Car-Like Mobile Robot along X axis

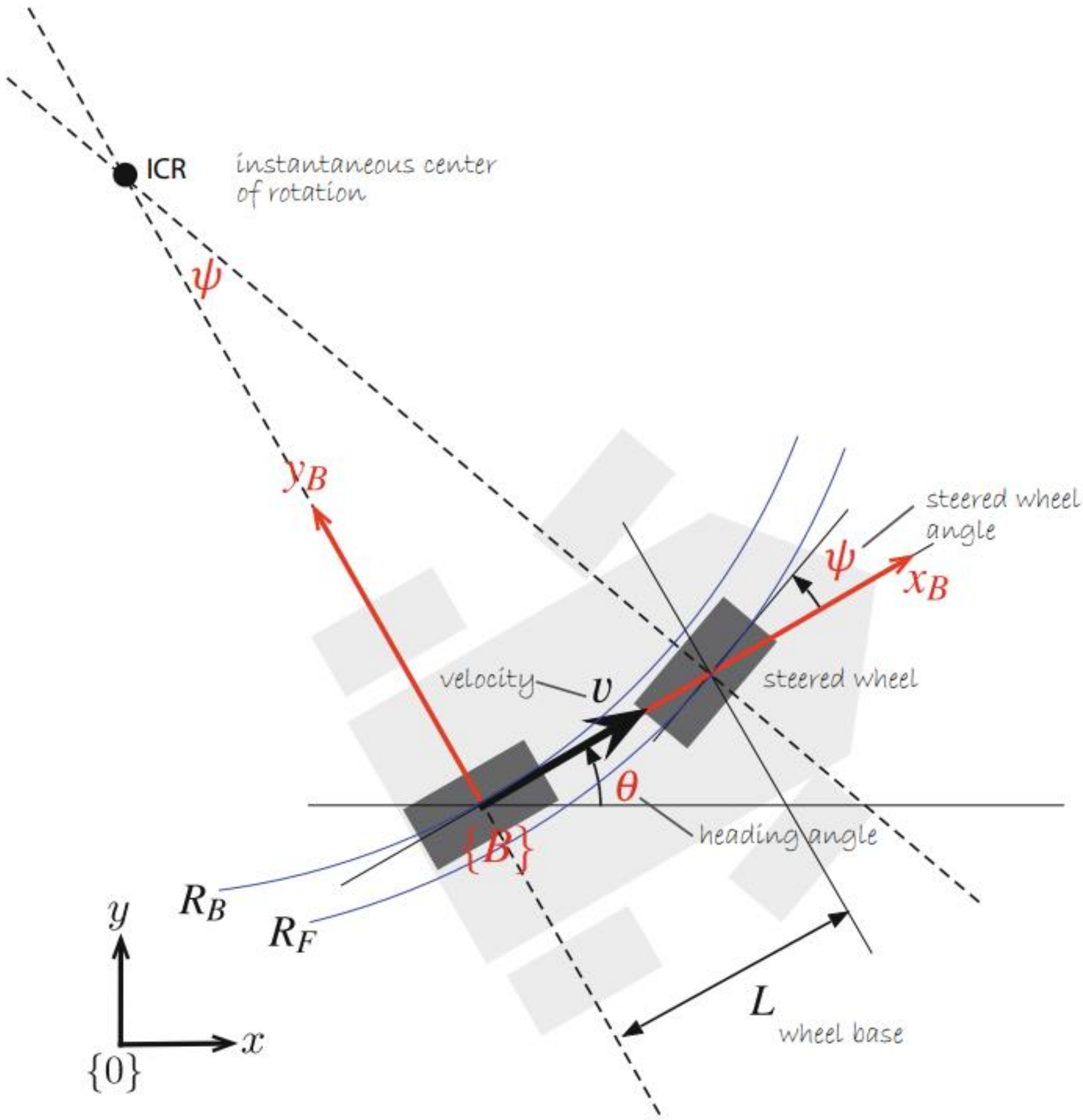


Fig. 1. Bicycle Model of Car-Like Mobile Robot

The 4-wheeled car is shown in light gray, and the equivalent two-wheeled bicycle model in dark gray. The vehicle's body frame is shown in red, and the world coordinate frame in black. The steered wheel angle is ψ with respect to the body frame, and the velocity of the back wheel, in the body's x-direction, is v. The two wheel axes are extended as dashed lines and intersect at the instantaneous center of rotation (ICR). The back and front wheels follow circular arcs around the ICR of radius R_B and R_F respectively.

The dashed lines show the direction along which the wheels cannot move, the lines of no motion, and these intersect at a point known as the Instantaneous Center of Rotation (ICR). The reference point of the vehicle, the origin of frame $\{B\}$, thus follows a circular path and its angular velocity is

$$\dot{\theta} = \frac{v}{R_B}$$

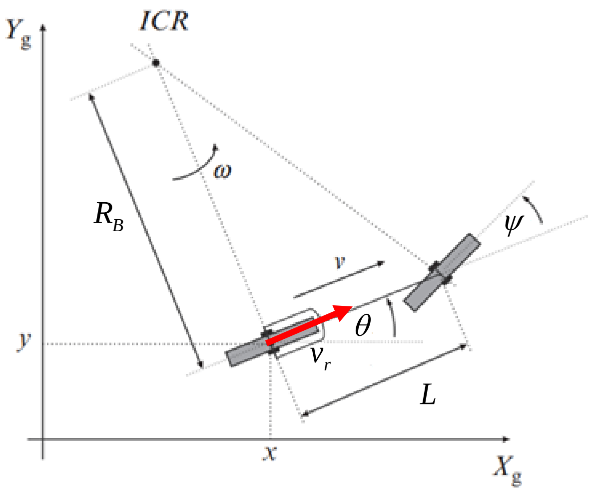
and by simple geometry the turning radius is

$$R_B = \frac{L}{\tan\psi} \qquad \left(\because \tan\psi = \frac{L}{R_B}\right)$$

where L is the length of the vehicle or the wheel base.

When a four-wheeled vehicle goes around a corner, the two steered wheels follow circular paths of slightly different radii and therefore the angles of the steered wheels ψ_L and ψ_R should be slightly different. This is achieved by the commonly used **Ackermann steering mechanism** that results in lower wear and tear on the tires. The driven wheels must rotate at different speeds on corners, so a differential gearbox is required between the motor and the driven wheels.

Front Steering and Rear Driving Model



$$R = \frac{L}{\tan\psi} \qquad \left(\because \tan\psi = \frac{L}{R}\right)$$

$$\dot{\theta} = \frac{v}{R_B} = \frac{v}{L/\tan\psi} = \frac{v}{L}\tan\psi$$

$v = v_r$ for real wheel driving

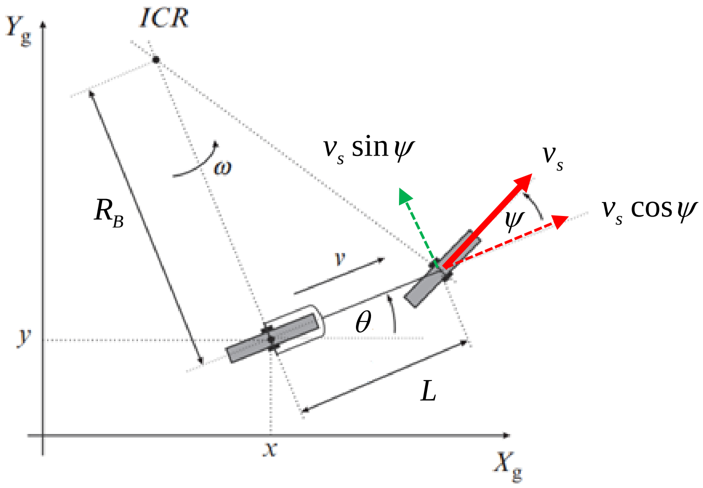
The velocity of the robot in the local frame is described by

$$v_x = \dot{x} = v = v_r$$
$$v_y = \dot{y} = 0$$
$$\dot{\theta} = \omega = \frac{v}{L}\tan\psi = \frac{v_r}{L}\tan\psi$$

The velocity of the robot in the world frame is described by

$$v_x = \dot{x} = v\cos\theta = v_r\cos\theta$$
$$v_y = \dot{y} = v\sin\theta = v_r\sin\theta$$
$$\dot{\theta} = \frac{v}{L}\tan\psi$$

Front Steering and Front Driving Model



$v = v_s \cos\psi$ for front wheel driving

The velocity of the robot in the local frame is described by

$$v_x = \dot{x} = v_s \cos\psi$$
$$v_y = \dot{y} = 0$$
$$\dot{\theta} = \omega = \frac{v_s \cos\psi}{L / \tan\psi} = \frac{v_s}{L} \cos\psi \tan\psi = \frac{v_s}{L} \sin\psi$$

The velocity of the robot in the world frame is described by

$$v_x = \dot{x} = v \cos\theta = v_s \cos\psi \cos\theta$$
$$v_y = \dot{y} = v \sin\theta = v_s \cos\psi \sin\theta$$
$$\dot{\theta} = \frac{v}{R} = \frac{v_s \cos\psi}{L / \tan\psi} = \frac{v_s}{L} \cos\psi \tan\psi = \frac{v_s}{L} \sin\psi$$