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ANTICIPATORY DRIVERS IN THE NAGEL−SCHRECKENBERG-MODEL

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The Nagel–Schreckenberg-model (NaSch-model) describes macroscopic features of real traffic very well. However, the characterization of a single car driver's behavior in some details is not realistic, e.g., the NaSch-driver calculates his/her distance to the car in front from the position this car has just in the very moment and ignores that it could move further in the next time step. This behavior is rarely found in real traffic. Normally, a driver estimates the speed of the car in front, takes as well a certain braking distance into consideration and keeps distance accordingly. As an answer to this demand, the second rule of the NaSch-model is modified in the following.

Keywords: Traffic; anticipatory; Nagel–Schreckenberg-model; cellular automata.

1. Introduction — The Original Model

First of all, we have a short look at the characteristics of the classic Nagel–Schreckenberg-model (NaSch-model).[1] According to the features of cellular automata,[2] the NaSch-model is a discrete model with homogeneous interactions: a single-lane street is divided into identical cells either occupied by a car or not. The cell number defines the location of the car, integer velocities of 0 up to a maximum velocity $v_{\max}$ are permitted. The NaSch-model is given by the following rules:

(1) If a car has not yet reached its maximum velocity, its velocity is incremented.
(2) If there is the danger to collide with the car in front, the driver brakes in order to avoid the accident, such that the car only reaches the cell behind the actual cell of the car in front.

(3) The driver additionally brakes slightly with a small probability p_{brake}.
(4) The car moves forward with the determined velocity.

A time step of the model consists of applying these rules to all cars and updating their velocities and positions. To get a parallel update of all cars, first the velocities of all cars are determined according to the first three rules. Then the positions of the cars are updated using the last rule. The endings of the street are put together such that the system is closed to a ring in order to reduce finite size effects and to provide a large number of time steps for the simulation. Although the model is determined by only four rules, it is able to simulate real traffic dynamics[1] quite well. A time step can be identified with one second and the length of a cell with 7.5 m, such that simulation results match measurements from the real traffic behavior. Rule (3) provides some randomization in the otherwise deterministic NaSch-model.

2. Anticipatory Drivers

In the classic NaSch-model a driver presumes that the car in front could stop without delay and without braking distance. As a consequence, there are no accidents. The behavior of the drivers, however, does not seem to be quite natural. In real life, a driver estimates the front car's velocity for the following time step and will as well assume a certain braking probability. In this paper, this instance will be discussed in a slightly abstract model, in which no accidents can happen, either. Summarizing these conditions, the following rules are to be applied:

(1) **Acceleration:**

$$\text{if} \quad v_i < v_{\max} \quad \text{then} \quad v_i = v_i + 1\,.$$

(2) **Braking:** let gap be the distance between a car and the car in front of it:

$$\text{gap} = x_{i+1} - x_i\,.$$

Now the future minimum velocity w of the car in front has to be determined: for this purpose we set $w = v_{i+1}$ and then apply the first three rules of the NaSch-model to w (i.e., the anticipatory driver assumes that the driver in front acts according to the NaSch-model):

(a) **Acceleration:**

$$\text{if} \quad w < v_{\max} \quad \text{then} \quad w = w + 1\,.$$

(b) **Braking:**

$$g = x_{i+2} - x_{i+1}$$

$$\text{if} \quad w \geq g \quad \text{then} \quad w = g - 1\,.$$

(c) **Deterministic Randomization:** the car in front could brake additionally, therefore, it is assumed to brake:

$$\text{if} \quad w > 0 \quad \text{then} \quad w = w - 1\,.$$

In addition to the actual spatial gap, the minimum future velocity w of the car in front is considered by the anticipatory driver:

$$\text{if} \quad v_i \geq \text{gap} + w \quad \text{then} \quad v_i = \text{gap} + w - 1\,.$$

(3) **Randomization**: with a small probability p_{brake} the driver brakes slightly. Whether he/she actually brakes or not is determined by means of a random number generator rnd:

$$\text{if} \quad \text{rnd} < p_{\text{brake}} \quad \text{and} \quad v_i > 0 \quad \text{then} \quad v_i = v_i - 1\,.$$

(4) **Motion**:

$$x_i = x_i + v_i\,.$$

As the future minimum velocity of the car directly in front is considered, we call the drivers who act according to the rules above anticipatory.

3. Computational Results

The following diagrams show the results of our simulations based on the NaSch-model and on the model of the anticipatory drivers. We used $p_{\text{brake}} = 0.05$ and $1 \leq v_{\max} \leq 10$. The system was initialized with the cars randomly put on the cells and performing 500 time steps in order to "equilibrate" the system. The average was taken over 10 000 measurements. One has to be careful when applying usual parameters of simulations of the classic NaSch-model to the model of the anticipatory drivers: in contrast to the original model, it is not sufficient to work with a system size of 10 000 cells in order to avoid finite size effects. We had to increase the system size up to 100 000 cells in order not to get large peaks in the traffic flow diagram for anticipatory drivers at large $v_{\max}$.

First of all, we have a look at the two so-called fundamental diagrams. Figure 1 shows the mean velocity $\langle v \rangle$ of all cars versus vehicle density ρ, for various maximum

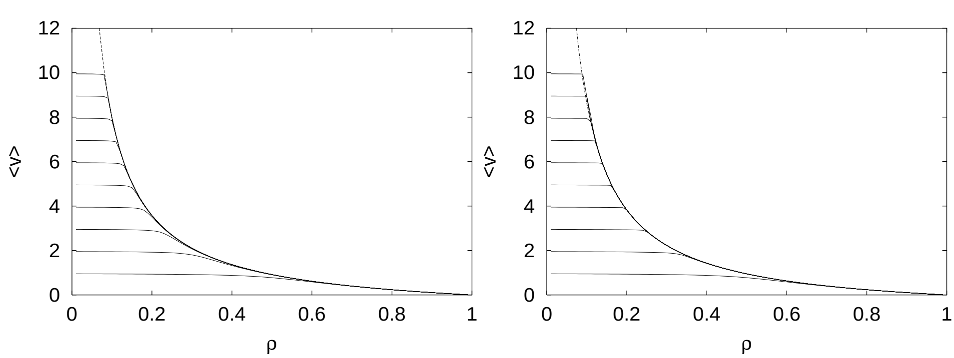

Fig. 1. Mean velocity $\langle v \rangle$ versus vehicle density ρ, for various maximum velocities $v_{\max}$, left: results for the classic Nagel–Schreckenberg-model, right: results for anticipatory drivers.

velocities $v_{\max}$ ($1 \leq v_{\max} \leq 10$), both for the classic model and for the anticipatory drivers. In both cases, $\langle v \rangle$ nearly reaches the value of $v_{\max}$ for small ρ. There is even a plateau for small ρ, i.e., all cars can nearly drive at $v_{\max}$ without disturbing each other. The mean velocity decays at increasing density, which takes place even earlier the higher the maximum velocity is set. At high densities the traffic finally breaks down. Additionally, there is a dashed line for $v_{\max} \to \infty$, which obviously serves as a limiting case for the NaSch-model: all curves for the different maximum velocities bend off the dashed line and reach a plateau with decreasing ρ. The graphics for the anticipatory drivers seem to indicate that the curve for $v_{\max} \to \infty$ is not a border line anymore, the curves for $v_{\max} = 8$, 9, 10 cut it due to more extended plateaus. However, this difference gets larger if using a smaller system size, such that we conclude for infinite systems, that the curve for $v_{\max} \to \infty$ is again a border line as in the original model. This border line is steeper than that for the original NaSch-model.

The second fundamental diagram, traffic flow $\langle \phi \rangle$ versus density ρ, is shown in Fig. 2. The progression of the flow in the case of the anticipatory drivers shows as well a similarity to its analogue of the original model. $\langle \phi \rangle$ vanishes in the limiting cases $\rho \to 0$ (no car on the street) and $\rho \to 1$ (no empty cell, such that no car can drive forward). $\langle \phi \rangle$ increases at lower vehicle densities ρ in a linear way with ρ, which corresponds to the plateaus of $\langle v \rangle$ in Fig. 1. Again a dashed line is shown for the limiting case that there is no maximum velocity, from which all curves for the various $v_{\max}$ bend off. The dashed line shows a roughly linear decrease of $\langle \phi \rangle$. $\langle \phi \rangle$ is symmetric around $\rho = 0.5$ for $v_{\max} = 1$, the flow of the "holes" at a car density of $1 - \rho$ (i.e., "hole density" ρ) is equal to the flow of the cars at a density of ρ in this case. The peak of the traffic flow is transferred to smaller densities but larger values for increasing $v_{\max}$. Indeed, at higher maximum velocities there can be observed a pointed peak in case of the anticipatory drivers instead of a rounded one, which is seen in the graphics of the original model. These pointed peaks stick out over the curve for $v_{\max} \to \infty$. This is again assumed to be an effect of the still

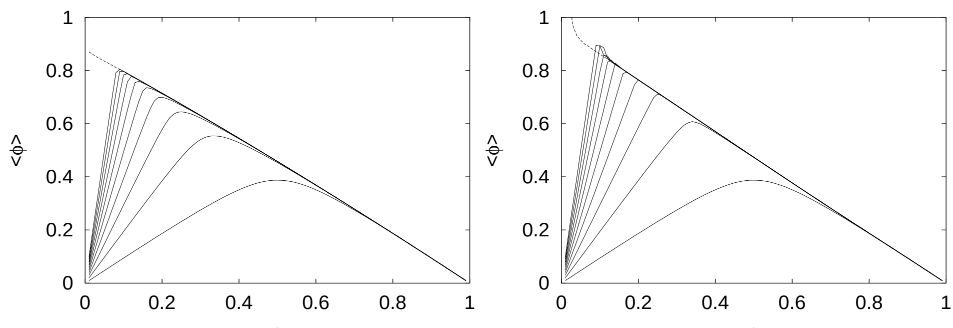

Fig. 2. Traffic flow $\langle \phi \rangle$ versus vehicle density ρ, for various maximum velocities $v_{\max}$, left: results for the classic Nagel–Schreckenberg-model, right: results for anticipatory drivers.

too small system size since the peaks of $\langle \phi \rangle$ above the curve for $v_{\max} \to \infty$ get larger if using a smaller system size. Surveying the case of the system without maximum velocity, it is obvious that at falling densities the curve rises at first steeper in case of the anticipatory drivers than its analogue of the original model. Thus, it does not tend towards 1 but towards 2 at quite small densities ρ. This result can be explained more easily by the case of exactly one resp. two cars on the lane: in the classic model, the single car considers itself as car in front, so that even without any restrictions of speed it can only reach a maximum velocity, which is one minor the number of cells. Therefore, in very big systems the flow $\langle \phi \rangle$ tends towards 1 for $\rho \to 0$. In the model of the anticipatory drivers, the driver in front is assumed to behave like a NaSch-driver, such that the traffic flow can only reach a value of 2 at $\rho \to 0$. All in all, one gets a larger flow for the anticipatory drivers than for the NaSch-drivers.

Besides these two so-called fundamental diagrams of the mean velocity and the traffic flow, we have a look at the fraction s of people, which are called speedometer sinners or speeders. In Germany, everybody learns the inofficial rule in a driving school that the distance to the car in front measured in meters should be at least half the number the speedometer shows in kilometers per hour (km/h) for safety reasons. As the cell is 7.5 meters long and a time step has a duration of 1 second, the integer velocities $0, 1, 2, \ldots, v_{\max}$ are given by multiples of 27 km/h. Thus, it shall be checked whether the gap multiplied with 15 is smaller than the velocity multiplied with 27 at the end of each time step. The results for $\langle s \rangle$ are shown in Fig. 3. The curve for $v_{\max} = 1$ is asymmetric, it peaks at $\rho = 2/3$. The peaks for the various $v_{\max}$ are shifted to smaller ρ for larger $v_{\max}$ with $\rho_{\text{peak}} \approx 0.63 \times v_{\max}^{-0.9}$ in case of the original model and $\rho_{\text{peak}} \approx 0.63 \times v_{\max}^{-0.85}$ for the anticipatory drivers. All curves bend off the limiting curve $v_{\max} \to \infty$, with their peaks being above this limiting curve. Comparing the graphics for the speeders among the anticipatory drivers with those for the original NaSch-model, one sees at first sight that the ratio $\langle s \rangle$ is identical for $v_{\max} = 1$, its peak is larger for the anticipatory drivers at

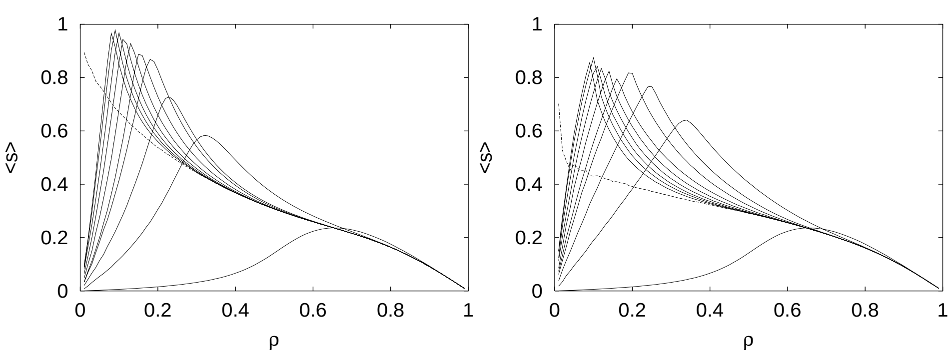

Fig. 3. Fraction $\langle s \rangle$ of speeders versus vehicle density ρ, for various maximum velocities $v_{\max}$, left: results for the classic Nagel–Schreckenberg-model, right: results for anticipatory drivers.

$v_{\max} = 2$ and $v_{\max} = 3$, whereas it is smaller for $v_{\max} \geq 4$. Despite of that, the curves are rather similar.

Let us again consider the special case $v_{\max} = 1$. In this case, the results are identical for the original model and the anticipatory drivers. This identity is caused by the deterministic randomization rule 2c of the anticipatory drivers, because of that the anticipatory driver always assumes the car in front to stand still if $v_{\max} = 1$, what the driver in the original NaSch-model also does.

4. Multiply Anticipatory Drivers

The question remains whether a larger flow than above can be achieved if the drivers are even more anticipatory. Let us introduce an anticipation parameter a. We set $a = 0$ for drivers obeying the NaSch-rules since they do not consider the velocity of the car in front. The anticipatory drivers calculate a minimum future velocity of the car in front depending on its actual velocity and the spatial gap to the predecessing car, such that we set here $a = 1$. One can extend this game a step further: if an anticipatory driver does not consider the driver in front to be a NaSch-driver but to be an anticipatory driver with $a = 1$ (i.e., the driver in front is assumed to

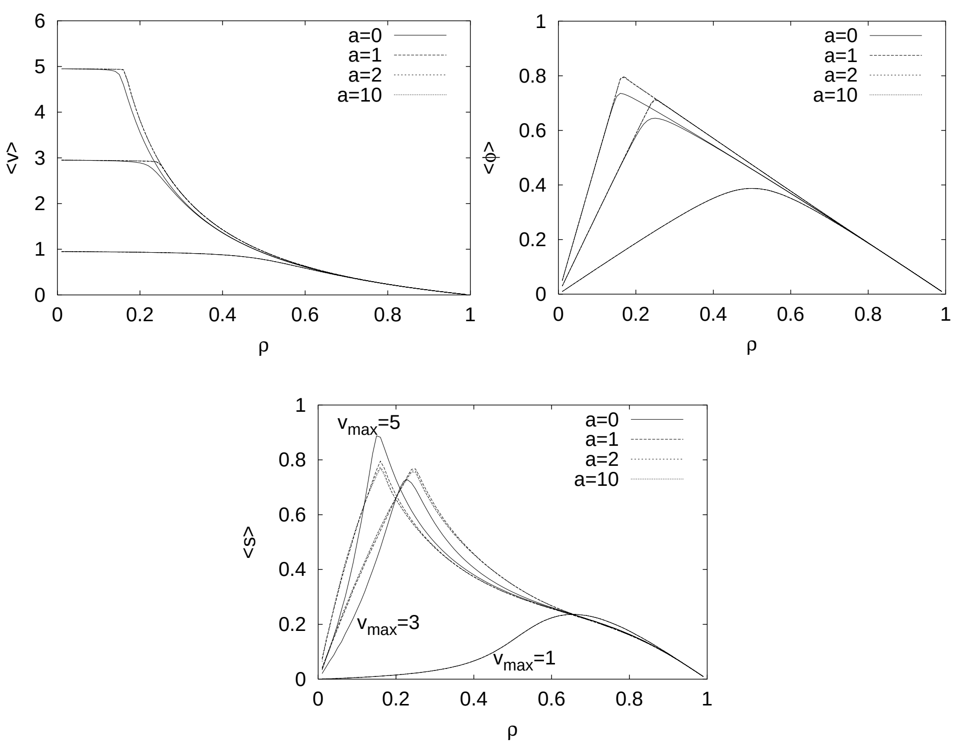

Fig. 4. Mean velocity $\langle v \rangle$, traffic flow $\langle \phi \rangle$, and fraction $\langle s \rangle$ of speeders against vehicle density ρ for the maximum velocities $v_{\max} = 1, 3, 5$ for NaSch-drivers ($a = 0$), anticipatory drivers ($a = 1$), and multiply anticipatory drivers ($a = 2$ and $a = 10$).

assume the driver in front of him/her to be a NaSch-driver), then we speak of an even more anticipatory driver and set $a = 2$ for such an intelligent driver. This approach can recursively be extended: we can define a multiply anticipatory driver of any positive anticipation parameter a, who assumes the driver in front to be an anticipatory driver with anticipation parameter $a - 1$.

Figure 4 shows a comparison of the results for the mean velocity $\langle v \rangle$, the traffic flow $\langle \phi \rangle$, and the fraction of speeders $\langle s \rangle$ for the anticipation parameters $a = 0$, 1, 2, 10. Nearly no additional flow can be achieved if the degree of anticipation is extended over $a = 1$, the curves for $\langle v \rangle$ are rather the same for $a \geq 1$. The largest differences can be seen (when zooming in) at the peaks of $\langle \phi \rangle$. The peaks of $\langle s \rangle$ get slightly smaller for $v_{\max} > 1$ if a is increased from 1 to larger values.

5. Conclusion

In this paper, we slightly varied the original Nagel–Schreckenberg-model and introduced anticipatory drivers who do not only consider the actual spatial gap between them and their predecessors but also calculate a minimum future velocity of the driver in front. Thus, the traffic flow can be significantly enlarged in a certain range of the traffic density. Any further anticipation does not lead to a further increase of the traffic flow.

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