The aim of this milestone is to identify the influence of the “n” and “dt” parameters when integrating the Cauchy problem for orbits. The one-step integration will be calculated using the Euler, inverse Euler, Crank-Nicolson and Runge-Kutta (order 4) time schemes.

The time schemes, as well as the function to integrate the Cauchy problem, can be found in other python files: Cauchy_Problem.py and Time_Schemes.py, so as to allow the Milestone_2.py program to run faster.

Please refer to the GIT repository, Milestone_2, to visualize the Python codes listed above.

After running the program, with initial conditions:

$$U\ (0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$n\ \equiv number\ of\ iterations = \ 1000$$

$$dt\ \ \equiv time\ interval = 0.1\ [s]$$

The program could not finish, as the newton solutions for both Euler inverse and Crank-Nicolson did not converge.

Modifying the time interval to $dt\ \ \equiv time\ interval = 0.01\ [s]$, but leaving the rest of the parameters the same, Figure 1 is obtained. It becomes clear that Euler and inverse Euler methods, do not give an output of a closed orbit, contrary to Crank-Nicolson and Runge-Kutta (order 4).

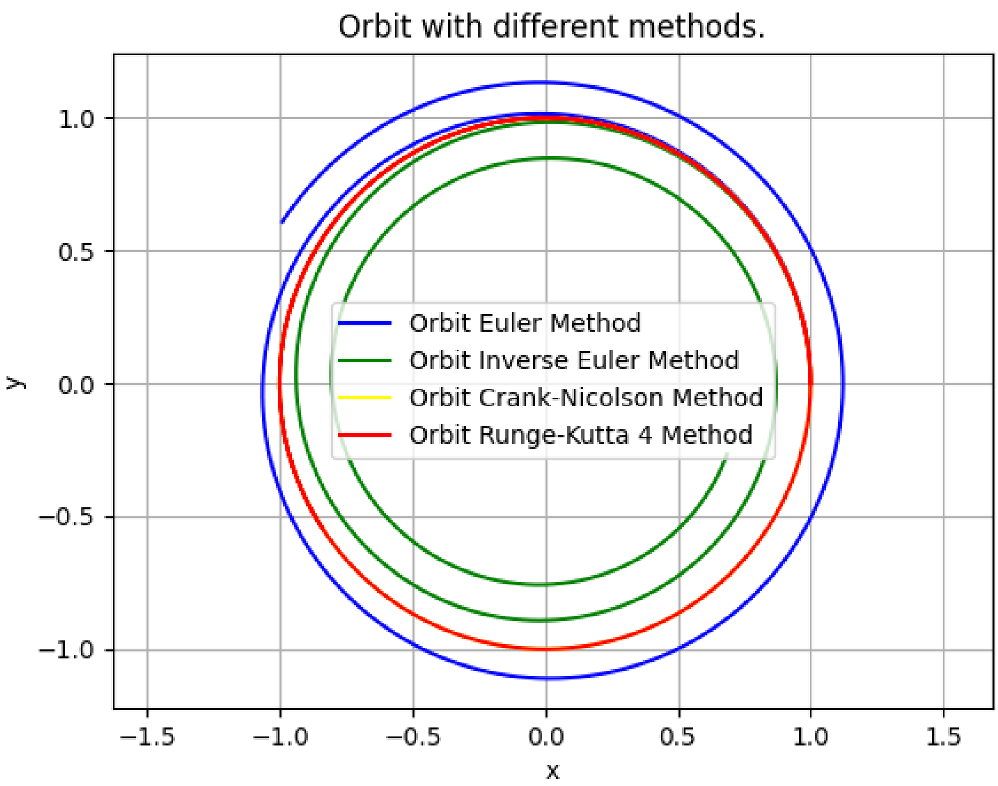

Figure 1: Orbit for n=1000, dt=0.01 [s].

In order to obtain that closed orbit with Euler and inverse Euler, the time interval is decreased: $dt \equiv time\ interval = 0.001\ [s]$. Figure 2 is obtained. It becomes clear that, when decreasing the time interval, more iterations will be needed in order to obtain enough data to plot the orbits. Euler and inverse Euler methods converge to a closed orbit when iterations are increased and time interval is decreased. This can be inferred from Figures 3 and 4.

In the end, a closed orbit, the desirable one, equal to the one obtained with the Crank-Nicolson and Runge-Kutta (order 4) time schemes is obtained with the Euler and inverse Euler time schemes (Figure 4) is obtained. The parameters "n" and "dt" are much different to the initial ones: a number of iterations considerably higher (from 1000 to 10000), and a decreased time interval (dt=0.001 [s]).

From all of this, the conclusion is that the Euler and inverse Euler methods require a higher amount of calculations compared to the Crank-Nicolson and Runge-Kutta (order 4) methods.

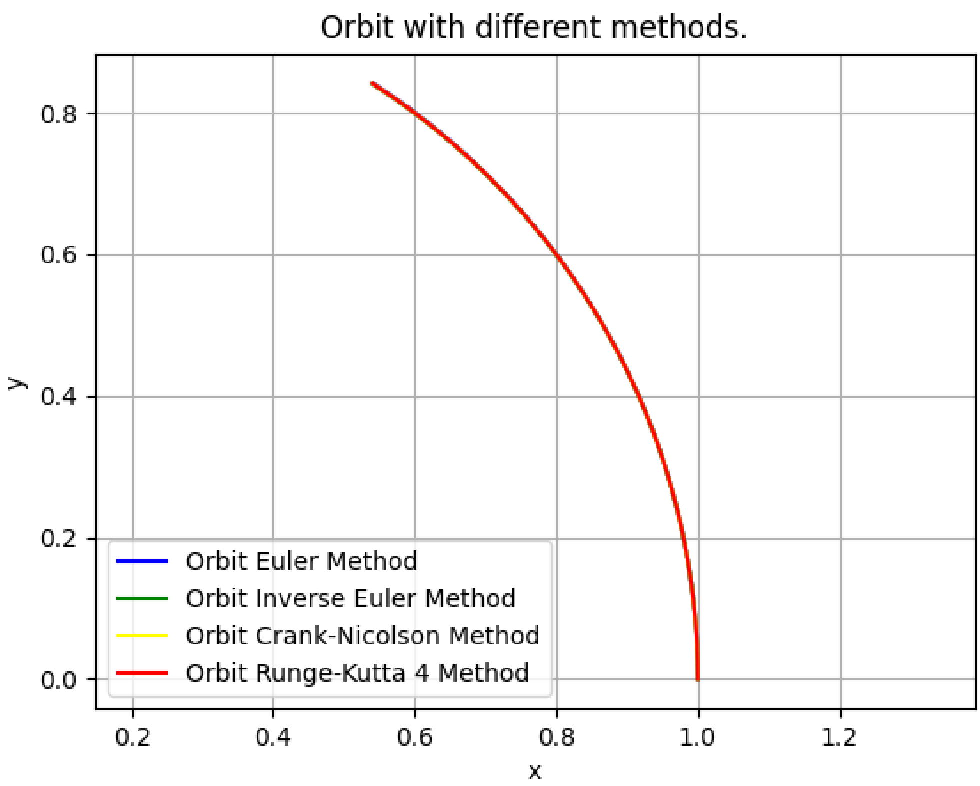

Figure 2: Orbit for n=1000, dt=0.001 [s].

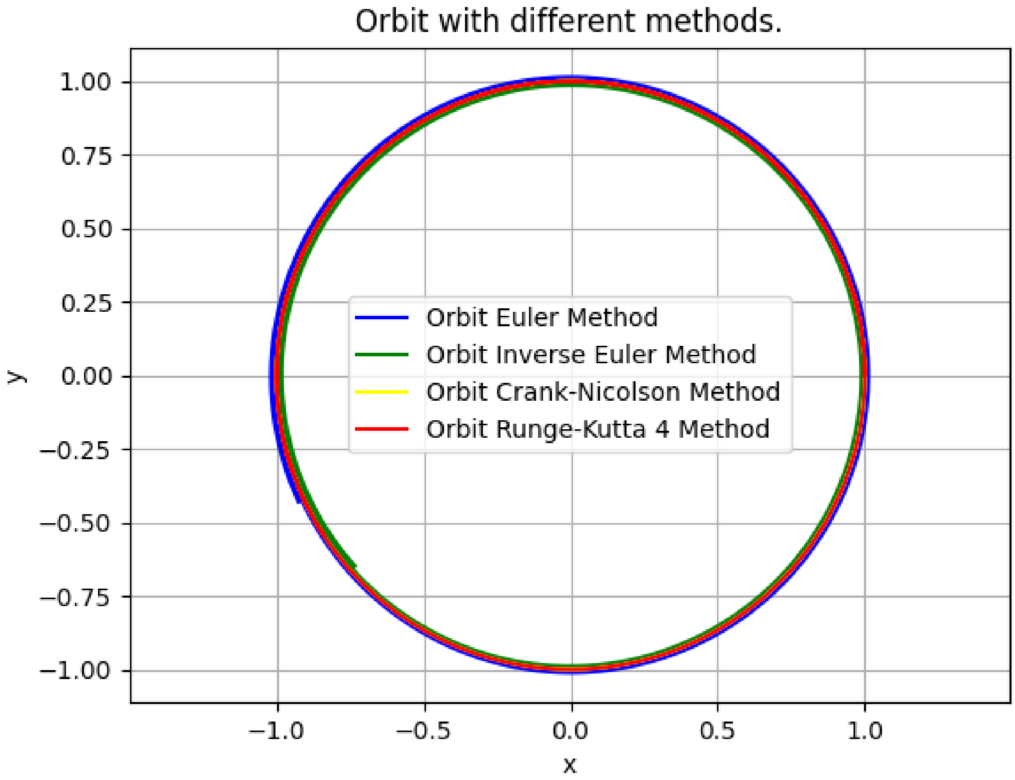

Figure 3: Orbit for n=10000, dt=0.001 [s].

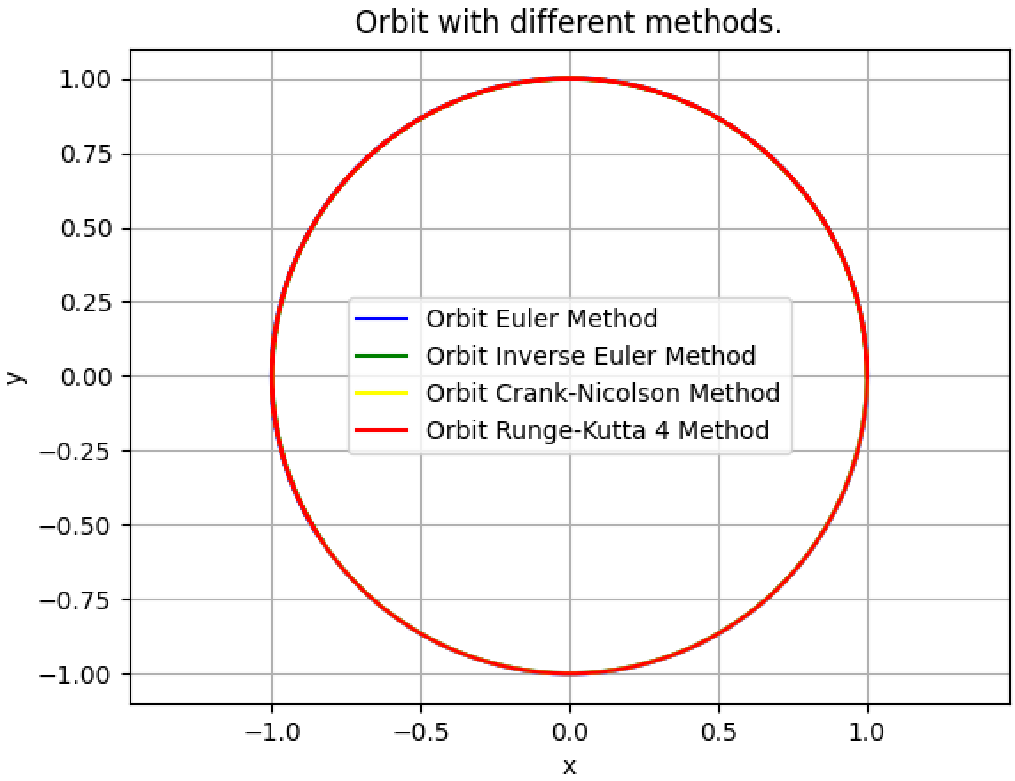

Figure 4: Orbit for n=100000, dt=0.0001 [s].