

**Department of Mechatronic Engineering**

**2019-2020 Academic Year**

**Fifth Year**

**Second Semester Examination**

**Marking Scheme of McE-52066 Sensors for Mechatronic System**

**Date: 24.3.2022 (THU)**

**Time: 9:00 to 12:00 noon**

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Ques No.	Solution	Marks
1.(a.)	i. Pulse count, pulse frequency ii. The null position iii. bandwidth iv. electrically insulating material v. the output signals	
1.(b.)	i. 1. Proximity Sensor ii. 4. Both 1 and 2 are correct iii. 2. Vary unequally depending on the core position iv. 4. All of the above v. 2. Decrease vi. 2. Displacement of a contact slider on a resistance vii. 3. Passive transducer viii. 1. Ferromagnetic  ix. 3. seeback effect x. 3. Thermocouple xi. 1. Mutual inductance xii. 1. Encoder xiii. 3. Digital  xiv. 4. Acceleration xv. 1. Shaft encoder	

Ques No.	Solution	Marks
2.(a.)	<div data-bbox="495 310 1104 682" data-label="Diagram"> </div> <p data-bbox="284 745 397 787"><u>Solution</u></p> <p data-bbox="284 808 917 903"> <math>M = 10 \text{ kg}, \quad K = 10 \text{ N/m}, \quad \text{and} \quad B = 2 \text{ N/m/s}</math>  <math>m = 5 \text{ gm}, \quad b = 0.05 \text{ N/m/s}</math> </p> <p data-bbox="284 976 1096 1018">The equation of free motion of the simple oscillator is given by</p> $M\ddot{y} + B\dot{y} + Ky = 0, \quad \dots\dots\dots (i)$ <p data-bbox="284 1144 1226 1249">where <math>y</math> denotes the displacement of the mass from the static equilibrium position. This equation is of the form</p> $\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0, \quad \dots\dots\dots (ii)$ <p data-bbox="284 1417 1112 1522">         where <math>\omega_n</math> = the undamped natural frequency of the oscillator and  <math>\zeta</math> = the damping ratio       </p> <p data-bbox="284 1585 738 1627">By direct comparison of (i) and (ii),</p> $\omega_n = \sqrt{\frac{K}{M}} \quad \text{and} \quad \zeta = \frac{B}{2\sqrt{MK}}.$ $\omega_d = \sqrt{1 - \zeta^2} \omega_n \quad \text{for } 0 < \zeta < 1.$	<p data-bbox="1347 310 1437 378">10 Marks</p> <p data-bbox="1347 1039 1437 1102">5 Marks</p> <p data-bbox="1347 1333 1437 1396">5 marks</p>

$$\omega_d = \sqrt{\left(1 - \frac{B^2}{4MK}\right) \frac{K}{M}}$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{10}{10}} = 1$$

$$\hat{\omega}_n = \sqrt{\frac{K}{M+m}} = \sqrt{\frac{10}{10+0.005}} = \sqrt{0.9995} = 0.99975$$

$$\begin{aligned} \text{Percentage error} &= \left[ \frac{\hat{\omega}_n - \omega_n}{\omega_n} \right] \times 100 \% = \left[ \frac{0.99975 - 1}{1} \right] \times 100 \% \\ &= 0.025\% \end{aligned}$$

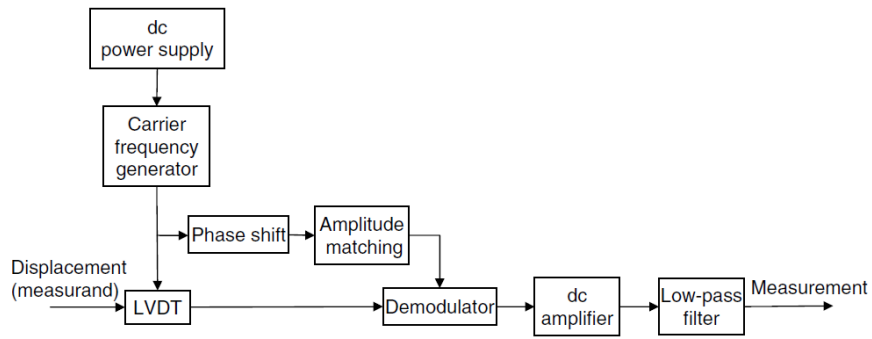
2.(b)

#### Solution

Digital signals (or digital representation of information) have several advantages in comparison with analog signals.

1. Digital signals are less susceptible to noise, disturbances, or parameter variation in instruments because data can be generated, represented, transmitted, and processed as binary words consisting of bits, which possess two identifiable states.
2. Complex signal processing with very high accuracy and speed is possible through digital means (hardware implementation is faster than software implementation).
3. High reliability in a system can be achieved by minimizing analog hardware components.
4. Large amounts of data can be stored using compact, high-density data storage methods.
5. Data can be stored or maintained for very long periods of time without any

	<p>drift or disruption by adverse environmental conditions.</p> <p>6. Fast data transmission is possible over long distances with no attenuation and with less dynamic delays, compared to analog signals.</p> <p>7. Digital signals use low voltages (e.g., 0–12 V DC) and low power.</p> <p>8. Digital devices typically have low overall cost.</p> <p>-----</p>	
3.(a)	<p><u>Solution</u></p> <p>Two methods are commonly used to interpret the crude output signal from a differential transformer: rectification and demodulation. Block diagram representations of these two procedures are given in Figure.</p> <p>In the first method (<b>rectification</b>) the ac output from the differential transformer is rectified to obtain a dc signal. This signal is amplified and then low-pass filtered to eliminate any high-frequency noise components. The amplitude of the resulting signal provides the transducer reading. In this method, phase shift in the LVDT output has to be checked separately to determine the direction of motion.</p> <p>In the second method (<b>demodulation</b>) the carrier frequency component is rejected from the output signal by comparing it with a phase-shifted and amplitude-adjusted version of the primary (reference) signal. Note that phase shifting is necessary because, as discussed earlier, the output signal is not in phase with the reference signal. The result is the modulating signal (proportional to <math>x</math>), which is subsequently amplified and filtered.</p> <div style="text-align: center;"> <pre> graph LR     PS[dc Power supply] --&gt; CFG[Carrier frequency generator]     D[Displacement measurand] --&gt; LVDT[LVDT]     CFG -- Primary excitation --&gt; LVDT     LVDT -- Secondary output --&gt; RC[Rectifier circuit]     RC --&gt; DA[dc amplifier]     DA --&gt; LPF[Low-pass filter]     LPF --&gt; M[Measurement]           </pre> <p>(a) Rectification</p> </div>	

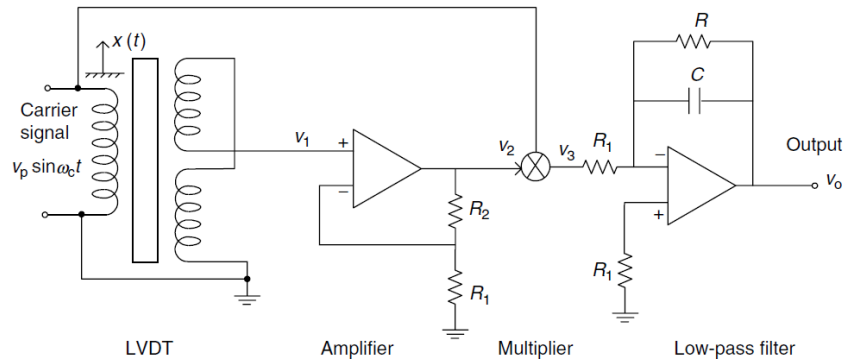


(b) Demodulation

Figure :Signal-conditioning methods for a differential transformer  
(a) Rectification and (b) Demodulation

3.(b)

### Solution



Potentials at the + and - terminals of the op-amp are nearly equal. Also, currents through these leads are nearly zero. (These are the two common assumptions used for an op-amp) Then, the current balance at node A gives,

$$\frac{V_2 - V_1}{R_2} = \frac{V_1}{R_1}$$

$$V_2 = k V_1$$

$$k = \frac{R_1 + R_2}{R_1}, \text{ amplifier gain}$$

Loss-pass filter: Since the + lead of the op-amp has approximately zero potential (ground), the voltage at point B is also approximately zero. The current balance for node B gives

$$\frac{V_3}{R_1} + \frac{V_0}{R} + C \dot{V}_0 = 0$$

$$R \frac{dV_0}{dt} + V_0 = -\frac{R}{R_1} V_3$$

$R = RC =$  filter time constant

The transfer function of the filter is,

$$\frac{V_0}{V_3} = -\frac{k_0}{(1+rs)}$$

the filter gain,

$$k_0 = \frac{R}{R_1}$$

In the frequency domain,

$$\frac{V_0}{V_3} = -\frac{k_0}{(1+rj\omega)}$$

Finally, neglecting the phase shift in the LVDT,

$$V_1 = V_p r x(t) \sin \omega_c t,$$

$$V_2 = V_p r k x(t) \sin \omega_c t,$$

$$V_3 = V_p^2 r k x(t) \sin^2 \omega_c t,$$

$$V_3 = \frac{V_p^2 r k}{2} [1 - \cos 2 \omega_c t]$$

The carrier signal will be filtered out by the low-pass filter with an appropriate cutoff frequency. Then,

$$V_0 = \frac{V_p^2 r k_0}{2} x(t) \quad \text{###}$$

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4.(a)

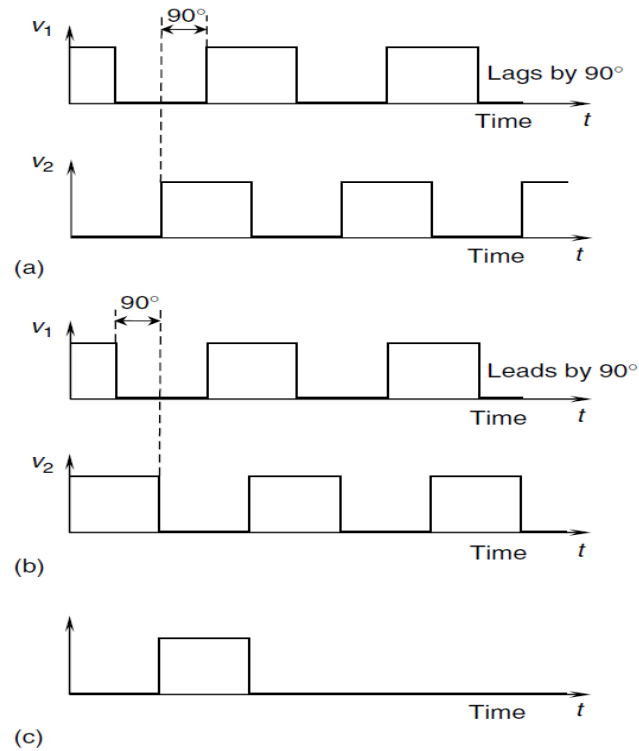


Figure: Shaped pulse signals from an incremental encoder(a) for clockwise rotation; (b) for counterclockwise rotation; (c) reference pulse signal

This logic for direction detection should be clear from Figure (.a) and Figure(b.). Another scheme can be given for direction detection. In this case, firstly detect a high level (logic high or binary 1) in signal v2 and then check whether the edge in signal v1 rises or falls during this period. As shown in Figure (a.) and Figure(b.), the following logic applies:

If rising edge in v1 when v2 is logic high ) cw rotation

If falling edge in v1 when v2 is logic high ) ccw rotation:

#### Solution

Speed = 1 rev/s

With 300 windows, we have 300 pulses/s

i. Pulse-counting method

$$\text{Counting period} = \frac{1}{20} \text{ Hz} = 0.05 \text{ s}$$

$$\text{Pulse count (in 0.1 s)} = 300 \times 0.05 = 15$$

$$\text{Percentage resolution} = 1/15 \times 100\% = 6.67\%$$

4.(b)

ii. Pulse-timing method

At 300 pulses/s,

$$\text{Pulse period} = 1/300 \text{ s} = 3.33 \times 10^{-3} \text{ s}$$

With a 20 MHz clock,

$$\text{Clock count} = 20 \times 10^6 \times 3.33 \times 10^{-3} = 66.6 \times 10^3$$

$$\begin{aligned} \text{Percentage resolution} &= \frac{1}{66.6 \times 10^3} \times 100\% \\ &= 0.0015\% \end{aligned}$$

Speed= 100 rev/s

N = 300 windows, pulses = 30,000 pulses/s

i. Pulse-counting method

$$\text{Pulse count (in 0.05s)} = 30,000 \times 0.05 = 1500$$

$$\text{Percentage resolution} = \frac{1}{1500} \times 100\% = 0.067\%$$

ii. Pulse-timing method

At 30,000 pulses/s,

$$\text{Pulse period} = \frac{1}{30000} \text{ s} = 3.33 \times 10^{-5}$$

With a 10 MHz clock,

$$\text{Clock count} = 20 \times 10^6 \times 3.33 \times 10^{-5} = 666$$

$$\text{Percentage resolution} = \frac{1}{666} \times 100\% = 0.15\%$$

Speed (rev/s)	Pulse_Counting Method(%)	Pulse_Timing Method(%)
1.0	6.67	0.0015
10	0.067	0.15

Therefore, pulse counting method is more suitable for measuring high speeds. In the pulse-timing method, the resolution degrades with speed, and hence it is more suitable for measuring low speeds.



5.(a)	<p style="text-align: center;"><u>Demodulation of the resolver</u></p> <p>As for differential transformers (i.e., LVDT and RVDT) transient displacement signals of a resolver can be extracted by demodulating its (modulated) outputs. As usual, this is accomplished by filtering out the carrier signal, thereby extracting the modulating signal. The two output signals <math>v_{o1}</math> and <math>v_{o2}</math> of a resolver are termed quadrature signals. Suppose that the carrier (primary) signal is <math>v_{ref} = v_a \sin \omega t</math>.</p> <p>The induced quadrature signals are:</p> $v_{o1} = av_a \cos \theta \sin \omega t,$ $v_{o2} = av_a \sin \theta \sin \omega t.$ <p>Multiply each quadrature signal by <math>v_{ref}</math> to get</p> $v_{m1} = v_{o1}v_{ref} = av_a^2 \cos \theta \sin^2 \omega t = \frac{1}{2}av_a^2 \cos \theta [1 - \cos 2\omega t]$ $v_{m2} = v_{o2}v_{ref} = av_a^2 \sin \theta \sin^2 \omega t = \frac{1}{2}av_a^2 \sin \theta [1 - \cos 2\omega t]$ <p>Since the carrier frequency <math>\omega</math> should be about 10 times the maximum frequency content of interest in the angular displacement <math>\theta</math>, a low-pass filter can be used with a cutoff set at <math>\omega/10</math> to remove the carrier components in <math>v_{m1}</math> and <math>v_{m2}</math>.</p> $v_{f1} = \frac{1}{2}av_a^2 \cos \theta,$ <p>This gives the demodulated outputs ..... (i)</p> $v_{f2} = \frac{1}{2}av_a^2 \sin \theta \text{ .....(ii)}$ <p>Note that Equation (i) and Equation (ii) provide both <math>\cos \theta</math> and <math>\sin \theta</math>, and hence magnitude and sign of <math>\theta</math>.</p>	
5.(b)	<p><b><u>Solution</u></b></p> <p>Sensors can be used in a control system in several ways:</p> <ol style="list-style-type: none"> <li>1. To measure the system outputs for feedback control.</li> <li>2. To measure some types of system inputs (unknown inputs, disturbances, etc.) for feedforward control.</li> <li>3. To measure output signals for system monitoring, diagnosis, evaluation, parameter adjustment, and supervisory control.</li> </ol>	

	<p>4. To measure input and output signal pairs for system testing and experimental modeling (i.e., for system identification).</p> <p>-----</p> <p><u>Solution</u></p> <p>Shaft Encoder</p> <p>5.(c) Any transducer that generates a coded (digital) reading of a measurement can be termed as an encoder. Shaft encoders are digital transducers that are used for measuring angular displacements and angular velocities. Applications of these devices include motion measurement in performance monitoring and control of robotic manipulators, machine tools, industrial processes (e.g., food processing and packaging, pulp and paper), digital data storage devices, positioning tables, satellite mirror positioning systems, vehicles, and rotating machinery such as motors, pumps, compressors, turbines, and generators. High resolution (which depends on the word size of the encoder output and the number of pulses generated per revolution of the encoder), high accuracy (particularly due to noise immunity and reliability of digital signals and superior construction), and relative ease of adoption in digital control systems (because transducer output can be read as a digital word), with associated reduction in system cost and improvement of system reliability, are some of the relative advantages of digital transducers in general and shaft encoders in particular, in comparison with their analog counterparts.</p> <p>-----</p>	

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