Ground State Topological Properties of Interacting Spin-Orbit Coupled Bosons

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Introduction

- Quantum matter has been known to have **topological properties**, properties that are robust against disturbances (e.g., deformation, impurities).

- These properties were famously shown through the discovery of the integer and fractional quantum Hall effects (IQHE/FQHE).

- FQHE shows that **particle interactions** can drive topological phenomena.

- Recently, spin-orbit (SO) coupled bosons in 2D lattices have been found to exhibit topological properties without needing an external magnetic field.

Introduction

- Understanding interacting bosonic systems could provide insight into bosonic **topological insulators**.

- This project was initiated by Nathan Ngo, who developed the initial framework. My work this summer builds on and extends his work.

Background

- **Topological insulators:** Materials insulating in the bulk but conducting at the surface.

- In recent studies, systems of SO coupled bosons have been proposed to behave as topological insulators and potentially hold unique topological properties that stem from the SO coupling.

- This contrasts previously observed topological insulators (where the carriers are fermions).

- Previous studies of these bosonic systems have relied on the common unrealistic assumption that particles on the lattice don’t interact.

- **This project will aim to examine the properties of a system’s ground state for a small number of SO-coupled particles on a square lattice to determine how particle interactions influence the topological properties of the ground state.**.

Background

Why do we care?

- The ability to model these boson systems on a quantum scale computationally and comprehend their topological properties serves as the foundation for the understanding of these new topological insulators and their abilities.

- This ultimately paves the path for the potential construction of bosonic topological insulators along with their promising applications, such as to the potential for advancement in transistor technology, or to developments in sensing technology.

Approach

1. Build on a Python computational program developed by Nathan Ngo by extending it from non-interacting few-particle systems to interacting multi-particle systems (*initial scope was 3 particles, we ended up reducing to 2*).

2. Analyze ground states and topological signatures (Chern number).

Single Particle Systems Studied

1. 1D Lattice With Cyclic Boundary Conditions
2. 2D Lattice With Cyclic Boundary Conditions
3. 1D Lattice With Spin-½ , However No Spin Flips Are Allowed When Tunneling
4. 2D Lattice With Spin-½ , However No Spin Flips Are Allowed When Tunneling
5. 1D Lattice With Spin-½ , Spin Flips Allowed When Tunneling
6. 2D Lattice With Spin-½ , Spin Flips Allowed When Tunneling
7. 2D Lattice With Spin-½ , Spin Flips Allowed When Tunneling, and the tunneling terms t, t_z, t_{so}, m_z included

System 1: 1D Lattice With Cyclic Boundary Conditions

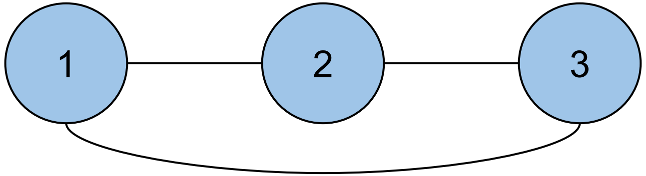

Figure 1 - A 1D lattice of three nodes with cyclic boundary conditions

- Hopping allowed both directions
- $n > 2$

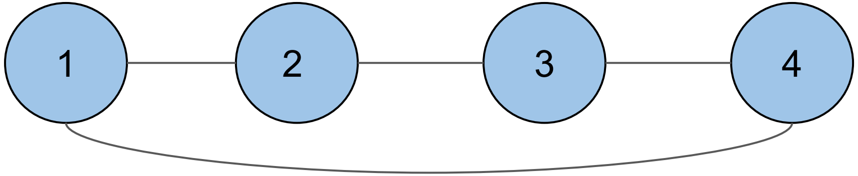

Figure 2 - A 1D lattice of four nodes with cyclic boundary conditions

- Each program generates system’s Hamiltonian and calculates eigenvalues and eigenvectors

System 2: 2D Lattice With Cyclic Boundary Conditions

$$H_x \otimes I_y + I_x \otimes H_y$$

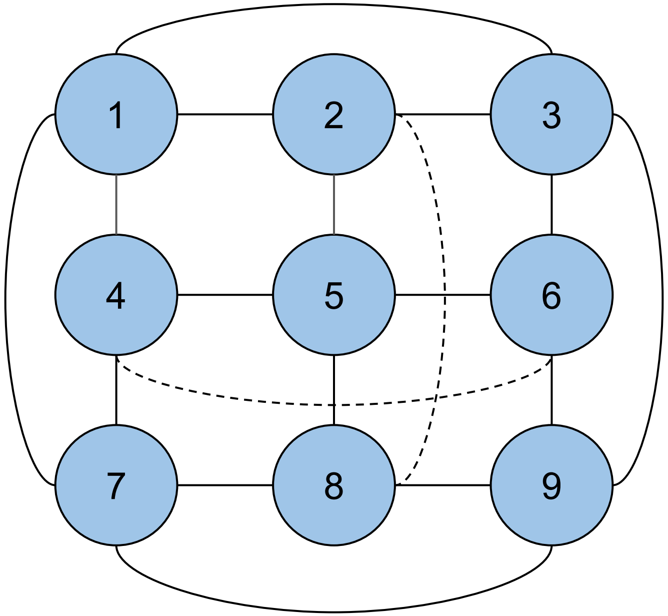

Figure 3 - A 2D lattice of nine nodes with cyclic boundary conditions

System 3: 1D Lattice With Spin-½ , However No Spin Flips Are Allowed When Tunneling

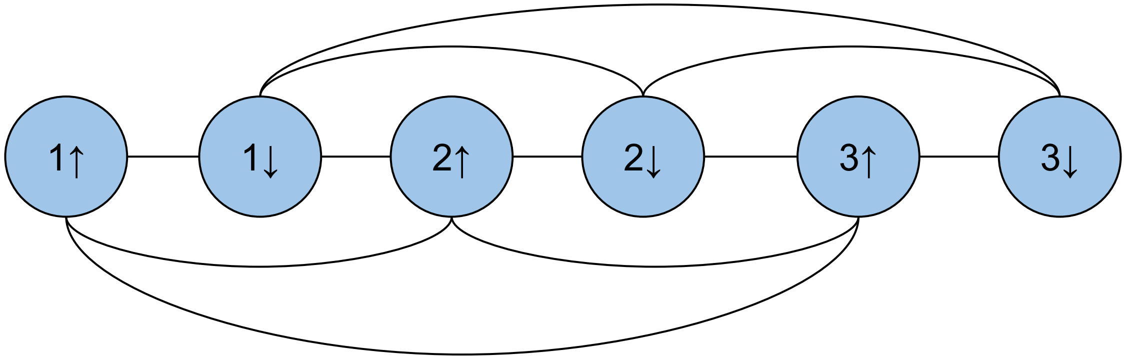

Figure 4 - A 1D lattice of three nodes with spin-½, cyclic boundary conditions, and no spin direction changes allowed when hopping

Remaining Systems

- System 4: 2D Lattice With Spin-½ , However No Spin Flips Are Allowed When Tunneling

- System 5: 1D Lattice With Spin-½ , Spin Flips Allowed When Tunneling

- System 6: 2D Lattice With Spin-½ , Spin Flips Allowed When Tunneling

These were hard to draw.

System 7: 2D Lattice With Spin-½ , Spin Flips Allowed When Tunneling, and the tunneling terms t, t_z, t_{so}, m_z included

$$
\begin{aligned}
H_{hop} =& -t \sum_{j_x, j_y, s} (b^\dagger_{j_x+1, j_y, s} b_{j_x, j_y, s} + b^\dagger_{j_x, j_y+1, s} b_{j_x, j_y, s} + H.c.) \\
H_{zhop} =& t_z \sum_{j_x, j_y} (b^\dagger_{j_x+1, j_y, \uparrow} b_{j_x, j_y, \uparrow} + b^\dagger_{j_x, j_y+1, \uparrow} b_{j_x, j_y, \uparrow} \\
& - b^\dagger_{j_x+1, j_y, \downarrow} b_{j_x, j_y, \downarrow} - b^\dagger_{j_x, j_y+1, \downarrow} b_{j_x, j_y, \downarrow} + H.c.) \\
H_{so} =& t_{so} \sum_{j_x, j_y} (b^\dagger_{j_x+1, j_y, \uparrow} b_{j_x, j_y, \downarrow} + b^\dagger_{j_x+1, j_y, \downarrow} b_{j_x, j_y, \uparrow} \\
& - b^\dagger_{j_x, j_y+1, \uparrow} b_{j_x, j_y, \downarrow} - b^\dagger_{j_x, j_y+1, \downarrow} b_{j_x, j_y, \uparrow} \\
& - i b^\dagger_{j_x+1, j_y+1, \uparrow} b_{j_x, j_y, \downarrow} + i b^\dagger_{j_x+1, j_y+1, \downarrow} b_{j_x, j_y, \uparrow} \\
& + i b^\dagger_{j_x+1, j_y-1, \uparrow} b_{j_x, j_y, \downarrow} - i b^\dagger_{j_x+1, j_y-1, \downarrow} b_{j_x, j_y, \uparrow} + H.c.) \\
H_{det} =& m_z \sum_{j_x, j_y} (b^\dagger_{j_x, j_y, \uparrow} b_{j_x, j_y, \uparrow} - b^\dagger_{j_x, j_y, \downarrow} b_{j_x, j_y, \downarrow})
\end{aligned}
$$

Two Particle Systems Studied

1. 2D lattice with two *distinguishable* particles hopping on the lattice, again with Spin-½ , Spin Flips Allowed When Tunneling, and the tunneling terms t, t_z, t_{so}, m_z included.

2. 2D lattice with two *indistinguishable* particles hopping on the lattice, again with Spin-½ , Spin Flips Allowed When Tunneling, and the tunneling terms t, t_z, t_{so}, m_z included (symmetrized two-boson system).
    - $B=Q^T AQ$

Sanity Checks After Code System Structure Was Finalized

Check 1: Checking the eigenvalues of the two-particle Hamiltonian (for distinguishable bosons) to see if the eigenvalues are all sums of two eigenvalues from the one-particle Hamiltonian.

Check 2: Verifying that if Check 1 holds then the corresponding eigenvectors for the two-particle case should be tensor products of the two associated single-particle eigenvectors.

Both held (yay!)

Indexing and Interaction Preparation

- Developed indexing scheme for the Hamiltonian of the 2D two-particle symmetrized bosonic system.

- Ensured ability to identify states where both bosons occupy the same site, along with identifying which spin state each boson is in.

- Prepared framework for adding interaction terms (on-site interaction weights).

Current Status and Next Steps

Current progress:

- Constructed Hamiltonians up to the symmetrized two-boson case.
- Finalized indexing structure for easy use to simulate interactions between bosons.
- Built the framework, *actual* results come next!

Next steps:

- Add interaction terms.
- Analyze resulting ground state structure.
- Explore topological signatures (Chern Number) and compare to the non-interacting multiparticle system case to see if topological characteristics of the ground state changed.