Financial Econometrics

Performance Evaluation and Portfolio Construction

for USA mutual funds

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$$S_i = \frac{E(R_i) - r_f}{\sigma_i}$$

Introduction

In this project the goal is to analyze the monthly returns of USA mutual funds for the period 01/1998 – 07/2019, based on the returns of some factors. The first part of the process has to do with Performance Evaluation, where different performance measures will be used to assess the performance of the funds. After that, I will construct optimal portfolios using different approaches and evaluate them. The series I must analyze are from 51 to 90 (40 funds in total).

Performance Evaluation

I am interested in analyzing the mutual funds for the initial period that starts 01/1998 and ends 07/2016. Then I will evaluate their performance for the last three years (i.e., 08/2016-07/2019), which is the out-of-sample period. For this period, I will construct equally weighted portfolios for the out-of-sample period based on the top 20% selected funds. To assess the performance of these funds, I will use different performance evaluation measures.

The initial period is 223 months and the out-of-sample period lasts 36 months. Since I need to construct portfolios based on the top 20% funds, I will keep only the best 8 funds. I also compute the in-sample fund and factor returns, as well as the out-of-sample returns of the funds, in order to estimate each measure.

First, I will give a short description of each measure and how I computed them, and I will present the results after.

Sharpe ratio

The Sharpe performance measure, which is known as the reward-to-volatility ratio, considers the expected returns as well as the total investment risk σ_i. It is written as:

$$S_i = \frac{E(R_i) - r_f}{\sigma_i}$$

First, I compute the mean and standard deviation of the returns and then the Sharpe Ratio. Then I sort it in ascending order, in order to keep the top 8 funds. Based on these, I construct the equally weighed portfolios by estimating their mean and cumulative returns for the out-of-sample period.

Treynor ratio

It is like the Sharpe ratio; however, it considers the systematic risk β_i (or security beta), and not the total investment risk. The Treynor ratio indicates the risk premium per unit of risk.

$$T_i = \frac{E(R_i) - r_f}{\beta_i}$$

To compute this ratio, I estimated the β_i's, so I ran a linear regression with the funds' returns as the response variable and the market factor as the explanatory variable. I keep the β_i's for each fund and then compute the ratio. The rest of the procedure is the same as the Sharpe ratio.

Sortino ratio

This ratio is estimated using a reference value called the Minimal acceptable return. As this reference value I used the mean return of each fund. Here the downside risk δ_i is used, which is the risk that follows the fall of a stock or in this case the negative returns of the funds.

$$SoRi_i = \frac{E(R_i) - R_{MAR}}{\delta_i}$$

The downside risk is given by the formula: $\sqrt{\frac{1}{T}\sum_{t=1}^{T} \min{(0, R_{i,t} - R_{MAR})^2}}$.

I computed the δ_i's of each fund using this formula and then with mean returns I estimated the Sortino ratio. The rest of the steps as the same as before.

Jensen's Alpha

In order to estimate the Jensen's Alpha, I considered the following models:

a. Single factor model
b. Multiple regression models

The single factor model is written as:

$$R_{i,t} - r_f = a_i + \beta_i\left(R_{M,t} - r_f\right) + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} \sim N(0, \sigma_i^2)$$

For the market factor I used the S&P 500 return index.

Here I run a regression model for each fund using the market factor as the explanatory variable and I keep the a_i's, i.e., the intercepts of each model. Based on these a_i's, I find the top 8 funds, after I sort them. Again, I construct the equally weighted portfolios by estimating the mean and cumulative returns of the out-of-sample period.

For the multiple regression models, I used the backward elimination technique to find the best regressors for each fund. The backward selection method starts with all the predictors in the model, iteratively removes the least contributive predictors, and stops when all the predictors are statistically significant. The best model is chosen by the AIC criterion.

The models are written in the form:

$$Y_t = a + \beta_i X_{1,t} + \cdots + \beta_k X_{k,t} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

For each fund, the backward elimination method found different regressors. I kept the a's from each fund and I used them to construct the portfolios.

Results

Now I will present the results and comment on them.

The top funds for each performance measure (in descending order) are shown in the table below.

	Performance Measures				
	Sharpe ratio	Treynor ratio	Sortino ratio	Jensen's alpha (single)	Jensen's alpha (multiple)
Top Funds	10	10	10	10	16
	16	33	16	33	18
	8	16	8	16	1
	33	8	33	8	34
	1	1	1	1	31
	23	29	23	29	32
	29	23	29	23	22
	18	32	18	32	4

Table 1: Top funds for each performance measure

We can see that for all the measures except the Jensen's alpha for the multiple regression models, the top fund is the 10^{th}. The Sharpe and Sortino ratio have the same top funds, and the same happens for the Treynor ratio and Jensen's alpha for the single factor model. This means that the measures with the same top funds will have the same cumulative returns. Only Jensen's alpha for the multiple factor models has some different funds that the other measures have not chosen. We can also see that this measure has greater cumulative returns than the others (see Figure 1). Jensen's alpha for the single factor model (orange) and the Treynor ratio (magenta) overlap, and the same applies to the Sharpe(black) and Sortino ratios (blue).

Even though the multiple factors Jensen's alpha starts with negative returns and stays lower than the other measures for some period, it quickly rises above them and gains greater cumulative returns. Single factor Jensen's alpha and Treynor perform poorly compared to the other measures. Perhaps multiple factors Jensen's alpha has such great cumulative returns because it considers more factors that can affect each fund's returns and not only the market factor, making it a more flexible performance measure.

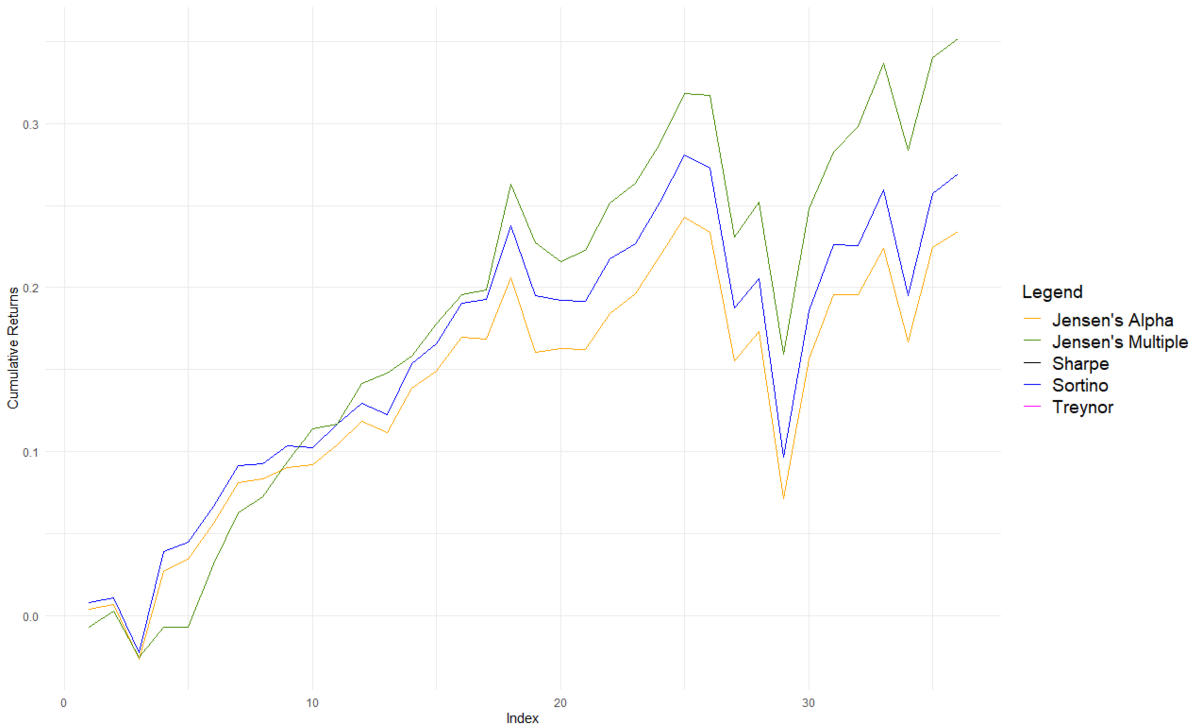

Figure 1: Cumulative returns for each performance measure

Portfolio Construction

The second part of this project is about portfolio construction. The goal is to construct minimum and mean variance portfolios.

The minimum and mean variance portfolios are of the form

$$\min_{w} \frac{1}{2} V\left(R_{p,t}\right) = \frac{1}{2} w' \Sigma_t w \qquad\qquad \min_{w} \frac{1}{2} V\left(R_{p,t}\right) = \frac{1}{2} w' \Sigma_t w$$

$$w_i \geq 0,\ \sum_{i=1}^{n} w_i = 1 \qquad\qquad w_i \geq 0,\ \sum_{i=1}^{n} w_i = 1$$

$$E\left(R_{p,t}\right) \geq r_{target}$$

respectively.

To estimate the mean vector and the covariance matrix of returns I used the following approaches:

a. Sample estimate of mean vector and covariance matrix
b. Single index model
c. Multivariate multiple regression model
d. Constant Conditional Correlation (CCC) model

For both portfolio constructions (minimum and mean variance), I assume to be in a long position since I want the portfolio weights to be positive.

For the mean variance portfolio, I need a target return, which will act as a minimum threshold for the expected fund returns. It will be on a monthly basis, since I have monthly returns, and the value I chose is $r_{target} = 0.005$. The target return is not the same for each situation, so it must be adjusted each time. Considering the data, I had to analyze, and their mean returns I decided that a target return of 0.005 would be a good threshold.

I evaluated the constructed portfolios based on their realized returns, their cumulative returns and the Conditional Sharpe Ratio (CSR) for the out-of-sample period (36 months).

For each of the above approaches the procedure is almost the same. After I estimate the expected returns and construct the covariance matrix of the returns, I use them to solve the quadratic programming problem (which is the formula of the two different portfolio constructions). After that, I find the portfolio weights for each fund and calculate the out-of-sample returns as well as the cumulative returns and the CSR.

For method (a) I used the mean and covariance of the in-sample fund returns. Methods (b) and (c) are computed using the market factor in the single index model and all the factors in the multiple factor model. Here I estimated the model coefficients (with regression), in order to compute the expected returns and covariance matrices. For method (d) the covariance matrix is calculated assuming that the correlation is constant throughout the series.

The results for each method are presented below.

Sample estimate of mean vector and covariance matrix

As it is shown the Minimum Variance portfolio has greater mean and cumulative returns than the Mean Variance, and the Conditional Sharpe Ratio is also greater. So, we can assume that the minimum variance portfolio is more optimal than the other.

	Mean return	**Volatility**	**Cumulative Return**	**CSR**	**# of funds in the portfolio**
Minimum Variance	0.0076	0.0338	0.2746	0.2238	2
Mean Variance	0.0052	0.0416	0.1873	0.1161	7

Table 2: Evaluation of constructed portfolios using sample estimates

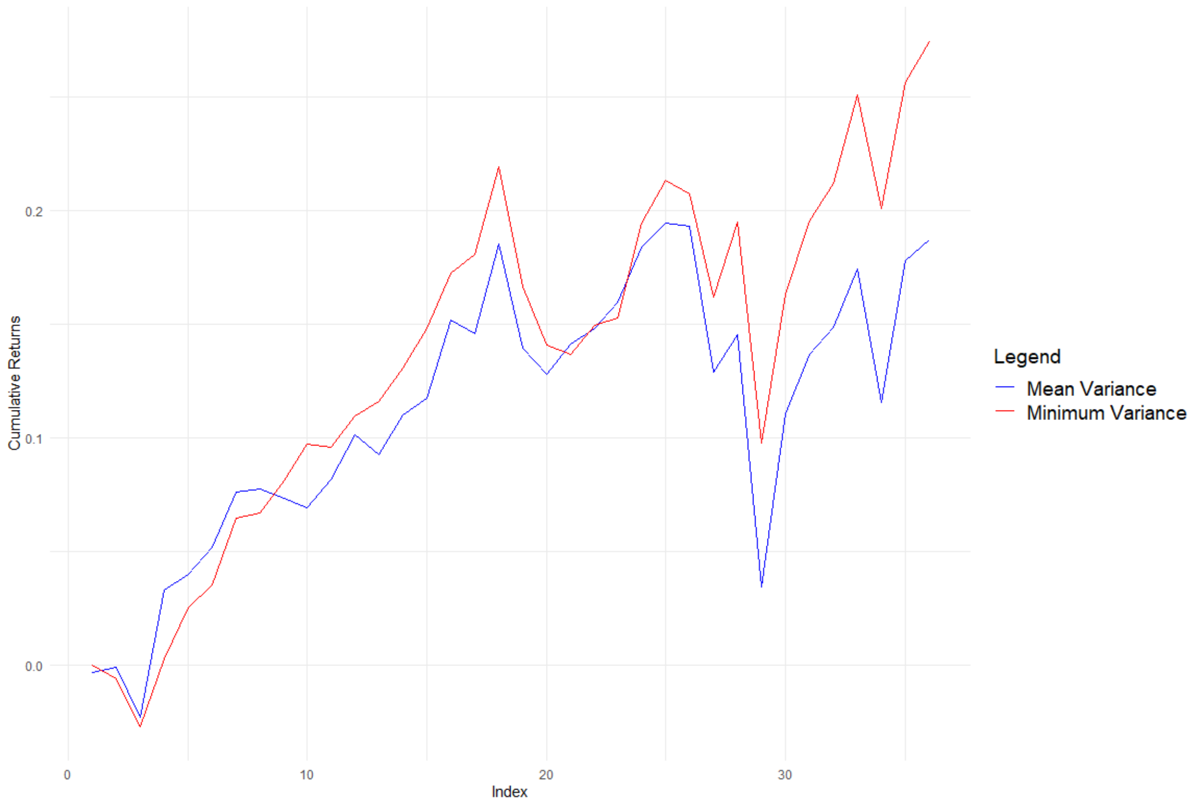

Figure 2: Cumulative returns using sample estimates

Single index model

Here the Minimum Variance portfolio is again more optimal than the Mean Variance one, as it is seen on the table below.

	Mean return	**Volatility**	**Cumulative Return**	**CSR**	**# of funds in the portfolio**
Minimum Variance	0.0073	0.0334	0.2635	0.2177	3
Mean Variance	0.0053	0.0404	0.1892	0.1189	7

Table 3: Evaluation of constructed portfolios using the Single Index model

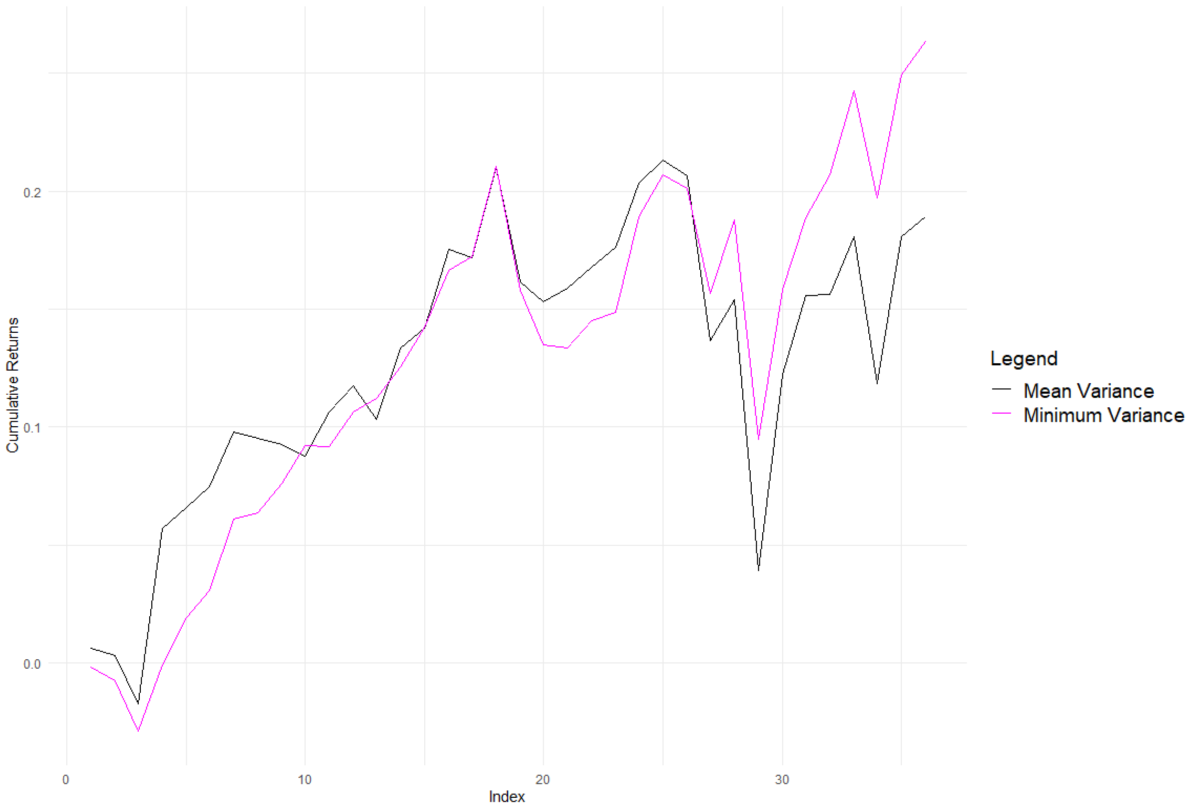

Figure 3: Cumulative returns using the Single Index model

Multivariate multiple regression model

For the multivariate full factor model the results are again the same as with the other methods.

	Mean return	**Volatility**	**Cumulative Return**	**CSR**	**# of funds in the portfolio**
Minimum Variance	0.0074	0.0336	0.2649	0.2172	3
Mean Variance	0.0047	0.0414	0.1703	0.1052	10

Table 4: Evaluation of constructed portfolios using the Full Factor model

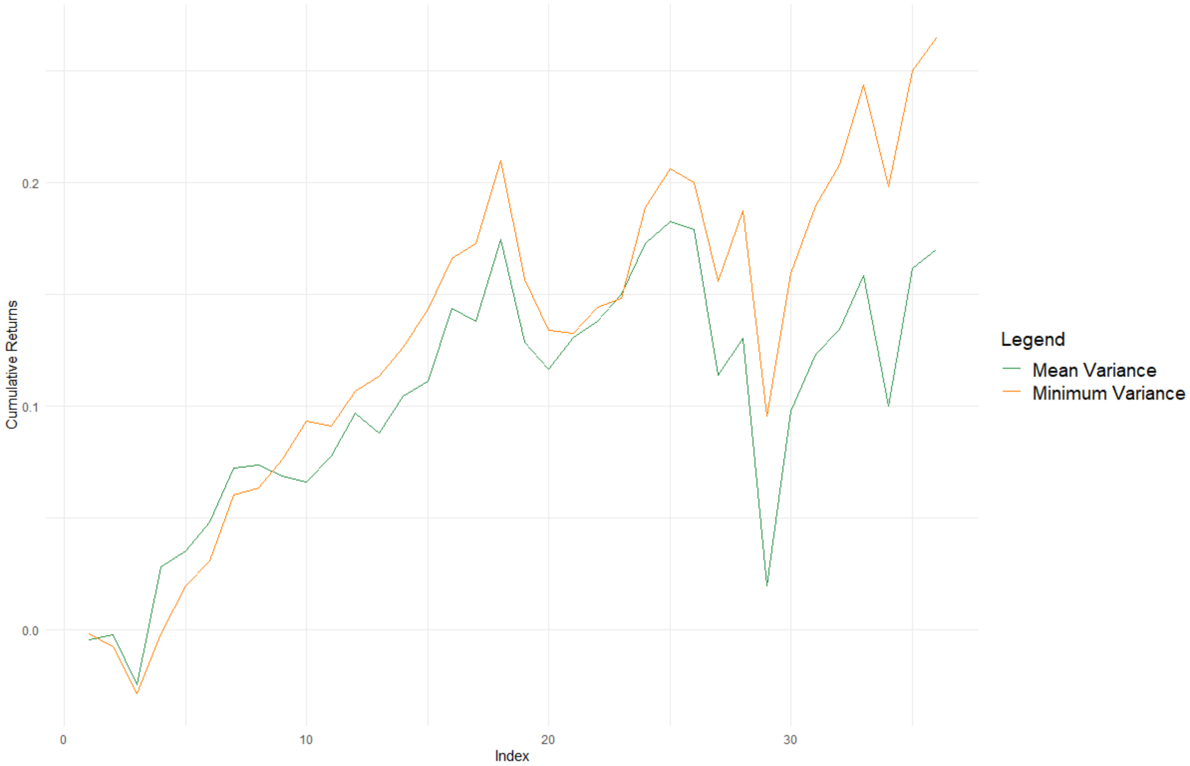

Figure 4: Cumulative returns using the Full Factor model

Constant Conditional Correlation (CCC) model

To construct optimal portfolios using this model, I first estimated each asset's variance with univariate GARCH(1,1) models. Then I computed the estimated residuals based on which I estimate the constant correlation matrix, I order to find the covariance matrix of the returns. The rest of the calculations are the same as with all the other different methods.

	Mean return	**Volatility**	**Cumulative Return**	**CSR**	**# of funds in the portfolio**
Minimum Variance	0.0064	0.0253	0.2290	0.2786	16
Mean Variance	0.0064	0.0253	0.2293	0.2789	16

Table 5: Evaluation of constructed portfolios using the CCC model

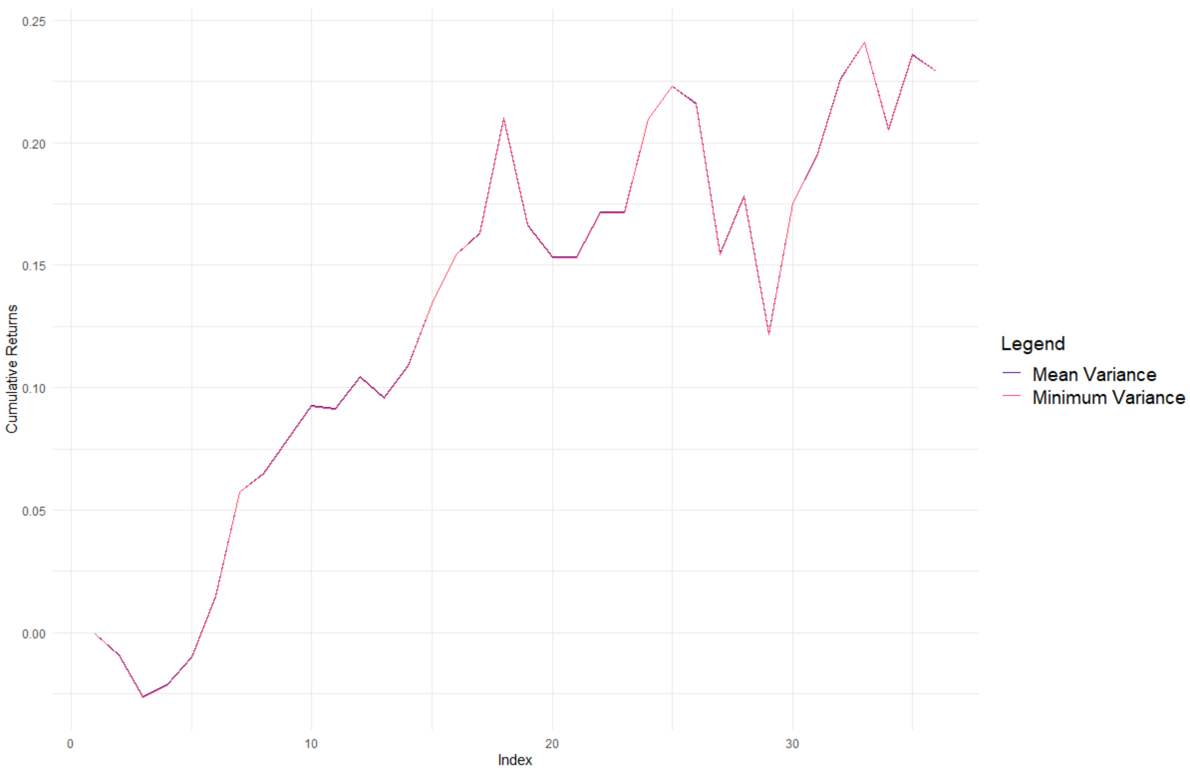

Figure 5: Cumulative returns using the CCC model

The number of funds in each portfolio is lower for the Minimum Variance portfolio than the funds used in the Mean Variance portfolio (except for the CCC model). This means that the Minimum Variance weights are divided between only a few funds, which implies that there are funds that are "strong" enough on their own and able to bring large returns.

In the first three methods, the Minimum Variance portfolio is more optimal than the Mean Variance one. In the CCC model however we can see that the Mean Variance portfolio is only slightly better than the other. The fact that I assume constant correlation seems to affect the construction of the two portfolios, resulting in almost the same returns.

If we were to compare all the different methods, the CCC model seems to result in a greater Conditional Sharpe Ration even though the mean returns are lower than those of the other three methods (for the Minimum Variance Portfolio).