

МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РОССИЙСКОЙ ФЕДЕРАЦИИ  
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## **ДОМАШНЯЯ РАБОТА**

**по дисциплине**

**«Теория формальных языков и методы трансляции»**

**Вариант 2.14**

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$$L = \{((a, b)^2)^k \cdot ((b, c)^2)^m : \forall k > 0, m \geq 0, k, m \in \mathbb{Z}\} \quad (1)$$

(2)

## 1 Определение типа языка L

Язык ф-л. (1) является регулярным. Докажем это, пользуясь замкнутостью класса регулярных языков.

1. Множества  $\{a\}, \{b\}, \{c\}$  являются регулярными по определению;

2. Множества

$$\{a\} \cup \{b\} = \{a, b\} \quad (3)$$

$$\{b\} \cup \{c\} = \{b, c\} \quad (4)$$

регулярны, так как объединение регулярных множеств — регулярное множество

3. Множества

$$S_1 = \{a, b\}\{a, b\} \quad (5)$$

$$S_2 = \{b, c\}\{b, c\} \quad (6)$$

регулярны, поскольку конкатенация регулярных множеств — регулярное множество

4. Множества

$$S_1^+ = S_1 S_1^* \quad (7)$$

$$S_2^* \quad (8)$$

регулярны, поскольку итерация регулярного множества — регулярное множество и конкатенация регулярных множеств — регулярное множество

5. Конкатенация регулярных множеств — регулярное множество, а потому:

$$S_3 = S_1^+ \cdot S_2^* \quad (9)$$

есть регулярное множество.

## 2 Регулярный язык

### 2.1 Приведите искомого множества к регулярному виду

Регулярное множество:

$$(\{a, b\} \cdot \{a, b\})^+ \cdot (\{b, c\} \cdot \{b, c\})^* \quad (10)$$

## 2.2 Построение регулярного выражения для искомого регулярного множества

$$p = ((a + b)(a + b))^+((b + c)(b + c))^* \quad (11)$$

## 2.3 Получение регулярной грамматики

### 2.3.1 Построение левостолбчатой и правостолбчатой грамматик

$$p = \underbrace{\left( \underbrace{\left( \underbrace{\underbrace{a}_{1} + \underbrace{b}_{2}}_{9} \right) \cdot \left( \underbrace{\underbrace{a}_{3} + \underbrace{b}_{4}}_{10} \right)}_{13} \right)^+}_{15} \cdot \underbrace{\left( \underbrace{\left( \underbrace{\underbrace{b}_{5} + \underbrace{c}_{6}}_{11} \right) \cdot \left( \underbrace{\underbrace{b}_{7} + \underbrace{c}_{8}}_{12} \right)}_{14} \right)^*}_{16} \quad (12)$$

$$\begin{aligned} G_1 &= \left( \begin{array}{l} \{S_1\}, \Sigma, \\ \{ S_1 \rightarrow a \} \end{array}, S_1 \right), G_2 = \left( \begin{array}{l} \{S_2\}, \Sigma, \\ \{ S_2 \rightarrow b \} \end{array}, S_2 \right) \\ G_3 &= \left( \begin{array}{l} \{S_3\}, \Sigma, \\ \{ S_3 \rightarrow a \} \end{array}, S_3 \right), G_4 = \left( \begin{array}{l} \{S_4\}, \Sigma, \\ \{ S_4 \rightarrow b \} \end{array}, S_4 \right) \\ G_5 &= \left( \begin{array}{l} \{S_5\}, \Sigma, \\ \{ S_5 \rightarrow b \} \end{array}, S_5 \right), G_6 = \left( \begin{array}{l} \{S_6\}, \Sigma, \\ \{ S_6 \rightarrow c \} \end{array}, S_6 \right) \\ G_7 &= \left( \begin{array}{l} \{S_7\}, \Sigma, \\ \{ S_7 \rightarrow b \} \end{array}, S_7 \right), G_8 = \left( \begin{array}{l} \{S_8\}, \Sigma, \\ \{ S_8 \rightarrow c \} \end{array}, S_8 \right) \\ G_9 &= \left( \begin{array}{l} \{S_9, S_1, S_2\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \rightarrow S_1|S_2 \\ S_1 \rightarrow a \\ S_2 \rightarrow b \end{array} \right\} \end{array}, S_9 \right), G_{10} = \left( \begin{array}{l} \{S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_{10} \rightarrow S_3|S_4 \\ S_3 \rightarrow a \\ S_4 \rightarrow b \end{array} \right\} \end{array}, S_{10} \right) \\ G_{11} &= \left( \begin{array}{l} \{S_{11}, S_5, S_6\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \rightarrow S_5|S_6 \\ S_5 \rightarrow b \\ S_6 \rightarrow c \end{array} \right\} \end{array}, S_{11} \right), G_{12} = \left( \begin{array}{l} \{S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{12} \rightarrow S_7|S_8 \\ S_7 \rightarrow b \\ S_8 \rightarrow c \end{array} \right\} \end{array}, S_{12} \right) \\ G'_{13} &= \left( \begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \rightarrow S_1|S_2 \\ S_1 \rightarrow a, S_2 \rightarrow b \\ S_{10} \rightarrow S_3|S_4 \\ S_3 \rightarrow S_9a \\ S_4 \rightarrow S_9b \end{array} \right\} \end{array}, S_{10} \right) \\ G''_{13} &= \left( \begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \rightarrow S_1|S_2 \\ S_1 \rightarrow aS_{10} \\ S_2 \rightarrow bS_{10} \\ S_{10} \rightarrow S_3|S_4 \\ S_3 \rightarrow a, S_4 \rightarrow b \end{array} \right\} \end{array}, S_9 \right) \end{aligned}$$

$$\begin{aligned}
G'_{14} &= \left( \left\{ \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_{11} \rightarrow S_5|S_6 \\ S_5 \rightarrow b, S_6 \rightarrow c \\ S_{12} \rightarrow S_7|S_8 \\ S_7 \rightarrow S_{11}b \\ S_8 \rightarrow S_{11}c \end{array} \right\}, S_{12} \right), G''_{14} = \left( \left\{ \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_{11} \rightarrow S_5|S_6 \\ S_5 \rightarrow bS_{12} \\ S_6 \rightarrow cS_{12} \\ S_{12} \rightarrow S_7|S_8 \\ S_7 \rightarrow b, S_8 \rightarrow c \end{array} \right\}, S_{11} \right) \\
G'_{15} &= \left( \left\{ \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_9 \rightarrow S_1|S_2 \\ S_1 \rightarrow S_{15}a|a \\ S_2 \rightarrow S_{15}b|b \\ S_{10} \rightarrow S_3|S_4 \\ S_3 \rightarrow S_9a \\ S_4 \rightarrow S_9b \\ S_{15} \rightarrow S_{10} \end{array} \right\}, S_{15} \right), G''_{15} = \left( \left\{ \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_9 \rightarrow S_1|S_2 \\ S_1 \rightarrow aS_{10} \\ S_2 \rightarrow bS_{10} \\ S_{10} \rightarrow S_3|S_4 \\ S_3 \rightarrow aS_{15}|a \\ S_4 \rightarrow bS_{15}|b \\ S_{15} \rightarrow S_9 \end{array} \right\}, S_{15} \right) \\
G'_{16} &= \left( \left\{ \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_{11} \rightarrow S_5|S_6 \\ S_5 \rightarrow S_{16}b|b \\ S_6 \rightarrow S_{16}c|c \\ S_{12} \rightarrow S_7|S_8 \\ S_7 \rightarrow S_{11}b \\ S_8 \rightarrow S_{11}c \\ S_{16} \rightarrow S_{12}|\varepsilon \end{array} \right\}, S_{16} \right), G''_{16} = \left( \left\{ \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{l} S_{11} \rightarrow S_5|S_6 \\ S_5 \rightarrow bS_{12} \\ S_6 \rightarrow cS_{12} \\ S_{12} \rightarrow S_7|S_8 \\ S_7 \rightarrow bS_{16}|b \\ S_8 \rightarrow cS_{16}|c \\ S_{16} \rightarrow S_{11}|\varepsilon \end{array} \right\}, S_{16} \right) \\
G'_{17} &= \left( \left\{ \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{ll} S_9 \rightarrow S_1|S_2 & S_{11} \rightarrow S_5|S_6 \\ S_1 \rightarrow S_{15}a|a & S_5 \rightarrow S_{16}b|S_{15}b \\ S_2 \rightarrow S_{15}b|b & S_6 \rightarrow S_{16}c|S_{15}c \\ S_{10} \rightarrow S_3|S_4 & S_{12} \rightarrow S_7|S_8 \\ S_3 \rightarrow S_9a & S_7 \rightarrow S_{11}b \\ S_4 \rightarrow S_9b & S_8 \rightarrow S_{11}c \\ S_{15} \rightarrow S_{10} & S_{16} \rightarrow S_{12}|S_{15} \end{array} \right\}, S_{16} \right) \\
G''_{17} &= \left( \left\{ \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \right. \right. \\
&\quad \left. \left. \begin{array}{ll} S_9 \rightarrow S_1|S_2 & S_{11} \rightarrow S_5|S_6 \\ S_1 \rightarrow aS_{10} & S_5 \rightarrow bS_{12} \\ S_2 \rightarrow bS_{10} & S_6 \rightarrow cS_{12} \\ S_{10} \rightarrow S_3|S_4 & S_{12} \rightarrow S_7|S_8 \\ S_3 \rightarrow aS_{15}|aS_{16} & S_7 \rightarrow bS_{16}|b \\ S_4 \rightarrow bS_{15}|bS_{16} & S_8 \rightarrow cS_{16}|c \\ S_{15} \rightarrow S_9 & S_{16} \rightarrow S_{11}|\varepsilon \end{array} \right\}, S_{15} \right)
\end{aligned}$$

### 2.3.2 Приведение грамматики

#### 1. Проверка пустоты

- Для левостроительной грамматики  $G'_{17}$

$$\begin{aligned}
C_0 &= \emptyset \\
C_2 &= \{S_9\} \cup C_1 = \{S_1, S_2, S_9\} \\
C_3 &= \{S_3, S_4, S_9\} \cup C_2 = \{S_1, S_2, S_3, S_4, S_9\} \\
C_4 &= \{S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \\
C_5 &= \{S_3, S_4, S_9, S_{10}, S_{15}\} \cup C_4 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}
\end{aligned}$$

Так как

$$S = S_{16} \in C_{11} \implies L(G'_{17}) \neq \emptyset \quad (13)$$

- Для правостроительной грамматики  $G''_{17}$

$$\begin{aligned}
C_0 &= \emptyset \\
C_1 &= \{S_7, S_8, S_{16}\} \cup C_0 = \{S_7, S_8, S_{16}\} \\
C_2 &= \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \\
C_3 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \cup C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}
\end{aligned}$$

Так как

$$S = S_{15} \in C_6 \implies L(G''_{17}) \neq \emptyset \quad (14)$$

## 2. Удаление бесполезных символов

- Для левوليнейной грамматики  $G'_{17}$

$$\begin{aligned}
C_0 &= \emptyset \\
C_2 &= \{S_9\} \cup C_1 = \{S_1, S_2, S_9\} \\
C_3 &= \{S_3, S_4, S_9\} \cup C_2 = \{S_1, S_2, S_3, S_4, S_9\} \\
C_4 &= \{S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \\
C_5 &= \{S_3, S_4, S_9, S_{10}, S_{15}\} \cup C_4 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph
\end{aligned}$$

Бесполезных символов нет, следовательно, грамматика  $G'_{17}$  не изменилась.

- Для праволинейной грамматики  $G''_{17}$

$$\begin{aligned}
C_0 &= \emptyset \\
C_1 &= \{S_7, S_8, S_{16}\} \cup C_0 = \{S_7, S_8, S_{16}\} \\
C_2 &= \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \\
C_3 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \cup C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_6 = \aleph
\end{aligned}$$

Бесполезных символов нет, следовательно, грамматика  $G''_{17}$  не изменилась.

## 3. Удаление недостижимых символов

- Для левостроительной грамматики  $G'_{17}$

$$C_0 = \{S_{16}\}$$

$$C_1 = \{S_{12}, S_{16}\} \cup C_0 = \{S_{12}, S_{16}\}$$

$$C_2 = \{S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_7, S_8, S_{12}, S_{16}\}$$

$$C_3 = \{S_7, S_8, S_{11}, S_{12}, S_{16}\} \cup C_2 = \{S_7, S_8, S_{11}, S_{12}, S_{16}\}$$

$$C_4 = \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{16}\} \cup C_3 = \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{16}\}$$

$$C_5 = \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_4$$

$$= \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{15}, S_{16}, b, c\}$$

$$C_6 = \{S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_5$$

$$= \{S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\}$$

$$C_7 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_6$$

$$= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\}$$

$$C_8 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_7$$

$$= \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\}$$

$$C_9 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_8$$

$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\}$$

$$C_{10} = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_9$$

$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\}$$

$$C_{11} = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_{10}$$

$$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} = \Sigma \cup \aleph$$

Недостижимых символов нет, следовательно, грамматика  $G'_{17}$  не изменилась.



- Для праволинейной грамматики  $G''_{17}$

$$\begin{aligned}
C_0 &= \{S_{15}\} \\
C_1 &= \{S_9\} \cup C_0 = \{S_9, S_{15}\} \\
C_2 &= \{S_1, S_2, S_9\} \cup C_1 = \{S_1, S_2, S_9, S_{15}\} \\
C_3 &= \{S_1, S_2, S_9, S_{10}\} \cup C_2 = \{S_1, S_2, S_9, S_{10}, S_{15}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}, a, b\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}, a, b\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} = \Sigma \cup \aleph
\end{aligned}$$

Недостижимых символов нет, следовательно, грамматика  $G''_{17}$  не изменилась.

#### 4. Удаление пустых правил

- Для леволинейной грамматики  $G'_{17}$

$$\begin{aligned}
C_0 &= \emptyset \\
C_1 &= \emptyset \cup C_0 = \emptyset = C_0
\end{aligned}$$

Пустых правил нет, следовательно, грамматика  $G'_{17}$  не поменялась.

- Для праволинейной грамматики  $G'''_{17}$

$$\begin{aligned}
C_0 &= \{S_{16}\} \\
C_1 &= \emptyset \cup C_0 = \{S_{16}\}
\end{aligned}$$

Итоговая грамматика  $G'''_{18}$  без пустых правил и после добавления новых примет вид

$$G'''_{18} = \left( \left\{ S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16} \right\}, \Sigma, \left\{ \begin{array}{ll} S_9 \rightarrow S_1 | S_2 & S_{11} \rightarrow S_5 | S_6 \\ S_1 \rightarrow a S_{10} & S_5 \rightarrow b S_{12} \\ S_2 \rightarrow b S_{10} & S_6 \rightarrow c S_{12} \\ S_{10} \rightarrow S_3 | S_4 & S_{12} \rightarrow S_7 | S_8 \\ S_3 \rightarrow a S_{15} | a S_{16} | a & S_7 \rightarrow b S_{16} | b \\ S_4 \rightarrow b S_{15} | b S_{16} | b & S_8 \rightarrow c S_{16} | c \\ S_{15} \rightarrow S_9 & S_{16} \rightarrow S_{11} \end{array} \right\}, S_{15} \right)$$

## 5. Удаление цепных правил

- Строим последовательность множеств  $\aleph_i^X$  для левостолбчатой грамматики  $G'_{17}$

$$\begin{aligned}
 & \left\{ \begin{array}{l} \aleph_0^{S_0} = \{S_0\} \\ \aleph_1^{S_0} = \{S_0\} \end{array} \right\} \Rightarrow \aleph^{S_0} = \emptyset \left\{ \begin{array}{l} \aleph_0^{S_1} = \{S_1\} \\ \aleph_1^{S_1} = \{S_1\} \end{array} \right\} \Rightarrow \aleph^{S_1} = \emptyset \\
 & \left\{ \begin{array}{l} \aleph_0^{S_2} = \{S_2\} \\ \aleph_1^{S_2} = \{S_2\} \end{array} \right\} \Rightarrow \aleph^{S_2} = \emptyset \left\{ \begin{array}{l} \aleph_0^{S_3} = \{S_3\} \\ \aleph_1^{S_3} = \{S_3\} \end{array} \right\} \Rightarrow \aleph^{S_3} = \emptyset \\
 & \left\{ \begin{array}{l} \aleph_0^{S_4} = \{S_4\} \\ \aleph_1^{S_4} = \{S_4\} \end{array} \right\} \Rightarrow \aleph^{S_4} = \emptyset \left\{ \begin{array}{l} \aleph_0^{S_5} = \{S_5\} \\ \aleph_1^{S_5} = \{S_5\} \end{array} \right\} \Rightarrow \aleph^{S_5} = \emptyset \\
 & \left\{ \begin{array}{l} \aleph_0^{S_6} = \{S_6\} \\ \aleph_1^{S_6} = \{S_6\} \end{array} \right\} \Rightarrow \aleph^{S_6} = \emptyset \left\{ \begin{array}{l} \aleph_0^{S_7} = \{S_7\} \\ \aleph_1^{S_7} = \{S_7\} \end{array} \right\} \Rightarrow \aleph^{S_7} = \emptyset \\
 & \left\{ \begin{array}{l} \aleph_0^{S_8} = \{S_8\} \\ \aleph_1^{S_8} = \{S_8\} \end{array} \right\} \Rightarrow \aleph^{S_8} = \emptyset \\
 & \left\{ \begin{array}{l} \aleph_0^{S_9} = \{S_9\} \\ \aleph_1^{S_9} = \{S_1, S_2, S_9\} \\ \aleph_2^{S_9} = \{S_1, S_2, S_9\} \end{array} \right\} \Rightarrow \aleph^{S_9} = \{S_1, S_2\} \\
 & \left\{ \begin{array}{l} \aleph_0^{S_{10}} = \{S_{10}\} \\ \aleph_1^{S_{10}} = \{S_3, S_4, S_{10}\} \\ \aleph_2^{S_{10}} = \{S_3, S_4, S_{10}\} \end{array} \right\} \Rightarrow \aleph^{S_{10}} = \{S_3, S_4\} \\
 & \left\{ \begin{array}{l} \aleph_0^{S_{11}} = \{S_{11}\} \\ \aleph_1^{S_{11}} = \{S_5, S_6, S_{11}\} \\ \aleph_2^{S_{11}} = \{S_5, S_6, S_{11}\} \end{array} \right\} \Rightarrow \aleph^{S_{11}} = \{S_5, S_6\} \\
 & \left\{ \begin{array}{l} \aleph_0^{S_{12}} = \{S_{12}\} \\ \aleph_1^{S_{12}} = \{S_7, S_8, S_{12}\} \\ \aleph_2^{S_{12}} = \{S_7, S_8, S_{12}\} \end{array} \right\} \Rightarrow \aleph^{S_{12}} = \{S_7, S_8\} \\
 & \left\{ \begin{array}{l} \aleph_0^{S_{15}} = \{S_{15}\} \\ \aleph_1^{S_{15}} = \{S_{10}, S_{15}\} \\ \aleph_2^{S_{15}} = \{S_3, S_4, S_{10}, S_{15}\} \\ \aleph_3^{S_{15}} = \{S_3, S_4, S_{10}, S_{15}\} \end{array} \right\} \Rightarrow \aleph^{S_{15}} = \{S_3, S_4, S_{10}\} \\
 & \left\{ \begin{array}{l} \aleph_0^{S_{16}} = \{S_{16}\} \\ \aleph_1^{S_{16}} = \{S_{12}, S_{15}, S_{16}\} \\ \aleph_2^{S_{16}} = \{S_7, S_8, S_{10}, S_{12}, S_{15}, S_{16}\} \\ \aleph_3^{S_{16}} = \{S_3, S_4, S_7, S_8, S_{10}, S_{12}, S_{15}, S_{16}\} \\ \aleph_4^{S_{16}} = \{S_3, S_4, S_7, S_8, S_{10}, S_{12}, S_{15}, S_{16}\} \end{array} \right\} \Rightarrow \aleph^{S_{16}} = \{S_3, S_4, S_7, S_8, S_{10}, S_{12}, S_{15}\}
 \end{aligned}$$

Множество правил  $P'_{18}$  содержит все правила грамматики  $G'_{17}$  кроме цепных:

$$P'_{18} = \left\{ \begin{array}{ll} S_1 \rightarrow S_{15}a|a & S_5 \rightarrow S_{16}b|S_{15}b \\ S_2 \rightarrow S_{15}b|b & S_6 \rightarrow S_{16}c|S_{15}c \\ S_3 \rightarrow S_9a & S_7 \rightarrow S_{11}b \\ S_4 \rightarrow S_9b & S_8 \rightarrow S_{11}c \end{array} \right\}$$

С добавлением новых правил, опираясь на соотношение вида

$$P'_{18} = P'_{18} \cup \{ (B \rightarrow \alpha) | \forall (A \rightarrow \alpha) \in P, A \in \aleph^B \},$$

то есть

$$P'_{18} = P'_{18} \cup \left\{ \begin{array}{ll} S_9 \rightarrow S_{15}a|a|S_{15}b|b & S_{10} \rightarrow S_9a|S_9b \\ S_{11} \rightarrow S_{16}b|S_{15}b|S_{16}c|S_{15}c & S_{12} \rightarrow S_{11}b|S_{11}c \\ S_{15} \rightarrow S_9a|S_9b & S_{16} \rightarrow S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}$$

Таким образом, результирующая грамматика  $G'_{18}$  примет следующий вид

$$G'_{18} = \left( \begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_1 \rightarrow S_{15}a|a & S_5 \rightarrow S_{16}b|S_{15}b \\ S_2 \rightarrow S_{15}b|b & S_6 \rightarrow S_{16}c|S_{15}c \\ S_3 \rightarrow S_9a & S_7 \rightarrow S_{11}b \\ S_4 \rightarrow S_9b & S_8 \rightarrow S_{11}c \\ S_9 \rightarrow S_{15}a|a|S_{15}b|b & S_{10} \rightarrow S_9a|S_9b \\ S_{11} \rightarrow S_{16}b|S_{15}b|S_{16}c|S_{15}c & S_{12} \rightarrow S_{11}b|S_{11}c \\ S_{15} \rightarrow S_9a|S_9b & S_{16} \rightarrow S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}, S_{16} \end{array} \right)$$

- Строим последовательность множеств  $\aleph_i^X$  для праволинейной грамматики  $G''_{18}$

$$\begin{aligned}
& \left\{ \begin{array}{l} \aleph_0^{S_0} = \{S_0\} \\ \aleph_1^{S_0} = \{S_0\} \end{array} \right\} \Rightarrow \aleph^{S_0} = \emptyset \quad \left\{ \begin{array}{l} \aleph_0^{S_1} = \{S_1\} \\ \aleph_1^{S_1} = \{S_1\} \end{array} \right\} \Rightarrow \aleph^{S_1} = \emptyset \\
& \left\{ \begin{array}{l} \aleph_0^{S_2} = \{S_2\} \\ \aleph_1^{S_2} = \{S_2\} \end{array} \right\} \Rightarrow \aleph^{S_2} = \emptyset \quad \left\{ \begin{array}{l} \aleph_0^{S_3} = \{S_3\} \\ \aleph_1^{S_3} = \{S_3\} \end{array} \right\} \Rightarrow \aleph^{S_3} = \emptyset \\
& \left\{ \begin{array}{l} \aleph_0^{S_4} = \{S_4\} \\ \aleph_1^{S_4} = \{S_4\} \end{array} \right\} \Rightarrow \aleph^{S_4} = \emptyset \quad \left\{ \begin{array}{l} \aleph_0^{S_5} = \{S_5\} \\ \aleph_1^{S_5} = \{S_5\} \end{array} \right\} \Rightarrow \aleph^{S_5} = \emptyset \\
& \left\{ \begin{array}{l} \aleph_0^{S_6} = \{S_6\} \\ \aleph_1^{S_6} = \{S_6\} \end{array} \right\} \Rightarrow \aleph^{S_6} = \emptyset \quad \left\{ \begin{array}{l} \aleph_0^{S_7} = \{S_7\} \\ \aleph_1^{S_7} = \{S_7\} \end{array} \right\} \Rightarrow \aleph^{S_7} = \emptyset \\
& \left\{ \begin{array}{l} \aleph_0^{S_8} = \{S_8\} \\ \aleph_1^{S_8} = \{S_8\} \end{array} \right\} \Rightarrow \aleph^{S_8} = \emptyset \\
& \left\{ \begin{array}{l} \aleph_0^{S_9} = \{S_9\} \\ \aleph_1^{S_9} = \{S_1, S_2, S_9\} \\ \aleph_2^{S_9} = \{S_1, S_2, S_9\} \end{array} \right\} \Rightarrow \aleph^{S_9} = \{S_1, S_2\} \\
& \left\{ \begin{array}{l} \aleph_0^{S_{10}} = \{S_{10}\} \\ \aleph_1^{S_{10}} = \{S_3, S_4, S_{10}\} \\ \aleph_2^{S_{10}} = \{S_3, S_4, S_{10}\} \end{array} \right\} \Rightarrow \aleph^{S_{10}} = \{S_3, S_4\} \\
& \left\{ \begin{array}{l} \aleph_0^{S_{11}} = \{S_{11}\} \\ \aleph_1^{S_{11}} = \{S_5, S_6, S_{11}\} \\ \aleph_2^{S_{11}} = \{S_5, S_6, S_{11}\} \end{array} \right\} \Rightarrow \aleph^{S_{11}} = \{S_5, S_6\} \\
& \left\{ \begin{array}{l} \aleph_0^{S_{12}} = \{S_{12}\} \\ \aleph_1^{S_{12}} = \{S_7, S_8, S_{12}\} \\ \aleph_2^{S_{12}} = \{S_7, S_8, S_{12}\} \end{array} \right\} \Rightarrow \aleph^{S_{12}} = \{S_7, S_8\} \\
& \left\{ \begin{array}{l} \aleph_0^{S_{15}} = \{S_{15}\} \\ \aleph_1^{S_{15}} = \{S_9, S_{15}\} \\ \aleph_2^{S_{15}} = \{S_1, S_2, S_9, S_{15}\} \\ \aleph_3^{S_{15}} = \{S_1, S_2, S_9, S_{15}\} \end{array} \right\} \Rightarrow \aleph^{S_{15}} = \{S_1, S_2, S_9\} \\
& \left\{ \begin{array}{l} \aleph_0^{S_{16}} = \{S_{16}\} \\ \aleph_1^{S_{16}} = \{S_{11}, S_{16}\} \\ \aleph_2^{S_{16}} = \{S_5, S_6, S_{11}, S_{16}\} \\ \aleph_3^{S_{16}} = \{S_5, S_6, S_{11}, S_{16}\} \end{array} \right\} \Rightarrow \aleph^{S_{16}} = \{S_5, S_6, S_{11}\}
\end{aligned}$$

Множество правил  $P''_{19}$  содержит все правила грамматики  $G''_{18}$  кроме цепных:

$$P''_{19} = \left\{ \begin{array}{ll} S_1 \rightarrow aS_{10} & S_5 \rightarrow bS_{12} \\ S_2 \rightarrow bS_{10} & S_6 \rightarrow cS_{12} \\ S_3 \rightarrow aS_{15}|aS_{16}|a & S_7 \rightarrow bS_{16}|b \\ S_4 \rightarrow bS_{15}|bS_{16}|b & S_8 \rightarrow cS_{16}|c \end{array} \right\}$$

С добавлением новых правил, опираясь на соотношение вида

$$P''_{19} = P'_{19} \cup \{(B \rightarrow \alpha) | \forall (A \rightarrow \alpha) \in P, A \in \aleph^B\},$$

то есть

$$P''_{19} = P''_{19} \cup \left\{ \begin{array}{ll} S_9 \rightarrow aS_{10}|bS_{10} & S_{10} \rightarrow aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b \\ S_{11} \rightarrow bS_{12}|cS_{12} & S_{12} \rightarrow bS_{16}|b|cS_{16}|c \\ S_{15} \rightarrow aS_{10}|bS_{10} & S_{16} \rightarrow bS_{12}|cS_{12} \end{array} \right\}$$

Таким образом, результирующая грамматика  $G'_{18}$  примет следующий вид

$$G''_{19} = \left( \begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_1 \rightarrow aS_{10} & S_5 \rightarrow bS_{12} \\ S_2 \rightarrow bS_{10} & S_6 \rightarrow cS_{12} \\ S_3 \rightarrow aS_{15}|aS_{16}|a & S_7 \rightarrow bS_{16}|b \\ S_4 \rightarrow bS_{15}|bS_{16}|b & S_8 \rightarrow cS_{16}|c \\ S_9 \rightarrow aS_{10}|bS_{10} & S_{10} \rightarrow aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b \\ S_{11} \rightarrow bS_{12}|cS_{12} & S_{12} \rightarrow bS_{16}|b|cS_{16}|c \\ S_{15} \rightarrow aS_{10}|bS_{10} & S_{16} \rightarrow bS_{12}|cS_{12} \end{array} \right\}, S_{15} \end{array} \right)$$

Так как при удалении пустых правил и цепных правил лево- и праволинейной грамматик произошло их изменение, то необходимо повторить удаление бесполезных и недостижимых символов.

#### 6. Удаление бесполезных символов грамматик $G'_{18}$ и $G''_{19}$

- Для леволинейной грамматики  $G'_{18}$

$$\begin{aligned} C_0 &= \emptyset \\ C_1 &= \{S_1, S_2, S_9\} \cup C_0 = \{S_1, S_2, S_9\} \\ C_2 &= \{S_3, S_4, S_{10}, S_{15}, S_{16}\} \cup C_1 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}\} \\ C_3 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}\} \cup C_2 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}\} \\ C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_3 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\ C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph \end{aligned}$$

Бесполезных символов нет, следовательно, грамматика  $G'_{18}$  не изменилась.

- Для праволинейной грамматики  $G''_{19}$

$$\begin{aligned} C_0 &= \emptyset \\ C_1 &= \{S_3, S_4, S_7, S_8, S_{10}, S_{12}\} \cup C_0 = \{S_3, S_4, S_7, S_8, S_{10}, S_{12}\} \\ C_2 &= \{S_1, S_2, S_5, S_6, S_9, S_{11}, S_{15}, S_{16}\} \cup C_1 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\ C_3 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_2 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph \end{aligned}$$

Бесполезных символов нет, следовательно, грамматика  $G''_{19}$  не изменилась.

7. Удаление недостижимых символов грамматик  $G'_{18}$  и  $G''_{19}$

- Для левوليнейной грамматики  $G'_{18}$

$$\begin{aligned} C_0 &= \{S_{16}\} \\ C_1 &= \{S_9, S_{11}, a, b, c\} \cup C_0 = \{S_9, S_{11}, S_{16}, a, b, c\} \\ C_2 &= \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \cup C_1 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \\ C_3 &= \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \cup C_2 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \end{aligned}$$

Строим результирующую грамматику  $G'_{19}$  без недостижимых символов

$$\begin{aligned} \aleph'_{19} &= \aleph'_{18} \cap C_3 = \{S_9, S_{11}, S_{15}, S_{16}\} \\ \Sigma'_{19} &= \Sigma'_{18} \cap C_3 = \{a, b, c\} \\ P'_{19} &= \{(A \rightarrow \alpha) | \forall (A \rightarrow \alpha) \in P'_{18}, A \in \aleph'_{19}, \alpha \in (\Sigma'_{19} \cup \aleph'_{19})^*\} = \\ &= \left\{ \begin{array}{ll} S_9 \rightarrow S_{15}a|a|S_{15}b|b & S_{11} \rightarrow S_{16}b|S_{15}b|S_{16}c|S_{15}c \\ S_{15} \rightarrow S_9a|S_9b & S_{16} \rightarrow S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\} \\ S'_{19} &\equiv S_{16} \end{aligned}$$

Таким образом, результирующая грамматика  $G'_{19}$  примет вид

$$G'_{19} = \left( \begin{array}{l} \{S_9, S_{11}, S_{15}, S_{16}\}, \{a, b, c\}, \\ \left\{ \begin{array}{ll} S_9 \rightarrow S_{15}a|a|S_{15}b|b & S_{11} \rightarrow S_{16}b|S_{15}b|S_{16}c|S_{15}c \\ S_{15} \rightarrow S_9a|S_9b & S_{16} \rightarrow S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}, S_{16} \end{array} \right)$$

- Для праволинейной грамматики  $G''_{19}$

$$\begin{aligned} C_0 &= \{S_{15}\} \\ C_1 &= \{S_{10}, b\} \cup C_0 = \{S_{10}, S_{15}, b\} \\ C_2 &= \{S_{10}, S_{15}, S_{16}, a, b\} \cup C_1 = \{S_{10}, S_{15}, S_{16}, a, b\} \\ C_3 &= \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_2 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \\ C_4 &= \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_3 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \end{aligned}$$

Строим результирующую грамматику  $G''_{20}$  без недостижимых символов

$$\begin{aligned} \aleph''_{20} &= \aleph''_{19} \cap C_4 = \{S_{10}, S_{12}, S_{15}, S_{16}\} \\ \Sigma''_{20} &= \Sigma''_{19} \cap C_4 = \{a, b, c\} \\ P''_{20} &= \{(A \rightarrow \alpha) | \forall (A \rightarrow \alpha) \in P''_{19}, A \in \aleph''_{20}, \alpha \in (\Sigma''_{20} \cup \aleph''_{20})^*\} = \\ &= \left\{ \begin{array}{ll} S_{10} \rightarrow aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b & S_{12} \rightarrow bS_{16}|b|cS_{16}|c \\ S_{15} \rightarrow aS_{10}|bS_{10} & S_{16} \rightarrow bS_{12}|cS_{12} \end{array} \right\} \\ S''_{20} &\equiv S_{15} \end{aligned}$$

Таким образом, результирующая грамматика  $G''_{20}$  примет вид

$$G''_{20} = \left( \begin{array}{l} \{S_{10}, S_{12}, S_{15}, S_{16}\}, \{a, b, c\}, \\ \left\{ \begin{array}{ll} S_{10} \rightarrow aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b & S_{12} \rightarrow bS_{16}|b|cS_{16}|c \\ S_{15} \rightarrow aS_{10}|bS_{10} & S_{16} \rightarrow bS_{12}|cS_{12} \end{array} \right\}, S_{15} \end{array} \right)$$

### 2.3.3 Построение конечного автомата для приведенной грамматики

#### 1. Приведение к автоматному виду