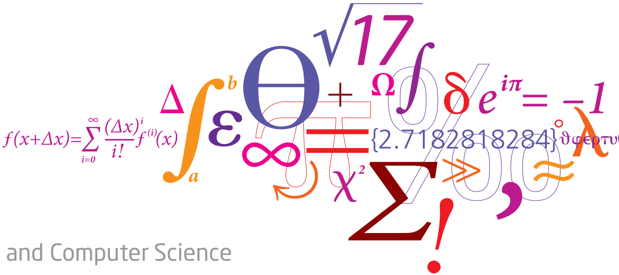
02450: Introduction to Machine Learning and Data Mining

Association mining

Jes Frellsen

DTU Compute, Technical University of Denmark (DTU)



DTU Compute
Department of Applied Mathematics and Computer Science

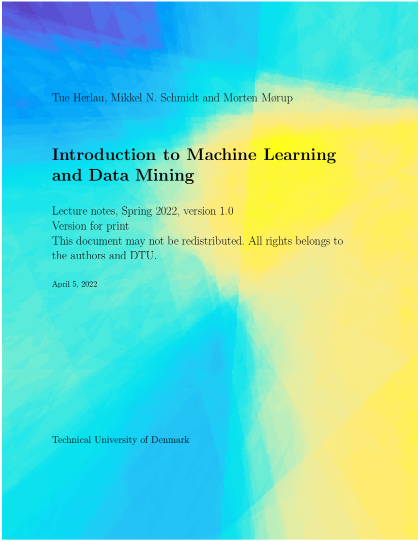
Today

Feedback Groups of the day:

Teitur Helgi Skúlason, Felix Moser, Chaoyang Zhang, Vivien Váradi, Marius Næraa-Nicolajsen, Adrian Sylwester Los, Viktor Kjer Steffensen, Katja Refsgaard Norsker, Nicolò Sguerso, Swastik Singh, Christian Francesco Notarmaso Pone, Jonas Hyldgaard Langager, Kåre Appel Mondrup, Philip Vestergaard-Laustsen, Karrar Adam Mahdi, Marcus Kristian Nielsen, Casper Gerhard Sonne, Martin Alexander Sørensen, Maximilian Stalzer, Bella Strandfort, Arnheidur Sveinsdottir, Moiz Abu Talib, Antoine Tissot, Ambrus Török, David Tromp, Li Chia Tung, Ioannis Tzanas

Reading material:

Chapter 21



Tue Herlau, Mikkel N. Schmidt and Morten Mørup

Introduction to Machine Learning and Data Mining

Lecture notes, Spring 2022, version 1.0
Version for print
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April 5, 2022

Technical University of Denmark

Lecture Schedule

1. Introduction
   31 January: C1

Data: Feature extraction, and visualization

2. Data, feature extraction and PCA
   7 February: C2, C3
3. Measures of similarity, summary statistics and probabilities
   14 February: C4, C5
4. Probability densities and data visualization
   21 February: C6, C7

Supervised learning: Classification and regression

5. Decision trees and linear regression
   28 February: C8, C9
6. Overfitting, cross-validation and Nearest Neighbor
   7 March: C10, C12 **(Project 1 due before 13:00)**
7. Performance evaluation, Bayes, and Naive Bayes
   14 March: C11, C13
8. Artificial Neural Networks and Bias/Variance
   21 March: C14, C15
9. AUC and ensemble methods
   28 March: C16, C17

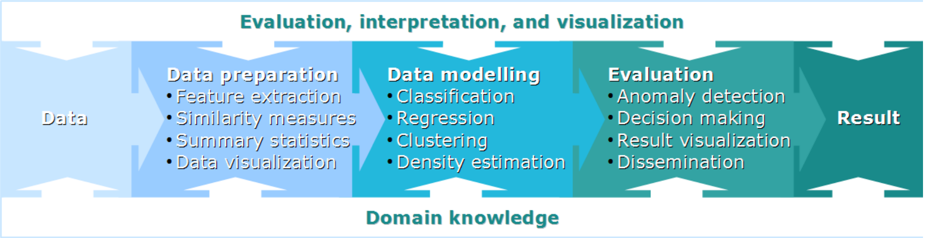
Unsupervised learning: Clustering and density estimation

10. K-means and hierarchical clustering
    11 April: C18
11. Mixture models and density estimation
    18 April: C19, C20 **(Project 2 due before 13:00)**
12. **Association mining**
    25 April: C21

Recap

13. Recap and discussion of the exam
    2 May: C1-C21

Online 24/7 help: Discussion Forum/Piazza
Streaming & Videos: `https://panopto.dtu.dk/`
Online exercises: MS Teams



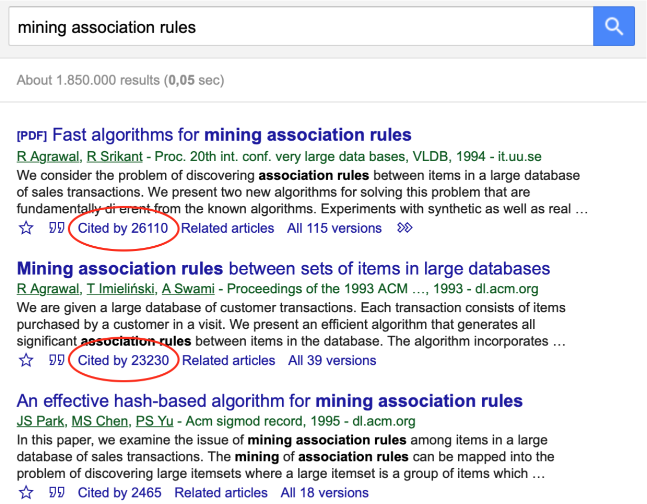
Learning Objectives

- Calculate support and confidence of association rules
- Describe the Apriori algorithm for association mining and how it is used for efficient estimation of association rules

Association rule discovery: Definition

- Given a set of **records**
  - Each containing a number of **items from a set**
- **Goal:** Produce dependency rules
  - Predict the occurence of an item based on occurences of other items

Association Mining



About 1.850.000 results (**0,05** sec)

[PDF] Fast algorithms for **mining association rules**

R Agrawal, R Srikant - Proc. 20th int. conf. very large data bases, VLDB, 1994 - it.uu.se

We consider the problem of discovering **association rules** between items in a large database of sales transactions. We present two new algorithms for solving this problem that are fundamentally different from the known algorithms. Experiments with synthetic as well as real …

☆ 99 Cited by 26110 Related articles All 115 versions »

Mining association rules between sets of items in large databases

R Agrawal, T Imieliński, A Swami - Proceedings of the 1993 ACM …, 1993 - dl.acm.org

We are given a large database of customer transactions. Each transaction consists of items purchased by a customer in a visit. We present an efficient algorithm that generates all significant **association rules** between items in the database. The algorithm incorporates …

☆ 99 Cited by 23230 Related articles All 39 versions

An effective hash-based algorithm for **mining association rules**

JS Park, MS Chen, PS Yu - Acm sigmod record, 1995 - dl.acm.org

In this paper, we examine the issue of **mining association rules** among items in a large database of sales transactions. The **mining** of **association rules** can be mapped into the problem of discovering large itemsets where a large itemset is a group of items which …

☆ 99 Cited by 2465 Related articles All 18 versions

Source: Google Scholar (November, 2020)

Association rule discovery: Example

Market basket analysis

Training set

1. {Bread, Soda, Milk}
2. {Beer, Bread}
3. {Beer, Soda, Diaper, Milk}
4. {Beer, Bread, Diaper, Milk}
5. {Soda, Diaper, Milk}

Rules discovered

{Milk} ► {Soda}
{Diaper, Milk} ► {Beer}

Market basket data

- Representation as

Transaction table

ID	Items
1	Bread, Soda, Milk
2	Beer, Bread
3	Beer, Soda, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Soda, Diaper, Milk

Data matrix

ID	Bread	Soda	Milk	Beer	Diaper
1	1	1	1	0	0
2	1	0	0	1	0
3	0	1	1	1	1
4	1	0	1	1	1
5	0	1	1	0	1

Association analysis, rules and support

- **Itemset**
  - For example {Bread, Soda, Milk}, {Milk, Diaper}, {}

- **Support** for an itemset **X**
  - Percentage of transactions that contain **X**

- **Association rule**
  - Expression of the form: **X ▶Y** if X then Y
    where **X** and **Y** are disjoint item sets

- **Support** for an association rule **X ▶Y**
  - Percentage of transactions that contain **X** ∪ **Y**

$$s(X \to Y) = \frac{\sigma(X \cup Y)}{N} = P(X, Y)$$

Quiz 1: Support (Spring 2018)

	x_1^L	x_1^H	x_2^L	x_2^H	x_3^L	x_3^H	x_4^L	x_4^H	x_5^L	x_5^H	x_6^L	x_6^H
O1	1	0	1	0	1	0	1	0	1	0	1	0
O2	0	1	0	1	0	1	0	1	0	1	0	1
O3	1	0	0	1	1	0	1	0	1	0	1	0
O4	1	0	1	0	1	0	0	1	0	1	1	0
O5	0	1	1	0	1	0	1	0	1	0	1	0
O6	0	1	0	1	0	1	0	1	0	1	0	1
O7	0	1	1	0	1	0	0	1	0	1	0	1
O8	1	0	1	0	1	0	1	0	0	1	0	1
O9	0	1	0	1	1	0	1	0	0	1	0	1
O10	1	0	0	1	0	1	0	1	0	1	1	0

Table 1: The ten first observations of the airline safety dataset binarized considering the attribute x_1–x_6.

We consider a dataset of airline safety binarized according to the median value. Values below median is referred to with the superscript L and above the median value using the superscript H. In Table 1 is given the first 10 observations O1–O10. Consider the association rule:

$$\{x_2^H, x_3^H, x_4^H, x_5^H\} \rightarrow \{x_6^H\}.$$

What is the support of the rule?

A. 0.0 %

[✓ B. 20.0 %]

C. 66.7 %

D. 100.0 %

E. Don't know.

Association analysis, confidence

- **Itemset**
  - For example {Bread, Soda, Milk}, {Milk, Diaper}, {}

- **Support** for an itemset **X**
  - Percentage of transactions that contain **X**

- **Association rule**
  - Expression of the form: **X** ► **Y**
    where **X** and **Y** are disjoint item sets

- **Support** for an association rule **X** ► **Y**
  - Percentage of transactions that contain **X** È **Y**

$$s(X \to Y) = \frac{\sigma(X \cup Y)}{N} = P(X, Y)$$

- **Confidence** for an association rule **X** ► **Y**
  - Percentage of transactions containing **X** that also contain **Y**

$$c(X \to Y) = \frac{\sigma(X \cup Y)}{\sigma(X)} = \frac{P(Y, X)}{P(X)} = P(Y|X)$$

Quiz 2: Confidence (Spring 2018)

	x_1^L	x_1^H	x_2^L	x_2^H	x_3^L	x_3^H	x_4^L	x_4^H	x_5^L	x_5^H	x_6^L	x_6^H
O1	1	0	1	0	1	0	1	0	1	0	1	0
O2	0	1	0	1	0	1	0	1	0	1	0	1
O3	1	0	0	1	1	0	1	0	1	0	1	0
O4	1	0	1	0	1	0	0	1	0	1	1	0
O5	0	1	1	0	1	0	1	0	1	0	1	0
O6	0	1	0	1	0	1	0	1	0	1	0	1
O7	0	1	1	0	1	0	0	1	0	1	0	1
O8	1	0	1	0	1	0	1	0	0	1	0	1
O9	0	1	0	1	1	0	1	0	0	1	0	1
O10	1	0	0	1	0	1	0	1	0	1	1	0

Table 1: The ten first observations of the airline safety dataset binarized considering the attribute x_1–x_6.

We again consider the airline safety data and the rule

$$\{x_2^H, x_3^H, x_4^H, x_5^H\} \rightarrow \{x_6^H\}.$$

What is the confidence of the rule?

A. 0.0 %

B. 20.0 %

[✓ C.] 66.7 %

D. 100.0 %

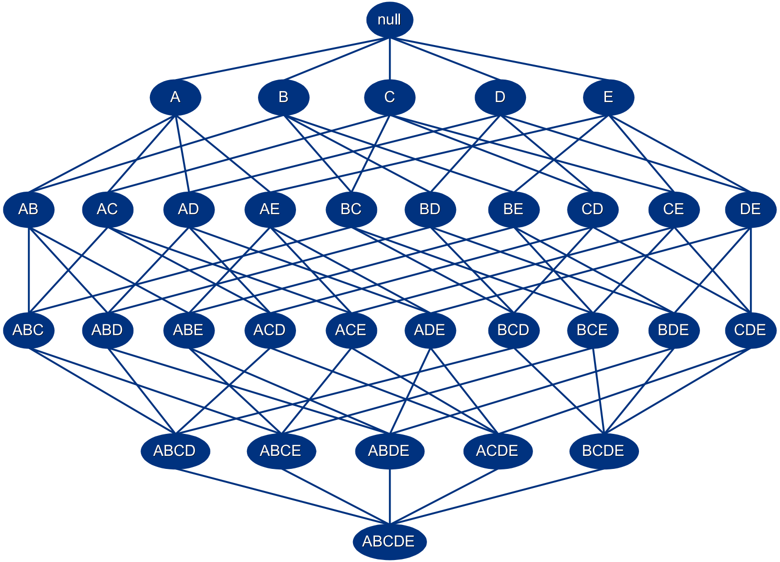
E. Don't know.

Association rule mining

- Find all association rules that have
  - **Support** ≥ *minsup*
  - **Confidence** ≥ *minconf*

- Approach
  - **Frequent itemset generation**
    - Generate a list of all **itemsets** with **Support** ≥ *minsup*
  - **Association rule generation**
    - Generate all **association rules** with **Confidence** ≥ *minconf*

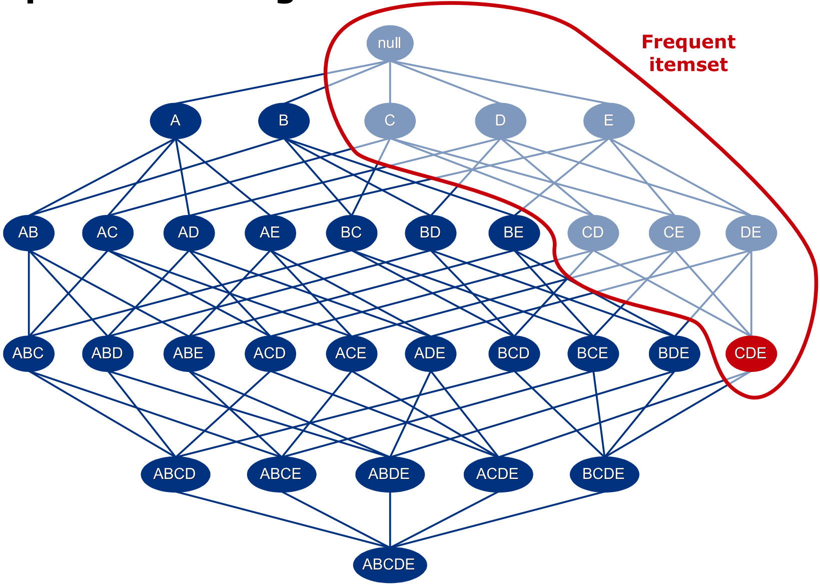
Frequent itemset generation



How many different itemsets can be created for a problem with a total of D items?

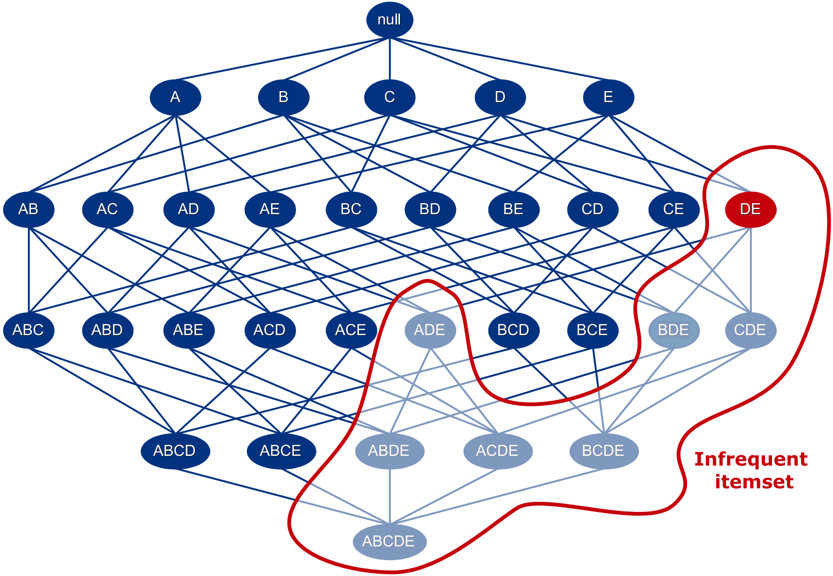
This itemset latice representation is adapted from:
“Introduction to Data Mining” by Pang-Ning Tan, Michael Steinbach, Vipin Kumar

Frequent itemset generation



If an itemset is frequent, then all of its subsets must also be frequent

Frequent itemset generation



If an itemset is infrequent, then all of its supersets must also be infrequent

The Apriori Algorithm

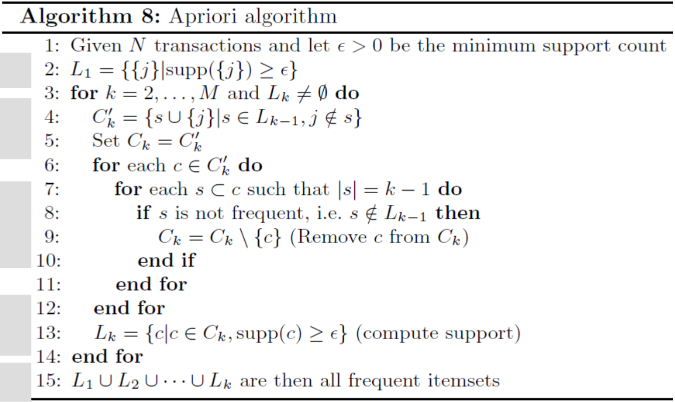
Find all 1-itemsets

Generate k-itemsets by merging single items to the k-1-itemsets

Remove all the generated itemsets for which subsets are not part of the k-1-itemsets

Keep remaining k-itemsets with enough support.

Output all frequent itemsets



Algorithm 8: Apriori algorithm

```
Given $N$ transactions and let $\epsilon > 0$ be the minimum support count
$L_1 = \{\{j\} | \text{supp}(\{j\}) \geq \epsilon\}$
for $k = 2, \ldots, M$ and $L_k \neq \emptyset$ do
    $C'_k = \{s \cup \{j\} | s \in L_{k-1}, j \notin s\}$
    Set $C_k = C'_k$
    for each $c \in C'_k$ do
        for each $s \subset c$ such that $|s| = k - 1$ do
            if $s$ is not frequent, i.e. $s \notin L_{k-1}$ then
                $C_k = C_k \setminus \{c\}$ (Remove $c$ from $C_k$)
            end if
        end for
    end for
    $L_k = \{c | c \in C_k, \text{supp}(c) \geq \epsilon\}$ (compute support)
end for
$L_1 \cup L_2 \cup \cdots \cup L_k$ are then all frequent itemsets
```

Quiz 3: A-priori (Fall 2018)

We will consider a binary dataset consisting of the $M = 6$ features f_1, f_2, f_3, f_4, f_5, f_6. We wish to apply the Apriori algorithm to find all itemsets with support greater than $\varepsilon = 0.15$. Suppose at iteration $k = 3$ we know that:

$$L_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Recall the key step in the Apriori algorithm is to construct L_3 by first considering a large number of candidate itemsets C_3', and then rule out some of them using the downwards-closure principle thereby saving many (potentially costly) evaluations of support. Suppose L_2 is given as above, which of the following itemsets does the Apriori algorithm *not* have to evaluate the support of?

A. $\{f_2, f_3, f_4\}$

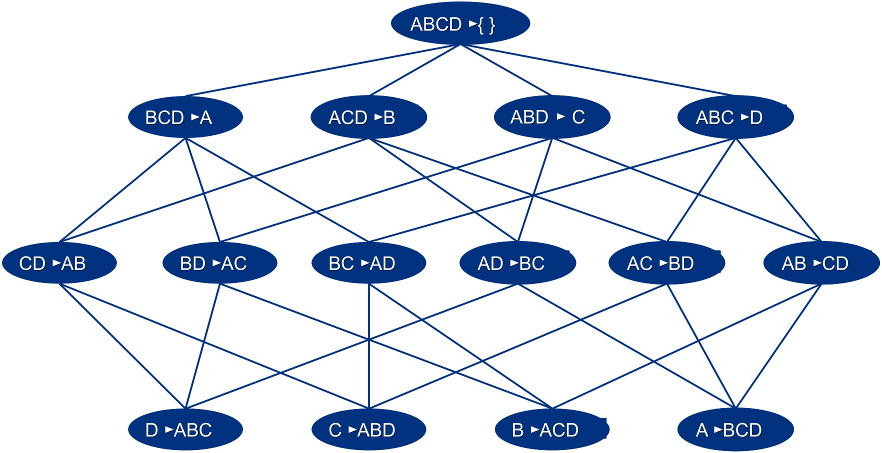
B. $\{f_1, f_2, f_6\}$

C. $\{f_2, f_3, f_6\}$

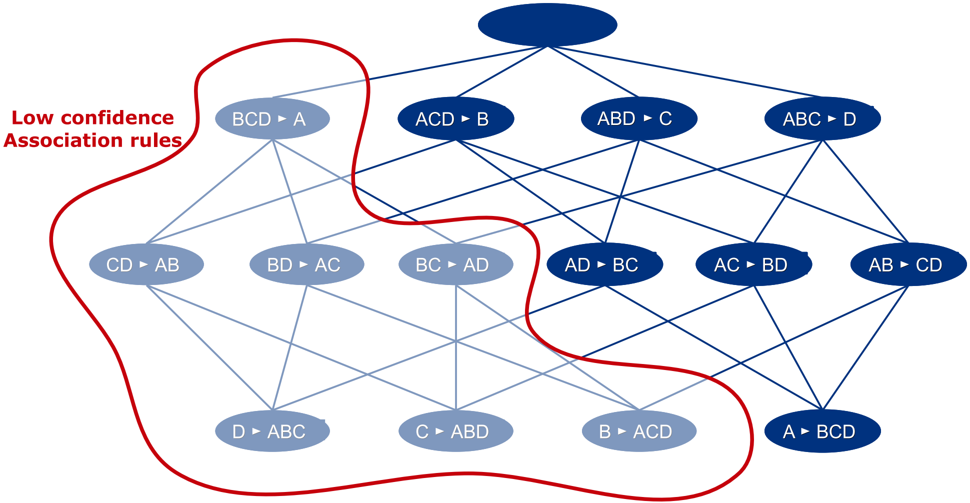
[✓ D.] $\{f_1, f_3, f_4\}$

E. Don't know.

Association rule generation



Association rule generation



Results for market basket example

Itemset	Support
Milk	80%
Bread	60%
Soda	60%
Beer	60%
Diaper	60%
Diaper Milk	60%
Soda Milk	60%
Bread Beer	40%
Bread Milk	40%
Soda Diaper	40%
Beer Diaper	40%
Beer Milk	40%
Soda Diaper Milk	40%
Beer Diaper Milk	40%

Association rule	Support	Confidence
{} ► Milk	80%	80%
Soda ► Milk	60%	100%
Diaper ► Milk	60%	100%
Soda, Diaper ► Milk	40%	100%
Beer, Diaper ► Milk	40%	100%
Beer, Milk ► Diaper	40%	100%

ID	Bread	Soda	Milk	Beer	Diaper
1	1	1	1	0	0
2	1	0	0	1	0
3	0	1	1	1	1
4	1	0	1	1	1
5	0	1	1	0	1

- How can we do association mining for continuous data?

$$\mathbf{X}= \begin{array}{ccc}
\textbf{Attribute 1} & \textbf{Attribute 2} & \textbf{Attribute 3} \\
0.3689 & 0.9827 & 0.6999 \\
0.4607 & 0.7302 & 0.6385 \\
0.9816 & 0.3439 & 0.0336 \\
0.1564 & 0.5841 & 0.0688 \\
0.8555 & 0.1078 & 0.3196 \\
0.6448 & 0.9063 & 0.5309 \\
0.3763 & 0.8797 & 0.6544 \\
0.1909 & 0.8178 & 0.4076 \\
0.4283 & 0.2607 & 0.8200 \\
0.4820 & 0.5944 & 0.7184 \\
0.1206 & 0.0225 & 0.9686 \\
0.5895 & 0.4253 & 0.5313 \\
0.2262 & 0.3127 & 0.3251 \\
0.3846 & 0.1615 & 0.1056 \\
0.5830 & 0.1788 & 0.6110 \\
0.2518 & 0.4229 & 0.7788 \\
0.2904 & 0.0942 & 0.4235 \\
0.6171 & 0.5985 & 0.0908 \\
0.2653 & 0.4709 & 0.2665 \\
0.8244 & 0.6959 & 0.1537
\end{array}$$

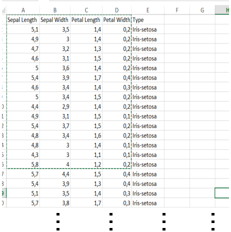
Binarize data according to percentiles

AttributeNames=	Attribute 1	Attribute 2	Attribute 3
	0.3689	0.9827	0.6999
	0.4607	0.7302	0.6385
	0.9816	0.3439	0.0336
	0.1564	0.5841	0.0688
	0.8555	0.1078	0.3196
	0.6448	0.9063	0.5309
	0.3763	0.8797	0.6544
	0.1909	0.8178	0.4076
	0.4283	0.2607	0.8200
X=	0.4820	0.5944	0.7184
	0.1206	0.0225	0.9686
	0.5895	0.4253	0.5313
	0.2262	0.3127	0.3251
	0.3846	0.1615	0.1056
	0.5830	0.1788	0.6110
	0.2518	0.4229	0.7788
	0.2904	0.0942	0.4235
	0.6171	0.5985	0.0908
	0.2653	0.4709	0.2665
	0.8244	0.6959	0.1537

AttributeNamesBin=	Attribute 1 0-50 %	Attribute 1 50-100 %	Attribute 2 0-33.3 %	Attribute 2 33.3-66.7%	Attribute 2 66.7-100 %	Attribute 3 0-50%	Attribute 3 50-100%
	1	0	0	0	1	0	1
	0	1	0	0	1	0	1
	0	1	0	1	0	1	0
	1	0	0	1	0	1	0
	0	1	1	0	0	1	0
	0	1	0	0	1	0	1
	1	0	0	0	1	0	1
	1	0	0	0	1	1	0
	0	1	1	0	0	0	1
Xbinary=	0	1	0	1	0	0	1
	1	0	1	0	0	0	1
	0	1	0	1	0	0	1
	1	0	1	0	0	1	0
	1	0	1	0	0	1	0
	0	1	1	0	0	0	1
	1	0	0	1	0	0	1
	1	0	1	0	0	1	0
	0	1	0	1	0	1	0
	1	0	0	1	0	1	0
	0	1	0	0	1	1	0

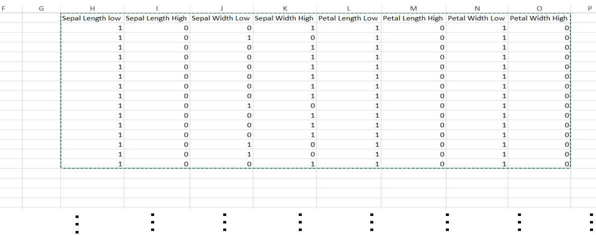
Recap of association rule discovery on Iris data

X=



Sepal Length	Sepal Width	Petal Length	Petal Width	Type
5,1	3,5	1,4	0,2	Iris-setosa
4,9	3	1,4	0,2	Iris-setosa
4,7	3,2	1,3	0,2	Iris-setosa
4,6	3,1	1,5	0,2	Iris-setosa
5	3,6	1,4	0,2	Iris-setosa
5,4	3,9	1,7	0,4	Iris-setosa
4,6	3,4	1,4	0,3	Iris-setosa
5	3,4	1,5	0,2	Iris-setosa
4,4	2,9	1,4	0,2	Iris-setosa
4,9	3,1	1,5	0,1	Iris-setosa
5,4	3,7	1,5	0,2	Iris-setosa
4,8	3,4	1,6	0,2	Iris-setosa
4,8	3	1,4	0,1	Iris-setosa
4,3	3	1,1	0,1	Iris-setosa
5,8	4	1,2	0,2	Iris-setosa
5,7	4,4	1,5	0,4	Iris-setosa
5,4	3,9	1,3	0,4	Iris-setosa
5,1	3,5	1,4	0,3	Iris-setosa
5,7	3,8	1,7	0,3	Iris-setosa
⋮	⋮	⋮	⋮	

Xbinary=



Sepal Length low	Sepal Length High	Sepal Width Low	Sepal Width High	Petal Length Low	Petal Length High	Petal Width Low	Petal Width High
1	0	0	1	1	0	1	0
1	0	1	0	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	1	0	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	0	1	0	1	0	1	0
1	0	1	0	1	0	1	0
1	0	0	1	1	0	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Example of association rules where y is also appended to Xbinary, i.e. [Xbinary Y], i.e. Y has three columns indicating which of the three flowers the observation belongs to.

{Petal length Low, Petal Width low, Iris Setosa} -> {Sepal Length Low}
(Conf 100, sup 33)

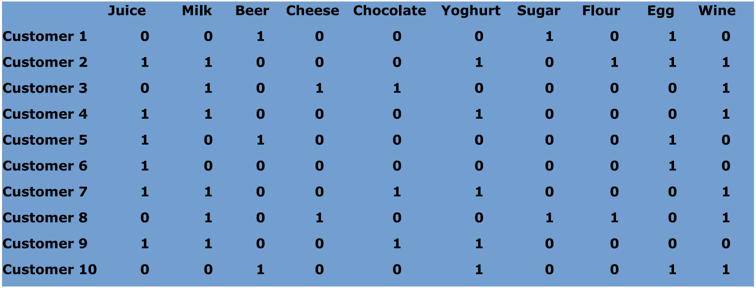
{Sepal Length Low, Petal Width low, Iris Setosa} -> {Petal length Low}
(Conf 100, sup 33)

{Sepal length Low, Petal length Low, Iris Setosa} -> {Petal Width Low}
(Conf 100, sup 33)

{Sepal length Low, Sepal Width High, Petal length Low, Petal Width Low} -> {Iris Setosa}
(Conf 100, sup 28)

Quiz 4: A-priori (Bonus)

Consider the following dataset consisting of 10 transactions



	Juice	Milk	Beer	Cheese	Chocolate	Yoghurt	Sugar	Flour	Egg	Wine
Customer 1	0	0	1	0	0	0	1	0	1	0
Customer 2	1	1	0	0	0	1	0	1	1	1
Customer 3	0	1	0	1	1	0	0	0	0	1
Customer 4	1	1	0	0	0	1	0	0	0	1
Customer 5	1	0	1	0	0	0	0	0	1	0
Customer 6	1	0	0	0	0	0	0	0	1	0
Customer 7	1	1	0	0	1	1	0	0	0	1
Customer 8	0	1	0	1	0	0	1	1	0	1
Customer 9	1	1	0	0	1	1	0	0	0	0
Customer 10	0	0	1	0	0	1	0	0	1	1

Find all itemsets with support greater than 35% (i.e. found in four or more transactions). How many are there?

A. 7

B. 9 %

C. 11 %

D. 13 %

E. Don't know.

Practicals

- Post-test on DTU Learn: Quizzes > Post test
  - Already open, closes Sunday at midnight
  - Similar to pre-test
  - We will present the results next week
- Next week (and beyond)
  - Recap of the course
  - Mock exam ... and other exam sets
- ...and beyond
  - Assistance via Piazza (limited TA availability, so help each other out)
  - Feedback on project 2 on DTU Learn (before the exam)

Resources

https://towardsdatascience.com Alternative guide to association rule learning (https://towardsdatascience.com/association-rules-2-aa9a77241654)

http://www.cse.msu.edu Key reference for association rule learning, "Fast algorithms for mining association rules" (Agrawal & Srikan) (http://www.cse.msu.edu/~cse960/Papers/MiningAssoc-AgrawalAS-VLDB94.pdf)

https://rakesh.agrawal-family.com Other key reference "Mining association rules between sets of items in large datatabases" (Agrawal et. al.) (https://rakesh.agrawal-family.com/papers/sigmod93assoc.pdf)