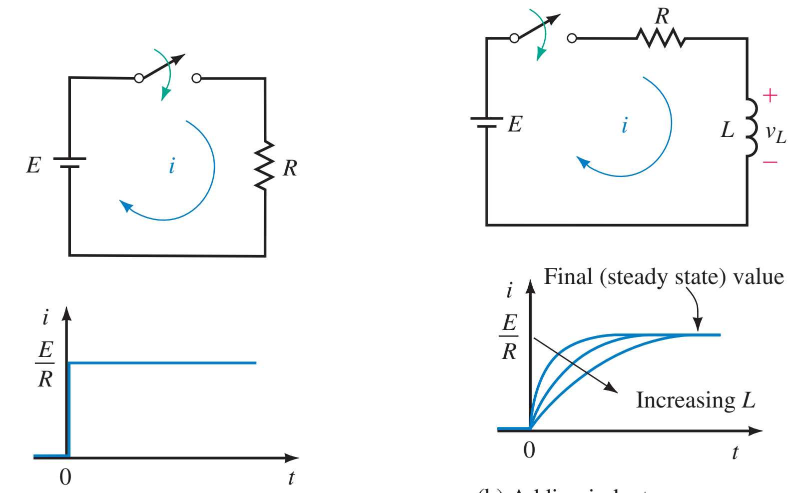
The continuity statement for inductor current has a sound mathematical basis. Recall, induced voltage is proportional to the rate of change of current. In calculus notation,

$$v_L = L\frac{di}{dt}$$

This means that the faster the current changes, the larger the induced voltage. If inductor current could change from one value to another instantaneously as in Figure 14–1(a), its rate of change (i.e., *di*/*dt*) would be infinite and hence the induced voltage would be infinite. But infinite voltage is not possible. Thus, we conclude that inductor current cannot change instantaneously.

Continuity of Current

As Figure 14–1(b) illustrates, *current through an inductance cannot change instantaneously, that is, it cannot jump abruptly from one value to another, but must be continuous at all values of time.* This observation is known as the statement of **continuity of current for inductance** (see Notes). You will find this statement of great value when analyzing circuits containing inductance. We will use it numerous times in what follows.



(a) No transient occurs in a purely resistive circuit

(b) Adding inductance causes a transient to appear. R is held constant here.

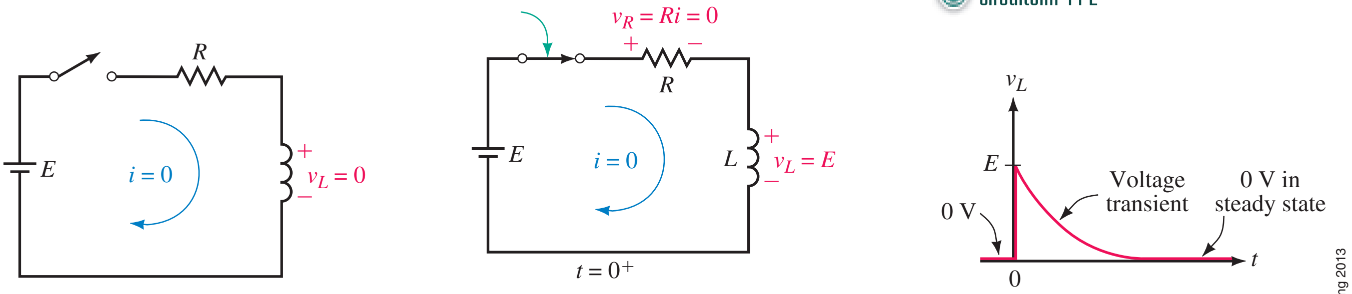
CircuitSim 14-1

FIGURE 14–1 Transient due to inductance. Adding inductance to a resistive circuit as in (b) slows the current rise and fall, thus creating a transient.

Inductor Voltage

Now consider inductor voltage. With the switch open as in Figure 14–2(a), the current in the circuit and voltage across L are both zero. Now close the switch. Immediately after the switch is closed, the current is still zero (since it cannot change instantaneously). Since $v_R = Ri$, the voltage across R is also zero and thus the full source voltage appears across L as shown in (b). The inductor voltage therefore jumps from 0 V just before the switch is closed to E volts just after. It then decays to zero, since, as we saw in Chapter 13, the voltage across inductance is zero for steady state dc. This is indicated in (c).

CircuitSim 14-2



(a) Circuit before switch is closed. Current $i = 0$

(b) Circuit just after the switch has been closed. Current is still equal to zero. Thus, $v_L = E$.

(c) Voltage across L

FIGURE 14–2 Voltage across L.

Open-Circuit Equivalent of an Inductance

Consider again Figure 14–2(b). Note that just after the switch is closed, the inductor has voltage across it but no current through it. It therefore momentarily looks like an open circuit. This is indicated in Figure 14–3. This observation is true in general, that is, *an inductor with zero initial current looks like an open circuit at the instant of switching.* (In Section 14.3, we extend this statement to include inductors with nonzero initial currents.)

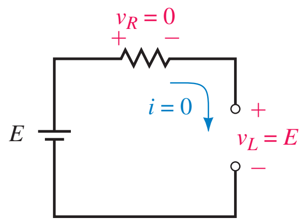


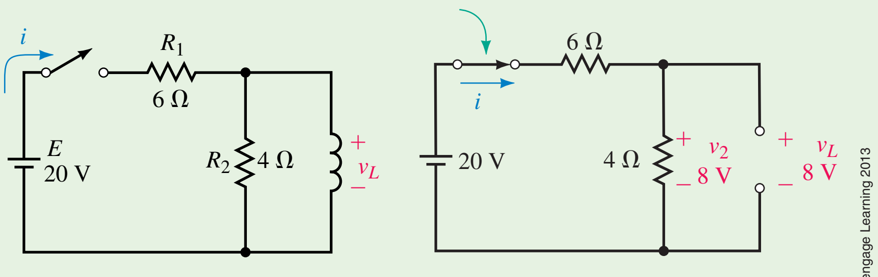
FIGURE 14–3 Inductor with zero initial current looks like an open circuit at the instant the switch is closed.

Initial Condition Circuits

Voltages and currents in circuits immediately after switching must sometimes be calculated. These can be determined with the aid of the open-circuit equivalent. By replacing inductances with open circuits, you can see what a circuit looks like just after switching. Such a circuit is called an **initial condition circuit.**

EXAMPLE 14–1

A coil and two resistors are connected to a 20-V source as in Figure 14–4(a). Determine source current i and inductor voltage v_L at the instant the switch is closed.



(a) Original circuit (b) Initial condition network

FIGURE 14–4

Solution Replace the inductance with an open circuit. This yields the network shown in (b). Thus, $i = E/R_T = 20\text{ V}/10\ \Omega = 2\text{ A}$, and the voltage across R_2 is $v_2 = (2\text{ A})(4\ \Omega) = 8\text{ V}$. Since $v_L = v_2$, $v_L = 8$ volts as well.

CircuitSim 14-3

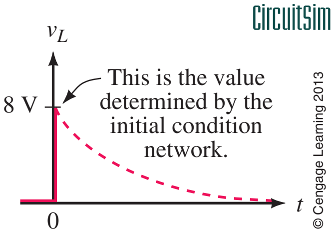


FIGURE 14–5 The initial condition network yields only the value at $t = 0^+$ s.

Initial condition circuits yield voltages and currents only at the instant of switching, that is, at $t = 0^+$ s. Thus, the value of 8 V calculated in Example 14–1 is only a momentary value as illustrated in Figure 14–5. Sometimes such an initial value is all that you need. In other cases, you need the complete solution. This is considered next, in Section 14.2.

PRACTICE PROBLEMS 1

Determine all voltages and currents in the circuit of Figure 14–6 immediately after the switch is closed and in steady state.

CircuitSim 14-4

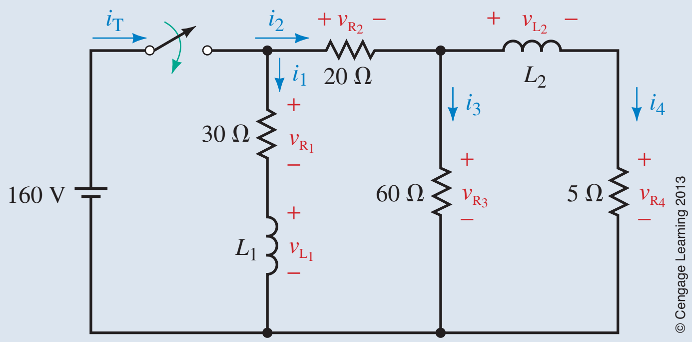


FIGURE 14–6

Answers
Initial: $v_{R_1} = 0$ V; $v_{R_2} = 40$ V; $v_{R_3} = 120$ V; $v_{R_4} = 0$ V; $v_{L_1} = 160$ V; $v_{L_2} = 120$ V; $i_T = 2$ A; $i_1 = 0$ A; $i_2 = 2$ A; $i_3 = 2$ A; $i_4 = 0$ A

Steady State: $v_{R_1} = 160$ V; $v_{R_2} = 130$ V; $v_{R_3} = v_{R_4} = 30$ V; $v_{L_1} = v_{L_2} = 0$ V; $i_T = 11.83$ A; $i_1 = 5.33$ A; $i_2 = 6.5$ A; $i_3 = 0.5$ A; $i_4 = 6.0$ A

14.2 Current Buildup Transients

Current

We will now develop equations to describe voltages and current during energization. Let us start with current. Consider Figure 14–7. Kirchhoff's voltage law (KVL) yields

$$v_L + v_R = E \quad \textbf{(14–1)}$$

Substituting $v_L = L\,di/dt$ and $v_R = Ri$ into Equation 14–1 yields

$$L\frac{di}{dt} + Ri = E \quad \textbf{(14–2)}$$

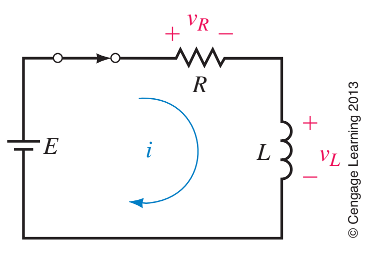


FIGURE 14–7 KVL yields $v_L + v_R = E$.

Equation 14–2 can be solved using basic calculus in a manner similar to what we did for *RC* circuits in Chapter 11. The result is

$$i = \frac{E}{R}(1 - e^{-Rt/L}) \quad \text{(A)} \quad \textbf{(14–3)}$$

where R is in ohms, L is in henries, and t is in seconds. Equation 14–3 describes current buildup. Values of current at any point in time can be found by direct substitution as we illustrate next. Note that E/R is the final (steady state) current since the inductor looks like a short circuit to steady state dc (recall Section 13.7).

EXAMPLE 14–2

For the circuit of Figure 14–7, suppose $E = 50$ V, $R = 10\ \Omega$, and $L = 2$ H:

a. Determine the expression for *i*.
b. Compute and tabulate values of *i* at $t = 0^+$, 0.2, 0.4, 0.6, 0.8, and 1.0 s.
c. Using these values, plot the current.
d. What is the steady state current?

Solution

a. Substituting the values into Equation 14–3 yields

$$i = \frac{E}{R}(1 - e^{-Rt/L}) = \frac{50\text{ V}}{10\ \Omega}(1 - e^{-10t/2}) = 5(1 - e^{-5t}) \text{ amps}$$

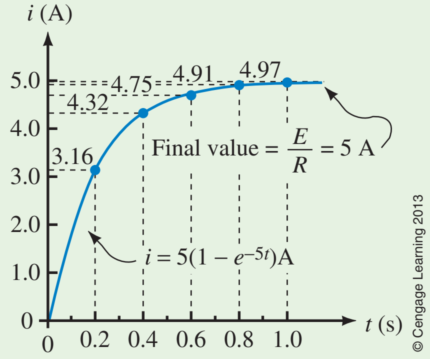
TABLE 14–1

Time	Current
0	0
0.2	3.16
0.4	4.32
0.6	4.75
0.8	4.91
1.0	4.97

b. At $t = 0^+$ s, $i = 5(1 - e^{-5t}) = 5(1 - e^0) = 5(1 - 1) = 0$ A.
At $t = 0.2$ s, $i = 5(1 - e^{-5(0.2)}) = 5(1 - e^{-1}) = 3.16$ A.
At $t = 0.4$ s, $i = 5(1 - e^{-5(04)}) = 5(1 - e^{-2}) = 4.32$ A.
Continuing in this manner, you get Table 14–1.

c. Values are plotted in Figure 14–8. Note that this curve looks exactly like the curves we determined intuitively in Figure 14–1(b).

d. Steady state current is $E/R = 50\text{ V}/10\ \Omega = 5$ A. This agrees with the current plot of Figure 14–8.



CircuitSim 14-5

FIGURE 14–8 Current buildup transient.

Circuit Voltages

With i known, circuit voltages can be determined. Consider voltage v_R. Since $v_R = Ri$, when you multiply R times Equation 14–3, you get

$$v_R = E(1 - e^{-Rt/L}) \quad \text{(V)} \tag{14–4}$$

Note that v_R has exactly the same shape as the current. Now consider v_L. Voltage v_L can be found by subtracting v_R from E as per Equation 14–1:

$$v_L = E - v_R = E - E(1 - e^{-Rt/L}) = E - E + Ee^{-Rt/L}$$

Thus,

$$v_L = Ee^{-Rt/L} \tag{14–5}$$

An examination of Equation 14–5 shows that v_L has an initial value of E at $t = 0^+$ s and then decays exponentially to zero. This agrees with our earlier observation in Figure 14–2(c).

EXAMPLE 14–3

Repeat Example 14–2 for voltage v_L.

Solution

a. From Equation 14–5,

$$v_L = Ee^{-Rt/L} = 50e^{-5t} \text{ volts}$$

b. At $t = 0^+$ s, $v_L = 50e^{-5t} = 50e^0 = 50(1) = 50$ V.
At $t = 0.2$ s, $v_L = 50e^{-5(0.2)} = 50e^{-1} = 18.4$ V.

At $t = 0.4$ s, $v_L = 50e^{-5(0.4)} = 50e^{-2} = 6.77$ V.

Continuing in this manner, you get Table 14–2.

c. The waveform is shown in Figure 14–9.

d. Steady state voltage is 0 V, as you can see in Figure 14–9.

TABLE 14–2

Time (s)	Voltage (V)
0	50.0
0.2	18.4
0.4	6.77
0.6	2.49
0.8	0.916
1.0	0.337

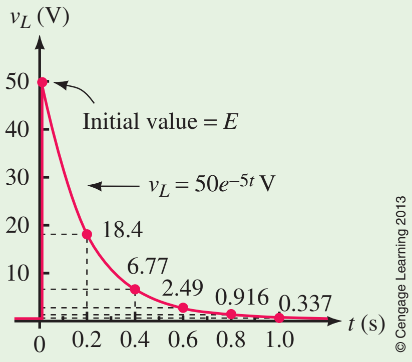


FIGURE 14–9 Inductor voltage transient.

CircuitSim 14-6

PRACTICE PROBLEMS 2

For the circuit of Figure 14–7, with $E = 80$ V, $R = 5$ kΩ, and $L = 2.5$ mH:

a. Determine expressions for i, v_L, and v_R.

b. Compute and tabulate values at $t = 0^+$, 0.5, 1.0, 1.5, 2.0, and 2.5 μs.

c. At each point in time, does $v_L + v_R = E$?

d. Plot i, v_L, and v_R using the values computed in (b).

Answers

a. $i = 16(1 - e^{-2 \times 10^6 t})$ mA; $v_L = 80e^{-2 \times 10^6 t}$ V; $v_R = 80(1 - e^{-2 \times 10^6 t})$

b.

t (μs)	v_L (V)	i_L (mA)	v_R (V)
0	80	0	0
0.5	29.4	10.1	50.6
1.0	10.8	13.8	69.2
1.5	3.98	15.2	76.0
2.0	1.47	15.7	78.5
2.5	0.539	15.9	79.5

c. Yes

d. i and v_R have the shape shown in Figure 14–8, while v_L has the shape shown in Figure 14–9, with values according to the table shown in b.

Time Constant

In Equations 14–3 to 14–5 L/R is the time constant of the circuit.

$$\tau = \frac{L}{R} \quad \text{(s)} \tag{14–6}$$

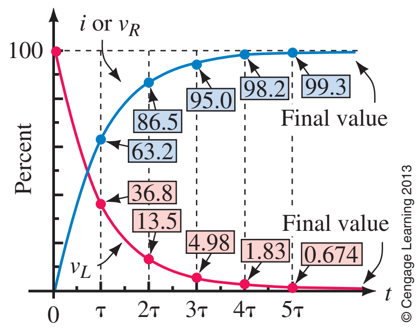


FIGURE 14–10 Universal time constant curves for the *RL* circuit.

Note that τ has units of seconds. (This is left as an exercise for the student.) Equations 14–3, 14–4, and 14–5 may now be written as

$$i = \frac{E}{R}(1 - e^{-t/\tau}) \quad \text{(A)} \tag{14–7}$$

$$v_L = Ee^{-t/\tau} \quad \text{(V)} \tag{14–8}$$

$$v_R = E(1 - e^{-t/\tau}) \quad \text{(V)} \tag{14–9}$$

Curves are plotted in Figure 14–10 versus time constant. As expected, transitions take approximately 5τ; thus, *for all practical purposes, inductive transients last five time constants.*

EXAMPLE 14–4

In a circuit where $L = 2$ mH, transients last 50 μs. What is R?

Solution Transients last five time constants. Thus, $\tau = 50\ \mu\text{s}/5 = 10\ \mu\text{s}$. Now $\tau = L/R$. Therefore, $R = L/\tau = 2\ \text{mH}/10\ \mu\text{s} = 200\ \Omega$.

EXAMPLE 14–5

For an *RL* circuit, $i = 40(1 - e^{-5t})$ A and $v_L = 100e^{-5t}$ V.

a. What are E and τ?
b. What is R?
c. Determine L.

Solution

a. From Equation 14–8, $v_L = Ee^{-t/\tau} = 100e^{-5t}$. Therefore, $E = 100$ V and $\tau = \frac{1}{5} = 0.2$ s.
b. From Equation 14–7,

$$i = \frac{E}{R}(1 - e^{-t/\tau}) = 40(1 - e^{-5t}).$$

Therefore, $E/R = 40$ A and $R = E/40\ \text{A} = 100\ \text{V}/40\ \text{A} = 2.5\ \Omega$.

c. $\tau = L/R$. Therefore, $L = R\tau = (2.5)(0.2) = 0.5$ H.

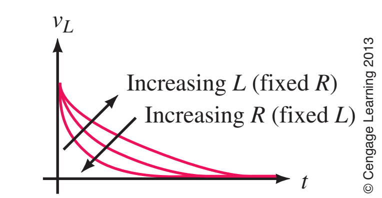


FIGURE 14–11 Effect of R and L on transient duration.

You can use the universal time constant curves to solve problems. (Be sure to convert curve percentages to a decimal value first, e.g., 63.2% to 0.632.) To illustrate, consider the problem of Examples 14–2 and 14–3. From Figure 14–10 at $t = \tau = 0.2$ s, $i = 0.632E/R$ and $v_L = 0.368E$. Thus, $i = 0.632(5\ \text{A}) = 3.16$ A and $v_L = 0.368(50\ \text{V}) = 18.4$ V as we found earlier.

The effect of inductance and resistance on transient duration is shown in Figure 14–11. The larger the inductance, the longer the transient for a given resistance. Resistance has the opposite effect: for a fixed inductance, the larger the resistance, the shorter the transient. [This is not hard to understand. As R increases, the circuit looks more and more resistive. If you get to a point where inductance is negligible compared with resistance, the circuit looks purely resistive, as in Figure 14–1(a), and virtually no transient occurs.]

IN-PROCESS LEARNING CHECK 1

(Answers are at the end of the chapter.)

1. For the circuit of Figure 14–12, the switch is closed at $t = 0$ s.
   a. Determine expressions for v_L and i.
   b. Compute v_L and i at $t = 0^+$, 10 μs, 20 μs, 30 μs, 40 μs, and 50 μs.
   c. Plot curves for v_L and i.
2. For the circuit of Figure 14–7, $E = 85$ V, $R = 50\ \Omega$, and $L = 0.5$ H. Use the universal time constant curves to determine v_L and i at $t = 20$ ms.
3. For a certain RL circuit, transients last 25 s. If $L = 10$ H and steady state current is 2 A, what is E?
4. An RL circuit has $E = 50$ V and $R = 10\ \Omega$. The switch is closed at $t = 0$ s. What is the current at the end of 1.5 time constants?

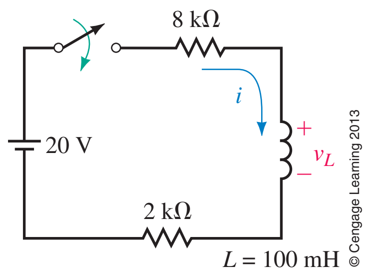


FIGURE 14–12

14.3 Interrupting Current in an Inductive Circuit

We now look at what happens when inductor current is interrupted. Consider Figure 14–13. At the instant the switch is opened, the field begins to collapse, which induces a voltage in the coil. If inductance is large and current is high, a great deal of energy is released in a very short time, creating a huge voltage that may damage equipment and create a shock hazard. (This induced voltage is referred to as an **inductive kick.**) For example, abruptly breaking the current through a large inductor (such as a motor or generator field coil) can create voltage spikes up to several thousand volts, a value large enough to draw long arcs as indicated in Figure 14–13. Even moderate-sized inductances in electronic systems can create enough voltage to cause damage if protective circuitry is not used.

CircuitSim 14-7

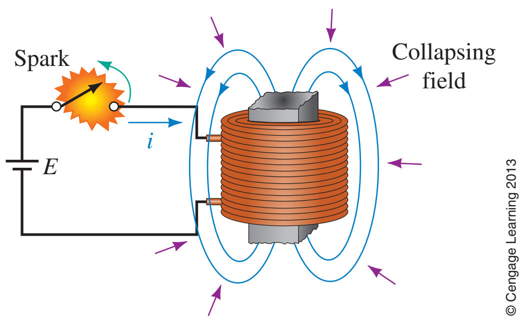


FIGURE 14–13 The sudden collapse of the magnetic field when the switch is opened causes a large induced voltage across the coil. (Several thousand volts may result.) The switch arcs over due to this voltage.

The dynamics of the switch flashover are not hard to understand. When the field collapses, the voltage across the coil rises rapidly. Part of this voltage appears across the switch. As the switch voltage rises, it quickly exceeds the breakdown strength of air, causing a flashover between its contacts. Once struck,

NOTES...

The intuitive explanation here has a sound mathematical basis. Recall, back EMF (induced voltage) across a coil is given by

$$v_L = L\frac{di}{dt} \approx L\frac{\Delta i}{\Delta t}$$

where Δi is the change in current and Δt is the time interval over which the change takes place. When you open the switch, current begins to drop immediately toward zero. Since Δi is finite, and $\Delta t \rightarrow 0$ the ratio $\Delta i/\Delta t$ is very large and thus, the voltage across L rapidly rises to a very large value, causing a flashover to occur. After the flashover, current has a path through which to decay and thus Δt, although small, no longer approaches zero. The result is a large but finite voltage spike across L.

the arc is easily maintained, as it creates ionized gases that provide a relatively low-resistance path for conduction. As the contacts spread apart, the arc elongates and eventually extinguishes as the coil energy is dissipated and coil voltage drops below that required to sustain the arc—see Notes.

There are several important points to note here:

1. Flashovers, as in Figure 14–13, are generally undesirable. However, they can be controlled through proper engineering design. (One way is to use a discharge resistor, as in the next example; another way is to use a diode or a varistor, as you will see in your electronics course.)
2. On the other hand, the large voltages created by breaking inductive currents have their uses. One is in the ignition system of automobiles, where current in the primary winding of a transformer coil is interrupted at the appropriate time by a control circuit to create the spark needed to fire the engine.
3. It is not possible for us to rigorously analyze the circuit of Figure 14–13 because the resistance of the arc changes as the switch opens. However, the main ideas can be established by studying circuits using fixed resistors. (Here, we assume the switch does not arc over.)

The Basic Ideas

We begin with the circuit of Figure 14–14. Assume the switch is closed and the circuit is in steady state. Since the inductance looks like a short circuit [Figure 14–15(a)], its pre-switching current is $i_L = 120\text{ V}/30\ \Omega = 4\text{ A}$.

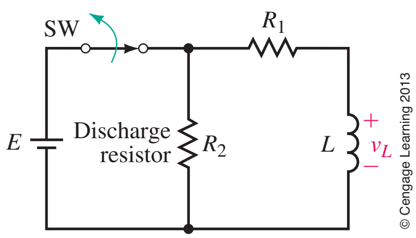
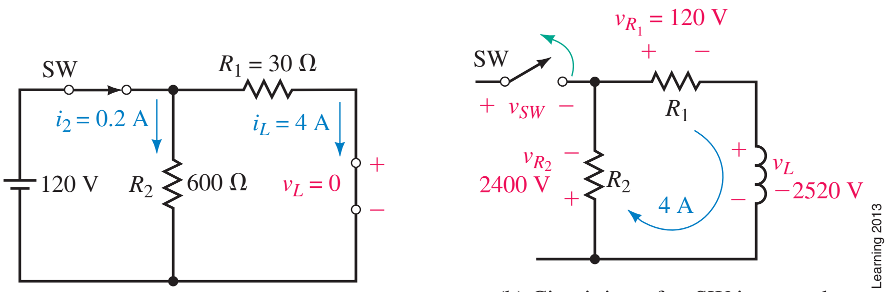


FIGURE 14–14 Discharge resistor R_2 helps limit the size of the induced voltage.



(a) Circuit just before the switch is opened

(b) Circuit just after SW is opened. Since coil voltage polarity is opposite to that shown, v_L is negative.

FIGURE 14–15 Circuit of Figure 14–14 immediately before and after the switch is opened. Coil voltage jumps from 0 V to −2520 V for this example.

CircuitSim 14-8

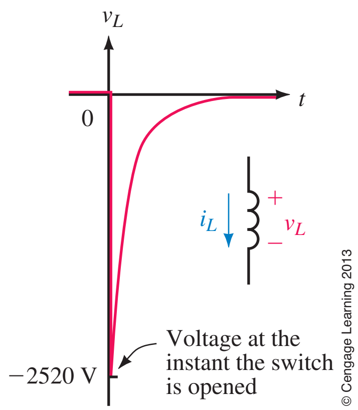


FIGURE 14–16 Voltage spike for the circuit of Figure 14–14. This voltage is more than 20 times larger than the source voltage.

Now open the switch. Just prior to opening the switch, $i_L = 4$ A; therefore, just after opening the switch, it must still be 4 A. As indicated in (b), this 4 A passes through resistances R_1 and R_2, creating voltages $v_{R_1} = 4\text{ A} \times 30\ \Omega = 120\text{ V}$ and $v_{R_2} = 4\text{ A} \times 600\ \Omega = 2400\text{ V}$ with the polarity shown. From KVL, $v_L + v_{R_1} + v_{R_2} = 0$. Therefore, at the instant the switch is opened,

$$v_L = -(v_{R_1} + v_{R_2}) = -2520 \text{ volts}$$

appears across the coil, yielding a negative voltage spike as in Figure 14–16. Note that this spike is more than 20 times larger than the source voltage. As we see in the next section, the size of this spike depends on the ratio of R_2 to R_1; the larger the ratio, the larger the voltage.

Consider again Figure 14–15. Note that current i_2 changes abruptly from 0.2 A just prior to switching to −4 A just after. This is permissible, however, since i_2 does not pass through the inductor, and only currents through inductance cannot change abruptly.

PRACTICE PROBLEMS 3

Figure 14–16 shows the voltage across the coil of Figure 14–14. Make a similar sketch for the voltage across the switch and across resistor R_2. Hint: Use KVL to find v_{SW} and v_{R_2}.

Answer
v_{SW}: With the switch closed, $v_{SW} = 0$ V; When the switch is opened, v_{SW} jumps to 2520 V, then decays to 120 V. v_{R_2}: Identical in shape to Figure 14–16 except that v_{R_2} spikes at −2400 V instead of −2520 V.

Inductor Equivalent at Switching

Figure 14–17 shows the current through L of Figure 14–15. Because the current is the same immediately after switching as it is immediately before, it is constant over the interval from $t = 0^-$ s to $t = 0^+$ s. Since this is true in general, we see that *an inductance with an initial current looks like a current source at the instant of switching*. Its value is the value of the current at switching, Figure 14–18. As indicated in Figure 14–17, this is $I_0 = i_L(0^+)$. In any given problem, you can use either representation, but depending on what you are focusing on, you might choose one over the other.

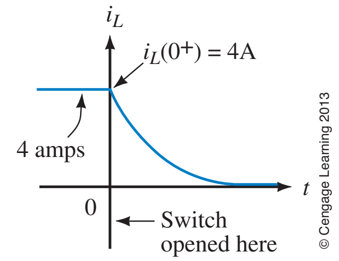
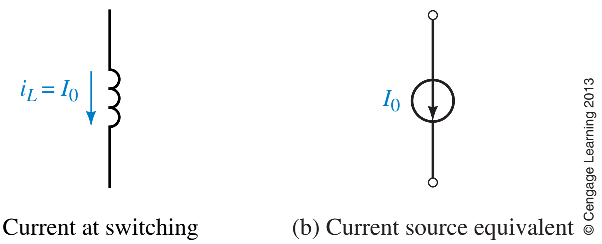


FIGURE 14–17 Inductor current for the circuit of Figure 14–15. Note that initial current $i_L(0^+)$ is often represented by the symbol I_0. In this notation, $I_0 = 4$ A.



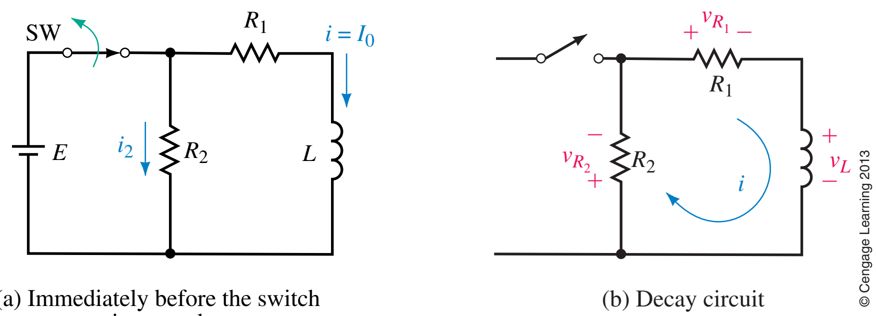
(a) Current at switching (b) Current source equivalent

FIGURE 14–18 An inductor carrying current looks like a current source at the instant of switching. You may see either representation (a) or representation (b) used in practice.

14.4 De-Energizing Transients

We now look at equations for the **de-energizing transient** voltages and currents described in the previous section.

Consider Figure 14–19(a). Let the current in the inductor before the switch is opened (the initial current) be denoted as I_0 amps. Now open the switch as in (b). KVL yields $v_L + v_{R_1} + v_{R_2} = 0$. Substituting $v_L = Ldi/dt$, $v_{R_1} = R_1 i$, and



(a) Immediately before the switch is opened (b) Decay circuit

FIGURE 14–19 Circuit for studying decay transients.

NOTES...

As an aid to help you understand and solve the problems of this chapter, note that the transient portions of inductive transients (like capacitive transients) are always exponential in form, varying according to either $e^{-t/\tau}$ or $1 - e^{-t/\tau}$, and have the general shape of either Figure 14–9 or Figure 14–8. If you remember this and understand basic principles, you won't have to rely so much on the memorization of formulas. This is what a lot of experienced people do.

$v_{R_2} = R_2 i$ yields $L di/dt + (R_1 + R_2)i = 0$. Using calculus, it can be shown that the solution is

$$i = I_0 e^{-t/\tau'} \quad \text{(A)} \tag{14–10}$$

where

$$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2} \quad \text{(s)} \tag{14–11}$$

is the time constant of the discharge circuit. If the circuit is in steady state before the switch is opened, initial current $I_0 = E/R_1$ and Equation 14–10 becomes

$$i = \frac{E}{R_1} e^{-t/\tau'} \quad \text{(A)} \tag{14–12}$$

EXAMPLE 14–6

For Figure 14–19(a), assume the current has reached steady state with the switch closed. Suppose that $E = 120$ V, $R_1 = 30\ \Omega$, $R_2 = 600\ \Omega$, and $L = 126$ mH:

a. Determine I_0.
b. Determine the decay time constant.
c. Determine the equation for the current decay.
d. Compute the current i at $t = 0.5$ ms.

CircuitSim 14-9

Solution

a. Consider Figure 14–19(a). Since the circuit is in a steady state, the inductor looks like a short circuit to dc. Thus, $I_0 = E/R_1 = 4$ A.
b. Consider Figure 14–19(b). $\tau' = L/(R_1 + R_2) = 126\text{ mH}/630\ \Omega = 0.2$ ms.
c. $i = I_0 e^{-t/\tau'} = 4e^{-t/0.2\text{ ms}}$ A.
d. At $t = 0.5$ ms, $i = 4e^{-0.5\text{ ms}/0.2\text{ ms}} = 4e^{-2.5} = 0.328$ A.

Now consider voltage v_L. Using calculus, it can be shown that

$$v_L = V_0 e^{-t/\tau'} \tag{14–13}$$

where V_0 is the voltage across L just after the switch is opened. Letting $i = I_0$ in Figure 14–19(b), you can see that $V_0 = -I_0(R_1 + R_2) = -I_0 R_T$. Thus, Equation 14–13 can also be written as

$$v_L = -I_0 R_T e^{-t/\tau'} \tag{14–14}$$

Finally, if the current has reached steady state before the switch is opened, $I_0 = E/R_1$, and Equation 14–14 becomes

$$v_L = -E\left(1 + \frac{R_2}{R_1}\right) e^{-t/\tau'} \tag{14–15}$$

Note that v_L starts at V_0 volts (which is negative) and decays to zero as shown in Figure 14–20.

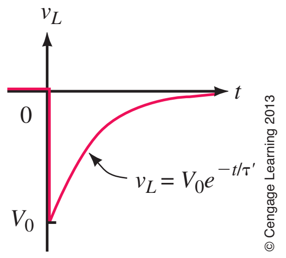


FIGURE 14–20 Inductor voltage during decay phase. V_0 is negative.

Now consider the resistor voltages. Each is the product of resistance times current (Equation 14–10). Thus,

$$v_{R_1} = R_1 I_0 e^{-t/\tau'} \tag{14–16}$$

and

$$v_{R_2} = R_2 I_0 e^{-t/\tau'} \tag{14–17}$$

CircuitSim 14-10

If current has reached steady state before switching, these become

$$v_{R_1} = E e^{-t/\tau'} \tag{14–18}$$

and

$$v_{R_2} = \frac{R_2}{R_1} E e^{-t/\tau'} \tag{14–19}$$

Substituting the values of Example 14–6 into these equations, we get for the circuit of Figure 14–19 $v_L = -2520e^{-t/0.2\text{ ms}}$ V, $v_{R_1} = 120e^{-t/0.2\text{ ms}}$ V and $v_{R_2} = 2400e^{-t/0.2\text{ ms}}$ V. These can also be written as $v_L = -2520e^{-5000t}$ V and so on, if desired.

Decay problems can also be solved using the decay portion of the universal time constant curves shown in Figure 14–10.

EXAMPLE 14–7

The circuit of Figure 14–21 is in steady state with the switch closed. Use Figure 14–10 to find i_L and v_L at $t = 2\ \tau$ after the switch is opened.

Solution Before the switch is opened, the inductor looks like a short circuit and its current is thus $E/R_1 = 120\text{ V}/40\ \Omega = 3$ A. Current just after the switch is opened will be the same. Therefore, $I_0 = 3$ A. At $t = 2\ \tau$, current will have decayed to 13.5% of this initial value. Therefore, $i_L = 0.135I_0 = 0.405$ A and $v_L = -(R_1 + R_1)i = -(60\ \Omega)(0.405\text{ A}) = -24.3$ V. [Alternately, $V_0 = -(3\text{ A})(60\ \Omega) = -180$ V. At $t = 2\ \tau$, this has decayed to 13.5%. Therefore, $v_L = 0.135(-180\text{ V}) = -24.3$ V as above.]

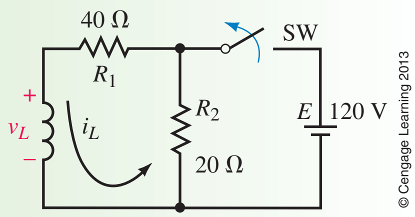


FIGURE 14–21

CircuitSim 14-11

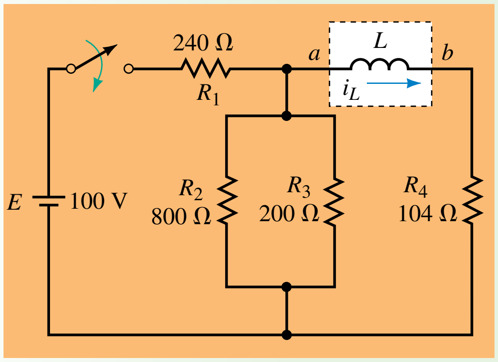
14.5 More Complex Circuits

The equations developed so far apply only to circuits of the forms of Figures 14–7 or 14–19. Fortunately, many circuits can be reduced to these forms using circuit reduction techniques such as series and parallel combinations, source conversions, Thévenin's theorem, and so on.

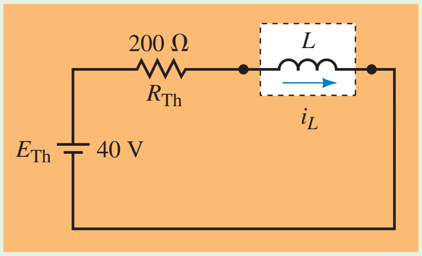
EXAMPLE 14–8

Determine the expression for i_L for the circuit of Figure 14–22(a) if $L = 5$ H. Compute current at $t = 10$ ms and 30 ms.

Solution The circuit can be reduced to its Thévenin equivalent (b) as you saw in Chapter 11 (Section 11.5). For this circuit, $\tau = L/R_{\text{Th}} = 5\text{ H}/200\ \Omega = 25$ ms. Now apply Equation 14–7. Thus,



(a) Circuit



(b) Thévenin equivalent

FIGURE 14–22

CircuitSim 14-12

$$i_L = \frac{E_{Th}}{R_{Th}}(1 - e^{-t/\tau}) = \frac{40}{200}(1 - e^{-t/25\text{ ms}}) = 0.2\,(1 - e^{-40t}) \quad \text{(A)}$$

At $t = 10$ ms, $i_L = 0.2(1 - e^{-(40)(0.010)}) = 0.2(1 - e^{-0.4}) = 65.9$ mA.

Similarly, at $t = 30$ ms, $i_L = 139.8$ mA.

EXAMPLE 14–9

For the circuit of Example 14–8, at what time does current reach 0.12 amps?

Solution

$$i_L = 0.2(1 - e^{-40t}) \quad \text{(A)}$$

Thus,

$$\begin{aligned} 0.12 &= 0.2(1 - e^{-40t}) \quad \text{(Figure 14–23)} \\ 0.6 &= 1 - e^{-40t} \\ e^{-40t} &= 0.4 \end{aligned}$$

Taking the natural log of both sides,

$$\begin{aligned} \ln e^{-40t} &= \ln 0.4 \\ -40t &= -0.916 \\ t &= 22.9 \text{ ms} \end{aligned}$$

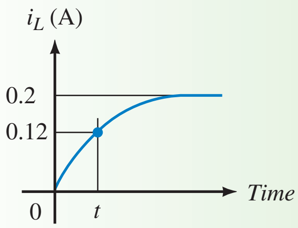


FIGURE 14–23

PRACTICE PROBLEMS 4

1. For the circuit of Figure 14–22, compute i_L and v_{R4} at $t = 50$ ms.
2. For the circuit of Figure 14–22, let $E = 120$ V, $R_1 = 600\ \Omega$, $R_2 = 3\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, $R_4 = 100\ \Omega$, and $L = 0.25$ H:
   a. Determine i_L and sketch it.
   b. Determine v_L and sketch it.

3. Let everything be as in Problem 1 except L. If $i_L = 0.12$ A at $t = 20$ ms, what is L?

Answers

1. 0.173 A, 17.99 V ≈ 18.0 V
2. a. $160(1 - e^{-2000t})$ mA; b. $80e^{-2000t}$ V. i_L climbs from 0 to 160 mA with the waveshape of Figure 14–1(b), reaching steady state in 2.5 ms. v_L looks like Figure 14–2(c). It starts at 80 V and decays to 0 V in 2.5 ms.
3. 7.21 H

CircuitSim 14-13

A Note about Time Scales

Until now, we have considered energization and de-energization phases separately. When both occur in the same problem, we must clearly define what we mean by time. One way to handle this problem (as we did with RC circuits) is to define $t = 0$ s as the beginning of the first phase and solve for voltages and currents in the usual manner, then shift the time axis to the beginning of the second phase, redefine $t = 0$ s, solve the second part, and then use continuity considerations to link the two solutions together. This is illustrated in Example 14–10. Note that only the first time scale is shown explicitly on the final waveform plot.

EXAMPLE 14–10

Refer to the circuit of Figure 14–24:

a. Close the switch at $t = 0$ and determine equations for i_L and v_L.
b. At $t = 300$ ms, open the switch and determine equations for i_L and v_L during the decay phase.
c. Determine voltage and current at $t = 100$ ms and at $t = 350$ ms.
d. Sketch i_L and v_L. Mark the points from (c) on the sketch.

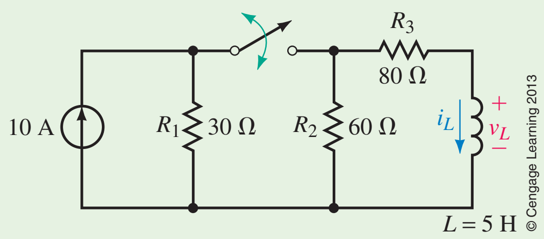


FIGURE 14–24

CircuitSim 14-14

Solution

a. Convert the circuit to the left of L to its Thévenin equivalent. As indicated in Figure 14–25(a), $R_{Th} = 60\|30 + 80 = 100\ \Omega$. From (b), $E_{Th} = V_2$, where

$$V_2 = (10\text{ A})(20\ \Omega) = 200\text{ V}$$

The Thévenin equivalent circuit is shown in Figure 14–26(a). $\tau = L/R_{Th} = 5\text{ H}/100\ \Omega = 50$ ms. Thus, during current buildup,

$$i_L = \frac{E_{Th}}{R_{Th}}(1 - e^{-t/\tau}) = \frac{200}{100}(1 - e^{-t/50\text{ ms}}) = 2\,(1 - e^{-20t}) \quad \text{A}$$

$$v_L = E_{Th}e^{-t/\tau} = 200\,e^{-20t} \quad \text{V}$$

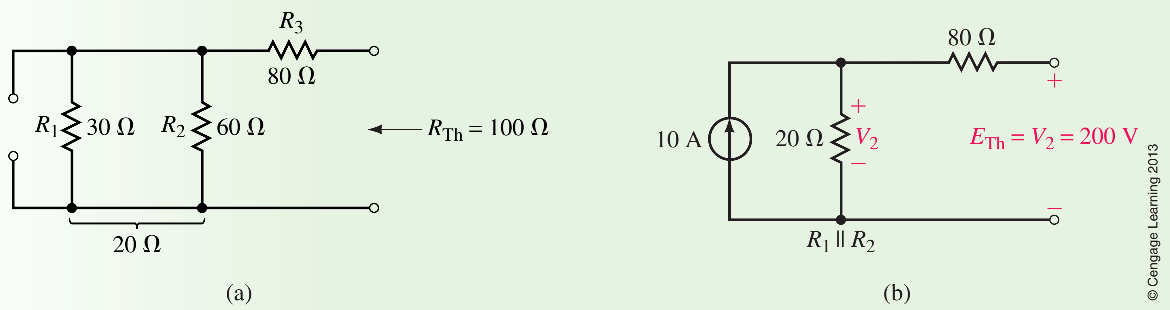


FIGURE 14–25

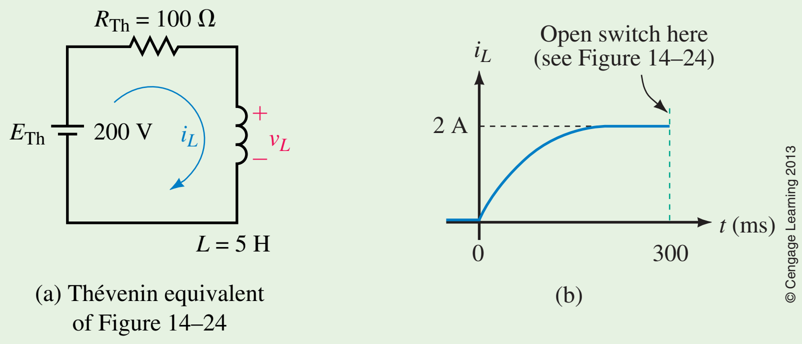


FIGURE 14–26 Circuit and current during the buildup phase.

b. Current buildup is sketched in Figure 14–26(b). Since $5\tau = 250$ ms, current is in steady state when the switch is opened at 300 ms. Thus, $I_0 = 2$ A. When the switch is opened, current decays to zero through a resistance of $60 + 80 = 140\ \Omega$ as shown in Figure 14–27. Thus, $\tau' = 5\text{ H}/140\ \Omega = 35.7$ ms. If $t = 0$ s is redefined as the instant the switch is opened, the equation for the decay is

$$i_L = I_0 e^{-t/\tau'} = 2e^{-t/35.7\text{ ms}} = 2e^{-28t} \quad \text{A}$$

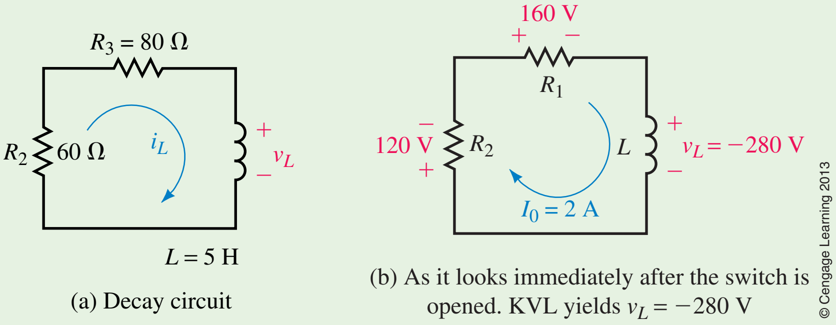


FIGURE 14–27 The circuit of Figure 14–24 as it looks during the decay phase.

Now consider voltage. As indicated in Figure 14–27(b), the voltage across L just after the switch is open is $V_0 = -280$ V. Thus,

$$v_L = V_0 e^{-t/\tau'} = -280e^{-28t} \quad \text{V}$$

c. You can use the universal time constant curves at $t = 100$ ms since 100 ms represents 2τ. At 2τ, current has reached 86.5% of its final value. Thus, $i_L = 0.865(2\text{ A}) = 1.73$ A. Voltage has fallen to 13.5%. Thus, $v_L = 0.135(200\text{ V}) = 27.0$ V. Now consider $t = 350$ ms: Note that this is 50 ms into the decay portion of the curve. However, since 50 ms is not a multiple of τ', it is difficult to use the curves. Therefore, use the equations. Thus,

$$i_L = 2\text{ A } e^{-28(50\text{ ms})} = 2\text{ A } e^{-1.4} = 0.493\text{ A}$$
$$v_L = (-280\text{ V})e^{-28(50\text{ ms})} = (-280\text{ V})e^{-1.4} = -69.0\text{ V}$$

d. The preceding points are plotted on the waveforms of Figure 14–28.

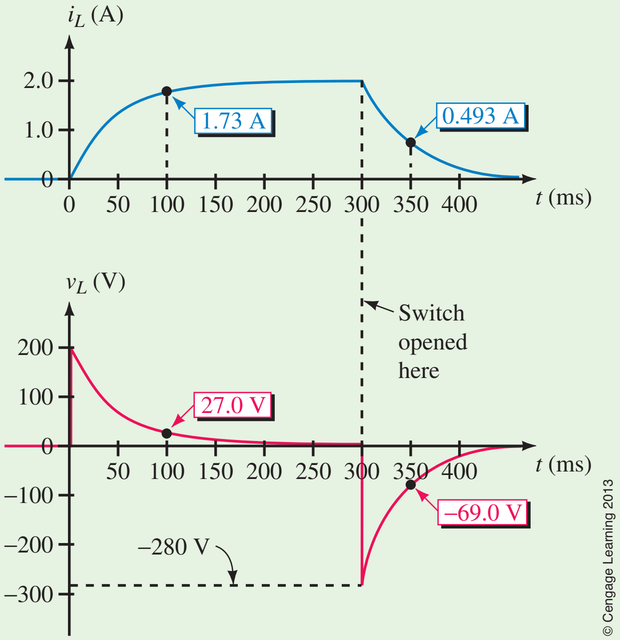
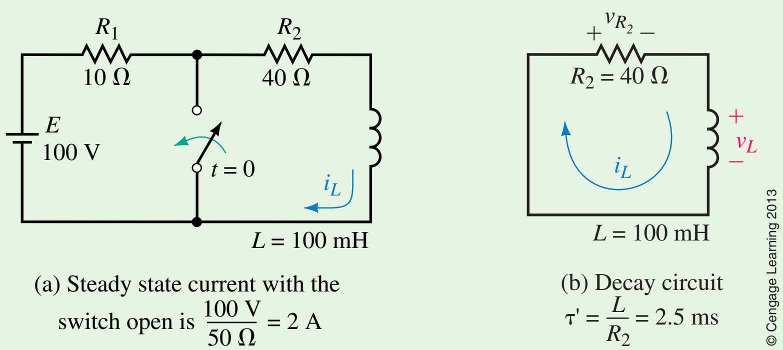


FIGURE 14–28

The basic principles that we have developed in this chapter permit us to solve problems that do not correspond exactly to the circuits of Figure 14–7 and 14–19. This is illustrated in the following example.

EXAMPLE 14–11

The circuit of Figure 14–29(a) is in steady state with the switch open. At $t = 0$ s, the switch is closed.



CircuitSim 14-15

FIGURE 14–29

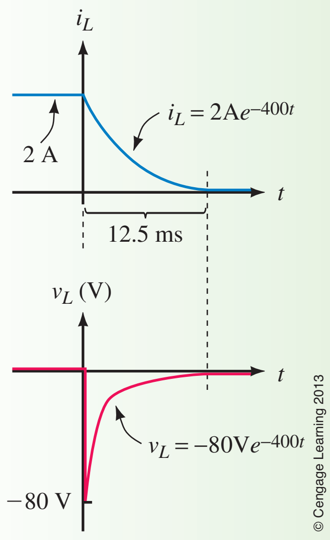


FIGURE 14–30

a. Sketch the circuit as it looks after the switch is closed and determine τ'.
b. Determine current i_L at $t = 0^+$ s.
c. Determine the expression for i_L.
d. Determine v_L at $t = 0^+$ s.
e. Determine the expression for v_L.
f. How long does the transient last?
g. Sketch i_L and v_L.

Solution

a. When you close the switch, you short out the E-R_1 branch, leaving the decay circuit of Figure 14–29(b). Thus, $\tau' = L/R_2 = 100$ mH/40 Ω = 2.5 ms.
b. In steady state with the switch open, $i_L = I_0 = 100$ V/50 Ω = 2 A. This is the current just before the switch is closed. Therefore, just after the switch is closed, i_L will still be 2 A (Figure 14–30).
c. i_L decays from 2 A to 0. From Equation 14–10, $i_L = I_0e^{-t/\tau'} = 2e^{-t/2.5\text{ ms}} = 2e^{-400t}$ A.
d. At $t = 0^+$, KVL yields $v_L = -v_{R_2} = -R_2I_0 = -(40\ \Omega)\ (2\text{A}) = -80$ V. Thus, $V_0 = -80$ V.
e. v_L decays from −80 V to 0. Thus, $v_L = V_0e^{-t/\tau'} = -80e^{-400t}$ V.
f. Transients last $5\tau' = 5(2.5\text{ ms}) = 12.5$ ms.

NOTES...

For Further Investigation

Additional examples may be found on our Web site. Go to cengagebrain.com and follow the links to *For Further Investigation* and select *Advanced Studies in Transients.*

14.6 *RL* Transients Using Computers

Multisim

As a first example, let us use Multisim to compute and plot inductor current and voltage waveforms for the circuit of Figure 14–22(a)—see Notes 1 and 2. Now build the circuit of Figure 14–31 on your screen. (To make the connections to

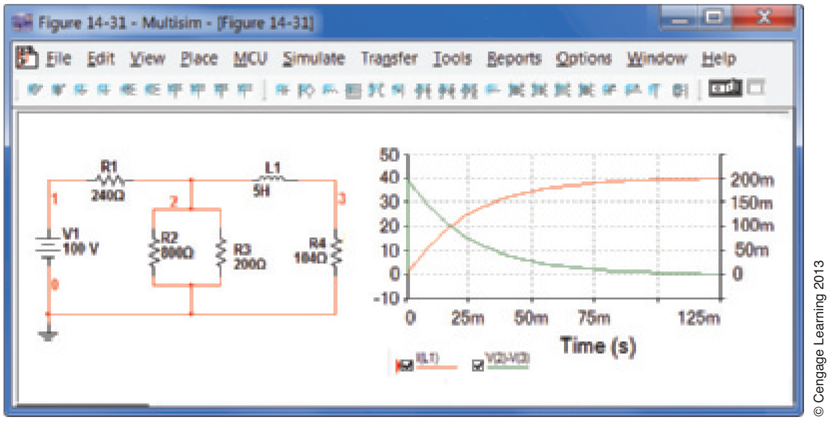


FIGURE 14–31 This is the circuit of Figure 14–22(a). Note that you do not need a switch since you initiate the transient by software.

NOTES...

Multisim

1. Because you use software to initiate the transient, you do not need the switch shown in Figure 14–22(a)—thus, it has been omitted from Figure 14–31.
2. If you are a Multisim 9 or 10 user, go to our Web site at cengagebrain.com and follow the links to *Special Instructions for Multisim 9 and 10 Users*.
3. Only basic steps are given for the following computer examples, as procedures here are similar to those of Chapter 11. If you need help, refer back to Chapter 11 or to Appendix A.
4. Multisim plots usually appear as colored traces on a black background. You can reverse the color as you did in Chapter 11.
5. Multisim scopes are grounded automatically, so an external ground is not necessary.
6. For Figure 14–32(a), to find the switch, click *Place/Components,* and from the Basic group, select *Switch/SPST,* then click *OK*.
7. To find the oscilloscope, right-click *View* and make sure *Instruments* is selected. Use your mouse to find the scope icon in the resulting tool bin on your screen.
8. If switch S1 freezes, click an empty spot on your workspace screen to unfreeze it.
9. If the ON/OFF switch is not shown, right-click *View,* then select *Simulation Switch* (or *Simulation* if you are a legacy software user).

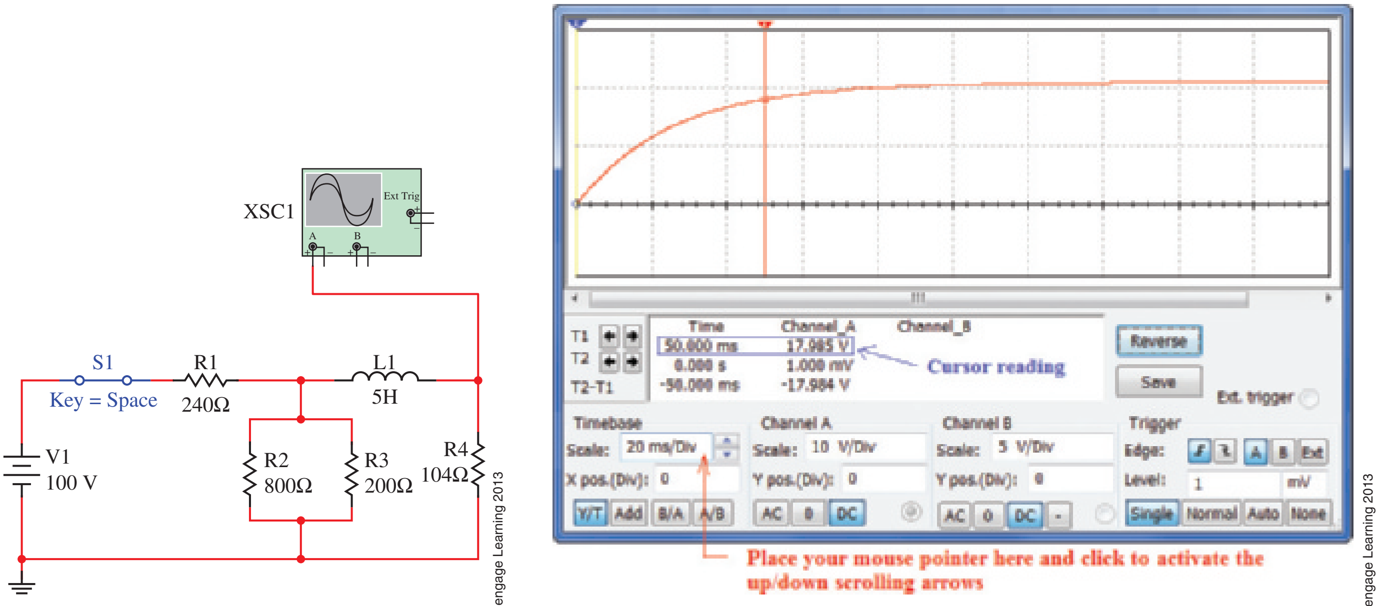
the R_2/R_3 branch, you require junction dots. Click *Place/Junction* and place the dots where you need them.) Click *Simulate/Analyses/Transient Analyses,* and in the dialog box, enter **0.125** for TSTOP and set the initial conditions to zero. Click the Output tab, highlight *I*(L1), then click *Add*. Now you need to create an expression for the voltage across the inductor. Click *Add* Expression, double-click *V*(2), press the minus key on your keyboard, double-click *V*(3), then click *OK*. This creates the expression *V*(2)-*V*(3). (You should recognize this as the voltage across inductor L1.) Click *Simulate*. Voltage and current waveforms appear. Following the procedure that you learned in Chapter 11, add a new scale for the current waveform. Your waveforms should now look like those of Figure 14–31. Click the current curve (to select it), activate the cursors (click the cursor icon), then drag the red cursor into the display area and right-click it. Select *Set Y Value,* type in **0.12,** then click *OK*. From the cursor display, you should see that the corresponding time is 22.9 ms (which agrees with our analytic solution to Example 14–9). Click the voltage curve, then use the cursor to measure voltage at $t = 20$ ms. Compare with the answer of Practice Problems 5.

PRACTICE PROBLEMS 5

Using the circuit of Figure 14–22(b), analytically determine the equation for v_L. Compute voltage at $t = 20$ ms.

Using the Multisim Oscilloscope

Create the circuit of Figure 14–32. (Use a switch—see Note 6.) Add the oscilloscope (see Notes 5 and 7). After you have wired it in, double-click the oscilloscope, and using your mouse, activate the scrolling arrows (as indicated



(a) Test circuit (b) Multisim oscilloscope screen

FIGURE 14–32 This is again the circuit of Figure 14–22(a). Here we have used a switch to initiate the transient.

in Figure 14–32), then set the time base to 20 ms/Div. Similarly, set Channel A to 10 V/Div and Level to 1 mV. Select Trigger Edge buttons as indicated. Select *Single,* then on your workspace screen, click the ON/OFF simulate switch to ON—see Note 9. Press the spacebar to close switch S1—see Note 8. Before the trace reaches the edge of the screen, click the simulate switch to OFF. Position the cursor at t = 50 ms and read voltage. You should get 17.99 V as we did in Practice Problems 4, Question 1.

PSpice

As a first example, use PSpice to solve for inductor voltage and current for the circuit of Figure 14–45, end-of-chapter Problem 21.

- Build the circuit on the screen as in Figure 14–33 (see Notes 1 and 2). Click the *New Profile* icon and name the file. In the *Simulations Settings* box, select transient analysis, set *TSTOP* to **90ms,** then click *OK*.

NOTES...

PSpice

1. Since the process here is similar to that of Chapter 11, only abbreviated instructions are given.
2. Because you use software to initiate the transient, you don't need the switch that is shown in Figure 14–45 of Problem 21.

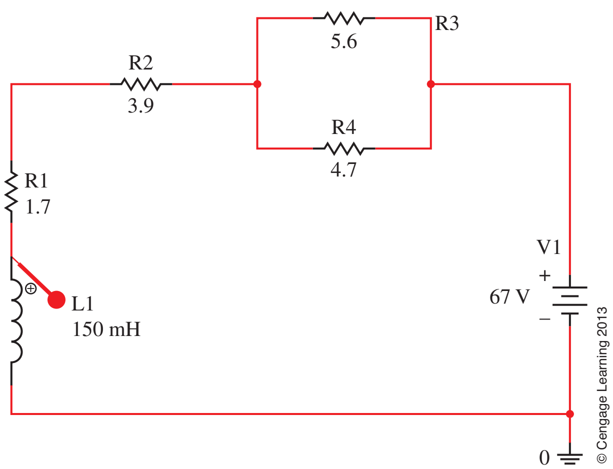


FIGURE 14–33 The voltage marker probe automatically creates the voltage trace of Figure 14–34. As is detailed in the text, you add the current trace later.

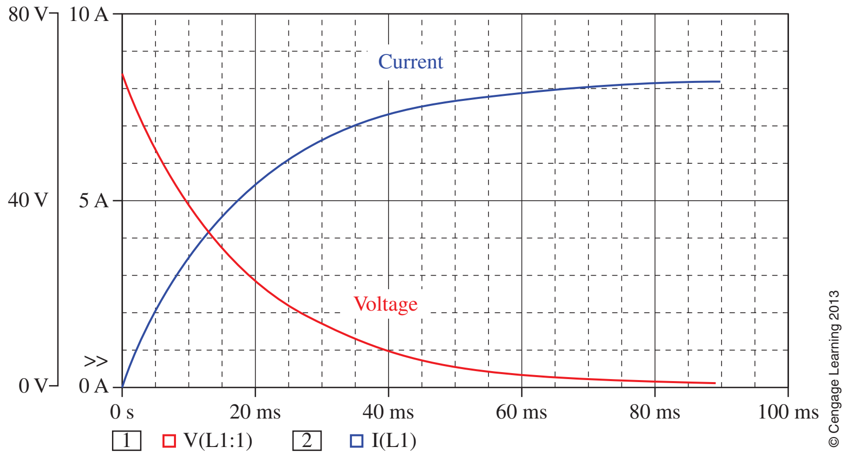


FIGURE 14–34 Inductor voltage and current for the circuit of Figure 14–33.

- Click the *Run* icon. When simulation is complete, you should have the voltage curve of Figure 14–34 on your screen.
- To add the current curve, click *Plot, Add Y Axis, Trace, Add Trace,* then double-click *I*(*L*1).
- Using a cursor, measure current at $t = 18.4$ ms. (You should get an answer of 5.19 A.)
- Measure voltage at $t = 18.4$ ms. Confirm your result using Equation 14–13.

EXAMPLE 14–12

Consider the circuit of Figure 14–24, Example 14–10. The switch is closed at $t = 0$ and opened 300 ms later. Prepare a PSpice analysis of this problem and determine v_L and i_L at $t = 100$ ms and at $t = 350$ ms.

Solution PSpice doesn't have a switch that both opens and closes. However, you can simulate such a switch by using two switches as in Figure 14–35 (see Note 1). Create the circuit on your screen using IDC for the current source. Double-click *TOPEN* of switch U2 and set it to **300ms.** Click the *New Profile* icon and name the file. In the *Simulation Settings* box, select transient analysis then type in a value of **0.6** for *TSTOP*. Run the simulation—see Note 2 and Figure 14–36—then compare to the current curve of Figure 14–28. Next, create a second Y-axis, then add the voltage trace V(L1:1). Using the cursor, read values at $t = 100$ ms, and 350 ms. You should get approximately 27 V and 1.73 A at $t = 100$ ms, and -69 V and 493 mA at $t = 350$ ms, as in Example 14–10.

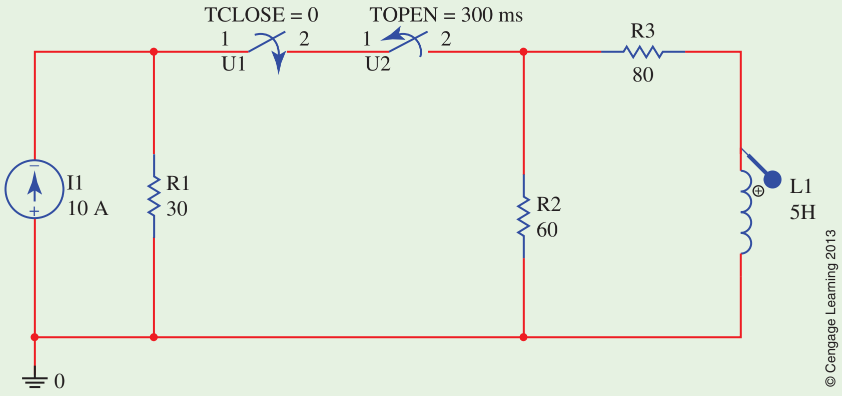


FIGURE 14–35 Simulating the circuit of Example 14–10. Two switches are used to model the closing and opening of the switch of Figure 14–24.

NOTES...

1. For the circuit of Figure 14–35, you need to use switches in your simulation circuit in order to simulate switching at two distinct times.
2. If you get an error message, "Sw_tClose used by X_U1 is undefined" (or similar), go to Appendix A for instructions on how to deal with it.

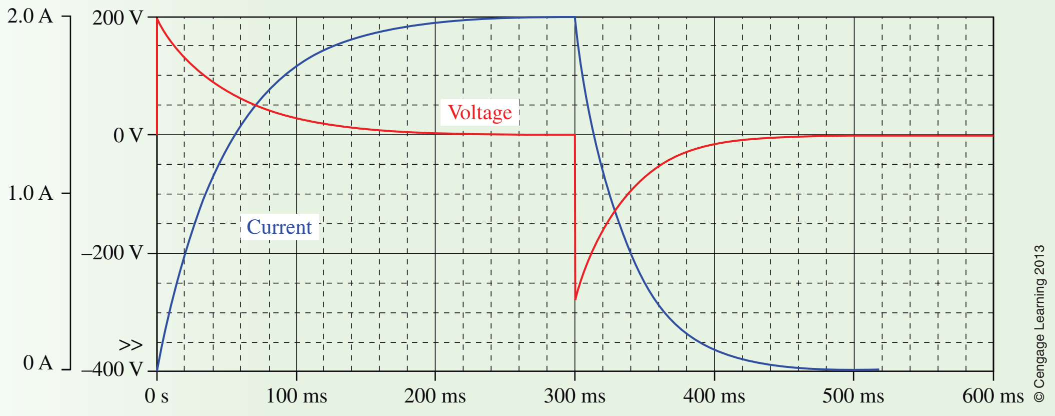
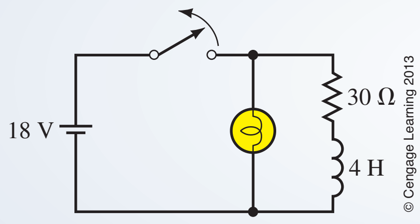


FIGURE 14–36 Inductor voltage and current for the circuit of Figure 14–35.

Putting It into Practice



The first sample of a new product that your company has designed has an indicator light that fails. (Symptom: When you turn a new unit on, the indicator light comes on as it should. However, when you turn the power off and back on, the lamp does not come on again.) You have been asked to investigate the problem and design a fix. You acquire a copy of the schematic and study the portion of the circuit where the indicator lamp is located. As shown in the accompanying figure, the lamp is used to indicate the status of the coil; the light is to be on when the coil is energized and off when it is not. Immediately, you see the problem, solder in one component and the problem is fixed. Write a short note to your supervisor outlining the nature of the problem, explaining why the lamp burned out and why your design modification fixed the problem. Note also that your modification did not result in any substantial increase in power consumption (i.e., you did not use a resistor). Note: This problem requires a diode. If you have not had an introduction to electronics, you may not be able to do this problem.

Problems

14.1 Introduction

1. a. What does an inductor carrying no current look like at the instant of switching?
   b. For each circuit of Figure 14–37, determine i_S and v_L immediately after the switch is closed.
2. Determine all voltages and currents in Figure 14–38 immediately after the switch is closed.
3. Repeat Problem 2 if L_1 is replaced with an uncharged capacitor.

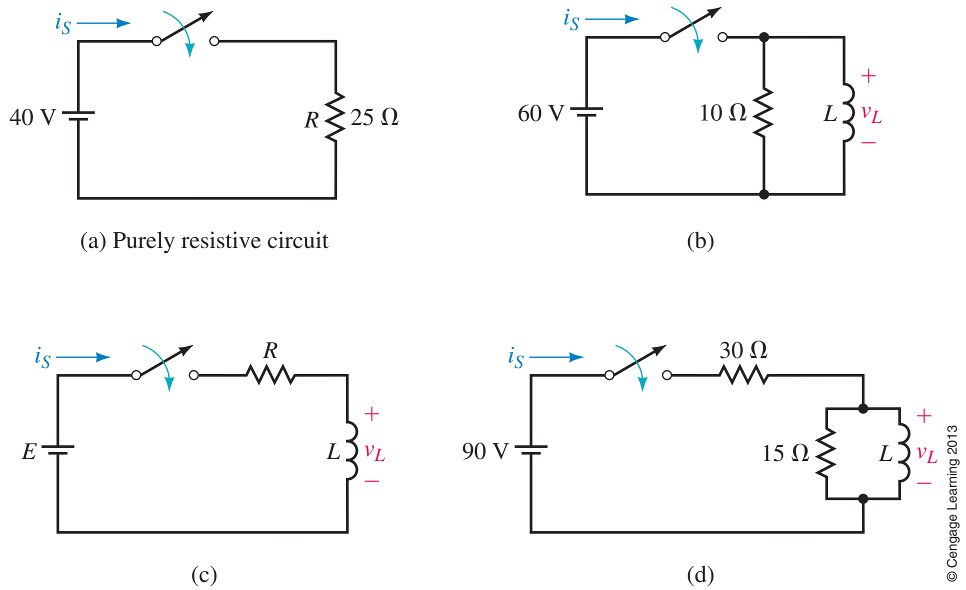


FIGURE 14–37 No value is needed for L here as it does not affect the solution.

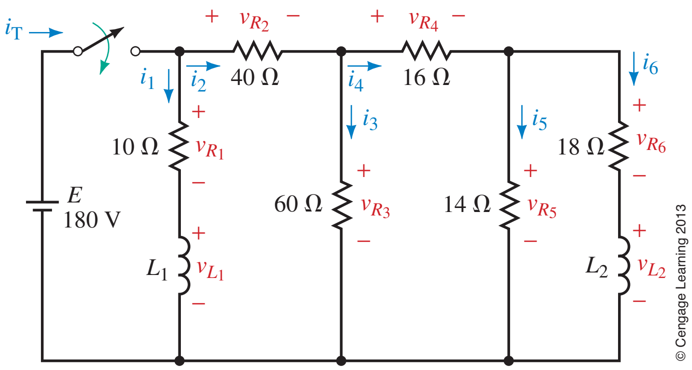


FIGURE 14–38

14.2 Current Buildup Transients

4. a. If $i_L = 8(1 - e^{-500t})$ A, what is the current at $t = 6$ ms?
   b. If $v_L = 125e^{-500t}$ V, what is the voltage v_L at $t = 5$ ms?
5. The switch of Figure 14–39 is closed at $t = 0$ s.
   a. What is the time constant of the circuit?
   b. How long is it until current reaches its steady value?
   c. Determine the equations for i_L and v_L.
   d. Compute values for i_L and v_L at intervals of one time constant from $t = 0$ to 5 τ.
   e. Sketch i_L and v_L. Label the axis in τ and in seconds.
6. Close the switch at $t = 0$ s and determine equations for i_L and v_L for the circuit of Figure 14–40. Compute i_L and v_L at $t = 1.8$ ms.
7. Repeat Problem 5 for the circuit of Figure 14–41 with $L = 4$ H.
8. For the circuit of Figure 14–39, determine inductor voltage and current at $t = 50$ ms using the universal time constant curve of Figure 14–10.

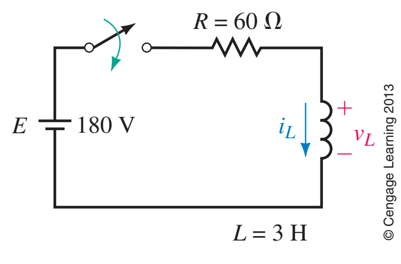


FIGURE 14–39

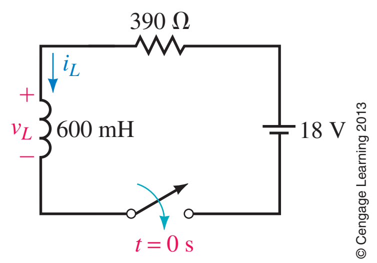


FIGURE 14–40

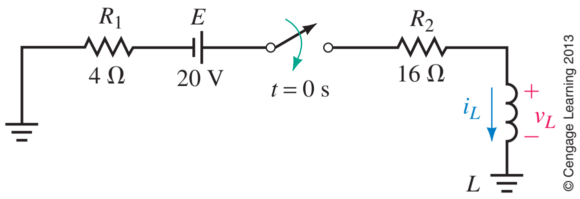


FIGURE 14–41

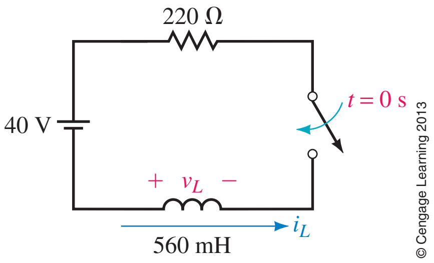


FIGURE 14–42

9. Close the switch at $t = 0$ s and determine equations for i_L and v_L for the circuit of Figure 14–42. Compute i_L and v_L at $t = 3.4$ ms.
10. Using Figure 14–10, find v_L at one time constant for the circuit of Figure 14–42.
11. For the circuit of Figure 14–1(b), the voltage across the inductance at the instant the switch is closed is 80 V, the final steady state current is 4 A, and the transient lasts 0.5 s. Determine E, R, and L.
12. For an RL circuit, $i_L = 20(1 - e^{-t/\tau})$ mA and $v_L = 40e^{-t/\tau}$ V. If the transient lasts 0.625 ms, what are E, R, and L?
13. For Figure 14–1(b), if $v_L = 40e^{-2000t}$ V and the steady state current is 10 mA, what are E, R, and L?

14.4 De-Energizing Transients

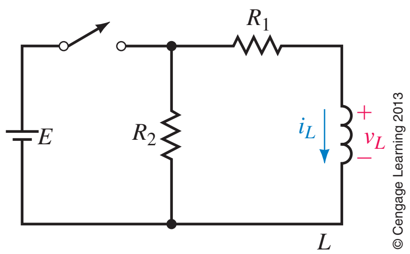


FIGURE 14–43

14. For Figure 14–43, $E = 80$ V, $R_1 = 200\ \Omega$, $R_2 = 300\ \Omega$, and $L = 0.5$ H.
    a. When the switch is closed, how long does it take for i_L to reach steady state? What is its steady state value?
    b. When the switch is opened, how long does it take for i_L to reach steady state? What is its steady state value?
    c. After the circuit has reached steady state with the switch closed, it is opened. Determine equations for i_L and v_L.
15. For Figure 14–43, $R_1 = 20\ \Omega$, $R_2 = 230\ \Omega$, and $L = 0.5$ H, and the inductor current has reached a steady value of 5 A with the switch closed. At $t = 0$ s, the switch is opened.
    a. What is the decay time constant?
    b. Determine equations for i_L and v_L.
    c. Compute values for i_L and v_L at intervals of one time constant from $t = 0$ to $5\ \tau$.
    d. Sketch i_L and v_L. Label the axis in τ and in seconds.
16. Using the values from Problem 15, determine inductor voltage and current at $t = 3\tau$ using the universal time constant curves shown in Figure 14–10.
17. Given $v_L = -2700\ \text{V}e^{-100t}$. Using the universal time constant curve, find v_L at $t = 20$ ms.
18. For Figure 14–43, the inductor voltage at the instant the switch is closed is 150 V and $i_L = 0$ A. After the circuit has reached steady state, the switch is opened. At the instant the switch is opened, $i_L = 3$ A and v_L jumps to -750 V. The decay transient lasts 5 ms. Determine E, R_1, R_2, and L.

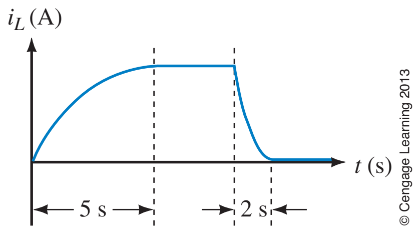


FIGURE 14–44

19. For Figure 14–43, $L = 20$ H. The current during buildup and decay is shown in Figure 14–44. Determine R_1 and R_2.
20. For Figure 14–43, when the switch is moved to energization, $i_L = 2$ A $(1 - e^{-10t})$. Now open the switch after the circuit has reached steady state and redefine $t = 0$ s as the instant the switch is opened. For this case, $v_L = -400\ \text{V}e^{-25t}$. Determine E, R_1, R_2, and L.

14.5 More Complex Circuits

21. For the coil of Figure 14–45, $R_\ell = 1.7\ \Omega$ and $L = 150$ mH. Determine coil current at $t = 18.4$ ms.

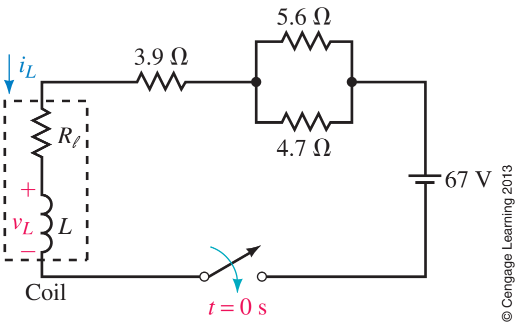


FIGURE 14–45

22. Refer to Figure 14–46:
    a. What is the energizing circuit time constant?
    b. Close the switch and determine the equation for i_L and v_L during current buildup.
    c. What is the voltage across the inductor and the current through it at $t = 20\ \mu$s?
23. For Figure 14–46, the circuit has reached steady state with the switch closed. Now open the switch.
    a. Determine the de-energizing circuit time constant.
    b. Determine the equations for i_L and v_L.
    c. Find the voltage across the inductor and current through it at $t = 17.8\ \mu$s using the equations determined previously.
24. Repeat part (c) of Problem 23 using the universal time constant curves shown in Figure 14–10.
25. a. Repeat Problem 22, parts (a) and (b) for the circuit of Figure 14–47.
    b. What are i_L and v_L at $t = 25$ ms?

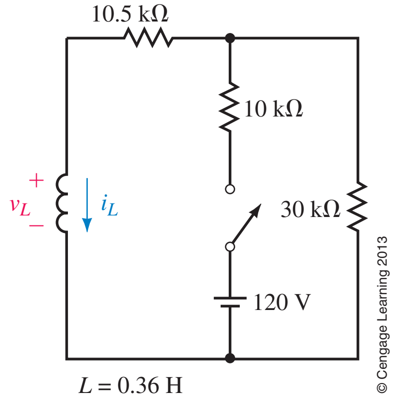


FIGURE 14–46

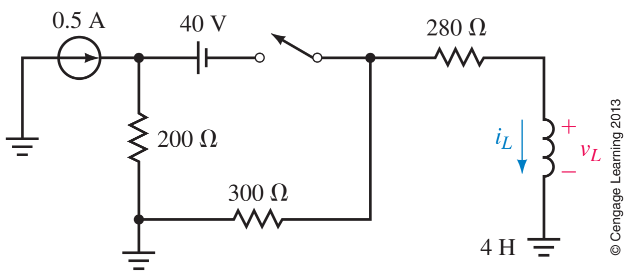


FIGURE 14–47

26. Repeat Problem 23 for the circuit of Figure 14–47, except find v_L and i_L at $t = 13.8$ ms.
27. An unknown circuit containing dc sources and resistors has an open-circuit voltage of 45 volts. When its output terminals are shorted, the short-circuit current is 0.15 A. A switch, resistor, and inductance are connected (Figure 14–48). Determine the inductor current and voltage 2.5 ms after the switch is closed.
28. The circuit of Figure 14–49 is in steady state with the switch in position 1. At $t = 0$, it is moved to position 2, where it remains for 1.0 s. It is then moved to position 3, where it remains. Sketch curves for i_L and v_L from

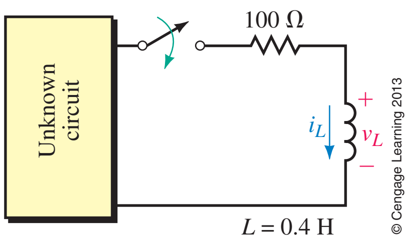


FIGURE 14–48

$t = 0^-$ until the circuit reaches steady state in position 3. Compute the inductor voltage and current at $t = 0.1$ s and at $t = 1.1$ s.

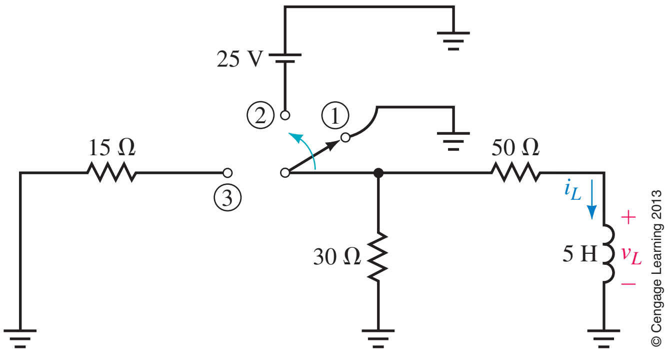


FIGURE 14–49

14.6 *RL* Transients Using Computers

Multisim PSPICE 29. The circuit of Figure 14–46 is in steady state with the switch open. At $t = 0$, it is closed and remains closed. Graph the voltage across L and find v_L at 20 μs using the cursor.

Multisim PSPICE 30. For the circuit of Figure 14–47, close the switch at $t = 0$ and find v_L at $t = 10$ ms. (For PSpice, use current source IDC.)

Multisim PSPICE 31. For Figure 14–6, let $L_1 = 30$ mH and $L_2 = 90$ mH. Close the switch at $t = 0$ and find the current in the 30 Ω resistor at $t = 2$ ms.

Multisim PSPICE 32. For Figure 14–41, let $L = 4$ H. Solve for v_L and i_L. Using the cursor, measure values at $t = 200$ ms and 500 ms. (Multisim users: You may have to create a separate vertical axis for the current waveform.)

PSPICE 33. We solved the circuit of Figure 14–22(a) by reducing it to its Thévenin equivalent. Using PSpice, analyze the circuit in its original form and plot the inductor current. Check a few points on the curve by computing values according to the solution of Example 14–8 and compare to values obtained from screen.

PSPICE 34. The circuit of Figure 14–46 is in steady state with the switch open. At $t = 0$, the switch is closed. It remains closed for 150 μs and is then opened and left open. Compute and plot i_L and v_L. With the cursor, determine values at $t = 60$ μs and at $t = 165$ μs.

ANSWERS TO IN-PROCESS LEARNING CHECKS

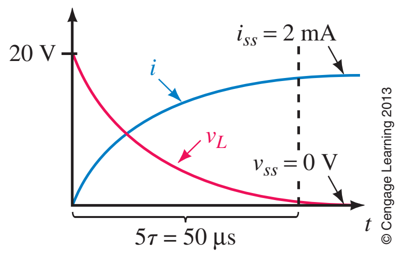
IN-PROCESS LEARNING CHECK 1

1. a. $20e^{-100\,000t}$ V; $2(1 - e^{-100\,000t})$ mA

b.

t(μs)	v_L(V)	i_L(mA)
0	20	0
10	7.36	1.26
20	2.71	1.73
30	0.996	1.90
40	0.366	1.96
50	0.135	1.99

c.



2. 11.5 V; 1.47 A
3. 4 V
4. 3.88 A