Simulating traffic flow using cellular automata

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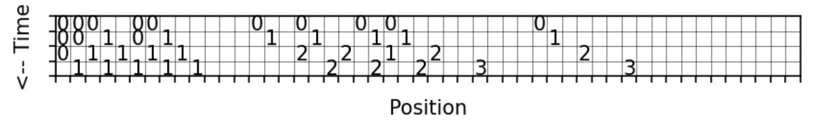
Presentation overview

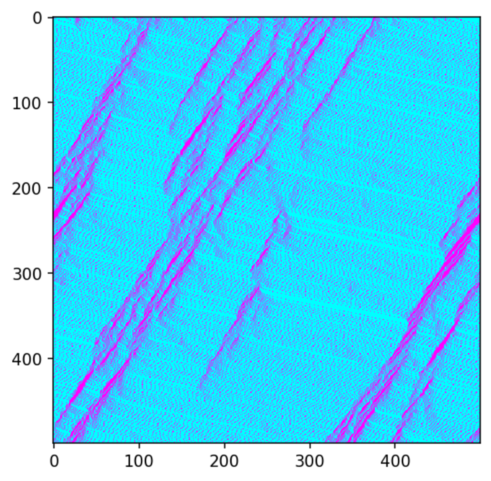
1. Model design: Nagel Schreckenberg
2. Motivation and research questions
3. Analysis and results:
    3.1 How to measure traffic (flow and clusters)
    3.2 Effects of varying the car density
    3.3 Effects of varying the maximum velocity
    3.4 Effects of varying the maximum acceleration
    3.5 Effects of varying the maximum braking
4. Conclusions

The model: Nagel-Schreckenberg



Rules:

1. Velocity
   $v \mapsto \min\{v+1, v_{max}\}$
2. If car in v spaces ahead, car brakes enough to be directly behind the first car ahead.
3. Velocity
   $v \mapsto \max\{0, v-1\}$ with probability p
4. Position is updated by v steps



The model: Nagel-Schreckenberg

Motivation:

Rule 184

Easy to conceptualise

Computationally simple

Default parameters:

Road length $L = 500$ cells $(2.5km)$

$\leftrightarrow$ 1 car is 1 cell $= 7m$

Timesteps $t_{max} = 1000$

$\leftrightarrow$ 1 timestep $= 1s$

Maximum velocity $v_{max} = 5\,\frac{cells}{timestep} = 126\,\frac{km}{h}$

Density $\rho = 0.13$

Probability of random braking $p = 0.2$

Maximum braking $=$ Maximum acceleration $= 1$

$\leftrightarrow$ corresponds to $7\,\frac{m}{s^2}$

Research questions

Main question:

Which initial conditions of parameters lead to congestion? Is there a phase transition for when traffic emerges, and if so what is the critical point?

Parameters:

1. Density
2. Maximum velocity
3. Braking probability
4. Slow braking vs sharp braking
5. Slow acceleration vs sharp acceleration

Main hypothesis:

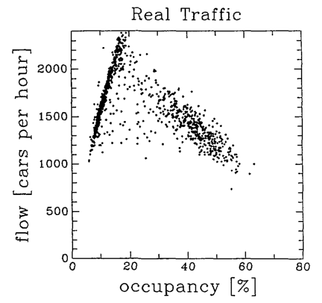
We expect to see a phase transition when varying each parameter. There exists a critical value, below which traffic flows smoothly, but beyond which congestion increases.

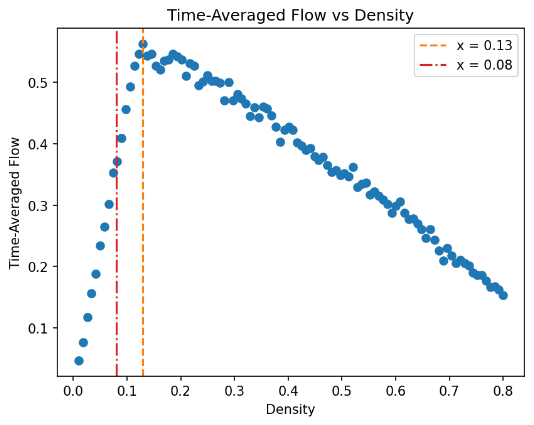
Flow vs Density

The time-averaged flow between fixed cells i and i+1:

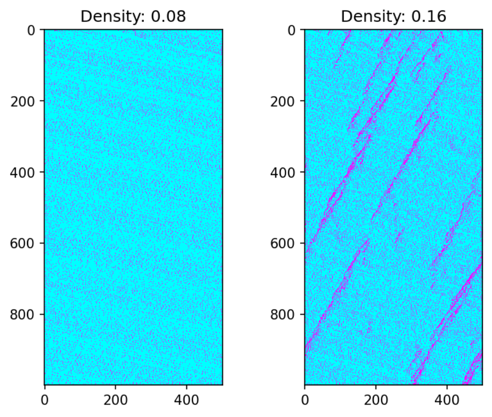
$$\frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{i,i+1}(t)$$

$$n_{i,i+1} = \begin{cases} 0 & \text{if no car passes} \\ 1 & \text{if a car passes} \end{cases}$$



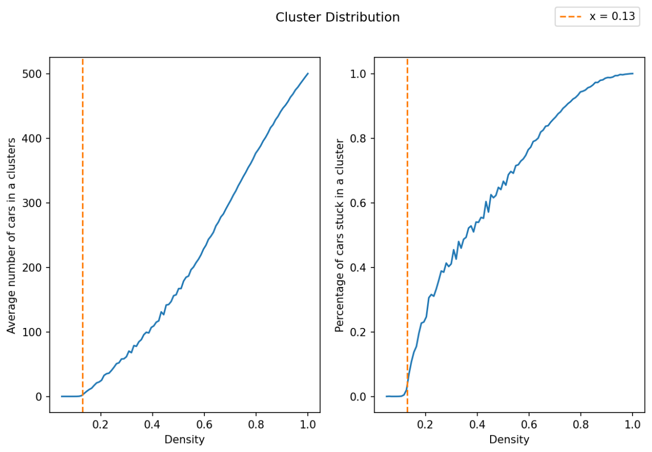


Phase transition in density

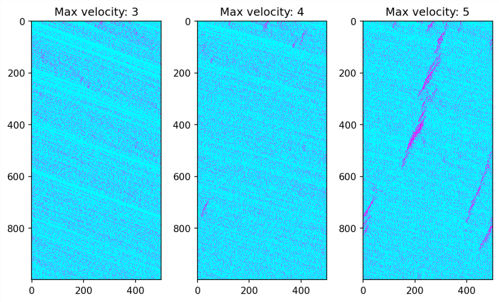


Number of cars in traffic

At our critical density, 0.13, the number of cars stuck in traffic suddenly starts to increase.

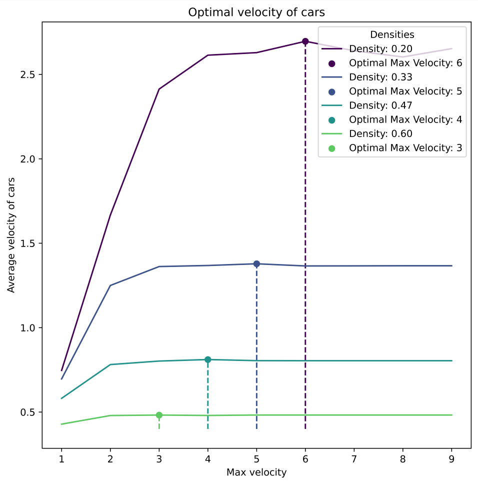


Maximum velocity



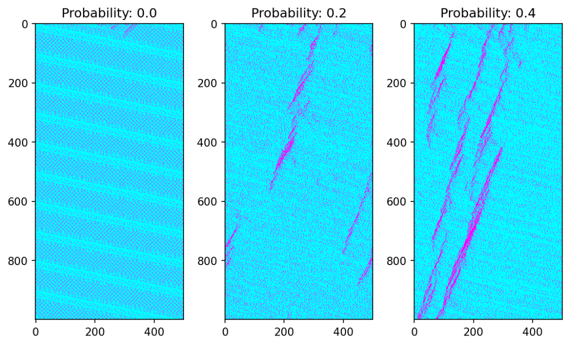
Maximum velocity is discrete $\implies$ no continuous phase transition. For densityn $\rho = 0.13$, we find a "critical velocity" of $v_{max} = 4$.

Optimal velocity

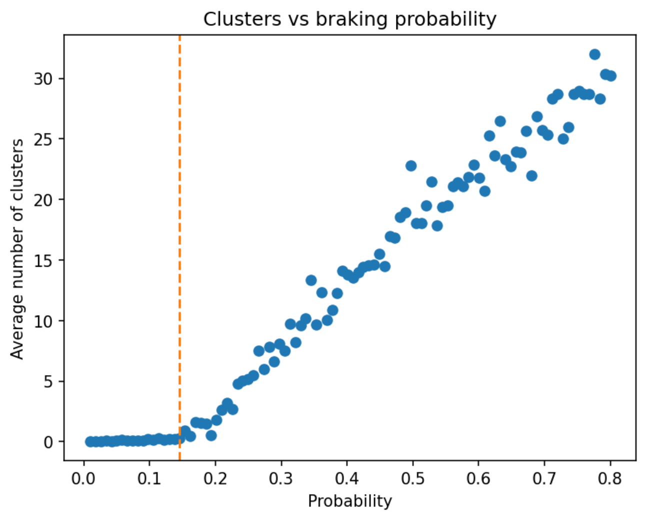


Correlation between ρ and v_{max}. Implies existence of "optimal" velocity.

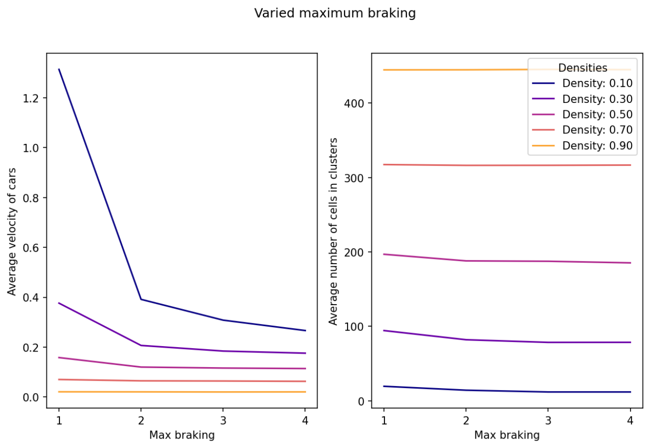
Braking probability



Phase transition in braking

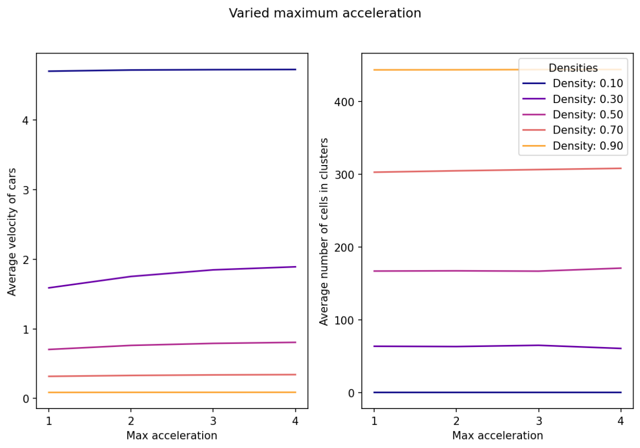


Maximum braking



Note: Braking is sampled from $U(0, max_{brake})$. Sharp breaking leads to lower velocities, but no change in the number of cars in traffic.

Maximum acceleration



Note: Accelerating is sampled from $U(0, max_{accel})$

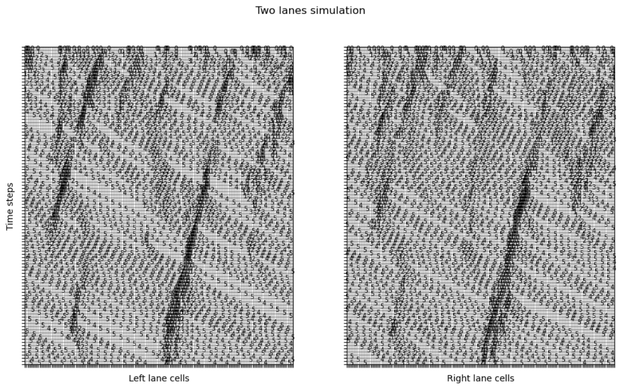
Conclusions

- The model undergoes a phase transition in flow under ρ
- A phase transition in number of clusters under p
- A discrete "phase transition" occurs under v_{max}
- No transition under max_{brake} and max_{accel}

Further exploration

- Open road/system where new cars can come go
- A more continuous model
- Different road types
- A 2-dimensional model

Two lanes



Two-lane Nagel-Schreckenerg model

End

Thank you for listening!

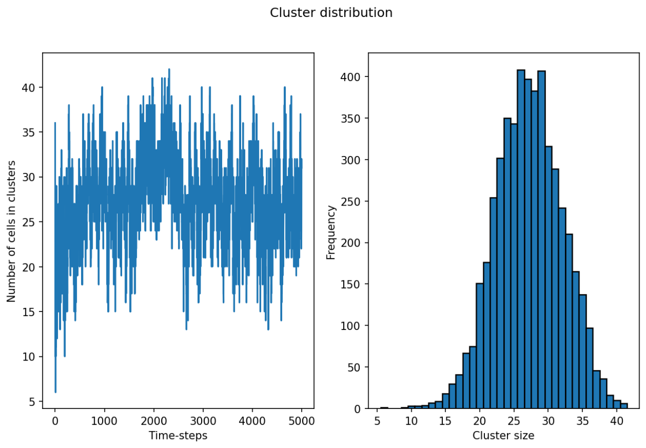
References

Nagel, Kai en Michael Schreckenberg (1992). **"A cellular automaton model for freeway traffic".** In: *Journal de physique I* 2.12, p. 2221–2229.

Rickert, Marcus e.a. (1996). **"Two lane traffic simulations using cellular automata".** In: *Physica A: Statistical Mechanics and its Applications* 231.4, p. 534–550.

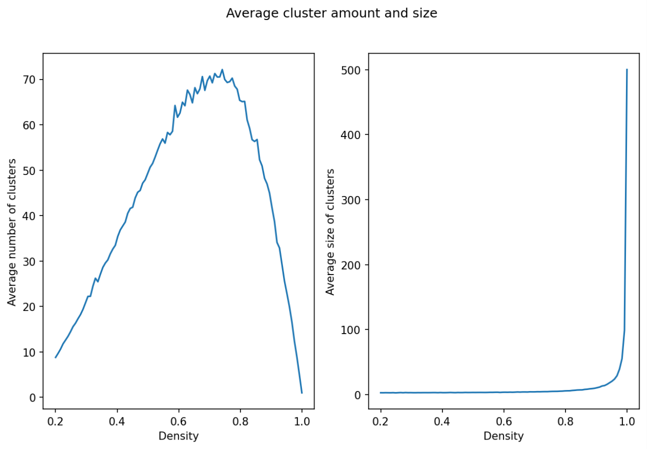
Cluster distribution

Another way of measuring traffic: a cluster = a traffic jam



Over time, in a single simulation, the traffic only fluctuates

Number of traffic jams



After reaching a peak of clusters, the traffic jams become fewer again since their size keeps increasing.