



Ecuación principal

$$F_a(t) = F_z(t) + F_L(t) = F_C(t) + F_R(t)$$

$$F_z(t) = \frac{P_a(t) - P_p(t)}{Z}$$

$$F_C(t) = C \frac{\partial P_p(t)}{\partial t}$$

$$F_L(t) = \frac{1}{L} \int [P_a(t) - P_p(t)] dt$$

$$F_R(t) = \frac{P_p(t)}{R}$$

Procedimiento algebraico

$$\frac{P_a(t)}{Z} - \frac{P_p(t)}{Z} + \frac{1}{L} \int [P_a(t) - P_p(t)] dt = C \frac{\partial P_p(t)}{\partial t} + \frac{P_p(t)}{R}$$

Transformada de Laplace

$$\frac{P_a(s)}{Z} - \frac{P_p(s)}{Z} + \frac{P_a(s) - P_p(s)}{LS} = CS P_p(s) + \frac{P_p(s)}{R}$$

$$\left(\frac{1}{Z} + \frac{1}{LS} \right) P_a(s) = \left(CS + \frac{1}{R} + \frac{1}{Z} + \frac{1}{LS} \right) P_p(s)$$

$$\frac{P_p(s)}{P_a(s)} = \frac{\left(\frac{1}{Z} + \frac{1}{LS} \right)}{\left(CS + \frac{1}{R} + \frac{1}{Z} + \frac{1}{LS} \right)} = \frac{\frac{Z+LS}{ZLS}}{\frac{CRZLS^2 + ZLS + RLS + RZ}{RZLS}}$$

$$= \frac{(Z+LS)RZLS}{(ZLS)(CRZLS^2 + ZLS + RLS + RZ)} = \frac{RZ + RLS}{CRZLS^2 + (ZL + RL)S + RZ}$$

Función de transferencia

$$\frac{P_p(s)}{P_a(s)} = \frac{RZ + RLS}{CRZLS^2 + (ZL + RL)S + RZ}$$

Error en estado estacionario

$$e(s) = \lim_{s \rightarrow 0} s P_a(s) \left[1 - \frac{P_p(s)}{P_a(s)} \right]$$

$$e(s) = \lim_{s \rightarrow 0} s \left(\frac{1}{s} \right) \left[1 - \frac{RLs + Rz}{CLRzs^2 + (Lz + RL)s + Rz} \right]$$

$$e(s) = 1 - \frac{Rz}{Rz} = 0V$$

Estabilidad en lazo abierto

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$$

$$a = CLRz \quad b = Lz + RL \quad c = Rz$$

$$\lambda_{1,2} = \frac{-(Lz + RL) \pm \sqrt{(Lz + RL)^2 - 4(CLRz)(Rz)}}{2CLRz}$$

El sistema tiene una respuesta estable porque $\text{Re} \lambda_{1,2} < 0$

Modelo de ecuaciones integrodiferenciales

$$P_p(t) \left(\frac{1}{R} + \frac{1}{z} \right) = \frac{P_a(t)}{z} + \frac{1}{L} \int [P_a(t) - P_p(t)] dt - \frac{C \partial P_p(t)}{\partial t}$$

$$P_p(t) = \left(\frac{P_a(t)}{z} + \frac{1}{L} \int [P_a(t) - P_p(t)] dt - \frac{C \partial P_p(t)}{\partial t} \right) \frac{zR}{z + R}$$