# 02610
# Optimization and Data Fitting

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2 September 2024

# Information

**Textbook:**

- Numerical Optimization
  by Jorge Nocedal and Stephen J. Wright
- Lecture notes and slides offered online

**Location:**

- Lectures: Building 308 - Auditorium 12
- Exercises: Building 308 - Databar 009 & 017

**Structure:**

- Lecture: 13:00 - 15:00 (except week 5 and 9)
- Exercises: 15:00 - 17:00

**Assessment:**

- Assessment is based on 2 homework assignments and written exam with weights: 15%+15%+70%.
- Homework assignments are carried out in small groups (at most 3 students per group), and must be subjected to individualization.
- Written exam (paper&pen): 3 hours.
- Grading will be based on the 7-scale.

# What is “Optimization”?

- “Optimization” comes from the same root as “optimal”, which means best. When you optimize something, you are “making it the best”.
- But “best” can vary. Both maximizing and minimizing are types of optimization problems.
- Optimization problems:
- $\max_{\boldsymbol{x}} f(\boldsymbol{x}) = -\min_{\boldsymbol{x}} -f(\boldsymbol{x})$.

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- Optimization problems:

| | |
|---|---|
| minimize or maximize | *objective* |
| by choosing | *variables* |
| subject to | *constraints* |

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$$\begin{aligned} \min_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) \\ \text{subject to} \quad & c_i(\boldsymbol{x}) = 0 \text{ for } i \in \mathcal{E} \\ & c_i(\boldsymbol{x}) \geq 0 \text{ for } i \in \mathcal{I} \end{aligned}$$

  - $\boldsymbol{x}$ is the vector of *variables*, also called *unknowns* or *parameters*.
  - $f$ is the objective function.
  - $c_i$ are *constraint* functions, and $\mathcal{E}$ and $\mathcal{I}$ are sets of indices for equality and inequality constraints, respectively.
  - *Feasible set:* The set of all possible $\boldsymbol{x}$, i.e., the points satisfy all constraints.
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# Type of optimization problems

- Some problems have constraints and some do not.
  - Unconstrained optimization problems: We would have $\mathcal{E} = \mathcal{I} = \emptyset$.
  - Constrained optimization problems: Constraints play an essential role. For example: in X-ray tomography, we may need $\boldsymbol{x} \geq 0$.
- There can be one variable or many.
- Variable can be discrete or continuous.
- Systems can be deterministic (specific causes produce specific effects) or stochastic (involve randomness/ probability).

# Type of optimization problems

- Some problems have constraints and some do not.
  - Unconstrained optimization
- There can be one variable or many.
  - Univariate optimization problems: We have $x \in \mathbb{R}$.
  - Multivariate optimization problems: We have $\boldsymbol{x} \in \mathbb{R}^n$ with $n > 1$.
- Variable can be discrete or continuous.
- Systems can be deterministic (specific causes produce specific effects) or stochastic (involve randomness/ probability).

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- There can be one variable or many.
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- Variable can be discrete or continuous.
  - Discrete optimization problems: $\boldsymbol{x}$ only can be a few certain numbers. For example: In QR code restoration, we need $\boldsymbol{x} \in \{0, 1\}$.
  - Continuous optimization problems: The components of $\boldsymbol{x}$ are allowed to be real numbers.
- Systems can be deterministic (specific causes produce specific effects) or stochastic (involve randomness/ probability).

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- Systems can be deterministic (specific causes produce specific effects) or stochastic (involve randomness/ probability).
  - Stochastic optimization problems: The model changes along the time. For example, in economic and financial planning models.
  - Deterministic optimization problems: The model is completely known and fixed.

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  - Deterministic optimization

# Why do we need to know "Optimization"?

Optimization is very useful for many applications spanning a large number of fields. Here are a few examples:

- Design (e.g. automotive, aerospace, biomechanical)
- Manufacturing
- Control
- Transportation
- Signal and image processing
- Finance
- Data fitting

Optimization is also often used in our daily life:

- When you are thinking which courses to choose in this semester ...
- When you are choosing a new phone or plan ...
- When you are planning for the weekend ...

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# Course content

- Mathematical preliminaries and fundamentals of unconstrained optimization (chap 2)
- Line search methods (chap 3)
- Trust-region methods (chap 4)
- Quasi-Newton methods (chap 6)
- Linear least squares data fitting (notes)
- Nonlinear least squares data fitting (chap 10)
- Exponential data fitting (notes)
- Data fitting but not using least squres (notes)
- Derivative free optimization (chap 9)
- Large-scale unconstrained optimization (chap 7)
- Conjugate gradient methods (chap 5)
- Introduction to constrained optimization (chap 12)