(e) $\int \frac{x+1}{x^3-x^2-6x} \Rightarrow \frac{x+1}{x(x^2-x-6)} \Rightarrow \frac{x+1}{x(x^2+2x-3x-6)} \Rightarrow \frac{x+1}{x(x(x+2)-3(x+2))}$

$\frac{x+1}{x(x+2)(x-3)} \Rightarrow \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3} \Rightarrow \frac{x+1}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3} \Rightarrow$

$x+1 = (x+2)(x-3)A + x(x-3)B + x(x+2)C \Rightarrow$

$x+1 = Ax^2 - Ax - 6A + Bx^2 - 3Bx + Cx^2 + 2Cx \Rightarrow$

$x+1 = Ax^2 + Bx^2 + Cx^2 - Ax - 3Bx + 2Cx - 6A \Rightarrow$

$x+1 = (A+B+C)x^2 + (-A-3B+2C)x - 6A \Rightarrow$

$\begin{cases} 1 = -6A \\ 1 = -A-3B+2C \\ 0 = A+B+C \end{cases}$

$1 = -6A \Rightarrow A = -1/6$

$1 = 1/6 - 3B + 2C \Rightarrow C = \frac{3B - 1/6 + 1}{2} \Rightarrow C = 4/15$

$0 = -1/6 + B + \left(\frac{3B - 1/6 + 1}{2}\right) \Rightarrow B = -1/10$

$\frac{-1/6}{x} + \frac{-1/10}{x+2} + \frac{4/15}{x-3} \Rightarrow -\frac{1/6}{x} - \frac{1/10}{x+2} + \frac{4}{15(x-3)} \Rightarrow -\frac{1}{6x} - \frac{1}{10(x+2)} + \frac{4}{15(x-3)} \Rightarrow$

$-\int \frac{1}{6x}\,dx - \int \frac{1}{10(x+2)}\,dx + \int \frac{4}{15(x-3)}\,dx \Rightarrow -\frac{1}{6}\ln(|x|) - \frac{1}{10}\ln(|x+2|) + \frac{4}{15}\ln(|x-3|)$

$-\frac{1}{6}\ln|x| - \frac{1}{10}\ln|x+2| + \frac{4}{15}\ln|x-3| + C \Rightarrow$

$$-\frac{1}{6}\ln|x| + \frac{4}{15}\ln|x-3| - \frac{1}{10}\ln|x+2| + C$$

(f) $\int \frac{x^2}{(x-1)^3}\,dx$; $t = x-1 \Rightarrow \int \frac{t^2+2t+1}{t^3} \Rightarrow \int \frac{t^2}{t^3} + \frac{2t}{t^3} + \frac{1}{t^3}\,dt$

$\int \frac{1}{t} + \frac{2}{t^2} + \frac{1}{t^3}\,dt \Rightarrow \int \frac{1}{t}\,dt + \int \frac{2}{t^2}\,dt + \int \frac{1}{t^3}\,dt \Rightarrow$

$\ln(|t|) - \frac{2}{t} - \frac{1}{2t^2} \Rightarrow \ln(|x-1|) - \frac{2}{x-1} - \frac{1}{2(x-1)^2} \Rightarrow$

$\ln|x-1| - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + C \Rightarrow \frac{-1}{2(x-1)^2} - \frac{2}{x-1} + \ln|x-1| + C$

① $\int \frac{5x^2+3x-2}{x^3+2x} \Rightarrow \frac{5x^2+3x-2}{x(x+2)} \Rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} \Rightarrow$

$\frac{5x^2+3x-2}{x(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} \Rightarrow 5x^2+3x-2 = x(x+2)A + (x+2)B + Cx^2 \Rightarrow$

$5x^2+3x-2 = Ax^2+2Ax+Bx+2B+Cx^2 \Rightarrow 5x^2+3x-2 = Ax^2+Cx^2+2Ax+Bx+2B \Rightarrow$

$5x^2+3x-2 = (A+C)x^2+(2A+B)x+2B$

$$\begin{cases} -2 = 2B \\ 3 = 2A+B \\ 5 = A+C \end{cases}$$

$-2 = 2B \Rightarrow B = -1$

$3 = 2A-1 \Rightarrow A = 2$

$5 = 2+C \Rightarrow C = 3$

$\frac{2}{x} + \frac{-1}{x^2} + \frac{3}{x+2} \Rightarrow \frac{2}{x} - \frac{1}{x^2} + \frac{3}{x+2} \Rightarrow \int \frac{2}{x}dx - \int \frac{1}{x^2}dx + \int \frac{3}{x+2}dx \Rightarrow$

$2\ln(|x|) + \frac{1}{x} + 3\ln(|x+2|) \Rightarrow 2\ln(|x|) + \frac{1}{x} + 3\ln(|x+2|) + C \Rightarrow$

$$3\ln|x+2| + 2\ln|x| + \frac{1}{x} + C$$

⑧ $\int \frac{9x^2-28x+12}{x^3-5x^2+6x} \Rightarrow \frac{3x^2-28x+12}{x(x^2-2x-3x+6)} \Rightarrow \frac{3x^2-28x+12}{x(x(x-2)-3(x-2))} \Rightarrow$

$\frac{3x^2-28x+12}{x(x-2).(x-3)} \Rightarrow \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-3} \Rightarrow$

$\frac{3x^2-28x+12}{x(x-2).(x-3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-3} \Rightarrow$

$3x^2-28x+12 = (x-2).(x-3)A + x.(x-3)B + x(x-2)C \Rightarrow$

$3x^2-28x+12 = Ax^2-5Ax+6A+Bx^2-3Bx+Cx^2-2Cx \Rightarrow$

$3x^2-28x+12 = Ax^2+Bx^2+Cx^2-5Ax-3Bx-2Cx+6A \Rightarrow$

$3x^2-28x+12 = (A+B+C)x^2+(-5A-3B-2C)x+6A \Rightarrow$

$$\begin{cases} 12 = 6A \\ -28 = -5A-3B-2C \\ 3 = A+B+C \end{cases}$$

$12 = 6A \Rightarrow A = \frac{12}{6} \Rightarrow A = 2$

$-28 = -5.2-3B-2C \Rightarrow B = \frac{-2C+18}{3} \Rightarrow B = 4$

$3 = \underset{1}{\frac{2}{1}} + \frac{(-2C+18)}{3} + \underset{1}{\frac{C}{1}} \Rightarrow C = 3$

$\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-3} \Rightarrow \frac{2}{x} + \frac{4}{x-2} + \frac{3}{x-3} \Rightarrow \int \frac{2}{x}dx + \int \frac{4}{x-2}dx + \int \frac{3}{x-3}dx \Rightarrow$

$2\ln|x| + 4\ln|x-2| + 3\ln|x-3| \Rightarrow$

$2\ln|x| + 4\ln|x-2| + 3\ln|x-3| + C \Rightarrow$

$2\ln|x| + 4\ln|x-2| + 3\ln|x-3| + C$