МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РОССИЙСКОЙ ФЕДЕРАЦИИ
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ДОМАШНЯЯ РАБОТА
по дисциплине
«Теория формальных языков и методы трансляции»
Вариант 2.14

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Вариант 2.14

$$L = \left\{ ((a,b)^2)^k \cdot ((b,c)^2)^m : \forall k > 0, m \geq 0, \, k, m \in \mathbb{Z} \right\} \tag{1}$$

$$\tag{2}$$

1 Определение типа языка L

Язык ф-л. (1) является регулярным. Докажем это, пользуясь замкнутостью класса регулярных языков.

1. Множества $\{a\}, \{b\}, \{c\}$ являются регулярными по определению;

2. Множества

$$\{a\} \cup \{b\} = \{a, b\} \tag{3}$$
$$\{b\} \cup \{c\} = \{b, c\} \tag{4}$$

   регулярны, так как объединение регулярных множеств — регулярное множество

3. Множества

$$S_1 = \{a, b\}\{a, b\} \tag{5}$$
$$S_2 = \{b, c\}\{b, c\} \tag{6}$$

   регулярны , поскольку конкатенация регулярных множеств — регулярное множество

4. Множества

$$S_1^+ = S_1 S_1^* \tag{7}$$
$$S_2^* \tag{8}$$

   регулярны, поскольку итерация регулярного множества — регулярное множество и конкатенация регулярных множеств — регулярное множество

5. Конкатенация регулярных множеств — регулярное множество, а потому:

$$S_3 = S_1^+ \cdot S_2^* \tag{9}$$

   есть регулярное множество.

2 Регулярный язык

2.1 Приведите искомого множества к регулярному виду

Регулярное множество:

$$\{a, b\} \cdot \{a, b\}^* \cdot \{b, c\}^* \tag{10}$$

2.2 Построение регулярного выражения для искомого регулярного множества

$$p = ((a+b)(a+b))^{+}((b+c)(b+c))^{*} \tag{11}$$

2.3 Получение регулярной грамматики

2.3.1 Построение леволинейной и праволинейной грамматик

$$p = \left(\left(\underbrace{\underbrace{a}_{1} + \underbrace{b}_{2}}_{9} \cdot \underbrace{\underbrace{a}_{3} + \underbrace{b}_{4}}_{10} \right)^{+} \cdot \underbrace{\left(\underbrace{\underbrace{b}_{5} + \underbrace{c}_{6}}_{11} \cdot \underbrace{\underbrace{b}_{7} + \underbrace{c}_{8}}_{12} \right)}_{14} \right)^{*}}_{17} \tag{12}$$

$$G_1 = \left(\begin{array}{c} \{S_1\}, \Sigma, \\ \{\ S_1 \to a\ \} \end{array}, S_1 \right), G_2 = \left(\begin{array}{c} \{S_2\}, \Sigma, \\ \{\ S_2 \to b\ \} \end{array}, S_2 \right)$$

$$G_3 = \left(\begin{array}{c} \{S_3\}, \Sigma, \\ \{\ S_3 \to a\ \} \end{array}, S_3 \right), G_4 = \left(\begin{array}{c} \{S_4\}, \Sigma, \\ \{\ S_4 \to b\ \} \end{array}, S_4 \right)$$

$$G_5 = \left(\begin{array}{c} \{S_5\}, \Sigma, \\ \{\ S_5 \to b\ \} \end{array}, S_5 \right), G_6 = \left(\begin{array}{c} \{S_6\}, \Sigma, \\ \{\ S_6 \to c\ \} \end{array}, S_6 \right)$$

$$G_7 = \left(\begin{array}{c} \{S_7\}, \Sigma, \\ \{\ S_7 \to b\ \} \end{array}, S_7 \right), G_8 = \left(\begin{array}{c} \{S_8\}, \Sigma, \\ \{\ S_8 \to c\ \} \end{array}, S_8 \right)$$

$$G_9 = \left(\begin{array}{c} \{S_9, S_1, S_2\}, \Sigma, \\ \left\{ \begin{array}{c} S_9 \to S_1|S_2 \\ S_1 \to a \\ S_2 \to b \end{array} \right\} \end{array}, S_9 \right), G_{10} = \left(\begin{array}{c} \{S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{c} S_{10} \to S_3|S_4 \\ S_3 \to a \\ S_4 \to b \end{array} \right\} \end{array}, S_{10} \right)$$

$$G_{11} = \left(\begin{array}{c} \{S_{11}, S_5, S_6\}, \Sigma, \\ \left\{ \begin{array}{c} S_{11} \to S_5|S_6 \\ S_5 \to b \\ S_6 \to c \end{array} \right\} \end{array}, S_{11} \right), G_{12} = \left(\begin{array}{c} \{S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{c} S_{12} \to S_7|S_8 \\ S_7 \to b \\ S_8 \to c \end{array} \right\} \end{array}, S_{12} \right)$$

$$G'_{13} = \left(\begin{array}{c} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{c} S_9 \to S_1|S_2 \\ S_1 \to a, S_2 \to b \\ S_{10} \to S_3|S_4 \\ S_3 \to S_9 a \\ S_4 \to S_9 b \end{array} \right\} \end{array}, S_{10} \right) G''_{13} = \left(\begin{array}{c} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{c} S_9 \to S_1|S_2 \\ S_1 \to a S_{10} \\ S_2 \to b S_{10} \\ S_{10} \to S_3|S_4 \\ S_3 \to a, S_4 \to b \end{array} \right\} \end{array}, S_9 \right)$$

$$G'_{14} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5 | S_6 \\ S_5 \to b, S_6 \to c \\ S_{12} \to S_7 | S_8 \\ S_7 \to S_{11}b \\ S_8 \to S_{11}c \end{array} \right\}, S_{12} \end{array} \right), G''_{14} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5 | S_6 \\ S_5 \to bS_{12} \\ S_6 \to cS_{12} \\ S_{12} \to S_7 | S_8 \\ S_7 \to b, S_8 \to c \end{array} \right\}, S_{11} \end{array} \right)$$

$$G'_{15} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1 | S_2 \\ S_1 \to S_{15}a | a \\ S_2 \to S_{15}b | b \\ S_{10} \to S_3 | S_4 \\ S_3 \to S_9 a \\ S_4 \to S_9 b \\ S_{15} \to S_{10} \end{array} \right\}, S_{15} \end{array} \right), G''_{15} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1 | S_2 \\ S_1 \to aS_{10} \\ S_2 \to bS_{10} \\ S_{10} \to S_3 | S_4 \\ S_3 \to aS_{15} | a \\ S_4 \to bS_{15} | b \\ S_{15} \to S_9 \end{array} \right\}, S_{15} \end{array} \right)$$

$$G'_{16} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5 | S_6 \\ S_5 \to S_{16}b | b \\ S_6 \to S_{16}c | c \\ S_{12} \to S_7 | S_8 \\ S_7 \to S_{11}b \\ S_8 \to S_{11}c \\ S_{16} \to S_{12} | \varepsilon \end{array} \right\}, S_{16} \end{array} \right), G''_{16} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5 | S_6 \\ S_5 \to bS_{12} \\ S_6 \to cS_{12} \\ S_{12} \to S_7 | S_8 \\ S_7 \to bS_{16} | b \\ S_8 \to cS_{16} | c \\ S_{16} \to S_{11} | \varepsilon \end{array} \right\}, S_{16} \end{array} \right)$$

$$G'_{17} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_9 \to S_1 | S_2 & S_{11} \to S_5 | S_6 \\ S_1 \to S_{15}a | a & S_5 \to S_{16}b | S_{15}b \\ S_2 \to S_{15}b | b & S_6 \to S_{16}c | S_{15}c \\ S_{10} \to S_3 | S_4 & S_{12} \to S_7 | S_8 \\ S_3 \to S_9 a & S_7 \to S_{11}b \\ S_4 \to S_9 b & S_8 \to S_{11}c \\ S_{15} \to S_{10} & S_{16} \to S_{12} | \varepsilon \end{array} \right\}, S_{16} \end{array} \right)$$

$$G''_{17} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_9 \to S_1 | S_2 & S_{11} \to S_5 | S_6 \\ S_1 \to aS_{10} & S_5 \to bS_{12} \\ S_2 \to bS_{10} & S_6 \to cS_{12} \\ S_{10} \to S_3 | S_4 & S_{12} \to S_7 | S_8 \\ S_3 \to aS_{15} | aS_{16} & S_7 \to bS_{16} | b \\ S_4 \to bS_{15} | bS_{16} & S_8 \to cS_{16} | c \\ S_{15} \to S_9 & S_{16} \to S_{11} | \varepsilon \end{array} \right\}, S_{15} \end{array} \right)$$

2.3.2 Приведение грамматики

1. Проверка пустоты

- Для леволинейной грамматики G'_{17}

$$C_0 = \varnothing$$
$$C_1 = \{S_5, S_6, S_9\} \cup C_0 = \{S_1, S_2, S_5, S_6, S_9, S_{16}\}$$
$$C_2 = \{S_3, S_4, S_5, S_6, S_9, S_{11}\} \cup C_1 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{11}, S_{16}\}$$
$$C_3 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}\} \cup C_2 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{16}\}$$
$$C_4 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_3 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}$$
$$C_5 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_4 = \aleph$$

Так как

$$S = S_{16} \in C_5 \implies L(G'_{17}) \neq \varnothing \tag{13}$$

- Для праволинейной грамматики G''_{17}

$$C_0 = \varnothing$$
$$C_1 = \{S_3, S_4, S_7, S_8, S_{12}\} \cup C_0 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\}$$
$$C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}\} \cup C_1 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\}$$
$$C_3 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}\} \cup C_2 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\}$$
$$C_4 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\}$$
$$C_5 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}$$
$$C_6 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_5 = \aleph$$

Так как

$$S = S_{15} \in C_6 \implies L(G''_{17}) \neq \varnothing \tag{14}$$

2. Удаление бесполезных символов

- Для леволинейной грамматики G'_{17}

$$C_0 = \varnothing$$
$$C_1 = \{S_5, S_6, S_9\} \cup C_0 = \{S_1, S_2, S_5, S_6, S_9, S_{16}\}$$
$$C_2 = \{S_3, S_4, S_5, S_6, S_9, S_{11}\} \cup C_1 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{11}, S_{16}\}$$
$$C_3 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}\} \cup C_2 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{16}\}$$
$$C_4 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_3 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}$$
$$C_5 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_4 = \aleph$$

Бесполезных символов нет, следовательно, грамматика G'_{17} не изменилась.

- Для праволинейной грамматики G''_{17}

$$C_0 = \varnothing$$
$$C_1 = \{S_3, S_4, S_7, S_8, S_{12}\} \cup C_0 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\}$$
$$C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}\} \cup C_1 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\}$$
$$C_3 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}\} \cup C_2 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\}$$
$$C_4 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\}$$
$$C_5 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}$$
$$C_6 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 =$$
$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_5 = \aleph$$

Бесполезных символов нет, следовательно, грамматика G''_{17} не изменилась.