Lecture1.2 - Linearity

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Definition

A linear function $f(x)$ is one with the property

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

Problems

1. Prove that $f(x) = \beta x$ is linear.

$$\begin{aligned} f(x) &= \beta x \\ f(ax_1 + bx_2) &= \beta(ax_1 + bx_2) \\ &= a\beta x_1 + b\beta x_2 \\ &= \beta a x_1 + \beta b x_2 \\ &= af(x_a) + bf(x_2) \end{aligned}$$

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

$$qed.$$

2. Prove that $f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ is *not* linear.

$$\begin{aligned} f(x) &= \beta_0 + \beta_1 x + \beta_2 x^2 \\ f'(x) &= \beta_1 x + \beta_2 * 2x_2 \end{aligned}$$

$$f''(x) = \beta_2 * 2$$

This result does not show linearity. However, as a model is linear if the beta coefficients are linear, we will look at the coefficients instead of the predictors:

$$f(\beta) = \beta_0 + \beta_1 x_1 + \beta x_2^2$$

$$f'(\beta) = 1 + x_1 + x_2^2$$

$$f''(\beta) = 0$$

$$qed.$$

The model $f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ is linear.