→ Sección 1.6.6

2. Demuestre:

(a). $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$.

(b). $\operatorname{sen}(3\alpha) = 3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)$.

Empleando la fórmula de De Moivre

$$\cos(3\alpha) + i\operatorname{sen}(3\alpha) = [\cos(\alpha) + i\operatorname{sen}(\alpha)]^3$$

$$= \cos^3(\alpha) + 3\cos^2(\alpha)\cdot i\operatorname{sen}(\alpha) + 3\cos(\alpha)\, i\operatorname{sen}^2(\alpha) + i\operatorname{sen}(\alpha)^3$$

$$= \cos^3(\alpha) + (3\cos^2(\alpha)\operatorname{sen}(\alpha))\, i - 3\cos(\alpha)\operatorname{sen}^2(\alpha) - i\operatorname{sen}^3(\alpha)$$

$$= \underbrace{[\cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)]}_{\text{Parte real}} + i\underbrace{[3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)]}_{\text{Parte imaginaria}}$$

Si igualamos la parte real con la real y la imaginaria con la imaginaria

$\rightarrow \cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$ ✓

$i\operatorname{sen}(3\alpha) = i[3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)]$ ✓

$\rightarrow \operatorname{sen}(3\alpha) = 3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)$

5. Encuentre las raíces de.

(a). $(2i)^{1/2}$

$z = 2i \rightarrow z = 2e^{i\pi/2}$

$z^{1/2} = 2^{1/2} e^{i\left(\frac{\frac{\pi}{2} + 2\pi k}{2}\right)}$ } 2 raíces

$k = 0 \rightarrow z_1^{1/2} = \sqrt{2}\, e^{i(\pi/4 + \pi 0)} \; \boxed{= \sqrt{2}e^{i(\pi/4)}}$

$k = 1 \rightarrow z_2^{1/2} = \sqrt{2}\, e^{i(\pi/4 + \pi)} \; \boxed{= \sqrt{2}e^{i(5\pi/4)}}$

(b). $(1 - \sqrt{3}i)^{1/2} \longrightarrow |z| = \sqrt{1^2 + (-\sqrt{3})^2} = 2$, $\alpha = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{1}{3}\pi)$

$z = 2e^{i(-1/3)\pi}$

$z^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\frac{\pi}{3} + 2\pi k}{2}\right)}$

$k = 0 \rightarrow z_1^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\pi/3 + 2\cdot 0\pi}{2}\right)} = \underline{\sqrt{2}\, e^{i(-\pi/6)}}$

$k = 1 \rightarrow z_2^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\pi/3 + 2\cdot 1\cdot \pi}{2}\right)} = \underline{\sqrt{2}\, e^{i(5\pi/6}}$

(c). $(-1)^{1/3}$ $\quad \theta = \pi$

$z = -1 = 1e^{i\pi}$

$z^{1/3} = 1^{1/3} e^{i\left(\frac{\pi + 2\pi k}{3}\right)}$

$k = 0 \rightarrow z_1^{1/3} = 1 \cdot e^{i\left(\frac{\pi + 0}{3}\right)} = \underline{e^{i(\pi/3)}}$

$k = 1 \rightarrow z_2^{1/3} = 1 \cdot e^{i\left(\frac{\pi + 2\pi}{3}\right)} = \underline{e^{i(\pi)}} = \underline{(-1)}$ ✓

$k = 2 \rightarrow z_3^{1/3} = 1 \cdot e^{i\left(\frac{\pi + 4\pi}{3}\right)} = \underline{e^{i(5\pi/3)}}$

(d). $8^{1/6} \rightarrow z=8 \quad , |z|=8| \quad , \theta=0$

$z = 8e^{i0}$

$z = 8e^{i(0+2\pi k)}$

$z^{1/6} = 8^{1/6} e^{i\left(\frac{0+2\pi k}{6}\right)}$

$k=0 \rightarrow z_1^{1/6} = 8^{1/6} e^{i\left(\frac{0+0}{6}\right)} = 8^{1/6} e^{0} = \boxed{8^{1/6}}$

$k=1 \rightarrow z_2^{1/6} = 8^{1/6} e^{i\left(\frac{0+2\pi}{6}\right)} = \boxed{8^{1/6} e^{i(\pi/3)}}$

$k=2 \rightarrow z_3^{1/6} = 8^{1/6} e^{i\left(0+\frac{4\pi}{6}\right)} = \boxed{8^{1/6} e^{i(2/3)\pi}}$

$k=3 \rightarrow z_4^{1/6} = 8^{1/6} e^{i(0+6/6\,\pi)} = \boxed{8^{1/6} e^{i(\pi)}}$

$k=4 \rightarrow z_5^{1/6} = 8^{1/6} e^{i\left(0+\frac{8\pi}{6}\right)} = \boxed{8^{1/6} e^{i(4/3)\pi}}$

$k=5 \rightarrow z_6^{1/6} = 8^{1/6} e^{i\left(0+\frac{10\pi}{6}\right)} = \boxed{8^{1/6} e^{i(5/3)\pi}}$

(e). $(-8-8\sqrt{3}\,i)^{1/4}$

$z = \sqrt{(-8)^2 + (8\sqrt{3})^2} = 16$

$\theta = \tan^{-1}\left(\frac{-8\sqrt{3}}{-8}\right) = \frac{1}{3}\pi \longrightarrow \alpha = \pi + \frac{1}{3}\pi = 4/3\,\pi$

$z = 16\, e^{i(4\pi/3 + 2\pi k)}$

$z^{1/4} = 16^{1/4} e^{i\left(\frac{\frac{4\pi}{3}+2\pi k}{4}\right)}$

$k=0 \rightarrow z_1^{1/4} = 16^{1/4} e^{i(\pi/3)} = \boxed{2e^{i(\pi/3)}}$

$k=1 \rightarrow z_2^{1/4} = 16^{1/4} e^{i\left(\frac{\frac{4\pi}{3}+2\pi}{4}\right)} = \boxed{2e^{i(5/6)\pi}}$

$k=2 \rightarrow z_3^{1/4} = 16^{1/4} e^{i\left(\frac{\frac{4\pi}{3}+4\pi}{4}\right)} = \boxed{2e^{i(4/3)\pi}}$

$k=3 \rightarrow z_4^{1/4} = 16^{1/4} e^{i\left(\frac{\frac{4\pi}{3}+6\pi}{4}\right)} = \boxed{2e^{i(11/6)\pi}}$

Escaneado con CamScanner