On the Empirical Identification of Time Preferences in Discrete Choice Dynamic Programming Models

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1 Introduction

The class of discrete choice dynamic programming (DCDP) models has been used to explore the potential effect of policy changes in a host of different domains, including the impact of increased access to contraception on teen pregnancies (Arcidiacono et al., 2012), term limits and lower salaries on the career decisions of politicians (Diermeier et al., 2005), and tuition subsidies or other monetary incentives on educational attainment in both developed and developing countries (Keane and Wolpin, 1997; Todd and Wolpin, 2006). Indeed, a unique payoff of structural econometric models is the possibility to simulate model outcomes in counterfactual environments, thus generating predictions useful to answer what-if questions that may be relevant for policy-making purposes.

Counterfactual analysis relies on the explicit characterization of the mechanism that links the agents' preferences to the outcomes of interest, which in turn relies on modelling choices within and beyond the domain of economic theory. Economically-motivated assumptions are needed to characterize the problem faced by the agent and the preferences she is endowed with, as well as the processes which are to be considered exogenous. Extra-theoretic assumptions, for instance on the distribution of shocks to choice-specific payoffs, are often needed to keep the solution and estimation of the model tractable.

The vast majority of DCDP models assume the agents to be endowed with exponential time preferences, such that their intertemporal utility can be represented by the sum of instantaneous utilities weighted by a discount factor, usually denoted by δ, which is constant across periods, therefore implying time consistent preferences. The discount factor is meant to summarize all the psychological reasons that influence intertemporal choice, which may include, for example, uncertainty, habit formation, changing tastes or impulsiveness, besides "pure" time preferences. Whether time preferences are best expressed via a unitary construct is up for debate (see, for example, Frederick et al., 2002).

Introduced by Samuelson (1937), exponential discounting is routinely taken as an assumption in problems of intertemporal choice, but its realism has been questioned starting from Samuelson himself, who made no claims about the normative nor the descriptive validity of the functional form he was proposing. In fact, there exist a large literature providing evidence that people[1] discount time in an hyperbolic rather than exponential fashion: That is, they discount time more heavily over shorter horizons than over longer horizons ("present bias"). Among the behavioral patterns usually interpreted as evidence of hyperbolic discounting there is the tendency to prefer

[1] And pigeons (Ainslie and Herrnstein, 1981).

a smaller-sooner rewards rather than a larger-later one, and the phenomenon commonly known as "preference reversal", for which the preference relation between two rewards later in time may reverse in favor of the more proximate as time passes (Thaler, 1981).

Multiple functional forms have been proposed to account for hyperbolic discounting. A formulation which captures this qualitative evidence and it is widely used, mostly for its tractability, is the $\beta - \delta$ model, initially proposed by Phelps and Pollak (1968) to study intergenerational altruism and later popularized by Laibson and co-authors in the context of consumption-saving behavior (see, for example, Angeletos et al., 2001). This formulation implies a declining discount rate between period t and period $t + 1$, determined by both β and δ, which should be respectively interpreted as a parameter capturing present bias and the long-run discount factor. However, the discount rate between any two future periods is constant, and determined solely by δ. Exponential discounting is therefore nested within the $\beta - \delta$ model for β equal to 1.

In principle, exponential discounting is not an assumption needed for the tractability of a DCDP model. However, assuming hyperbolic discounting complicates issues of model identification and introduces changes in the solution of the model. The nature of these changes depends on whether the agent is aware of her own present bias, or her "degree of sophistication".

The literature on time-inconsistent preferences usually models an agent as a collection of many autonomous selves, one for each period. The period-t self controls the decision of the agent for the current period, taking into account her perception of the future selves' decisions. A sophisticated agent correctly perceives, at each point in time, his next period self to have a present bias parameter β. A partially naïve agent underestimates his future selves' present bias, while in the extreme case a completely naïve agent believes his future selves to act time-consistently. Since they hold false beliefs about the behavior of future selves, partially and completely naïve agents tend to revisit the plans they initially made, which results in the solution of a DCDP model being dynamically inconsistent. The interplay between inconsistency in decision-making and present bias is of special interest for public policies, as different degrees of present bias and self-awareness may imply different optimal policies, for instance with respect to the provision of commitment devices.

The last decade has seen a growing interest in integrating models of hyperbolic discounting within the DCDP framework. Many of these attempts have been concerned with assessing the relevance of behavioral responses and utility losses due to time-inconsistent preferences, typically benchmarked against the exponential discounting model, for the evaluation of social policies. However, testing the exponential versus the hyperbolic model can be problematic, as the actions of time-consistent and time-inconsistent agents can be observationally equivalent, especially when commitment devices are not available (for instance with respect to life-cycle consumption in absence of credit card borrowing; see Laibson et al., 1998).

Theoretical results on identification with quasi-hyperbolic discounting formalize the intuition that time preferences can be recovered comparing the behavior of agents that only differ in their "futures", where future in this context usually refers to the evolution of the state space. If agents are otherwise identical, differences in their behavior are solely determined by how they discount the utility stream from future periods.

Moreover, a small literature exist where dynamic models with hyperbolic discounting are estimated after imposing specific parametric restrictions, or where identification is aided by particular features of the data. Models of labor supply and job search seem particularly suited to overcome issues of identification, as they involve intertemporal tradeoffs between monetary and non-monetary upfront costs and benefits that will realize later. Hyperbolic discounting could then explain patterns in the data which reflect a tendency to postpone costly activities, such as looking for a job or pursuing an education. Indeed, many empirical applications exploit data on the take-up of unemployment benefits or other social security benefits targeted to vulnerable groups. A concern is then whether the findings may generalize to different groups, as there is evidence correlating high time-discounting and self-control issues with poverty (Lawrance, 1991; Banerjee and Mullainathan, 2010; Bernheim et al., 2013).

In this thesis I study the empirical identification of time preference parameters in a model of occupational choice where agents can face exogenous restrictions on their employment possibilities. Such restrictions should aid identification, because they do not affect the per-period utility function but matter for choice: Future-oriented agents take the restrictions into account when deciding on their level of education. The same identification strategy is used in a setting with exponential discounters and in a setting where agents discounts quasi-hyperbolically and are completely naïve.

The remaining of this thesis is organized as follows. Section II discusses theoretical results on and empirical approaches to time preferences identification, after introducing the framework of DCDP models and their solution, with special attention to the case of a completely naïve agent. Section III presents the simulation of two models of occupational choice based on Keane and Wolpin (1994), implemented with the open-source software *respy* (Gabler and Raabe, 2020), and describes the identification strategy used to recover the time-preference parameters in the two cases. Section IV uses the Method of Simulated Moment to assess whether the time preferences parameters are empirically identified, and explore the consequences of model misspecification for counterfactual predictions on the effect of a tuition subsidy. Section V concludes.

2 Identification

2.1 Economic framework

Discrete choice dynamic models can be seen as an extension of the static discrete choice framework where the agents additionally take into account the impact of their current actions on future welfare, and where the latent variable that determines choice contains either past choices or unobservables that are serially correlated (Keane et al., 2011). Introducing a link among past, present and future choices implies assumptions on the agents' behavior. These assumptions are related to the agents' expectations about the evolution of unobservables and to the agents' time preferences, that is, their preference for immediate utility over delayed utility.

In the general setting, time is discrete and indexed by t, with the time horizon T being either finite or infinite. Agents are indexed by i and have preferences, defined over a sequence of states of the world, from period $t = 0$ until period $t = T$. The state of the world at period t for individual i has two components: a vector of state variables h_{it} that is known at period t, and an alternative d_{it} chosen by the agent at period t that belongs to the finite, discrete set of mutually exclusive alternatives $\mathcal{D} = d_1, d_2, ...d_K$.

State variables h can be either observed or unobserved by the econometrician. Observed state variables are denoted by $x \in \mathcal{X} = \{x_1, x_2, ..., x_N\}$: The set $\mathcal{X}$ is finite, it may include time-invariant agent's characteristics and, for notational simplicity, includes time t. The vector of state variables $\epsilon \equiv (\epsilon_1, \epsilon_2, ...\epsilon_K) \in \mathcal{E}^K$, which is distributed continuously over $\mathcal{R}^K$, is unobserved by the econometrician but known to the agent at time t. It is interpreted as a vector of alternative-specific random shocks associated with choosing an alternative d from $\mathcal{D}$. Other state variables unobserved to the econometrician may reflect unobserved heterogeneity in the population (see Keane and Wolpin (1997) for an example of a finite mixture model).

In period t, agent i observes the vector of state variables h_{it} and chooses an alternative d, receiving a choice-specific instantaneous utility denoted by $u_d(h_{it}) \equiv u_d(x_{it}, \epsilon_{it})$. Usually, the instantaneous utility function is assumed to be additively separable, such that $u_d(x_{it}, \epsilon_{it}) = u_d(x_{it}) + \epsilon_{dit}$. After the agent has taken his or her decision, the state of the world updates and the process repeats itself.

Agents are assumed to have rational beliefs about the state variables' transition probabilities: Agents' beliefs coincide with the true transition probabilities of the state variables and can be modelled via a Markov transition distribution function, denoted by $Q(h_{i,t+1}|d_{it}, h_{it})$. This assumption can be relaxed whenever separate data on agents' beliefs, in addition to data on choices and states, are available for estimation.

Moreover, agents are assumed to be future-oriented: They maximize their discounted expected lifetime utility, rather than their instantaneous utility. Formally, if the agent discounts the future exponentially, he or she faces a dynamic programming problem where the objective function is:

$$\mathbb{E}(\sum_{j=0}^{T-t} \delta^j u(d_{i,t+j}, h_{i,t+j}) \mid d_{it}, h_{it}) \tag{2.1}$$

The problem can be formulated as a Markov decision process, where agents implement an optimal decision rule, that is, a function $p(h)$ dictating which action the agents should take in each period, given the state of the world.

The optimal decision rule can be recovered invoking Bellman's principle of optimality, which assumes that the optimal decision rule will be used in all periods, and solving recursively for the *value function*, that is, the expected total discounted per-period utilities under a decision rule $p(h)$, from period t onwards.

If the agent discounts future periods in a quasi-hyperbolic fashion, the maximization problem of the agent becomes less straight-forward, since the behavior of time-inconsistent agents is analysed as if a single individual consisted of many autonomous selves, one for each period. Each period-t self maximizes his current discounted expected lifetime utility, while the future selves control the subsequent decisions.

Fang and Wang (2015) derive the equilibrium for a partially naïve agent, whose period-t self believes that, beginning next period, her future selves will behave optimally with a present-bias factor of $\tilde{\beta} \in [\beta, 1]$. Here, we focus on the nested case of completely naïve agent, who believes that his future selves will behave time-consistently starting next period. The solution of the model is treated as the equilibrium outcome of an intra-personal game, where the selves at different periods are the players and only the action of the current self are observed.

A *feasible strategy* $\sigma_t(h_t)$ for a self in period t represents the self's choice over the alternatives given the state of the world. A *strategy profile* for all selves, $\boldsymbol{\sigma} \equiv \{\boldsymbol{\sigma_t}\}_{t=1}^{T}$ represents the action of each self in all possible states and under all possible realizations of the shock vector.

For any strategy profile, $\boldsymbol{\sigma}_k^+ \equiv \{\boldsymbol{\sigma_t}\}_{t=k}^{T}$ is the *continuation strategy profile* from period k to the terminal period T. Define the self's expected continuation utility under her long-run preferences, the state of the world h_t, and the continuation value $\boldsymbol{\sigma}_t^+$ as:

$$V_t(h_t, \boldsymbol{\sigma}_t^+) = u_t(\sigma_t, h_t) + \delta E[V_{t+1}(h_{t+1}, \boldsymbol{\sigma}_{t+1}^+) \mid h_t, \sigma_t] \tag{2.2}$$

Then, the *perceived continuation strategy profile* for a completely naïve agent is:

$$\tilde{\sigma}_t(h_t) = \underset{d\in D}{\operatorname{argmax}}\{u_t(d,h_t) + \delta E[V_{t+1}(h_{t+1}, \tilde{\boldsymbol{\sigma}}^{+}_{t+1}) \mid h_t, d]\} \tag{2.3}$$

as the agent anticipates her future selves to behave according to δ only. Given this perception, the best response *perception-perfect strategy profile* for the agent is:

$$\sigma^*_t(h_t) = \underset{d\in D}{\operatorname{argmax}}\{u_t(d,h_t) + \beta\delta E[V_{t+1}(h_{t+1}, \tilde{\boldsymbol{\sigma}}^{+}_{t+1}) \mid h_t, d]\} \tag{2.4}$$

The strategy profile $\tilde{\boldsymbol{\sigma}}$ is the agent's unobserved perception of what her future selves will do, under the assumption that her future selves won't suffer from present bias, while the strategy profile $\boldsymbol{\sigma}^*$, in contrast, generates the actions observed in the data. The two strategy profiles coincide for an exponential discounter, as her expectations correctly predicts behavior, however they do not coincide for a completely naïve agent, since despite her expectations in no period t the agent behaves time-consistently (nor learns her true preferences). As a result, the optimal decision rule that solves the model is time-inconsistent.

2.2 Set-up for identification

Time preference parameters in structural models of dynamic discrete choices are underidentified (see Rust, 1994), which is especially problematic for counterfactual analysis, since time preferences are an important ingredient to make any statement about the behavioral response of the agents to a policy intervention. A significant body of research has focused on how to achieve identification via specific restrictions on some of the model's state variables (which are usually economically motivated) or via parametric restrictions, for instance on the distribution of the payoff shocks (which usually lack economic content).

In the following, I focus on the identification of time-preference parameters. The set-up for identification follows Magnac and Thesmar (2002), who analyse non-parametric identification in the case of a discrete choice dynamic model with exponential discounting, with and without unobserved heterogeneity. I start by defining the structure of the model, denoted by θ, as

$$\theta = \{\delta, G(\cdot), \{u_d(x_{it}), V_d(x_{i,t+1}) : d \in \mathcal{D}, x_{it} \in \mathcal{X}, x_{i,t+1} \in \mathcal{X}\}\}, \tag{2.5}$$

where δ denotes the agent's long-term discount factor; $G(\cdot)$ the distribution of the alternative-specific random shocks; $u_d(x_{it})$ the agent's instantaneous utility function, and $V_d(x_{i,t+1})$ the long-run value function. These objects summarize the current and future behavior of the agents, with the rational beliefs assumption ensuring that the agent's beliefs over the Markov process of the alternative-specific shocks can be represented by the distribution $G(\cdot)$.

Given any structure $\theta \in \Theta$, where Θ is the set of all possible structures, the model predicts $\hat{P}_d(x;\theta)$, that is, the probability that an agent will choose alternative $d \in \mathcal{D}$ in state $x \in \mathcal{X}$. This behavioral prediction is the *reduced form* of structure θ. An underidentification problem arises whenever the same reduced form is consistent with multiple different structures. Formally, two structures are observationally equivalent if they have the same reduced form:

$$\hat{P}_d(x;\theta) = \hat{P}_d(x;\theta') \text{ for all } d \in \mathcal{D}, x \in \mathcal{X} \tag{2.6}$$

A model is said to be identified if and only if for any $\theta, \theta' \in \Theta, \theta = \theta'$ if they are observationally equivalent. In other words, to achieve identification it is necessary that identical predictions imply identical structures of the model. It is not possible to estimate consistently a model's parameters if the model is underidentified.

2.3 Identification with exponential discounting

Theoretical results on identification of time preferences usually rely on restrictions that exclude some state space variables which may matter for choice — for instance, because they influence an exogenous process that in turn determines the evolution of the state space — from the instantaneous utility function.

The intuition is that information on time preferences can be captured whenever different states shift the expected discounted future utilities, but not the instantaneous utilities: Then, the extent to which comparable agents take different decisions under different states can be used to infer how much weight they place on their future. Completely myopic agents, for instance, will tend to take the same course of action regardless of the state they experience, as only the instantaneous utility from choosing an alternative influence their decision-making process.

This intuition relies on two important assumptions: That different states are salient to non-myopic agents, such that experiencing one state rather than another affects behavior, and that agents experiencing different states are comparable. The latter assumption implies that unobserved heterogeneity makes this identification argument weaker, although, at least for a model with exponential discounting, identification may still be achieved after imposing further restrictions, for example on the transition probabilities of the unobserved types (Magnac and Thesmar, 2002).

To ensure that the agents' heterogeneity is fully observed, all the exclusion restrictions discussed below require some version of Rust (1994) conditional independence, which also rules out any unobserved persistence in the wage shocks.

2.3.1 Exclusion restriction on current value functions. Magnac and Thesmar (2002) show that the utility functions in each alternative cannot be (nonparametrically) identified unless $G(\cdot)$ and δ are set and one alternative's utility is normalized. The result relies on assuming additive separability of the utility function, agents' rational expectations on transition probabilities, and a version of Rust (1994) conditional independence, which requires all unobservable state variables to be independent. In the case with unobserved heterogeneity, the degree of underidentification of the model is even higher.

When heterogeneity is fully observed, Magnac and Thesmar (2002) provide an exclusion restriction to point identify the discount factor. Such exclusion restriction requires the existence of two observed state variables that provide, for each alternative, the same *current value function*, which measures the difference between the expected values of two sequences of choices and therefore depends on both instantaneous utilities and expected discounted future utilities. Formally:

Proposition 1. ***(Exclusion restriction in Magnac and Thesmar, 2002)***
Consider two state variables, $x_1, x_2 \in \mathcal{X}$, such that $x_1 \neq x_2$. An exclusion restriction of the form

$$\exists d \in \mathcal{D} s.t. U_d(x_1) = U_d(x_2)$$

together with a rank condition, can be used to identify the discount factor δ.

Where $U_d(\cdot)$ is the alternative specific current value function. This exclusion restriction is sufficient for point identification of the discount factor δ if coupled with a rank condition ensuring enough variation in the expected discounted future utilities, so that experiencing one of the two states rather that the other actually induces a change in the decision problem of the agent.

2.3.2 Exclusion restriction on primitive utilities. As noted in Abbring and Daljord (2020), the exclusion restriction by Magnac and Thesmar is of difficult economic interpretation, as it is hard to think of two state variables that give rise to different expected discounted future utilities, but have the same current value function. In the same paper, the authors introduce an exclusion restriction on primitive utility that requires the same assumptions taken in Magnac and Thesmar (2002) but is easier to verify empirically:

Proposition 2. ***(Exclusion restriction in Abbring and Daljord, 2020)***
Consider a pair of known choices $d_1, d_2 \in \mathcal{D}$ and a pair of known states $x_1, x_2 \in \mathcal{X}$, with either $d_1 \neq d_2, x_1 \neq x_2$, or both. Then, the discount factor δ can be set-identified provided that:

$$u_{d_1}(x_1) = u_{d_2}(x_2)$$

The result can be extended to the general case where the value $u_{d_1}(x_1) - u_{d_2}(x_2)$ is known but different from zero. Together with a rank condition similar to the one found in Magnac and Thesmar (2002), this exclusion restriction leads to moment conditions that allow to set-identify the discount factor, once values close to 1 are excluded. Point identification can be achieved only after assuming further restrictions, typically economically motivated.

2.3.3 Exclusions restriction on choice probabilities. Another approach is found in Schneider (2020). In Schneider's setting, agents may face a restricted choice set, in certain periods and with a certain probability, $\pi(d, x)$, which depends either stochastically or deterministically on the agent's previous choice and previous state. Examples can be negative demand shocks that lower the probability of receiving a job offer in labor supply models.

Exploiting exogenous variation in these probabilistic restrictions allows for point-identification of the discount factor. Formally:

Proposition 3. ***(Exclusion restriction in Schneider (2020))***
Consider two states, x_1 and x_2, and two choices, d_1 and d_2. Assume that the following three conditions are satisfied:

$$u_{d_1}(x_1) = u_{d_1}(x_2) \text{ and } u_{d_2}(x_1) = u_{d_2}(x_2),$$
$$Q(x|d, x_1) = Q(x|d, x_2) for d \in d_1, d_2 \text{ and } x \in X,$$
$$\pi(d_1, x_1) < \pi(d_1, x_2)$$

Then, the discount factor δ can be point identified.

These three conditions, together with a rank condition, imply that restriction probabilities must differ between state x_1 and x_2, while both state transition probabilities and the instantaneous utility given by alternatives d_1 and d_2 respectively must not.

The theoretical result does not rely on the stationarity assumption, nor on normalizing the utility of one of the alternatives which, although standard in the literature, may affect the model's counterfactual predictions (Norets and Tang, 2013; Kalouptsidi et al., 2016).

Fundamental assumptions are utility being additive separable, and payoff shocks being continuously distributed, independent across periods and independent across alternatives. Moreover, state transition probabilities, choice restriction probabilities and *genuine choice probabilities* need to be known for at least two periods.

Genuine choice probabilities reflect the agent's preferences over the restricted and the unrestricted choice sets, given the current state, while observed choice probabilities reflect both preference and possible choice restrictions. Schneider provides conditions that allow to uniquely determine the genuine choice probabilities, given that observed choice probabilities and restriction probabilities are known.

2.4 Identification with quasi-hyperbolic discounting

Under certain conditions, theoretical results on the identification of time preferences with exponential discounting can be extended to the (quasi-)hyperbolic setting.

2.4.1 Sophisticated agents. In the context of a labor supply model, Fang and Silverman (2006) show that, given standard data for 3 or more periods, an hyperbolic discounting model for a sophisticated agent can be distinguished from an exponential discounting model without making parametric assumptions on the distribution of the stochastic shocks to payoffs. With "standard data" they mean a data set consisting of an infinite number of individuals with observations on the experience level and choices for all individuals at each period; the welfare benefit level; and the accepted wages of those who work. However, the argument does not apply for unobserved heterogeneity (Fang and Silverman, 2009).

Abbring et al. (2018) extend his exclusion restriction on primitive utilities to a quasi-hyperbolic setting with sophisticated agents, imposing two exclusion restrictions (which involve two *pair* of states) on instantaneous utilities. They additionally evaluate the performance of their identification strategy in finite samples.

They show that, given at least three periods of data, while δ and β are theoretically separately identified and their product is quite precisely recovered from observed choices, it is difficult to disentangle the two values during estimation with finite samples. Holding the other time-preference parameter constant, the criterion functions for β and δ does not display a basin around the minimum, but a "banana shaped through" similar to the pattern shown in the model of life-cycle consumption in Laibson et al. (2007).

2.4.2 Partially naïve agents. To my knowledge, the only paper attempting to establish a theoretical result on the identification of time preferences for partially naïve agents is Fang and Wang (2015). The sophisticated and completely naïve cases are nested. The structure of the model for a partially naïve hyperbolic discounter becomes:

$$\theta = \{\beta, \tilde{\beta}, \delta, G, \{u_d(x_ti), Z_d(x_{i,t+1}), V_d(x_{i,t+1}) : d \in \mathcal{D}, x_{it} \in \mathcal{X}, x_{i,t+1} \in \mathcal{X}\}\}, \tag{2.7}$$

where the time-preference parameters β, $\tilde{\beta}$, and δ denote respectively the agent's present-bias, *naïveté*'s parameter and long-term discount factor, $Z_d(x_{i,t+1})$ denotes the current self's perception of the choice probability of the next period's self, and $V_d(x_{i,t+1})$ denotes the perceived long-run value function, which is never observed in the data. The other terms are as in (2.5).

In the case of a completely naïve agent, $\tilde{\beta} = 1$ and consequently $Z_d(x_{i,t+1}) = V_d(x_{i,t+1})$: The agent believes his next period's self will discount the future utility streams exponentially.

Assume a data structure such that: (i) The choice probabilities $P_i(x)$ for all $i \in \mathcal{X}$ are observed for all $x \in \mathcal{X}$; (ii) The transition probabilities $q(x_{t+1} \mid x_t, d_t)$ are observed for all $(x_t, x_{t+1}) \in \mathcal{X}^2$, all $d \in \mathcal{D}$, and (iii) At least two periods of the above data are observed.

Moreover, assume stationarity, additive separability, conditional independence, and an extreme-value distribution for the payoff shocks. Under these assumptions and the data structure described above, Fang and Wang (2015) obtain (via an application of Hotz and Miller (1993) choice probability inversion) a system of equations that relates the transition probabilities and the conditional choice probabilities to the structure of the model.

It is then possible to identify $u_d(x_t), Z_d(x_{t+1}), V_d(x_{t+1}) : d \in \mathcal{D}, x_t \in \mathcal{X}, x_{t+1} \in \mathcal{X}$ for a given set of discount factors $\beta, \tilde{\beta}, \delta$. In other words, it is possible to identify, from the observed data, values of $u_d(x_t), Z_d(x_{t+1}), V_d(x_{t+1}) : d \in \mathcal{D}, x_t \in \mathcal{X}, x_{t+1} \in \mathcal{X}$ such that given values of $\beta, \tilde{\beta}, \delta$ are consistent with the data.

The second result is more problematic. Assume additionally the existence of a variable that does not directly affect $u_d(\cdot)$ for all $d \in \mathcal{D}$, but affects the transition of state variables and therefore may matter for choice. Formally, assume the following exclusion restriction:

Proposition 4. ***(Exclusion restriction in Fang and Wang, 2015)***
There must exist variables $(x_1, x_2) \in \mathcal{X}^2$ with $x_1 \neq x_2$, but:

i. For all $d \in \mathcal{D}, u_d(x_1) = u_d(x_2)$;

ii. For some $d \in \mathcal{D}, Q(x_{t+1} \mid x_{1,t}, d_t) \neq Q(x_{t+1} \mid x_{2,t}, d_t)$.

Then, all parameters in the model are generically identified.

Fang and Wang claim that it is possible to exploit the vector of variables in the state space satisfying the exclusive restriction to generically[2] identify all the parameters in the model. In particular, if x_e is the vector of state variables satisfying the restriction, at least three cross-x_e restrictions are needed to identify $\beta, \tilde{\beta}$ and δ, i.e., a minimum of four different points in the support $\mathcal{X}_e$ of state variables that satisfy the exclusion restriction.

The intuition is the following: Given two state vectors x_1 and x_2 that only differ in the exclusive restriction component, for $\beta, \tilde{\beta}$ and δ to be consistent with the true values, the identified values $u_d(x_1)$ and $u_d(x_2)$ must be equal. Therefore, individuals having different choice probabilities at state x_1 and x_2 reveal information about their time-preference parameters, as the exclusive restriction constrains $u_d(x_1)$ and $u_d(x_2)$ to be equal.

[2]"For almost all data sets generated by the assumed hyperbolic discounting model", Fang and Wang (2015).

Fang and Wang (2015) additionally present a model on mammography decisions as an empirical application of their result. Chan (2017) and Haan et al. (2020) follow Fang and Wang's result, despite in both cases the exclusion restrictions relevant to identification restrict the choice sets in certain periods, instead of inducing different state transition probabilities.

However, Abbring and Daljord (2020) show that the generic identification result by Fang and Wang is void, because it does not preclude an identified model from being rationalized by any parameter vector, or no parameter vector at all. Besides, they note that Fang and Wang's notion of generic identification is imprecise, as it is defined on the data space: The result claims that, for a very small subset of the *data sets* generated by the model, identification of the model may fail. Such subset cannot be characterized, since Fang and Wang set up the model as a system of nonlinear equations without characterizing its empirical content.

In standard applications, such as Ekeland et al. (2004), (generic) model identifiability is a property of the parameter vector, therefore identification of the model is defined on the parameter space: It may fail for certain values of the parameter vector. An additional remark is that generic identification is a weak result, since the models for which identification fail could be economically relevant.

2.4.3 Empirical approaches to identification. The theoretical results discussed above, when valid, rely on assumptions about (the absence of) unobserved heterogeneity that may prove too restrictive in empirical applications. In particular, conditional independence rules out serially correlated shocks and unobserved types, while a large literature points to the importance of stickiness in wage dynamics (Campbell and Kamlani, 1997; Le Bihan et al., 2012; Barattieri et al., 2014) and of unobserved ability in human capital accumulation (Weiss, 1995; Keane and Wolpin, 1997; Belzil and Hansen, 2002). Moreover, these restrictive assumptions are usually derived to prove identification for short panel data, where many agents are observed for two or three periods. In empirical applications agents are often observed for many more than two or three periods, a feature that may aid identification.

A small list of empirical papers estimate dynamic models with hyperbolic discounting making informal arguments for identification that sometimes rely on the same intuition underlying the theoretical results discussed above, with exogenous policies being exploited as a source of variation in the agents' expected discounted utilities.

As an example, Chan (2017) presents a model of work-welfare decision with potentially time-inconsistent agents who are randomly assigned to two different welfare policies, with and without time limits on welfare benefits. Treatment-control behavioral differences reflected in the two groups' choice probabilities (conditional on the value of welfare use) are then informative of the agents' time preferences, since different welfare time limits affect the state transition probabilities but not the per-period utility function.

Haan et al. (2020) make similar use of variation in job protection regimes in a model of female labor supply with potentially time-inconsistent and fully naïve agents. In each period t, the agent chooses whether not to work, work part-time, or work full-time, conditional on having received a job offer. Women protected by different job protection regimes experience different job offer probabilities after childbirth. The job offer probability is then used as exclusion restriction, as it does affect future employment possibilities (by restricting future choices) but does not affect the flow utilities. In period t, comparing groups of women that differ in their employment probability in period $t + 1$ and further in the future in period $t + n$ can help identify the one-period-ahead discount factor $\beta\delta$ and the long-run discount factor δ respectively.

Identification can also rest on specific features of the data interpreted as pointing to inconsistent behavior, and/or on parametric restrictions on some components of the model.

Paserman (2008) presents a model of job search with (quasi-)hyperbolic discounting, sophisticated agents and endogenous search effort where time preferences identification is aided by both intrinsic features of the model and functional form assumptions. The role of β in the search process is particularly important, since present bias affects the search effort decision directly and the reservation wage only indirectly, via the continuation values. Once estimated the other model parameters using data on accepted wages, β can be recovered from data on unemployment duration, as a long average unemployment duration must imply little search effort, which can be interpreted as evidence of β being low. The model is non-stationary, as unemployment benefits have time limits and the composition of the sample varies over time. This helps disentangle parameters whose effect would be indistinguishable in a stationary environment (for instance, the present bias and the value of being unemployed). Observed heterogeneity is an additional aid, as multiple observable type of agents can provide multiple moments to be used for estimation. Finally, a key element for identification is the functional form assumed for the wage offer distribution.

In Fang and Silverman (2009), who model the work-welfare decision of a single parent, three patterns in the data can be interpreted as evidence of time-inconsistent behavior: Very low level of work when young, relatively high levels of work when older, and substantial returns to experience in the labor market. Observing many periods for the typical member of the sample, as well as taking parametric assumptions with respect to the joint distribution of the payoffs shocks and the modelling of the stigma from switching into welfare and its decay over time, presumably aid identification.

Tarozzi and Mahajan (2011) model the take-up of anti mosquito devices in India. The model allows for coexisting (partially) unobserved types which differ in their time preferences. In this case, identification exploits detailed information present in the data: In particular, elicited subjective beliefs on the probability of falling sick with malaria and a dummy variable for time preference reversal, used as a proxy to distinguish unobserved types. However, identification is complicated by the heterogeneity in time preferences. Some of the assumptions invoked are standard, such as transition probabilities being Markov, conditional independence, additive utility function, and choice probabilities being directly observed, while others are more restrictive and related to the distribution of agents' beliefs.

3 Model

3.1 Time preferences in models of occupational choice

To investigate empirical identification of time preference parameters, I adapt parametrization 3 of the occupational choice model in Keane and Wolpin (1994). The reasons to use this model are both theoretical and practical.

In human capital theory, time preferences matter for occupational choice because people with high discount rates should invest less in their human capital (Mincer, 1958). A large literature empirically relates time-preferences to specific education decisions, such as dropping-out from school (Oreopoulos, 2007), and to general lifetime outcomes, such as educational achievements, occupational choices and measures of long-term quality of life (see Koch et al., 2015 for a review). As an example, Golsteyn et al. (2014) study the relationship between impatience and school performance, as well as long-run social and economic outcomes that include labor supply and lifetime income: In line with the theoretical prediction, they find a substantial negative correlation and show that early human capital investment mediates between time preferences and long-run outcomes.

Investigating the time preferences of the agents may be especially important when assessing the impact of educational reforms aimed at increasing years spent in education. Kemptner and Tolan (2018) study how much the functional form of time preferences in a model of educational choice matters for policy evaluation, using data from the German Socio-Economic Panel. They find evidence of time inconsistent behavior and, when simulating policies that affect tuition costs, they find that the exponential and hyperbolic specifications predict different effects on educational outcomes.

On the practical side, all the three parametrizations in Keane and Wolpin (1994) ensures that experience in education is an investment good. When a college tuition subsidy is introduced, agents respond to a decrease in the cost of education by staying in school longer: In Keane and Wolpin's counterfactual simulation, a subsidy of 2000 USD increases final experience in education by 1.67 years on average under parametrization 3, where the return to education is the highest. This result suggests that agents will react accordingly to a increase in the option value in education, which in my version of the model is varied exogenously to aid identification, as explained in Subsection **??**. The cost and value of education being salient for the agents is very important for the argument underlying identification.

3.2 Basic model

The basic set-up is the following. In each period t, the agent chooses among four mutually exclusive alternatives: work in either occupation A or occupation B, continue education, or remain at home (for leisure or home production). The per-period utility functions are given by

$$\begin{aligned}
u^A(t) &= w^A(x_t^E, x_t^A, x_t^B; \alpha^A)e^{\varepsilon_t^A} \\
u^B(t) &= w^B(x_t^E, x_t^A, x_t^B; \alpha^B)e^{\varepsilon_t^B} \\
u^E(t) &= \gamma_0 - \gamma_1 I(x_t^E \geq 12) - \gamma_2(1 - E_{t-1}) + \varepsilon_t^E \\
u^H(t) &= \theta + \varepsilon_t^H,
\end{aligned} \tag{3.1}$$

where E and H refer to the education and home alternatives, respectively. x_t^E, x_t^A and x_t^B indicate the number of periods the agent has already spent in education, occupation A and occupation B in period t, that is, the agent's experience in each alternative. Note that experience cannot be accumulated in the home alternative. α_A and α_B are parameter vectors associated with the wage functions $w^A(\cdot)$ and $w^B(\cdot)$. Experience in any of the two occupation has a positive return to the same occupation, but only experience in occupation A has a positive return both for occupation A and for occupation B (a possible interpretation is that agents can develop more general skills in occupation A). Wage offers are always non-negative.

The utility function of education includes γ_0, the consumption value of education which can be of either sign; γ_1, the college tuition cost, where I an indicator function equal to one if the agent has completed high school and zero otherwise; and γ_2, the "adjustment cost" if the choice in $t-1$ was not education. Importantly, the agents' decision-making process is not framed as an optimal stopping problem where school is an absorbing state: Individuals can go back to school in any period, although they face the adjustment cost and they can't stay in school for more than 10 periods during their life cycle. θ is the (mean) value of the home alternative. The model abstracts from observed and unobserved heterogeneity: Agents are identical, and their different life cycle paths differ solely because of different shock draws.

Note that the payoff shocks enter the utility functions for the home and education alternative linearly, while they enter the utility functions for the two occupation multiplicatively. Moreover, the shocks are serially independent, but are correlated across alternatives: In particular, shocks in the two occupations are positively correlated and shocks between the home and education alternative are negatively correlated. This violates conditional independence and is the only serious departure from the theoretical framework used to develop the exclusion restrictions in Section 2.

To aid identification, I make a few departures from this basic model.

3.2.1 Model with exponential discounting. The model with exponential discounting is identical to parametrization 3 of Keane and Wolpin (1994), but I add a time-invariant observable characteristic that determines whether an agent faces restrictions in his choice set. Restricted agents can choose occupation B only if they have accumulated at least 14 periods of experience in education (all agents have 10 years of education in period 0). Unrestricted agents face no restrictions on their choice set. Each agent has equal probability to experience the restriction or not, therefore in a randomly drawn sample roughly 50% of the agents will be restricted and roughly 50% will not.

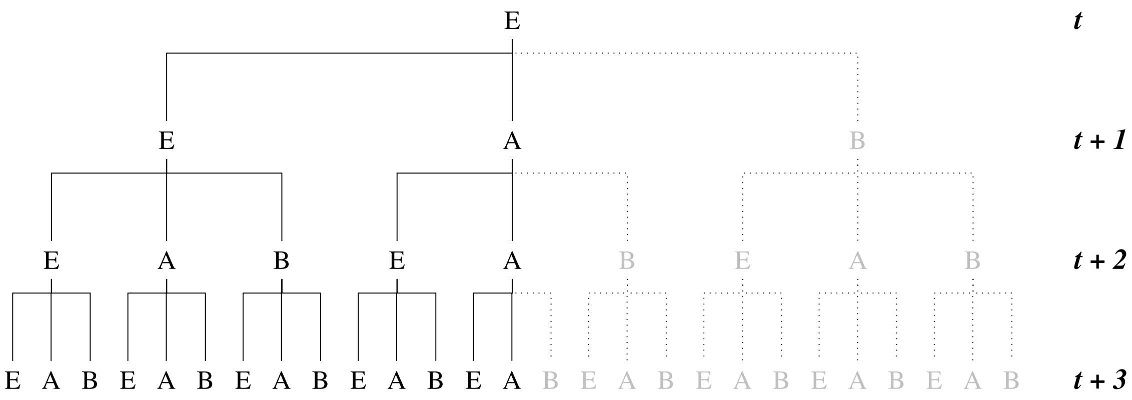
3.2.2 Model with hyperbolic discounting. In the model with hyperbolic discounting agents are completely naïve with respect to their own time preferences, which are described by the discount factor δ and the present bias β (the naïvete parameter $\tilde{\beta}$ is assumed to be 1). Agents face again a restricted choice set depending on time-invariant, randomly assigned observable characteristics: agents can be unrestricted, and thus choose any alternative in any period, they can face a restriction on occupation B, or they can face both a restriction in occupation A and a restriction in occupation B. The restriction on occupation A and B require agents to have completed at least 12 and 14 years of education respectively.

3.3 Identification strategy

Experiencing a restriction on one or more occupational choices does not enter the instantaneous utility function from choosing to stay in school, but it raises the option value of education and therefore its expected discounted stream of utility. The degree to which agents adjust their educational decision should be informative of their time preferences.

The identification argument for the model with exponential discounting is similar to the one found in Schneider (2020). The probability of being restricted is deterministic and depends on the previous choice and on the current state of the world (Figure 3.1). The state of the world evolves similarly for restricted and unrestricted agents, as they are not prevented from accumulating education and on-the-job experience in A, which has the same return in both occupations. The same argument is extended for the model with hyperbolic discounting and completely naïve agents. Since the time preference parameters to be identified are now two, there are three different state variables that generate variation in the agents' occupation possibilities. Note that the additional restriction on A is particularly binding, because it forces agents to stay out of the labor force until they have chosen education for at least two periods. Staying at home in early periods

Figure 3.1: Decision Tree with Restricted Choice Set



Notes. *Effect on the choice set of a choice restriction, if the agent has 13 period of experience in education in t. For simplicity, the home alternative is removed. Choosing the education alternative in any period* $t+n$ *allows the agent to choose occupation B starting from period* $t+n+1$. *An unrestricted agent can choose any alternative in any period.*

is then highly suggestive of present bias, as the option value of education becomes very high. Finally, the long optimization horizon (agents are observed for 40 consecutive periods) should be an additional aid to identification.

3.4 Simulated data

The "observed" data are two datasets with 10,000 agents observed for 40 periods, simulated using the software *respy* after I implemented the solution and simulation of the model for a naïve hyperbolic discounter[3].

The parametrizations used to generate the datasets are shown in Table A1 in the Appendix. The choice probabilities of the agents, conditional on the policy they experience, are illustrated in Figure 3.2. Summary statistics on the conditional choice probabilities and on the wages of those who work are shown in Table 3.1 and Table 3.2 respectively.

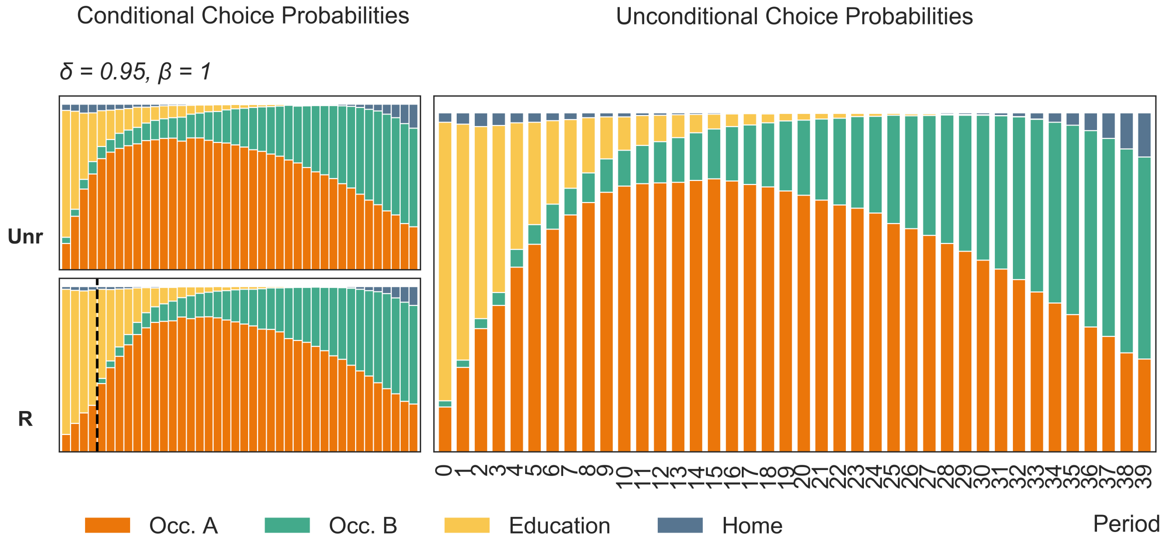
At the beginning of the life cycle the choice is among education, home, and occupation A, since occupation B provides large returns only when agents have accumulated experience in education and/or occupation A. The utility from education is low, and it becomes negative if agents go to college. Agents who face one or two restrictions tend to stay in education longer than those who don't (Table 3.1) regardless of their time preferences, as the choice restrictions increase the option value of education. As a result, the mean wage of restricted agents is higher (Table 3.2), because each additional year of education provides a positive return.

Hyperbolic discounters choose the home alternative more often than exponential discounters,

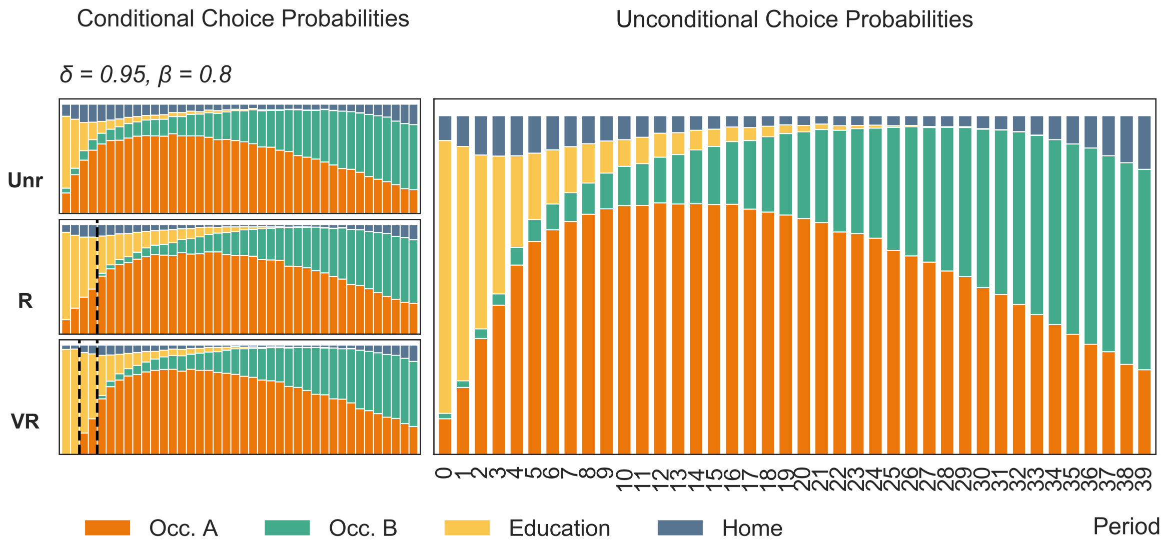
[3]See `https://github.com/OpenSourceEconomics/respy/pull/347`

Figure 3.2: Choice Probabilities

(a) Model with Exponential Discounting



(b) Model with Hyperbolic Discounting



Notes: *Model of occupational choice based on Keane and Wolpin (1994), one dataset with 10,000 simulated agents and 40 observed periods. The figure shows choice probabilities for unrestricted agents (**UNR**), restricted agents who face one restriction on occupation B (**R**) and very restricted agents who face one restriction on A and one on B (**VR**). In (a), the dashed line indicates the first period in which agents can potentially choose occupation B, as the initial conditions of the model ensures that all agents start with 10 years of education. In (b), the dashed lines indicate the first period in which agents can potentially choose occupation A and occupation B, after 12 and 14 years of education respectively.*

Table 3.1: Conditional Choice Probabilities

	Exponential discounting		Hyperbolic discounting		
	No restriction	One restriction	No restriction	One restriction	Two restrictions
Occ. A	0.61	0.59	0.54	0.57	0.56
Occ. B	0.27	0.25	0.31	0.26	0.27
Education	0.09	0.14	0.06	0.12	0.12
Home	0.02	0.02	0.09	0.05	0.05
Observations	5105.00	4895.00	3429.00	3205.00	3366.00

Table 3.2: Distribution of Observed Wages

	Exponential discounting		Hyperbolic discounting		
	No restrictions	One restriction	No restrictions	One restriction	Two restriction
Mean	48841.81	50948.94	47595.00	49361.78	49850.32
Standard Deviation	56658.72	57465.12	53619.12	54421.62	54746.45
Observations	5105.00	4895.00	3429.00	3205.00	3366.00

especially at the beginning of the life cycle and even when they experience the additional restriction on occupation A. A lower present bias makes education less attractive, as the future gains are discounted more heavily, so agents tend to choose to stay at home whenever the wage shock to occupation A is low.

Note that the exponential discounters who face restrictions stay longer in education and delay their entrance in the job market, while the share of agents shifting to the home alternative, which mostly happens at the end of the life cycle, is nearly unchanged. The entrance in the job market is similarly delayed for hyperbolic discounters, but the share of those who choose the home alternative in the initial periods decreases, which is consistent with the idea that hyperbolic discounter are more sensitive to educational incentives (because they stay in education shorter than optimal in the first place).

3.4.1 Assessing the validity of the choice restrictions. Choice restrictions need to be salient to the decision-making process of the agents, a requirement which is addressed by a precise rank condition in theoretical results and by economically-motivated analysis in empirical applications.

In Haan et al. (2020), exogenous variation in the length of job protection for working mothers provides observable state variables that matter for choice, without directly entering the per-

Table 3.3: Effect of Choice Restriction on Years of Education

	Years of education ($t = 10$)		Years of education ($t = 40$)	
	Exponential	Hyperbolic	Exponential	Hyperbolic
Intercept	12.85***	11.9 ***	13.59***	12.29***
	(0.03)	(0.03)	(0.03)	(0.03)
Restriction	2.08***	1.84***	2.13***	2.6 ***
	(0.04)	(0.04)	(0.04)	(0.04)
Double restriction		2.4 ***		2.65***
		(0.04)		(0.04)
Observations	10 000.0	10 000.0	10 000.0	10 000.0
R^2	0.25	0.3	0.25	0.45
Adj. R^2	0.25	0.3	0.25	0.45
Residual Std. Error	1.78	1.58	1.86	1.38
F Statistic	3441.22***	2224.25***	3299.03***	3340.6 ***
Note:			***p<0.01;**p<0.05;*p<0.1	

period utility function. The authors show that women who enjoyed longer period of job protection took significantly longer career breaks than those who didn't, which is taken as evidence of the policies being salient. In Kemptner and Tolan (2018), the argument for identification exploits educational reforms that, the authors argue, increased the probabilities of students gaining a degree during their time spent in school, without influencing the per-period utility of choosing to stay in school. Therefore, to test the salience of these educational reforms it is sufficient to check whether being part of a cohort affected by the reforms increases the transition probabilities from actual to "successful" years of education, that lead to a degree.

In the two revisited version of Keane and Wolpin (1994), the choice restrictions should influence the agents' educational decisions, particularly at the beginning of the life cycle. Therefore, it is natural to check whether being exposed to increasingly more restrictive policy raises, on average, the agents' years of education. The regression results in Table 3.3 show that experiencing one or more choice restrictions increases the experience in education. This difference is persistent between unrestricted and restricted agents. However, in the dataset where agents are hyperbolic discounters, the difference between agents who experience one and two restrictions is relatively large at the beginning of the life cycle but it reduces significantly by the last observed period.

4 Results

During the estimation process the econometrician chooses a method to fit the candidate model to the observed data, constructs the associated criterion function, and chooses the optimization algorithm to recover the vector of model parameters estimates.

If a structural model's parameters are identified, it should be possible to consistently estimate them. However, the criterion function may be non-smooth, have multiple local minima, be discontinuous or non-differentiable. The (multidimensional) optimization problem may be further complicated by the presence of constraints, which are common in structural economic models: As an example, in the model presented here, the probabilities to experience a certain restriction regime need to sum up to 1, and the variance-covariance matrix of the payoff shocks needs to be positive definite and symmetric.

All these features complicate both solving the optimization problem and making inferences about the estimated parameters.

Moreover, carrying out the model estimation on a single dataset may not be particularly informative in this setting, as sampling variation would be neglected. On the other hand, estimation on many samples is time-consuming and computationally expensive, especially for a relatively complex model with a long planning horizon like the one presented here.

For these reasons, the following section focuses on investigating the performance of the chosen criterion function to find out whether the true model may be consistently estimated on the datasets previously described, while stopping short of the real estimation process.

4.1 Practical identification

In practice, whether the time preference parameter(s) can be identified from the data depends on the curvature of the criterion function. Consider the problem of estimating a model with exponential discounting on observational data: If the criterion is computed as a function of δ only, while all the other parameters are set a their respective estimates, we need the criterion to exhibit a unique minimum (or maximum) between 0 and 1 to recover δ.

Here, I mainly focus on the practical identification of the time preference parameters, the true values being $\delta = 0.95$ and $\beta = 0.8$, and set all the other parameters at their true, rather than estimated, values. This overcomes the hurdles of the real estimation process, simplifies the exposition and aids the visualization of the results. However, it represents a "best case scenario" where practical identification is not complicated by inaccurate estimates of the other model parameters.

I use the Method of Simulated Moment (MSM), a simulation-based estimation method introduced by McFadden (1989). The MSM estimator is the parameter vector that minimizes the weighted distance between the empirical set of moments, computed from the observed dataset, and the set of simulated moments generated by the candidate structural model. Formally, the MSM criterion function is:

$$\Delta(\theta) = [f - \tilde{f}(\theta)]' W^{-1} [f - \tilde{f}(\theta)], \tag{4.1}$$

where f represents the vector of empirical moments, $\tilde{f}(\theta)$ is the vector of simulated moments, and W is a positive definite weighting matrix.

In this setting, the candidate structural model with exponential (hyperbolic) discounting θ coincides with the true data-generating process of the "observed" dataset with exponential (hyperbolic) discounting described in Section 3.4. However, the empirical moments won't perfectly coincide with the simulated moments because of random variation in the payoff shocks, which induces variation in the agents' choice probabilities. This implies that the MSM criterion function is not necessarily minimized at the true parameter vector for a given finite sample.

The moments used for estimation are per-period choice probabilities, conditional on restriction regime, and the per-period wage profile (mean and standard deviation) of those who work, again conditional on restriction regime. The weighting matrix is a diagonal matrix where the weights are the inverse variances of the observed sample moments, computed via a bootstrapping procedure.

4.1.1 Model with exponential discounting. Figure 4.1 represents the criterion function, evaluated at different values of the discount factor δ, holding all the other parameters at their true value. The global evaluation exploits the theoretical bounds of the δ parameter. The same exercise is repeated for other selected parameters in Figure 4.2 and for some elements of the variance-covariance matrix of the shocks in Figure 4.3.

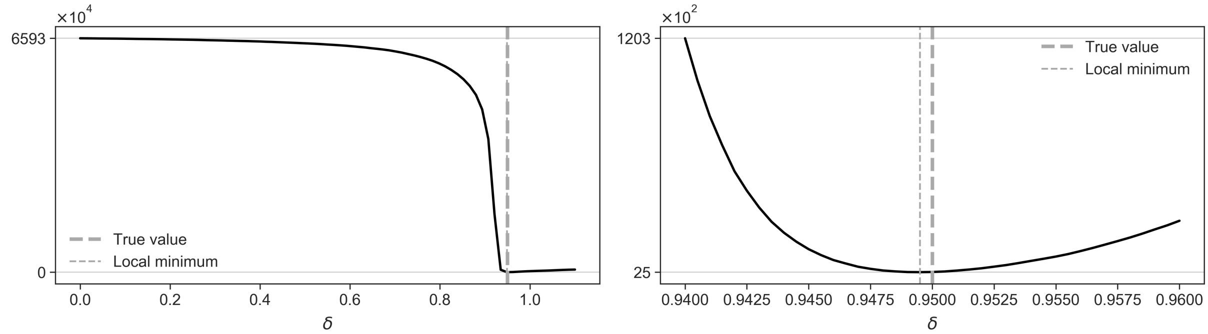
The criterion function displays a global minimum at $\delta = 0.949$ and has a minimum close to the true value for the parameters that enter the wage function for occupation A (return to education and return to experience in occupation A), while is relatively flatter for those parameters entering the wage of occupation B and for most of the elements and for most elements of the variance-covariance matrix of the shocks. In the latter cases, a closer look reveals that the criterion function is particularly non-smooth and displays multiple local minima, which may require fixing or imposing tight bounds on such parameters during the estimation process.

4.1.2 Model with hyperbolic discounting. Figure 4.4 represents the criterion function evaluated at different values of the time preference parameters β and δ, with all the other parameters

Figure 4.1: Univariate Distribution of Discount Factor,

Model with Exponential Discounting

(a) Global Evaluation **(b)** Local Evaluation



Notes: *Evaluations of Method of Simulated Moment's criterion function for model with exponential discounting based on Keane and Wolpin (1994). The criterion function is evaluated for different values of the discount factor, while all the other parameters are fixed at their true values.*

are fixed at their true value. The same exercise is repeated for other selected parameters and elements of the variance-covariance matrix of the shocks (Figure A2 and Figure A3 in the Appendix). The results are similar to those discussed above.

Figure 4.5 plots the criterion function for many combinations of β and δ, making the optimization problem two-dimensional. The darker the area, the lower the value of the criterion function associated with the corresponding combination of β and δ. All the other parameters are again held at their true values.

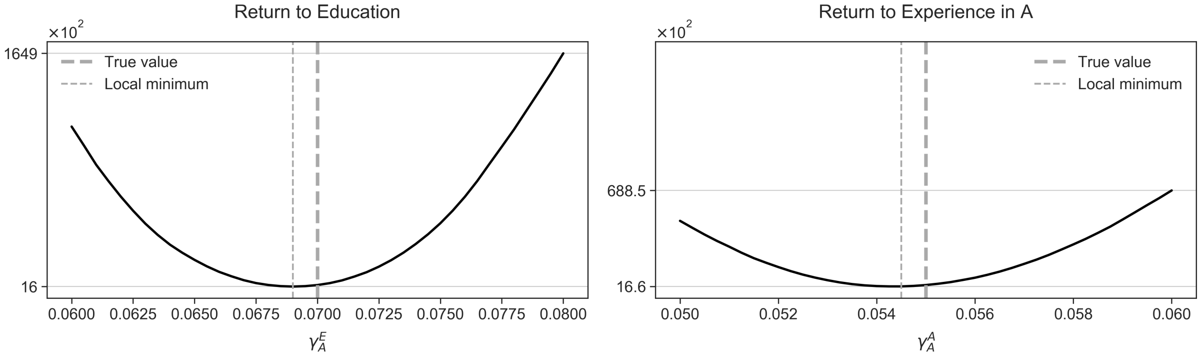
The global minimum is at $\delta = 0.948$ and $\beta = 0.83$ and there is no basin around the minimum. The criterion function takes similar values for combinations of the time preference parameters where β is underestimated with respect to the true value and δ is overestimated, and vice versa. The range of values of δ associated with a relatively low value of the criterion function is tighter than the range of values of β.

This pattern is similar to the "banana-shaped through" found in Abbring et al. (2018) and Laibson et al. (2007), which consider models that are different from the one presented here. In particular, Abbring et al. (2018) use a simple model of binary choice based on Rust (1987) to assess whether their exclusion restrictions are sufficient to achieve identification in a simulated dataset. The estimation routine is based on a minimum-distance estimator. The data are observed only for three periods, which is the theoretical minimum number of periods required for identification, while the number of simulated agents is large (1 million).

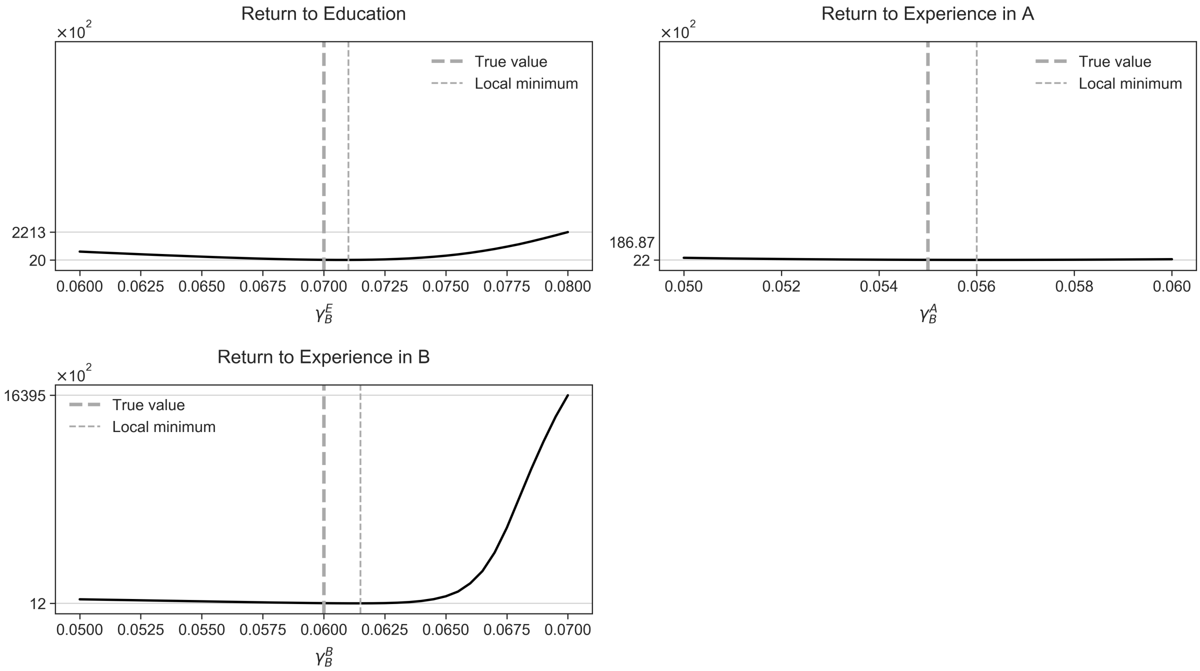
Laibson et al. (2007) estimate a complex model of labor supply using data on retirement, credit

Figure 4.2: Univariate Distribution of Wage Parameters,
Model with Exponential Discounting

(a) Occupation A



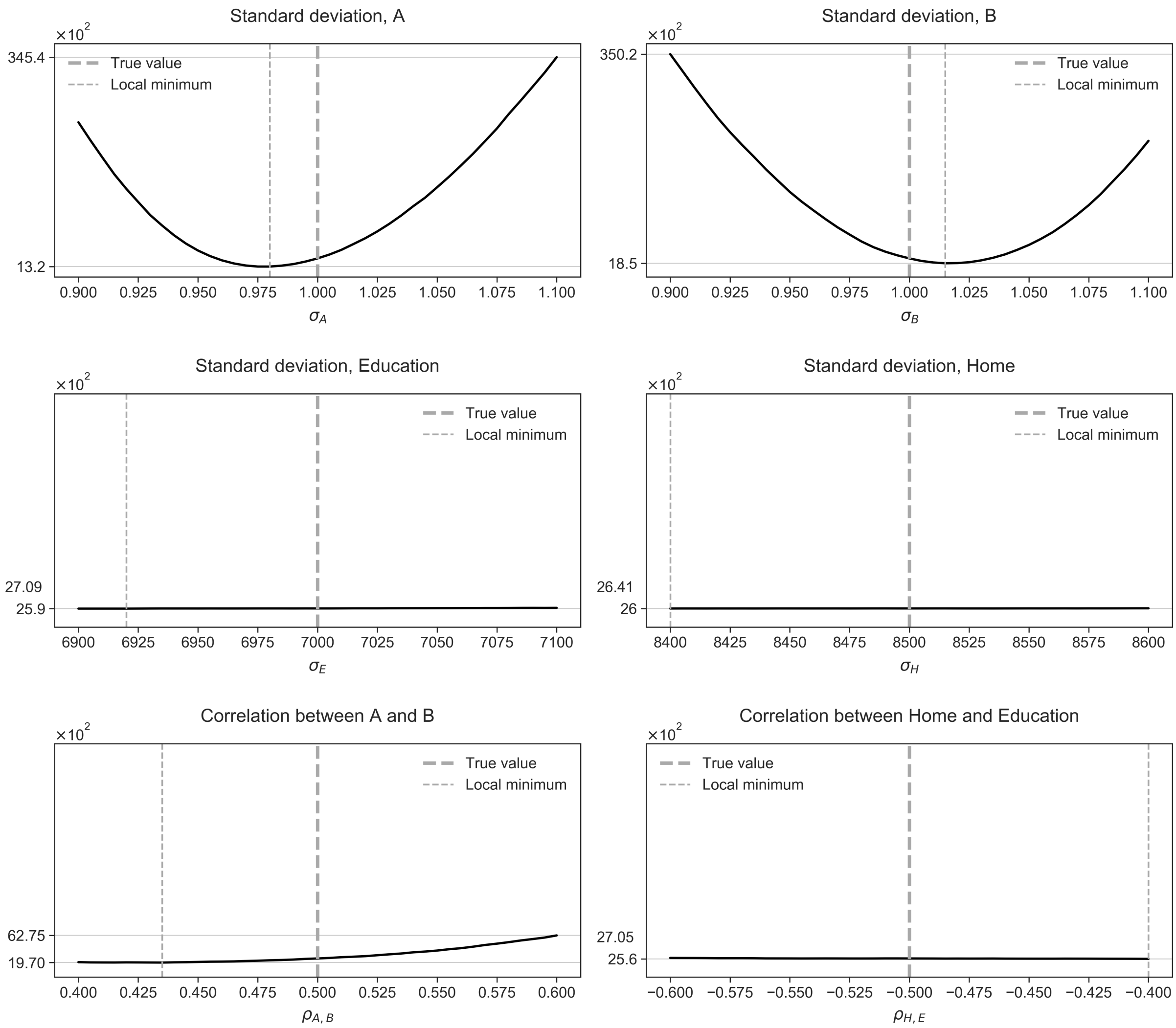
(b) Occupation B



***Notes**: Evaluations of Method of Simulated Moment's criterion function for model with exponential discounting based on Keane and Wolpin (1994). All parameters besides the x-axis parameter are fixed at their true values.*

Figure 4.3: Univariate Distribution of Variance-Covariance Matrix of Shocks,

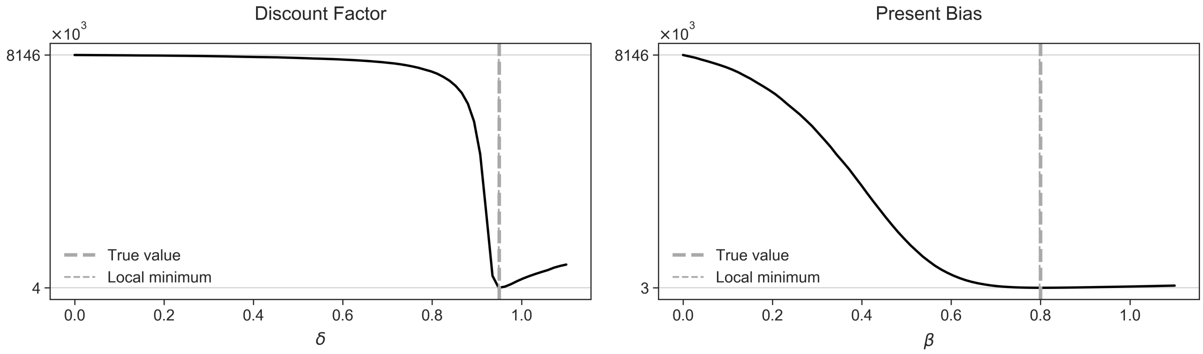
Model with Hyperbolic Discounting



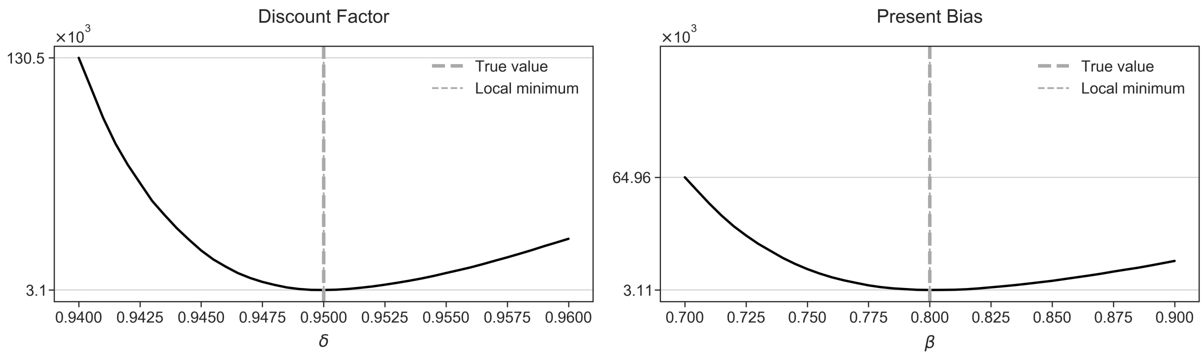
Notes: *Evaluations of Method of Simulated Moment's criterion function for model with hyperbolic discounting based on Keane and Wolpin (1994). All parameters besides the x-axis parameter are fixed at their true values.*

Figure 4.4: Univariate Distribution of Time Preference Parameters,
Model with Hyperbolic Discounting

(a) Global Evaluation



(b) Local Evaluation



***Notes**: Evaluations of Method of Simulated Moment's criterion function for model with hyperbolic discounting based on Keane and Wolpin (1994). The criterion function is evaluated for different values of the discount factor (figures on the left) and of the present bias (figures on the right) respectively, while all the other parameters are fixed at their true values.*

card borrowing, wealth accumulation and consumption-income co-movement via the Method of Simulated Moments. They note that the criterion function is strictly convex in the region they consider ($0.2 < \beta < 1$ and $0.93 < \delta < 0.99$) and rises whenever both the two parameters rise or fall, a description that fits Figure 4.5.

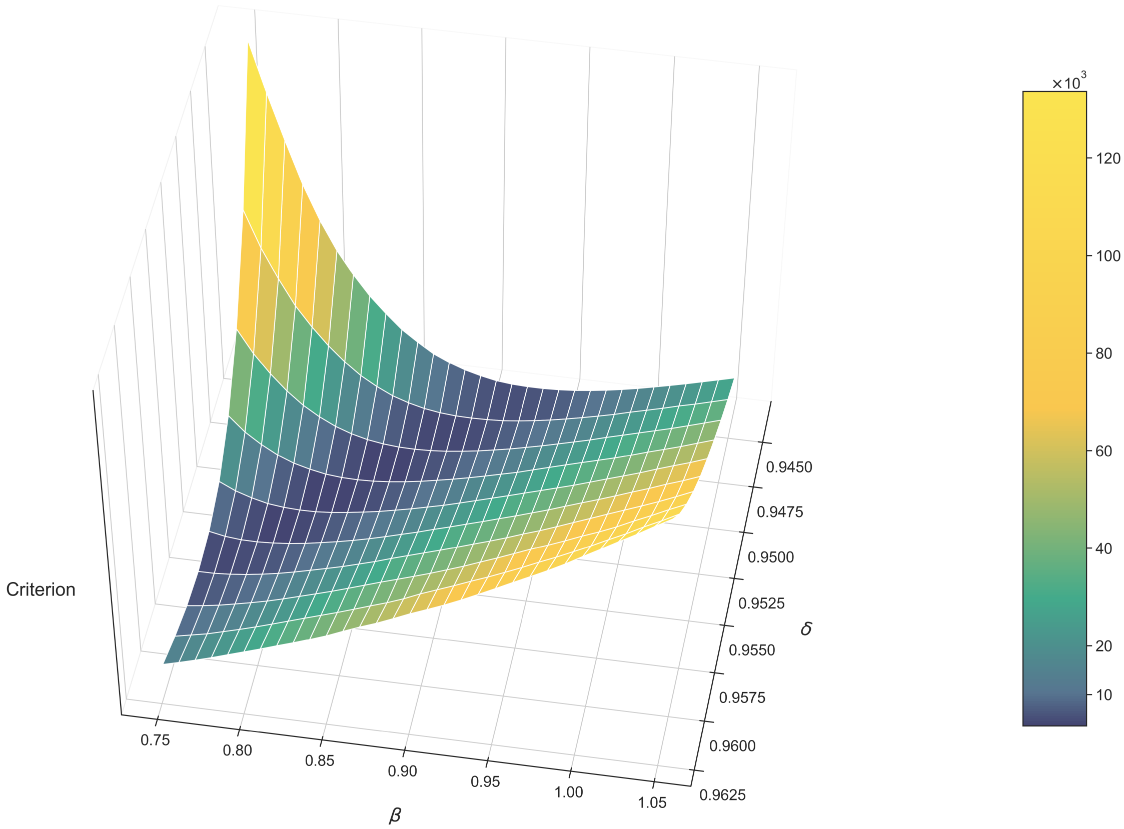
When β is fixed at 1, the criterion function attains the lowest value for $\delta = 0.938$, which is consistent with the intuition that a misspecified exponential model fitted on hyperbolic dataset needs a lower discount factor to rationalize the data.

Figure 4.5 points at the practical difficulty of disentangling β and δ in estimation which is mentioned elsewhere in the literature. A relevant question is then whether different combinations of the two parameters produce similar life-cycle patterns and similar counterfactuals.

The following sections focus on the specification with hyperbolic discounting, first comparing the life-cycle pattern observed for unrestricted agents to that predicted by various models where

Figure 4.5: Heatmap Criterion for Time Preference Parameters,

Model with Hyperbolic Discounting



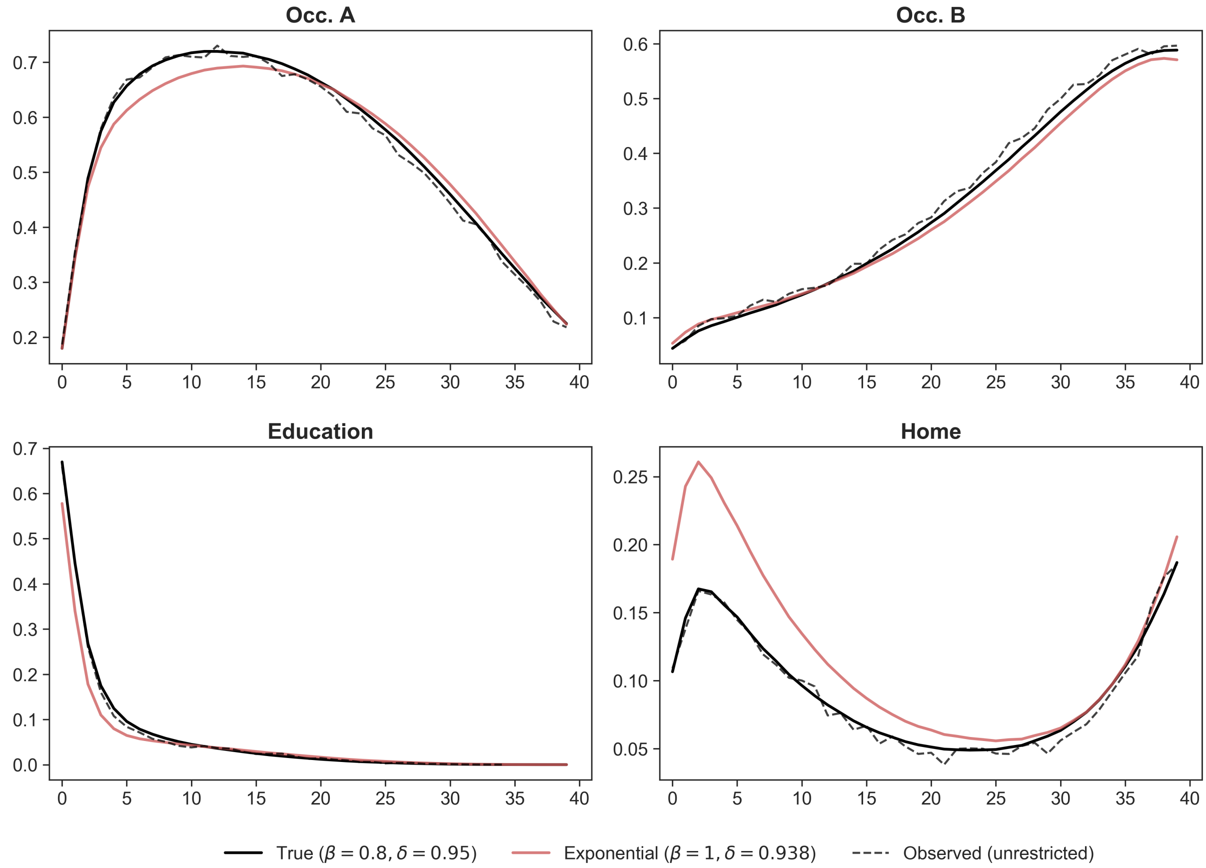
***Notes**: Evaluations of Method of Simulated Moment's criterion function for model with hyperbolic discounting based on Keane and Wolpin (1994), for different value of discount factor and present bias.*

the time preferences are misspecified, and then exploring the sensitivity of counterfactual predictions to model misspecification.

4.2 Predicted life-cycle pattern

Figure 4.6 compares the average choice probabilities pattern predicted by the true hyperbolic model and by the misspecified exponential model respectively, in absence of choice restrictions. The dotted line shows the choice probabilities for the unrestricted agents in the "observed" data. The average pattern predicted by the true hyperbolic model is shown in Figure 4.6 . The share of agents in education drops quickly in the first few periods (0.668 in $t = 0$, 0.431 in $t = 1$, 0.251 in $t = 2$, 0.156 in $t = 2$). No more than 4% and 1% of the agents in each period chose education after $t = 10$ and $t = 20$ respectively. The share of individuals in occupation B grows slowly and constantly during the life-cycle, while the share of individuals in occupation A grows

Figure 4.6: Life-Cycle Pattern, True Model vs. Misspecified Models



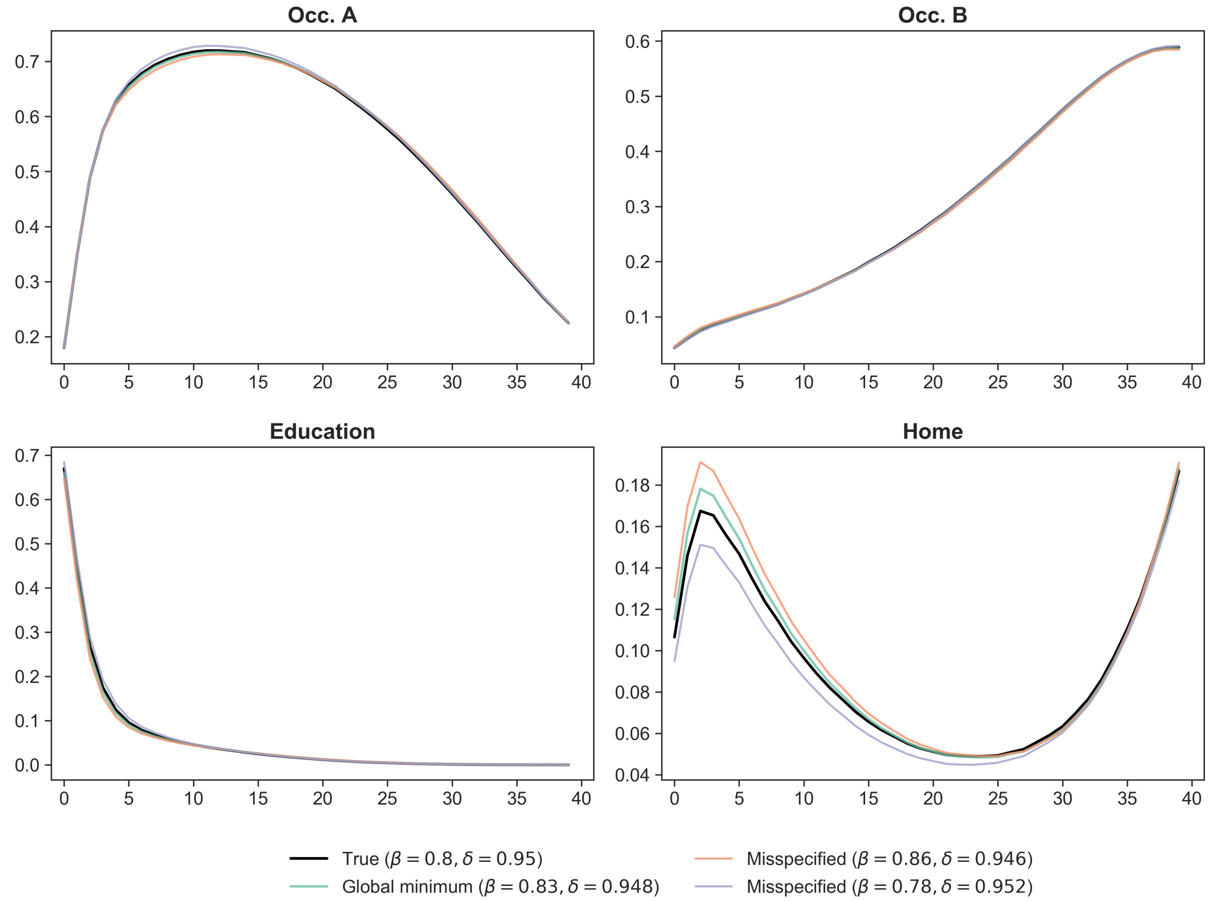
***Notes**: Average share of individuals in each occupation in each period, as predicted by different models. The average is computed over 100 datasets, each with 10,000 agents observed for 40 periods.*

quickly in the first 5 observed periods (from 20% to nearly 64%), peaks between the 10th and the 15th period and then declines steadily. The share of agents who choose home increases during the first period (from 10% to nearly 17%), declines steadily until the 25th period, and increases again at the end of the life-cycle in what Keane and Wolpin (1994) characterize as "voluntary retirement".

The misspecified model with exponential discounting compensates the absence of present bias with a lower discount factor, which is set at 0.938, the value of δ that minimized the criterion function when β is set to 1. All the other parameters are set at the true model values. With respect to the true model, the exponential one predicts a lower share of agents in education in the first 10 periods, a higher share of agents choosing occupation A in the first half of the life cycle, and a substantially similar pattern in occupation B. The share of agents choosing home, which is high and growing in the first 5 periods (from 19% to 24%), remains higher than the hyperbolic counterpart until the 30th period.

Figure 4.6 compares the average choice probabilities predicted by the true model with those

Figure 4.6: Life-Cycle Pattern, True Model vs. Misspecified Models (continued)

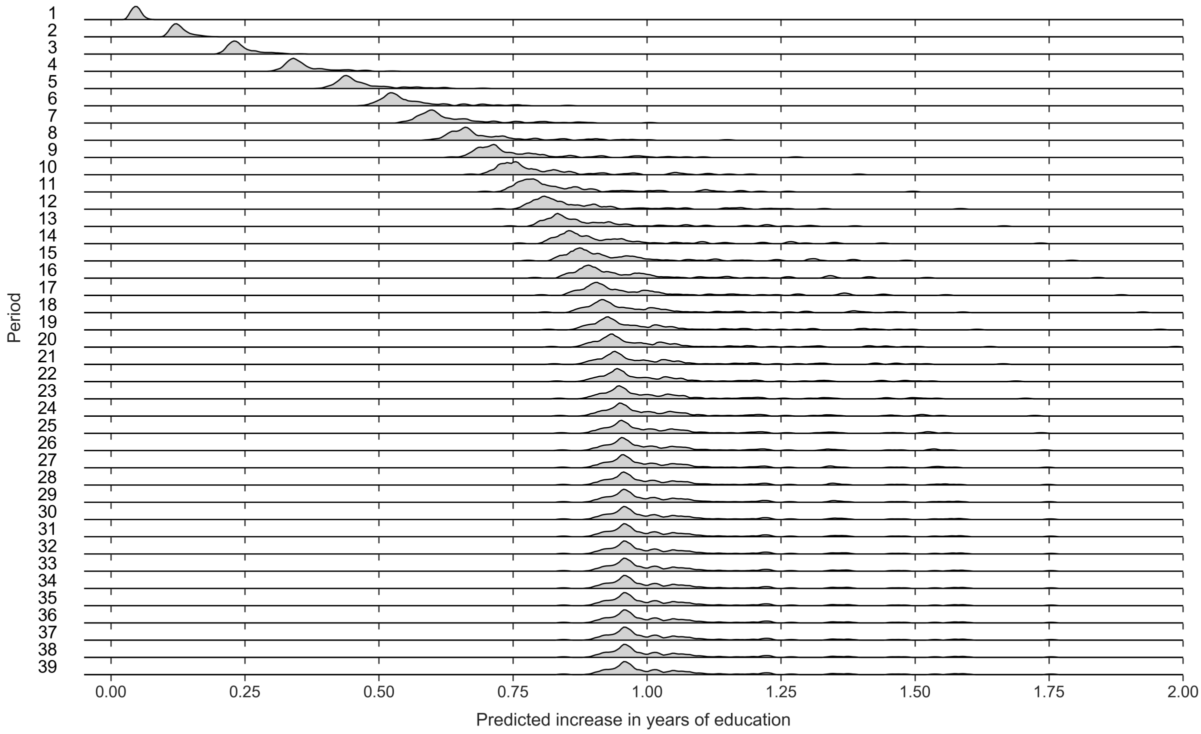


Notes: *Average share of individuals in each occupation in each period, as predicted by different models. The average is computed over 100 datasets, each with 10,000 agents observed for 40 periods.*

predicted by models where both β and δ are misspecified. I choose a few combinations of present bias and discount factor values in the dark area of Figure 4.5, which have criterion values close to that achieved by the true combination of parameters.

The average predictions for occupation A and occupation B are virtually identical. The model where β is closer to unity than in the true value (0.86 vs. 0.8) while δ is lower (0.946 vs. 0.95) predicts an higher share of agents at home when the home choice is peaking, around the 5th observed period, and a lower share of agents in education in the same interval. Vice versa, when β is set at 0.78 and δ, to compensate, is set at 0.952, the average share of agents at home is lower, while the share of agents in education is higher. This seems to suggest that the discount factor influences the trade-off between home and education at the beginning of the life cycle more than the present bias, which is not surprising given that the planning horizon is long and the costs of education are very low compared to the long-term benefits.

Figure 4.7: Predicted Effect of Tuition Subsidy on Experience in Education



***Notes**: Effect of 2000 USD tuition subsidy on experience in education, in each period, as predicted by the true model (*$10,000$ *agents,* 40 *observed periods).*

4.3 Counterfactual predictions

This section studies how the introduction of a 2000 USD tuition subsidy affects final and per-period experience in education. Figure 4.7 shows the distribution of the true subsidy effect on years of education in each period, for an example dataset. Periods are indicated on the y-axis, while the x-axis indicates the increase in years of education with respect to a counterfactual dataset where the subsidy is not implemented. All agents start with 10 periods of education in period 0, which is omitted.

The figure shows that the distribution of the true subsidy effect is decidedly skewed to the right, in each period. The subsidy is mostly effective in the first 10 periods of the life cycle, which is graphically represented by the per-period distributions shifting to the right of the plot. In the last 20 periods the distributions are stacked vertically, which means that agents have substantially stopped to accumulate experience in education.

The distribution in period 39 (last row of Figure 4.7 can be collapsed to single numbers if the quantity of interest is simply the average change in final years of education. Table 4.1 compares a few summary statistics derived from the distribution of the tuition subsidy effect on final years of education, as predicted by the true model and the misspecified models. The distribution is

Table 4.1: Effect of tuition subsidy on average years of education

	Mean	Median	SD	Min.	Max.
True ($\beta = 0.8, \delta = 0.95$)	1.071	0.989	0.211	0.844	2.216
Exponential ($\beta = 1, \delta = 0.938$)	0.779	0.765	0.082	0.594	1.207
Global minimum ($\beta = 0.83, \delta = 0.948$)	1.042	0.974	0.199	0.812	2.191
Misspecified ($\beta = 0.86, \delta = 0.946$)	0.999	0.938	0.178	0.767	2.066
Misspecified ($\beta = 0.78, \delta = 0.952$)	1.125	1.034	0.224	0.911	2.242

generated by 100 simulated datasets with sampling variation, and the standard deviation should be interpreted as reflecting the simulation error.

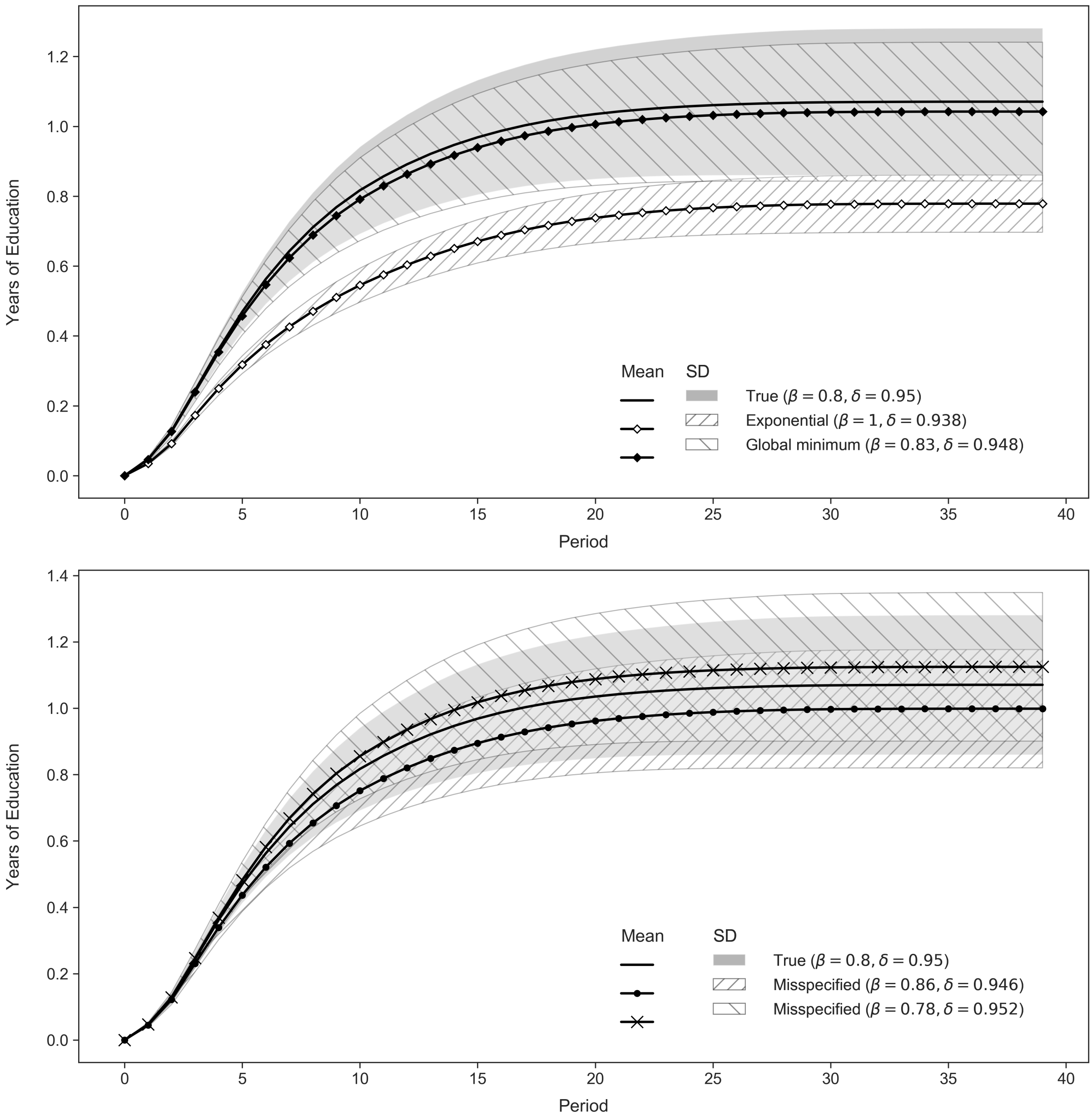
The true model with hyperbolic discounting predicts that agents spend on average about 1.07 additional years in education. The median is lower (0.99) because of the distribution's right tail, as seen in Figure 4.7. The model with exponential discounting predicts an increase of 0.78 years, with a tighter standard deviation. The average predicted effect is lower than the average true model's effect whenever the discount factor is lower than 0.95, despite the present bias being higher than 0.8, and vice versa when the discount factor is higher than 0.95. Again, the discount factor, rather than the present bias, appears to be the main driver of educational decisions.

Figure 4.8 shows the average effect of the tuition subsidy on experience in education, in each period, again computed on 100 datasets with sampling variation. It confirms that, in all models, the tuition subsidy is associated with the largest increase in years of education at the beginning of the life cycle, and that the lower the discount factor the lower the average predicted effect in each period. The shaded areas visually represent the simulation error, which is largely overlapping whenever the model is hyperbolic and smaller for the model with exponential discounting. The latter feature can be explained by the decision rule of naïve agents being dynamically inconsistent and more sensitive to random payoff shocks.

Kemptner and Tolan (2018) estimate both an exponential and an hyperbolic model on the same German Socio-Economic Panel data and similarly, when studying the effect of a student grant on successful years of education, find that the behavioral adjustment of hyperbolic discounters is larger. In their educational choice model hyperbolic discounters care less about achieving successful years of education with respect to exponential discounters, and are more likely to drop out shortly before achieving a degree. Therefore, hyperbolic discounters realize higher gains from staying in education longer.

In the model presented here, there is no distinction between successful and actual years of ed-

Figure 4.8: Counterfactual Predictions, True Model vs. Misspecified Models



***Notes**: Effect of tuition subsidy of 2000 USD on average years of education in each period, predicted by true model and misspecified models. The average is computed over 100 datasets, each with 10,000 agents observed for 40 periods.*

ucation. The trade-off for hyperbolic discounters is initially between home and education and, later, between education and occupation A. The simulated data show that hyperbolic discounter are more likely to choose the home alternative early in their life cycle: A tuition subsidy draws to education those at the margin. Unsurprisingly, agents react to the introduction of a tuition subsidy and to the introduction of a (deterministic) restriction in their choice set in the same way, that is, spending more time in education. Indeed, a choice restriction raises the option value of education, while the tuition subsidy lowers its per-period cost.

5 Conclusion

This thesis assesses the practical identification of the time preference parameters in a discrete choice dynamic model of occupational choice, after introducing empirically-motivated exclusion restrictions that influence the size of the agents' choice set. In particular, agents may or may not face future employment restrictions that depend deterministically on their educational choices. The intuition is that comparing the behavioral response of similar agents to different (expected) futures may reveal information on their time preferences.

In the literature, both theoretical arguments for identification and empirical identification strategies exploit, with mixed results, variables that leave the per-period utility function unaffected, while being relevant to the agents' decisions. Formal identification is elusive for the general case of a partially naive hyperbolic discounter, may not lead to consistently estimated parameters in practice, and often requires assumptions that may be too restrictive in empirical applications. On the other hand, empirical identification often requires the data to have specific features, or additional parametric assumptions about certain components of the model.

Here, practical identification is studied on two simulated datasets, generated respectively by two models of occupational choice based on Keane and Wolpin (1994), via the Method of Simulated Moments. In the first dataset, agents are exponential discounters, therefore their attitude towards the future is solely described by their discount factor, δ. In the second dataset, the agents are hyperbolic discounters and are completely naive with respect to their own time preferences. Their time preferences are described by the discount factor and the present bias, β.

I simplify the problem by focusing only on the practical identification of the time preference parameters, while all the other parameters are set to their true values. Additional results repeat the exercise for other selected parameters, that enter either the wage functions or the variance-covariance matrix of the payoff shocks. This approach has an obvious limitation: It does not address the impact on identification of additional issues that may occur during the estimation process, such as the inaccurate estimates of the other parameter.

When the problem is one-dimensional, that is, the criterion function is computed as a function of δ or β only, the criterion appears to be smooth and displays a (global) minimum close to the true value of the time preference parameters for both simulated datasets, which is an encouraging, although preliminary, result.

For the second dataset, I address the two-dimensional problem of recovering δ and β simultaneously, which is visualized by means of a heat map of the criterion function. The combination of parameters that jointly minimize the criterion function slightly underestimates the discount factor and overestimates the present bias. Interestingly, the pattern around the minimum resembles the "banana-shaped through" that appears elsewhere in the literature: The values of the criterion function are similar for combination of parameters that underestimate the discount factor and overestimate the present bias, and vice versa. This suggests difficulties in disentangling the two parameters during estimation.

Do combinations of time preference parameters belonging to the "banana-shaped through" generate significantly different predictions? Again assuming that all other parameters are fixed at their true values, I compare an arbitrary counterfactual outcome and the life-cycle patterns predicted by the "true" model (that generates the second dataset) and a few misspecified versions of the same model. I additionally include the misspecified model with exponential discounting which comes closer to matching the data. When the present bias is fixed at 1, that is, the corner case equivalent to exponential discounting, the discount factor that minimizes the criterion function is lower than the "true" δ, which is consistent with economic intuition.

The results show that all the models with hyperbolic discounting considered predict, on average, a substantially similar life-cycle pattern. The counterfactual predictions for the effect of a tuition of subsidy on years of education are also overlapping. The model with exponential discounting predicts more agents at home and in occupation A, and fewer agents choosing education. Moreover, the behavioral adjustment of exponential discounters to the introduction of a subsidy is smaller than that of hyperbolic discounters, which is again consistent with economic intuition. Finally, the discount factor appears to be more relevant for educational choice than the present bias.

This work has some obvious limitations, which may be overcome with higher computational power: Besides bypassing the estimation procedure and related issues, practical identification is investigated n only on two simulated datasets and on a few selected parameters. Therefore, the results should be seen as partial and preliminary. Nevertheless, these partial results appear to be consistent with economic intuition, and similar patterns are observed elsewhere in the literature on the identification of time preferences. Further research on the sensitivity of model predictions to misspecification may be useful, as estimating the parameters separately seems to be a recurring problem in empirical research.

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Appendix

Table A1: Model with Exponential Discounting – Parametrization

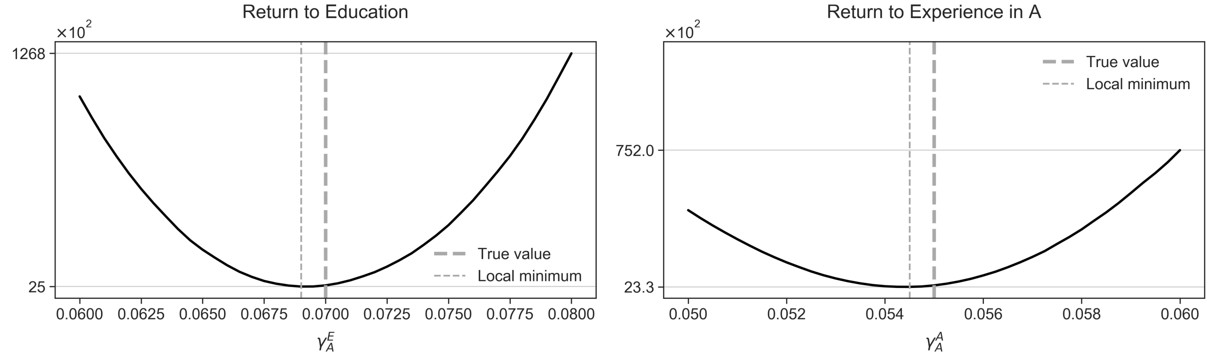
Parameter	Value	Comment
Time preference parameters		
δ	0.95	Discount factor
β	1	Present bias
Occupation A		
α_0^A	8	Log of rental price
α_1^A	0.07	Return to an additional year of schooling
α_2^A	0.055	Return to same sector experience
α_3^A	0	Return to same sector, quadratic experience
α_4^A	0	Return to other sector experience
α_5^A	0	Return to other sector, quadratic experience
Occupation B		
α_0^B	7.9	Log of rental price
α_1^B	0.07	Return to an additional year of schooling
α_2^B	0.06	Return to same sector experience
α_3^B	0	Return to same sector, quadratic experience
α_4^B	0.055	Return to other sector experience
α_5^B	0	Return to other sector, quadratic experience
Education		
γ_0	5000	Constant reward for choosing education
γ_1	-5000	Cost of going to college (tuition, etc.)
γ_2	-20000	Penalty for going back to school
Home		
Θ	21500	Constant reward of home alternative

Standard Deviation/Correlation Matrix		
σ_0	1	Standard deviation for A
σ_1	1	Standard deviation for B
σ_2	7000	Standard deviation for E
σ_3	8500	Standard deviation for H
$\rho_{1,0}$	0.5	Correlation between B and A
$\rho_{2,0}$	0	Correlation between E and A
$\rho_{2,1}$	0	Correlation between E and B
$\rho_{3,0}$	0	Correlation between H and A
$\rho_{3,1}$	0	Correlation between H and B
$\rho_{3,2}$	-0.5	Correlation between H and E
Probabilities		
π_R	0.5	Probability of experiencing choice restriction on B
π_{UNR}	0.5	Probability of not experiencing choice restriction
$\pi_{E,t-1}$	1	Probability that the first lagged choice is education
$\pi_{E=10,t=0}$	1	Probability that the initial level of education is 10

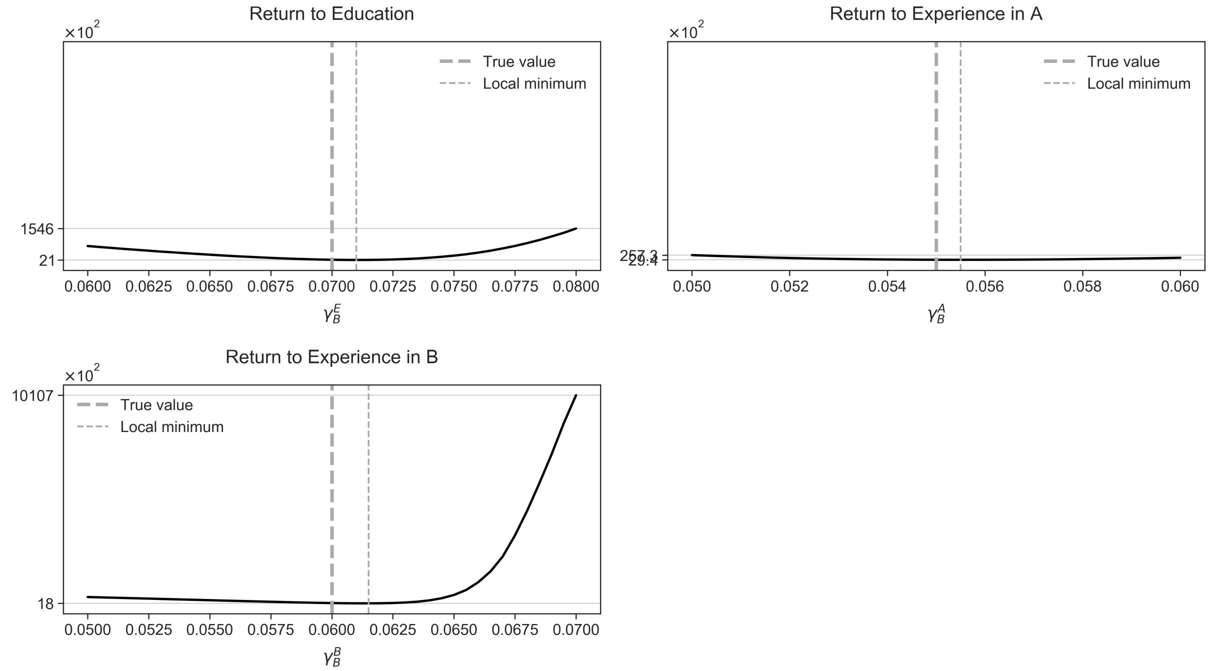
***Notes**: The parametrization of the model with hyperbolic discounting is nearly identical: β is set to 0.8 and $\pi_{UNR} = 0.34$, $\pi_R = \pi_{VR} = 0.33$, where π_{VR} is the probability of facing choice restrictions on both A and B.*

Figure A2: Univariate Distribution of Wage Parameters,
Model with Hyperbolic Discounting

(a) Occupation A

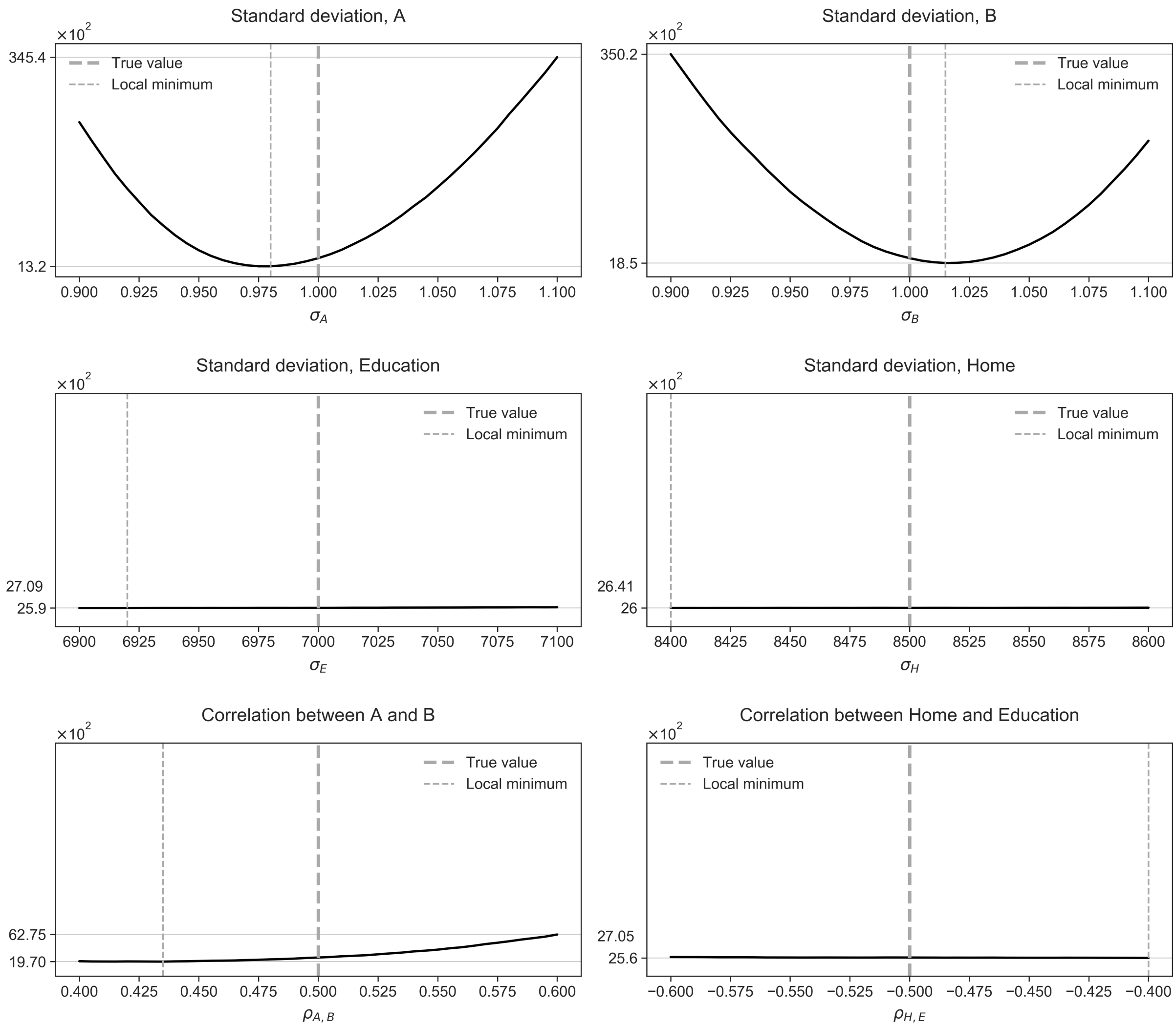


(b) Occupation B



***Notes**: Evaluations of Method of Simulated Moment's criterion function for model with hyperbolic discounting based on Keane and Wolpin (1994). All parameters besides the x-axis parameter are fixed at their true values.*

Figure A3: Univariate Distribution of Variance-Covariance Matrix of Shocks, Model with Hyperbolic Discounting



***Notes**: Evaluations of Method of Simulated Moment's criterion function for model with hyperbolic discounting based on Keane and Wolpin (1994). All parameters besides the x-axis parameter are fixed at their true values.*