МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РОССИЙСКОЙ ФЕДЕРАЦИИ
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ДОМАШНЯЯ РАБОТА
по дисциплине
«Теория формальных языков и методы трансляции»
Вариант 2.14

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Вариант 2.14

$$L = \left\{((a,b)^2)^k \cdot ((b,c)^2)^m \colon \forall k > 0, m \geq 0,\ k,m \in \mathbb{Z}\right\} \tag{1}$$
$$\tag{2}$$

1 Определение типа языка L

Язык ф-л. (1) является регулярным. Докажем это, пользуясь замкнутостью класса регулярных языков.

1. Множества $\{a\}, \{b\}, \{c\}$ являются регулярными по определению;
2. Множества

$$\{a\} \cup \{b\} = \{a,b\} \tag{3}$$
$$\{b\} \cup \{c\} = \{b,c\} \tag{4}$$

   регулярны, так как объединение регулярных множеств — регулярное множество
3. Множества

$$S_1 = \{a,b\}\{a,b\} \tag{5}$$
$$S_2 = \{b,c\}\{b,c\} \tag{6}$$

   регулярны , поскольку конкатенация регулярных множеств — регулярное множество
4. Множества

$$S_1^+ = S_1 S_1^* \tag{7}$$
$$S_2^* \tag{8}$$

   регулярны, посколько итерация регулярного множества — регулярное множество и конкатенация регулярных множеств — регулярное множество
5. Конкатенация регулярных множеств — регулярное множество, а потому:

$$S_3 = S_1^+ \cdot S_2^* \tag{9}$$

   есть регулярное множество.

2 Регулярный язык

2.1 Приведите искомого множества к регулярному виду

Регулярное множество:

$$(\{a,b\} \cdot \{a,b\})^+ \cdot (\{b,c\} \cdot \{b,c\})^* \tag{10}$$

2.2 Построение регулярного выражения для искомого регулярного множества

$$p = ((a+b)(a+b))^{+}((b+c)(b+c))^{*} \tag{11}$$

2.3 Получение регулярной грамматики

2.3.1 Построение леволинейной и праволинейной грамматик

$$p = \underbrace{\underbrace{\left(\underbrace{\left(\underbrace{\underbrace{a}_{1}+\underbrace{b}_{2}}_{9}\right)\cdot\left(\underbrace{\underbrace{a}_{3}+\underbrace{b}_{4}}_{10}\right)}_{13}\right)^{+}}_{15}\cdot\underbrace{\left(\underbrace{\left(\underbrace{\underbrace{b}_{5}+\underbrace{c}_{6}}_{11}\right)\cdot\left(\underbrace{\underbrace{b}_{7}+\underbrace{c}_{8}}_{12}\right)}_{14}\right)^{*}}_{16}}_{17} \tag{12}$$

$$G_1 = \left(\begin{array}{l} \{S_1\}, \Sigma, \\ \{\ S_1 \to a\ \}, S_1 \end{array} \right), G_2 = \left(\begin{array}{l} \{S_2\}, \Sigma, \\ \{\ S_2 \to b\ \}, S_2 \end{array} \right)$$

$$G_3 = \left(\begin{array}{l} \{S_3\}, \Sigma, \\ \{\ S_3 \to a\ \}, S_3 \end{array} \right), G_4 = \left(\begin{array}{l} \{S_4\}, \Sigma, \\ \{\ S_4 \to b\ \}, S_4 \end{array} \right)$$

$$G_5 = \left(\begin{array}{l} \{S_5\}, \Sigma, \\ \{\ S_5 \to b\ \}, S_5 \end{array} \right), G_6 = \left(\begin{array}{l} \{S_6\}, \Sigma, \\ \{\ S_6 \to c\ \}, S_6 \end{array} \right)$$

$$G_7 = \left(\begin{array}{l} \{S_7\}, \Sigma, \\ \{\ S_7 \to b\ \}, S_7 \end{array} \right), G_8 = \left(\begin{array}{l} \{S_8\}, \Sigma, \\ \{\ S_8 \to c\ \}, S_8 \end{array} \right)$$

$$G_9 = \left(\begin{array}{l} \{S_9, S_1, S_2\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1 | S_2 \\ S_1 \to a \\ S_2 \to b \end{array} \right\}, S_9 \end{array} \right), G_{10} = \left(\begin{array}{l} \{S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_{10} \to S_3 | S_4 \\ S_3 \to a \\ S_4 \to b \end{array} \right\}, S_{10} \end{array} \right)$$

$$G_{11} = \left(\begin{array}{l} \{S_{11}, S_5, S_6\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5 | S_6 \\ S_5 \to b \\ S_6 \to c \end{array} \right\}, S_{11} \end{array} \right), G_{12} = \left(\begin{array}{l} \{S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{12} \to S_7 | S_8 \\ S_7 \to b \\ S_8 \to c \end{array} \right\}, S_{12} \end{array} \right)$$

$$G'_{13} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1 | S_2 \\ S_1 \to a, S_2 \to b \\ S_{10} \to S_3 | S_4 \\ S_3 \to S_9 a \\ S_4 \to S_9 b \end{array} \right\}, S_{10} \end{array} \right) G''_{13} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1 | S_2 \\ S_1 \to a S_{10} \\ S_2 \to b S_{10} \\ S_{10} \to S_3 | S_4 \\ S_3 \to a, S_4 \to b \end{array} \right\}, S_9 \end{array} \right)$$

$$G'_{14} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5|S_6 \\ S_5 \to b, S_6 \to c \\ S_{12} \to S_7|S_8 \\ S_7 \to S_{11}b \\ S_8 \to S_{11}c \end{array} \right\}, S_{12} \end{array} \right), G''_{14} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5|S_6 \\ S_5 \to bS_{12} \\ S_6 \to cS_{12} \\ S_{12} \to S_7|S_8 \\ S_7 \to b, S_8 \to c \end{array} \right\}, S_{11} \end{array} \right)$$

$$G'_{15} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1|S_2 \\ S_1 \to S_{15}a|a \\ S_2 \to S_{15}b|b \\ S_{10} \to S_3|S_4 \\ S_3 \to S_9a \\ S_4 \to S_9b \\ S_{15} \to S_{10} \end{array} \right\}, S_{15} \end{array} \right), G''_{15} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}\}, \Sigma, \\ \left\{ \begin{array}{l} S_9 \to S_1|S_2 \\ S_1 \to aS_{10} \\ S_2 \to bS_{10} \\ S_{10} \to S_3|S_4 \\ S_3 \to aS_{15}|a \\ S_4 \to bS_{15}|b \\ S_{15} \to S_9 \end{array} \right\}, S_{15} \end{array} \right)$$

$$G'_{16} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5|S_6 \\ S_5 \to S_{16}b|b \\ S_6 \to S_{16}c|c \\ S_{12} \to S_7|S_8 \\ S_7 \to S_{11}b \\ S_8 \to S_{11}c \\ S_{16} \to S_{12}|\varepsilon \end{array} \right\}, S_{16} \end{array} \right) G''_{16} = \left(\begin{array}{l} \{S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{l} S_{11} \to S_5|S_6 \\ S_5 \to bS_{12} \\ S_6 \to cS_{12} \\ S_{12} \to S_7|S_8 \\ S_7 \to bS_{16}|b \\ S_8 \to cS_{16}|c \\ S_{16} \to S_{11}|\varepsilon \end{array} \right\}, S_{16} \end{array} \right)$$

$$G'_{17} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_9 \to S_1|S_2 & S_{11} \to S_5|S_6 \\ S_1 \to S_{15}a|a & S_5 \to S_{16}b|S_{15}b \\ S_2 \to S_{15}b|b & S_6 \to S_{16}c|S_{15}c \\ S_{10} \to S_3|S_4 & S_{12} \to S_7|S_8 \\ S_3 \to S_9a & S_7 \to S_{11}b \\ S_4 \to S_9b & S_8 \to S_{11}c \\ S_{15} \to S_{10} & S_{16} \to S_{12}|S_{15} \end{array} \right\}, S_{16} \end{array} \right)$$

$$G''_{17} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_9 \to S_1|S_2 & S_{11} \to S_5|S_6 \\ S_1 \to aS_{10} & S_5 \to bS_{12} \\ S_2 \to bS_{10} & S_6 \to cS_{12} \\ S_{10} \to S_3|S_4 & S_{12} \to S_7|S_8 \\ S_3 \to aS_{15}|aS_{16} & S_7 \to bS_{16}|b \\ S_4 \to bS_{15}|bS_{16} & S_8 \to cS_{16}|c \\ S_{15} \to S_9 & S_{16} \to S_{11}|\varepsilon \end{array} \right\}, S_{15} \end{array} \right)$$

2.3.2 Приведение грамматики

1. Проверка пустоты

- Для леволинейной грамматики G'_{17}

$$
\begin{aligned}
C_0 &= \varnothing \\
C_2 &= \{S_9\} \cup C_1 = \{S_1, S_2, S_9\} \\
C_3 &= \{S_3, S_4, S_9\} \cup C_2 = \{S_1, S_2, S_3, S_4, S_9\} \\
C_4 &= \{S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \\
C_5 &= \{S_3, S_4, S_9, S_{10}, S_{15}\} \cup C_4 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}
\end{aligned}
$$

Так как

$$S = S_{16} \in C_{11} \Longrightarrow L(G'_{17}) \neq \varnothing \tag{13}$$

- Для праволинейной грамматики G''_{17}

$$
\begin{aligned}
C_0 &= \varnothing \\
C_1 &= \{S_7, S_8, S_{16}\} \cup C_0 = \{S_7, S_8, S_{16}\} \\
C_2 &= \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \\
C_3 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \cup C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\}
\end{aligned}
$$

Так как

$$S = S_{15} \in C_6 \Longrightarrow L(G''_{17}) \neq \varnothing \tag{14}$$

2. Удаление бесполезных символов

   - Для леволинейной грамматики G'_{17}

$$
\begin{aligned}
C_0 &= \varnothing \\
C_2 &= \{S_9\} \cup C_1 = \{S_1, S_2, S_9\} \\
C_3 &= \{S_3, S_4, S_9\} \cup C_2 = \{S_1, S_2, S_3, S_4, S_9\} \\
C_4 &= \{S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \\
C_5 &= \{S_3, S_4, S_9, S_{10}, S_{15}\} \cup C_4 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{15}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{15}\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph
\end{aligned}
$$

     Бесполезных символов нет, следовательно, грамматика G'_{17} не изменилась.

   - Для праволинейной грамматики G''_{17}

$$
\begin{aligned}
C_0 &= \varnothing \\
C_1 &= \{S_7, S_8, S_{16}\} \cup C_0 = \{S_7, S_8, S_{16}\} \\
C_2 &= \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_3, S_4, S_7, S_8, S_{12}, S_{16}\} \\
C_3 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \cup C_2 = \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{12}, S_{16}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_3 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{16}\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = C_6 = \aleph
\end{aligned}
$$

     Бесполезных символов нет, следовательно, грамматика G''_{17} не изменилась.

3. Удаление недостижимых символов

- Для леволинейной грамматики G'_{17}

$$
\begin{aligned}
C_0 &= \{S_{16}\} \\
C_1 &= \{S_{12}, S_{16}\} \cup C_0 = \{S_{12}, S_{16}\} \\
C_2 &= \{S_7, S_8, S_{12}, S_{16}\} \cup C_1 = \{S_7, S_8, S_{12}, S_{16}\} \\
C_3 &= \{S_7, S_8, S_{11}, S_{12}, S_{16}\} \cup C_2 = \{S_7, S_8, S_{11}, S_{12}, S_{16}\} \\
C_4 &= \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{16}\} \cup C_3 = \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{16}\} \\
C_5 &= \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_4 \\
&= \{S_5, S_6, S_7, S_8, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \\
C_6 &= \{S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_5 \\
&= \{S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \\
C_7 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_6 \\
&= \{S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \\
C_8 &= \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_7 \\
&= \{S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, b, c\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} = \Sigma \cup \aleph
\end{aligned}
$$

Недостижимых символов нет, следовательно, грамматика G'_{17} не изменилась.

- Для праволинейной грамматики G''_{17}

$$
\begin{aligned}
C_0 &= \{S_{15}\} \\
C_1 &= \{S_9\} \cup C_0 = \{S_9, S_{15}\} \\
C_2 &= \{S_1, S_2, S_9\} \cup C_1 = \{S_1, S_2, S_9, S_{15}\} \\
C_3 &= \{S_1, S_2, S_9, S_{10}\} \cup C_2 = \{S_1, S_2, S_9, S_{10}, S_{15}\} \\
C_4 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}\} \cup C_3 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}\} \\
C_5 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}, a, b\} \cup C_4 \\
&= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}, a, b\} \\
C_6 &= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \cup C_5 \\
&= \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \\
C_7 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \cup C_6 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}, a, b\} \\
C_8 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \cup C_7 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \\
C_9 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \cup C_8 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b\} \\
C_{10} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_9 \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \\
C_{11} &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_{10} \\
&= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}, a, b, c\} = \Sigma \cup \aleph
\end{aligned}
$$

  Недостижимых символов нет, следовательно, грамматика G''_{17} не изменилась.

4. Удаление пустых правил

   - Для леволинейной грамматики G'_{17}

$$
\begin{aligned}
C_0 &= \varnothing \\
C_1 &= \varnothing \cup C_0 = \varnothing = C_0
\end{aligned}
$$

     Пустых правил нет, следовательно, грамматика G'_{17} не поменялась.

   - Для праволинейной грамматики G''_{17}

$$
\begin{aligned}
C_0 &= \{S_{16}\} \\
C_1 &= \varnothing \cup C_0 = \{S_{16}\}
\end{aligned}
$$

     Итоговая грамматика G''_{18} без пустых правил и после добавления новых примет вид

$$
G''_{18} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_9 \to S_1 | S_2 & S_{11} \to S_5 | S_6 \\ S_1 \to aS_{10} & S_5 \to bS_{12} \\ S_2 \to bS_{10} & S_6 \to cS_{12} \\ S_{10} \to S_3 | S_4 & S_{12} \to S_7 | S_8 \\ S_3 \to aS_{15} | aS_{16} | a & S_7 \to bS_{16} | b \\ S_4 \to bS_{15} | bS_{16} | b & S_8 \to cS_{16} | c \\ S_{15} \to S_9 & S_{16} \to S_{11} \end{array} \right\}, S_{15} \end{array} \right)
$$

5. Удаление цепных правил

- Строим последовательность множеств $\aleph_i^X$ для леволинейной грамматики G'_{17}

$$\left.\begin{array}{l}\aleph_0^{S_0}=\{S_0\}\\ \aleph_1^{S_0}=\{S_0\}\end{array}\right\}\Longrightarrow \aleph^{S_0}=\varnothing \qquad \left.\begin{array}{l}\aleph_0^{S_1}=\{S_1\}\\ \aleph_1^{S_1}=\{S_1\}\end{array}\right\}\Longrightarrow \aleph^{S_1}=\varnothing$$

$$\left.\begin{array}{l}\aleph_0^{S_2}=\{S_2\}\\ \aleph_1^{S_2}=\{S_2\}\end{array}\right\}\Longrightarrow \aleph^{S_2}=\varnothing \qquad \left.\begin{array}{l}\aleph_0^{S_3}=\{S_3\}\\ \aleph_1^{S_3}=\{S_3\}\end{array}\right\}\Longrightarrow \aleph^{S_3}=\varnothing$$

$$\left.\begin{array}{l}\aleph_0^{S_4}=\{S_4\}\\ \aleph_1^{S_4}=\{S_4\}\end{array}\right\}\Longrightarrow \aleph^{S_4}=\varnothing \qquad \left.\begin{array}{l}\aleph_0^{S_5}=\{S_5\}\\ \aleph_1^{S_5}=\{S_5\}\end{array}\right\}\Longrightarrow \aleph^{S_5}=\varnothing$$

$$\left.\begin{array}{l}\aleph_0^{S_6}=\{S_6\}\\ \aleph_1^{S_6}=\{S_6\}\end{array}\right\}\Longrightarrow \aleph^{S_6}=\varnothing \qquad \left.\begin{array}{l}\aleph_0^{S_7}=\{S_7\}\\ \aleph_1^{S_7}=\{S_7\}\end{array}\right\}\Longrightarrow \aleph^{S_7}=\varnothing$$

$$\left.\begin{array}{l}\aleph_0^{S_8}=\{S_8\}\\ \aleph_1^{S_8}=\{S_8\}\end{array}\right\}\Longrightarrow \aleph^{S_8}=\varnothing$$

$$\left.\begin{array}{l}\aleph_0^{S_9}=\{S_9\}\\ \aleph_1^{S_9}=\{S_1,S_2,S_9\}\\ \aleph_2^{S_9}=\{S_1,S_2,S_9\}\end{array}\right\}\Longrightarrow \aleph^{S_9}=\{S_1,S_2\}$$

$$\left.\begin{array}{l}\aleph_0^{S_{10}}=\{S_{10}\}\\ \aleph_1^{S_{10}}=\{S_3,S_4,S_{10}\}\\ \aleph_2^{S_{10}}=\{S_3,S_4,S_{10}\}\end{array}\right\}\Longrightarrow \aleph^{S_{10}}=\{S_3,S_4\}$$

$$\left.\begin{array}{l}\aleph_0^{S_{11}}=\{S_{11}\}\\ \aleph_1^{S_{11}}=\{S_5,S_6,S_{11}\}\\ \aleph_2^{S_{11}}=\{S_5,S_6,S_{11}\}\end{array}\right\}\Longrightarrow \aleph^{S_{11}}=\{S_5,S_6\}$$

$$\left.\begin{array}{l}\aleph_0^{S_{12}}=\{S_{12}\}\\ \aleph_1^{S_{12}}=\{S_7,S_8,S_{12}\}\\ \aleph_2^{S_{12}}=\{S_7,S_8,S_{12}\}\end{array}\right\}\Longrightarrow \aleph^{S_{12}}=\{S_7,S_8\}$$

$$\left.\begin{array}{l}\aleph_0^{S_{15}}=\{S_{15}\}\\ \aleph_1^{S_{15}}=\{S_{10},S_{15}\}\\ \aleph_2^{S_{15}}=\{S_3,S_4,S_{10},S_{15}\}\\ \aleph_3^{S_{15}}=\{S_3,S_4,S_{10},S_{15}\}\end{array}\right\}\Longrightarrow \aleph^{S_{15}}=\{S_3,S_4,S_{10}\}$$

$$\left.\begin{array}{l}\aleph_0^{S_{16}}=\{S_{16}\}\\ \aleph_1^{S_{16}}=\{S_{12},S_{15},S_{16}\}\\ \aleph_2^{S_{16}}=\{S_7,S_8,S_{10},S_{12},S_{15},S_{16}\}\\ \aleph_3^{S_{16}}=\{S_3,S_4,S_7,S_8,S_{10},S_{12},S_{15},S_{16}\}\\ \aleph_4^{S_{16}}=\{S_3,S_4,S_7,S_8,S_{10},S_{12},S_{15},S_{16}\}\end{array}\right\}\Longrightarrow \aleph^{S_{16}}=\{S_3,S_4,S_7,S_8,S_{10},S_{12},S_{15}\}$$

Множество правил P'_{18} содержит все правила грамматики G'_{17} кроме цепных:

$$P'_{18}=\left\{\begin{array}{ll}S_1\to S_{15}a|a & S_5\to S_{16}b|S_{15}b\\ S_2\to S_{15}b|b & S_6\to S_{16}c|S_{15}c\\ S_3\to S_9a & S_7\to S_{11}b\\ S_4\to S_9b & S_8\to S_{11}c\end{array}\right\}$$

С добавлением новых правил, опираясь на соотношение вида

$$P'_{18} = P'_{18} \cup \left\{ (B \to \alpha) | \forall (A \to \alpha) \in P, A \in \aleph^B \right\},$$

то есть

$$P'_{18} = P'_{18} \cup \left\{ \begin{array}{ll} S_9 \to S_{15}a|a|S_{15}b|b & S_{10} \to S_9a|S_9b \\ S_{11} \to S_{16}b|S_{15}b|S_{16}c|S_{15}c & S_{12} \to S_{11}b|S_{11}c \\ S_{15} \to S_9a|S_9b & S_{16} \to S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}$$

Таким образом, результирующая грамматика G'_{18} примет следующий вид

$$G'_{18} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_1 \to S_{15}a|a & S_5 \to S_{16}b|S_{15}b \\ S_2 \to S_{15}b|b & S_6 \to S_{16}c|S_{15}c \\ S_3 \to S_9a & S_7 \to S_{11}b \\ S_4 \to S_9b & S_8 \to S_{11}c \\ S_9 \to S_{15}a|a|S_{15}b|b & S_{10} \to S_9a|S_9b \\ S_{11} \to S_{16}b|S_{15}b|S_{16}c|S_{15}c & S_{12} \to S_{11}b|S_{11}c \\ S_{15} \to S_9a|S_9b & S_{16} \to S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}, S_{16} \end{array} \right)$$

- Строим последовательность множеств $\aleph_i^X$ для праволинейной грамматики G''_{18}

$$\left\{ \begin{array}{l} \aleph_0^{S_0} = \{S_0\} \\ \aleph_1^{S_0} = \{S_0\} \end{array} \right\} \Longrightarrow \aleph^{S_0} = \varnothing \quad \left\{ \begin{array}{l} \aleph_0^{S_1} = \{S_1\} \\ \aleph_1^{S_1} = \{S_1\} \end{array} \right\} \Longrightarrow \aleph^{S_1} = \varnothing$$

$$\left\{ \begin{array}{l} \aleph_0^{S_2} = \{S_2\} \\ \aleph_1^{S_2} = \{S_2\} \end{array} \right\} \Longrightarrow \aleph^{S_2} = \varnothing \quad \left\{ \begin{array}{l} \aleph_0^{S_3} = \{S_3\} \\ \aleph_1^{S_3} = \{S_3\} \end{array} \right\} \Longrightarrow \aleph^{S_3} = \varnothing$$

$$\left\{ \begin{array}{l} \aleph_0^{S_4} = \{S_4\} \\ \aleph_1^{S_4} = \{S_4\} \end{array} \right\} \Longrightarrow \aleph^{S_4} = \varnothing \quad \left\{ \begin{array}{l} \aleph_0^{S_5} = \{S_5\} \\ \aleph_1^{S_5} = \{S_5\} \end{array} \right\} \Longrightarrow \aleph^{S_5} = \varnothing$$

$$\left\{ \begin{array}{l} \aleph_0^{S_6} = \{S_6\} \\ \aleph_1^{S_6} = \{S_6\} \end{array} \right\} \Longrightarrow \aleph^{S_6} = \varnothing \quad \left\{ \begin{array}{l} \aleph_0^{S_7} = \{S_7\} \\ \aleph_1^{S_7} = \{S_7\} \end{array} \right\} \Longrightarrow \aleph^{S_7} = \varnothing$$

$$\left\{ \begin{array}{l} \aleph_0^{S_8} = \{S_8\} \\ \aleph_1^{S_8} = \{S_8\} \end{array} \right\} \Longrightarrow \aleph^{S_8} = \varnothing$$

$$\left\{ \begin{array}{l} \aleph_0^{S_9} = \{S_9\} \\ \aleph_1^{S_9} = \{S_1, S_2, S_9\} \\ \aleph_2^{S_9} = \{S_1, S_2, S_9\} \end{array} \right\} \Longrightarrow \aleph^{S_9} = \{S_1, S_2\}$$

$$\left\{ \begin{array}{l} \aleph_0^{S_{10}} = \{S_{10}\} \\ \aleph_1^{S_{10}} = \{S_3, S_4, S_{10}\} \\ \aleph_2^{S_{10}} = \{S_3, S_4, S_{10}\} \end{array} \right\} \Longrightarrow \aleph^{S_{10}} = \{S_3, S_4\}$$

$$\left\{ \begin{array}{l} \aleph_0^{S_{11}} = \{S_{11}\} \\ \aleph_1^{S_{11}} = \{S_5, S_6, S_{11}\} \\ \aleph_2^{S_{11}} = \{S_5, S_6, S_{11}\} \end{array} \right\} \Longrightarrow \aleph^{S_{11}} = \{S_5, S_6\}$$

$$\left\{ \begin{array}{l} \aleph_0^{S_{12}} = \{S_{12}\} \\ \aleph_1^{S_{12}} = \{S_7, S_8, S_{12}\} \\ \aleph_2^{S_{12}} = \{S_7, S_8, S_{12}\} \end{array} \right\} \Longrightarrow \aleph^{S_{12}} = \{S_7, S_8\}$$

$$\left\{ \begin{array}{l} \aleph_0^{S_{15}} = \{S_{15}\} \\ \aleph_1^{S_{15}} = \{S_9, S_{15}\} \\ \aleph_2^{S_{15}} = \{S_1, S_2, S_9, S_{15}\} \\ \aleph_3^{S_{15}} = \{S_1, S_2, S_9, S_{15}\} \end{array} \right\} \Longrightarrow \aleph^{S_{15}} = \{S_1, S_2, S_9\}$$

$$\left\{ \begin{array}{l} \aleph_0^{S_{16}} = \{S_{16}\} \\ \aleph_1^{S_{16}} = \{S_{11}, S_{16}\} \\ \aleph_2^{S_{16}} = \{S_5, S_6, S_{11}, S_{16}\} \\ \aleph_3^{S_{16}} = \{S_5, S_6, S_{11}, S_{16}\} \end{array} \right\} \Longrightarrow \aleph^{S_{16}} = \{S_5, S_6, S_{11}\}$$

Множество правил P''_{19} содержит все правила грамматики G''_{18} кроме цепных:

$$P''_{19} = \left\{ \begin{array}{ll} S_1 \to aS_{10} & S_5 \to bS_{12} \\ S_2 \to bS_{10} & S_6 \to cS_{12} \\ S_3 \to aS_{15} | aS_{16} | a & S_7 \to bS_{16} | b \\ S_4 \to bS_{15} | bS_{16} | b & S_8 \to cS_{16} | c \end{array} \right\}$$

С добавлением новых правил, опираясь на соотношение вида

$$P''_{19} = P''_{19} \cup \left\{ (B \to \alpha) | \forall (A \to \alpha) \in P, A \in \aleph^B \right\},$$

то есть

$$P''_{19} = P''_{19} \cup \left\{ \begin{array}{ll} S_9 \to aS_{10}|bS_{10} & S_{10} \to aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b \\ S_{11} \to bS_{12}|cS_{12} & S_{12} \to bS_{16}|b|cS_{16}|c \\ S_{15} \to aS_{10}|bS_{10} & S_{16} \to bS_{12}|cS_{12} \end{array} \right\}$$

Таким образом, результирующая грамматика G'_{18} примет следующий вид

$$G''_{19} = \left(\begin{array}{l} \{S_9, S_1, S_2, S_{10}, S_3, S_4, S_{15}, S_{11}, S_5, S_6, S_{12}, S_7, S_8, S_{16}\}, \Sigma, \\ \left\{ \begin{array}{ll} S_1 \to aS_{10} & S_5 \to bS_{12} \\ S_2 \to bS_{10} & S_6 \to cS_{12} \\ S_3 \to aS_{15}|aS_{16}|a & S_7 \to bS_{16}|b \\ S_4 \to bS_{15}|bS_{16}|b & S_8 \to cS_{16}|c \\ S_9 \to aS_{10}|bS_{10} & S_{10} \to aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b \\ S_{11} \to bS_{12}|cS_{12} & S_{12} \to bS_{16}|b|cS_{16}|c \\ S_{15} \to aS_{10}|bS_{10} & S_{16} \to bS_{12}|cS_{12} \end{array} \right\}, S_{15} \end{array} \right)$$

Так как при удалении пустых правил и цепных правил лево- и праволинейной грамматик произошло их изменение, то необходимо повторить удаление бесполезных и недостижимых символов.

6. Удаление бесполезных символов грамматик G'_{18} и G''_{19}

   - Для леволинейной грамматики G'_{18}

$$\begin{aligned} C_0 &= \varnothing \\ C_1 &= \{S_1, S_2, S_9\} \cup C_0 = \{S_1, S_2, S_9\} \\ C_2 &= \{S_3, S_4, S_{10}, S_{15}, S_{16}\} \cup C_1 = \{S_1, S_2, S_3, S_4, S_9, S_{10}, S_{15}, S_{16}\} \\ C_3 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}\} \cup C_2 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_9, S_{10}, S_{11}, S_{15}, S_{16}\} \\ C_4 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_3 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\ C_5 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_4 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph \end{aligned}$$

     Бесполезных символов нет, следовательно, грамматика G'_{18} не изменилась.

   - Для праволинейной грамматики G''_{19}

$$\begin{aligned} C_0 &= \varnothing \\ C_1 &= \{S_3, S_4, S_7, S_8, S_{10}, S_{12}\} \cup C_0 = \{S_3, S_4, S_7, S_8, S_{10}, S_{12}\} \\ C_2 &= \{S_1, S_2, S_5, S_6, S_9, S_{11}, S_{15}, S_{16}\} \cup C_1 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \\ C_3 &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} \cup C_2 \\ &= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{15}, S_{16}\} = \aleph \end{aligned}$$

     Бесполезных символов нет, следовательно, грамматика G''_{19} не изменилась.

7. Удаление недостижимых символов грамматик G'_{18} и G''_{19}

   - Для леволинейной грамматики G'_{18}

$$
\begin{aligned}
&C_0 = \{S_{16}\}\\
&C_1 = \{S_9, S_{11}, a, b, c\} \cup C_0 = \{S_9, S_{11}, S_{16}, a, b, c\}\\
&C_2 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \cup C_1 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\}\\
&C_3 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\} \cup C_2 = \{S_9, S_{11}, S_{15}, S_{16}, a, b, c\}
\end{aligned}
$$

   Строим результирующую грамматику G'_{19} без недостижимых символов

$$
\begin{aligned}
&\aleph'_{19} = \aleph'_{18} \cap C_3 = \{S_9, S_{11}, S_{15}, S_{16}\}\\
&\Sigma'_{19} = \Sigma'_{18} \cap C_3 = \{a, b, c\}\\
&P'_{19} = \{(A \to \alpha) | \forall (A \to \alpha) \in P'_{18}, A \in \aleph'_{19}, \alpha \in (\Sigma'_{19} \cup \aleph'_{19})^*\} =\\
&\quad = \left\{ \begin{array}{ll} S_9 \to S_{15}a|a|S_{15}b|b & S_{11} \to S_{16}b|S_{15}b|S_{16}c|S_{15}c\\ S_{15} \to S_9a|S_9b & S_{16} \to S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}\\
&S'_{19} \equiv S_{16}
\end{aligned}
$$

   Таким образом, результирующая грамматика G'_{19} примет вид

$$
G'_{19} = \left(\begin{array}{l} \{S_9, S_{11}, S_{15}, S_{16}\}, \{a, b, c\},\\ \left\{ \begin{array}{ll} S_9 \to S_{15}a|a|S_{15}b|b & S_{11} \to S_{16}b|S_{15}b|S_{16}c|S_{15}c\\ S_{15} \to S_9a|S_9b & S_{16} \to S_{11}b|S_{11}c|S_9a|S_9b \end{array} \right\}, S_{16} \end{array} \right)
$$

   - Для праволинейной грамматики G''_{19}

$$
\begin{aligned}
&C_0 = \{S_{15}\}\\
&C_1 = \{S_{10}, b\} \cup C_0 = \{S_{10}, S_{15}, b\}\\
&C_2 = \{S_{10}, S_{15}, S_{16}, a, b\} \cup C_1 = \{S_{10}, S_{15}, S_{16}, a, b\}\\
&C_3 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_2 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\}\\
&C_4 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\} \cup C_3 = \{S_{10}, S_{12}, S_{15}, S_{16}, a, b, c\}
\end{aligned}
$$

   Строим результирующую грамматику G''_{20} без недостижимых символов

$$
\begin{aligned}
&\aleph''_{20} = \aleph''_{19} \cap C_4 = \{S_{10}, S_{12}, S_{15}, S_{16}\}\\
&\Sigma''_{20} = \Sigma''_{19} \cap C_4 = \{a, b, c\}\\
&P''_{20} = \{(A \to \alpha) | \forall (A \to \alpha) \in P''_{19}, A \in \aleph''_{20}, \alpha \in (\Sigma''_{20} \cup \aleph''_{20})^*\} =\\
&\quad = \left\{ \begin{array}{ll} S_{10} \to aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b & S_{12} \to bS_{16}|b|cS_{16}|c\\ S_{15} \to aS_{10}|bS_{10} & S_{16} \to bS_{12}|cS_{12} \end{array} \right\}\\
&S''_{20} \equiv S_{15}
\end{aligned}
$$

   Таким образом, результирующая грамматика G''_{20} примет вид

$$
G''_{20} = \left(\begin{array}{l} \{S_{10}, S_{12}, S_{15}, S_{16}\}, \{a, b, c\},\\ \left\{ \begin{array}{ll} S_{10} \to aS_{15}|aS_{16}|a|bS_{15}|bS_{16}|b & S_{12} \to bS_{16}|b|cS_{16}|c\\ S_{15} \to aS_{10}|bS_{10} & S_{16} \to bS_{12}|cS_{12} \end{array} \right\}, S_{15} \end{array} \right)
$$

2.3.3 Построение конечного автомата для приведенной грамматики

1. Приведение к автоматному виду Все правила в заданной грамматике имеют вид

$$P'_{19} \subset \{A \to Bx | x \colon A, B \in \aleph, x \in \Sigma\}$$

для леволинейной грамматики, а для праволинейной

$$P''_{20} \subset \{A \to xB | x \colon A, B \in \aleph, x \in \Sigma\}$$

А это в свою очередь значит, по построению, что правила данных грамматик G'_{19} и G''_{20} удовлетворяют определению автоматной грамматики, а, значит, изменение данных грамматик не производится.

2. Построение конечных автоматов $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ и $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ для автоматных грамматик G'_{19} и G''_{20}.

   - Построение автомата $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ для леволинейной грамматики производится следующим образом:
     - Множество состояний состоит из именуемых нетерминалы состояний;
     - Добавляется новое состояние — начальное (на наименование действуют соглашения по наименованию нетерминалов грамматик)

     Таким образом

$$Q_1 = \aleph'_{19} \cup \{H\} = \{H, S_9, S_{11}, S_{15}, S_{16}\}$$

     Начальное состояние:

$$q_1 \equiv H$$

     Множество заключительных состояний содержит целевой символ исходной грамматики

$$F = \{S_{16}\}$$

     Множество переходов:

$$\begin{array}{lll} \delta_1(S_{15}, a) = \{S_9\} & \delta_1(S_{15}, b) = \{S_9, S_{11}\} & \delta(S_{15}, c) = \{S_{11}\} \\ \delta_1(S_{16}, b) = \{S_{11}\} & \delta_1(S_{16}, c) = \{S_{11}\} & \\ \delta_1(S_9, a) = \{S_{15}, S_{16}\} & \delta_1(S_9, b) = \{S_{15}, S_{16}\} & \\ \delta_1(S_{11}, b) = \{S_{16}\} & \delta_1(S_{11}, c) = \{S_{16}\} & \\ \delta_1(H, a) = \{S_9\} & \delta_1(H, b) = \{S_9\} & \end{array}$$

   - Построение автомата $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ для леволинейной грамматики производится следующим образом:
     - Множество состояний состоит из именуемых нетерминалы состояний;
     - Добавляется новое состояние — заключительное (на наименование действуют соглашения по наименованию нетерминалов грамматик)

Таким образом

$$Q_2 = \aleph''_{20} \cup \{F\} = \{F, S_10, S_{12}, S_{15}, S_{16}\}$$

Начальное состояние — состояние, соответствующее целевому символу исходной грамматики:

$$q_2 \equiv S_{15}$$

Множество заключительных состояний будет содержать новое состояние

$$F_2 = \{F\}$$

Множество переходов:

$$\begin{array}{ll} \delta_2(S_{16}, b) = \{S_{12}\} & \delta_2(S_{16}, c) = \{S_{12}\} \\ \delta_2(S_{12}, b) = \{S_{16}, F\} & \delta_2(S_{12}, c) = \{S_{16}, F\} \\ \delta_2(S_{10}, a) = \{S_{15}, S_{16}, F\} & \delta_2(S_{10}, b) = \{S_{15}, S_{16}, F\} \\ \delta_2(S_{15}, a) = \{S_{10}\} & \delta_2(S_{15}, b) = \{S_{10}\} \end{array}$$

На этом построение конечных автоматов по автоматным грамматикам заканчивается

3. Построение диаграммы состояний автомата M